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DECISION SUPPORT METHODS FOR LARGE-SCALE FLEXIBLE TRANSMISSION EXPANSION PLANNING

Tesis para la obtención del grado de Doctor

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A mi tía Chilina. Querías dejar buen recuerdo, y desde luego lo dejaste.

A mis padres y a mi hermana. Los mejores padres del mundo y la mejor hermana. Mamá, me decías cuando estaba pensando en volverme: "Imagínate, qué felicidad, en Madrid toda la familia, tú haciendo tu doctorado y disfrutándolo y yo preparándote los táper que más te apetezcan." Ha sido completamente cierto, con mi pobre padre todos los días a las 8 quedando conmigo para traerme esos táper deliciosos.

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Acronyms

- AC: Alternating Current
- BOWF: Barrow Offshore Wind Farm
- CCGT: Combined Cycle Gas Turbine
- DC: Direct Current
- ENTSO-e: European Network of Transmission System Operators for electricity
- EU: European Union
- GEP: Generation Expansion Planning
- HVDC: High Voltage Direct Current
- IV: Intrinsic Value
- LCC: Line Commutated Converter
- LP: Linear Programming
- MINLP: Mixed-Integer Non-Linear Programming
- MIP: Mixed-Integer Programming
- MIS: Minimal Infeasible Subsystems
- OO: Ordinal Optimization
- OPC: Ordered Performance Curve
- OV: Option Value
- OWF: Offshore Wind Farm
- OWL: Offshore Windfarm Layout optimizer
- PCC: Point of Common Coupling
- PCI: Progressive Contingency Incorporation
- QP: Quadratic Programming
- RES: Renewable Energy Sources
- ROV: Real Options Valuation
- SAA: Sample Average Approximation
- SP: Stochastic Programming
- TEP: Transmission Expansion Planning
- TEPES: Transmission Expansion Planning for an Electric System
- TSO: Transmission System Operator
- TYNDP: Ten Year Network Development Plan
- VSC: Voltage Source Converter

In a Nutshell

Power systems are immersed in deep changes that will shape their future for the coming decades. The key to this transformation will be the shift from a fossil fuel system to a low-carbon economy, which will involve the extensive installation of renewable generation. Although, in general, traditional plants are placed at the planner's discretion, the location of these renewable power plants depends on the abundance of the resource they use. In the case of wind and solar power, these high-potential areas are typically far from the highly populated regions where demand concentrates and are often poorly interconnected to the rest of the system. Therefore, the development of the transmission network will be necessary to accommodate this new generation.

In addition, the long permitting process involved in transmission expansion means that, while new generation can be installed in one to three years for some technologies, a new transmission line typically takes ten years from planning to full deployment. This implies that transmission expansion decisions must anticipate new generation developments. The liberalization of generation markets introduces further complications, as companies take independent decisions and there is no binding generation expansion plan to guide transmission investments. Then, Transmission Expansion Planning (TEP) must incorporate the uncertainties inherent to generation expansion. Consequently, it is necessary to develop methods to propose robust, flexible plans that are able to adapt to the evolution of the system as the future unfolds. In addition, the integration of markets results in planning increasingly large areas where it is no longer possible to rely on the experience of a TSO (Transmission System Operator) to understand the structure of the network and provide functions such as the identification of potentially interesting candidate transmission lines. Objective, automatic techniques to perform these functions would be greatly beneficial. Finally, the long-term planning of large systems results in computationally complex problems for which the currently used optimization techniques can fall short.

Identifying these challenges is the starting point and inspiration of this thesis, which we structure as an organized response in the form of a series of innovative techniques. These contributions are rigorous, academically sound but also have a special emphasis on being implementable for large-scale, real power systems. Academic and realistic case studies demonstrate the applicability of the developed techniques, but they are not the focus of this thesis. Case studies based on general TEP or the OWF problem have been used depending on where the technique was considered most beneficial. Case studies should only be valued as an illustration of the contributions rather than for their results themselves.

Finally, this thesis focuses on centralized TEP, where a TSO or group of TSOs take medium to long-term expansion decisions considering market operation for certain different futures for generation expansion, fuel prices or demand. We model these uncertainties as externally provided scenarios. The dynamics of generation expansion or the interactions between fuel and power prices are out of the scope of this thesis. Likewise, we model system operation in a centralized way. Similarly, although the ramifications of regulatory decisions are extremely interesting in planning decisions and other related issues such as transmission cost allocation; this thesis does not focus on the regulatory aspects of power transmission.

This document is organized as follows. First, Chapter 1 presents our diagnosis of the challenges that make TEP such an interesting –and complex- problem in the current environment. These challenges are the main lines around which we articulate this thesis. Then, we review the vast amount of literature that deals with this subject in Chapter 2. The two optimization models that form the basis of this thesis are described in Chapter 3. Chapter 4, Chapter 5, and Chapter 6 describe the main contributions of our work, which cover modeling, problem resolution and the interpretation of results. Finally, Chapter 7 extracts conclusions and outlines future lines of research.

Chapter 1

Motivation. The Current Challenges of Power Network Design

1.1. The Essential Role of Transmission in the Power Systems of the Future

Electric power transmission is the long-distance transfer of electricity from generation plants to substations, from where the distribution network delivers it to individual consumers. The transmission grid is therefore the basic infrastructure that enables the large physical power flows that make the power system possible (Figure 1-1).

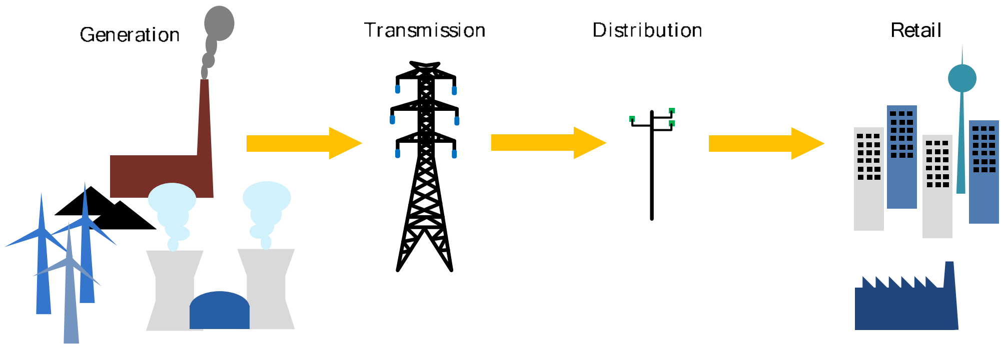


Figure 1-1 – Transmission within power systems.

The transmission grid imposes physical constraints to the power flows that traverse it (Kirchhoff's Laws) and therefore to generation and demand. The main consequence of this is that not all dispatch solutions are feasible. Consequently, Transmission Expansion Planning (TEP), that is, "**deciding which new lines will enable the system to satisfy forthcoming loads with the required degree of reliability**" [1], is one of the key strategic decisions in power systems. Table 1-1 provides some context on the decisions that concern the transmission network, with TEP being the most important, highest-impact problem.

	Strategic	Tactical	Operational
Liberalized Network Activities	Merchant line investment.	Valuation of transmission rights and network contracts. Network contracting.	Transmission rights trading.
Centralized Network Activities	Evaluation of transmission investments (particularly, interconnections). Evaluation of the impact of TEP on market agents. **Transmission Expansion Planning.**	Network adequacy assessment. Network maintenance. Network cost allocation. Determination of payments for network use.	Introduction of network constraints into the economic dispatch. Determination of nodal prices and losses. Network operation.

Table 1-1 – Decisions that concern the transmission network.

Transmission investments are very capital intensive and have extremely long useful lives (of up to 40 years), so transmission investment decisions have a long-standing impact on the power system as a whole. For instance, ENTSO-e members have a joint budget over EUR 100 bn for their 2012- 2022 investments [2].

Country	EUR bn	Country	EUR bn
Austria	1.1	Ireland	3.9
Belgium	1.9	Latvia	0.4
Bosnia-Herzegovina	0.0	Lithuania	0.7
Bulgaria	0.2	Luxemburg	0.3
Croatia	0.2	Montenegro	0.4
Czech Republic	1.7	Netherlands	3.3
Cyprus	0.0	Norway	6.5
Denmark	1.4	Poland	2.9
Estonia	0.3	Portugal	1.5
Finland	0.8	Romania	0.7
France	8.8	Serbia	0.2
FYROM	0.1	Slovakia	0.3
Germany	30.1	Slovenia	0.3
Greece	0.3	Spain	4.8
Hungary	0.1	Sweden	2.0
Iceland	0.0	Switzerland	1.7
Italy	7.1	United Kingdom	19.0
Total			**104**

Table 1-2 – 2012-2022 Transmission investment budget for ENTSO-e members.

This long-term impact on the power system is particularly relevant for the integration of new generation. In some cases, it is only necessary to connect a new power plant to the grid, which is relatively inexpensive compared to the long-distance connections and wide-range reinforcements that are required when generation is located in remote areas. This is increasingly the case given that renewable generation largely determines its location based on the availability of the resource, particularly wind and solar, which often abound in remote areas far away from the main demand centers.

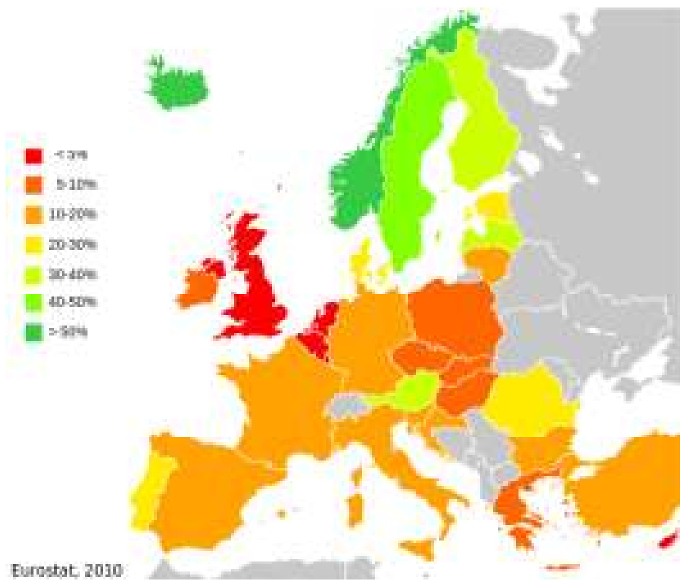


Figure 1-2 – Percentage of energy generated by renewable sources per country as of 2010 [3].

The European Union has set very aggressive emission reduction targets, establishing a 20% reduction in greenhouse gases with respect to 1990 levels by 2020 and endorsing an objective of 80% reductions and 100% clean electricity by 2050 [4, 5]. Although considerable amounts of renewable power have been installed in the past couple of decades, most of the member countries are still far from these targets [3], as shown in Figure 1-2. Therefore, vast amounts of new generation are expected to be built in the medium term future.

A substantial part of this new renewable generation will probably materialize in the form of coordinated, remarkably large projects. Some particularly interesting examples of this kind of projects are:

- Desertec is an initiative proposed by a German-led consortium of companies that aims at installing a large amount of renewable generation (over 20 GW) in the Sahara desert and its surroundings, with the aim of exporting part of the generated energy to Europe (Figure 1-3) [6].

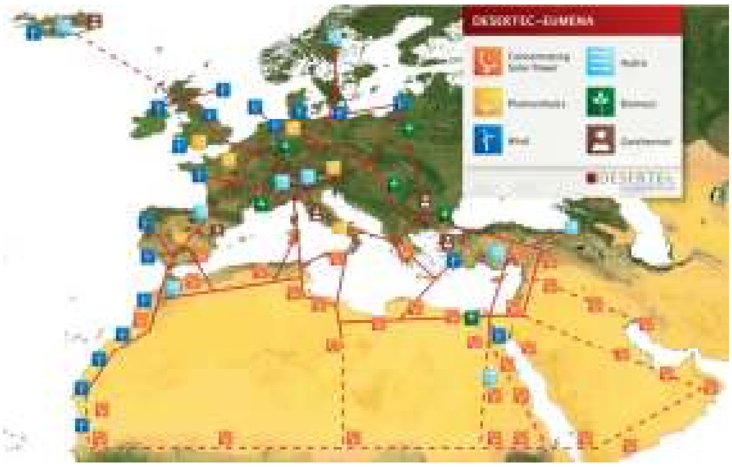


Figure 1-3 – Desertec vision [6].

- MedGrid, a French-based project, contemplates the exploitation of the renewable potential of the Mediterranean region in a coordinated way, installing over 20 GW of new capacity and supporting energy exchanges among the Mediterranean countries (Figure 1-4) [7]. MedRing proposes a similar concept [8].

Figure 1-4 – MedGrid vision [7].

Offshore wind seems set to play a crucial role in Europe's future energy landscape. At sea, winds are abundant and stable, and the shallow waters of the North and Irish Seas allow for the relatively inexpensive installation of the turbines [9]. In addition, Offshore Wind Farm (OWF) projects can avoid the public opposition and some of the environmental concerns raised by their onshore counterparts. The expanding pipeline of European projects has already installed 5.3 GW of offshore wind power [10], including the impressive London Array (Figure 1-5), (1 GW). Other colossal projects such as Dogger Bank (9 GW) [11] are attracting remarkable interest.

Figure 1-5 –London Array Map [12].

These vast plants are not conceived as isolated projects. Projects such as the Offshore Grid Initiative focus on creating an offshore network that links neighboring countries and to which large wind farms can be connected [13], as shown in Figure 1-6.

Figure 1-6 – Vision map of the Offshore Grid Initiative [13].

All these ambitious projects depend critically on the transmission network. The necessary expansion can emerge in the form of isolated reinforcements, an extensive HVDC overlay to the existing network or an entirely new grid. The latter case, known as *greenfield expansion*, applies to the particularly interesting case of offshore grid design, where no existing network can be used as the starting point for the developments.

The design of these large-scale network expansions poses considerable challenges that make this problem extremely attractive from an academic perspective. This thesis aspires to address these challenges, developing new techniques for large-scale TEP, regarding the design of offshore networks and large OWFs as a highly relevant particular case.

1.2. The New Challenges of Transmission Expansion Planning

Having such a deep impact on the power system as a whole, it is not surprising that TEP has been studied in an academic context for decades. It has long been recognized that the uncertainties present in the problem, together with its combinatorial nature, constitute a considerable burden to its resolution [14]. However, relatively recent changes have substantially increased this complexity. This section presents our diagnosis of these challenges.

Deregulation amplifies the uncertainties traditionally present in the problem. Generation Expansion Planning (GEP), which was once centrally performed, is now a decision taken privately by the companies participating in the generation market. This is especially important considering that, while building a power plant can take around one to three years for some technologies, transmission projects have a much longer lead-time. The permitting processes are increasingly difficult and, together with growing environmental concerns, can severely delay transmission projects, often for more than ten years. Therefore, TEP must anticipate generation investments, but there is no longer a binding, coordinated generation expansion plan that can guide transmission decisions.

In addition, the establishment of regional markets and the increase of cross-border flows mean that planning must deal with much larger areas. For instance, the Ten-Year Network Development Plan (TYNDP) from ENTSO-e deals with the regions that compose Europe [15] (albeit from a bottom-up approach) and projects such as Desertec or MedGrid focus on the Mediterranean area as a whole. Working with such extensive regions implies larger, more difficult to solve problems. In addition, the TSO's expert knowledge, which used to play a prominent role in national planning studies (notably, in proposing the candidate investments considered in the problem), is not available for regional planning as no single TSO has experience on the whole area. This brings additional difficulties to the planning process.

The challenges brought by these relatively recent developments articulate along three distinct lines around which this thesis is built:

- Efficient resolution,
- Flexibility and robustness,
- Structure identification.

1.2.1. Efficient Resolution

The extensive areas involved in regional TEP, together with the vast uncertainties involved, mean that some optimization techniques can be impractical in this context. This thesis develops formulation and algorithmic enhancements to the state-of-the-art methods in order to make accessible the large problem sizes currently involved in TEP studies.

This thesis structures problem resolution in a master-subproblem architecture based on **Benders' decomposition**, which is especially suitable for TEP and offshore network design given the characteristics of the problems.

We perform a comparative study of the different acceleration techniques available for Benders' decomposition, evaluating their potential computational savings. In addition, we develop some new acceleration techniques. ***Semi-relaxed cuts*** **deal with large numbers of discrete decision variables. A Progressive Contingency Incorporation Algorithm evaluates the effect of component failures on system reliability efficiently.** We illustrate these contributions with case studies based on TEP and OWF design. The enhancements reduce computation times by one to two orders of magnitude in the case studies developed.

1.2.2. Flexibility and Robustness

Several sources of uncertainty have a deep impact on transmission expansion decisions:

- Generation expansion, that is, new generation installed,
- Demand,
- Fuel prices, including coal, gas and carbon prices,
- Renewable energy production,
- Hydro inflows,
- Contingencies.

Some of these uncertainties are static risks, such as load levels, hydro inflows, or contingencies. A static probability distribution, if properly adjusted, can describe them. These are referred to as *random* uncertainties and can be represented by means of stochastic scenarios. The second group, known as *nonrandom* uncertainties, includes generation expansion or fuel and carbon prices. The factors that affect them evolve, so it is not possible to estimate them directly using past data. Dynamic snapshots, often based on expert opinion, are used to describe these possible future evolutions.

We will define an *expansion plan* as a set of values for the relevant investment decisions. That is, a plan specifies which investments should be made at each stage of the planning horizon [1]. By contrast, an *expansion strategy* will represent a set of values for the decisions that is conditional on the previous uncertain outcomes. In the TEP problem, projects can be put on hold or abandoned depending on the development of events and the TSO can request permits for more transmission lines than the ones that are completed. Having this into account, TEP should propose path-dependent strategies rather than fixed plans [16].

The uncertainties present in the problem, particularly generation expansion, have such a deep impact on transmission that it is essential to develop *flexible* strategies that adapt dynamically to the unveiling of events. In addition, scenario definition is often subjective and incomplete, so ideally investments should serve their purpose even if the scenarios are slightly different. That is, they should be *robust* with respect to the scenario definition. This thesis aims at developing explicit methods to obtain flexible strategies and robust expansion plans.

The following example illustrates these concepts (Figure 1-7). There is one demand area, two regions where new generation might be installed (G1 and G2), and an intermediate area I. All regions are already interconnected and planning considers only reinforcements between areas.

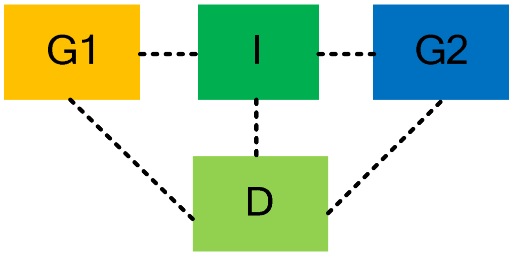


Figure 1-7 – Map of possible connections.

The TSO studies several plans. The first one installs capacity to link region G1 to the demand center. The second one does the same with G2 and the third one with the intermediate region. Figure 1-8 and Table 1-3 describe the plans and their associated costs.

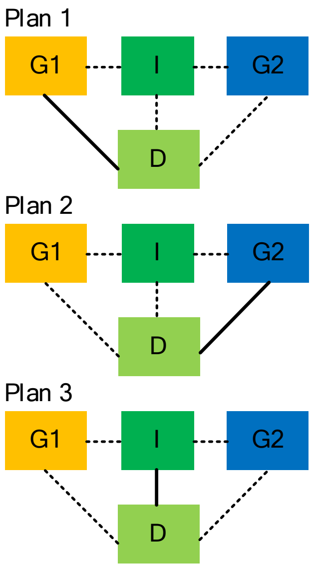


Figure 1-8 – Example plans.

	GEP		Cost Associated to Plans			Regret Associated to Plans		
Scenario #	G1	G2	Plan 1	Plan 2	Plan 3	Plan 1	Plan 2	Plan 3
1	100	10	50	200	100	0	150	50
2	10	100	200	50	100	150	0	50

Table 1-3 – Costs and regrets associated to plans.

A robustness criterion selects Plan 3, given that it presents a maximum regret of 50 compared to the 150 of the other options. Thus, the TSO would build the interconnection between the intermediate area and the demand center. This decision depends heavily on the definition of scenarios. It is desirable to find plans that serve their purpose even when there are slight modifications in the scenarios.

We can now study the same problem from a dynamic perspective, introducing two time stages. We define some expansion strategies as opposed to static expansion plans. It should be noted that these strategies have been proposed just as examples and are in no way implied by the expansion plans defined in Figure 1-8.

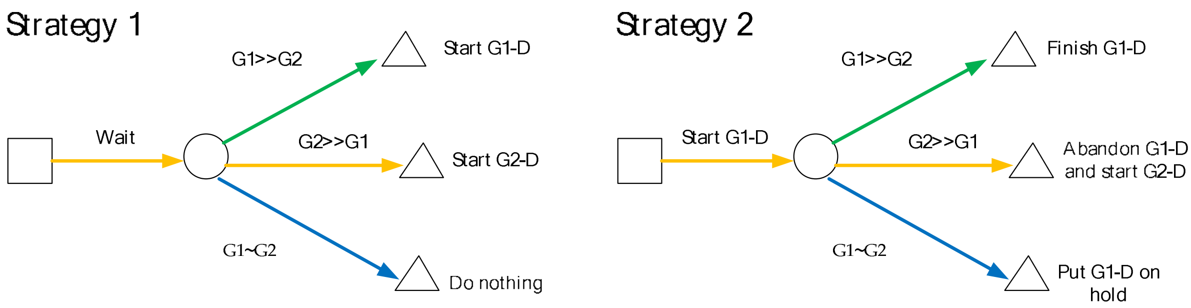


Figure 1-9 – Example expansion strategies.

Strategy 1 does not take any action at Period 1. At Period 2, if one of the generators seems much more probable than the other it will start the relevant corridor. Strategy 2 takes the view that G1 will be built. If, on the second period, it discovers that the bet is not correct, it will abandon the project and take a different direction, either building I-D or G2-D.

We can evaluate regret in a path-dependent way. We propose the following path-dependent scenarios or *dynamic snapshots* as an example. It should be noted that costs are provided as an example and are in no way implied by the definition of strategies. According to them, Strategy 1 is more flexible than Strategy 2, as it adapts better to the different possible scenarios:

Scenario #	Stage 1	Stage 2	Strategy 1	Strategy 2
1	G1~G2	G1~G2	10	100
2	G1~G2	G1>>G2	50	10
3	G1~G2	G2>>G1	50	500
4	G1>>G2	G1>>G2	10	5
5	G1>>G2	G1~G2	50	90
6	G2>>G1	G2>>G1	10	501
7	G2>>G1	G1~G2	50	110

Table 1-4 – Costs associated to the strategies for different dynamic snapshots.

Like the example above, many approaches limit themselves to the worst-case scenario and therefore minimize the maximum regret (such as Robust Optimization [17, 18]). By contrast, this thesis takes a perspective based on Stochastic Optimization, incorporating to the decision process all the information regarding the different scenarios and their probabilities.

We aim at developing flexible expansion strategies that are able to adapt to the future as it unfolds. In addition, we need expansion plans that are robust with respect to the definition of scenarios.

This thesis develops **methods to obtain flexible expansion strategies**. In particular, we apply **Real Options Valuation to the TEP problem**. An option is the right (but not the obligation) to buy or sell a given asset at a pre-specified price at a future date. Real Options applies financial valuation techniques to project management decisions and assesses not only the value of investments but also their potential for adapting to different situations. The developed approach uses this framework to identify transmission lines with a high potential and for which permits should be requested, analyze their main value drivers and find the essential investments of an expansion plan.

In addition, we propose a **regularization approach for Stochastic Optimization** which produces robust solutions that are able to perform relatively well despite modifications in the definition of scenarios. We achieve this through *shrinkage*, a widely used technique in statistical estimation that modifies an estimator to make it more similar to a known, robust solution. We extend this technique to the resolution of Stochastic Optimization problems with either an unmanageably large number of scenarios or an ambiguous scenario definition. The solution to partial descriptions of the problem is shrunk towards a robust benchmark to make it less dependent on the particular scenario tree.

We illustrate these approaches with case studies based on the Spanish power system.

1.2.3. Structure Identification

The complexities of the problem and its combinatorial nature make it impossible to solve it directly having into account all possible transmission investments. Therefore, the candidate investment proposal formerly carried out by the TSO played the crucial role of reducing an unmanageably large search space. However, there is now no single TSO with experience of the whole system under planning. In addition, any manual, heuristic process would necessarily come with limitations that could result in a suboptimal final solution.

1.2.3.1. Candidate Investment Proposal and Candidate Management

As seen above, automatic techniques for the identification of promising candidate investments are necessary for TEP in large areas. These should be systematic and rigorous in order to reduce the feasible search region while preserving all potentially good solutions. In addition, in the cases where the list of interesting investments is too large, it would be beneficial to resort to similarly automatic procedures to handle the search space, focusing on the most promising areas. In particular, it is often the case that a just few candidate lines have the largest impact on the solution, constituting the core of the expansion plan and largely determining its overall goodness. Identifying this reduced group of decisive network investments can assist in the design of the plan as well as in establishing project priorities.

1.2.3.2. Identification of the Rationale for Investments and their Relationships

The TSO used to propose investments based on a clear purpose. That is, if a high-demand area needed a stronger connection to the rest of the network, he would propose several alternative corridors to do so. Similarly, the TSO could project some choices of transmission lines to integrate some forecasted new renewable generation. Therefore, each possible investment responded to a specific network need. However, when automatic mechanisms perform candidate proposal, this information about investment rationale is lost.

In addition, the interactions among investments are particularly important in TEP. Transmission lines can be parts of longer transmission routes, so that their full functionality depends on installing all the constituents of the route. Conversely, several lines can serve a similar purpose with respect to power flows. Deciding to build more than one of these substitute lines could be suboptimal, as the benefit from the additional investment would be very limited. Discovering these complementary or substitute sets of investments simplifies the problem considerably and enhances the understanding of its structure. This information would support the definition of projects, that is, transmission lines that share a common purpose.

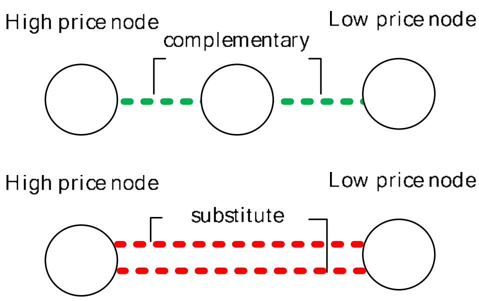


Figure 1-10 – Complementary and substitute investments.

This thesis proposes automatic mechanisms for **candidate discovery and the management of large search spaces**. The proposed approach uses sensitivities that can be calculated relatively easily from power flows and intermediate solutions of the TEP optimization problem. In addition, a candidate analysis method uses power flow data to **disentangle the relationships between candidate transmission lines** and identify complementary and alternative sets of investments, so that this information can support further decisions. We illustrate this with case studies based on the Spanish power system.

Moreover, we combine Ordinal Optimization (OO), a metaheuristic that identifies relatively good solutions efficiently, with Mixed Integer Programming (MIP) and apply it to network design. We use **OO to extract valuable information about the structure of the solution**. Then, MIP finds the best expansion plan among the ones that share the structure identified as favorable. We apply this to the OWF design problem, which exhibits a high degree of structure.

1.3. Solving Realistic Problems

This thesis originates from the challenges of current complex, large-scale transmission expansion problems. These real TEP applications are quite different in size from the academic test cases that usually appear in the literature. In order to be realistic, considered problem sizes should accommodate of the order of hundreds or thousands of nodes and lines. System operation should be modeled with the possibility of considering a DCLF and some sort of losses approximation. However other considerations such as short-circuit currents or voltage stability are not usually taken into account at the planning stage, as their complexity would generally be too burdensome for optimization purposes. The general approach seems to be to include them in subsequent analyses and use identified weaknesses in the design as additional constraints for a new resolution of the TEP problem.

The academic and realistic case studies in this thesis have been developed only to demonstrate the applicability of the developed techniques. They should only be valued as an illuminating example rather than for their results themselves.

Some of the contributions have been implemented in models with a clear practical orientation:

- TEPES (Transmission Expansion Planning for an Electric System) applied in projects such as Desertec [6], Medgrid [7] or Beyond 2020 [19].
- OWL (Offshore Windfarm Layout optimizer), developed as part of this thesis.

This work has also benefitted from experience in TEP-related projects:

- e-Highway, a project supported by the European Union aimed at developing a methodology to support the long-term planning of the European transmission network [20].
- The Renewable Grid Initiative, which promotes the development of the European transmission network for the integration of 100% renewable electricity [21].

1.4. Areas of Knowledge Involved

We have addressed the identified challenges using tools from very different backgrounds:

- Stochastic optimization techniques, in particular, Benders' decomposition, take advantage of the structure of the problem in order to solve it efficiently.
- We apply project finance tools, in particular, Real Options Valuation, to generate flexible expansion strategies. We use mathematical finance to price the modeled financial instruments.
- We extend statistical estimation techniques (shrinkage) to the stochastic optimization problem with the aim of generating robust solutions.
- We apply non-classical optimization methods, namely OO, to extract problem structure and complement and accelerate classical methods.

Chapter 2

State of the Art. Models and Techniques.

2.1. A Long-Studied Problem

Given the long-lived impact of transmission expansion decisions, it is not surprising that a vast amount of literature addresses this problem from different angles, with the first works appearing more than forty years ago [1]. This section is an effort to summarize decades of research, with the limitations this implies. First, we review the different modeling options in the problem. After that, we present the solution methodologies that have been used to tackle it. Then, we discuss some notes on the inclusion of additional criteria into the problem, which are particularly useful in practical planning studies. Precisely practical planning provides our perspective for the assessing these models and methods: we will assume that we perform large-scale TEP (hundreds or thousands of nodes) on a general-purpose computer. Finally, we describe the peculiarities of OWF network design, an especially relevant particular case.

2.2. Modeling Transmission Expansion

Transmission expansion is by nature a multi-stage problem where the planner takes decisions at several time horizons. In addition, investment decisions have an impact not only on the operation of the system but also on other factors such as its dynamic behavior or regulatory issues such as the possibility of market power exercise. The literature adopts different perspectives with respect to the inclusion of these effects, taking a wide spectrum of simplifying assumptions. This section reviews the most important modeling decisions in TEP.

2.2.1. Treatment and Scope of Uncertainties

Traditionally, the literature classifies uncertainties in two different groups [22]:

- Random uncertainties are those which arise from repeated deviations of parameters that can be described by means of a probability distribution. This probability distribution can be estimated from past observations. This type of uncertainty is usually linked to the short-term and is known as *risk*. Hourly demand profiles or hydro inflows constitute random uncertainties.
- Nonrandom uncertainty describes the non-repeatable evolution of parameters and, therefore, they cannot be modeled from past data. It is usually linked to the long-term. Generation expansion is the single most important example of these uncertainties. Although a probability distribution does not exist in a strict sense, we can nonetheless build scenarios and assess their likelihood.

In a random uncertainty context, expected cost represents the realized average profit & loss (P&L) after a sufficiently large number of repetitions of a decision, where the alternative scenarios compensate their positive and negative returns. However, nonrandom uncertainties only happen a single time, with no possibility for compensation. That is, generation expansion decisions for 2050 will only happen once, as there will be just one future for 2050. Once transmission expansion decisions have been made and 2050 has arrived, the rest of alternative futures are irrelevant. Therefore, it can be argued that methods that base their results on expected cost are not necessarily appropriate in a nonrandom context. Concepts such as flexibility and robustness are key in this environment.

The main tools applied for the treatment of uncertainties in transmission expansion are:

- Stochastic optimization, which incorporates random uncertainties directly in the decision process to optimize expected objective value [23-26]. The recent reference [27] quantifies the cost of ignoring uncertainty instead of solving the stochastic problem. The recent paper [28] performs a stochastic optimization of transmission under market and regulatory uncertainties. The paper [29] (under review) performs the joint optimization of generation and transmission and proposes to use Jensen's inequality to calculate efficient bounds for problems with a large number of scenarios. In a similar spirit, probabilistic load flows take into account the whole load-time profile in a single load-flow calculation [30].
- Robust optimization and other related techniques focus on a worst-case scenario analysis and minimize maximum regret [22, 31, 32].
- Fuzzy decision analysis deals with the outcomes of different scenarios in an analogous way to the multiple attributes defined in a multi-criteria problem. It identifies non-dominated solutions and works with the decision maker to analyze the relative importance of the objectives [33, 34].

Many works have focused on deterministic forecasts, ignoring uncertainty. Among the most commonly incorporated risks, we can find demand, generation costs, hydro inputs and generation expansion. A particularly relevant example is the inclusion of reliability considerations, where an N-1 criterion (which considers the failures of single components) is often used [23, 35].

2.2.2. Decision Dynamics

TEP is a multi-stage problem that implements long-term decisions in phases, with clear milestones where it re-evaluates decisions in the light of the revealed uncertainties. However, the complexity of this dynamic nature has lead most studies to focus on simplifications of the problem:

- Static. A vast majority of research studies consider only a snapshot of the future system at a particular moment. This is known in the literature as the Static Transmission Expansion Planning problem (STEP) [36-38].
- Sequential static. Widely used, it consists in modeling several time horizons, taking into account that any investments made will be available from their deployment date to the end of the planning scope. Sequential static planning can be carried out forward (moving in time from the closest to the furthest time horizon) or backwards (starting with the final year and moving towards the present) [23, 30, 39, 40].
- Dynamic planning. It keeps the full dynamic complexity of the problem. Given the computational complexity associated with this option, most research has focused on very small case studies or used heuristic methods with no optimality guarantee [26,

41]. Very interestingly, some references have suggested that TEP can be a promising area for the use of real options theory. However, applications have been limited until this moment [42, 43].

2.2.3. Market Considerations

As explained, market considerations or the regulatory implications of transmission are outside the scope of this thesis. However, we consider it interesting to review the main approaches taken.

Most regulators acknowledge that only centralized planning results in building all necessary transmission investments:

- The TSO proposes plans that the regulator later approves, possibly introducing competitive bidding to assign their construction.
- The regulator awards transmission licenses to private companies and considers them a monopoly, usually with remuneration based on a price index scheme.

In some cases, we can find examples of the following, which are not intended to substitute but rather to complement centralized planning:

- Mixed planning developed with the collaboration of market agents and regulatory institutions. For instance, coalitions of users can propose reinforcements that the regulator later approves. The remuneration of the project can either be regulated or bear the risk of investment.
- Market planning, where the expansion is performed at the discretion of market agents following price signals. Merchant lines, private investments that perceive a profit based on collected congestion rent, are the perfect example of this kind of regimes.

Given that maximizing aggregate social welfare in a market context is equivalent to minimizing operation cost, most works carry out centralized TEP with centralized cost-based operation, even in liberalized generation markets [36, 44]. Mostly in the cases where the focus is the medium-term expansion of the system, some references consider centralized TEP in a competitive generation market [33, 45]. Reference [46] presents a sophisticated approach where both competition and uncertainty are incorporated to the GEP problem. Finally, there has been some work done on decentralized expansion [47-49] and the impact of coalitions [50].

Therefore, this thesis is consistent with most of the literature on TEP when it considers centralized TEP with centralized cost-based system operation.

2.2.4. Mono vs. Multi-criteria Studies

The most basic approaches regard cost as the only objective included in the optimization. The factors usually considered are:

- Investment cost of the transmission assets added to the network.
- In a centralized operation context:

- Operation cost, defined as the average generation cost across scenarios.
- Adequacy penalties, introduced to avoid solutions where demand is not satisfied in normal operating conditions. For some of the simplest studies, the only objective considered is investment cost, but restrictions on adequacy are imposed to ensure acceptable service levels.
- Expected Energy Not Supplied (EENS) and other reliability indices penalize plans with a poor behavior under system contingencies.

- In a generation market context:
  - Aggregate social welfare is the main objective considered. It can incorporate elements for the objectives described above, such as EENS penalties.
  - Facilitating competition.

In addition, other relevant objectives are social acceptance of new corridors, environmental impact, renewable generation integration, congestion cost reduction, impact on system stability, flattening of nodal prices, financial effort required for the project and geopolitical risk.

Most of the reviewed works limit their focus to a single objective that integrates the attributes considered. Within a multi-criteria context, the most widely used tools are Fuzzy Decision Theory [26], Goal Programming [51] and Analytic Hierarchy Process [52].

2.2.5. Technical Grid Modeling Options

The literature presents different levels of detail in the technical description of the grid. With respect to power flow resolution, relatively simple transportation models (which only take into account Kirchhoff's First Law) reduce the computational requirements of the optimization [53]. Linearized DC power flows or hybrid models (where only existing lines abide Kirchhoff's Second Law) are widely applied [54, 55]. More sophisticated grid modeling options usually evaluate a given transmission plan rather than performing optimal transmission expansion. Some applications incorporate the nonlinearities of the AC power flow [56]. Most studies ignore losses, although a few take them into account either in an approximate or exact way [57-59]. Finally, some analyses consider the use of HVDC technology [43, 60-62] or other devices such as FACTS [43].

Table 2-1 summarizes the overall modeling options in TEP, highlighting the ones that this thesis considers.

	Model			
Uncertainty	Ignored	Stochastic Optimization	Robust Optimization	Fuzzy Decision Analysis
Decision dynamics	Static	Sequential Static	Dynamic	
Market	Centralized TEP with cost-based generation	Centralized TEP with generation market	Decentralized TEP in generation market	

Table 2-1 – Modeling options in TEP.

This thesis models a DC power flow and incorporates stochasticity on demand, renewable generation, hydro inflows and component failures through scenario trees, although not all these options are present in all case studies. We base our approach on Stochastic Optimization and develop applications of real options and regularization to obtain flexible expansion strategies and robust plans.

2.3. Solution Techniques Applied to TEP

In the long history of this problem, the literature has applied a wide range of techniques to its resolution. Within an optimization framework, we can find an array of classical and non-classical approaches that have been applied to TEP.

2.3.1. Classical Methods and Some Notes on Problem Formulation

The first papers that formalize the TEP problem do so from a perspective of linear programming (LP), where efficient simplex-based algorithms could find optimal solutions in affordable times for the computers available at the time [53]. These approaches ignore the discrete nature of investment in order to avoid the computational burden of integer variables. Linear formulations can accommodate transportation power-flow models. In addition, most of these works consider only deterministic forecasts for the uncertainties involved and focus on finding optimal plans for those. These simplifications are still useful in a context where the problem is too large to use a discrete model.

Quadratic Programming models approximate losses quadratically from a DC power flow, such as in the model used in reference [61], although it must be noted that this reference does not perform TEP.

Mixed Integer Programming (MIP) acknowledges the discrete nature of investment (a transmission line can either be installed or not) [63, 64].

In a handful of cases, Non Linear Programming and Mixed-Integer Non Linear Programming are used to accommodate the nonlinearities of an AC power flow. The additional computational burden can only be solved in relatively small networks, and is only justified when some effects that cannot be accurately captured with the DCPF need to be taken into account (e.g. reactive power or losses) [51].

The explicit exploitation of problem structure in stochastic models has prompted the appearance of studies based on stochastic decomposition such as Benders' decomposition [54, 65], column generation [66], or Lagrangean Relaxation [30]. Reference [53] compares alternative decomposition approaches.

2.3.2. Non-Classical Methods

Transmission expansion studies have reflected the development of non-classical methods [67]. The complexity of the problem explains this popularity, as the guarantee of a global optimum is traded off for more affordable computation times.

Metaheuristic algorithms improve a solution iteratively, usually including some form of random evolution. Genetic Algorithms replicate the principles of Darwinian evolution to solve optimization problems. They are currently a widely popular approach when solving combinatorial problems. This has been reflected in TEP applications [68]. Simulated Annealing draws a parallel between the thermodynamical evolution of a slowly cooling system and the optimization process [69]. Swarm intelligence mimics the collective dynamics of self-organized systems [40], with Ant Colony Optimization being a particular application of these principles [70]. Differential Evolution improves a solution pool by combining candidates among them [35]. Constructive Heuristic Algorithms refine solution proposals locally [71]. Artificial Immune systems focus on mutation rather than recombination processes [58]. Ordinal Optimization exploits the lower computational requirements of finding relatively good solutions as opposed to a single optimal one [72, 73]. General artificial intelligence techniques such as Artificial Neural Networks have been also been incorporated to TEP studies [41].

Other relatively little-known algorithms add diversity to the landscape of techniques that have been used in TEP. Examples of this are invasive weed optimization [74], random leap frog algorithms [75], path relinking [76], mosquitoes mating strategies [77] or hybrids of previously presented approaches [78].

2.3.3. Automatic Search for a Plan Based on the Application of Heuristic Rules

In addition to optimization, TEP has been tackled using automatic searches. This approach relies on a set of pre-defined rules that are applied to generate a suitable expansion plan. There is no a-priori guarantee of the goodness of the final solution or the soundness of the rules applied. However, the relatively simple layout of these tools compared to optimization allows including a high level of modeling detail.

Some works apply general local search methods that are not specific to the TEP problem. Some of the most interesting examples are GRASP (Greedy Randomized Adaptive Search Procedure) [55] and Tabu Search (which avoids already visited directions by adding them to a restricted or *tabu* list) [79].

The application of rules to guide the expansion is also widespread. Particularly, greedy local searches directed by sensitivity analyses are rather popular [80, 81]. Expert Systems can extract expansion rules by generalizing information from a set of sample systems [82]. These techniques can easily result in solutions that are far from optimal, as they reflect the necessary limited expertise of the planner. Expert Systems generalize the behavior of smaller systems and, therefore, can miss effects and structures that arise only in larger systems. Notably, although this method might detect suitable reinforcements, it might miss large *supergrid* investments, resulting in suboptimal solutions. This risk increases as the size of the area under study grows.

Table 2-2 summarizes these solution techniques, highlighting the ones this thesis applies:

<table>
<tr><th></th><th colspan="4">Solution methods</th></tr>
<tr><td rowspan="3">Classical</td><td>Linear Programming</td><td>Quadratic Programming</td><td>Mixed-Integer Programming</td><td>Mixed-Integer Non-Linear Programming</td></tr>
<tr><td colspan="4">Equilibrium Models</td></tr>
<tr><td colspan="4">Decomposition Techniques</td></tr>
<tr><td rowspan="4">Non- classical</td><td colspan="2">Expert Systems</td><td colspan="2">Sensitivity Analyses</td></tr>
<tr><td>Tabu Search</td><td colspan="3">Greedy Randomized Adaptive Search</td></tr>
<tr><td>Genetic Algorithms</td><td>Differential Evolution</td><td>Simulated Annealing</td><td>Swarm</td></tr>
<tr><td>Artificial Immune Systems</td><td>Constructive Heuristic Algorithm</td><td colspan="2">Ordinal Optimization</td></tr>
</table>

Table 2-2 – Solution techniques applied to TEP.

We apply Benders' decomposition to a MIP model to exploit the structure of the problem. In addition, we develop the first published application (to our knowledge) of decomposition techniques to offshore wind farm design.

This thesis also applies non-classical methods, in particular, Ordinal Optimization. We combine this technique with MIP to create a hybrid that extracts problem structure and exploits it to find a near-optimal solutions efficiently.

We conduct sensitivity analyses to identify potentially interesting investments and reduce problem size.

2.4. Some Notes on Practical TEP Projects

Solution methods impose limitations on the level of modeling detail and the complexity of the criteria included in the analysis. Therefore, not all the considerations that are relevant for a practical TEP problem can be included in the optimization. The most straightforward way of reconciling these approaches is to iterate between two different modules:

- A planning module uses one or more of the techniques described in 2.3 to generate an expansion plan or, preferably, a set of quasi-optimal solutions.
- An evaluation module recalculates system operation including all the additional criteria needed for the appropriate evaluation of plans, considering also the ones that the optimization problem could not include, such as dynamic behavior, impact

on market power structure or environmental impact. This module detects possible problems. The planner interprets them as additional constraints and adds them to a subsequent execution of the planning module. In addition, techno-economical indicators calculated in the evaluation module can identify potentially interesting candidate investments. When working with several quasi-optimal plans, the best solution can be selected according to the additional criteria.

This setting complements the advantages of optimal planning with the possibility of integrating all relevant criteria. More details can be found in the report we prepared for project e-Highway [20, 83].

2.5. The Particularities of Offshore Networks. Offshore Wind Farm Design.

Offshore wind seems set to play a crucial role in Europe's future energy landscape. At sea, winds are abundant and stable. In addition, the shallow waters of the North and Irish Seas allow installing turbines inexpensively and can avoid some of the environmental concerns and public opposition raised by their onshore counterparts. As of 2012, 5.3 GW of offshore wind power had been installed in Europe [10], including farms as large as the London Array (1 GW). Projects such as the Offshore Grid Initiative focus on creating an offshore network that links neighboring countries and to which large wind farms could connect [13]. Either as a large project stretching across several countries or as the connection of a single wind farm to the onshore network, the development of an offshore transmission grid is necessary for the integration of this new generation into the system.

Offshore network expansion has some particularities that differenciate it from other TEP problems:

- Offshore network planning is a *greenfield* expansion, that is, there is no existing network to take as the starting point for the developments.
- This implies considerable additional difficulties that are compounded by the fact that there is often a great deal of symmetry present in the layouts, so that many solutions are roughly equivalent. This makes the search for optimal solutions much harder.
- Reliability is especially important in this problem, as repair times at sea are remarkably long. This motivates the explicit introduction of reliability scenarios and redundant connections.
- The use of isolated cables, with a high reactance, combined with long distances to shore, means that considering HVDC connections is especially important in these applications. In addition, the cost of transformers and converters, which is minor in general TEP problems, has a large impact on the total cost of an OWF.

This thesis contemplates offshore network planning and the design of OWFs as a particularly interesting case of TEP. We present some context for this problem in the following sections.

2.5.1. Recent Trends and Current Situation

The increasing interest in offshore wind is resulting in larger and larger plants, such as the London Array with 1 GW or Greater Gabbard in the UK with 504 MW. Furthermore, forthcoming projects will be even larger. Dogger Bank in the North Sea will accommodate 9 GW of generating power, and there are over 10 more projects over 1 GW in the UK and Ireland. This expansion is attracting remarkable amounts of investment. According to EWEA, the offshore wind industry received a total investment of up to EUR 4.7 bn in 2012 [10].

However, there are still some engineering issues that should be overcome for offshore wind to reach its full potential. According to a recent study [84], the main drivers for cost reduction in the coming years will be the up-scaling of turbines (possibly with innovative designs [85]), component standardization and the improvement of vessel utilization rates. These factors are expected to lead to a cost reduction in the range of 25-40% by 2020. In addition, other needs for improvement are already being researched. Submarine cables need further refinement; their lead coating tends to not endure mechanical wear as needed, and the industry would welcome higher-rated 3xXLPE cables [86, 87]. Most importantly, multi-terminal HVDC would allow a more efficient transmission and operation of the huge wind farms planned. Floating foundations would make it possible to install OWFs even in deep waters and therefore harvest winds in areas where now such developments are prohibitively expensive [86]. Given the attention that the offshore wind industry is receiving nowadays, there are countless research projects (in some cases very close to production stages) that push the limits of the current technology. For instance, the Deepwind project by the Technical University of Denmark is developing a 20 MW vertical axis floating turbine [88]. KiteGen is an Italian company that attempts to exploit the stronger winds at higher altitudes with kite-like generators without a solid foundation [89]. Researchers at CalTech [90] are investigating the potential of imitating the fluid dynamics of fish schools to minimize wake effects in wind farms. Although it is not clear whether any of these blueprints will someday become the industry standard, they throw light upon the challenges inherent to offshore wind farms components and layout optimization.

2.5.2. Designing an OWF. The General Problem.

An OWF is a project of considerable complexity with extensive implications between the different perspectives of the design problem.

- *Macrositing* is the definition of the wider area where the wind farm will be located. Different considerations must be taken into account [91], such as wind regimes, water depth, and environmental impact factors such as distance to shore (which correlates to visual impact) electromagnetic field effects, barrier effects or expected bird collision rates [92, 93].
- *Micrositing* studies the specific placement of turbines within the wind farm. Wind generators should collect as much energy as possible incorporating the fluid dynamic interactions between the turbines (*wake effects*), which diminish the energy effectively collected [94].

- The electrical layout is already a key factor in onshore wind farm design, and it is even more important for offshore plants, where it amounts to around 20% of the total cost [95]. Cables, transformers and other electrical and electronic equipment that constitute the farm's collector and transmission systems are critical for operation and reliability.
- Other objectives can be incorporated. For instance, optimal control is also an interesting area that enables the farm to participate in ancillary services such as voltage and both active and reactive power control. Indeed, in some countries, regulation is pushing for this to become standard [96]. Several studies develop strategies for optimal management of ancillary services [97, 98]. In addition, repair times are much longer offshore and costs are much higher. Estimates show that maintenance costs amount to around 30% of the total energy generation cost [9, 99]. Therefore, maintenance strategies must be carefully studied. The standard is to pay service visits to each turbine every six months and perform corrective maintenance when needed [100]. Experiences with condition monitoring techniques in OWF have not been conclusive yet [99].

We can solve most of these objectives independently, in the same order as introduced above. First, the area where the farm will be installed is selected. Then, the turbines are placed and the electrical layout determined. Once the design is complete, the best strategy for operation and control is developed.

Substantial interactions couple turbine micrositing and the electrical layout problem. Placing turbines sparsely in order to minimize wake effects increases the cost of the collection system. Conversely, packing the turbines closely reduces cable costs but increases wake effects, which results in decreased energy production. However, this effect decreases dramatically if we consider that there is a limited area allocated to the farm. In this case, micrositing tries to pack the turbines as efficiently as possible into the given area. In turn, the electrical layout optimization designs the most efficient collection and transmission system for that specific turbine placement. This has led most of the reviewed studies to work using a decoupled strategy, which is somehow analogous to the independent consideration of GEP and TEP in most studies.

In the following sections, we discuss the electrical layout problem in more detail.

2.5.3. The Electrical Layout Problem

As mentioned above, the electrical system of an OWF amounts to around 20% of the total cost and affects deeply the farm's efficiency and reliability. It is therefore essential to find a good design.

Repair times and costs are much higher at sea. Moreover, getting access to the plant can be even impossible during the winter months or under stormy weather. For this reason, failures often take several months to be repaired, with substantial economic losses arising from not being able to evacuate the produced power. Therefore, an optimization that takes into account reliability can bring substantial operation and maintenance cost savings.

The collector system gathers the energy from the turbines. Often, a few standard configurations are used in the design of collector systems [95, 101-103]. The main standard configurations are shown in Figure 2-1.

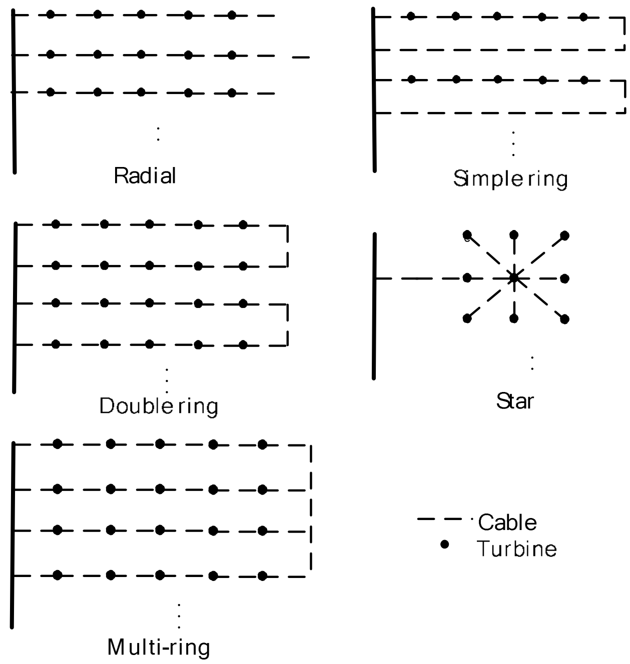


Figure 2-1 – Standard collector configurations.

- The *radial* collection system is the simplest of the possible assortments. Several turbines connect to the same feeder. It has been the most widely applied design because of its low cost and easy control [104]. However, the lack of redundant elements means that its reliability is poor, as any fault will potentially prevent all upstream turbines from selling their power. In addition, that is the design with the most losses.
- The *single-sided ring* or *simple ring* sets an additional cable from the last turbine in a string back to the collection hub. This cable should be able to carry the total power generated by the string. It is the most expensive of the standard assortments, doubling the cost of a radial layout [95]. However, it is also the most reliable and the one with the least losses.
- The *double-sided ring* or *double ring* tries to overcome the cost disadvantages of the single-sided ring by using the cable of the neighbor string as the redundant circuit. If a fault occurs, the string would need to deliver the full power of two strings. Therefore, the cable rating should double the power of a single row. This option is around 60% more expensive than a radial design [95].
- The *star* design was proposed as an attempt to reduce cable ratings (as all the cables from its center to the arms would just need to bear the power coming from one turbine). However, cables can be longer and the switchgear more complex, so the cost advantage depends on the specific case under study. This configuration also offers a high level of reliability, with only failures of the cable that connects the center to the collector hub affecting more than one turbine.
- The *multi-ring* configuration was conceived as a way of dividing the power generated in a faulted string among the rest of the rows, so that the capacity does not need to be upgraded as in the double-sided ring. This design has around 25%

less losses than a typical radial array, is more reliable and its cost is only around 20% higher [95].

The main properties of the standard collector options as described above are summarized in Table 2-3.

	Investment cost	Losses
Radial	1.00	1.00
Single-sided ring	2.10	0.54
Double-sided ring	1.58	0.81
Star	0.97	1.01
Multi-ring	1.18	0.76

Table 2-3 – Collector characteristics relative to the radial configuration.

Even though preliminary studies of standard designs are useful, both cost and reliability are so dependent on the particular geometry that the optimal solution is normally a hybrid of these architectures. What is more, even if the turbine layout is symmetric, the optimal layout can be asymmetric. An example of this appears in one of the articles published as result of this thesis [105]. For this reason, it is necessary to solve the full electrical layout optimization problem rather than selecting a standard collector design. This layout appears in Figure 4-2 (Chapter 4).

The transmission system takes the power generated to shore, bringing it to the main network at the Point of Common Coupling (PCC). Its architecture determines the number and location of offshore substations or converters, and the cables used. The technological option used is the single most important decision. There are several alternatives [106]:

- MVAC. This architecture is the simplest one, gathering the cables from the collector system and taking them together to the onshore network. Transmitting in MV means that losses will be high, and therefore this approach will only be appropriate for small plants relatively close to shore.
- HVAC. Almost all the OWFs in operation rely on this transmission system [104], where a power transformer elevates voltage up to 170 kV therefore reducing losses [107]. As distance to shore and power increase (which is the current trend) so does the need for reactive power compensation. Therefore, it is expected that this architecture will not be used in the large projects currently under planning [107, 108].
- LCC HVDC. The classic version of HVDC (thyristor-based Line-Commutated Converters) can transmit large amounts of power at voltages as high as 800 kV. It is a mature technology that relies on large converters both onshore and offshore. These include transformers, converters and filters, as well as an auxiliary power set [101]. Their cost is only competitive at higher distances to shore. In addition, they decouple the OWF and the onshore grid so that failures do not propagate and operation can be performed at frequencies other than the synchronous one. They also enable active and reactive power control [109]. However, this technology has the disadvantage of requiring a voltage source in order to provide commutation for the thyristors and that the generally used control systems are unable to function

when connected to a weak grid. The practical application of this technology has been limited for these reasons.

- VSC HVDC. Voltage Source Converter-based HVDC is based on self-commutated switches, mainly Insulated Gate Bipolar Transistors (IGBTs). Pulse Width Modulation (PWM) is widely used, although more sophisticated strategies have been proposed [110]. VSC provides better and independent active and reactive power control in each converter, being able to operate in all the four quadrants of the PQ-plane (that is, either consuming or producing active or reactive power). This makes it possible to connect the OWF to a weak grid. Furthermore, it is relatively easier to implement multi-terminal schemes and the system as a whole is more compact than traditional HVDC [111].

In general, it is possible to determine which transmission system is most suitable for a given OWF just by taking into account its rated power and distance to shore [112, 113]. According to these calculations, HVAC is only efficient for distances under 140 km and rated powers under 200 MW. VSC performs better in the rest of cases if the rated power is below 300 MW. Over 300 MW, component limitations become relevant and LCC HVDC is the preferred option. Table 2-4 summarizes these considerations.

	Low power (<200 MW)	**Medium power (200 MW<Power<300 MW)**	**High power (>300 MW)**
Short distance (< 140 km)	HVAC	VSC HVDC	LCC HVDC
Long distance (> 140 km)	VSC HVDC	VSC HVDC	LCC HVDC

Table 2-4 – Transmission system options depending on rated power and distance to shore [112, 113].

More innovative options are under research, such as operating the farm at medium frequency, which would allow for much cheaper transformers [102, 103]. Other projects consider a MV DC grid [114] or variable-frequency mini-grids [115].

As part of this thesis, **a State of the Art review OWF network design has been performed and published** (and it is the first one in this topic, to our knowledge):

S. Lumbreras and A. Ramos, "Offshore Wind Farm Electrical Design: A Review", Wind Energy 16 (3): 459-473 April 2013 10.1002/we.1498.

In addition, the importance of considering HVDC transmission has been acknowledged by the development of **a compact model for the technological choice of HVDC**.

2.6. Quick Guide to the Rest of this Document

The rest of this document is devoted to presenting the main contributions of this thesis. We develop contributions in modeling, problem resolution and interpretation of results.

First, Chapter 3 describes the optimization models that were used for TEP and OWF design. The contributions here are summarized below.

- We develop a compact model for AC and DC transmission.
- We publish the first state of the art review on OWF design.

Then, we detail our work, organized around the three basic challenges identified in current transmission planning:

- Efficient Resolution (Chapter 4), where our main contributions are:
  - We develop the first published application (to our knowledge) of decomposition strategies to OWF design.
  - We review of acceleration techniques applied to Benders' decomposition.

The recent report [116] deals with the joint optimization of the transmission network and other supply resources.

  - We proposed *Semi-relaxed cuts*, a novel type of cut that accelerates the convergence of Benders' decomposition when there are binary decision variables in the master problem.
  - We also propose a Progressive Contingency Incorporation (PCI) algorithm, which solves efficiently stochastic optimization problems where scenarios describe element failures.
- Flexibility and Robustness (Chapter 5):
  - We apply Real Options Valuation to TEP to identify transmission lines with a high potential for bringing operating cost savings to the system.
  - We develop a regularization approach to Stochastic Programming applied to TEP. Shrinkage makes the solutions to the stochastic problem more robust with respect to changes in the definition of scenarios.
- Structure identification (Chapter 6):
  - We propose a candidate discovery method, which uses sensitivity information to identify interesting candidate lines.
  - We develop a candidate management algorithm which focuses the search for optimal solutions on the most promising candidates.
  - Candidate analysis. This technique extracts the complementary and substitute relationships among transmission investments.
  - We create a Hybrid algorithm combining Ordinal Optimization (OO) and Mixed Integer Programming (MIP). This algorithm uses OO to identify the features that characterize relatively good solutions. Then, it applies MIP to find the best solution within the ones that share these features.

However, there is a considerable overlap in these challenges as well as in the purposes of our contributions. For instance, we used some of the structure identification techniques to accelerate problem resolution, so they could also be included under “Efficient Resolution”. We classify the developed contributions according to their primary aim and discuss other relevant implications. In addition, each of the chapters of the core of this thesis includes a “Chapter Takeaways” section that gives a brief context and highlights the most relevant contributions.

Finally, we extract conclusions and outline interesting future lines of research.

Chapter 3

TEP and OWF Design Models

3.1. Introduction

This chapter introduces the basic optimization models used in this thesis. The description included in this section does not intend to be exhaustive. It is rather a stylized model; we leave out some of the details (such as losses approximation) when their inclusion does not benefit a clear understanding of the contributions.

Figure 3-1 – Entso-e map of the European Transmission Network [117].

We present two different models, one for general TEP (in section 3.2) and another one for OWF layout design (in section 3.3). The particularities of OWF design lead us to build a specific model for this special case. The main differences between the two problems are:

- General TEP focuses on installing transmission lines. In OWFs, the selection of transformers, converters and offshore substations is also key for the performance of a layout. The specific notation we develop for the OWF problem reflects this difference.
- As discussed, repair times and costs are much higher at sea: it is necessary to rent a specialized ship and crew and organize a trip to the plant in order to perform a repair. What is more, getting access to the OWF can be even impossible during the winter months or under stormy weather. For this reason, failures often take several months to be repaired, with the subsequent economic losses arising from not being able to evacuate the produced power. Therefore, reliability is especially important in this context. It is therefore interesting to develop electrical layouts that can withstand transmission contingencies, for which redundant connections can be particularly useful. Consequently, we include these elements explicitly in our notation.
- The only uncertainties we consider in the OWF design problem are generation scenarios (resulting from different wind speeds) and transmission contingencies. We define them explicitly, which also allows us to explore the effect of different scenario tree arrangements (Chapter 4).
- TEP is, in most contexts, a *brownfield* expansion in the sense that there is a starting network that we should take as the initial point of the optimization. However, in

OWF design there is no such starting network and we perform a greenfield expansion instead. This has deep implications for the computational complexity of the problem, which we tackle by introducing additional constraints.

3.2. Basic Transmission Expansion Planning Model

The TEP problem selects the optimal network additions that minimize the sum of first-stage (investment) and second-stage (operation) costs. We define a Mixed-Integer Programming (MIP) formulation that accommodates a DC Power Flow (DCPF). The model considers uncertain scenarios that can describe different situations for demand, generation expansion, fuel prices, hydro inflows or renewable.

In general, we use lower case letters for indices and variables, and upper case for parameters.

3.2.1. Indices

i, j: nodes,

c: cable types,

t: discrete time periods when lines can be installed. The first period (the decision period) and the last period in the analysis will be represented as D, T. The time elapsed between two periods will be referred to as Et_{t_1,t_2}, with $t_2 > t_1$.

el, cl: existing and candidate lines,

ω_t: scenarios.

3.2.2. Parameters

EL_{ijc}: parameter that determines existing lines {0,1}. If a line exists, $El_{ijc} = 1$.

CL_{ijc}: parameter that determines candidate lines {0,1}. If a line is a candidate, $Cl_{ijc} = 1$.

$Df_{t_2 t_1}$: discount factor to evaluate a future cash flow corresponding to date t_1 in value terms of a closer date t_2 [p.u.]. It is calculated as $Df_{t_1 t_2} = \mathrm{e}^{-\rho Et_{t_1 t_2}}$.

ρ: discount rate [1/ yr],

P^{ω_t}: probability of a scenario,

Dur^{ω_t}: duration of a scenario [h],

Ic_{ijc}: investment cost of a circuit [M€],

$D_{ti}^{\omega_t}$: demand at each node [MW],

$Pnsc$: penalty for power not served [M€/MWh],

$Gc_i^{\omega_t}$: generation cost [M€/MWh],

X_{ijc}: reactance of a circuit [p.u.],

$\overline{F}_{tijc}^{\omega_t}, \overline{G}_{ti}^{\omega_t}, \underline{G}_{ti}^{\omega_t}$: flow and generation limits [MW],

M: big-M parameter.

3.2.3. Variables

3.2.3.1. First-stage variables

x_{tijc}: cumulative decision to install a candidate investment {0,1},

$invc_t$: total investment cost.

3.2.3.2. Second-stage variables

$g_{ti}^{\omega_t}$: power generated at a node [MW],

$opc_t^{\omega_t}$: operation cost [M€],

$pns_{ti}^{\omega_t}$: power not served [MW],

$f_{tijc}^{\omega_t}$: flow through a circuit [MW],

$\theta_{ti}^{\omega_t}$: voltage angles at nodes [rad].

3.2.4. Objective Function

The objective function minimizes total expected cost, composed of investment cost and operation expenses:

$$\min \sum_t Df_{tD} invc_t + \sum_{t,\omega_t} Df_{tD} P^{\omega_t} Dur^{\omega_t} opc_t^{\omega_t} \ . \qquad (1)$$

3.2.5. Constraints

3.2.5.1. First-stage constraints

The investment cost of a plan is calculated adding the costs of the installed lines:

$$invc_t = \sum_{ijc \in cl} Ic_{ijc} \left(x_{tijc} - x_{t-1,ijc} \right), \tag{2}$$

Once a line has been installed, it is available for the rest of the planning scope:

$$x_{tijc} \geq x_{(t-1),ijc} \,. \tag{3}$$

3.2.5.2. Second-stage Constraints

We define operation cost as the sum of generation cost and penalties for power not served:

$$opc_t^{\omega_t} = \sum_i (g_{ti}^{\omega_t} Gc_i^{\omega_t} + pns_{ti}^{\omega_t} Pnsc) \,. \tag{4}$$

We model a DCPF. Kirchhoff's First Law establishes power balances at every node and scenario. Incoming flows minus outgoing flows plus generated power equal demand minus power not served.

$$\sum_{jc/(El_{jic}+Cl_{jic})>0} f_{tjic}^{\omega_t} - \sum_{jc/(El_{ijc}+Cl_{ijc})>0} f_{tijc}^{\omega_t} + g_{ti}^{\omega_t} = D_{ti}^{\omega_t} - pns_{ti}^{\omega_t} \qquad \forall i, \omega_t \,. \tag{5}$$

Kirchhoff's Second Law or Voltage Law is linearized in the DCPF formulation:

$$f_{tijc}^{\omega_t} = \frac{\theta_{ti}^{\omega_t} - \theta_{tj}^{\omega_t}}{X_{ijc}} \qquad ijc \in el \,, \tag{6}$$

$$-M\left(1 - x_{tijc}\right) \leq f_{tijc}^{\omega_t} - \frac{\theta_{ti}^{\omega_t} - \theta_{tj}^{\omega_t}}{X_{ijc}} \leq M\left(1 - x_{tijc}\right), \qquad ijc \in cl \,, \tag{7}$$

Generation and flow limits:

$$\begin{aligned} -\overline{F}_{tijc}^{\omega_t} \leq f_{tijc}^{\omega_t} \leq \overline{F}_{tijc}^{\omega_t}, \qquad & ijc \in el \\ -\overline{F}_{tijc}^{\omega_t} x_{tijc} \leq f_{tijc}^{\omega_t} \leq \overline{F}_{tijc}^{\omega_t} x_{tijc}, \quad & ijc \in cl \end{aligned} , \tag{8}$$

$$\underline{G}_{ti}^{\omega_t} \leq g_{ti}^{\omega_t} \leq \overline{G}_{ti}^{\omega_t} \,, \tag{9}$$

$$0 \leq pns_{ti}^{\omega_t} \leq D_{ti}^{\omega_t} \,. \tag{10}$$

Each possible scenario ω_t assigns a value to each of the uncertain parameters that define the system $\omega_t = (\omega_{yt}^0, y \in Y)$, given some $\omega_{yt}^0 \in [\underline{\omega}_{yt}^0, \overline{\omega}_{yt}^0]$, where:

y: set that contains all the uncertainty sources considered in the problem, such as generation expansion, demand, fuel prices, renewable production or hydro inputs.

3.3. Model for OWF Layout Design

The OWF design problem finds the best architecture for the collector and transmission systems of an OWF. It defines the cables, transformers and converters installed. The starting data are characteristics and positions of the individual wind turbines, the point of common coupling to the grid (PCC) and the possible locations of offshore substations (the result of a micrositing problem), as well as the cable, transformer and converter types and their features.

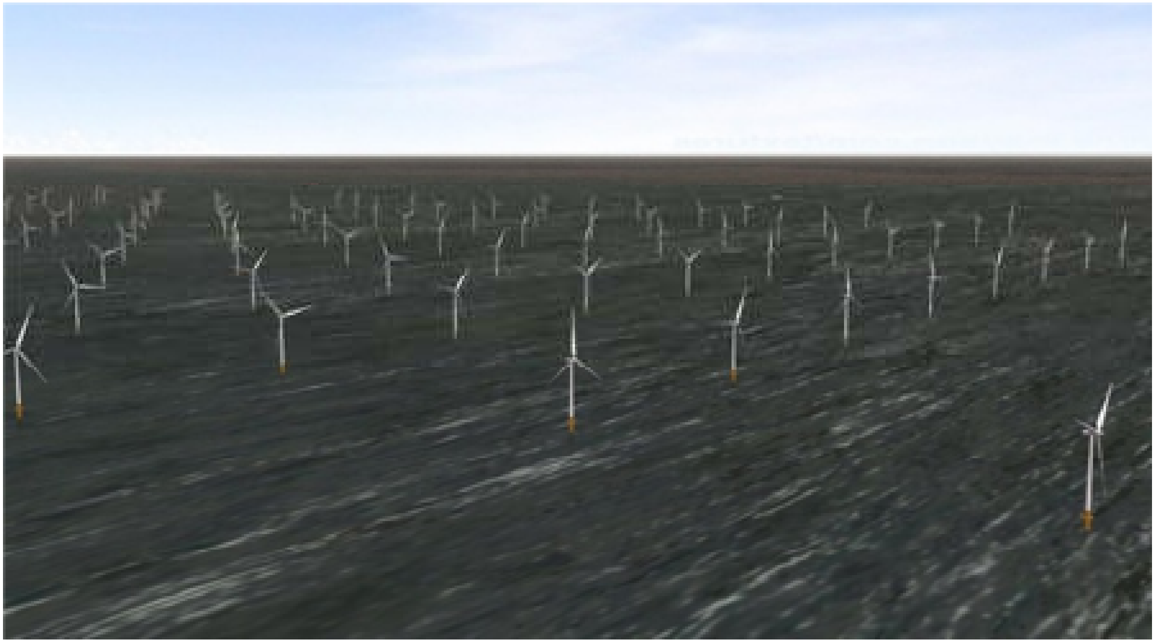


Figure 3-2 – Conceptual design of the London Array [12].

The main differences with respect to general TEP design are:

- Transformers and converter stations have a profound impact on the overall design of the layout. Hence, we include explicitly transformers/converter stations and constraints to impose voltage consistency on the selected equipment.
- Reliability is especially important in the OWF problem. We include it explicitly in the formulation by means of contingency scenarios and modeling redundant cables.
- The electrical layout is designed and built once. Therefore, it is a static problem where the scenarios are not time-dependent.
- In a greenfield expansion such as the one performed in OWF design, there are potentially many layouts with the same objective value; that is, the problem presents a high level of degeneracy. This is even more so when the turbines are placed in a symmetrical arrangement. Many cables have exactly the same investment cost (for instance, linking the turbines either in rows or in columns) and therefore many layouts (combinations of installed cables) will have the same investment cost too. Benders' decomposition will have to investigate many of these solutions and

therefore the problem will need more iterations to converge. We introduce additional constraints to tackle this issue.

For the sake of brevity, we define only the elements that are different from the ones used for the general TEP model.

3.3.1. Indices

tt: transformer or converter type. Similarly to the way we deal with voltage levels, we develop a compact notation where transformers and converters are both considered voltage-changing devices. The constraints ensure that all equipment is consistent with its rated voltage.

$ttac(tt), ttdc(tt)$: transformers and converters respectively.

$ac(c), dc(c)$: sets of AC and DC cables,

$vldc(vl), vlwt(vl)$: parameters that correspond to DC equipment and wind turbines {0,1},

r: redundancy level (several elements of the same kind can be installed in parallel, being marked with redundancy levels of r_1, r_2 and so on),

ω': wind generation scenario,

ω'': system state scenario, which defines contingencies in the collector and the transmission systems.

3.3.2. Parameters

α: depreciation parameter, calculated for a French amortization schedule (equal payments) as:

$$\alpha = \frac{\rho(1+\rho)^n}{(1+\rho)^n - 1}.$$

$D_i^{\omega'}$: demand,

$|wt|$: total number of turbines,

Di_{ij}: distance between any two nodes [m],

$Cp_c, Cc_c, Cx_c, C\lambda_c, C\mu_c$: rated power, cost, reactance, failure rate and repair rate of a certain cable type [MW, M€/km, p.u./km, failures/(km year) and repairs/year],

$Tp_{tt}, Tc_{ct}, Tvl_{tt}, Tvh_{tt}, TX_{tt}, T\lambda_{tt}, T\mu_{tt}$: rated power, cost, lower voltage, higher voltage, reactance, failure rate and repair rate of transformer or converter [MW, M€, kV, kV, p.u., failures/year and repairs/year],

Wt_i, Cp_i, Ps_i: parameters that describe the positions of wind turbines, a PCC or suitable locations for offshore substations {0,1},

$Btv_{tt,vs,vl}$: parameters that describe the correspondence between transformer or converter types and their respective voltage levels {0,1},

$Cvl_{c,vl}$: parameters that describe the correspondence between cable types and their respective voltage levels {0,1},

$G^{\omega'}$: turbine output by scenario [MW],

$\overline{G} = \max_{\omega'} G^{\omega'}$: rated power of turbines [MW],

$Dur^{\omega'}$: period duration of wind scenario [h],

$P^{\omega''}$: probability of system state [p.u.],

$Fc_{ijcr}^{\omega''}$: failure in a given system state of the cable between two points of a certain cable type and redundancy. The parameter takes a value of 1 if the element fails in that state and 0 otherwise {1, 0},

$Ft_{ij,tt,r,vs}^{\omega''}$: failure in a given system state of a transformer. The parameter takes a value of 1 if the element is failed on that state and 0 otherwise {1, 0},

$PCur$: penalty for curtailment.

3.3.3. Variables

3.3.3.1. First-stage variables

$invc$: investment cost [M€]. We assume that the complete layout is deployed at the time when the plant is completed, so that in this case investment cost does not depend on time.

$v_{i,vl}$: selection of voltage level used for a node {0,1},

os_i: installation of a substation at a given node {0,1},

x_{ijcr}: installation of a cable between two given nodes, of a certain cable type and with a specific redundancy level {0,1},

$tf_{ij,tt,r,vs}$: installation of a transformer or converter on the line linking two nodes, using equipment of a certain type and with a specified redundancy and position {0,1}.

3.3.3.2. Second-stage variables

$opc^{\omega'\omega''}$: operation cost [M€],

$f_{ijcr}^{\omega'\omega''}$: power flow between two nodes in a specific scenario [MW],

$\theta_i^{\omega'\omega''}$: voltage angle of a node in a specific scenario [rad],

$cur_i^{\omega'\omega''}$: wind curtailment at a turbine (power that cannot be delivered) [MW],

$pns^{\omega'\omega''}$: total energy not served [MW].

3.3.4. Objective Function

The model minimizes total annualized cost as the sum of the annual depreciated amount corresponding to investment cost and operation cost. A linear approximation of losses is included in some of the case studies, but, for the sake of simplicity, it has not been included in this section.

$$\min \alpha invc + \sum_{\omega'\omega''} P^{\omega''} Dur^{\omega'} opc^{\omega'\omega''} \ . \tag{11}$$

3.3.5. Constraints

3.3.5.1. First-stage constraints

Total investment cost considers cables and transformers or converters:

$$invc \ = \sum_{ijcr} x_{ijcr} Cc_c Di_{ij} + \sum_{ij,tt,r,vs} tf_{ij,tt,r,vs} Tc_{tt} \ , \tag{12}$$

We can install transformers only in substations and between two connected nodes:

$$\begin{aligned} os_i &\geq \sum_{j,tt,r,vs} tf_{ij,tt,r,vs}, \\ \sum_{c,r'} x_{ijcr'} &\geq tf_{ij,tt,r,vs} \end{aligned} \quad . \tag{13}$$

We impose that can use only one type of transformer or converter for every position. This assumption is widespread in realistic settings:

$$\begin{aligned} \sum_{tt} tf_{ij,tt,r_1,vs} &\leq 1, \\ \sum_{vs} tf_{ij,tt,r_1,vs} &\leq 1 \end{aligned} . \tag{14}$$

Similarly, we impose that if redundant cables are used, they must be of the same type:

$$\sum_{c} x_{ijcr_1} \leq 1 \,. \tag{15}$$

Redundancy levels of cables and transformers or converters are used in ascending order, so a redundancy level can only be used if the previous ones have been assigned:

$$\begin{aligned} x_{ijcr} &\leq x_{ijcr'}, \\ tf_{ij,tt,r,vs} &\leq tf_{ij,tt,r',vs} \end{aligned} \qquad \forall r > r' \,. \tag{16}$$

One of the contributions of this thesis is a compact formulation for HVDC. We base it on the following constraints, which ensure equipment is used consistently taking into account its rated voltage.

Every node must have a voltage level assigned:

$$\sum_{vl} v_{i,vl} = 1 \,. \tag{17}$$

We can only install transformers that are consistent with the voltage of the host node. As we can install only transformers of one type, it is sufficient to impose this condition for the first of the redundancy levels, r_1:

$$\sum_{tt,vs/Tvl_{tt,vs,vl}=1} tf_{ij,tt,r_1,vs} \leq 1 - v_{i,vl} \,. \tag{18}$$

Similarly, we allow only cables that have the same rated voltage as the nodes (or transformers) they connect:

$$\begin{aligned} x_{ijcr_1} &\leq \sum_{vl/Cvl_c=1} v_{i,vl} + \sum_{\substack{vl/Cvl_c=1 \\ tt,vs,vs'/ \\ vs \neq vs'}} tf_{ij,tt,r_1,vs} \left(Btvl_{tt,vs',vl} - Btvl_{tt,vs,vl} \right) \\ x_{ijcr_1} &\leq \sum_{vl/Cvl_c=1} v_{i,vl} + \sum_{\substack{vl/Cvl_c=1 \\ tt,vs,vs'/ \\ vs \neq vs'}} tf_{ij,tt,r_1,vs} \left(Btvl_{tt,vs',vl} - Btvl_{tt,vs,vl} \right) \end{aligned} . \tag{19}$$

3.3.5.2. Second-stage constraints

The goodness of a layout is defined in terms of the expected amount of energy that it can evacuate. This delivered power is the maximum generating capacity available minus curtailment. Therefore, we construct our objective function by adding investment cost and curtailment penalties.

$$opc^{\omega'\omega''} = PCur \sum_{\omega'\omega'' i} cur_i^{\omega'\omega''} \, . \tag{20}$$

Operation cost minimizes curtailed power assuming a constant penalty per MWh for all scenarios. This would be consistent with the wind farm receiving a feed-in tariff.

We model system operation with a DCPF. Kirchhoff's First Law establishes that, at every node, incoming flows plus generation (if the node has a turbine) equals outward flows plus curtailment (if the node has a turbine) and plus demand. Although there is no real demand present at the OWF, we capture its operation by introducing an artificial demand at the CPP that is always able to absorb the maximum power generated at each scenario. That is:

$$D_i^{\omega'} \begin{cases} \left| wt \right| G^{\omega'} & CP_i = 1 \\ 0 & CP_i = 0 \end{cases} . \tag{21}$$

$$\sum_{jcr} f_{jicr}^{\omega'\omega''} + Wt_i G^{\omega'} = \sum_{jcr} f_{ijcr}^{\omega'\omega''} + cur_{i/Wt_i=1}^{\omega'\omega''} + D_i^{\omega'} \, . \tag{22}$$

Kirchhoff's Second Law. This law applies only to AC cables and transformers:

$$-M\left(1 - x_{ijcr}\right) \geq f_{ijcr}^{\omega'\omega''} - \frac{\theta_i^{\omega'\omega''} - \theta_j^{\omega'\omega''}}{CX_{ijcr}} \geq -M\left(1 - x_{ijcr}\right), ijcr \in cl, c \in cac \, . \tag{23}$$

$$-M\left(1 - tf_{ij,tt,r,vs}\right) \geq f_{ijcr}^{\omega'\omega''} - \frac{\theta_i^{\omega'\omega''} - \theta_j^{\omega'\omega''}}{TX_{tt}} \geq -M\left(1 - tf_{ij,tt,r,vs}\right), tt \in ttac \, . \tag{24}$$

Cables respect their rated powers:

$$-x_{ijcr}(1 - Fc_{ijcr}^{\omega''})Cp_c \leq f_{ijcr}^{\omega'\omega''} \leq x_{ijcr}(1 - Fc_{ijcr}^{\omega''})Cp_c \tag{25}$$

Transformers and converters respect their rated powers. These constraints are active only if equipment is installed.

$$f_{ijcr}^{\omega'\omega''} - G^{\omega'} \left| wt \right| (1 - \sum_{tt,vs} tf_{ij,tt,r_1,vs}) \leq \sum_{tt,r,vs} tf_{ij,tt,r,vs} Tp_{tt} \left(1 - Ft_{ij,tt,r,vs}^{\omega''}\right). \tag{26}$$

$$f_{ijcr}^{\omega'\omega''} + G^{\omega'} \left| wt \right| (1 - \sum_{tt,vs} tf_{ij,tt,r_1,vs}) \geq -\sum_{tt,r,vs} tf_{ij,tt,r,vs} Tp_{tt} \left(1 - Ft_{ij,tt,r,vs}^{\omega''}\right). \tag{27}$$

Another problem that is characteristic of OWF layout is a very large search space. Turbine arrangements (resulting from micrositing) are often highly symmetric. This, together with the lack of initial connections (the OWF problem is a greenfield expansion), leads to a remarkably large, highly degenerate problem. In order to reduce the feasible region, we introduce some additional constraints (that is, we *strengthen* the problem). These constraints should restrict as many layouts as possible while preserving the optimal solution. The developed additional constraints impose that the farm should be able to evacuate its rated power if no contingencies occur. This is a reasonable criterion that reduces considerably the search space of the problem.

All turbines must be connected somewhere in order to be able to evacuate at least their rated power. Similarly, all couples of turbines must be connected to a different node (outside the couple) so that they can evacuate at least their combined power. The same argument holds for all the possible combination groups $gwt(i)$ of one, two, three, four,… $\left|wt\right|$ nodes where there is a turbine, $Wt_i = 1$. The number of turbines included in each of these sets is given by $\left|gwt\right|$ (so that, for every group of pairs of turbines $\left|gwt\right| = 2$, for the threesomes of turbines $\left|gwt\right| = 3$ and so on). For each of these groups, a constraint ensures that they can evacuate their rated power by establishing that there must be enough capacity linking the nodes of group $i \in gwt$ to the exterior $i \notin gwt$.

$$\begin{aligned} &\sum_{jcr} x_{ijcr} \geq 1 \qquad \forall i \,/\, Wt_i = 1 \\ &\sum_{\substack{ijcr/\\ i \in gwt(wt)\\ j \notin gwt(wt)}} x_{ijcr} Cp_c \geq \left|gwt\right| \overline{G} \end{aligned} \quad . \tag{28}$$

In addition, the cables connecting the offshore substation to shore must be able to withstand the total generation power of the farm:

$$\sum_{\substack{ijct/\\ Ps_i CP_j > 0}} x_{ijcr} Cp_c \geq \left|wt\right| \overline{G} \,. \tag{29}$$

We impose an equivalent constraint to transformers/converters. This will only be active if there is a transformer installed.

$$\sum_{ij,tt,r} tf_{ij,tt,r,vs} Tp_{tt} \geq \left|wt\right| \overline{G} - M(1 - \sum_{ij,tt,vs} tf_{ij,tt,r_1,vs}) \,. \tag{30}$$

3.4. Chapter Takeaways

This chapter introduces concisely the models used in this thesis. We develop two different models, one for general TEP and a second one which acknowledges the particular characteristics of OWF design.

The TEP model selects the optimal network additions that minimize the sum of investment and operation costs. A DCPF is used for system operation, which includes uncertain scenarios for generation expansion, fuel prices, hydro inflows, demand or renewable generation.

The OWF design problem defines the best electrical layout for the collector system (which links the turbines among them) and the transmission system (which links them to the onshore network), including offshore substations. Uncertain scenarios for available wind power and contingencies are modeled explicitly. In addition, it takes into account transformers and converter stations explicitly and ensures that voltages are used consistently

Chapter 4

Efficient Resolution

4.1. We Have Work Cut Out For Us…

The complexity inherent to TEP and the OWF design problem means that some optimization techniques can be impractical in this context. Moreover, realistic planning involves solving many instances of the same problem. Not all the relevant objectives can be introduced in the optimization problem (for instance, voltage stability or short-circuit currents), so the usual approach is iterative. First, an optimal expansion is obtained. Then, it is evaluated using more detailed models and introducing different considerations. If any infeasibilities are detected, they are expressed as new constraints or modifications to the starting optimization problem, which is solved again. In the case of OWF layout, the complexity of the problem leads to solving micrositing first, the electrical layout afterwards. The relationships between these two problems leads to an iterative framework too, where several options for micrositing are tested, with the OWF electrical layout being solved for each of them.

Having this into account, these problems demand not only the possibility of solving large-scale cases, but also affordable computation times. In this thesis, we develop algorithmic enhancements to the state-of-the-art methods in order to solve the large problem sizes currently involved in TEP and OWF studies.

We exploit the structure of the problem applying Benders' decomposition, a technique that divides a two-stage stochastic linear problem into two parts, a master problem and a subproblem that are solved iteratively until convergence. We develop the first published application (to our knowledge) of decomposition strategies to the OWF problem. In addition, in order to develop an efficient implementation of Benders' decomposition, we perform a comparative study of the different techniques that have been applied to accelerate it, a study that was missing in the literature. We then evaluate the potential time savings that these techniques can bring to TEP.

Furthermore, we develop some new acceleration techniques ourselves. Semi-relaxed cuts are a new kind of cut that deals more efficiently with large numbers of discrete variables in the master problem. We also propose a Progressive Contingency Incorporation algorithm to evaluate the effect of component failures efficiently. These enhancements reduce computation times by up to two orders of magnitude in the case studies developed.

The first application of decomposition strategies to the OWF layout problem, which includes free topologies and the consideration of HVDC transmission, was published as:

- S. Lumbreras and A. Ramos, "Optimal design of the electrical layout of an offshore wind farm applying decomposition strategies", IEEE Transactions on Power Systems 28 (2): 1434-1441, May 2013 10.1109/TPWRS.2012.2204906.

The review on Benders' decomposition acceleration techniques and semi-relaxed cuts were presented at:

- S. Lumbreras, A. Ramos, "Transmission expansion planning using an efficient version of Benders' decomposition. A case study", PowerTech 2013. Grenoble, France, 16-20 June 2013.

- S. Lumbreras, A. Ramos, “Improvements to Benders’ decomposition. A practical evaluation using a transmission expansion planning problem”, 13th Trans-Atlantic Doctoral Conference. London, United Kingdom, 9-11 May 2013.

They have also been submitted for publication:

- S. Lumbreras, A. Ramos ,“Optimal Transmission Expansion Planning using Benders' Decomposition. Acceleration Techniques“, submitted to JORS last April.

The PCI algorithm was published in the paper:

- S. Lumbreras, A. Ramos and S. Cerisola, “A Progressive Contingency Incorporation Approach for Stochastic Optimization Problems ”, IEEE Transactions on Power Systems 28 (2): 1452-1460, May 2013 10.1109/TPWRS.2012.2225077 [118].

This section describes these developments. First, section 4.2 introduces Benders’ decomposition and reviews the existing acceleration techniques and the proposed semi-relaxed cuts. Section 4.3 details our application of Benders’ decomposition to OWF design. Section 4.4 applies the reviewed acceleration techniques, as well as semi-relaxed cuts, to the general TEP problem. Section 4.5 describes the Progressive Contingency Incorporation algorithm developed to consider efficiently the failures of elements in stochastic design problems. We prove its optimality and show its potential advantages in a case study based on OWF design. Finally, section 4.6 presents conclusions.

4.2. Benders’ Decomposition

4.2.1. The Basics

Benders’ decomposition is recognized as one of the most important techniques in the optimization domain in general and in Stochastic Optimization in particular [119-121]. This method is also known as *primal decomposition* (because the *master* problem fixes variables in the *subproblem*), *L-shaped decomposition* (because of the shape of the constraint matrix) or *recourse decomposition* (because the master problem assigns directly the resource decisions to the subproblem). It has been applied extensively to the resolution of two-stage stochastic linear problems.

The two-stage stochastic linear problem is defined conventionally in its *complete problem* form as [122]:

$$\min_{x,y^{\omega}} c^T x + \sum_{\omega\in\Omega} d^T y^{\omega} \,, \tag{31}$$

$$Ax = b \,, \tag{32}$$

$$Tx + Wy^{\omega} = h^{\omega} \,, \tag{33}$$

$$x, y^{\omega} \geq 0 \,, \tag{34}$$

where:

x: first-stage variables,

y^{ω}: second-stage variables,

c, d: first and second-stage costs.

Eq. (32) represents first-stage constraints and Eq. (33) corresponds to the constraints that link both stages (known as *tender constraints*).

Benders' decomposition divides the problem into two parts. The *master problem* represents the first stage and some conditions derived from the second stage known as *cuts* [123], which provide a lower bound on second stage costs. The *subproblem* solves the second stage for the particular values of first-stage decisions defined by the master problem. This valid solution of the subproblem provides an upper bound for the optimal value. In addition, we use the information obtained from the subproblem to improve the description of the second stage that is included in the master problem by including a new cut. We solve both problems iteratively until we reach convergence.

The problem can also be interpreted as:

$$\begin{aligned} & \min_{x} c^{T} x + \sum_{\omega \in \Omega} \theta^{\omega}(x) \\ & s.t. \quad Ax = b \\ & \qquad\; x \geq 0 \end{aligned} \quad , \tag{35}$$

where $\theta^{\omega}(x)$ represents the second-stage objective function as a function of the first-stage decisions, that is, the recourse function. This can be expressed as:

$$\begin{aligned} & \theta^{\omega}(x) = \min_{y^{\omega}} d^{T} y^{\omega} \\ & s.t. \quad W y^{\omega} = h^{\omega} - Tx \qquad : \pi^{\omega} \\ & \qquad\; y^{\omega} \geq 0 \end{aligned} \; . \tag{36}$$

where π^{ω} represent the dual variables of the constraints. In Benders' decomposition, problem (35) is known as the *complete master problem* and problem (36) as the *subproblem.*

Expressed in dual form, the subproblem would be:

$$\begin{aligned} & \theta^{\omega}(x) = \max_{\pi^{\omega}} (h^{\omega} - Tx)^{T} \pi^{\omega} \\ & s.t. \quad W^{T} \pi^{\omega} \leq d \end{aligned} \; . \tag{37}$$

The optimal solution of the problem above must coincide with one of its vertices. The problem can therefore be solved by enumeration:

$$\theta^{\omega}(x\ ) = \max \left\{ (h^{\omega} - Tx)^{T} \pi^{wl} \right\} \quad l = 1,\dots,\upsilon \,, \tag{38}$$

where l denotes the existing vertices up to the total number of vertices, υ.

Therefore, if all the vertices of the feasible region were known it would be possible to build the *complete master problem* and solve it directly:

$$\begin{aligned}
&\min_{x,\theta^{\omega}} c^T x + \sum_{\omega\in\Omega} \theta^{\omega} \\
&s.t. \quad Ax = b \\
&\qquad \theta^{\omega} \geq (h^{\omega} - Tx)^T \pi^{wl} \quad l = 1,\ldots,\upsilon, \forall\omega \\
&\qquad x \geq 0
\end{aligned} \quad . \tag{39}$$

However, instead of obtaining the complete master problem, Benders' decomposition builds the description of the recourse function iteratively, one vertex at a time. The *relaxed master problem* is defined as:

$$\begin{aligned}
&\min_{x,\theta^{\omega}} c^T x + \sum_{\omega\in\Omega} \theta^{\omega} \\
&s.t. \quad Ax = b \\
&\qquad \theta^{\omega} \geq (h^{\omega} - Tx)^T \pi^{wl} \quad l = 1,\ldots,j, \forall\omega \\
&\qquad x \geq 0
\end{aligned} \quad , \tag{40}$$

where l denotes the iterations up to the current iteration j This can also be expressed in the following form, which is the most commonly used expression to define the master problem:

$$\begin{aligned}
&\min_{x,\theta^{\omega}} c^T x + \sum_{\omega\in\Omega} \theta^{\omega} \\
&s.t. \quad Ax = b \\
&\qquad \theta^{\omega} \geq f^{\omega l} + \pi^{wlT} T(x^l - x) \quad l = 1,\ldots,j, \forall\omega \\
&\qquad x \geq 0
\end{aligned} \quad , \tag{41}$$

where $f^{\omega l}$ represents the optimal second-stage cost for the first-stage decisions x^l proposed by the master problem evaluated in scenario ω. The optimal second stage cost for iteration j is obtained by solving the subproblem:

$$\begin{aligned}
f^{\omega j} &= \min_{y^{\omega}} d^T y^{\omega} \\
&\quad Wy^{\omega} = h^{\omega} - Tx^j \qquad : \pi^{\omega j} \; . \\
&\quad y^{\omega} \geq 0
\end{aligned} \tag{42}$$

Figure 4-1 shows a schematic representation of Benders' decomposition.

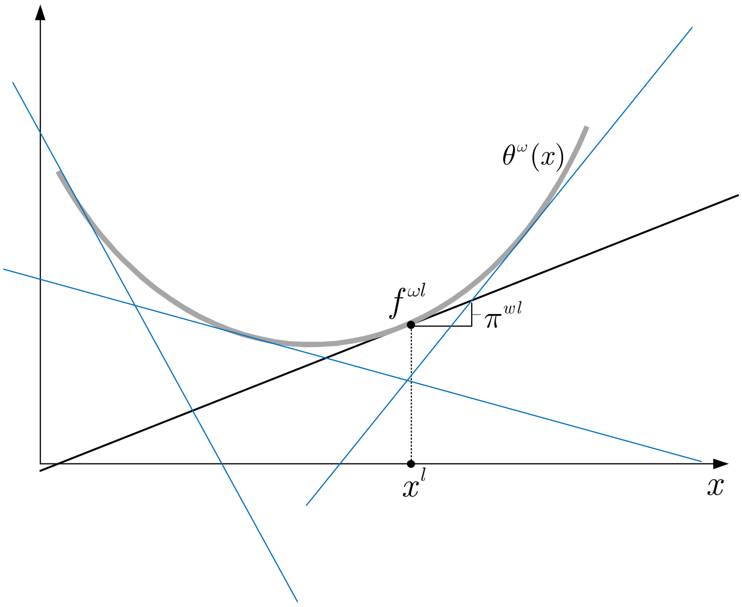


Figure 4-1 – Graphical representation of Benders' decomposition. The recourse function in the two-stage linear problem is a piecewise linear convex function, represented as a smooth convex function here for the sake of simplicity.

Because the whole definition of the problem is linear, the recourse function $\theta^{\omega}(x)$ is piecewise linear (convex) with respect to the first-stage decisions x. This convexity means that θ^{ω} represents a lower bound on second-stage costs. Thus, a lower bound on the optimal value can be obtained from the current master problem:

$$\underline{Z}^{j} = \min c^{T} x^{j} + \sum_{\omega \in \Omega} \theta^{\omega} \, . \tag{43}$$

Conversely, the subproblem minimizes second-stage costs for the first-stage decisions proposed by the master problem at the current iteration j, x^{j}. This constitutes a feasible solution for the problem. The best of the solutions obtained until the current iteration provides an upper bound on the optimal value:

$$\overline{Z}^{j} = \min_{l=1,2,\ldots,j} \left\{ c^{T} x^{j} + \sum_{\omega \in \Omega} f^{\omega j} \right\}. \tag{44}$$

The algorithm terminates when the upper and lower bound converge under a given relative optimality tolerance:

$$\left| \frac{\overline{Z}^{j} - \underline{Z}^{j}}{\overline{Z}^{j}} \right| \leq \varepsilon \, . \tag{45}$$

If the problem has an *incomplete* recourse (that is, some of the master problem solutions are not feasible in the second stage) the cut structure needs to be adapted to generate what is known as *feasibility cuts*. If the subproblem is infeasible (dual unbounded), it is modified so that the sum of the infeasibilities is minimized:

$$\begin{aligned} \min_{y^\omega, v^{+\omega}, v^{-\omega}} \; & f^{\omega j} := e^T v^{+\omega} + e^T v^{-\omega} \\ s.t. \quad & W y^\omega + I v^{+\omega} - I v^{-\omega} = h^\omega - T x^j \qquad : \pi^{\omega j}, \\ & y^\omega, v^{+\omega}, v^{-\omega} \geq 0 \end{aligned} \tag{46}$$

where e, I represent a vector of ones and the identity matrix. The result of this problem is used to build a feasibility cut, which does not add information about second-stage costs but rather eliminates infeasible first-stage solutions.

The formulation of both types of cuts (optimality and feasibility cuts) is unified under the following expression:

$$\begin{aligned} \min_{x, \theta^\omega} \; & c^T x + \sum_\omega \theta^\omega \\ s.t. \quad & Ax = b \\ & \delta^{\omega l} \theta^\omega \geq f^{\omega l} + \pi^{\omega l T} T \left(x^l - x \right) \quad l = 1, ..., j \\ & x \geq 0 \end{aligned}, \tag{47}$$

where $\delta^{\omega l}$ takes the values 1 for optimality and 0 for feasibility cuts.

The initial scope for Benders' decomposition was restricted to linear problems that could have integer variables in the master problem. However, early after the method was first proposed, Generalized Benders Decomposition (GBD) [124] extended it to convex subproblems in general. Other similar algorithms have been proposed by adding modifications to the master problem or the subproblem of GBD, such as [125].

Benders' decomposition has been applied in a wide variety of fields. The topological optimization of networks [126] and robust spanning tree construction [127] have been solved using this method. There are also applications of this technique to fixed-charge network design problems [128], scheduling [129, 130] and related problems such as aircraft routing [131], the traveling salesman problem [132], locomotive assignment [133] or distribution system management [134, 135]. Within computer and communications architecture, Benders' decomposition has been applied to network design [136] and spare capacity allocation [137]. Manufacturing system design has also been approached from this perspective [138]. There are also applications of this method to finance, in particular to portfolio optimization [139, 140].

Within power systems, Benders' decomposition has been applied to problems as diverse as generation expansion planning [53, 141-143], transmission expansion planning [25, 54] and distribution system design [144], hydrothermal co-ordination [145], the unit commitment problem [146], SO2 compliance planning [147] or distributed utility planning [148].

As can be seen from the formulation, the subproblem that we formulate for TEP is a linear problem and the master problem is mixed-integer, so we can apply Benders' decomposition

4.2.2. When is Benders' Decomposition Efficient?

Benders' decomposition is likely to yield time savings when one or both of the following conditions are met:

- First-stage variables (also known as *complicating variables*) increase notably the difficulty of the problem. That is, the problem becomes much easier to solve if first-stage variables temporarily take some fixed values. In addition, the possibility of decoupling the second-stage scenarios and solving them individually or in groups (typically in parallel) is especially interesting.
- The master problem and the subproblem have a different nature. Decomposition enables the application of the technique that is best suited for each [121], resulting in improved efficiency. A particularly relevant case arises when first-stage decisions are discrete and second-stage decisions are continuous. This results in the application of MIP algorithms for the master problem such as Branch & Cut, while the subproblem can benefit from the efficient application of LP solvers.

Both conditions apply to TEP and OWF design:

- The number of first-stage variables (which correspond to the candidate transmission lines) is much smaller than the number of second-stage variables (which describe all power flows thorough candidate and existing transmission lines at each scenario). Moreover, some case studies have so many scenarios that the resulting problem can be too large for direct resolution.
- The master problem and the subproblem have a different nature, as the first stage deals with integer variables (investments) while the second stage is linear (power flows).

We show the stylized formulation for Benders' decomposition for the general TEP problem in section 4.2.3. In addition, we develop the first published application (to our knowledge) of decomposition strategies to OWF design, where the advantages of this method can be observed. Section 4.3 reports our main findings.

However, even if the two conditions for the suitability of the method are met, they are not sufficient to guarantee that the decomposition will be more efficient than direct resolution. What is more, usually after an initial phase where the upper and lower bounds converge very quickly, often the decomposition only converges very slowly. Therefore, the use of acceleration methods can be interesting in this context. Section 4.2.4 details the accelerating methods that can increase the efficiency of a particular implementation of Benders' decomposition, which we then apply to the OWF design and TEP problems.

4.2.3. TEP Formulation using Benders' Decomposition

This section describes the formulation for the master problem and the subproblem obtained when we apply Benders' decomposition to the TEP problem as modeled in 3.2. The master problem decides transmission investments taking into account operation costs based on a cutting plane approximation:

$$\begin{aligned}
&\min \sum_{t,ijc\in cl} Ic_{ijc} Df_{tD}(x_{tijc} - x_{t-1,ijc}) + \sum_{\omega\in\Omega} P^{\omega_t} Dur^{\omega_t} \Theta_2^{\omega_t} \\
&s.t.\quad \Theta_2^{\omega_t} \geq Z_2^{\omega_t l} + \sum_{tijc} \left[\overline{F}_{ijc}(\pi^{+\omega_t l}_{tijc} - \pi^{-\omega_t l}_{tijc}) - M(\rho^{+\omega_t l}_{tijc} - \rho^{-\omega_t l}_{tijc}) \right] (x^l_{ijc} - x_{ijc}), \\
&l = 1, \dots, j
\end{aligned} \tag{48}$$

where:

ω_t: scenarios,

$\Theta_2^{\omega_t}$: recourse variables for every scenario,

$\pi^{+\omega_t l}_{tijc}, \pi^{-\omega_t l}_{tijc}$: dual variables of the equations imposing maximum and minimum power flows through lines,

$\rho^{+\omega_t l}_{tijc}, \rho^{-\omega_t l}_{tijc}$: dual variables of the equations imposing Kirchhoff's Second Law,

x^l_{tijc}: first-stage variables selected by the master problem at iteration l.

It should be noted that, in this case, no feasibility cuts are needed. All master proposals are feasible at the second stage, although some solutions will be penalized for instance if there is ENS.

The subproblem calculates operation cost for the investment variables fixed by the master problem and obtains the parameters needed for a new cut:

$$Z_2^{\omega_t j} = \min_{g_i^{\omega_t}, pns_i^{\omega_t}} \sum_{ti} g_{ti}^{\omega_t} cg_{ti}^{\omega_t} + pns_{ti}^{\omega_t} Pnsc. \tag{49}$$

With all the remaining constraints being exactly the same as in Chapter 3.

This formulation is a stylized version of the one implemented in TEPES [149] and used in the developments of this thesis.

4.2.4. Accelerating Benders' Decomposition

In general, acceleration methods focus on reducing the computation time spent on either the master problem or the subproblem. Master problem resolutions can be slow because of size, the presence of integer variables or the addition of a large accumulated number of cuts. Alternatively, the subproblem can be burdensome because of a large number of constraints or stochastic scenarios. The rest of this section focuses on describing the solutions that have been proposed to deal with these issues, linking them to the particular issue they aim at tackling. The technique descriptions do not intend to be exhaustive but rather give a general idea of the concepts involved and refer the reader to the appropriate literature for more detailed information.

4.2.5. Master Problem Modification Techniques

Usually, the master problem requires more computational resources than the subproblem. This is due to size (in addition, the master problem grows with every added cut) and because it is typical to find discrete first-stage variables. Therefore, the need to accelerate master problem resolutions is widespread and a multitude of techniques is aimed at this issue. We classify them in two categories:

- Solution technique modifications, which change how the algorithm finds proposals for the first-stage variables.
- Modifications to Benders' cuts, which define alternative cuts, append additional ones or remove them.

These techniques have a double impact on solution time. Solution technique modifications reduce computation time per iteration. However, the number of iterations needed to find the optimal solution might increase. Which of these two effects has the largest impact depends on the particular problem. In particular, in the case of methods that return sub-optimal master proposals, the relationship between time savings and the distance to the optimal solution is especially important.

Similarly, modifications to Benders' cuts generally result in more efficient formulations that can be solved in shorter computation times. However, some time will be spent in building these more sophisticated cuts. Again, the dominant effect will depend on the particular problem under study.

4.2.5.1. Solution Technique Modifications

a. *Master Problem Relaxations. Semi-Relaxed Cuts.*

It is not necessary to use the optimal solution from the master problem to obtain a valid Benders' cut [150]. Any feasible first-stage solution will suffice. If obtaining an optimal solution is computationally slow, the use of relaxed solutions can result in efficiency improvements.

In particular, solving a MIP problem is notably more complicated than finding the solution for its linear relaxation. Some authors have proposed solving linear relaxations of the master problem to derive a first set of cuts [151, 152] and this strategy can be found in a variety of practical contexts [153, 154]. This kind of approaches provides with a relatively good starting description of the recourse function at a low computational cost.

We propose to start with the relaxed problem introducing the binary description of variables progressively, one variable at a time, so we generate semi-relaxed cuts. The discretizations take place once the decomposition has converged under a given tolerance, and the variable to be discretized is chosen to be the one closest to take the values 0 or 1. It should be noted that the discretization just imposes that the variable is binary (i.e. not relaxed). In any case does it impose the value that it should take. This method improves convergence and avoids stagnant master proposals [105].

b. *Sub-optimal Master Problem Solutions*

Given that all first-stage solutions are acceptable for generating Benders' cuts, some authors have resorted to using sub-optimal solutions for the master problem, obtained from solving the master problem with a relatively high optimality tolerance. This tolerance is reduced gradually in order to guarantee that the optimal solution is finally obtained.

Although more iterations might be needed, the computation time per master problem resolution can be greatly reduced. Which of these opposite effects dominates depends on the characteristics of the problem under study. Some particular strategies include using any newly discovered first-stage feasible solution for cut generation [134], every node evaluated in a Branch & Bound resolution of the master problem [155] or rounded linearized solutions [156].

The application of non-classical optimization techniques to the master problem can be advantageous as well. In many cases, these methods are able to find near-optimal solutions in affordable computation times. In addition, very often these algorithms have a relatively good behavior with growing problem size. For instance, the master problem can be solved by metaheuristics such as Genetic Algorithms [157] or any heuristic technique in general [158].

Another version of the use of sub-optimal master solutions is the *boxstep* method [159]. This technique consists in incorporating an additional constraint to the master problem to reduce its feasible region, so that the solution is found in a smaller number of iterations. If the additional constraint is active at the optimal solution, then it cannot be considered actually optimal. In that case, the decomposition is resumed relaxing the additional constraint by a pre-specified amount (the *step*).

c. *Regularization Approaches*

It has also been argued that the approximation of second-stage costs that is implicit in the recourse function is only valid for values relatively close to the already explored solutions. Extrapolating to solutions that are far from them necessarily leads to high errors and probably introduces instability in the search that results in damaged convergence. Some authors have proposed to limit the distance between master problem proposals and an *incumbent* solution that is defined as the best one found until that moment [160].

4.2.5.2. Modifications to Benders' Cuts

Another fundamental way to accelerate the master problem is to modify the definition of cuts. Some approaches focus on building cuts so that they bring the maximum possible amount of information about the recourse function. Others eliminate the cuts that are not likely to be active again. This section reviews the main approaches followed in this area.

a. *Cut Structure*

Approaches that place a single cut per iteration are known as *monocut* (50) , while the ones that add several of them are referred to as *multicut* (51) [161].

$$\begin{aligned}
&\min_x c^T x + \theta \\
&s.t. \quad Ax = b \\
&\qquad \theta \geq \sum_{\omega \in \Omega} (h^\omega - Tx)^T \pi^{\omega l} \quad l = 1,\ldots,j, \quad \forall \omega \\
&\qquad x \geq 0
\end{aligned} \quad , \tag{50}$$

$$\begin{aligned}
&\min_x c^T x + \sum_{\omega \in \Omega} \theta^\omega \\
&s.t. \quad Ax = b \\
&\qquad \theta^\omega \geq (h^\omega - Tx)^T \pi^{\omega l} \quad l = 1,\ldots,j, \quad \forall \omega \\
&\qquad x \geq 0
\end{aligned} \quad . \tag{51}$$

Scenario partitioning techniques explore the different possibilities of arranging the resolution of the different scenarios and assigning them to different cuts [162]. We partition the set of scenarios into subsets and cuts are generated for each subset.

$$\begin{aligned}
&\min_x c^T x + \sum_i \theta^i \\
&s.t. \quad Ax = b \\
&\qquad \theta^i \geq \sum_{\omega \in \Omega_i} (h^\omega - Tx)^T \pi^{\omega l} \quad l = 1,\ldots,j, \quad \forall \omega \\
&\qquad x \geq 0 \\
&\qquad \bigcup_i \Omega_i = \Omega \\
&\qquad \Omega_i \cap \Omega_j = \varnothing \qquad \forall i, j
\end{aligned} \quad . \tag{52}$$

More cuts define the recourse function more accurately, but also increase the computation time needed to solve the master problem. Therefore, it is usually desirable to test several alternatives before making this choice [163]. More sophisticated approaches assign scenarios to cuts dynamically, trying to optimize the amount of information available in them [164]. In addition, if some scenarios have a particularly high impact on the final solution, it can be interesting to extract them from the subproblem and insert them into the master problem, together with all their related constraints and variables. In this way, the master problem grows but gets an exact description of the second-stage cost of these high-impact scenarios since the very first iteration. We used this approach to solve the OWF layout problem [162].

b. *Minimal Infeasible Subsystems (MIS)*

In problems with both optimality and feasibility cuts, the appropriate selection of the latter might accelerate convergence notably. The usual method to choose the cut is to minimize the sum of infeasibilities as explained in 4.2.1. MIS provide an alternative method for the generation of feasibility cuts [165, 166]. MIS are defined as the smallest subset of constraints that result in an infeasibility. The calculation of these sets is in itself an NP-hard problem, but several alternative approaches have been proposed to generate cuts through MIS in an efficient way [167].

Feasibility cuts only provide information about the feasibility of first-stage decisions, but bring no information about second-stage cost. Therefore, in problems with more feasibility than optimality cuts it can be difficult to guide master proposals. In these cases, it is possible to generate constraints with the structure of optimality cuts from infeasible subproblems. Apart from originating the corresponding feasibility cut, a minimum number of constraints is relaxed in order to obtain a feasible subproblem, which is then solved to create an optimality cut [168].

c. *Non-Dominated Cuts*

A cut or constraint C^* is said to *dominate* or be stronger than another cut C, expressed as:

$$\begin{array}{ll} C: & z \geq f(u) + yg(u) \\ C^*: & z \geq f^*(u) + yg^*(u) \end{array} . \tag{53}$$

if:

$$f^*(u) + yg^*(u) \geq f(u) + yg(u) . \quad (54).$$

with an strict inequality for at least one value $y \in Y$.

Pareto-optimal cuts are constraints that are not dominated by any other. Adding dominated cuts to the master problem does not change the feasible region and increases size making problem resolution generally slower. Non-dominated cuts [169] have been applied in a variety of works [170]. In particular, when the subproblem is degenerated, there is more than one set of cuts that could be built from the same master solution. Some authors have suggested using the solution that generates a non-dominated cut, so that the information encoded in the master problem constraints is as efficient as possible [169].

For a feasible first-stage solution $\hat{x}$ (called a *core point*), the algorithm selects the solution of the subproblem that will generate a cut with the highest value when evaluated at the core point. That is, if we express the value of the cut at the core point as:

$$\pi^T (h - T\hat{x}) , \tag{55}$$

where $\pi \in \Pi_{opt}$ stands for the set of alternative dual solutions of the subproblem for the value of the first-stage decisions $\hat{x}$. The non-dominated cut will be the one that gives the highest value at the core point:

$$\pi^* = \max_{\pi} \pi^T (h - T\hat{x}) . \tag{56}$$

Some authors have highlighted potential difficulties in the implementation of this method, such as complications finding a core point and the fact that computational improvements cannot be guaranteed given that there is an additional linear problem to be solved at each iteration [131, 171]. Other similar techniques include *maximal cuts*, which are calculated by means of small perturbations of the right-hand-side of the subproblem [168].

d. *Covering Cuts*

It has been noted that Benders' cuts are often *low-density* in the sense that they involve only a small fraction of the decision variables. These cuts strengthen the problem only to a limited extent. It is desirable to generate cuts that cover the highest possible number of variables [172, 173]. When a master proposal is infeasible at the subproblem, a feasibility cut is generated under the form:

$$0 \geq f^{\omega l} + \pi^{\omega l T} T(x^l - x)\,. \tag{57}$$

The coefficients that affect first-stage variables are given by the product of the dual variables vector and the constraint matrix. The coefficient that affects a particular first-stage variable j is:

$$\left(\pi^{\omega l T} T\right)_j\,. \tag{58}$$

A cut is said to *α-cover* a given variable if the absolute value of the ratio of the variable coefficient divided by the maximum coefficient in the cut exceeds α. That is:

$$\left|\left(\pi^{\omega l T} T\right)_j\right| \geq \alpha \max_{j'} \left\{\left|\left(\pi^{\omega l T} T\right)_{j'}\right|\right\}. \tag{59}$$

A series of auxiliary subproblems is solved with the aim of generating cuts that cover all the first-stage variables. For a given variable j, not yet covered, bounds are imposed to the existing dual subproblem so that the coefficient corresponding to the variable results in a higher cover:

$$LB_j \leq \left(\pi^{\omega l T} T\right)_j \leq UB_j\,. \tag{60}$$

Where LB_j and UB_j represent the bounds that are imposed to the coefficient.

This process is repeated with the variables that have not yet appeared with large enough coefficients, adding the necessary number of cuts so that all variables are covered to the desired level. The increased number of cuts results in a larger master problem. However, the consequent reduction in the feasible region is reported to overweight this cost [173, 174].

e. *Inactive Cut Removal*

Keeping more cuts generally implies larger master problems and longer solution times. However, it has been shown that, often, large fractions of the existing cuts are not active. Cut removal can result in lighter master problems. However, some cuts might have to be generated more than once with the consequent impact in solution times. In addition, a mechanism to avoid the elimination of recurrent cuts is necessary to avoid cyclical solutions. A number of authors have followed this approach with notable time savings [53, 153].

4.2.6. Subproblem Modification Techniques

4.2.6.1. Scenario Structure

In multi-stage problems or problems with more than one source of stochasticity, the definition of a suitable scenario tree is not always straightforward. Scenario tree reduction and node aggregation techniques can reduce efficiently the size of the scenario tree [175, 176].

4.2.6.2. Bunching

Fixing first-stage variables allows solving the subproblems independently. Notably, parallel computing solves scenarios simultaneously with the consequent time savings [177].

However, it is possible to avoid solving some of the scenarios until optimality applying information about previously solved subproblems. Bunching [121] solves some of the scenarios using sensitivity information. It applies the optimal basis B obtained for one scenario to approximate others:

$$y^{\omega} = B^{-1}(h^{\omega} - Tx). \tag{61}$$

4.2.6.3. Specific Algorithms

In the cases where there are specific algorithms available for the subproblem, their application can result in significant time savings. For instance, this approach has been applied to a network design problem that uses a shortest-path algorithm instead of a regular LP solver [53].

4.2.6.4. Increasing Modeling Complexity

Another alternative is to solve an increasingly accurate version of the subproblem. For the cuts to be valid, approximations must give values below the exact cost. This technique can accelerate convergence if the solution of the approximations is very similar to the one of the exact model but takes a shorter computation time. A particularly interesting example of this technique is the use of increasingly detailed network representations in a TEP problem while keeping the cuts generated in the less accurate and faster stages [178]. Along the same lines, we propose the resolution of design problems where scenarios describe the possible failures of the installed components by solving a series of progressively more complex problems (4.5). A similar approach using increasingly complex master problems was proposed in [28].

4.2.6.5. Sub-optimal Subproblem Solutions

Any dual feasible solution of the subproblem (primal infeasible) will still yield valid cuts that can be used as a lower bound for second-stage costs. In the cases where the subproblems are large it can make sense to use sub-optimal dual solutions for cut generation (Zakeri's cuts) [179].

4.3. OWF Design with Benders' Decomposition

4.3.1.1. Introduction

We apply Benders' decomposition to the OWF design model described in 3.3. The formulation is very similar to the one for the TEP problem. Therefore, we omit it for the sake of simplicity. We solve several case studies in order to illustrate the efficiency of this approach, and also the economical benefits implied by optimizing the layout rather than using pre-established configurations.

The case studies include a real case representative of the wind farms currently in operation. The OWF chosen was Barrow Offshore Wind Farm (BOWF), which was the largest OWF when commissioned in 2006 by Centrica and Dong Energy. A detailed description of the plant and its components is included in [180]. We also consider a much larger case (525 MW, 75 turbines, which is larger than the largest case seen in the literature) as a representation of the large projects that will be committed in the future.

4.3.1.2. Results

Figure 4-2 compares the actual layout of BOWF to the result of our optimization. In the former, generators are linked to form four rows. The extremes of the strings are linked using higher rated cables due to the higher power they carry. The four extremes connect to the offshore platform, which hosts a 120 MVA transformer.

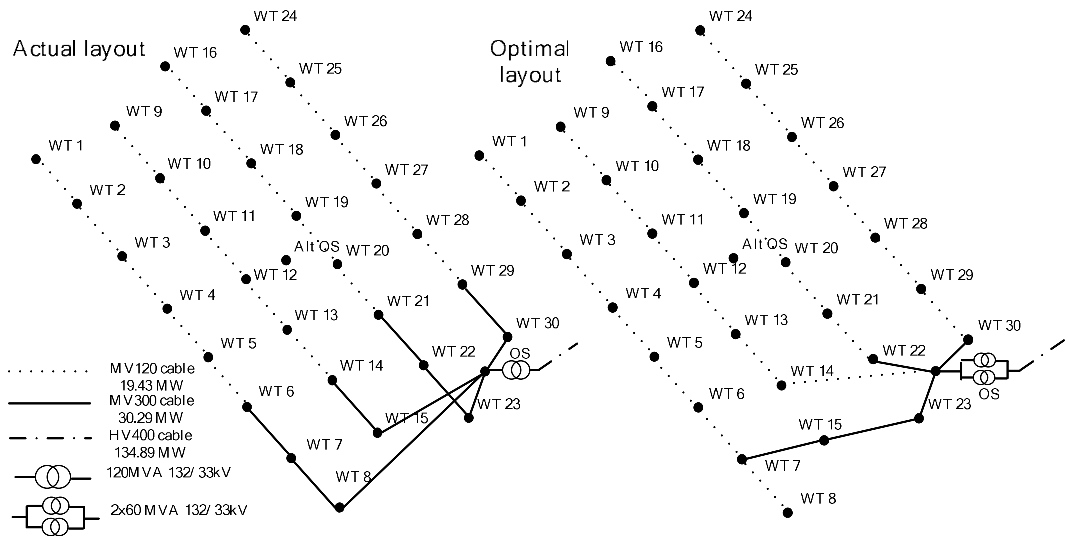


Figure 4-2 – Actual layout for BOWF vs. proposed layout.

The main differences between them are the use of two 60 MVA transformers instead of a single 120 MVA device, which improves reliability, and some breaks of symmetry close to the offshore substation that allow a more efficient use of cables. The objective values, split in investment cost and operation cost, can be seen in Table 4-1.

	Total (M€/y)	InvC (M€)	OpC (M€/y)
Actual layout	**2.28**	19.10	0.88
Optimized layout	**2.24**	18.69	0.86

Table 4-1 – Optimization results for the real size case example.

The optimized layout results in savings of 1.8% in total cost including 2.2% in investment cost. The proposed design allows using the more expensive cables more sparingly (only four links versus the ten present in the original layout). In addition, reliability improves mainly due to the redundancy in the transformer. The total realizable savings would have been of the order of 800,000 € during the life of the project, 410,000 € in the investment phase and the remaining amount to be recovered throughout the life of the farm. The analysis of the results was based only in a base case for the parameters, so they should be taken with caution. However, they show clearly that the optimal layout is in general different from any standard configuration and that there are cost savings achievable by optimization.

4.3.1.3. Problem Size and Algorithm Performance

Table 4-4 shows the main problem statistics for the cases solved: number of equations , continuous and binary variables, non-zero elements and integrality gap (the difference between the optimal solution to the discrete problem and its linearization, which is commonly used as a measure of the complexity of problems with discrete variables).

	Equations	Continuous variables	Binary variables	Non-zero elements	Integrality Gap
BOWF	42767	27042	102	147357	24.96%
Larger case	475685	300292	415	2219362	84.12%

Table 4-2 – Problem size.

Four different versions of Benders' decomposition were tested. Detailed explanations of these versions can be found in section 4.2.5.2.a.

The first three correspond to different ways of arranging the scenarios in the generated cuts:

- Both: one cut per system state (which describes contingencies) and wind scenario.
- System state: one cut per system state, which aggregates across wind scenarios.
- Wind scenario: one cut per wind scenario, which aggregates across system states.

The other one, system state aggregation, corresponds to adding the base case (no contingencies) to the master problem. This is also explained in more detail in section 4.2.5.2.a.

CPU times, expressed in seconds, were calculated using CPLEX 12.1 on GAMS on a PC at 2.80 GHz with 4 GB RAM running Microsoft XP 32 bits. The optimality tolerance was 0.1% in all cases.

In the case study based on BOWF, Benders' decomposition converges in only 4.3% of the time spent by the complete problem.

	Complete Problem	Benders' decomposition			
		Both	System state	Wind scenario	System state and wind scenario
Total	14055.5	597.6	880.7	672.5	124.3

Table 4-3 – Computation time for the case study (BOWF), in seconds.

It was not possible to perform a direct resolution of the larger case given its size. However, it was solved in only 225.1s using Benders' decomposition and the proposed PCI algorithm (as described in section 4.5). The final design can be seen in Figure 4-3. It combines a radial topology for each of the rows with a multi-ring at the extremes for enhanced reliability. Two offshore substations are installed. The main one is placed at the closest point to the CPP. The second one is placed at the center of the layout. The latter allows using lower rated (and cheaper) cables by joining the farthest half of the layout directly to the main substation.

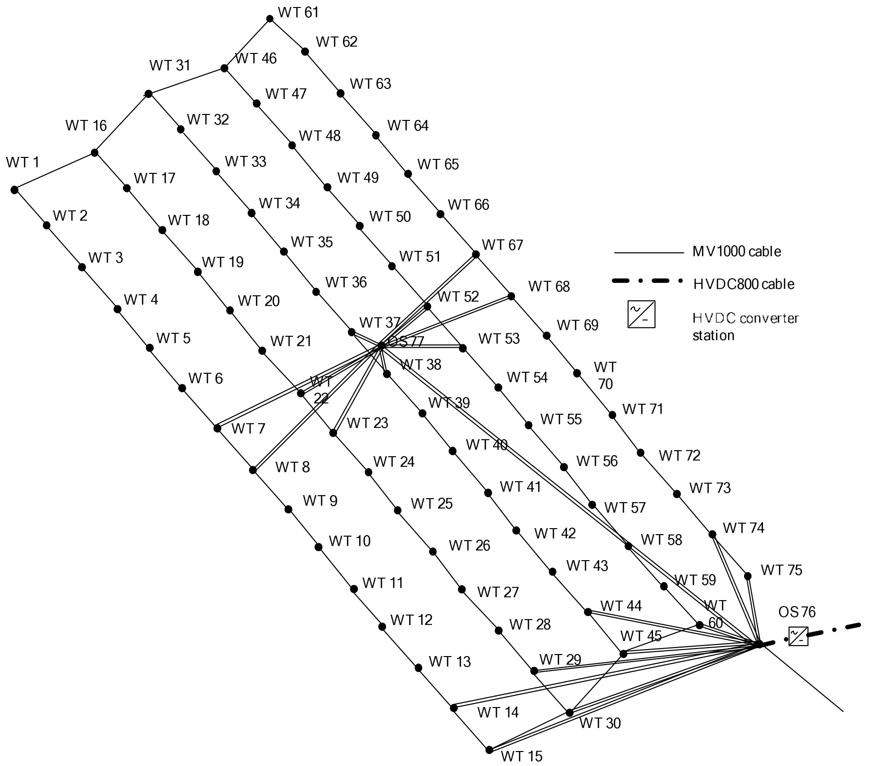


Figure 4-3 – Proposed layout for the larger case.

As can be seen from the results, Benders' decomposition is able to reduce efficiently computation time in the OWF design problem, with the best time savings reaching two orders of magnitude for the case study. Furthermore, the large case study could be solved only by the decomposition approach.

4.4. Accelerating Benders' Decomposition in the TEP Problem

We solve several TEP case studies in order to assess the performance of the proposed improvements with growing problem size. The cases are based on the 46N IEEE test system and the 87N IEEE test system [181]. Table 4-4 shows some statistics on the straightforward application of Benders' decomposition to the case studies. We consider a single scenario except in the 46N 100-scenario case, which explores the effect of a larger number of stochastic scenarios. We generated these scenarios from demand profiles calculated proportionally to the base case, stretched from 70% to 100% of the base-case load. The relative optimality tolerance was 0.1% for both the master problem and

Benders' decomposition in all cases. The number of equations corresponds to the last iteration of the decomposition. We report solution times of the master problem and the subproblem as an average value per iteration. As can be seen, most of the solution time is spent in the master problem, where decision variables are discrete. This is the main fact that will determine the selection of improvement techniques applied.

	Master				Subproblem			Decomposition		
Case	Eq	Var	D var	Time (s)	Eq	Var	Time (s)	It.	Time (s)	Time per it.(s)
46N	162	82	79	1.0	208	359	0.0	161	162.6	1.0
46N/ 100 sc	7402	181	79	9.7	208	359	1.2	75	820.6	10.9
87N	124	155	152	2.9	181	748	0.0	123	364.5	3.0

Table 4-4 – Case study statistics. "Eq" means "equations", "var" means "variable"s and "D var" means "discrete variables", "Time (s)" means" time in seconds" and "it" means iterations.

4.4.1.1. Selection of the Improvements to Benders Decomposition Applied to the Case Study

This section explains the rationale for the selection of the improvement methods that we apply to the case study. First, as explained, most of the decomposition time is spent in the master problem. In all cases, the total time invested in the subproblem is less than 12%. Therefore, even if the subproblem could be accelerated so much that virtually no time was spent on its resolution, the maximum achievable savings would be 12%. This discourages the application of subproblem modification methods in this case. Therefore, we test only master modification techniques. Within master problem modification techniques we find modifications to the solution technique and modifications to Benders' cuts. We explore both.

a. *Solution Technique Modifications*

The relatively slow resolution of the master problem is mainly due to the presence of binary variables. We observe this when solving the linearized problem. For instance; the largest case study contains 152 discrete variables and is solved in 2.9 s, while its linear relaxation is solved in less than 0.1 s (97% savings). Therefore, in an ideal situation where the cuts obtained were exactly the same, there would be potential savings up to 85% (This is, (1-12%)·97% = 85%) if we used a relaxation of the master problem. This leads us to test some alternative master problem relaxations:

- Solving a linear relaxation of the problem for a certain number of Benders' iterations to derive a first set of cuts (identified with the label *linear first* in the results table).
- Using semi-relaxed cuts (our proposed cuts that discretize variables progressively).

b. *Sub-Optimal Master Solutions*

As seen above, the potential savings if we can find a solution for the master more efficiently can reach up to 85% of total solution time. Therefore, we also consider suboptimal master solutions:

- Using sub-optimal master solutions obtained by setting a large relative optimality tolerance that decreases as the algorithm progresses. This mechanism is defined by the initial tolerance and the step by which the tolerance is reduced at each iteration. In order to prevent premature convergence, the tolerance will be adjusted so that it is always lower than the convergence ratio that has been obtained at the previous iteration:

$$\varepsilon Master_j = \min\left(\left|\frac{\overline{Z}_j - \underline{Z}_j}{\overline{Z}_j}\right|, \varepsilon Master_{j-1} - \varepsilon Step\right). \tag{62}$$

where:

$\varepsilon Master_j$: convergence ratio at current iteration j,

$\overline{Z}_j$: current upper bound, resulting from the best evaluated solution until the moment,

$\underline{Z}_j$: current lower bound, resulting from the most recent evaluation of the master problem.

$\varepsilon Step$: step by which the tolerance is reduced at each iteration,

This will be referred to as *inexact master* in the results section.

In order to assess the parameters that should be used for inexact master resolution, we present some considerations on the effect of tolerance on computation time. Increasing tolerance reduces computation time for the individual iterations. However, using suboptimal solutions can result in less efficient cuts and therefore more iterations.

Computation time savings per iteration that can be obtained by relaxing the tolerance can be assessed by solving several instances of the same problem for different tolerance values. Figure 4-4 shows how computation time varies with tolerance for the 87N case study.

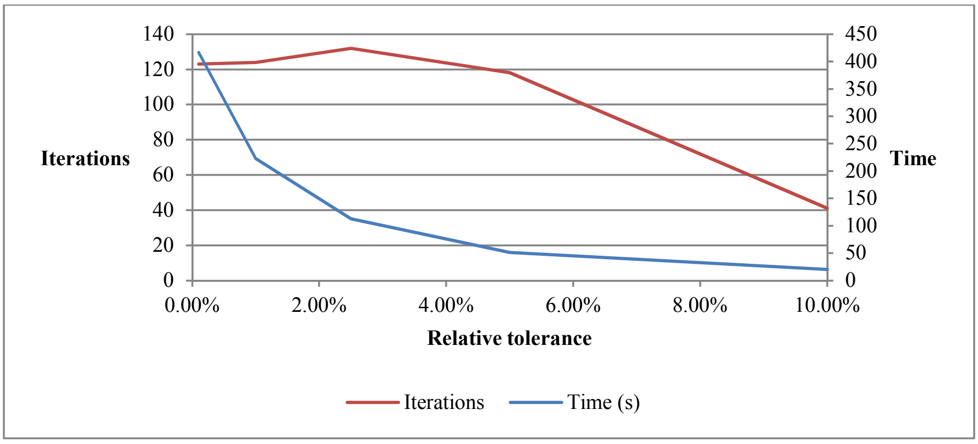


Figure 4-4 – Computation time vs. optimality tolerance for the master problem in the 87N example.

Relative optimality tolerances of around 2.5% seem to offer a good compromise. Therefore, it can be inferred that an inexact master method that spends most of its solution time using tolerance values of around 2.5% could accomplish notable time savings. However, as mentioned above, the decomposition might need more iterations to converge. This is also shown in Table 4-5, which shows total computation time and number of iterations for different optimality tolerances. The tolerance applied to Benders' decomposition was 0.1% in all cases.

Relative optimality tolerance (%)	Time (s)	Optimal solution (M€)	Distance from true optimal (%)	Benders' decomposition iterations
10.0%	20.4	141779.0	1.5%	41
5.0%	51.1	139853.5	1.0%	118
2.5%	112.6	139726.4	0.0%	132
1.0%	222.8	139726.4	0.0%	124
0.1%	364.5	139726.4	0.0%	123

Table 4-5 –Relative optimality tolerance for the master problem vs. solution time, optimal value and number of iterations for Benders' decomposition on the 87N case.

Solving the problem with an inexact master can result in a larger number of iterations (for instance, a tolerance of 2.5% needs 132 iterations to converge, which compares to only 124 for a tolerance of 1%). However, quicker iterations mean that the 2.5% value is still advantageous. It should be noted that a correct implementation of this method should adjust the master problem tolerance to be always lower than or equal to the current convergence ratio of Benders' decomposition. If this is not done, it is possible to arrive to convergence prematurely. We include an example with a relatively high starting value for the tolerance which is quickly reduced to match the current convergence ratio of the decomposition under the name *highest tolerance*.

In addition, we implement a boxstep method, where we set an additional constraint to limit maximum investment:

$$\sum_{t,ijc \in cl} Ic_{ijc} x_{tijc} \leq MaxInv \, . \tag{63}$$

where $MaxInv$, the maximum investment, has a starting value $MaxInv_0$ and is increased in steps of $\Delta MaxInv$ as needed. We select the values of these parameters to be relatively close to the optimal value but with sufficient variation in order to illustrate the possible effects of this technique. We use values of 85%, 100% and 115% of the total investment in the optimal solution (which was previously calculated), with step increments in the constraint of the equivalent of one transmission line in all cases.

c. *Alternative Techniques to Find Master Problem Solutions*

If there were more efficient methods to find the optimal solution, the time savings provided by the alternative solution technique would be translated directly to savings in the decomposition by a factor of 85%. However, we could not identify any specific solution algorithm for this particular case, so we do not consider this alternative.

d. *Modifications to Benders' Cuts*

Modifying Benders' cuts can be advantageous in two main cases: if there is an excessively large number of cuts or if many of them are feasibility cuts. The inspection of the cuts in the case study showed that the number of cuts was not excessive, so techniques such as cover cut generation or inactive cut removal are not advisable. In addition, the subproblem is feasible for all possible solutions of the master, so there are no feasibility cuts. Therefore, we implement none of the modifications to Benders' cuts except for semi-relaxed cuts as explained.

4.4.2. Relative Performance of the Improvements

We test the improvements for the three different problem sizes.

46N		Time (s)	% Savings
Benders' decomposition		156.6	
Linear first		144.6	8%
Semi relaxed cuts		105.8	32%
Inexact master	εMaster = 10% εStep = 0.1%	151.4	3%
Inexact master	εMaster = 2.5% εStep = 0%	111.3	29%
Inexact master	Highest tolerance	113.6	27%
Boxstep	Initial = 85% optimal	176.7	-13%
Boxstep	Initial = 100% optimal	77.9	50%
Boxstep	Initial = 115% optimal	170.2	-9%

Table 4-6 – 46N system. Relative performance of improvements.

46N (100 x scenarios)		Time (s)	% Savings
Benders' decomposition		820.6	
Linear first		844.5	-3%
Semi relaxed cuts		499.8	39%
Inexact master	εMaster = 10% εStep = 0.1%	702.0	14%
Inexact master	εMaster = 2.5% εStep = 0%	691.4	16%
Inexact master	Highest tolerance	689.1	16%
Boxstep	Initial = 85% optimal	3,141.0	-283%
Boxstep	Initial = 100% optimal	2,482.0	-202%
Boxstep	Initial = 115% optimal	1,570.4	-91%

Table 4-7 –. 46N system with 100 scenarios. Relative performance of improvements.

87N		Time (s)	% Savings
Benders' decomposition		364.5	
Linear first		374.7	-3%
Semi relaxed cuts		162.4	55%
Inexact master	εMaster = 5% εStep = 0.025%	106.5	71%
Inexact master	εMaster = 2.5% εStep = 0%	232.5	36%
Inexact master	Highest tolerance	237.1	35%
Boxstep	Initial = 85% optimal	193.0	47%
Boxstep	Initial = 100% optimal	135.8	63%
Boxstep	Initial = 115% optimal	551.3	-51%

Table 4-8 – 87N system. Relative performance of improvements.

As can be seen, the two versions of the 46N system studied (a single scenario and 100 scenarios respectively) benefit from the techniques tested in some cases. The linear-first approach does not seem to offer any consistent advantage. Inexact master approaches seem to be generally beneficial, reaching savings of 71%. Results tend to favor relatively high initial tolerance values that decrease slowly. Boxstep generated time savings that were considerable in some instances (up to 63%) but are inconsistent and depend heavily on the starting value given to the additional constraint. The approach we propose, semi-relaxed cuts, delivers notable time savings of up to 55% in a consistent manner.

4.4.3. Conclusions

Relatively simple techniques can generate substantial computation time reductions in Benders’ decomposition. We have reviewed the acceleration techniques available in the literature and classified them with respect to the component of the decomposition they modify: the master problem or the subproblem.

If most of the computation time is spent solving the master problem, the decomposition can be accelerated by the use of low-cost solutions to get cuts (such as relaxations, sub-optimal solutions or alternative techniques). In addition, if most cuts are feasibility cuts, Minimal Infeasible Subsystems can improve efficiency. Maximal Feasible Subsystems generate additional optimality cuts to guide master proposals. Alternatively, if the master problem is slow due to a large number of cuts that are not active, removing cuts can be an interesting alternative. More efficient definitions of cuts, such as non-dominated or cover cuts, can be useful as well.

If most of the computation time is spent solving the subproblem, an improved definition of the scenario tree or the cut structure can have a notable impact on efficiency. In addition, it is possible to apply specific algorithms, use increasingly accurate models or inexact cuts.

The TEP problem is particularly amenable to Benders’ decomposition and therefore has been solved extensively using this method. We describe the formulation of Benders’ decomposition applied to the problem and test some of the acceleration techniques in several case studies based on the IEEE 46N and the IEEE 87N test systems.

Given that most of the computation time spent on the subproblem was negligible, the improvement mechanisms suitable for TEP focused on modifications of the master problem. We solved a linear relaxation at the first iterations of the decomposition (linear-first), applied semi-relaxed cuts (which are one of the contributions of this thesis), inexact master resolution and a boxstep method. Using semi-relaxed cuts or inexact master resolution generated reductions in computation time in excess of 50%. This illustrates how acceleration techniques can accelerate implementations of Benders' decomposition in a relatively straightforward way. We presented our improved version of Benders' decomposition applied to TEP at the conference:

- S. Lumbreras, A. Ramos, "Transmission expansion planning using an efficient version of Benders' decomposition. A case study", PowerTech 2013. Grenoble, France, 16-20 June 2013.

4.5. A Progressive Contingency Incorporation Algorithm

Reliability is a key objective in many optimal design problems, including TEP and, even more so, OWF design. Very often, this criterion is incorporated through contingency evaluation scenarios [182]. This generates a special problem structure where the stochastic reliability scenarios describe the failure of specific individual elements. We develop a Progressive Contingency Incorporation (PCI) algorithm that takes advantage of this structure to increase efficiency. We apply this algorithm to TEP and OWF design. In the case of OWF design, which is particularly suited for the proposed approach, we reach time savings of two orders of magnitude.

This algorithm has been published as:

- S. Lumbreras, A. Ramos and S. Cerisola, "A Progressive Contingency Incorporation Approach for Stochastic Optimization Problems", IEEE Transactions on Power Systems 28 (2): 1452-1460, May 2013 10.1109/TPWRS.2012.2225077.

4.5.1. Introduction

Many optimal design problems incorporate reliability as one of their objectives and thus involve the evaluation of contingencies in one way or another. Reference [182] performs a literature review on system reliability optimization, highlighting applications in wide areas such as software development, systems with common mode failure, optimal assembly, maintenance policy or burn-in optimization. Many of these represent a two-stage optimization problem where both design and operation subject to failures must be taken into account simultaneously.

In particular, software development seems to be an area of increasing interest [183, 184]. These models represent the software system as an interaction between series and parallel modules where the optimal redundancy of each constituent is determined. Real-time applications, as in the aerospace industry are a good example of software systems where the need for robustness leads to this sort of optimization [185]. Electronic device design, where elements present different costs and failure rates and can be arranged in redundant architectures, has been subject to similar research [186]. Communication networks, with their special hub-based structure, are also optimized taking into account possible contingencies [126, 187, 188]. Supply chain networks have also been studied from a similar perspective [189, 190].

Within power systems, the design optimization problem with contingency evaluation appears in a number of different areas. GEP with reliability considerations acknowledges the effect of individual generator failures in the system [53, 143, 191, 192]. There are applications to TEP that consider stochastic element failures [16, 193]. In general, explicit enumeration is used to represent failures in specific transmission lines.

The optimized placement of switchgear [194, 195] and capacitors [196-198] has also been analyzed. More recently, the design of offshore wind farms (OWF), particularly the electrical layout considering element failures, is receiving increasing attention [199, 200].

We propose an algorithm that can improve computation times for problems that present this structure, where the design of a given architecture must be performed taking into account the possible failures of its elements. We propose a strategy that solves a reduced version of the problem where not all the possible contingencies are included. We add contingencies progressively until a specified condition is met. We show that the reduced problem and the original one have the same optima. As the reduced problem is smaller and generally much quicker to solve, considerable time savings can be derived from this approach. The Progressive Contingency Incorporation (PCI) algorithm is applicable to problems where uncertainty in element failures is incorporated as scenario enumeration in a stochastic programming model.

This section presents this algorithm. First, we define contingency structure for stochastic problems. Then, we define the reduced problem that will actually be solved by the algorithm and prove that they have the same optimal solution. A case study illustrates the high computation time savings achievable by this approach.

4.5.2. Stochastic Problems with a Contingency Structure

For the sake of simplicity, we will restrict our focus to a two-stage linear problem, although we can apply the results to a multi-stage problem as long as all stages present the same structure. The two-stage stochastic optimization problem P_Ω is defined as:

$$\begin{aligned}
\min_{x,y^\omega} \quad & z_\Omega = c^T x + \Big(1 - \sum_{\substack{\omega\in\Omega \\ \omega\neq\omega_0}} Pr^\omega\Big) d^T y^{\omega_0} + \sum_{\substack{\omega\in\Omega \\ \omega\neq\omega_0}} Pr^\omega d^T y^\omega \\
s.t. \quad & Ax = b \\
& T\,x + W\,y^\omega = h^\omega \\
& x \in \{0,1\}, y^\omega \geq 0, \omega \in \Omega
\end{aligned} \quad , \qquad (64)$$

where:

x: first stage or investment decisions (binary variables). For each investment proposal, the investment variable x_i takes a value of 1 when the element i has been installed and 0 when it has not.

c: investment costs,

$\omega \in \Omega$: second-stage scenarios describing contingencies. The base case (no failures) ω_0 must always be included in this subset, that is, $\omega_0 \in \Omega$.

Pr^ω: probability associated to each scenario. The probability that weights the base case equals one minus the sum of the probabilities of the contingencies:

$$Pr^{\omega_0} = 1 - \sum_{\omega \neq \omega_0} Pr^\omega , \tag{65}$$

$$\sum_\omega Pr^\omega = 1 . \tag{66}$$

y^ω: second stage or operation decisions for each scenario. In particular, y^{ω_0} represents the operation decisions for the base-case scenario (no contingencies).

d: operation costs.

The objective function reflects the first-stage or investment costs plus the expected operation cost taking into account the different stochastic scenarios. It is also useful to define, for any given feasible set of values for the first-stage decisions x, the problem that minimizes expected cost for the second stage. This is known as the recourse problem, $Q_\Omega(x)$:

$$\begin{aligned} Q_\Omega\left(x\right) = \min_{y^\omega} & (1 - \sum_{\substack{\omega \in \Omega \\ \omega \neq \omega_0}} Pr^\omega) d^T y^{\omega_0} + \sum_{\substack{\omega \in \Omega \\ \omega \neq \omega_0}} Pr^\omega d^T y^\omega \\ s.t. \quad & W y^\omega = h^\omega - Tx \\ & y^\omega \geq 0, \omega \in \Omega \end{aligned} . \tag{67}$$

The second-stage scenarios introduce a great deal of complexity. The base-case scenario describes operation with no outages. Every remaining scenario describes system operation under the loss of a single element. We define problems with contingency structure as the stochastic optimization problems where uncertainty arises from considering the failures of the elements that can be selected as investment decisions. This implies that the evaluated scenarios correspond to the failures of the elements that will be installed ($x_i = 1$) or not ($x_i = 0$) at the investment stage. In addition, the operation cost associated to a contingency scenario must be higher than or equal to the cost associated to the base case. We summarize this definition below.

- Second-stage scenarios describe the failures of the elements already in the system and the ones that can be installed at the first stage. Without loss of generality, we assume an N-1 framework [201]. This approach assumes that, for low probabilities of failure, the events where two or more elements fail at the same time can be neglected (hence the name; all the elements of the system are working except for one). This allows assigning a failure scenario to each of the possible investments. In this way, a scenario ω_i describes the failure of element i.

  All the scenarios of installed elements must be present in the optimization. However, if an element has not been installed, the scenario that describes its loss does not have any meaning. Any constraints or terms in the objective function that refer to it should have no impact. Therefore, the scenarios referring to elements that have not been installed have a behavior that is identical to the base case.

  That is, let $y^{\omega^*}(x)$ be the optimal values of y^{ω} in the recourse problem $Q_{\Omega}(x)$ for a specific feasible proposal of first stage decisions x:

$$\begin{aligned} y^{\omega^*}(x) &= \arg\min_{y^{\omega}} \sum_{\omega\in\Omega} Pr^{\omega} d^T y^{\omega} \\ s.t.\quad Wy^{\omega} &= h^{\omega} - Tx \\ y^{\omega} &\geq 0, \omega \in \Omega \end{aligned} \quad . \tag{68}$$

  Then, the decision of not installing a given element $x_i = 0$ implies that the operation stage of the scenario describing its failure will be identical to the base case. That is:

$$x_i = 0 \Rightarrow y^{\omega_i{}^*}(x) = y^{\omega_0{}^*}(x). \tag{69}$$

- Element failures increase operation costs. Any failure scenario has an associated term in the objective function that is larger than or equal to the base case. It should be stressed that this is a condition for the algorithm to be valid rather than a general statement about systems with element failures, although it is commonly true. That is, let $y^{\omega^*}(x)$ be the optimal values of y^{ω} in the recourse problem $Q_{\Omega}(x)$ defined for a specific feasible set of first stage decisions x. Then, for this optimal operation, the costs associated to the contingency scenarios must be always higher than or equal to the costs of the base case:

$$d^T y^{\omega^*}(x) \geq d^T y^{\omega_0{}^*}(x) \qquad \omega, \omega_0 \in \Omega . \tag{70}$$

Given the large number of variables (and therefore, of associated contingency scenarios) present in real-life problems, the resolution of the complete optimization model is usually very computationally intensive and sometimes unaffordable. Next, we define a reduced version of the optimization problem and present its links to the developed contingency structure.

4.5.3. Reduced Problem Definition

The proposed algorithm exploits the contingency structure of the problem. We define a reduced problem $P_{\Omega'}$ where only a subset of the contingency scenarios (the reduced subset Ω') is evaluated and the probability of the not-evaluated scenarios is added to the base case (so that probabilities still sum to 1):

$$\begin{aligned} \min_{x,y^\omega} z_{\Omega'} &= c^T x + (1 - \sum_{\substack{\omega\in\Omega' \\ \omega\neq\omega_0}} Pr^\omega) d^T y^{\omega_0} + \sum_{\substack{\omega\in\Omega' \\ \omega\neq\omega_0}} Pr^\omega d^T y^\omega \\ s.t.\quad & Ax = b \\ & Tx + Wy^\omega = h^\omega \\ & x \in \{0,1\}, y^\omega \geq 0, \omega \in \Omega' \end{aligned} \qquad (71)$$

Similarly to the recourse of the original problem $Q_\Omega(x)$, the recourse of the reduced problem $Q_{\Omega'}(x)$ is defined on a subset of the contingency scenarios Ω'.

We state two basic properties of the reduced problem:

- The recourse value of the complete problem is always greater than or equal to the recourse value of the reduced problem. Given that the cost associated with contingency scenarios is higher, and that the reduced problem includes necessarily less contingency scenarios, we have:

  $$Q_{\Omega'}(x) \leq Q_\Omega(x). \qquad (72)$$

- If the reduced set of scenarios includes the failure scenarios of all the elements that were installed at the first stage, the recourse value of the reduced problem is identical to the recourse value of the original problem.
  That is equivalent to say that the inclusion of scenarios that describe failures of elements that have not been installed can have no impact on the optimization. Only the contingency scenarios that describe failures of installed elements have an impact. Therefore, any reduced problems that include the contingency scenarios of every installed element will have the same value. If we have a given solution of the first stage x where the installed elements have $x = 1$, if all the scenarios describing its failure are included, that is:

  $$\tilde{\Omega} \subset \Omega', \quad where \quad \tilde{\Omega} = \bigcup_{i/x_i=1} \omega_i. \qquad (73)$$

  The definition of the recourse function of the reduced problem $Q_{\Omega'}(x)$ is:

  $$\begin{aligned} Q_{\Omega'}(x) &= \min_{y^\omega} (1 - \sum_{\substack{\omega\in\Omega' \\ \omega\neq\omega_0}} Pr^\omega) d^T y^{\omega_0} + \sum_{\substack{\omega\in\Omega' \\ \omega\neq\omega_0}} Pr^\omega d^T y^\omega \\ s.t.\quad & Wy^\omega = h^\omega - Tx \\ & y^\omega \geq 0, \omega \in \Omega' \end{aligned} \qquad (74).$$

  We can split the argument of the minimization into two parts, corresponding to the set of the scenarios that describe contingencies of the specified solution x (that is, $\tilde{\Omega}$)

and the remaining ones (i.e., $\omega_i \in \Omega' \cap \bar{\tilde{\Omega}}$), where a bar above a set represents negation, that is, the elements that do not belong to it.

$$\begin{aligned}&Q_{\Omega'}(x) = \min_{y^\omega}(1 - \sum_{\substack{\omega\in\Omega' \\ \omega\neq\omega_0}} Pr^\omega)d^T y^{\omega_0} + \sum_{\substack{\omega\in\Omega' \\ \omega\neq\omega_0}} Pr^\omega d^T y^\omega = \\ &\min_{y^\omega}(1 - \sum_{\substack{\omega\in\Omega' \\ \omega\neq\omega_0}} Pr^\omega)d^T y^{\omega_0} + \sum_{\substack{\omega\in\Omega'\cap\tilde{\Omega} \\ \omega\neq\omega_0}} Pr^\omega d^T y^\omega + \sum_{\substack{\omega\in\Omega'\cap\bar{\tilde{\Omega}} \\ \omega\neq\omega_0}} Pr^\omega d^T y^\omega\end{aligned} \quad (75).$$

If an element is not installed then its failure has no effect and the optimal second-stage variables in that failure scenario are identical to the base case:

$$x_i = 0 \Rightarrow y^{\omega_i *}(x) = y^{\omega_0 *}(x) \quad (76).$$

Substituting this in the definition of the problem, and then expanding the first term and rearranging, we get:

$$\begin{aligned}&Q_{\Omega'}(x) = \min_{y^\omega}(1 - \sum_{\substack{\omega\in\Omega' \\ \omega\neq\omega_0}} Pr^\omega)d^T y^{\omega_0} + \sum_{\substack{\omega\in\Omega'\cap\tilde{\Omega} \\ \omega\neq\omega_0}} Pr^\omega d^T y^\omega + \sum_{\substack{\omega\in\Omega'\cap\bar{\tilde{\Omega}} \\ \omega\neq\omega_0}} Pr^\omega d^T y^{\omega_0} = \\ &\min_{y^\omega}(1 - \sum_{\substack{\omega\in\Omega'\cap\tilde{\Omega} \\ \omega\neq\omega_0}} Pr^\omega - \sum_{\substack{\omega\in\Omega'\cap\bar{\tilde{\Omega}} \\ \omega\neq\omega_0}} Pr^\omega + \sum_{\substack{\omega\in\Omega'\cap\bar{\tilde{\Omega}} \\ \omega\neq\omega_0}} Pr^\omega d^T y^{\omega_0})d^T y^{\omega_0} + \sum_{\substack{\omega\in\Omega'\cap\tilde{\Omega} \\ \omega\neq\omega_0}} Pr^\omega d^T y^\omega\end{aligned} \quad (77).$$

Simplifying:

$$Q_{\Omega'}(x) = \min_{y^\omega}(1 - \sum_{\substack{\omega\in\Omega'\cap\tilde{\Omega} \\ \omega\neq\omega_0}} Pr^\omega)d^T y^{\omega_0} + \sum_{\substack{\omega\in\Omega'\cap\tilde{\Omega} \\ \omega\neq\omega_0}} Pr^\omega d^T y^\omega \quad (78).$$

As we have imposed the condition $\tilde{\Omega} \subset \Omega'$ we can simplify to the following:

$$Q_{\Omega'}(x) = \min_{y^\omega}(1 - \sum_{\substack{\omega\in\tilde{\Omega} \\ \omega\neq\omega_0}} Pr^\omega)d^T y^{\omega_0} + \sum_{\substack{\omega\in\tilde{\Omega} \\ \omega\neq\omega_0}} Pr^\omega d^T y^\omega \quad (79).$$

This does not depend on Ω' (the reduced set of scenarios) but only on $\tilde{\Omega}$ (the set of scenarios that describe failures of elements installed at the first stage). That is, the recourse value for all the reduced problems that include all the failure scenarios of the installed elements is the same. As the original problem includes all the scenarios, this value is identical to the recourse of the original problem:

$$\tilde{\Omega} \subset \Omega' \Rightarrow Q_{\Omega'}(x) = Q_\Omega(x) \quad (80).$$

4.5.4. Support for the Developed Algorithm

We base our algorithm on the following *scenario condition.*

Scenario condition:

Let us have a reduced set of scenarios Ω' and the optimal solution of the reduced problem $\hat{z}_{\Omega'}$, together with the optimal investment and operation variables $x^*, \left(y^\omega\right)^* \equiv \arg\min P_{\Omega'}, \omega \in \Omega'$.

The scenario condition defines the situation where all the scenarios that refer to the investment decisions installed in the optimal solution of the reduced problem are already included in the reduced subset of scenarios. That is:

$$\hat{\Omega} \subset \Omega, \qquad where \qquad \hat{\Omega} = \bigcup_i \omega_i \,/ x_i^* = 1 \,. \tag{81}$$

For instance, the scenario condition would be met if the reduced set of scenarios included the failures of elements *a* and *b*, and the optimal solution of the reduced problem installed element *a*. It would not be met if the optimal solution installed elements *b* and *c*.

It is important to note that, in order to check the scenario condition, only the reduced problem needs to be solved. It is not necessary to solve the original problem. This is the main idea for the developed algorithm.

Theorem 1:

If the scenario condition is met, the optimal solution of the reduced problem is also the optimal solution of the complete problem.

A proof of Theorem 1 follows, in italics. The reader can skip this section without damaging his understanding of the rest of the section.

Proof:

In general, we know that the evaluation of any given first-stage decisions in the recourse function of the complete problem is larger than or equal to the evaluation of those decisions in the recourse function of the reduced problem. This is true for all feasible first-stage solutions. Therefore, the optimal value of the complete problem $\hat{z}_{\Omega}$ *has to be larger than or equal to the optimal value of the reduced problem* $\hat{z}_{\Omega'}$*:*

$$\hat{z}_{\Omega'} \le \hat{z}_{\Omega} \,. \tag{82}$$

If the scenario condition is met, then we have that at least all the failure scenarios related to the installed elements are included in the reduced set. Thus, if x^* *represents the optimal first-stage decisions for the reduced problem, we have:*

$$\tilde{\Omega} \subset \Omega', \qquad where \qquad \tilde{\Omega} = \bigcup_{i / x_i^* = 1} \omega_i \,. \tag{83}$$

Therefore, the only difference between them are terms that describe failures of elements that were not installed and are thus meaningless. Then, both recourse functions must have the same value at the optimal of the reduced problem:

$$Q_{\Omega'}(x^*) = Q_{\Omega}(x^*) \,. \tag{84}$$

Adding the constant term that describes the cost of the first-stage, which is the same for both as it does not depend on the scenarios, we can get the optimal value of the reduced problem $\hat{z}_{\Omega'}$*:*

$$\hat{z}_{\Omega'} = c^T x^* + Q_{\Omega'}(x^*) = c^T x^* + Q_{\Omega}(x^*) \,. \tag{85}$$

Therefore, we have these two facts:

- *The objective value of the reduced problem must be lower than or equal to the objective value of the original problem. That is, there can be no combination of first-stage decisions that gives a lower cost than the optimal value of the reduced problem.*
- *If the scenario condition is met, then the original problem evaluated at x^* has the same objective value than the reduced problem evaluated at x^*.*

Therefore, if there can be no solution of the original problem with a better value and there is one solution with the same value (the same first-stage decisions) then both problems have necessarily the same optima.

$$\hat{z}_{\Omega'} = c^T x^* + Q_{\Omega'}(x^*) = \hat{z}_{\Omega}. \qquad (86)$$

This result is the core of the algorithm developed as detailed in the next section.

Optimizing the original, complete problem requires, assuming an N-1 criterion, to solve a problem of $N \times (v+1)$ variables and $r_1 + r_2 \times (N+1)$ restrictions, where N is the number of possible elements to install, v is the number of operation variables per scenario, r_1 is the number of first-stage restrictions and r_2 is the number of second-stage restrictions per scenario. In many problems, the amount of possible options is far larger than the subset of elements that will be installed. The larger the array of choices present in the model, the more scenarios need to be solved. The number of variables and constraints gets multiplied by that number of alternatives. However, most of the scenarios linked to those alternatives do not have an impact on the value of the objective function because they are not installed. This situation is very relevant for capacity expansion or network expansion planning problems.

We propose to solve the problem iteratively for an evolving subset of scenarios until the scenario condition is met. In addition, the objective function values found in the process are lower bounds of the optimal solution, therefore providing an increasing lower bound. We can calculate an upper bound by evaluating the problem with the next set of scenarios, usually a very quick step as it involves evaluation only and not optimization. We can describe the process as follows.

1. Initialize the reduced scenario set to contain the base case only. This base case contains the failures of the existing elements only:
   $\Omega_0 = w_0$.
2. Solve $x_l^*, y_l^{\omega*} \equiv \arg\min P_{\Omega_l}$
3. Update the reduced scenario set, adding the scenarios that refer to the elements that have been installed at the previous stage :
   $\Omega_{l+1} = \Omega_l \cup \tilde{\Omega}$
   $\tilde{\Omega} = \bigcup_{i/x_i=1} \omega_i$,
4. If $\Omega_{l+1} = \Omega_l$ (that is, all the scenarios referring to installed elements were already contained in the reduced scenario set so the scenario condition is met) then the algorithm has converged. Else, go to step 2.

Note that as $\Omega_l \subset \Omega_{l+1}$ (scenarios get added but never removed) the succession of intermediate solutions increases monotonically. This ensures convergence as there are only two possible outcomes:

- The subset Ω stabilizes at some point, so that the ending criterion is met:

$$\Omega_l = \Omega_{l-1}. \tag{87}$$

  This would mean that all the contingencies associated to the installed elements are included and therefore the solution is the actual optimum.

- Alternatively, the subset does not stabilize but rather grows until all the contingency scenarios have been incorporated. In this case the reduced problem would be exactly equal to the original definition of the problem:

$$\Omega_l = \Omega \Rightarrow P_{\Omega_l} = P_{\Omega}. \tag{88}$$

  In this extreme case the proposed algorithm would converge to the optimum too, although its application would not have been efficient.

In practice, given that contingency events have a low probability, we can expect the algorithm to converge in a very limited number of iterations, so that computation time savings are significant. These savings will be related to the difference in size between the number of investments made and the total available alternatives. The OWF layout problem in general considers a much higher number of decision variables than TEP. In addition, reliability is arguably more important in this case, with transmission contingencies taking longer times to be repaired. For this reason, the proposed approach represents a higher advantage for OWF design.

The potential savings will be reduced if more iterations are necessary. Although difficult to assess beforehand, it can be seen that problems with a higher level of degeneracy, with solutions that are very close in objective function value, will take more iterations to be solved. In general, the more subtle the differences between the solutions (especially concerning contingencies) and the more cost-indifferent they are, the longer the algorithm will take to converge.

We present several problems of different sizes in order to illustrate the performance of the algorithm. Turbine positions and wind inputs are given. The problem must determine the placement and type of cables. For the optimization problem, we use the model described in 3.3. The full description of the plant and the technical specifications of its elements can be found in [180]. We take the failure and repair rates from reference [202].

4.5.5. Results

First, we present a small example in detail to demonstrate the algorithm. We later present similar results for other instances of the problem.

Consistently with most layouts, we design the turbine layout as parallel rows, so there is a high level of symmetry and several configurations exhibit the same objective value. In principle, the algorithm would need to evaluate all of them with their associated contingency scenarios in order to prove the optimality of the final solution.

All CPU times were calculated using CPLEX 12.1 called from GAMS on a PC at 2.80 GHz with 4 GB RAM running Microsoft Windows XP 32 bits. The relative optimality tolerance was set to 0.1% both for CPLEX and for the Progressive Contingency Incorporation algorithm. We found consistent results for other tolerance levels.

a. *Detailed Execution of a Case Study*

The table below summarizes the execution run when solving a case study (27249 equations and 14517 total variables, 336 of which were discrete). This is not a case where the largest time savings can be expected: the number of first stage variables (the discrete ones) is small. In addition, the layout is highly symmetrical, as it generally happens in OWF layout optimization problems. This causes a high level of degeneracy in the objective function, which can result (if not controlled by the tolerance) in the algorithm exploring a large number of equivalent solutions. The objective value at each iteration, as well as the changes in the installed cables between consecutive iterations and the solution time are presented in Table 4-9.

	Value (M€)	Changes	Solution time (s)
Original problem	0.071290	-	**42.6**
PCI iteration 1	0.063248	-	0.2
PCI iteration 2	0.065959	6	0.5
PCI iteration 3	0.067587	4	1.0
PCI iteration 4	0.068784	6	2.0
PCI iteration 5	0.071290	6	7.3
PCI iteration 6	0.071290	6	8.3
PCI iteration 7	0.071290	2	10.4
Total			**29.7**

Table 4-9 – Detailed Execution Example.

As can be seen above, the Progressive Contingency Incorporation (PCI) algorithm achieves a modest 30% time savings with respect to the complete problem. The optimal solution was already found in iteration 5. This would represent 75% savings. Iterations from 5 to 7 are spent testing equivalent configurations with the same objective value.

Time savings vary with problem size and structure, so they cannot be easily estimated beforehand. We solve several case studies of different characteristics in order to illustrate the behavior of the algorithm further.

b. *Time Savings with respect to Wind Scenarios (Non-Contingency Related)*

Algorithm efficiency derives from the obliteration of scenarios that are not relevant according to the current selected configuration. Therefore, the more scenarios can be ignored, the more time savings this method will generate. This is illustrated by Table 4-10.

	Original problem			Progressive Contingency Incorporation				
Scenarios	Eq	Var	Time (s)	Eq	Var	PCI iterations	Time (s)	Savings (%)
4	2594	1695	0.2	1026	687	3	0.1	50.0%
8	5170	3351	0.5	2034	1335	3	0.2	60.0%
16	10322	6663	1.0	4050	2631	3	0.3	70.0%
32	20626	13287	2.4	8082	5223	3	0.4	83.3%
64	41234	26535	8.0	16146	10407	3	0.9	88.8%
128	82450	53031	47.7	32274	20775	3	3.7	92.2%

Table 4-10 – Performance versus number of non-contingency scenarios.

All the executions represent the same case study, with 36 discrete optimization variables optimized under an increasing number of wind scenarios, from 4 to 128, represented as the case study index. The number of equations and variables for the PCI algorithm correspond to the maximum incorporated equations and variables in the problem resolution.

c. *Time Savings with respect to the Number of First-Stage Variables*

Another advantage of PCI is that optimization time gets to some extent decoupled from the number of first stage possibilities. If a configuration (or set of configurations) turns out to be far better than the rest, the remaining ones will not be selected and therefore their corresponding contingency scenarios will never be incorporated to the optimization. Therefore, time savings should increase as the number of possible configurations for a given problem increase. This can be seen in Table 4-11 where the index column corresponds to the number of possible first-stage decisions.

	Original problem			Progressive Contingency Incorporation				
1st stage variables	Eq	Var	Time (s)	Eq	Var	PCI iterations	Time (s)	Savings (%)
36	2594	1695	0.2	1026	687	3	0.1	50.0%
48	3965	2487	0.8	1245	807	3	0.2	75.0%
60	6270	3807	6.3	1470	927	3	0.8	87.3%
72	8687	5151	25.9	1695	1047	3	1.3	95.0%
84	11482	6687	31.8	1914	1167	3	1.3	95.9%

Table 4-11 – Performance versus number of first stage variables.

d. *Time Savings with respect to Problem Size and Considerations about Degeneracy*

Because of the effects presented in the previous sections, larger problems tend to exhibit more time savings when solved with the PCI algorithm.

Original problem				Progressive Contingency Incorporation				
Active var	Eq	Var	Time (s)	Eq	Var	PCI iterations	Time (s)	Savings (%)
9	23702	15123	0.6	1302	1123	3	0.2	66.7%
11	32587	20303	9.4	2843	3051	4	0.5	94.7%
14	45921	27703	919.7	4505	2987	4	41.9	95.4%
15	57771	34663	36978.2	5043	3267	3	405.5	98.9%

Table 4-12 – Performance versus actual investments.

Table 4-12 illustrates this point. The number of installed elements in the optimal solution, which are represented by binary variables with a value of one, has been included as a figure related to problem size. In all cases, the number of discrete variables is constant and equal to 320.

However, larger problems often present higher degrees of degeneracy. In many cases, this degeneracy results from geometrical symmetry. This effect is remarkable in the OWF design problem. However, a good choice of the PCI tolerance parameter should avoid most of the redundant evaluations. This can also be seen in Table 4-12, as the number of iterations completed by the PCI algorithm does not increase with problem size.

Time savings are expected to deteriorate with the ratio between actually installed elements and elements available for installation, given that more scenarios should be added to the reduced problem. There is no guarantee that, in extreme cases, PCI will be more efficient than direct resolution. However, most real-life cases exhibit some correlation between the number of installed elements and resolution difficulty (that is, the more elements need to be installed, the more difficult the optimization tends to be). This effectively counteracts the impact of adding a larger set of scenarios, as can be seen in Table 4-13, where we represent problem complexity as integrality gap as usual in MIP optimization [123].

Integrality gap (%)	Active var	Original problem time (s)	PCI time(s)
4.2%	9	221.4	0.1
7.0%	10	238.3	0.2
9.7%	9	688.1	1.4
10.9%	12	527.6	2.2
12.4%	14	1218.0	3.6
14.9%	14	1360.9	5.8

Table 4-13 – Performance versus integrality gap.

In all cases problem sizes were the same (62954 equations and 37323 variables, with 320 discrete investment decisions), but installation costs and non-served energy costs varied.

4.5.6. Conclusions

Many optimal design problems present a special contingency structure in which the stochastic scenarios related to reliability are linked to the failures of individual elements. We propose to exploit this structure with the Progressive Contingency Incorporation algorithm described in this section. The algorithm works by introducing the scenarios corresponding to the failure of specific elements only when they have been selected for a previous first-stage solution. We present a proof of optimality. The PCI algorithm produces substantial time savings which grow with the number of total first-stage variables, non-contingency scenarios and problem complexity.

4.6. Chapter Takeaways

TEP and OWF design are highly complex problems that need efficient solution techniques. We exploit the structure of the problems applying Benders' decomposition, developing the first published application (to our knowledge) of decomposition strategies to OWF design. In addition, we review and compare the different techniques that have been developed to accelerate Benders' decomposition, a study that was missing in the literature. We then evaluate the potential time savings that these techniques can bring to TEP. Furthermore, we develop some new acceleration techniques. We propose *semi-relaxed cuts* to deal efficiently with large numbers of discrete variables in the master problem. In addition, we develop a Progressive Contingency Incorporation algorithm to evaluate the effect of component failures in an efficient way. These approaches reduce computation times by up to two orders of magnitude in the case studies developed.

The review on Benders' decomposition acceleration techniques, as well as semi-relaxed cuts, were presented at the conferences:

- S. Lumbreras, A. Ramos, "Transmission expansion planning using an efficient version of Benders' decomposition. A case study", PowerTech 2013. Grenoble, France, 16-20 June 2013.
- S. Lumbreras, A. Ramos, "Improvements to Benders' decomposition. A practical evaluation using a transmission expansion planning problem", 13th Trans-Atlantic Doctoral Conference. London, United Kingdom, 9-11 May 2013.

This has also been submitted for publication:

- S. Lumbreras, A. Ramos ,"Optimal Transmission Expansion Planning using Benders' Decomposition. Acceleration Techniques", submitted to JORS last April.

The first application of decomposition strategies (to our knowledge) to the OWF layout problem was published as:

- S. Lumbreras and A. Ramos, "Optimal design of the electrical layout of an offshore wind farm applying decomposition strategies", IEEE Transactions on Power Systems 28 (2): 1434-1441, May 2013 10.1109/TPWRS.2012.2204906.

We present the PCI algorithm in the paper:

- S. Lumbreras, A. Ramos and S. Cerisola, "A Progressive Contingency Incorporation Approach for Stochastic Optimization Problems ", IEEE Transactions on Power Systems 28 (2): 1452-1460, May 2013 10.1109/TPWRS.2012.2225077 [118].

Chapter 5

Flexibility and Robustness

5.1. Hard and Flexible

TEP is a problem characterized by the presence of large uncertainties. Some of these uncertainties, such as generation expansion or fuel prices are *dynamic* in the sense that they evolve with time. They are also known as *nonrandom* because it is not possible to use past data to fit a probability distribution to describe their future. Building a new line involves a long permitting process (around ten years), so TEP must anticipate the future evolution of these uncertainties. In particular, transmission expansion must foresee generation projects (where new investments can take less than three years to be completed). It is therefore extremely important to develop expansion strategies that are able to adapt with flexibility to the unveiling of uncertainties. Our concept is based on identifying the transmission lines with the highest potential for bringing operation savings in the future and starting the permitting process for them even though not all lines will be built finally when the permits are granted. We develop an approach based on Real Options Valuation (ROV) to evaluate the potential benefit of candidate lines and identify priority projects. We carry out a simplified interpretation of optionality in TEP projects and propose an approximation that enables a feasible evaluation of option value. The proposed technique is able to identify the candidate transmission lines with the highest potential and the ones that are essential to an expansion plan, as well as their main value drivers. We illustrate this with a realistic case study based on the Spanish system.

In addition, solving the Stochastic Problem (SP) for all the relevant scenarios in TEP can be too complex, so Sample Average Approximations (SAA) are often necessary. Moreover, obtaining accurate descriptions for the probability distributions of the uncertainties is extremely difficult and scenarios are often designed by experts based on their subjective judgment. In this situation, it is desirable to obtain transmission expansion plans that are relatively stable with respect to changes in the definition of the scenario tree. We propose to apply regularization to the SAA problem to obtain a more stable behavior. We do this by shrinking the SAA solution towards a robust benchmark. Shrinkage is a technique that has been widely used in statistical estimation. We reinterpret it to apply it in a SP context. In this case, we essentially modify the TEP solution to make it more similar to a selected benchmark plan. We present two different alternative techniques to apply shrinkage to the SAA problem: linear shrinkage and norm-constraint shrinkage. We develop two case studies to illustrate the advantages of the proposed approach, based on a academic and a realistic network. Shrinkage provides results that are consistently better in terms of average out-of-sample cost and decision error.

The Real Options approach for TEP is under review:

S. Lumbreras, A. Ramos, D. Bunn, M. Chronopoulos, "Real Options Valuation Applied to Dynamic Transmission Expansion Planning", under review at IEEE Transactions on Power Systems.

The regularization approach for SP will be submitted for publication as:

- S. Lumbreras, V. DeMiguel, A. Ramos, "Transmission Expansion Planning. A Regularized Stochastic Programming Approach", to be submitted for publication to Operations Research.

5.2. Real OptionsValuation Applied to TEP

5.2.1. Introduction

Envisaging the load flow consequences of all possible sequences of upgrades to an existing system, over a long horizon, with stochastic evolutions of generation expansion, demand and fuel prices, is a task of unmanageable dimensionality. Furthermore, from a welfare maximizing perspective, the joint optimization of generation and transmission expansion has always been difficult to handle, and even more so in an age of unbundled ownerships rather than central planning.

This pragmatic framework for analysis is perhaps most evident at the initial stages of TEP and the first phase of any expansion strategy: the decision to seek permits. The process of building a line usually involves a relatively long permitting process of up to 10 years, followed by 1-2 years of actual construction. In comparison, the lead-time to build a new power plant is much shorter, e.g. 1-3 years for a CCGT. This means that TEP must anticipate generation investments. However, new generation is a key element in the valuation of transmission investments. Given this dependency, transmission projects are often withdrawn when, after a long permitting process, forecasts have not materialized as anticipated.

Realistic, dynamic stochastic TEP is too complex to solve towards a comprehensive optimal solution and so approximation techniques are necessary. The permitting process is the longest stage in TEP, and so we decide to focus on the decision of which permits to request. As it would not be possible, at the outset, to seek permits simultaneously for all possible upgrades of the transmission system, evaluating permits based on the potential savings that transmission lines can bring is an attractive alternative. ROV provides an appealing approach to evaluate these permits. We provide a feasible solution technique for this and demonstrate its application in a realistic context.

5.2.2. Dynamic Uncertainty

Some uncertainties can be described with a probability distribution. They are known as *random* uncertainties. Examples of these are renewable production or hydro inflows. Other sources of uncertainty, such as generation expansion or fuel prices, cannot be captured accurately with a static probability distribution and are therefore more difficult to handle. They are known as dynamic or nonrandom uncertainties. In particular, the relationship between generation expansion planning (GEP) and TEP is complex and difficult to characterize. Some transmission investments are dependent on the installation of new generation and are worthless if the generation project fails to materialize. Therefore, their construction should be delayed until the generation project is sufficiently assured. Conversely, the generator will not be able to sell its power unless the transmission investments are completed. Therefore, it will not commit to the project without a reasonable guarantee that an adequate connection will be provided. This endogeneity is notably difficult to model in a competitive electricity market. Furthermore, even in a centralized setting, the joint resolution of GEP and TEP can be challenging for real-size systems given the large problem size. Some authors have attempted to model this interrelationship. For instance, reference [203] recently proposed a 3-level model that optimizes TEP, GEP and solves system operation. However, most studies consider that GEP and TEP take a leader and a follower role respectively. This work focuses on TEP entirely and, consistently with most of the literature, GEP is considered an input for transmission planning. This input is defined through uncertain scenarios. The recent report [116] deals with the joint optimization of the transmission network and other supply resources.

TEP is by nature a multi-stage problem. Decisions can be re-evaluated in light of the revealed uncertainties, namely generation expansion. However, the complexity of this dynamic nature has caused most studies to focus on simplifications of the problem. Most research works consider a static version of the problem [36]. A second group develops sequential static planning, with several time horizons [204]. A few studies carry out dynamic planning. Given its difficulty, most of the available research works have focused on very small case studies and used heuristic methods with no optimality guarantee. Some of the most relevant techniques that have been applied are Dynamic Programming (DP) [205] and metaheuristics [71]. ROV can be understood as a simplification of the dynamic problem.

5.2.3. A Dynamic TEP Model

In order to apply ROV and evaluate the potential benefits from investments in the future, we need to introduce a dynamic TEP formulation. The only difference with respect to the model in 3.2 is that now, first-stage decisions depend on the scenario. This is, hence, a dynamic TEP model.

5.2.1. Indices

i, j: nodes,

c: cable types,

t: discrete time periods when lines can be installed. The first period (the decision period) and the last period in the analysis will be represented as D, T. The time elapsed between two periods will be referred to as Et_{t_1,t_2}, with $t_2 > t_1$.

el, cl: existing and candidate lines,

ω_t: scenarios.

5.2.2. Parameters

EL_{ijc}: parameter that determines existing lines {0,1}. If a line exists, $El_{ijc} = 1$.

CL_{ijc}: parameter that determines candidate lines {0,1}. If a line is a candidate, $Cl_{ijc} = 1$.

Df_{t_2,t_1}: discount factor to evaluate a future cash flow corresponding to date t_1 in value terms of a closer date t_2 [p.u.]. It is calculated as $Df_{t_1 t_2} = \mathrm{e}^{-\rho E t_{t_1 t_2}}$.

ρ: discount rate [p.u.],

P^{ω_t}: probability of a scenario, which depends on the previous state ω_{t-1},

Dur^{ω_t}: duration of a scenario [h],

Ic_{ijc}: investment cost of a circuit [M€],

$D_{ti}^{\omega_t}$: demand at each node [MW],

$Pnsc$: penalty for power not served [M€/MWh],

$Gc_i^{\omega_t}$: generation cost [M€/MWh],

X_{ijc}: reactance of a circuit [p.u.],

$\overline{F}_{tijc}^{\omega_t}, \overline{G}_{ti}^{\omega_t}, \underline{G}_{ti}^{\omega_t}$: flow and generation limits [MW],

M: big-M parameters.

5.2.3. Variables

5.2.3.1. First-stage variables

$x_{tijc}^{\omega_t}$: cumulative decision to install a candidate investment {0,1}. The main difference of this model with respect to the one in Chapter 3 is that in this case investment decisions depend on the state,

$invc_t$: total investment cost.

5.2.3.2. Second-stage variables

$g_{ti}^{\omega_t}$: power generated at a node [MW],

$opc_t^{\omega_t}$: operation cost [M€],

$pns_{ti}^{\omega_t}$: power not served [MW],

$f_{tijc}^{\omega_t}$: flow through a circuit [MW],

$\theta_{ti}^{\omega_t}$: voltage angles at nodes [rad].

5.2.4. Objective Function

The objective function minimizes total expected cost, composed of investment cost and operation expenses:

$$\min \sum_t Df_{tD} invc_t^{\omega_t} + \sum_{t\omega_t} Df_{tD} P^{\omega_t} Dur^{\omega_t} opc_t^{\omega_t} . \tag{89}$$

5.2.5. Constraints

5.2.5.1. First-stage constraints

The investment cost of a plan is calculated adding the costs of the installed lines:

$$invc_t^{\omega_t} = \sum_{ijc \in cl} Ic_{ijc} \left(x_{tijc}^{\omega_t} - x_{t-1,ijc}^{\omega_t} \right), \tag{90}$$

Once a line has been installed, it is available for the rest of the planning scope. We will decide the transmission lines that are installed at a given time and they will be available for all the scenarios that describe the moments in the planning scope after the investment was made:

$$x_{tijc}^{\omega_t} \geq x_{(t-1),ijc}^{\omega_t} . \tag{91}$$

5.2.5.2. Second-stage Constraints

We define operation cost as the sum of generation cost and penalties for power not served:

$$opc_t^{\omega_t} = \sum_i (g_{ti}^{\omega_t} Gc_i^{\omega_t} + pns_{ti}^{\omega_t} Pnsc) . \tag{92}$$

We model a DCPF. Kirchhoff's First Law establishes power balances at every node and scenario. Incoming flows minus outgoing flows plus generated power equal demand minus power not served.

$$\sum_{jc/(El_{jic}+Cl_{jic})>0} f_{tjic}^{\omega_t} - \sum_{jc/(El_{ijc}+Cl_{ijc})>0} f_{tijc}^{\omega_t} + g_{ti}^{\omega_t} = D_{ti}^{\omega_t} - pns_{ti}^{\omega_t} \qquad \forall i, \omega_t . \tag{93}$$

Kirchhoff's Second Law or Voltage Law is linearized in the DCPF formulation:

$$f_{tijc}^{\omega_t} = \frac{\theta_{ti}^{\omega_t} - \theta_{tj}^{\omega_t}}{X_{ijc}} \qquad ijc \in el , \tag{94}$$

$$-M\left(1 - x_{tijc}^{\omega_t}\right) \le f_{tijc}^{\omega_t} - \frac{\theta_{ti}^{\omega_t} - \theta_{tj}^{\omega_t}}{X_{ijc}} \le M\left(1 - x_{tijc}^{\omega_t}\right), \qquad ijc \in cl , \tag{95}$$

Generation and flow limits:

$$\begin{array}{ll} -\overline{F}_{tijc}^{\omega_t} \le f_{tijc}^{\omega_t} \le \overline{F}_{tijc}^{\omega_t}, & ijc \in el \\ -\overline{F}_{tijc}^{\omega_t} x_{tijc}^{\omega_t} \le f_{tijc}^{\omega_t} \le \overline{F}_{tijc}^{\omega_t} x_{tijc}^{\omega_t}, & ijc \in cl \end{array}, \tag{96}$$

$$\underline{G}_{ti}^{\omega_t} \le g_{ti}^{\omega_t} \le \overline{G}_{ti}^{\omega_t} , \tag{97}$$

$$0 \le pns_{ti}^{\omega_t} \le D_{ti}^{\omega_t} . \tag{98}$$

Each possible system scenario ω_t assigns a value to each of the uncertain parameters that define the system $\omega_t = (\omega_{yt}^0, y \in Y)$, given some $\omega_{yt}^0 \in [\underline{\omega}_{yt}^0, \overline{\omega}_{yt}^0]$, where:

y: set that contains all the uncertainty sources considered in the problem, such as generation expansion, demand, fuel prices, renewable production or hydro inputs.

5.2.6. Real Options and the Value of Waiting to Invest

ROV applies option valuation techniques to decision making under uncertainty. Financial options are instruments which grant the holder the right (but not the obligation) to buy (in the case of a *call* option) or sell (in the case of a *put* option) a given *underlying* asset at a future date (the *expiry*) at a specified price (the *strike*) [206].The holder pays the *premium* in exchange for the option. The value of the option lies in the fact that the holder only decides whether he will buy or sell at the future date, when he knows the final value of the underlying asset. The expected payoffs (and hence the values) of a call and a put option are, respectively:

$$\begin{array}{l} \mathrm{E}[N\left(Y_T - K\right) Df_{T,D}]^+ \\ \mathrm{E}[N\left(K - Y_T\right) Df_{T,D}]^+ \end{array}, \tag{99}$$

where:

$\mathrm{E}\left[X\right]^+ = \mathrm{E}\left[\max(0, X)\right]$: denotes the expected value of the positive values of a random variable,

N: option notional (total invested amount),

K: strike,

Y_T: price of the underlying asset at expiry T,

$Df_{T,D}$: discount factor to value cash at expiry T at decision date D.

Option Value (OV) has two main components, intrinsic value and time value:

$$OV = IV + TV. \tag{100}$$

Intrinsic value (IV) captures net present value. This amounts to the expected payoff that the holder would receive if he exercised the option at a given moment. Time value (TV) reflects the potential for the payoff to increase before expiry. Consequently, time value is higher for longer option maturities and more volatile underlying assets. Time value and option value can only be positive.

Option valuation techniques assume a given model for the evolution of the underlying asset (in a financial context this often means a Geometric Brownian Motion, GBM) and obtain outputs such as option value or optimal exercise time. In some cases, these magnitudes can be calculated conveniently as closed-form solutions.

A real option is similarly defined as the right (but not the obligation) to engage in a business initiative such as expanding capacity, deferring investment or abandoning a project [207, 208]. This technique provides with a framework to analyze the value of flexibility in management decisions.

It has been proven that the solutions found using a ROV approach coincide exactly with those of dynamic optimization at least in the cases where myopic decisions are optimal [209]. This method is therefore an interesting simplification of dynamic problems [209].

There have been some applications of real options to power systems, in particular to GEP [209-211]. Some authors have also studied some specific aspects of transmission investments using this technique, such as the optionality embedded in selecting the size of interconnections [212], the optimal moment to invest [213, 214], or the potential of distributed generation or FACTS for deferring investment in transmission lines [43, 215]. We apply ROV to TEP, developing a feasible approximation for option value that can be applied to individual transmission lines and support the selection of which permits to request and what option lines are key for a given expansion plan.

5.2.7. Transmission Expansion Decisions

The expansion decisions that should be considered in TEP include:

- Adding a new circuit to an existing OHL (overhead line). This can take up to 1 year and can be undertaken relatively easily in most cases.
- Building a new line. This can take from one to two years and is the most expensive action in transmission expansion. The process of building an overhead line (OHL) consists of land clearing at the right-of-way, building temporary access roads, material delivering, foundation building, tower assembly and erection, conductor

stringing, inspection and site restoration [216]. These steps are represented in Figure 5-1. The relevant permits should be obtained before starting construction.

Figure 5-1 –Stages in the process of building a new OHL [216].

- Starting a permitting process for building a new line. This step is relatively inexpensive but it is the most time consuming, taking usually from 7 to 10 years and even up to 15 years if problems arise. In general, the more jurisdictional areas are crossed by the line, the longer this procedure will take. In addition, public opposition or environmental concerns can get a project rejected. A permit gives the right (but not the obligation) to build a line. It is possible to abandon permitted projects that once were in the medium-term plan if they are not optimal short-term decisions. However, it is not possible to initiate the permitting process for all the potentially interesting lines (which are prohibitively many).

Our work considers that initiating a permitting process creates a real option on investing on a transmission project at the future date when the permit is granted. ROV will be applied to calculate intrinsic value (expected operation cost savings minus investment cost) and option values (potential upside) for the candidate transmission investments considered. These magnitudes constitute easily interpretable information that can be used for decision support. In particular:

- Intrinsic value can be used to prioritize transmission lines according to their relative urgency. This is related to the candidate discovery methods we show in Chapter 6.

- Option value can be used to identify investments with a high potential upside. The permitting process should be initiated for the most promising investments, identified by high option values.
- It is possible to calculate the threshold levels of the uncertain parameters that would make a given investment attractive. These thresholds determine by how much fuel prices should rise or how much new generation should be installed so that a given investment is profitable.

5.2.8. Definition of Intrinsic and Option Value

Although ROV can be applied to other decisions in TEP, we focus on the optionality that is implicit in the longest stage, the permitting process. The simplified decision dynamics are:

D: decision period when permits are requested based on potential operation cost savings (option value).

P: time period when the permit is granted (assumed to be around ten years after the permit was requested). Once the permit has been granted, the decision to build the line or not is made. Lines with a sufficiently high intrinsic value (they bring high operation cost savings to the system) will be built.

B: time period when the line is already built and in operation. This is assumed to happen one year after the decision to build the line was taken and the investment cost was realized.

T: final time period in the analysis.

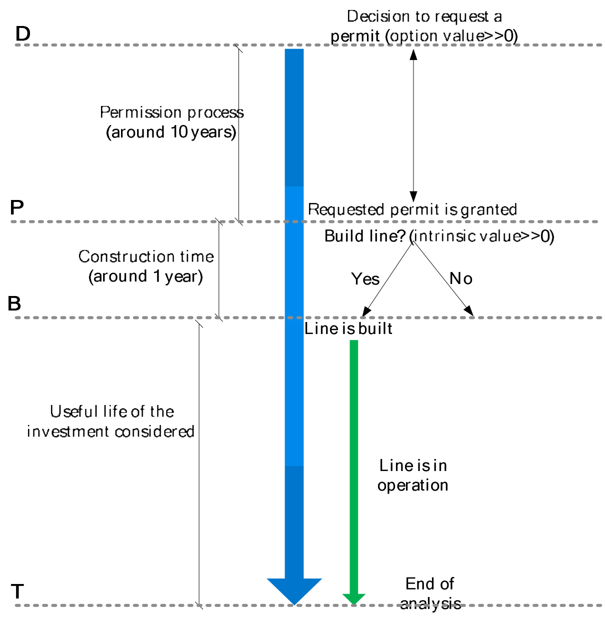


Figure 5-2 –Simplified investment dynamics in the TEP problem.

The intrinsic value of an investment is calculated as the expected value of operation cost savings that the investment would bring. In the case of building a line, this means the difference in operation costs between a base case where the line is not installed and the situation where the line is in operation, minus the investment cost:

$$IV_{ijc} = \mathrm{E}\left[\sum_{t=B}^{T} Df_{t,D} opc_t^{\omega_t/x_{t,ijc}^{\omega_t}=0} \sum_{t=B}^{T} Df_{t,D} opc_t^{\omega_t/x_{t,ijc}^{\omega_t}=1} - Ic_{ijc} Df_{P,D}\right], \tag{101}$$

where

$opc_t^{\omega_t/x_{t,ijc}^{\omega_t}=0}$: operation cost calculated considering the line ijc is not installed.

$opc_t^{\omega_t/x_{t,ijc}^{\omega_t}=1}$: operation cost calculated considering the line ijc is indeed installed.

We calculate option value at the starting date assuming the line is only built if doing so results in net savings.

$$OV_{ijc} = \mathrm{E}\left[\sum_{t=B}^{T} Df_{t,D} opc_t^{\omega_t/x_{t,ijc}^{\omega_t}=0} - \sum_{t=B}^{T} Df_{t,D} opc_t^{\omega_t/x_{t,ijc}^{\omega_t}=1} - Ic_{ijc} Df_{P,D}\right]^{+}. \tag{102}$$

It is important to note that, given that investment is discrete, the differences in operation costs are based on increments rather than on marginal information.

If an expansion plan has already been defined, a line can be canceled if it is not profitable enough once the permit has been granted. Option value reflects the possibility of canceling or deferring the project. In this case, we invert the calculation of expected savings:

$$OV_{ijc} = \mathrm{E}\left[\sum_{t=B}^{T} Df_{t,D} opc_t^{\omega_t/x_{t,ijc}^{\omega_t}=1} - \sum_{t=B}^{T} Df_{t,D} opc_t^{\omega_t/x_{t,ijc}^{\omega_t}=0} + Ic_{ijc} Df_{P,D}\right]^{+}. \tag{103}$$

5.2.9. Proposed Approximation

The exact evaluation of option value is complex, as it implies integrating costs across the probability distributions of all uncertain parameters, with each evaluated point demanding the resolution of optimal system operation. Therefore, a simplified procedure is necessary.

5.2.9.1. Operation Cost Approximation

The proposed operation cost approximation considers several sources of uncertainty y (such as demand or fuel prices). Given that the problem is a two-stage SP, once investment variables are fixed, changes in operation cost when only one of the uncertain parameters varies can be exactly described with a piecewise linear function [121]. For small changes in the uncertain parameters, we can approximate the changes in operation cost for a given scenario ω_t by a linear function. The sensitivity of operation cost with respect to every uncertain parameter is:

$$\delta_t^y = \frac{\partial opc_t^{\omega_t}}{\partial \omega_t^y}. \tag{104}$$

For the sake of simplicity, hatted expressions will represent the expected values of variables or parameters. For instance, $\hat{\omega}_t^{y0}$ represents the expected value of the uncertain parameter y at time t. The expected operation cost is therefore $\hat{o}pc_t^{\omega_t} = opc_t^{\omega_t}(\hat{\omega}_t^{y0})$.

A linear approximation of operation cost for a given scenario ω as a function of the uncertain parameters gives:

$$opc_t^{\omega_t}(Y_t^y) = \hat{o}pc_t^{\omega_t} + \sum_y \delta_t^y(\omega_t^{y0} - \hat{\omega}_t^{y0}). \tag{105}$$

It should be noted that δ_t^y, the sensitivities of operation cost with respect to the uncertainties, are marginal information that is readily available from solving the linear problem that represents system operation. For instance, the sensitivity of operation cost with respect to demand at each node corresponds to the dual variable of the energy balance equations.

5.2.9.2. Expressions for Intrinsic and Option Value

The expected value of investing in line $x_{D,ijc}$ (Intrinsic Value) can be calculated as the expected value of its operation cost savings (the difference in operation cost when the line is installed with respect to the base case where it is not installed) net of investment costs:

$$IV_{ijc} = \mathrm{E}\left[\sum_{t=B}^{T} Df_{t,D}\, opc_t^{\omega_t/x_{t,ijc}^{\omega_t}=0} - \sum_{t=B}^{T} Df_{t,D}\, \hat{o}pc_t^{\omega_t/x_{t,ijc}^{\omega_t}=1} - Ic_{ijc} Df_{P,D}\right]. \tag{106}$$

Option value captures the expected value of the investment conditional to it being positive at the time when the decision to build the line is made:

$$\begin{aligned} OV_{ijc}(x_{D,ijc}) = \mathrm{E}\Bigg[&\sum_{t=B}^{T} Df_{t,D}\left(\hat{o}pc_t^{\omega_t/x_{t,ijc}^{\omega_t}=0} - \hat{o}pc_t^{\omega_t/x_{t,ijc}^{\omega_t}=1}\right) + \\ &\sum_{t=B}^{T} Df_{t,D}\left(\sum_y \left(\delta_t^{y/x_{t,ijc}^{\omega_t}=0} - \delta_t^{y/x_{t,ijc}^{\omega_t}=1}\right)(\omega_t^{y0} - \hat{\omega}_t^{y0})\right) - Ic_{ijc} Df_{P,D}\Bigg]^{+}. \end{aligned} \tag{107}$$

These expressions will guide the rest of this section.

5.2.9.3. Description of the Uncertainties

For a given scenario, the uncertain parameters at time B are assumed to follow a normal distribution centered on its forecast $\hat{\omega}_t^{y0}$. We must adjust its standard deviation σ^y for time, therefore giving:

$$\sigma^{*y} = \sigma^y \sqrt{Et_{B,D}}. \tag{108}$$

This assumption is arguably good for uncertainty sources such as fuel prices or generation capacities where increments can be very small (such as wind or solar) [217]. It will not be as good when representing lumpier investments, such as nuclear capacity. In addition, it is consistent with a GBM when increments are small, a very frequent assumption in financial contexts and a very reasonable hypothesis when dealing with financial uncertainties such as fuel prices. Then, we have the normal distribution:

$$\omega_B^{y0} - \hat{\omega}_B^{y0} \approx \mathrm{N}(0, \sigma^{*y}). \tag{109}$$

5.2.9.4. Expressions for option value

Substituting in the option value expression, we obtain the following:

$$\begin{aligned} &OV_{ijc} \approx \mathrm{E}\Bigg[\sum_{t=B}^{T} Df_{t,D}\left(\hat{o}pc_t^{\omega_t/x_{t,ijc}^{\omega_t}=0} - \hat{o}pc_t^{\omega_t/x_{t,ijc}^{\omega_t}=1}\right) + \\ &\sum_{t=B}^{T} Df_{t,D}\left(\sum_y \left(\delta_{yt}^{x_{t,ijc}^{\omega_t}=0} - \delta_{yt}^{x_{t,ijc}^{\omega_t}=1}\right)\mathrm{N}(0,\sigma^{*y})\right) - Ic_{ijc} Df_{P,D}\Bigg] = \\ &\mathrm{E}\left[\mathrm{N}(\mu_{Total}, \sigma_{Total})\right]^{+} \end{aligned} \tag{110}$$

Where the parameters of the normal distribution describing the overall dynamics for the transmission line ijc are $\mu_{Total}, \sigma_{Total}$. For the sake of simplicity, the indices ijc have been omitted in the remaining of this section.

A linear combination of normal distributions, where each normal $\mathrm{N}(\mu_i, \sigma_i)$ has a weight w_i is defined as:

$$\mathrm{N}(\mu_{LC}, \sigma_{LC}) = \sum_i w_i \mathrm{N}(\mu_i, \sigma_i). \tag{111}$$

The mean of the combination, μ_{LC} , is:

$$\mu_{LC} = \sum_i w_i \mu_i . \tag{112}$$

The volatility of the combination, σ_{LC} , can be obtained as:

$$\sigma_{LC} = \sqrt{\sum_i w_i \sigma_i^{\,2} + \sum_{i,j\neq i} w_i w_j \sigma_i \sigma_j \rho_{ij}} . \tag{113}$$

It is convenient to define a constant correlation coefficient applicable to all pairs, known as *equicorrelation*:

$$\begin{aligned} \rho &= \sum_{i\neq j} c_{ij}\rho_{ij} \\ c_{ij} &= \frac{w_i w_j \sigma_i \sigma_j}{\sum_{i\neq j} w_i w_j \sigma_i \sigma_j} \end{aligned} . \tag{114}$$

Therefore, for the particular case of option value:

$$\mu_{Total} = \sum_{t=B}^{T}\sum_{\omega_t} Df_{t,D} P^{\omega_t}\left(\hat{o}pc_t^{\omega_t/x_{t,ijc}^{\omega_t}=0} - \hat{o}pc_t^{\omega_t/x_{t,ijc}^{\omega_t}=1}\right), \tag{115}$$

$$\sigma_{Total} = \left[\sum_{t=B}^{T} \sum_{\omega_t, y} P^{\omega_t} Df_{t,D} \left((\delta_t^{y/x_{t,ijc}^{\omega_t}=0} - \delta_t^{y/x_{t,ijc}^{\omega_t}=1}) \omega_B^{y0} \ \sigma^{*y} \right)^2 \right.$$
$$\left. + \sum_{t=B}^{T} \sum_{\omega_t, y} P^{\omega_t} Df_{t,D} \left[\begin{array}{l} \rho(\delta_t^{y/x_{t,ijc}^{\omega_t}=0} - \delta_t^{y/x_{t,ijc}^{\omega_t}=1}) \\ (\delta_t^{y'/x_{t,ijc}^{\omega_t}=0} - \delta_t^{y'/x_{t,ijc}^{\omega_t}=1}) \omega_B^{y0} \omega_B^{y'0} \end{array} \right] \right]^{1/2} . \tag{116}$$

Option value can be calculated as:

$$OV_{ijc} = \mathrm{E}\left[\mathrm{N}(\mu_{Total}, \sigma_{Total}) \right]^{+} . \tag{117}$$

This can be expressed as the expected value of a truncated normal probability distribution where only positive values are integrated. This can be calculated as:

$$\begin{array}{l} OV_{ijc} = \mathrm{E}\left[\mathrm{N}(\mu_{Total}, \sigma_{Total}) \right]^{+} = \\ \left[\mu_{Total} + \sigma_{Total} \dfrac{\phi(-\mu_{Total} \ / \ \sigma_{Total})}{1 - \Phi(-\mu_{Total} \ / \ \sigma_{Total})} \right] \left[1 - \Phi(-\mu_{Total} \ / \ \sigma_{Total}) \right]' \end{array} \tag{118}$$

where ϕ and Φ denote the standard normal probability density function and the standard cumulative distribution respectively. We use this result to approximate option value in a closed form and apply it in the case study. Another interesting result is the probability that a given investment will be carried out at a future date. This is referred to as the In-The-Moneyness or In-The-Money (ITM) Probability of the option, and can be calculated as:

$$ITMP_{ijc} = \mathrm{P}\left[\mathrm{N}(\mu_{Total}, \sigma_{Total}) \geq 0 \right] = 1 - \Phi(-\mu_{Total} \ / \ \sigma_{Total}) . \tag{119}$$

In addition, we can calculate the sensitivity of intrinsic value with respect to the uncertainties as:

$$Sens_{ijc}^{y} = \sum_{t=B}^{T} \sum_{\omega_t} Df_{t,D} P^{\omega_t} \left(\delta_t^{y/x_{t,ijc}^{\omega_t}=0} - \delta_t^{y/x_{t,ijc}^{\omega_t}=1} \right) . \tag{120}$$

We can identify the uncertainties y^* that have the largest impact on the value of a given transmission line as the ones with the largest absolute sensitivity $y_{ijc}^{*} = \arg\max_y \left| Sens_{ijc}^{y} \right|$.

The thresholds for having a positive intrinsic value assuming only one of the uncertainties varies can be calculated as:

$$Th_{ijc}^{y^*} = \frac{IV_{ijc}}{Sens_{ijc}^{y^*}} . \tag{121}$$

We can use these threshold values to analyze the robustness of a decision with respect to changes in the scenarios, which can be very useful for decision support. We show this in the case study.

5.2.10. Case Study

We illustrate the proposed approach with a realistic case study based on the Spanish power system. We implement our ROV approximation as an additional feature to the existing TEPES model, which is based on the model described in Chapter 3 [64]. We take transmission data as of 2008 from publicly available ENTSO-e and REE E-SIOS cases [218]. We use the same E-SIOS cases to build forecasts for a peak-demand scenario and generation capacities. The system is composed of 1084 nodes and 294 power plants (nuclear, coal, CCGTs, hydro, wind and solar) plus the 1505 lines and transformers that configure the existing transmission network. The network was stressed by introducing a security coefficient of 60% (that is, an effective reduction on the NTC of the line to 60% of its TTC). The case incorporates uncertain demand, generation capacities, fuel and carbon prices. For the sake of simplicity, we assume that demand and capacities grow equally at every node. We analyze historical data to extract values for the volatilities of the uncertain parameters. We use public values for fuel prices, extracting them from Bloomberg, using 12m futures for Rotterdam coal, UK gas prices (the most relevant European price indicators) and EU Allowances for carbon prices. We take historical installed capacities for wind, solar and gas generators from the system operator [219]. When possible, we use a 15 year-long data series. If there was not sufficient history, we used all the available data. We adjust annual volatility from historical data as the standard deviation among consecutive increments:

$$\begin{aligned} \sigma^y &= \sqrt{n^y \frac{1}{N^y} \sum_{t=1}^{t} \left(\omega_t^{y0} - \omega_{t-1}^{y0} - \mu^y \right)^2} \\ \mu^y &= \frac{1}{N^y} \sum_{t=1}^{t} \left(\omega_t^{y0} - \omega_{t-1}^{y0} \right) \end{aligned} , \qquad (122)$$

where:

N^y : number of observations,

n^y : number of observations that can be found in one year. For instance, if there is an observation every business day, $n^y = 260$.

The resulting parameters can be seen in Table 5-1.

	Annual volatility
Demand	4.1%
Generation capacities	
Wind	17.5%
Solar	57.7%
Gas	22.1%
Fuel and carbon prices	
Coal	24.9%
Gas	53.4%
Carbon	77.4%

Table 5-1 – Annual volatilities considered.

We use intrinsic and option value to establish investment priorities:

- Relatively high, positive intrinsic values signal transmission lines that are expected to bring immediate savings. Permits should be requested for them and they should already be treated as priority projects.
- Transmission lines with a low intrinsic value but a relatively high option value have the potential of bringing high savings in the future. Permits should be requested for them and their value should be monitored during the permitting process.

A total of 74 promising candidate lines were included in the study. The optimal expansion plan identified by TEPES installed 24 of these lines. Intrinsic and option values were calculated for all these candidates.

	Initial state of the network					Intermediate					Optimal plan				
Line	Included in plan?	Intrinsic Value	Option Value	ITM Prob	Priority	Included in plan?	Intrinsic Value	Option Value	ITM Prob	Priority	Included in plan?	Intrinsic Value	Option Value	ITM Prob	Priority
1		302.5	302.5	1.00	High		77.5	77.5	1.00	High	YES	-25.4	0.0	0.00	
3		306.3	306.3	1.00	High		88.1	88.1	1.00	High	YES	-0.7	3.2	0.47	
4		304.9	304.9	1.00	High		90.6	90.6	1.00	High		-5.3	6.5	0.41	
6		341.4	341.4	1.00	High		34.7	34.7	1.00	High	YES	-9.2	3.7	0.31	
17		308.2	308.2	1.00	High		86.6	86.6	1.00	High	YES	-20.1	0.0	0.01	
18		306.8	306.8	1.00	High		109.3	109.3	1.00	High		-5.0	5.8	0.40	
20		385.6	385.6	1.00	High	YES	-84.3	0.0	0.00		YES	-81.9	0.0	0.00	
25		343.0	343.0	1.00	High		61.5	61.5	1.00	High	YES	-67.2	0.0	0.00	
30		383.9	383.9	1.00	High		3.7	4.2	0.79		YES	-76.0	0.0	0.00	
31		346.2	346.2	1.00	High		20.2	20.4	0.97	High	YES	-4.7	8.4	0.43	
33		389.0	389.0	1.00	High		-0.7	2.7	0.47		YES	-1.4	9.0	0.48	
36		344.4	344.4	1.00	High		36.1	36.1	1.00	High	YES	-48.1	0.0	0.00	
40		419.1	419.1	1.00	High	YES	-159.6	0.0	0.00		YES	-125.9	0.0	0.00	
45		315.2	315.2	1.00	High		-3.9	1.4	0.30		YES	-56.3	0.5	0.04	
46		313.8	313.8	1.00	High	YES	-81.9	0.0	0.00			-5.3	8.9	0.43	
47		356.8	356.8	1.00	High		34.6	34.6	1.00	High	YES	-1.5	14.6	0.48	Low
51		219.8	219.8	1.00	High		-2.9	1.9	0.36		YES	-18.7	7.3	0.31	
52		218.7	218.7	1.00	High	YES	-92.9	0.0	0.00			-4.0	9.5	0.44	
55		348.3	348.3	1.00	High		-1.5	2.1	0.41		YES	-1.8	14.9	0.48	Low
57		348.8	348.8	1.00	High		43.2	43.2	1.00	High	YES	-2.3	14.1	0.48	Low
59		365.3	365.3	1.00	High		63.8	63.8	1.00	High	YES	-31.1	4.6	0.21	
60		364.0	364.0	1.00	High		63.4	63.4	1.00	High	YES	-75.6	0.0	0.00	
62		328.6	328.6	1.00	High		-1.0	1.3	0.41		YES	-7.6	6.8	0.38	
63		330.2	330.2	1.00	High		-2.9	1.9	0.36		YES	-16.1	9.7	0.35	
66		338.0	338.0	1.00	High		14.2	14.6	0.92	High	YES	-36.7	1.2	0.09	
67		316.8	316.8	1.00	High		-0.5	2.6	0.47		YES	-2.1	16.5	0.48	Low
69		342.6	342.6	1.00	High		19.4	19.7	0.94	High	YES	-16.2	10.1	0.35	Low
73		331.2	331.2	1.00	High		9.1	11.6	0.73	Medium	YES	-4.8	14.0	0.45	Low

Table 5-2 – Intrinsic value, option value and ITM probability for a subset of the candidate transmission lines studied (this is, not all lines tested are reported in the table).

As intrinsic and option values depend on operation cost, they also depend on the starting situation of the network. Intrinsic and option value depend on the base case of the network, and to illustrate this we consider three different base case situations in the case study:

- Initial state of the network, with no transmission additions.
- Assuming the optimal expansion plan is going to be fully deployed. For the 24 transmission lines actually included in the plan, the values refer to the option of not building them or deferring their construction. The optimal plan was obtained by TEPES. It should be noted that, given that there are synergies among lines, the optimal expansion plan does not correspond to merely installing the lines with a positive intrinsic value. This synergies are explored in section 6.2.5.
- An intermediate situation where only the most urgent investments are considered committed, and the others are only being studied. This is an interesting example as

often long-term TEP selects only some of the most important investments while leaving the rest of the plan undefined. This assumes only 4 lines are installed.

Table 5-2 shows some of the most interesting results. A threshold of 10 M€ (roughly equivalent to the investment cost of a single circuit, 400 kV 20 km line) has been established in order to filter investments. As expected, intrinsic and option values are much higher at the initial state of the network, where more improvements can be made. Conversely, if the base case is the optimal plan, option values are much lower. Moreover, by definition, all intrinsic values are negative (deviating from the optimal plan necessarily worsens costs). The intermediate situation presents intermediate results.

High option values point to investments that should be monitored and for which a permit should be requested. For the lines not included in the expansion plan, a high option value signals a permit that should be requested. For lines already included in the plan, the values refer to the option to cancel them or deferring their construction. In many cases, this value is very close to zero. This means that the line seems so profitable under the current conditions that it will be built almost certainly. For instance, deferring line 60 has an option value of 0, while deferring line 67 has an option value of 16.5 M€. The first kind of lines should be prioritized, while in the second case potential savings should be monitored.

In addition, all the sensitivities with respect to the uncertain parameters are available as intermediate calculations. In the example of line 67, the key uncertainties are gas prices (with a sensitivity of -2.42 M€/ 1€ increase in gas prices) and carbon prices (with a sensitivity of -2.24 M€/ 1€ increase in carbon prices). This means that, although that line is currently in the optimal expansion plan, it will be withdrawn if gas or carbon prices rise sufficiently. For instance, the threshold increase in gas prices before the line becomes unprofitable can be calculated as the intrinsic value of not building it divided by the sensitivity of intrinsic value with respect to gas prices. That is:

$$Th_{ijc}^{y^*} = \frac{IV_{ijc}}{Sens^{y^*}} = \frac{-2.1}{-2.42} = 0.87\,, \tag{123}$$

This means that if gas prices increase just slightly (by 0.87 €), then transmission line 67 will not be part of the optimal plan. The ITM probability represents the probability of the line being still profitable when the permit is granted. In this case, it was 47%. This means that there is roughly a 50/50 chance that this line will be built.

This example shows how some investments that form part of the optimal plan are so sensitive to the uncertainties that they have a high probability of not being completed. However, some others (such as the example of line 60, where deferring investment had an option value of 0) seem robust with respect to the uncertainties and their assumed volatilities. This information can make it easier for planners to identify the key investments and focus on them while requesting additional permits for other lines that could bring important savings in some scenarios.

5.2.11. Conclusions

TEP is challenged by the presence of long-term uncertainties. In particular, while lead times for installing generation expansion can be as short as 1-3 years in the case of CCGTs, the lengthy permitting process for a new transmission line can take around 10 years. This means that TEP must anticipate the future state of the system, in particular with respect to new generation capacity and fuel prices. This situation would make dynamic optimization appear attractive but the complexity of the problem generally means that such approaches are not viable for real-sized systems.

ROV is a useful technique when studying investment opportunities in an uncertain context. We interpret TEP from a real options perspective focusing on its longest stage, this is, the permitting process. Given that the exact evaluation of option value would be intractable, we develop a feasible approximation that can provide with intrinsic value (expected benefit) and option value (potential upside) for a list of candidate transmission lines. We illustrate the proposed approach with a realistic case study based on the Spanish power system. We use option value to identify the investments with a high potential and for which permits should be requested. Lines for which the option to defer investment is worthless and therefore will almost certainly be completed are labeled as priority projects. We use sensitivities to assess the robustness of any given transmission line with respect to the uncertain parameters as well as to establish approximate thresholds that would result in them being completed or withdrawn.

5.3. A Regularization Approach to Stochastic Programming

As explained, the long lead-times involved in transmission expansion mean that TEP decisions must anticipate generation investments, with the consequent risks involved: it is extremely difficult to assess the completion probability of these generation projects. Each proposal is so different from the others that previous history is hardly applicable. It can be argued that the only useful information in these cases is expert opinion. However, scenario trees based on expert opinion are necessarily imprecise, with vaguely defined scenarios and probabilities. In addition, although further studies can improve scenario analyses in other contexts, interviewing additional experts acquainted with the problem might not be possible. In this context, it would be interesting to generate expansion plans that are stable with respect to changes in the definition of the scenario tree. However, Stochastic Programming (SP) approaches in general ignore this risk and focus on expected cost entirely, so the optimal stochastic solution can be highly sensitive to the definition of scenarios.

Robust Optimization acknowledges that small data errors can lead to optimal solutions that are highly infeasible [18] and focuses on guaranteeing feasibility for the worst-case scenario [220]. However, the main issue TEP faces in this context is not avoiding infeasibility but rather ensuring a relatively good solution regardless of changes in the definition of scenarios.

Fuzzy decision analysis deals with the outcomes of different scenarios (or different definitions of the scenario tree) in an analogous way to the several attributes defined in a multi-criteria problem. It looks for non-dominated solutions and works with the decision maker to analyze the relative importance of the objectives [33]. Although this approach can evaluate the suitability of a given expansion plan, it cannot generate robust solutions on its own. In addition, it requires the manual intervention of the decision maker to select the best alternative plans.

Our approach originates from the need to find good solutions in terms of expected cost that are relatively stable with respect to changes in the definition of the scenario tree. We propose to regularize the SP solution in order to obtain an expansion plan with a more stable behavior with respect to the uncertainties.

Regularization in general refers to the process of introducing additional information to avoid overfitting or to solve an ill-posed problem. This additional information usually takes the form of a penalty for complexity. We extend this definition to the SP by penalizing solutions that depend too much on the scenario tree. This is assessed by comparing them to a *benchmark solution* that is considered robust in the sense that it does not depend on the scenario tree. For instance, this robust benchmark can be calculated as the solution of a deterministic version of the problem (such as the expected value problem).

In essence, we combine two different estimations for the optimal expansion plan. The first one is the solution of the SP, which, as the scenario tree built from expert opinion is necessarily incomplete, we interpret as a Sample Average Approximation (SAA) problem. The second one is the robust benchmark.

This technique has been widely applied in statistical estimation, where shrinkage or James-Stein estimators [221] improve the error of a given estimator by combining it with another one, typically with less variance. This kind of estimators is called *shrinkage* estimators as they “shrink” their estimate towards the low-variance estimator.

This technique has been applied to estimating the covariance matrix of a series of stock returns. The covariance matrix is key to build a minimum-variance portfolio, which is a frequently used investment strategy. The main reason for the success of the minimum-variance strategy is that the Markowitz portfolio (maximum mean-variance) has a usually very poor out-of-sample performance due to the inherent difficulty in accurately forecasting mean asset returns. In comparison, the minimum-variance portfolio has a much more stable behavior, but it is extremely dependent on the quality of the estimated covariances. A direct estimation of the covariance matrix involves calculating the sample historical covariances among stock pairs. However, this direct estimation can lead to substantial errors. Several approaches have been proposed in the literature to deal with this problem. Some authors use higher-frequency data, such as daily returns. Some others impose a certain factor structure to the matrix so that fewer elements are estimated. Regularization has been successfully applied to this problem [222]; extreme values in the covariance matrix, which might signal estimation errors, are avoided by shrinking the covariance matrix towards the identity matrix.

To our knowledge, shrinkage had never been applied to Stochastic Programming. We propose two alternative mechanisms to do so. *Linear shrinkage* creates a linear combination of the SAA solution and the robust benchmark. We prove the existence of an optimal shrinkage parameter and provide an expression for its value based on the characteristics of the SAA solution and the robust benchmark.

The second shrinkage approach we propose is *norm-constraint shrinkage.* It imposes a maximum distance between the SAA solution and the robust benchmark, so that the SP incorporates this constraint and automatically generates solutions within a given distance to the robust benchmark. As the optimal shrinkage parameter, the best distance parameter depends on the characteristics of the SAA solution and the robust benchmark. We select it by *cross-validation*, that is, we test several values of distances and select the one that gives the best empirical results.

We apply this method to two TEP case studies, an academic example and a realistic one. Both cases reflect the uncertainties present in large, long-term generation expansion projects as described above. The academic case study is based on the well-known Garver system [36]. The system is composed of 6 nodes where load and generation are installed. The case considers a situation where two areas have a high potential for new renewable generation and studies the impact of the two proposed shrinkage approaches.

A realistic case study based on the Spanish system illustrates the applicability of the method to real-sized systems. The case assumes that a large amount of renewable energy could be available for import from North Africa in the future. This is consistent with visions such as the Desertec Industrial Initiative [6]. Some of this energy would have to be exported to France through the North-East and North-West. The amount available for import and the export needs are uncertain. Shrinkage consistently generates solutions that are more stable with respect to these uncertainties and shows the suitability of the proposed approach for realistic TEP problems.

The rest of this section is organized as follows. First, SP and the SAA problem are defined. Then, we introduce the proposed linear and norm-constraint shrinkage approaches. Finally, we present the case studies and discuss conclusions.

5.3.1. Stochastic Programming and the Sample Average Approximation Problem

Stochastic Optimization deals with optimization under uncertainty, where the problem depends on parameters that follow a known probability distribution. The SP selects the best alternative from the feasible set in terms of expected value.

An especially relevant case is the two-stage problem, of which TEP and OWF design are particular examples. The problem minimizes expected total cost by selecting values for some first-stage variables without knowing which realization of the parameters will occur in the second stage. Then, second-stage decisions optimize costs for the particular realized values of the parameters. If costs and constraints are linear, the two-stage stochastic problem takes the form:

$$\begin{aligned} &\min_x c^T x + \mathrm{Q}(x) \\ &s.t. \quad Ax = b \\ &\qquad\; x \geq 0 \end{aligned} \quad , \tag{124}$$

where $\mathrm{Q}(x)$ represents the optimal value of the second stage problem for a given first-stage solution:

$$\mathrm{Q}(x) := \mathrm{E}\Big[\mathrm{Q}(x, \xi(\omega))\Big], \tag{125}$$

$$\begin{aligned} &\mathrm{Q}(x, \xi(\omega)) = \min_y d^T y \\ &s.t. \quad T^\omega x + W y^\omega = h^\omega \\ &\qquad y^\omega \geq 0 \end{aligned} \; . \tag{126}$$

In the formulation above, we have:

x: first-stage decisions, which can be either continuous or discrete.

c: costs of first-stage decisions.

y: second-stage decisions.

d: costs of second-stage decisions.

The relationship between W^ω and h^ω depends on $\omega \in \Omega$ The possible realizations $\xi^1, \xi^2, .., \xi^\Omega$ are referred to as *scenarios*. These scenarios have an associated probability, which we represent as P^{ω^k}. We will refer to the optimal solution of the two-stage stochastic linear problem as x^*.

Often, the scenario tree is too large for optimization and must be reduced to be tractable. In addition, if experts build the scenario tree they do so based on their subjective judgment and experience, which is necessarily incomplete. In both cases, the SP can be considered an approximation of the problem based on a reduced scenario tree. In particular, we will consider a reduced scenario tree as j random samples of the original, not necessarily known tree, containing s realizations of scenarios each:

$$\Omega = \left\{\omega^i\right\}, i = \left\{1, 2, 3, ..., s\right\}, \tag{127}$$

where $\mathrm{P}^{\omega^i} = \dfrac{1}{s}$ represents the probability applied to every sampled scenario.

The sample average approximation (SAA) problem solves the two-stage optimization problem for the reduced tree, and considers that the optimal value of the problem is the average of the individual results for the samples:

$$\min_x c^T x + \hat{\mathrm{Q}}(x), \tag{128}$$

$$\hat{\mathrm{Q}}(x) := \frac{1}{s} \sum_{i=1}^{s} \mathrm{Q}(x, \xi^i), \tag{129}$$

$$\begin{aligned} s.t. \quad & Ax = b \\ & x \geq 0 \end{aligned}, \tag{130}$$

where $\mathrm{Q}(x,\xi)$ represents the optimal second-stage cost for a given first-stage decision:

$$\mathrm{Q}(x,\xi(\omega)) = \min_{y} d^T y\,, \tag{131}$$

$$\begin{aligned} s.t. \quad & T^{\omega} x + W^{\omega} y = h^{\omega} \\ & y^{\omega} \geq 0 \end{aligned}. \tag{132}$$

The optimal first-stage solution to this problem, $\hat{X}$ (for the sake of clarity, we use upper case to represent random variables) converges exponentially quickly to the optimal solution of the original tree x^* as the number of scenarios in the scenario samples increases [223].

$$\lim_{s \to k} \frac{1}{k} \log\left[1 - \mathrm{P}(\hat{X} = x^*)\right] = -\beta\,, \tag{133}$$

where $\beta > 0$ depends on the characteristics of the problem, namely its condition number, with better-conditioned problems converging faster than ill-conditioned ones. This fact has led some authors to solve a series of SAA problems with increasingly large scenario samples until an estimation of the error $\left|\hat{\mathrm{Q}}(x) - \mathrm{Q}(x)\right|$ is below a certain threshold [224]. However, it is not always possible to increase the number of samples. This is often the case for scenario trees based on expert opinion, as it might be impossible to obtain new interviews with experts acquainted with the problem. In addition, even if we can generate a new sample, it is possible that the number of scenarios needed for a sufficiently good SAA solution is still larger than the optimization capabilities available. This situation is widespread in applications of Stochastic Programming. In particular, both conditions apply to TEP. First, increasing the number of scenario samples describing the future evolution of the power system is very difficult, as the availability of expert opinion is limited. In addition, the complexity of real systems limits the size of the scenario tree that optimization can handle.

Therefore, we focus on improving the SAA solution without increasing the number of samples, making the solution stable with respect to changes in the scenario samples. For this, we propose a regularization approach. In some Stochastic Optimization contexts, regularization is understood as a technique that accelerates problem resolution by limiting the distance between consecutively evaluated solutions [160], without affecting the final result. However, we refer to the term as in the field of statistical estimation, by which regularization introduces additional information to a problem with the aim of avoiding overfitting or solving an ill-posed problem. Usually, this additional information is expressed as a penalty for complexity. In the SAA problem, we penalize solutions that depend too much on the scenario samples. We assess this dependence by comparing the SAA solution to a benchmark solution that is robust in the sense that it does not depend on the scenario tree.

Therefore, the proposed method combines the information of two different estimations for the optimal expansion plan: the SAA solution and the robust benchmark. Similar techniques have been applied extensively in statistical estimation. Shrinkage or James-

Stein estimators [221] improve estimation error by combining a given estimator with another one, typically with a lower variance. These estimators are known as *shrinkage* estimators because they "shrink" their estimate towards the low-variance estimator. We develop two alternative shrinkage techniques for the SP: *linear shrinkage,* which uses a linear combination of the SAA solution and *norm-constraint shrinkage,* which imposes a maximum distance between the SAA solution and the robust benchmark. Linear shrinkage cannot be applied to problems with discrete variables directly; however, we develop an extension for discrete variables. Norm-constraint shrinkage can be applied to problems with both continuous and discrete variables.

5.3.2. Linear Shrinkage

We combine the SAA solution $\hat{X}$ and the robust benchmark x^b. The resulting estimation of the optimal first-stage decisions is:

$$\hat{X}^{est} = \alpha x^b + (1-\alpha)\hat{X}\,. \tag{134}$$

The literature offers some useful results for calculating the optimal shrinkage parameter α^* in shrinkage estimators when the estimated variable is a scalar [222]. We extend this result to multidimensional variables and provide an expression for the shrinkage parameter that minimizes the expected squared error of the SAA solution.

Theorem 1

We define estimation error as the norm (represented as $\|\ \|$) of the vector difference between the estimator $\hat{X}^{est}$ and the optimal solution x^*:

$$\mathrm{E}\left[\left\|\hat{X}^{est} - x^*\right\|^2\right]. \tag{135}$$

The linear combination parameter that minimizes estimated error is:

$$\alpha^* = \frac{B_1 + B_2 - C}{A + B_1 + B_2 - 2C}, \tag{136}$$

where:

$$\begin{aligned} A &:= \left\|x^b - x^*\right\|^2 \\ B_1 &:= \left\|\mathrm{E}\left[\hat{X}\right] - x^*\right\|^2 \\ B_2 &:= \mathrm{E}\left\|\hat{X} - \mathrm{E}\left[\hat{X}\right]\right\|^2 \\ C &:= \left\langle x^b - x^*, \mathrm{E}\left[\hat{X}\right] - x^*\right\rangle \end{aligned}\,. \tag{137}$$

We can interpret these magnitudes as:

A: squared bias of the benchmark solution,

B_1: squared bias of the stochastic problem solution,

B_2: variance of the stochastic problem solution,

C: inner product of the biases of the benchmark solution and the stochastic problem solution.

A proof for this theorem is provided below, italics. The reader can skip this section without any damage to the understanding of the rest of this chapter.

Proof

Our first goal is to express the expected square error of the linear shrinkage estimator as a function of the linear combination parameter. Then, we derive this expression with respect to alpha to find the extreme point. Expected square error takes the form:

$$\mathrm{E}\left[\left\|\hat{X}^{est} - x^*\right\|^2\right] = \mathrm{E}\left[\left\|\hat{X}^{est} - x^*\right\|^2\right] = \mathrm{E}\left[\left\|\alpha x^b + (1-\alpha)\hat{X} - x^*\right\|^2\right]. \qquad (138)$$

Adding and subtracting the same term, $(1-\alpha)\mathrm{E}\left[\hat{X}\right]$*, we find:*

$$\begin{aligned} &\mathrm{E}\left[\left\|\alpha x^b + (1-\alpha)\hat{X} - (1-\alpha)\mathrm{E}\left[\hat{X}\right] + (1-\alpha)\mathrm{E}\left[\hat{X}\right] - x^*\right\|^2\right] = \\ &= \mathrm{E}\left[\left\|(\alpha x^b + (1-\alpha)\mathrm{E}\left[\hat{X}\right] - x^*) + (1-\alpha)(\hat{X} - \mathrm{E}\left[\hat{X}\right])\right\|^2\right] \end{aligned}. \qquad (139)$$

We can expand the expression using $\left\|a+b\right\|^2 = \left\|a\right\|^2 + \left\|b\right\|^2 + 2\left\langle a,b\right\rangle$*, where* $\left\langle a,b\right\rangle$ *represents the inner product of the two vectors* a *and* b*:*

$$\begin{aligned} &\mathrm{E}\left[\left\|\alpha(x^b - x^*) + (1-\alpha)(\mathrm{E}\left[\hat{X}\right] - x^*)\right\|^2\right] + \mathrm{E}\left[\left\|(1-\alpha)(\hat{X} - \mathrm{E}\left[\hat{X}\right])\right\|^2\right] + \\ &+2\mathrm{E}\left[\left\langle \alpha x^b + (1-\alpha)\mathrm{E}\left[\hat{X}\right] - x^*, (1-\alpha)(\hat{X} - \mathrm{E}\left[\hat{X}\right])\right\rangle\right] \end{aligned}. \qquad (140)$$

After some algebraic manipulation, we arrive to:

$$\begin{aligned} &\alpha^2\left\|x^b - x^*\right\|^2 + (1-\alpha)^2\left\|\mathrm{E}\left[\hat{X}\right] - x^*\right\|^2 + (1-\alpha)^2\mathrm{E}\left[\left\|(\hat{X} - \mathrm{E}\left[\hat{X}\right])\right\|^2\right] + \\ &+2\left\langle \alpha(x^b - x^*), (1-\alpha)(\mathrm{E}\left[\hat{X}\right] - x^*)\right\rangle + 2\mathrm{E}\left[\left\langle \alpha(x^b - x^*), (1-\alpha)(\hat{X} - \mathrm{E}\left[\hat{X}\right])\right\rangle\right]. \\ &+2\mathrm{E}\left[\left\langle (1-\alpha)(\mathrm{E}\left[\hat{X}\right] - x^*), (1-\alpha)(\hat{x} - \mathrm{E}\left[\hat{X}\right])\right\rangle\right] \end{aligned} \qquad (141)$$

The inner product of a fixed vector and a vector with an expected value of zero is zero. Therefore, as $\left\|\mathrm{E}\left[\hat{X} - \mathrm{E}\left[\hat{X}\right]\right]\right\|^2 = 0$*, and we can eliminate the last two terms. Using the definitions of* A, B_1, B_2, C *, the expression for the error results in:*

$$\mathrm{E}\left[\left\|\hat{X}^{est} - x^*\right\|^2\right] = \alpha^2 A + (1-\alpha)^2 B_1 + (1-\alpha)^2 B_2 + 2\alpha(1-\alpha)C. \qquad (142)$$

Deriving with respect to α*, we obtain:*

$$\frac{d\left\|\hat{X}^{est} - x^*\right\|^2}{d\alpha} = 2\alpha A + 2(\alpha-1)B_1 + 2(\alpha-1)B_2 + 2(1-2\alpha)C, \qquad (143)$$

$$\frac{d^2\left\|\hat{X}^{est}-x^*\right\|^2}{d\alpha^2}=2A+2B_1+2B_2-4C\,. \tag{144}$$

Given that $\langle a,b\rangle=\|a\|\|b\|\cos(a,b)\leq\|a\|\|b\|$, *we can find an upper bound on the scalar product of the biases:*

$$\begin{aligned}&\left\langle x^b-x^*,\mathrm{E}\left[\hat{X}\right]-x^*\right\rangle\leq\left\|x^b-x^*\right\|\left\|\mathrm{E}\left[\hat{X}\right]-x^*\right\|,\\&C\leq\sqrt{AB_1}\end{aligned} \tag{145}$$

$$\frac{d^2\left\|\hat{x}^{est}-x^*\right\|^2}{d\alpha^2}\geq 2A+2B_1+2B_2-4\sqrt{AB_1}\,. \tag{146}$$

From Jensen's inequality we can infer:

$$\frac{A+B_1}{2}\geq\sqrt{AB_1} \tag{147}$$

Consequently, the second derivative of the squared error is necessarily positive and, therefore, the extreme point is a minimum:

$$\frac{d^2\left\|\hat{X}^{est}-x^*\right\|^2}{d\alpha^2}\geq 0\,. \tag{148}$$

The minimum is:

$$\frac{d\left\|\hat{X}^{est}-x^*\right\|^2}{d\alpha}=2\alpha A+(2\alpha-2)(B_1+B_2)+(2-4\alpha)C=0\,. \tag{149}$$

This results in the following optimal value:

$$\alpha^*=\frac{B_1+B_2-C}{A+B_1+B_2-2C}\,. \tag{150}$$

For the combination to bring any advantages, the optimal shrinkage parameter must take a value within the interval [0,1], with $\alpha^*=0$ meaning that the optimal estimation is actually the SAA solution and $\alpha^*=1$ meaning that the optimal estimation is the robust benchmark.

Proposition 1

The optimal linear combination parameter takes a value in the interval]0,1[if the following sufficient conditions are met:

$$A-B_1<2B_2\,. \tag{151}$$

$$A>B_1\,. \tag{152}$$

A proof of this proposition is provided below, in italics. The reader can skip this section without any damage to the understanding of the rest of the chapter.

Proof:

For the optimal value to represent a valid convex linear combination parameter it must be reached in the interval]0,1[. We obtain sufficient conditions for this. Given the described bound on C, the denominator is strictly positive:

$$A + B_1 + B_2 - 2C > 0. \quad (153)$$

The numerator is positive in the cases where:

$$\begin{gathered} B_1 + B_2 - C > 0 \\ C < B_1 + B_2 \end{gathered}. \quad (154)$$

Given that $C \leq \sqrt{AB_1} \leq \frac{A + B_1}{2}$*, a sufficient but not necessary condition for this is:*

$$\begin{gathered} C < \frac{A + B_1}{2} < B_1 + B_2 \\ A - B_1 \leq 2B_2 \end{gathered}. \quad (155)$$

This condition means that the robust benchmark must be sufficiently close to the optimal solution (the difference in the biases of the benchmark solution and the SAA solution must not be larger than twice the variance of the SAA solution). That is, the smaller the variance of the SAA solution, the better the robust benchmark should be for shrinkage to represent an advantage in terms of expected error.

The numerator is lower than the denominator in the following cases:

$$\begin{gathered} A + B_1 + B_2 - 2C > B_1 + B_2 - C \\ C < A \end{gathered}. \quad (156)$$

Given that $C \leq \sqrt{AB_1}$*, a sufficient (but not necessary) condition for this to happen is to have:*

$$A > B_1. \quad (157)$$

The first condition means that the robust benchmark must not be too far from the optimal solution. In particular, the difference of the biases of the robust benchmark and the SAA solution must not be larger than twice the variance of the SAA solution. That is, the smaller the variance of the SAA solution, the better the robust benchmark must be for linear shrinkage to represent an advantage. The second condition simply states that the bias of the SAA solution must be lower than the bias of the robust benchmark.

We estimate the optimal shrinkage parameter α^* by solving several instances of the SAA problem. Then, we evaluate them out-of-sample. The optimal solution is estimated as the SAA solution with the lowest out-of-sample cost. Assuming we use t samples of s scenarios $x^j, j = \{1,2,..,t\}$, which for the sake of simplicity will be referred to as $x^{stoch\ j}, j = \{1,2,..,t\}$.

The optimal solution is estimated as the one that provides the lowest out-of-sample cost.

The estimation of the optimal value of α^* then results:

$$\begin{aligned}
&\hat{A} := \left\| x^b - \hat{x}^{opt} \right\|^2, \\
&\hat{B}_1 := \left\| \mathrm{E}\left[\hat{x}^{stoch}\right] - \hat{x}^{opt} \right\|^2, \\
&\hat{B}_2 := \frac{1}{t-1} \sum_{j=1}^{j=t} \left\| \hat{x}_j^{stoch} - \mathrm{E}\left[\hat{x}^{stoch}\right] \right\|^2, \\
&\hat{C} := (x^b - \hat{x}^{opt})^T (\mathrm{E}\left[\hat{x}^{stoch}\right] - \hat{x}^{opt}).
\end{aligned} \tag{158}$$

$$\hat{\alpha}^* = \frac{\hat{B}_1 + \hat{B}_2 - \hat{C}}{\hat{A} + \hat{B}_1 + \hat{B}_2 - \hat{2}C}.. \tag{159}$$

Linear shrinkage is not applicable to problems with discrete variables. Given the prevalence of this kind of problems, which include TEP if discrete investments are considered, we propose to extend this definition to *discrete combinations.*

A linear combination x between vectors a and b, according to a linear combination parameter α is:

$$x = \alpha a + (1-\alpha) b\,. \tag{160}$$

This implies the following conditions on the norms of the vector differences between the starting vectors and the combination, as seen in Figure 5-3.

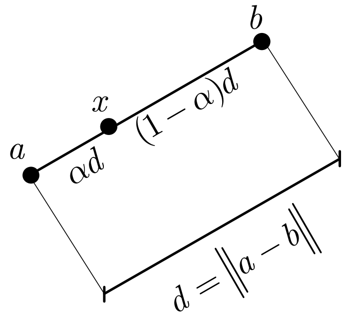


Figure 5-3 – Relationships between the norms of a linear combination and the individual vectors.

$$\begin{aligned}
\left\| x - a \right\| &= \alpha \left\| a - b \right\| \\
\left\| x - b \right\| &= (1-\alpha) \left\| a - b \right\|
\end{aligned}. \tag{161}$$

We use these conditions define the discrete combination. However, it is possible that no integral solution satisfies them. Therefore, we formulate an optimization problem that minimizes the deviations from these conditions:

$$\begin{aligned}
&\min d \\
&s.t \quad d \geq \varepsilon_1, d \geq \varepsilon_2 \\
&\qquad \varepsilon_1 \geq \left\| x - b \right\| - (1-\alpha) \left\| a - b \right\| \\
&\qquad \varepsilon_1 \geq (1-\alpha) \left\| a - b \right\| - \left\| x - b \right\|. \\
&\qquad \varepsilon_2 \geq \alpha \left\| a - b \right\| - \left\| x - a \right\| \\
&\qquad \varepsilon_2 \geq \left\| x - a \right\| - \alpha \left\| a - b \right\| \\
&\qquad x \in \left\{0,1\right\}
\end{aligned} \tag{162}$$

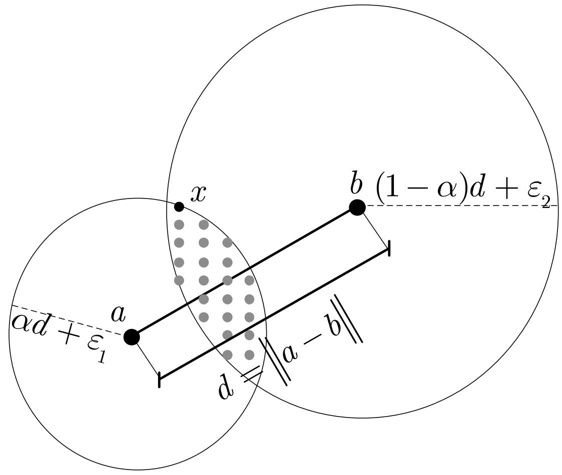


Figure 5-4 – Graphical representation of a discrete combination. It imposes the same relationships between the norms of the combination and the individual vectors than in the linear case. If there is no integer solution, it minimizes the deviations with respect to these conditions.

5.3.3. Norm-Constraint Shrinkage

Norm-constraint shrinkage guarantees that the estimator is not too far away from the robust benchmark by imposing a maximum distance, as represented in Figure 5-5:

$$\left\|\hat{x}^{est} - x^{b}\right\| < \delta\,. \tag{163}$$

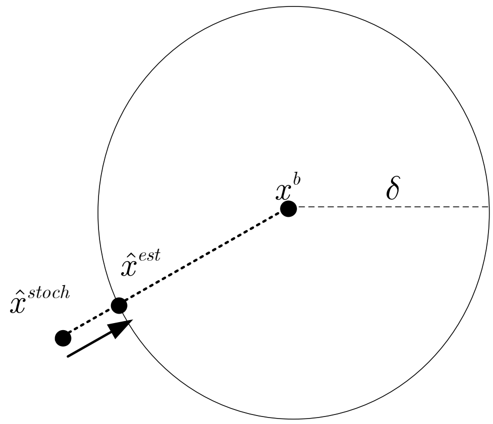


Figure 5-5 – Graphical representation of norm-constraint shrinkage. We limit the distance between the estimator and the robust benchmark.

We determine the optimal distance parameter by cross-validation, one of the most widely used methods for estimating prediction error. This technique estimates out-of-sample error as the average generalization error [225]. Cross-validation partitions the available data into two complementary subsets, the *training set* and the *test set*. We adjust the model using the training set and validate it using the test set. The parameters that give the best results over the test sets are the ones are selected.

We calculate out-of-sample performance as the cost of the solution obtained when optimizing the SAA problem over the training set of scenarios, evaluated in the scenarios of the test set. We define average out-of-sample performance as the average cost obtained across several of these training and test sets. Then, the optimal distance parameter is the one that provides the best average out-of-sample performance. A similar approach is followed in reference [226].

The rest of this chapter presents the developed case studies. First, an academic network illustrates the applicability of the proposed approach. Then, a case study based on the Spanish system demonstrates the potential of regularization applied in a realistic TEP context.

5.3.4. The Academic Network

The academic network is based on the well-known Garver system [36]. This network is composed of six nodes where load and generation can be located. The details of the network are described in the appendix. The case study describes a situation where there are two areas with a high potential for new renewable generation. However, how much new capacity will be installed at each area is uncertain. This situation is representative of real TEP studies where ambitious generation projects could materialize partly or in full.

To be able to explore the consequences of using scenario samples instead of the complete scenario tree, we will assume that we know the complete tree, although this is not the case in real problems. We model installed capacity as a uniform distribution:

$$\begin{array}{l} \overline{G}_2(u) = gb_2 \, \mathrm{U}[(1-mv)\,\mathrm{E}(\overline{G}_2),(1+mv)\,\mathrm{E}(\overline{G}_2)] \\ \overline{G}_3(u) = gb_3 \, \mathrm{U}[(1-mv)\,\mathrm{E}(\overline{G}_3),(1+mv)\,\mathrm{E}(\overline{G}_3)] \end{array}. \tag{164}$$

where:

$\mathrm{E}(\overline{G}_2),\mathrm{E}(\overline{G}_3)$: expected value of generation capacities. This takes a value of 200 MW in the case study.

mv: maximum variation. In order to study the impact of uncertainty, the case study considers several values for this parameter: 50% (low uncertainty), 100% (moderate uncertainty) and 150% (high uncertainty). We create 100 generation scenarios based on this distribution.

We solve a version of the problem that allows continuous investment as well as the discrete one. The use of continuous variables can be reasonable in TEP when the planner simplifies a large area into a reduced network. In these cases, individual decision variables do not represent individual transmission lines but aggregate capacities between areas.

In order to apply shrinkage to the problem, we must define the norm and the robust benchmark. We should develop the definition of distance having into account the characteristics of the problem as well. In general, we can use norms of the following form:

$$\left\|x^{*}-x^{b}\right\|_{n}=\left(\sum_{ijc}\left|x_{ijc}^{*}-x_{ijc}^{b}\right|^{n}\right)^{\frac{1}{n}}. \tag{165}$$

Where n determines the order of the norm. In order to keep the linearity of the problem, we will use $n=1$. If first-stage decisions are binary, this norm coincides with the Hamming distance.

However, if first-stage decisions are not equally important, we should capture their relative significance:

$$\left\|x^{*}-x^{b}\right\|_{n}=\left(\sum_{ijc}w_{ijc}\left|x_{ijc}^{*}-x_{ijc}^{b}\right|^{n}\right)^{\frac{1}{n}}. \tag{166}$$

In the case of TEP, not all first-stage decisions are equally important; some transmission lines are more expensive than others. What is more, lines that are more expensive are usually longer and with a higher capacity, so their impact on system operation is generally higher. Therefore, we take a weighted approach to the definition of distance, where weights correspond to first-stage costs:

$$w_{ijc}=Ic_{ijc}. \tag{167}$$

We use this definition of norm in both case studies.

The benchmark should be robust in the sense of not depending on the details of the scenario set. For instance, we can use the solution of a deterministic problem for the average scenario. In general, the most appropriate benchmark definition should have into account the particular characteristics of the problem under study. If underinvestment is preferred to overinvestment, the solution that provides with the minimum first-stage cost can be an interesting robust benchmark. We can calculate it as the solution of a deterministic version of the problem where the uncertain parameters take either their upper or lower bounds depending on what results in the lowest investment. Conversely, if the problem favors solutions with overinvestment, we can use the solution of a deterministic problem where the uncertain parameters take the bounds that result in the lowest operation costs (and highest investment). TEP does not favor any of these options clearly. What is more, which side is favored will depend, for instance, on the penalty value given to ENS or curtailment. If this parameter is very high, the robust benchmark should focus on situations with overinvestment in transmission lines (we make sure that ENS and curtailment will be low). If this parameter is low, then underinvestment would be preferred. As we do not know beforehand which of this situation is closer to the reality of the case studies, we test several alternatives for the robust benchmark:

Therefore, we test several alternatives for the robust benchmark:

- The optimal solution for a deterministic problem with minimum generation, referred to as *minimum-generation benchmark*.
- The optimal solution for a deterministic problem where the capacities take their expected value, referred to as *expected value benchmark*.
- The optimal solution for a deterministic problem where generators take their maximum capacity, referred to as *maximum-generation benchmark*.

We develop four instances of the case study to test linear and norm-constraint shrinkage for discrete and continuous variables. We use out-of-sample performance to evaluate the stability of the resulting transmission expansion plan with respect to changes in the scenario tree. We measure it in two different ways: as average expected cost and as average squared error of the plan compared to the optimal solution of the complete stochastic problem. In all cases, reported results correspond to 25 samples of 5 scenarios from a total of 100 scenarios.

Uncertainty	α	Minimum-generation benchmark Avg cost [M€]	Minimum-generation benchmark Avg sq error decision [p.u.]	Expected-generation benchmark Avg cost [M€]	Expected-generation benchmark Avg sq error decision [p.u.]	Maximum-generation benchmark Avg cost [M€]	Maximum-generation benchmark Avg sq error decision [p.u.]
Low	0	31.2271	1.1037	31.2271	1.1037	31.2271	1.1037
	0.01	31.1402	1.0711	31.1909	1.1046	31.147	1.1102
	0.05	30.8689	0.9788	31.049	1.115	30.8853	1.1903
	0.1	30.802	0.9493	30.89	1.1433	30.7637	1.4122
	0.2	31.1262	1.1761	30.6585	1.251	31.0107	2.2619
	0.3	31.5792	1.7842	30.5428	1.4268	31.5695	3.6527
	0.4	32.0763	2.7735	30.5169	1.6707	32.2773	5.5847
	0.5	32.5995	4.1441	30.5365	1.9826	33.0853	8.0579
	0.6	33.1308	5.896	30.5862	2.3627	33.9667	11.0722
	0.7	33.6645	8.0292	30.645	2.8108	34.8763	14.6277
	0.8	34.2006	10.5436	30.7156	3.327	35.7892	18.7243
	0.9	34.7381	13.4393	30.7931	3.9113	36.7022	23.3621
	0.95	35.0072	15.0301	30.8335	4.2289	37.1587	25.8839
	0.99	35.2224	16.3714	30.8667	4.4953	37.5239	27.9988
	1	35.2763	16.7163	30.8752	4.5636	37.6152	28.5411
Medium	0	59.0243	4.2518	59.0243	4.2518	59.0243	4.2518
	0.01	58.3274	4.2442	58.791	4.1659	58.4124	4.1521
	0.05	55.6796	4.2455	57.9164	3.8368	56.1125	3.8095
	0.1	52.6263	4.3183	56.9076	3.4583	53.4718	3.5076
	0.2	47.2948	4.7012	55.1975	2.8108	48.9433	3.3245
	0.3	43.071	5.4006	53.9528	2.3095	45.4084	3.7026
	0.4	39.9667	6.4164	53.1708	1.9541	42.9635	4.642
	0.5	37.8374	7.7488	52.9531	1.7449	41.5049	6.1425
	0.6	36.5163	9.3976	53.4417	1.6817	40.8096	8.2042
	0.7	35.9088	11.3628	54.4445	1.7646	40.6535	10.8271
	0.8	35.8066	13.6446	55.8502	1.9935	40.8264	14.0112
	0.9	36.0251	16.2428	57.7684	2.3685	41.5006	17.7565
	0.95	36.2219	17.6606	58.8892	2.6108	42.0966	19.8396
	0.99	36.4048	18.8517	59.7949	2.8309	42.5894	21.607
	1	36.4517	19.1575	60.0227	2.8896	42.7172	22.0629
High	0	71.612	6.3356	71.612	6.3356	71.612	6.3356
	0.01	70.93	6.1678	71.4764	6.2818	71.0278	6.3425
	0.05	68.2739	5.5254	71.0021	6.0848	68.7691	6.431
	0.1	65.0751	4.7873	70.5088	5.88	66.1042	6.6785
	0.2	59.1536	3.5277	69.9706	5.6086	61.2665	7.63
	0.3	53.7811	2.5568	70.0431	5.5217	57.1332	9.1903
	0.4	49.0836	1.8745	70.6306	5.6191	53.7694	11.3592
	0.5	45.265	1.481	71.8255	5.9009	51.4274	14.1369
	0.6	42.1805	1.3761	73.6697	6.3671	50.0206	17.5233
	0.7	39.8048	1.5599	76.0184	7.0177	49.5647	21.5183
	0.8	38.0989	2.0324	79.0167	7.8526	49.9567	26.1221
	0.9	37.1285	2.7935	82.636	8.8719	51.6129	31.3347
	0.95	36.9208	3.2824	84.6584	9.4507	53.444	34.1692
	0.99	36.9462	3.7254	86.3544	9.9469	55.1129	36.5464
	1	36.9787	3.8434	86.7824	10.0755	55.5334	37.1559

Table 5-3 – Garver system. Results for linear shrinkage with continuous variables. α represents the linear shrinkage intensity parameter.

Uncertainty	δ	Minimum-generation benchmark Avg cost [M€]	Minimum-generation benchmark Avg sq error decision [p.u.]	Expected-generation benchmark Avg cost [M€]	Expected-generation benchmark Avg sq error decision [p.u.]	Maximum-generation benchmark Avg cost [M€]	Maximum-generation benchmark Avg sq error decision [p.u.]
Low	INF	33.0618	8.362	33.0618	8.362	33.0618	8.362
	15	33.0618	8.362	33.0618	8.362	33.0618	8.362
	12	32.9726	6.3615	33.0618	8.362	33.1033	10.8026
	11	31.8718	2.8806	33.0618	8.362	33.1033	10.8026
	10	31.776	0	33.0773	7.0416	33.2429	13.0433
	9	31.776	0	33.0773	7.0416	33.3746	17.0041
	8	31.776	0	33.0773	7.0416	33.3746	17.0041
	7	34.7045	7.3216	32.5807	6.9215	33.3746	17.0041
	6	34.7044	7.0015	32.5807	6.9215	34.1646	26.0063
	5	35.1043	10.6022	32.5807	6.9215	34.1646	26.0063
	4	34.7761	13.003	31.8718	2.8806	34.1646	26.0063
	3	34.7761	13.003	32.0753	9.0019	36.8934	40.4101
	2	36.7767	17.0052	31.159	9.0019	37.1651	35.0092
	1	36.7767	17.0052	32.0753	9.0019	37.1651	35.0092
	0	36.7767	17.0052	32.0753	9.0019	37.1651	35.0092
Medium	INF	53.1546	10.2423	53.1546	10.2423	53.1546	10.2423
	15	53.1546	10.2423	53.1546	10.2423	53.2331	9.5222
	12	51.2999	11.3226	53.1546	10.2423	46.5679	7.0017
	11	47.9697	10.1223	53.1546	10.2423	45.4526	4.9212
	10	47.9713	12.0027	45.5836	9.9222	46.342	9.1624
	9	50.2142	12.3628	45.3298	9.2021	44.5629	8.4423
	8	43.9182	13.243	45.2737	8.3219	44.2607	10.0827
	7	40.9966	15.8838	43.4946	7.6017	46.5113	14.724
	6	40.896	15.5637	43.2208	7.0416	42.4094	15.0841
	5	41.8873	21.2449	43.3616	5.8013	43.2897	18.6854
	4	37.0565	21.5651	38.7091	5.6813	44.4678	23.4867
	3	37.0565	21.5651	35.7902	0.9603	47.3191	25.2871
	2	37.9521	26.007	35.6896	0.6402	44.5984	24.567
	1	37.9521	26.007	35.5258	0	65.1171	26.0073
	0	37.9521	26.007	35.5258	0	65.1171	26.0073
High	INF	60.7942	9.5976	60.7942	9.5976	60.7942	9.5976
	15	60.7942	9.5976	60.7942	9.5976	60.905	12.0832
	12	60.7268	7.8371	60.7942	9.5976	55.218	16.3244
	11	57.8834	8.3973	60.7942	9.5976	54.3118	16.5244
	10	53.9645	3.6184	55.8859	9.2375	52.0975	19.8053
	9	56.6793	4.2986	55.8505	8.4998	48.8213	20.5255
	8	50.1684	4.5386	55.7438	8.98	45.6179	21.9258
	7	46.2781	3.1606	52.4677	9.7002	47.7874	25.9269
	6	46.1914	2.8406	48.5683	7.762	44.712	26.6471
	5	47.8263	8.4824	48.8465	14.3811	44.9976	31.8888
	4	40.7422	8.1623	42.5819	13.5409	48.3324	35.3698
	3	40.7422	8.1623	39.8935	10.9225	49.7197	37.3303
	2	38.4791	13.0037	39.8068	10.6025	46.2657	36.6101
	1	38.4791	13.0037	43.1322	13.0033	91.4528	39.0106
	0	38.4791	13.0037	43.1322	13.0033	91.4528	39.0106

Table 5-4 – Garver system. Results for norm-constraint shrinkage and discrete variables. δ represents the maximum distance allowed between the solution and the robust benchmark.

			A	B1	B2	C	Alpha *
Continuous - Linear combination	Minimum-generation benchmark	Low uncertainty	17.00	0.32	0.81	-0.62	**0.09**
		Medium uncertainty	19.00	2.16	2.18	3.79	**0.03**
		High uncertainty	3.84	2.73	3.76	-2.13	**0.59**
	Expected-generation benchmark	Low uncertainty	4.56	0.32	0.81	1.13	**0.00**
		Medium uncertainty	2.89	2.16	2.18	-0.08	**0.60**
		High uncertainty	10.00	2.73	3.76	3.60	**0.31**
	Maximum-generation benchmark	Low uncertainty	29.00	0.32	0.81	1.29	**-0.01**
		Medium uncertainty	22.00	2.16	2.18	-0.87	**0.19**
		High uncertainty	37.00	2.73	3.76	6.53	**0.00**

Table 5-5 – Garver system. Optimal alpha parameter and intermediate magnitudes for linear shrinkage with continuous variables. These magnitudes are defined in Theorem 1.

Table 5-3 shows the results for linear shrinkage with continuous variables and Table 5-4 displays the results for norm-constraint shrinkage with discrete variables. Table 5-5 shows the calculation of the optimal shrinkage parameter for linear shrinkage with continuous variables. For the sake of brevity, the rest of the results appear in the appendix section. Also in the appendix section, we provide the optimal solution to the complete scenario tree for illustration purposes, although in general it is not possible to solve this problem. As can be observed, higher levels of uncertainty usually result in higher total costs.

The proposed approach achieves substantial reductions in decision error and expected cost. Both linear and norm-constraint shrinkage perform well, although norm-constraint shrinkage seems to provide slightly better results in terms of out-of-sample cost. The three benchmarks considered gave relatively good results, although the expected-generation benchmark is arguably more consistent across problem instances.

In addition, our estimation of the optimal shrinkage parameter fits nicely with the experimental results (to see this, compare our estimation in Table 5-5 and the best value obtained in Table 5-3). Higher values of the optimal shrinkage parameter (that is, a higher proportion of robust benchmark in the optimal estimator) seem to correspond to higher reductions in decision error. We predicted accurately the optimal shrinkage parameter for the three levels of uncertainty considered. The intermediate calculations for this optimal shrinkage parameter are interesting in themselves. As seen in Table 5-5, the bias of the SAA solution and its standard deviation grows with uncertainty, shrinkage will provide higher benefits in problems with higher uncertainty. On the contrary, the studied benchmarks behave differently with respect to uncertainty. The bias of the minimum-generation benchmark decreases quickly as uncertainty rises, while the maximum-generation benchmark displays the opposite behavior. Their quality as benchmarks therefore depends on the extreme scenarios. The expected-generation benchmark shows an intermediate behavior, so it is expected to perform better in a wider range of situations.

5.3.5. The Spanish Case Study

We develop a case study based on the Spanish network to demonstrate the proposed approach in a realistic context. We take transmission data as of 2008 from publicly available ENTSO-e and REE E-SIOS cases [218]. We use the same E-SIOS cases to build forecasts for a peak-demand scenario and generation capacities. The system is composed of 1084 nodes and 294 power plants (nuclear, coal, CCGTs, hydro, wind and solar). The existing network includes 1505 lines. The case assumes that a large amount of relatively inexpensive renewable energy could be available for import from North Africa. This is consistent with visions such as the Desertec Industrial Initiative). In addition, Spain would have to export an uncertain amount of renewable generation power to France through interconnections at the North-East and the North-West of the country. The amounts imported and exported would be:

$$\begin{aligned} I &= \mathrm{U}[0,\overline{I}] \\ E_1 &= \mathrm{U}[0,\overline{E_1}] \,, \\ E_2 &= \mathrm{U}[0,\overline{E_2}] \end{aligned} \tag{168}$$

where:

$\overline{I}$: maximum imported power. This takes a value of 5 GW in the case study,

$\overline{E_1}, \overline{E_2}$: maximum exported power though the North-East and the North-West. These take a value of 2.5 GW.

The results obtained from the Garver system led us to use the solution to the expected value problem as the robust benchmark. We obtained this solution solving the deterministic problem assuming that 5 GW were available for import and that the export needs were exactly 5 GW, equally split between North-East and North-West.

This case study studies a real, detailed network and so it considers specific transmission lines as opposed to corridors between areas. Therefore, the model should use discrete variables. As in the Garver system case study, reported results correspond to 25 samples of 5 scenarios from a total of 100 scenarios. Table 5-6 shows the results for discrete variables and norm-constraint shrinkage. Additional results can be found in the appendix section.

The case study considers an uncertain amount of power entering the country from the South and exiting at the North-East and North-West. These amounts vary across scenarios, so that different scenario samples in the SAA problem lead to different network reinforcement needs. If we understand a project as a set of transmission lines, possibly disconnected, that serve the same purpose, the network reinforcement needs in the case study essentially consist in strengthening corridors from South to North-East and from South to North-West. The robust benchmark is the optimal expansion plan for the expected value problem and installs a moderate amount of reinforcements along both power routes.

δ :	Avg cost [bln €]	Avg sq error decision [p.u.]
+INF	11.3	1.0007
8.9814	11.4	1.0072
6.736	10.6	0.8799
4.4907	10.7	0.8963
3.368	11.3	0.9992
2.2453	11.1	0.9517
2.0208	11.1	0.9517
1.7963	11.2	0.9814
1.5717	11.2	0.9814
1.3472	11.2	0.9814
1.1227	11.2	0.9814
0.8981	11.4	1.0128
0.6736	11.4	1.0128
0.4491	11.4	1.0128
0.2245	11.4	1.0128

Table 5-6 – Realistic Network. Results for norm-constraint shrinkage and discrete variables. δ represents the maximum distance allowed between the solution and the robust benchmark.

Regularization avoids extreme expansion plans with either very large or very low investment and favors balanced solutions with moderate reinforcements along both corridors. This leads to expansion plans that are considerably more robust with respect to changes in the scenarios: the regularized plans are up to 7% better than the SAA plans in terms of out-of-sample cost (10,600 vs. 11,300). This example shows that the proposed approach can indeed be useful when dealing with realistic TEP problems.

5.3.6. Conclusions

The scenario trees used to model the dynamic uncertainties that impact TEP are based largely on expert opinion. Therefore, it is desirable to obtain expansion plans that are relatively stable with respect to changes in the definition of scenarios.

We propose to regularize the SP to obtain solutions that are more robust by using shrinkage. Shrinkage estimators improve the error of a given estimator by combining it with another one, typically one with a lower variance. We shrink the SAA solution towards a robust benchmark that is relatively independent from the scenario tree. We propose two alternative techniques. Linear shrinkage creates a combination of the SAA solution and the robust benchmark. We prove the existence of an optimal shrinkage parameter and provide an expression for its value. Norm-constraint shrinkage imposes a maximum distance between the SAA solution and the robust benchmark. We select the best distance parameter by cross-validation.

We develop two TEP case studies to illustrate this approach, an academic network based on the well-known Garver system and a realistic one based on Spanish power system. Both cases are representative of the uncertainties inherent to new generation projects. We study continuous and discrete versions of the problem for both linear and norm-constraint shrinkage. The proposed approach results in expansion plans that are notably more robust with respect to changes in the scenarios, with average out-of-sample cost reductions of up to 7% in the realistic case study, which illustrates the usefulness of regularization in this context. Discrete problems benefit more from regularization than continuous ones. Although both shrinkage approaches produce improvements, norm-constraint shrinkage performs better in all the problem instances studied.

5.4. Chapter Takeaways

TEP is a problem characterized by the presence of large dynamic uncertainties, such as generation expansion or fuel prices. The long permitting process for building a new line (around 10 years) means that transmission expansion must anticipate the evolution of these uncertainties. Therefore, it is extremely important to develop flexible plans that are able to adapt to the unveiling of events. In addition, these uncertainties are modeled by expert opinion, which means that forecasts are necessarily limited. This makes it desirable to design expansion plans that are robust with respect to changes in the definition of scenarios.

We identify the transmission lines with the highest potential for bringing operation cost savings in the future. The permitting process for these promising transmission lines should be initiated, even though not all lines will be built once the permits are granted. We apply Real Options Valuation (ROV) to evaluate the potential benefit of candidate lines and identifying priority projects. We develop a simplified interpretation of optionality in TEP and calculate an approximation that enables to evaluate option value in realistic settings. We are able to identify the candidate transmission lines with the highest potential and the ones that are essential to an expansion plan, as well as their main value drivers. We illustrate this with a realistic case study based on the Spanish system.

In addition, we propose a technique to obtain expansion plans that are robust with respect to changes in the scenario tree. We regularize the SAA problem shrinking the solution towards a robust benchmark. We essentially modify the TEP solution to make it more similar to a plan that does not depend on the scenario tree. We develop two different alternatives to apply shrinkage to the SAA problem: linear shrinkage and norm-constraint shrinkage. We produce two case studies to illustrate the advantages of the proposed approach, based on an academic and a realistic network respectively. Shrinkage provides results that are consistently better in terms of average out-of-sample cost and decision error.

The Real Options approach for TEP is under review:

- S. Lumbreras, A. Ramos, D. Bunn, M. Chronopoulos, "Real Options Valuation Applied to Dynamic Transmission Expansion Planning", under review at IEEE Transactions on Power Systems.

The regularization approach for SP will be submitted for publication as:

- S. Lumbreras, V. DeMiguel, A. Ramos, "Transmission Expansion Planning. A Regularized Stochastic Programming Approach", to be submitted for publication to Operations Research.

Chapter 6

Structure Identification

6.1. A Matter of Structure

The study of the structure of the problem is essential in large-scale TEP. To begin with, the combinatorial nature of the problem makes it impossible to take into account all possible transmission investments. Therefore, the candidate investment proposal formerly carried out by the TSO played the crucial role of reducing an unmanageably large search space. However, there is currently no single TSO with experience of the whole system under planning. Hence, there is a need for automatic candidate proposal mechanisms. Moreover, when the TSO proposed investments, each candidate line was linked to a specific network need; the rationale for investments and their relationships was known. However, if we perform proposals automatically, this information is lost. We propose an automatic, objective method for candidate proposal, and a power-flow based approach for extracting the rationale and relationships among investments. We illustrate the proposed approach with a TEP case study based on the Spanish Power system.

In addition, although a TEP problem might consider a huge number of possible candidate investments, often just a few of them have the largest impact. Identifying these investments gives extremely interesting information about the structure of the network. We apply Ordinal Optimization (OO) with the aim of extracting the most significant investments in the optimization problem. Moreover, we exploit this information to solve larger problems in shorter computation times. For this, we develop a hybrid optimization algorithm that combines OO and MIP. We demonstrate the advantages of this approach in a case study based on OWF design.

We published the candidate proposal approach as:

- S. Lumbreras, A. Ramos, P. Sánchez, "Automatic Selection of Candidate Investments for Transmission Expansion Planning", International Journal of Electrical Power and Energy Systems, vol. 59, pp. 130-140, July 2014.

We describe the OO+MIP hybrid in:

- S. Lumbreras, A. Ramos, P. Sánchez, "Offshore Wind Farm Electrical Design using a Hybrid of Ordinal Optimization and Mixed Integer Programming", under review at Wind Energy.

6.2. Candidate Discovery

6.2.1. Introduction

We deal explicitly with the problem of proposing candidate lines (sometimes referred to as technical alternatives) for TEP. Candidate lines have been traditionally regarded as expert-provided system information. However, given the need to plan larger networks, identifying interesting candidates is an issue of increasing relevance and complexity. We propose a consistent method to tackle this problem.

First, an automatic and objective candidate discovery mechanism based on sensitivities proposes potentially interesting investments. Then, a candidate management strategy filters the list of candidates to keep problem size within tractable levels without compromising global optimality. Finally, a candidate analysis tool reveals the relationships among investments from a relatively fast and simple power flow study. This information can be interesting to support expansion decisions. We complement these theoretical developments with a realistic case study that illustrates the applicability of the method.

TEP is a complex problem with ramifications that extend from system operation, static reliability and dynamic stability analyses to environmental and political considerations. For this reason, when confronted with this problem, TSOs tend to structure the decision process in several stages.

- Several alternative expansion plans are proposed. These proposals can be generated manually by the planner or obtained by optimization. In the latter case, problem size makes it often necessary to simplify its scope by using a reduced model of the system and considering only some of the relevant objectives. The general approach seems to be to consider system operation cost and static reliability [193]. These two objectives are the ones that have arguably the largest impact on cost and can be efficiently incorporated into an optimization problem.
- The alternative expansion plans are evaluated, this time, across all the relevant objectives. These can include stability and short-circuit currents, flexibility, social acceptance or environmental impact [227].
  The decision maker selects his preferred expansion plan from the list of alternative expansion plans that score acceptably in his objectives. Multi-Criteria Decision Methods can be used to guide this selection [228].

When the alternative expansion plans are generated by optimization, the problem is generally understood as the selection of the best combination of individual investments from a pre-defined list of candidate lines. Usually, this list is considered part of the input data of the problem.

- Academic studies assume candidates are provided externally.
- Practical applications, usually carried out by the relevant TSO, make use of the TSO's own knowledge and experience about the system. Candidate lines are defined manually according to the planner's expertise.

However, this traditional approach can lead to fundamental problems that are becoming increasingly relevant.

First, it is possible that the planner's expertise fails to identify candidate lines with a better potential than the ones actually included in the list, leading to a suboptimal final solution.

In addition, market integration means that there is a trend to plan increasingly large geographical areas, where new generation (particularly renewable) can be located far away from demand areas [229]. TSOs have experience on their individual systems, which can be very large, as for instance in the NERC regions [230]. However, when planning several regions coordinately, most often, there is no entity familiar with the whole area, and therefore large investments stretching among several territories could be overlooked. These investments constitute the backbone of supergrid architectures and are therefore extremely relevant for TEP in large regions. This is manifest in studies such as ENTSO-e's Ten-Year Network Development Plan (TYNDP) [231], which performs a bottom-up approach where investments are proposed by TSOs in the scope of their individual regions and are integrated at a later stage. A similar problem arises when the object of TEP is a completely new region with no existing network. An interesting example of this is the design of large OWFs.

It is also important to note that the inherent computational complexity of optimal TEP (usually formulated as a discrete combinatorial problem) makes it necessary to limit the size of the candidate list in order to keep the problem tractable. It is therefore impossible to consider all the feasible investments as a way to avoid the candidate selection problem. What is more, in very large systems, even a reduced list of interesting candidates (as opposed to all feasible candidates) can be excessive for optimization.

We propose a method to deal with these issues:

- Candidate discovery demands automatic, objective methods rather than manual processes relying on individual or institutional expertise.
- The potentially large number of interesting candidates should be managed in order to keep problem sizes tractable.
- When candidates are proposed by the planner, they can be easily related to specific needs in the existing network (e.g. integrating new generation or reinforcing a congested corridor). The relationships among them are therefore understood (e.g. alternative circuits that serve the same function or lines that complement each other to create a longer corridor). However, if candidates are proposed automatically, this information is no longer available. It would be desirable to recover as much of this knowledge as possible and make it available for decision support.

These identified needs, which the literature has not dealt with explicitly, are the three main points around which we structure our approach. We propose an automatic candidate discovery method. In addition, we develop an algorithmic candidate reduction algorithm that keeps problem sizes tractable while guaranteeing optimality. We have incorporated this algorithm into a Benders' decomposition framework efficiently, so that existing cuts are easily updated to incorporate the newly discovered candidates. Finally, a candidate analysis mechanism elicits the relationships among candidates with the aim of supporting decision-making. After introducing the proposed approach, we describe the case studies and discuss results.

6.2.2. The Proposed Approach

We base our approach on three complementary and sequential algorithms:

- *Automatic candidate discovery* searches for potentially interesting candidate lines using sensitivity information.

- *Candidate management* reduces problem size to tractable levels without compromising global optimality. It deals with large numbers of integer variables as well as reducing computation time.
- *Candidate analysis* reveals the relationships among the investments analyzed.

We base the developed approach in the formulation for the TEP problem presented in 3.2 and using Benders' decomposition as explained in 4.2.

6.2.3. Automatic Candidate Discovery

In addition to the lines directly proposed by the planner, we would like to add all other potentially interesting candidates to the optimization problem. It is not possible to incorporate all the potential candidate lines (that is, all the possible pairs of nodes and type of cable). Of course, not all possible transmission lines are feasible or reasonable. Some lines are, for instance, too long for their capacity according to the planner's criteria (for instance, we can impose that no AC line can be longer than 1000 km). Other filters can impose, for instance, social or environmental criteria (such as that no line can cross a natural reserve). In addition, different alternatives for the structure of TEP can be imposed by controlling the length of the candidates: an incremental grid would be characterized by relatively short candidates while *supergrid* investments would present a longer length and higher capacity.

However, even after this step the possible number of candidate lines leads in general to an intractable problem. We define candidate discovery as the selection of a subset of these suitable lines that we deem promising. Automatic candidate discovery should be based on objective and readily available information, ideally element characteristics and power flow data. Candidate discovery should identify a set of investments with potential to reduce operation costs and produce savings in excess of their combined costs.

Some references [81, 232, 233] have used sensitivities to guide heuristics determining the next step in a local search among a pre-defined list of candidates. These sensitivities give an upper bound on the potential benefit of installing a new line. We then compare the potential benefit to investment cost, which is calculated as the product of line cost per km and length:

$$c_{ijc} = CC_c GD_{ij} \qquad (169),$$

where:

CC_c: cost of a cable type [M€/km],

GD_{ij}: distance between line extremes adjusted for geographical accidents [km]. It is possible to modify this distance to include other considerations in the analysis, for instance, increasing it for some lines to represent environmental costs or social concerns.

In order to ensure that we include all promising candidates, we need to ensure that our approximation of potential benefits is optimistic. In addition, we need to apply the mechanism iteratively until no more potential candidates appear. This is described in Figure 6-1.

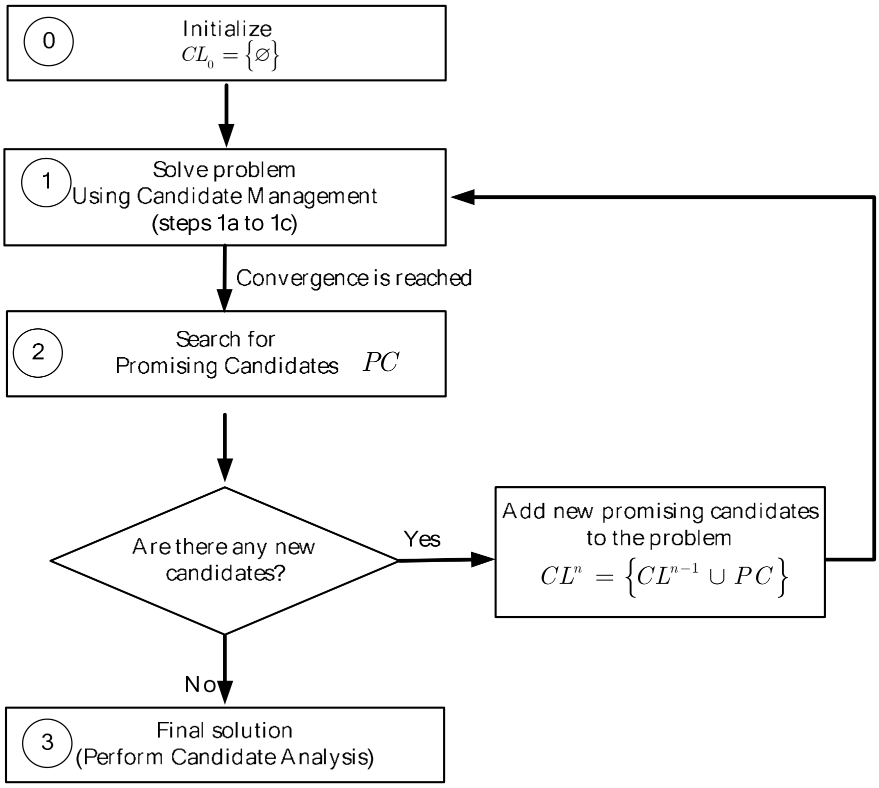


Figure 6-1 – Candidate discovery algorithm.

As the problem formulation is convex, we can use sensitivity information to derive an upper bound on benefits. In general, if Benders' decomposition is used, we can obtain cost sensitivities with respect to any transmission asset from the slopes of the cuts. In addition, if a transportation model is used, we get an upper bound on benefits from the current nodal price differences and the capacity of the line:

$$PB_{ijc} = \sum_{\omega} P^{\omega} D^{\omega} \left| \sigma_i^{\omega} - \sigma_j^{\omega} \right| \overline{F}_{ijc} \, , \tag{170}$$

where:

PB_{ijc}: potential benefit of installing a line [M€].

$\overline{F}_{ijc}$: rated power of the line [MW],

If a DCPF is used, a better approximation of the flow is given by the angle difference and the reactance of the circuit:

$$PB_{ijc} = \sum_{\omega} P^{\omega} D^{\omega} \left(\sigma_j^{\omega} - \sigma_i^{\omega} \right) \frac{\left(\theta_i^{\omega} - \theta_j^{\omega} \right)}{X_c GD_{ij}} \, , \tag{171}$$

where:

X_c: reactance per km of cable type c [pu/km].

Both calculations, the DCPF and the transportation one, require only information available from a power flow calculation of the existing network. It should be noted that all stochastic scenarios are taken into account in this calculation, in particular the different load levels and the reliability scenarios. Therefore, we can assess the potential benefit as the sum of these benefits per scenario weighted by the relevant probability and duration. This leads to recurrent congestions being weighted more than infrequent ones.

Once we have calculated the potential benefits, we identify promising candidates as the ones where the ratio between potential benefits and cost is higher than one:

$$BTC_{ijc} = \frac{PB_{ijc}}{ic_{ijc}} \geq 1. \tag{172}$$

For the sake of simplicity, we have only expressed these rates for transmission lines, which are arguably the most important type of transmission asset. However, we can build equivalent expressions for other types of transmission assets such as transformers. In addition and as explained above, we can obtain sensitivities from the slopes of Benders' cuts for any transmission asset in general.

It is interesting to note that, the more reinforcements the transmission network needs, the more promising candidates will appear. In case the promising candidate lines are too many, we can define a maximum number of promising candidates. The best candidates according to their potential benefit BTC_{ijc} will be included progressively.

Candidate discovery should include different cable types and technologies. In particular, this list should include the installation of new large-capacity HVDC links that could conform the backbone of a supergrid.

In order to ensure that we explore all potentially interesting candidates, we solve the TEP problem iteratively until no more promising candidates are identified. Figure 6-1 shows a schematic description of the candidate discovery process.

One of the advantages of using Benders' decomposition to solve the problem is that, if new candidate lines are added, we can still keep all the information about the problem that we have already calculated (namely, all the calculated Benders' cuts).

A cut gives a lower bound on second-stage cost as a linear function of first-stage variables. The cut is defined by a point $(x_{ijc}, Z_2^{\omega l})$ and some slopes $\pi^{+\omega l}_{ijc}, \pi^{-\omega l}_{ijc}$. The formulation of the master problem and the subproblem given in Chapter 3 is reproduced here for convenience.

$$\begin{aligned}
&\min_{x,\theta^\omega} c^T x + \sum_{\omega\in\Omega} \theta^\omega \\
&s.t. \quad Ax = b \\
&\qquad \theta^\omega \geq f^{\omega l} + \pi^{wlT} T(x^l - x) \quad l = 1,\ldots,j, \forall\omega' \\
&\qquad x \geq 0
\end{aligned} \tag{173}$$

$$\begin{aligned} f^{\omega j} &= \min_{y^\omega} d^T y^\omega \\ W y^\omega &= h^\omega - T x^j \qquad : \pi^{\omega j} \, . \\ y^\omega &\geq 0 \end{aligned} \tag{174}$$

The point relates a first-stage solution and its cost, and the slopes relate increments in first-stage variables with increments in second-stage cost. In classical Benders' decomposition, the first-stage solution used is the optimal solution of the master problem. The value used for the cost of this solution is its exact cost as calculated by the subproblem. The slopes that relate cost and first-stage variables are the exact marginal information calculated by the subproblem.

However, it is possible to build different cuts that are still valid, as long as the approximation of second-stage cost they provide is lower than or equal to the real cost:

- Any first- stage solution can be used to build the cut as long as the subproblem is used to evaluate second-stage cost and the slopes. This includes linearized solutions or suboptimal solutions (for instance, if we fix some first-stage variables).
- If any of the parameters of the cannot be obtained (for instance, the slopes), it is possible to assign them other values as long as we ensure that the approximation of second-stage cost that results from the cut is lower than or equal to the exact second-stage cost.

We refer the reader to the work in [121] for a more detailed description of Benders' decomposition and to [150] for a more detailed discussion on valid cuts.

The stylized formulation of Benders' decomposition applied to the TEP problem is reproduced below for the sake of convenience:

$$\begin{aligned} &\min \sum_{t,ijc \in cl} Ic_{ijc} Df_{tD} (x_{tijc} - x_{t-1,ijc}) + \sum_{\omega \in \Omega} P^{\omega_t} Dur^{\omega_t} \Theta_2^{\omega_t} \\ &s.t. \;\; \Theta_2^{\omega_t} \geq Z_2^{\omega_t l} + \sum_{tijc} \Big[\overline{F}_{tijc} (\pi^{+\omega_t l}_{tijc} - \pi^{-\omega_t l}_{tijc}) - M_{ijc} (\rho^{+\omega_t l}_{tijc} - \rho^{-\omega_t l}_{tijc}) \Big] (x^l_{ijc} - x_{ijc}), \\ &l = 1, \ldots, n \end{aligned} \tag{175}$$

where:

$\Theta_2^{\omega_t}$: recourse variables,

$\pi^{+\omega_t l}_{tijc}, \pi^{-\omega_t l}_{tijc}$: dual variables of the equations imposing maximum and minimum power flows through lines,

$\rho^{+\omega_t l}_{tijc}, \rho^{-\omega_t l}_{tijc}$: dual variables of the equations imposing Kirchhoff's Second Law,

x^l_{tijc}: first-stage variables selected by the master problem at iteration l .

The subproblem calculates operation cost for the investment variables fixed by the master problem and obtains the parameters needed for a new cut:

$$Z_2^{\omega_t n} = \min_{g_i^{\omega_t}, pns_i^{\omega_t}} \sum_i g_{ti}^{\omega_t} cg_{ti}^{\omega_t} + pns_{ti}^{\omega_t} Pnsc \, . \tag{176}$$

We can use the cuts calculated in previous iterations because the subproblem is the same in all the defined problems (system operation is identical). The only modification needed is to supplement all already calculated cuts with valid slopes for the new variables.

Calculating $\pi_{ijc}^{+\,\omega}, \pi_{ijc}^{-\,\omega}, \rho_{ijc}^{+\,\omega}, \rho_{ijc}^{-\,\omega}$ for the new candidates in all the already calculated cuts would imply solving again the subproblems for all previously calculated iterations, this time including the new candidates. However, we can avoid this if, instead of calculating the exact slopes, we use some conservative values that make sure that the cuts obtained are valid.

Any definition of the slopes that guarantees that the cut lies below the recourse function originates a valid Benders' cut. That is, operation cost evaluated using the cut must be lower than or equal to the true cost. A relatively straightforward way to introduce a new variable is to define the cut such that:

- If the new candidate $i'j'c'$ is not installed, the previously calculated cut is left unchanged.
- If the new candidate is installed indeed, the recourse function is equal to a known lower bound on total cost regardless of the remaining investment decisions. We have a lower bound that will always be valid: zero, as we know that cost must be positive in this problem.

A proposed value for the slopes using a lower bound of zero is:

$$\begin{aligned}&\pi^{+\omega l}_{i'j'c'} = \frac{1}{\overline{f}_{i'j'c'}}\left[Z_2^{\omega l} + \sum_{ijc} x^l_{ijc}\left(\overline{f}_{ijc}(\pi^{+\omega l}_{ijc} - \pi^{-\omega l}_{ijc}) - M_{ijc}(\rho^{+\omega l}_{ijc} - \rho^{-\omega l}_{ijc})\right)\right] \\ &\pi^{-\omega l}_{i'j'c'} = \rho^{+\omega l}_{i'j'c'} = \rho^{-\omega l}_{i'j'c'} = 0 \\ &ijc \in CL, ijc \neq i'j'c'\end{aligned} \quad . \qquad (177)$$

This expression enables adding new candidates while keeping all the previously accumulated information.

Similarly, the candidate management technique described in the next section can benefit from the decomposition structure as well.

Figure 6-1 describes the iterative application of candidate discovery, keeping all previously calculated cuts and using candidate management (described below) to solve the problem more efficiently if a large number of candidates are identified. The loop is repeated until there are no more promising candidates.

6.2.4. Candidate Management

Given that candidate discovery can result in an unmanageably large list, it is necessary to reduce problem size to tractable levels. The developed method reduces master problem size by temporarily filtering out some candidate lines (*restricting* them). We refer to this as *algorithmic candidate reduction*.

The candidates that will be restricted are the ones that have not appeared in a previous, simplified solution of the problem (a linear relaxation where fractional investments are allowed and which is generally calculated much quicker than the MIP version, referred to as stage 1a) . As the case study shows, a large fraction of the available candidates is never used, so that a substantial size reduction can be achieved from restricting those unused candidates (fixing their values to 0). Then, the reduced MIP problem is solved (1b). When we reach convergence for the reduced MIP problem, in order to prove optimality, we must solve the problem again including all potentially interesting candidates (1c).

Cuts derived from the linear relaxations and from the MIP resolution with some restricted candidates are valid cuts because they correspond to linearized and suboptimal first-stage solutions calculated with the same subproblem [134]. Therefore, we can keep all Benders' cuts calculated previously. The previously calculated cuts make it possible to solve the problem much more efficiently than if they were not included.

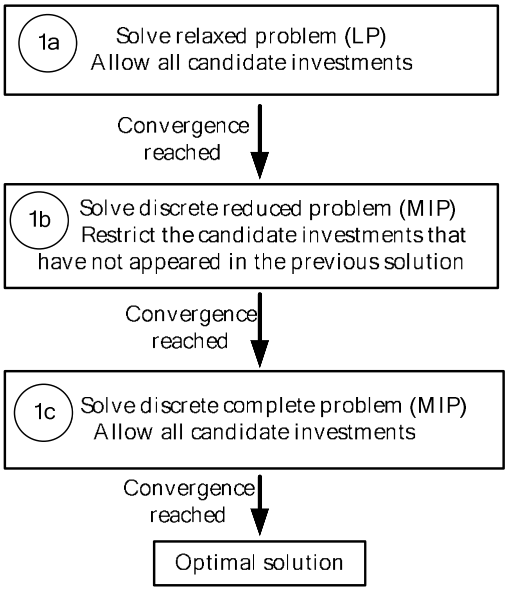


Figure 6-2 – Candidate management.

Time savings resulting from the application of this simple strategy can be seen in the case study section. However, if we identify a very large number of candidate investments, solving the MIP problem might not be possible. Then, candidate restriction (and, in extreme cases, only steps 1a and 1b) can be the only suitable alternative.

6.2.5. Candidate Analysis

If candidate discovery is performed manually, experts are able to link each candidate investment to a specific network need and therefore understand the relationships among investments. However, if the proposal is automatic, this explanation is lost. The developed candidate analysis method aims at providing this information in an efficient manner.

It is known that the benefit that can be obtained from some investments can be higher if they are installed together (positive synergy), or lower (negative synergy) [234]. Positive synergy usually corresponds to investments that form part of the same long-distance corridor, so the full benefit can only be obtained when all stretches are installed. That is, the investments should be deemed as complementary goods. Negative synergy is related to alternative ways of dealing with the same need; if one circuit is built to alleviate congestion between two given nodes and it is successful, adding extra capacity will not bring any additional advantage. That is, the investments are substitute, competitive goods.

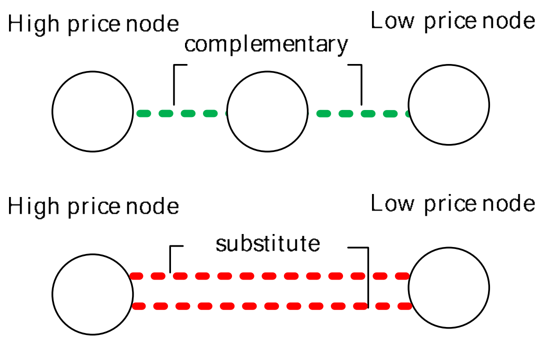


Figure 6-3 – Schematic illustration of positive and negative synergy between pairs of investments.

Positive synergy leads to diversifying investment (that is, investments bring more benefits if installed together) and negative synergy leads to concentrating investment (investments tend to not be installed together). This information can be useful to justify planning decisions within a transmission expansion project. We include some schematic examples of positive and negative synergy.

The identification of complementary and substitutability relationships allows defining transmission projects, that is, sets of transmission lines that are highly related and that should be analyzed as a whole.

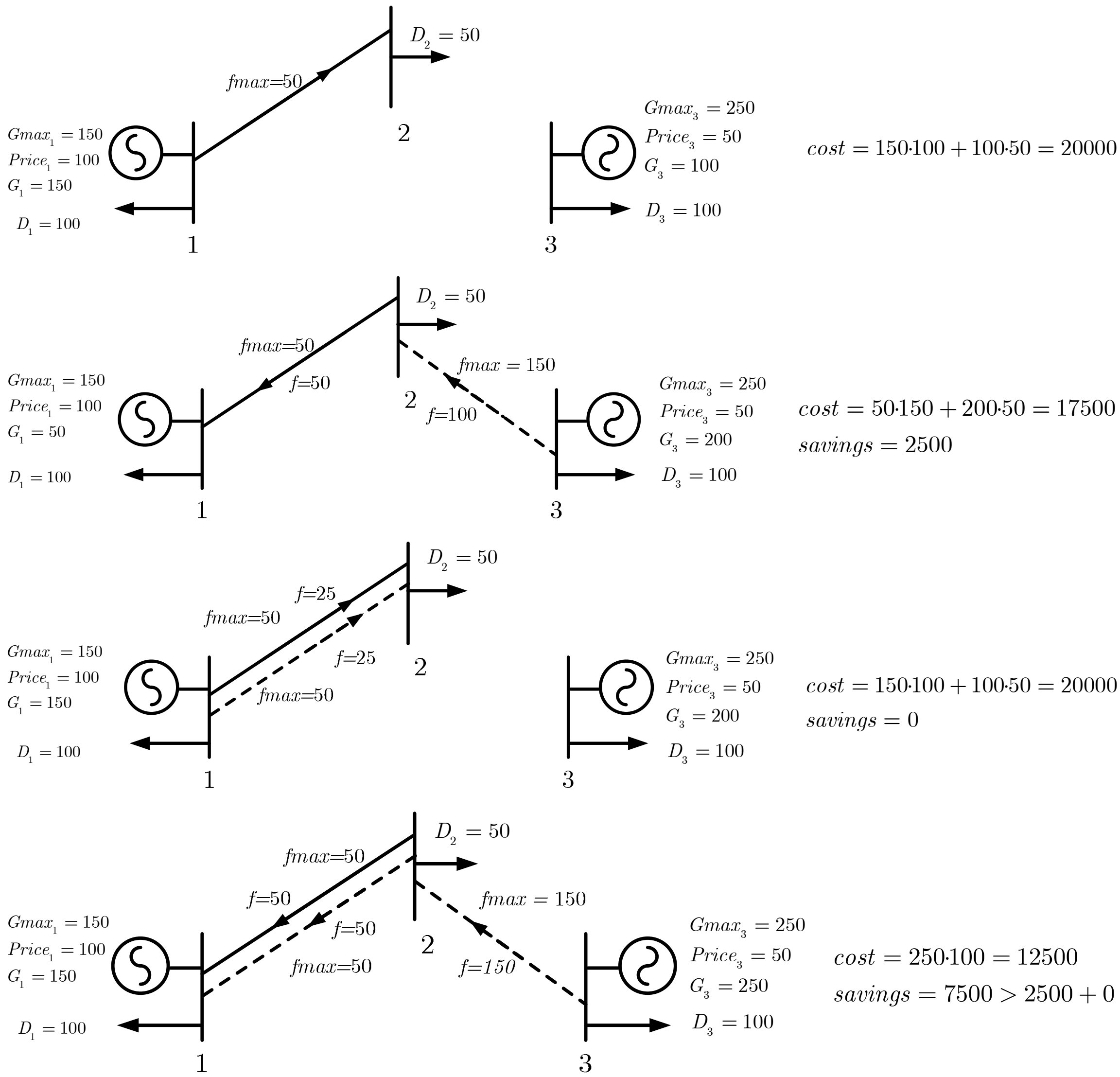
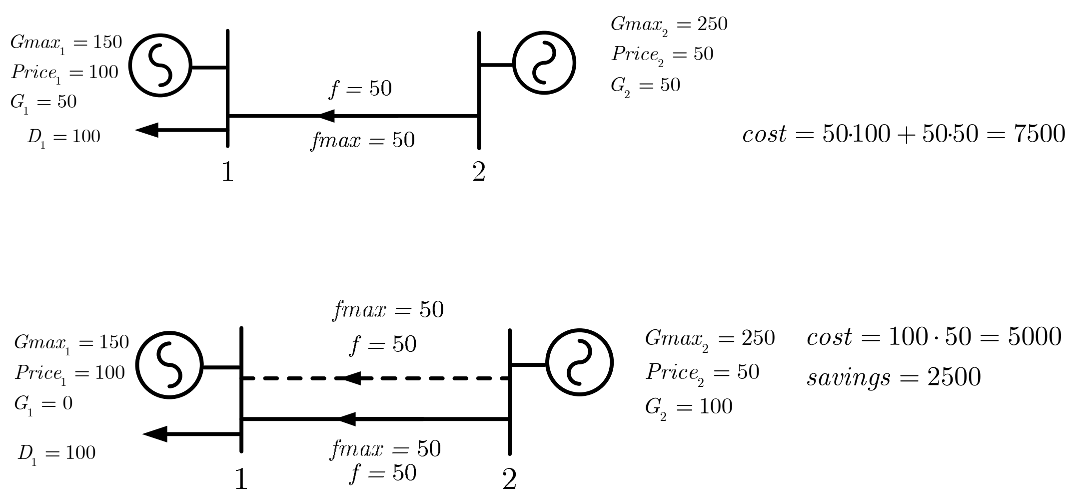


Figure 6-4 – Example of positive synergy (complementary assets). Combined savings are higher than the sum of individual savings, as a longer corridor appears, taking cheap energy from node 3 to node 1.



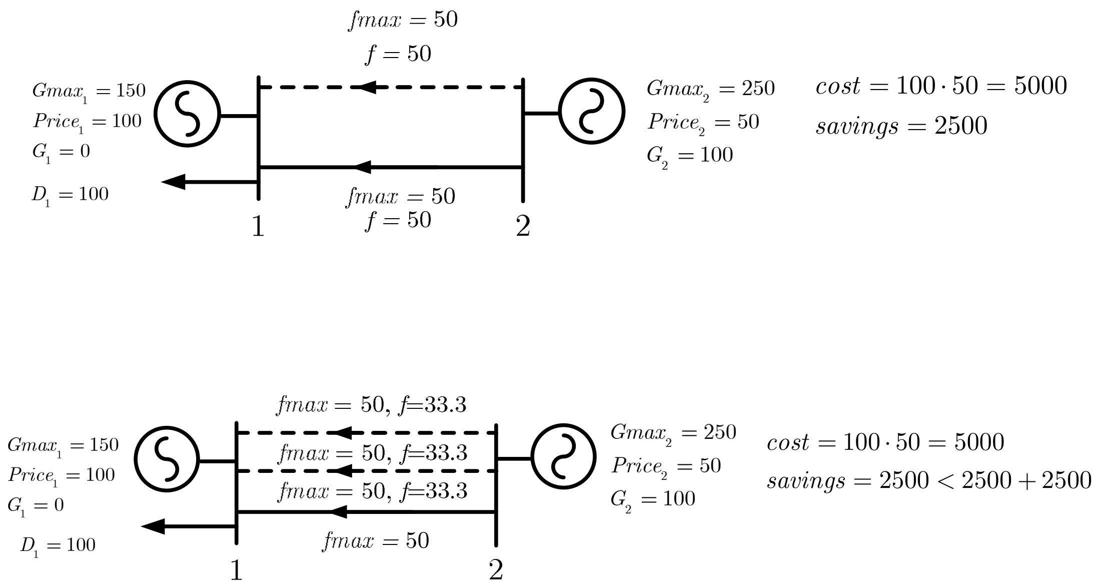


Figure 6-5 – Example of negative synergy (substitute assets). Savings in the combined case are lower than the sum of individual savings. Installing both lines gives no additional benefit as Node 1 can be completely supplied with cheap energy with just one of the lines.

It has been claimed [235] that it seems to be a common rule in complex systems that the synergetic relationships between pairs of variables reveal also the higher order relationships (groups of three, four, etc). This issue is remarkably important in genetics and as such has been extensively studied in this context [235], although this rule does not appear to be documented yet for other types of systems. In power systems, the fact that some sets of transmission lines appear to be related has been acknowledged in reference [236]. Therefore, we limit our approach to analyzing the relationships between pairs of lines. Although other alternatives are possible, the implementation we propose studies the variations in utilization (measured as power flow through the circuit) that a single line experiences when a second line is installed. The rest of the network is assumed to have the configuration that has been deemed optimal.

We calculate this variation as the percentage difference in power flow through a line ijc when a second line $i'j'c'$ is installed versus a case where the second line is not installed. Similar ratios have been used in the literature with the objective of assessing the effect of contingencies on the network [237]. We add these variations across scenarios weighted by their probability and duration in order to get a single measure:

$$\Delta f_{ijc,i'j'c'} = \sum_{\omega} P^{\omega} D^{\omega} \left(\frac{f^{\omega}_{ijc/x_{i'j'c'}=1}}{f^{\omega}_{ijc/x_{i'j'c'}=0}} - 1 \right). \tag{178}$$

- Investment pairs where this variation is positive tend to be used more when both of them are installed and are deemed complementary.

- Investments where this variation is negative tend to be used less when both of them are installed and are deemed substitutes.

Similarly to the approach in reference [234], a certain percentage threshold (e.g. 50%) is established in order to filter only the significant relationships. The only calculation needed is therefore a series of power flows (in particular, as many as analyzed candidates). That is generally affordable in terms of computational time for realistic systems.

6.2.6. Case Study

We develop a realistic case study based on the Spanish system in order to demonstrate the discovery, management and explanation methods. We implement the proposed techniques using TEPES [64].

The case study is based on the Spanish power system. We take transmission data as of 2008 from publicly available ENTSO-e REE E-SIOS cases [218]. We build forecasts for a peak-demand scenario and generation from the E-SIOS cases as well. We stress the system by decreasing all line capacities by 35% and importing 15GW of renewable generation from North Africa at Tarifa and exporting it to the rest of Europe through Barcelona and Bilbao. This kind of high renewable import scenario is currently being considered in different forms in several international initiatives [6]. The system is composed of 1084 nodes and 294 power plants (nuclear, coal, CCGTs, hydro, wind and solar). The existing transmission network is configured by 1505 lines and transformers. Figure 6-6 shows the 400 kV circuits. CPU times, expressed in seconds, were calculated using CPLEX 12.1 on GAMS in a PC at 2.80 GHz with 4 GB RAM running Microsoft XP 32 bits. The relative optimality tolerance was 0.1%.

Figure 6-6 – Initial state of the network for the Spanish case study.

The size of the initial candidate pool (in the sense of all possible lines that could link every pair of nodes with every cable type available) was 1.7 million. From them, automatic candidate discovery takes 6.6 s to select 4447 corridors as potentially interesting (which was lower than the maximum number of candidates we had imposed, which were 10000). The optimal solution installed 71 new lines. They consist of reinforcements along the routes from the import point to the export nodes.

Figure 6-7 – 400 kV lines installed in the optimal solution.

Figure 6-8 – Example of substitute investments identified.

Figure 6-9 – Example of complementary investments indentified.

The straightforward application of Benders' decomposition found the optimal solution in 1129 iterations and 14923.0 s. If algorithmic candidate management is applied, only 88 corridors are not restricted (i.e. 98% of all candidates are filtered out). None of these restricted candidates were subsequently selected for the optimal solution. Algorithmic candidate management reduces computation time to 5860.4 s, which represents 61% time savings. Our candidate analysis mechanism analyzed all the possible pairs of investments, which amounted to 2845 pairs. Candidate analysis identified 163 pairs of substitute investments and 16 pairs of complementary investments. We represent some examples for illustration in Figure 6-8 and Figure 6-9. Figure 6-8 represents two alternative ways of increasing network capacity at the South. Figure 6-9 shows a corridor that is made of four seemingly disconnected stretches that serve the same function, namely bringing power from South to North. This kind of relationship is not obvious from direct examination and its identification allows for better project management. For instance, the identification of complementary investments can lead to proposing other *supergrid* investments that comprise all the identified lines that are part of a corridor.

	Iterations	Time(s)
Solve relaxed problem (1a)	656	1784.8
Solve reduced discrete problem (1b)	140	278.3
Solve complete discrete problem (1c)	204	3797.3
Total	1000	5860.4

Table 6-1 – Computational performance of candidate management mechanism in the illustrative case study.

6.2.7. Conclusions

Proposing investment candidates for Transmission Expansion Planning is a relevant issue when large areas are studied or in cases where expert knowledge of the system is not available. We deal with this problem explicitly and propose a consistent and efficient method to tackle it, articulated around three main strategies:

An automatic *candidate discovery* mechanism objectively proposes potentially interesting investments based on sensitivity information.

Given that automatic candidate discovery can result in a very large number of candidates, a *candidate management* algorithm reduces problem size in order to keep it within tractable levels without losing optimality.

A *candidate analysis* mechanism analyzes the joint variations of flows. This reveals complementary and substitute groups of investments.

We test these techniques on a realistic case study that illustrates the applicability of our approach.

6.3. A Hybrid OO+MIP Algorithm Applied to Network Design Problems

In TEP and OWL problems, expansion plans can be highly complex, so identifying the most important investments can be extremely interesting for decision support. Furthermore, we take advantage of this structure to accelerate problem resolution by applying a OO+ MIP hybrid algorithm.

6.3.1. Introduction

Ordinal optimization (OO) was proposed in 1992 [238] and is now an established paradigm within soft computing techniques. OO lowers the demands of classical optimization and offers a statistical optimality guarantee instead of an absolute one. This means that OO can provide a solution in the top X% of solutions with a given probability P [239]. Both X and P can be chosen arbitrarily. On the contrary, MIP offers an absolute guarantee, which warrants a solution that is ensured to not be worse than X% in cost than the optimal one. Again, X can be arbitrarily chosen.

In OO, the group of solutions that are in the top X% of solutions is known as the *good enough subset*. OO returns several solutions (the *selected subset*) as a result. At least one of these solutions will belong to the good enough subset with probability P, $P(card(G \cap S) \geq k)$; where G represents the good enough subset, S is the selected subset and k is the alignment level (the number of elements of the selected subset that belong to the good enough subset).

The potential of OO can be appreciated intuitively by calculating a simple example: in 200 random samples, there is a 86.60% probability of having at least one solution in the top 1%: $1-(1-1\%)^{200}$. Similarly, there is almost a 100% probability of having at least one solution in the top 5%.

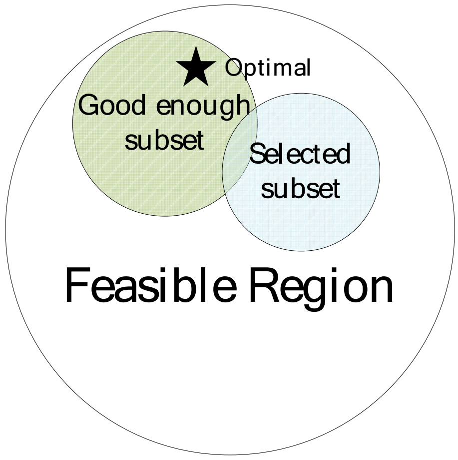


Figure 6-10 – OO searches for a subset of solutions that has some overlap with the good enough subset.

It should be noted that this sample size does not depend on the number of decision variables, unlike what happens with other techniques (for instance, the number of solutions that must be inspected to solve a MIP optimization problem). This is the main cause of the relatively benign behavior of this method with growing problem size. It has been proven that algorithms based on OO converge exponentially under certain conditions [240],[241]. We refer the reader to reference [242] for a detailed explanatory account of OO.

As with metaheuristics, OO is not guaranteed to return a local optimum, that is, there is a possibility that a small change in the solution will provide a better result. By analyzing the solutions that OO provides as the selected sample, we can check whether they have any common features. For instance, if all the solutions in the selected subset for an OWF problem have two 60 MVA transformers in the transmission system, we can infer that the transformer is a key factor in the economical viability of a solution. Moreover, two 60 MVA transformers seem to be the preferred option so, probably, the optimal solution will have them as well. In addition, if the decision maker can test additional components not originally included in the problem, it might be worth exploring other additional types of transformers.

OO has been welcomed in a variety of fields. Within power systems, it has solved an optimal power flow problem with discrete variables [243]. Similarly, optimal meter placement [244], power system control [245, 246] distributed generation capacity expansion [245, 247] and optimal bidding strategies [248] have been studied with OO. Moreover, there have been also some applications of this technique to TEP [72, 249].

We combine OO and MIP to search efficiently for good solutions to the OWF layout problem. Given the need to plan large OWFs, the technique should be able to deal with large problem sizes. In addition, this kind of problems is usually solved multiple times (for instance, testing several micrositing solutions as explained in Chapter 4). This makes it necessary to keep computation times as low as possible. An absolute optimality guarantee would be ideal, but given the large problem sizes, this requirement should be softened to having at least some kind of guarantee (such as the statistical one in OO). In addition, OO generates a good enough set that we can analyze in order to extract structure. This possibility is very attractive as well. Finally, it would be desirable that the returned solution is at least a local optimum.

The proposed algorithm combines OO and MIP in an attempt to benefit from their best features. First, we apply OO to the problem. OO returns the selected subset, where we search for common features. These common features are based on the fact that some variables or combinations of variables have a special importance in the problem. This has been observed, within a TEP context, in reference [236]. If common features exist, then we assume that good solutions have a structure consistent with them. We express the common features as constraints that we add to the problem. For instance, if all the solutions in the good enough subset have two 60 MVA transformers, the constraints impose that the transmission system must have two 60 MVA transformers as well. These constraints reduce the feasible region for the problem, so that we can solve it with MIP in a reduced computation time. This MIP problem returns a local optimum. However, the added constraints can result in this local optimum being different from the global optimal solution.

The hybrid inherits the statistical optimality guarantee and the explicit expression of solution structure from OO. We combine this with a *constrained optimality* guarantee brought by the subsequent application of MIP (that is, the solution is guaranteed to be the best one that abides the additional constraints). Table 6-6 displays a comparison of the characteristics of the proposed hybrid against MIP, metaheuristics (MH) and OO:

	MIP	MH	OO	OO+MIP
Global optimality guarantee	Absolute	None	Statistical	Statistical
Can deal with large problems	No	Yes	Yes	Yes
Extracts structure	No	No	No	Yes
Constrained optimality guarantee	Yes	No	No	Yes

Table 6-2 – Comparison of the characteristics of the main solution techniques considered.

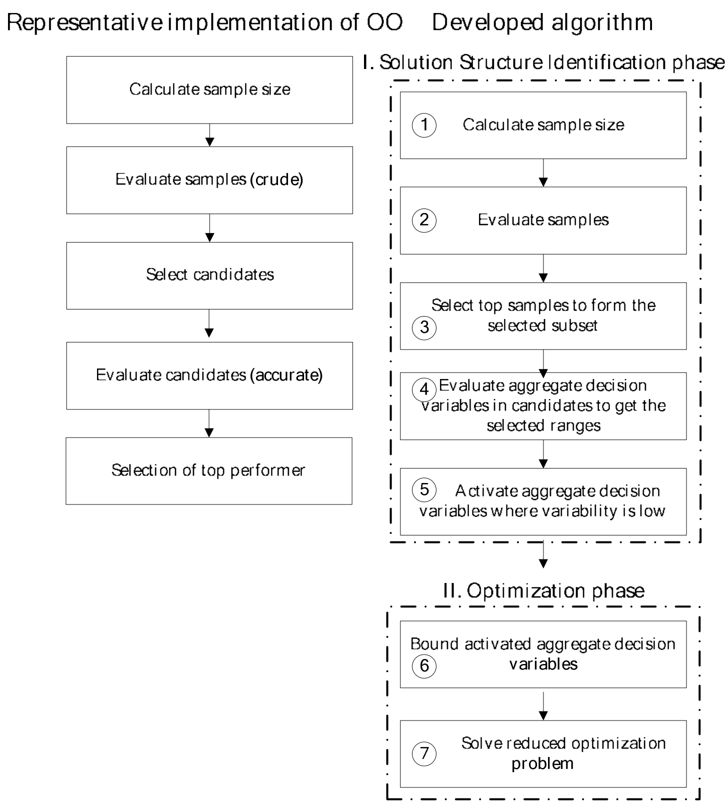


Figure 6-11 – Comparison of the structure of a representative implementation of OO versus the developed hybrid OO+MIP algorithm.

The steps in the process are:

I. OO: Solution structure identification phase

The first part of the algorithm generates the selected subset and analyzes their structure in order to identify common features.

1. **Calculate sample size**

We choose a target alignment of having at least one element from the top 1% of solutions with a 99% probability. A size of 500 samples satisfies this condition (actually, using the formulae explained in section 6.3.1 the resulting probability is 99.44%).

2. **Generate samples**

Each sample describes a possible layout for the OWF, with its cables, transformers or converter stations. We build random samples by generating a random value for each of the decision variables. In the case of the OWF layout problem, these decision variables define the cables in the collector system and the cables and transformers/converters in the transmission system. They are described as binary variables, so that a {0,1} is generated for each. Although we applied simple random sampling, more efficient techniques could be used, such as Latin Hypercube sampling. If a different sampling technique is used, the OPCs will change, but this does not impact the applicability of the method.

If the generated solution is infeasible, an auxiliary feasibility problem minimizes the deviations from that random solution while satisfying first stage constraints.

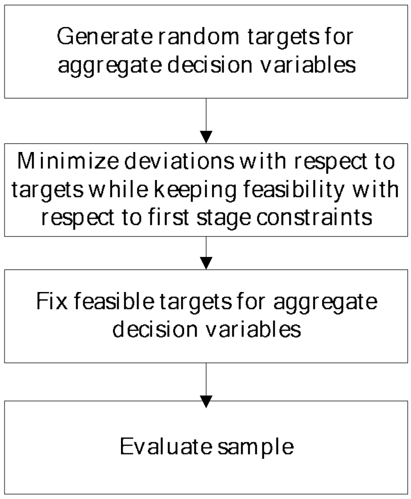


Figure 6-12 – Feasible sample generation.

3. Evaluate samples

Sample evaluation involves calculating the investment and operation costs associated with each of the generated layouts. We perform this evaluation by solving the model presented in 3.3 while keeping the decision variables fixed. This is a linear problem that can be solved relatively quickly.

a. Select the top samples to form the selected subset

The selected subset contains the samples solutions that have the lowest cost. In this case, we select the top 1 % performers in terms of total cost.

b. Analyze common features in the selected subset

Similarly to the measures used in reference [250] to guide a heuristic solution, the mechanism that looks for common features studies decision variables and aggregations of variables (this will be referred to as descriptive variables). The purpose of aggregating information is to make it easier to find structure. The different ways of aggregating variables identified in this problem are:

- aggregated capacity linking each node to the rest of the layout (referred to later as CP1),
- aggregated capacity linking each pair of nodes in the layout (CP2),
- number of cables linking each pair of nodes (RP2),
- total capacity linking a node to the rest (CP1),
- total number of cables linking a node to the rest (RP1),
- total power of the transformers installed,
- number of transformers installed.

As shown in the results, all these descriptive variables give good results in the case study.

The mechanism calculates the ranges taken by the descriptive variables in both the set of all samples and the selected subset. For instance, the total power of transformers installed might be between 0 and 500 MVA in the samples but 120 MVA exactly for the selected subset. Having a range in the selected subset (selected range) that is much narrower than its equivalent for the set of all samples indicates that this particular descriptive variable is important. The optimal solution will probably be within the selected range (in this example, the optimum will be assumed to have a total transformer power of 120 MVA). How much narrower the selected range must be in order to impose an additional constraint is determined by a threshold parameter.

Therefore, the only information that must be specified for a given implementation are the definition of the descriptive variables and the threshold parameters.

II. MIP: Optimization phase

The optimization phase makes use of the identified structure in order to focus on the most promising area of the feasible region.

6. **Impose the identified structure as constraints**

In the cases where the selected range of a certain descriptive variable is much smaller than the all-sample range, we add a constraint to the problem to ensure that the relevant descriptive variable takes a value within the selected range.

7. **Solve the reduced optimization problem**

The inclusion of additional constraints reduces the feasible region of the problem and allows finding the optimal solution in a relatively short computation time. The example below illustrates this. The following two figures show two different examples of Ordered Performance Curves (OPCs). OPCs represent the relative abundance of good and bad solutions when sampling randomly. The x-axis describes cost and the y-axis represents the probability density of obtaining a solution with a given objective value when sampling randomly.

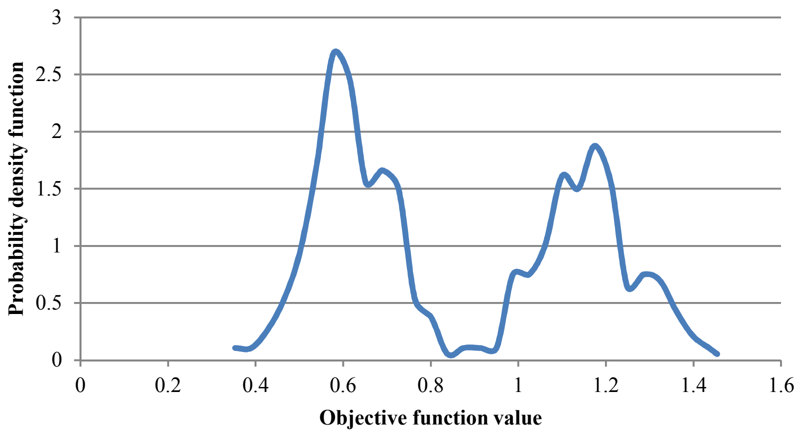


Figure 6-13 – OPC of small case example. Objective function value expressed in M€.

Figure 6-13 shows the OPC corresponding to the OWF design of a small case study [251]. The distribution is clearly bimodal. An examination of the solutions showed that each of the two halves of the distribution correspond to a different choice in the cable type that links the substation to shore. Therefore, this decision has a profound impact on total cost. We can use this information to improve algorithm performance.

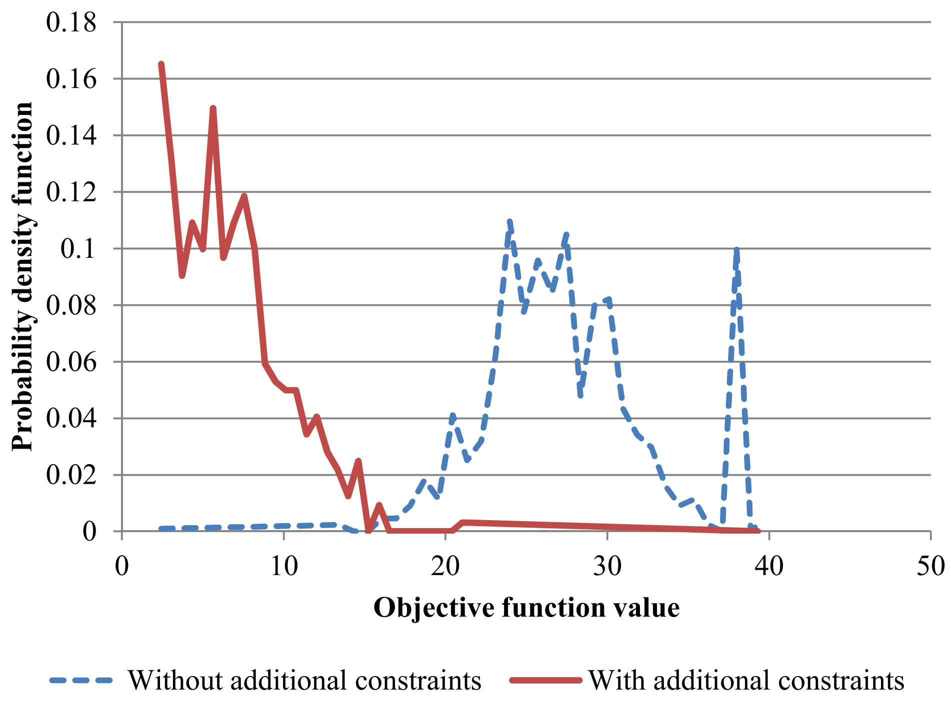


Figure 6-14 – OPC of real case study without additional constraints (right) and with the additional constraints (left).Objective function value expressed in M€.

Figure 6-14 displays two OPCs for the real case study; one missing the additional constraints described in 3.3.5.2 and the other including them. The fact that the OPC with the additional constraints is more benign, with more solutions with low objective values, demonstrates that their inclusion is strongly beneficial for the problem.

6.3.2. Case Study

We test the algorithm on a real case study based on Barrow Offshore Wind Farm. All the details of the plant, as well as the characteristics of the components considered, can be found in references [180, 199, 252, 252]. The model is coded in GAMS using CPLEX 12.4 for the optimizations, on a PC at 2.80GHz with 4GB RAM running Microsoft XP 32 bits.

	Equations	Continuous variables	Binary variables	Non-zero elements
Case study	42767	27042	102	147357

Table 6-3 – Problem size.

If a sample size of 500 is used, OO has a 99.44% chance of returning at least a value in the top 1% performers. In addition, the final optimization step improves this result, so that the final solution of the hybrid algorithm is expected to be better than the result of the OO stage. In particular, it will be a local optimum.

We tested the algorithm exhaustively in order to assess its robustness. We executed several problem instances for different descriptive variables and threshold levels.

We study all the different types of identified descriptive variables independently, as well as several possible values for the thresholds. In all cases, the number of samples calculated was equal to 500. We find that performance is robust with respect to both descriptive variables and threshold values.

	Direct resolution	Th= 0.999	Th= 0.99	Th= 0.9
Generation time (s)		0.9	0.6	0.9
OO time (s)		158.1	156.0	157.5
MIP time (s)		0.3	1.0	0.35
Total time (s)	1688.1	159.4	157.7	158.7
OO result (€ million)		2.1203	2.1203	2.1203
MIP result (€ million)	1.9985	2.0082	2.0082	1.9941

Table 6-4 – Performance vs. threshold values (Th).

	CP1+RPI	CP2+RP2P2	CP1+CP2	RP1+RP2
Generation time (s)	0.8	0.8	0.8	0.8
OO time (s)	158.8	157.8	159.6	155.8
MIP time (s)	159.2	158.1	160.0	156.2
Total time (s)	159.2	158.1	160.0	156.2
OO result (€ million)	2.1203	2.1203	2.1203	2.1203
MIP result (€ million)	1.9941	2.0082	2.0020	2.0105

Table 6-5 – Performance vs. descriptive variables.

In the next table, we present solution times and optimal values as a percentage of the direct optimization with a relative optimality tolerance of 1%. For reference, we obtain the true optimal, 1.99005, solving the problem with CPLEX 12.1 using MIP with a tolerance of 0.1%, taking 13327s. This problem was at the size limit of what can be solved using this technique.

As can be seen, results are notably robust with respect to algorithm parameters. In addition, the obtained layouts were all very similar to the optimal one, with only small differences in the cables close to the substation. The additional constraints mainly referred to the transmission system.

	Time	Distance to 0.01 optimal value	Distance to true optimal
MIP with 1% tolerance	100.00%	0.00%	0.43%
Th=0.999/all vars	9.45%	0.48%	0.91%
Th=0.99/all vars	9.34%	0.48%	0.91%
Th=0.9/all vars	9.40%	-0.22%	0.20%
Th=0.999/one point	9.43%	-0.22%	0.20%
Th=0.999/two points	9.36%	0.48%	0.91%
Th=0.999/capacity	9.47%	0.17%	0.60%
Th=0.999/redundancy	9.25%	0.60%	1.03%

Table 6-6 – MIP vs. OO+MIP.

Computation times were only several minutes in all cases, which allows solving the layout problem for as many times as needed. In addition, this shows that the OO+MIP hybrid would be able to deal also with larger layouts that might be too complex for other optimization techniques.

6.3.3. Conclusions

Ordinal Optimization (OO) is able to find solutions with a statistical guarantee (the solution will be in the top X% of solutions with a probability P).

We develop a novel hybrid technique that combines OO and MIP to benefit from their best features. First, we apply OO to obtain a subset of good solutions (the selected subset). We inspect these solutions for common features. In order to do this, we define some descriptive variables as either decision variables or aggregations of decision variables (for instance, the total transformer power installed). We identify a common feature when the range of values taken by a descriptive variable in the selected subset is much narrower than the range of values taken in the set of all samples. For instance, if transformer power is between 0 and 200 MVA in general but it is always 120 MVA in the selected subset of top solutions, we assume that the optimal solution has 120 MVA of transformer power.

We express the identified common features as constraints that we add to the MIP problem. They reduce its feasible region so that we can solve the problem faster. All types of descriptive decision variables give good results. In addition, in all the instances tested the solution found was either optimal or very close to the optimal one.

6.4. Chapter Takeaways

When planning large areas, there is no single TSO with experience of the whole system, so it is not possible to rely on its experience to propose interesting candidate investments. Therefore, automatic candidate proposal methods are necessary. In the same way, it is necessary to recover the rationale for investments and their relationships. We propose an automatic, objective method for candidate proposal, an algorithm that manages large lists of candidate investments and an approach for extracting the rationale and relationships among investments. We illustrate the proposed techniques with a TEP case study based on the Spanish Power system.

In addition, we apply OO to identify the transmission decisions with the highest impact and exploit this information by means of a novel OO+MIP algorithm that solves the problem efficiently. We demonstrate the advantages of this approach in a case study based on OWF design.

We published our candidate proposal approach as:

- S. Lumbreras, A. Ramos, P. Sánchez, "Automatic Selection of Candidate Investments for Transmission Expansion Planning", International Journal of Electrical Power and Energy Systems, vol. 59, pp. 130-140, July 2014.

We describe the OO+MIP hybrid in:

- S. Lumbreras, A. Ramos, P. Sánchez, "Offshore Wind Farm Electrical Design using a Hybrid of Ordinal Optimization and Mixed Integer Programming", under review at Wind Energy.

Chapter 7

Conclusions and Further Research

7.1. Conclusions

As explained, power systems experience a deep transformation towards more sustainable models. These changes require the installation of large amounts of renewable generation. Unlike traditional generation technologies, where location can be decided at the planner's discretion, renewable power plants are projected based on the abundance of the resource they use. For wind and solar power, this often means relatively distant regions, often poorly interconnected to the rest of the system. What is more, in the case of offshore wind, the whole network must be designed, with no starting grid to reinforce. Expanding the existent network will be fundamental to accommodate this new generation and hence for the success of these projects.

The design of these large-scale network expansions presents considerable challenges that make this problem extremely interesting.

Building a new transmission line involves a complex permitting process that can take over ten years. This means that transmission expansion decisions must anticipate the evolution of uncertainties such as fuel prices or, more importantly, the generation expansion decisions of competitive players in a deregulated environment. Therefore, it is necessary to develop robust expansion plans and flexible strategies that can adapt to the evolution of the system as the future unfolds.

Moreover, market integration and the increase of cross-border flows leads to planning increasingly large areas. In these cases, there is no single TSO with experience of the whole region under planning. The TSO traditionally performed functions such as proposing the candidate transmission lines to consider in the optimization. Therefore, it is necessary to create automatic, objective techniques to fulfill this function.

Finally, the planning of large systems results in complex problems where existing optimization techniques can fail.

This thesis is born as an attempt to address these challenges. We present contributions in modeling, problem resolution and the interpretation of results. We develop new techniques and illustrate them with case studies. For this, we take tools from classical and non-classical optimization (such as Benders' decomposition and Ordinal Optimization), project finance (in particular, ROV) or statistical estimation (namely, shrinkage).

The focus of this thesis is the development of new methods; case studies should not be valued in themselves but only as illustrations. In addition, we assume TEP is performed centrally with cost-based operation, which as explained is a reasonable assumption in this context. The introduction of market considerations in the transmission network was out of the scope of this thesis. In addition and although the ramifications of regulatory decisions are extremely interesting in TEP and other related issues such as transmission cost allocation, this lies beyond our scope as well.

Chapter 1 introduces TEP and diagnoses the main challenges it faces in the current environment.

Chapter 2 reviews the main approaches to TEP that appear in the literature, focusing on the modeling options and the solution techniques applied. We also describe the problem of offshore network design as an especially relevant particular case. We published the first review (to our knowledge) of this topic as:

- S. Lumbreras and A. Ramos, "Offshore Wind Farm Electrical Design: A Review", Wind Energy 16 (3): 459-473 April 2013 10.1002/we.1498.

Chapter 3 presents stylized versions of the two optimization models used in this thesis. The first one deals with general TEP. The second one is a particularization for offshore network design, which has some singular characteristics that lead us to deal with it specifically. In particular, we develop a compact model for the consideration of HVDC transmission in OWF design. This model was published in:

- S. Lumbreras and A. Ramos, "Optimal design of the electrical layout of an offshore wind farm applying decomposition strategies", IEEE Transactions on Power Systems 28 (2): 1434-1441, May 2013 10.1109/TPWRS.2012.2204906.

The rest of the chapters deal with the contributions we made around the identified challenges. Chapter 4 describes the developments aimed at efficient resolution, Chapter 5 deals with flexibility and robustness and Chapter 6 explores structure identification techniques in TEP problems.

7.1.1. Efficient Resolution

Large-scale TEP is a highly complex problem that demands efficient solution techniques. We exploit its structure applying Benders' decomposition. In addition, we develop the first application of decomposition strategies (to our knowledge) to OWF design.

Then, in order to build an efficient implementation of this method, we review the techniques developed to accelerate Benders' decomposition, a study that was missing in the literature. Then, we apply these techniques to TEP and compare their potential savings.

Moreover, we develop some new acceleration techniques of our own. We propose *semi-relaxed cuts* to deal with large numbers of discrete variables in the master problem. In addition, we develop a Progressive Contingency Incorporation algorithm to evaluate the effect of component failures in an efficient way. These enhancements reduce computation times by up to two orders of magnitude in the case studies developed.

The first application of decomposition strategies (to our knowledge) to the OWF layout problem was published as:

- S. Lumbreras and A. Ramos, "Optimal design of the electrical layout of an offshore wind farm applying decomposition strategies", IEEE Transactions on Power Systems 28 (2): 1434-1441, May 2013 10.1109/TPWRS.2012.2204906.

The review on acceleration techniques applied to Benders' decomposition, as well as semi-relaxed cuts, were presented at the conferences:

- S. Lumbreras, A. Ramos, "Transmission expansion planning using an efficient version of Benders' decomposition. A case study", PowerTech 2013. Grenoble, France, 16-20 June 2013.
- S. Lumbreras, A. Ramos, "Improvements to Benders' decomposition. A practical evaluation using a transmission expansion planning problem",13th Trans-Atlantic Doctoral Conference. London, United Kingdom, 9-11 May 2013.

This has also been submitted for publication:

- S. Lumbreras, A. Ramos ,"Optimal Transmission Expansion Planning using Benders' Decomposition. Acceleration Techniques", submitted to JORS last April.

The PCI algorithm is described in the paper:

- S. Lumbreras, A. Ramos and S. Cerisola, "A Progressive Contingency Incorporation Approach for Stochastic Optimization Problems ", IEEE Transactions on Power Systems 28 (2): 1452-1460, May 2013 10.1109/TPWRS.2012.2225077.

7.1.2. Flexibility and Robustness

TEP is challenged by the presence of large dynamic uncertainties, such as generation expansion. The long permitting process involved in building a new line means that transmission expansion decisions must anticipate the evolution of these uncertainties. Therefore, it is extremely important to develop flexible strategies that can adapt to the future as it unfolds. In addition, these uncertainties are usually assessed by expert opinion, which means that forecasts are necessarily limited. This makes it desirable to design robust plans that are able to perform relatively well in spite of changes in the definition of scenarios.

We apply Real Options Valuation (ROV) to determine the potential benefit of candidate lines and identify priority projects. We derive a simplified interpretation of optionality in TEP projects that evaluates option value in realistic settings. We identify the candidate transmission lines with the highest potential and the ones constitute the core of an expansion plan, as well as their main value drivers. We illustrate this with a case study based on the Spanish system.

We also develop a technique to apply regularization to the Stochastic Optimization problem in order to obtain expansion plans that are robust with respect to changes in the scenario tree. Our approach "shrinks" the Sample Average Approximation (SAA) solution towards a robust benchmark, that is, we modify the TEP solution to make it more similar to a plan that does not depend on the scenario tree. We develop two different alternatives to apply shrinkage to the SAA problem: linear shrinkage and norm-constraint shrinkage. We illustrate the advantages of the proposed approach with two case studies based on an academic case and on the Spanish system. Shrinkage produces expansion plans that are consistently better in terms of average out-of-sample cost and decision error.

An article that is currently under review details our ROV approach for TEP:

- S. Lumbreras, A. Ramos, D. Bunn, M. Chronopoulos, "Real Options Valuation Applied to Dynamic Transmission Expansion Planning", under review at IEEE Transactions on Power Systems.

We will submit for publication a paper describing our shrinkage technique as:

- S. Lumbreras, V. DeMiguel, A. Ramos, "Transmission Expansion Planning. A Regularized Stochastic Programming Approach", to be submitted to Operations Research.

7.1.3. Structure Identification

Automatic candidate proposal methods are necessary to substitute the formerly available experience of the TSO. These automatic mechanisms should also recover the rationale for investments and their relationships. We develop an automatic, objective method for candidate proposal, an algorithm that manages large lists of candidate investments and a technique to disentangle the relationships among them. We illustrate our approach with a case study based on the Spanish system.

In addition, we apply Ordinal Optimization (OO) to the identification of the transmission decisions with the highest impact. Moreover, we make use of this information to solve the problem more efficiently by means of a novel OO+MIP algorithm. We demonstrate the advantages of our method in a case study based on OWF design, where computation time is reduced by a factor of ten.

We published our candidate proposal approach as:

- S. Lumbreras, A. Ramos, P. Sánchez, "Automatic Selection of Candidate Investments for Transmission Expansion Planning", International Journal of Electrical Power and Energy Systems, vol. 59, pp. 130-140, July 2014.

We describe the OO+MIP hybrid in:

- S. Lumbreras, A. Ramos, P. Sánchez, "Offshore Wind Farm Electrical Design using a Hybrid of Ordinal Optimization and Mixed Integer Programming", under review at Wind Energy.

7.2. Further Research

While working on this thesis, we encountered some additional questions and interesting ideas for future work. The study of architectural alternatives, such as the so-called supergrid, could be undertaken in an explicit way. In addition, the discovered relationships among investments could be further exploited to define transmission expansion projects. This section summarizes these lines of further research.

7.2.1. Solving OWF Design and Micrositing

Solving micrositing (the positioning of wind turbines) together with the electrical design is an interesting problem that presents a high level of complexity.

7.2.2. Efficient Resolution.

7.2.2.1. Formal Study of Acceleration Techniques

In this thesis, the acceleration techniques proposed for Benders' decomposition have been studied from a practical, case-study based perspective. It would be interesting to tackle this issue from a theoretical perspective, so that their underlying mechanisms can be understood.

7.2.2.2. Simultaneous Decomposition.

In this thesis, we defined *descriptive variables* as single variables and aggregations of variables that have a special significance for a given problem. In the case of TEP, they represent variables or aggregations of variables that have a deep impact on the total cost associated to a given expansion plan. We used OO to identify them and proposed an OO+MIP hybrid algorithm to exploit them.

We would like to go one step further and use these descriptive variables within our Benders' decomposition scheme in what we refer to as a *simultaneous decomposition*. On top of the classical Benders' cuts, we would generate cuts in the space of the descriptive variables, therefore working in two spaces of variables simultaneously. As the descriptive variables are the ones that have the largest impact in the solution, simultaneous decomposition should solve the problem more efficiently than classical decomposition.

We would implement this in a relatively straightforward way:

- Descriptive variables can be built by either expert opinion or identified using information from the problem. They are defined as a function of simple variables in the master problem. That is, we include their definition as first-stage constraints.
- Two types of Benders' cuts are added with each iteration:
    - The classical ones.
    - A second set of cuts which only depend on the descriptive variables. The slopes with respect to the descriptive variables are calculated as aggregations of the slopes with respect to simple variables, so there are no complex calculations associated with the second set of cuts.

The space of descriptive variables has a much smaller number of variables. In general, the convergence of Benders' decomposition is linked to the number of *not-obvious* first-stage variables. With *not-obvious*, we mean the ones where the relationship between the variable and total cost is not straightforward, so that the decomposition has to spend a substantial number of iterations modeling this relationship. This opposes to the behavior of *obvious* variables, where just a few iterations suffice for determining clearly that the variable will or will not be present at the optimal solution.

Depending on the particular characteristics of the problem, the number of iterations necessary to discriminate obvious and not-obvious variables might vary. Simultaneous decomposition would provide this information directly, so that we should expect time savings that could be considerable depending on the problem.

In addition, because descriptive variables all have a considerable impact on cost, the slopes of Benders' cuts will be more meaningful in general. Therefore, the cuts are expected to have a better cover, which usually leads to improved convergence (see section 4.2.5.2.d).

Of course, searching for the appropriate descriptive variables is an intricate issue in its own. We have already used OO to identify the most relevant descriptive variables. Some other interesting ideas are presented in 7.2.4.

7.2.3. Flexibility and Robustness

We find two main extensions to our proposed approaches that would be worth exploring.

7.2.3.1. Extending our Real Options Approach to Projects and Portfolios

In this thesis, we have developed an interpretation of optionality for individual transmission lines. We propose to extend it to TEP *projects*, that is, highly related sets of lines. As explained in Chapter 6, the synergetic relationships among transmission lines define complementary and substitute investments. We define a *transmission project* as a set of investments whose performance is relatively independent from all other projects.

We can identify the relationships among investments using the candidate analysis tool as described in 6.2.5. Defining the relevant projects is equivalent to partitioning the set of investments into subsets among which there are no meaningful complementarity or substitution relationships. Then, all possible combinations of the assets within a subset are formulated as combined investments. The intrinsic and option values for these combined investments can be calculated independently. That is, total cost is roughly separable in terms of projects.

In addition, we propose to study flexibility from a *portfolio* perspective by means of sensitivity analysis at an aggregate level. This could answer the questions: *How prepared are we to respond to the uncertainties? What are the weaknesses, the blind spots of our expansion strategy?*

We will refer to the transmission licenses that we have already requested as our expansion portfolio. We need to make sure that our portfolio contains investments that can face, to a certain extent, all possible scenarios for the uncertainties. We can assess that by means of sensitivities.

That is, let's assume that, in our portfolio, we have some transmission lines with a very high positive sensitivity with respect to gas prices (that is, these transmission lines will bring benefits to the system if gas prices rise). Then, we need to include others with the opposite behavior, so that if gas prices fall we will have projects that would be valuable in that situation as well. We can progressively add lines from the set of candidate investments to make sure that all possible future evolutions are covered, that is, that we are *hedged* against the uncertainties. This concept is very similar to what we would do to manage the risk of a portfolio of financial options.

7.2.3.2. Extending Regularization to Several Robust Benchmarks

When we applied regularization, we shrunk the SAA solution towards a robust benchmark. We obtained this benchmark as the solution of the problem for the expected value of the uncertainties or for some scenario that was deemed particularly relevant for a given problem. In some cases, several of these benchmarks worked well. We would like to explore the possibility of using several of these benchmarks simultaneously, shrinking the SAA solution towards all of them.

We propose to obtain these benchmarks as the first-stage solutions for particularly relevant scenarios. We expect extreme scenarios to originate similarly extreme benchmarks whose impact will be diluted when several of them are used.

7.2.4. Structure Identification

The definition of transmission projects and expansion portfolios requires a relatively deep understanding of the network structure, namely:

- Sets of complementary and alternative investments,
- Definition of relevant descriptive variables.

We proposed candidate analysis to acknowledge complementary and substitutability relationships. In addition, we applied OO to identify high-impact descriptive variables. However, we have not exploited yet another interesting source of information about structure: Benders' cuts themselves.

We advanced this idea in the conference paper:

- S. Lumbreras, A. Ramos, "Transmission expansion planning using an efficient version of Benders' decomposition. A case study", PowerTech 2013. Grenoble, France, 16-20 June 2013.

Benders' cuts reveal part of the problem structure. Some of the transmission lines are either installed or not across iterations. These are *obvious* investments, where it is relatively clear whether they will appear or not in the optimal solution. The lines that most certainly will be installed have sometimes been referred to as *robust corridors.* Some others keep oscillating, pointing to equivalent solutions in the problem where it is particularly difficult to discern differences in cost (that is, they are *not-obvious* investments). If some of these oscillating variables tend to appear together in the evaluated solutions, we could consider them complementary. The opposite case will happen with alternative investments. Benders' cuts are therefore a valuable source of information in themselves that seems worth exploring.

Additional Materials

Node	Demand [MW]	Maximum Generation [MW]	Generation cost [USD/MWh]
1	80	150	50
2	240		
3	40	uncertain	10
4	160		
5	240		
6		uncertain	10

Table 0-1 – Academic network. Generation and load data.

From	To	Existing vs. Candidate lines	X [p.u]	NTC [MW]	Cost [M$]
1	2	E	0.4	100	2
1	3	C	0.38	100	1.9
1	4	E	0.6	80	3
1	5	E	0.2	100	1
1	6	C	0.68	70	3.4
2	3	E	0.2	100	1
2	4	E	0.4	100	2
2	5	C	0.31	100	1.55
2	6	C	0.3	100	1.5
3	4	C	0.59	82	2.95
3	5	E	0.2	100	1.
3	6	C	0.48	100	2.4
4	5	C	0.63	75	3.15
4	6	C	0.3	100	1.5
5	6	C	0.61	78	3.05

Table 0-2 – Academic network. Existing and candidate lines. Plans can install identical copies of these lines up to a limit of four copies.

		Continuous investment			Discrete investment		
From	To	Minimum-generation benchmark	Expected-generation benchmark	Maximum-generation benchmark	Minimum-generation benchmark	Expected-generation benchmark	Maximum-generation benchmark
1	5	2.25	0.4		3	1	
2	3			0.45			
2	6		1	2		1	2
3	5			0.4			1
4	6		1	1.25		1	2
Cost with low uncertainty [M€]		35.27	30.88	37.62	36.77	32.08	37.17
Cost in moderate uncertainty [M€]		36.45	60.02	42.72	37.95	35.53	65.11
Cost with high uncertainty [M€]		36.98	86.78	55.53	38.47	43.13	91.45

Table 0-3 – Academic network. Benchmark solutions.

From	To	Low variability	Medium variability	High variability
1	5		0.42	1.16
2	6	1.19	1.26	1.02
3	5	0.40		
4	6	0.17	0.42	0.55
Cost [M€]		30.19	31.75	34.36

Table 0-4 – Academic network. Stochastic solutions.

			A	B1	B2	C	Alpha *
Continuous - Linear combination	Minimum-generation benchmark	Low uncertainty	17.00	0.32	0.81	-0.62	**0.09**
		Medium uncertainty	19.00	2.16	2.18	3.79	**0.03**
		High uncertainty	3.84	2.73	3.76	-2.13	**0.59**
	Expected-generation benchmark	Low uncertainty	4.56	0.32	0.81	1.13	**0.00**
		Medium uncertainty	2.89	2.16	2.18	-0.08	**0.60**
		High uncertainty	10.00	2.73	3.76	3.60	**0.31**
	Maximum-generation benchmark	Low uncertainty	29.00	0.32	0.81	1.29	**-0.01**
		Medium uncertainty	22.00	2.16	2.18	-0.87	**0.19**
		High uncertainty	37.00	2.73	3.76	6.53	**0.00**

Table 0-5 – Academic network. Optimal alpha parameter and intermediate magnitudes for linear shrinkage with continuous variables.

Uncertainty	α	Minimum-generation benchmark Avg cost [M€]	Minimum-generation benchmark Avg sq error decision [p.u.]	Expected-generation benchmark Avg cost [M€]	Expected-generation benchmark Avg sq error decision [p.u.]	Maximum-generation benchmark Avg cost [M€]	Maximum-generation benchmark Avg sq error decision [p.u.]
Low	0	31.2271	1.1037	31.2271	1.1037	31.2271	1.1037
	0.01	31.1402	1.0711	31.1909	1.1046	31.147	1.1102
	0.05	30.8689	0.9788	31.049	1.115	30.8853	1.1903
	0.1	30.802	0.9493	30.89	1.1433	30.7637	1.4122
	0.2	31.1262	1.1761	30.6585	1.251	31.0107	2.2619
	0.3	31.5792	1.7842	30.5428	1.4268	31.5695	3.6527
	0.4	32.0763	2.7735	30.5169	1.6707	32.2773	5.5847
	0.5	32.5995	4.1441	30.5365	1.9826	33.0853	8.0579
	0.6	33.1308	5.896	30.5862	2.3627	33.9667	11.0722
	0.7	33.6645	8.0292	30.645	2.8108	34.8763	14.6277
	0.8	34.2006	10.5436	30.7156	3.327	35.7892	18.7243
	0.9	34.7381	13.4393	30.7931	3.9113	36.7022	23.3621
	0.95	35.0072	15.0301	30.8335	4.2289	37.1587	25.8839
	0.99	35.2224	16.3714	30.8667	4.4953	37.5239	27.9988
	1	35.2763	16.7163	30.8752	4.5636	37.6152	28.5411
Medium	0	59.0243	4.2518	59.0243	4.2518	59.0243	4.2518
	0.01	58.3274	4.2442	58.791	4.1659	58.4124	4.1521
	0.05	55.6796	4.2455	57.9164	3.8368	56.1125	3.8095
	0.1	52.6263	4.3183	56.9076	3.4583	53.4718	3.5076
	0.2	47.2948	4.7012	55.1975	2.8108	48.9433	3.3245
	0.3	43.071	5.4006	53.9528	2.3095	45.4084	3.7026
	0.4	39.9667	6.4164	53.1708	1.9541	42.9635	4.642
	0.5	37.8374	7.7488	52.9531	1.7449	41.5049	6.1425
	0.6	36.5163	9.3976	53.4417	1.6817	40.8096	8.2042
	0.7	35.9088	11.3628	54.4445	1.7646	40.6535	10.8271
	0.8	35.8066	13.6446	55.8502	1.9935	40.8264	14.0112
	0.9	36.0251	16.2428	57.7684	2.3685	41.5006	17.7565
	0.95	36.2219	17.6606	58.8892	2.6108	42.0966	19.8396
	0.99	36.4048	18.8517	59.7949	2.8309	42.5894	21.607
	1	36.4517	19.1575	60.0227	2.8896	42.7172	22.0629
High	0	71.612	6.3356	71.612	6.3356	71.612	6.3356
	0.01	70.93	6.1678	71.4764	6.2818	71.0278	6.3425
	0.05	68.2739	5.5254	71.0021	6.0848	68.7691	6.431
	0.1	65.0751	4.7873	70.5088	5.88	66.1042	6.6785
	0.2	59.1536	3.5277	69.9706	5.6086	61.2665	7.63
	0.3	53.7811	2.5568	70.0431	5.5217	57.1332	9.1903
	0.4	49.0836	1.8745	70.6306	5.6191	53.7694	11.3592
	0.5	45.265	1.481	71.8255	5.9009	51.4274	14.1369
	0.6	42.1805	1.3761	73.6697	6.3671	50.0206	17.5233
	0.7	39.8048	1.5599	76.0184	7.0177	49.5647	21.5183
	0.8	38.0989	2.0324	79.0167	7.8526	49.9567	26.1221
	0.9	37.1285	2.7935	82.636	8.8719	51.6129	31.3347
	0.95	36.9208	3.2824	84.6584	9.4507	53.444	34.1692
	0.99	36.9462	3.7254	86.3544	9.9469	55.1129	36.5464
	1	36.9787	3.8434	86.7824	10.0755	55.5334	37.1559

Table 0-6 – Academic network. Results for linear shrinkage with continuous variables. α represents the linear shrinkage intensity.

	Minimum-generation benchmark	Expected-generation benchmark	Maximum-generation benchmark
Low Uncertainty	1%	2%	1%
Medium Uncertainty	39%	10%	31%
High Uncertainty	48%	2%	31%

Table 0-7 – Academic network. Best reduction achieved in average cost with linear shrinkage and continuous variables.

	Minimum-generation benchmark	Expected-generation benchmark	Maximum-generation benchmark
Low Uncertainty	14%	0%	0%
Medium Uncertainty	0%	60%	22%
High Uncertainty	78%	13%	0%

Table 0-8 – Academic network. Best reduction achieved in average squared decision error with linear shrinkage and continuous variables.

Uncertainty	α	Minimum-generation benchmark		Expected-generation benchmark		Maximum-generation benchmark	
		Avg cost [M€]	Avg sq error decision [p.u.]	Avg cost [M€]	Avg sq error decision [p.u.]	Avg cost [M€]	Avg sq error decision [p.u.]
Low	**0**	33.0618	8.362	33.0618	8.362	33.0618	8.362
	0.01	32.3047	2.2006	32.4551	8.0817	33.662	17.8442
	0.05	32.3047	2.2006	32.4551	8.0817	33.662	17.8442
	0.1	32.3047	2.2006	32.4551	8.0817	33.662	17.8442
	0.2	32.224	2.2406	32.4551	8.0817	33.662	17.8442
	0.3	32.6678	4.001	32.4551	8.0817	33.6345	18.8045
	0.4	32.8198	3.6409	31.9316	4.681	33.5338	22.7656
	0.5	34.3806	7.6015	31.9316	4.681	33.5338	22.7656
	0.6	34.9406	9.8421	31.9316	4.681	34.3011	26.0065
	0.7	34.9763	12.0032	32.0633	8.6418	36.0849	31.7681
	0.8	35.1364	11.2032	32.0753	9.0019	36.565	33.2086
	0.9	36.1368	11.2442	32.0753	9.0019	37.1651	35.0092
	0.95	36.7767	17.0052	32.0753	9.0019	37.1651	35.0092
	0.99	36.7767	17.0052	32.0753	9.0019	37.1651	35.0092
	1	36.7767	17.0052	32.0753	9.0019	37.1651	35.0092
Medium	**0**	53.1546	10.2423	53.1546	10.2423	53.1546	10.2423
	0.01	39.8884	15.5238	41.9659	2.5607	44.6186	9.0023
	0.05	39.8884	15.5238	41.9659	2.5607	44.6186	9.0023
	0.1	39.8884	15.5238	41.9659	2.5607	44.6186	9.0023
	0.2	39.8884	15.5238	41.9659	2.5607	44.5858	8.3621
	0.3	41.4115	16.204	41.0764	2.2006	42.9125	8.5624
	0.4	41.7666	18.7245	38.661	2.0405	41.5869	8.9225
	0.5	41.7163	18.5644	38.661	2.0405	43.5448	13.8838
	0.6	39.5129	21.5651	36.0331	0.8802	43.3447	18.6452
	0.7	37.4255	20.8055	35.9828	0.7201	43.4608	23.6868
	0.8	37.7066	20.646	35.5258	0	48.5461	25.2072
	0.9	37.9404	25.647	35.5258	0	64.1302	25.8473
	0.95	37.9521	26.007	35.5258	0	65.1171	26.0073
	0.99	37.9521	26.007	35.5258	0	65.1171	26.0073
	1	37.9521	26.007	35.5258	0	65.1171	26.0073
High	**0**	60.7942	9.5976	60.7942	9.5976	60.7942	9.5976
	0.01	45.7566	5.8017	48.759	14.7412	51.5461	15.1641
	0.05	45.7566	5.8017	48.759	14.7412	51.5461	14.8439
	0.1	45.7566	5.8017	48.759	14.7412	51.5461	14.8439
	0.2	45.7566	5.8017	49.1664	14.021	51.5079	18.9251
	0.3	46.2275	5.6816	47.5283	14.3811	46.8917	21.7257
	0.4	46.2275	5.6816	43.9862	14.221	45.4835	23.1662
	0.5	46.1842	5.5216	43.9862	14.221	46.6254	24.4865
	0.6	43.7331	7.9623	43.4092	13.3634	45.7026	30.2082
	0.7	39.6338	8.9627	43.7733	12.4832	47.9852	34.6495
	0.8	38.7367	10.3232	43.1322	13.0033	61.9191	36.8902
	0.9	38.4894	12.2836	43.1322	13.0033	79.6521	38.3705
	0.95	38.4791	13.0037	43.1322	13.0033	91.4528	39.0106
	0.99	38.4791	13.0037	43.1322	13.0033	91.4528	39.0106
	1	38.4791	13.0037	43.1322	13.0033	91.4528	39.0106

Table 0-9 – Academic network. Results for linear shrinkage and discrete variables. α represents the linear shrinkage intensity.

	Minimum-generation benchmark	Expected-generation benchmark	Maximum-generation benchmark
Low Uncertainty	3%	3%	0%
Medium Uncertainty	30%	33%	22%
High Uncertainty	37%	29%	25%

Table 0-10 – Academic network. Best reduction achieved in average cost with linear shrinkage and discrete variables.

	Minimum-generation benchmark	Expected-generation benchmark	Maximum-generation benchmark
Low Uncertainty	74%	44%	0%
Medium Uncertainty	0%	100%	18%
High Uncertainty	42%	0%	0%

Table 0-11 – Academic network. Best reduction achieved in average squared decision error with linear shrinkage and discrete variables.

Uncertainty	δ	Minimum-generation benchmark		Expected-generation benchmark		Maximum-generation benchmark	
		Avg cost [M€]	Avg sq error decision [p.u.]	Avg cost [M€]	Avg sq error decision [p.u.]	Avg cost [M€]	Avg sq error decision [p.u.]
Low	**INF**	31.2271	1.1037	31.2271	1.1037	31.2271	1.1037
	15	31.2271	1.1037	31.2271	1.1037	31.2271	1.1037
	12	31.1299	1.0912	31.1253	1.0098	31.1307	1.093
	11	31.1299	1.0912	31.1299	1.0912	31.1188	1.0718
	10	31.1144	0.9729	31.1299	1.0912	31.3004	1.1218
	9	30.8563	0.5837	31.1299	1.0912	31.7404	1.3595
	8	30.7599	0.3547	31.1299	1.0912	32.0999	2.929
	7	32.8044	0.8971	31.1299	1.0912	31.3206	6.2722
	6	36.2756	1.9922	31.1307	1.093	31.9208	11.3971
	5	41.7415	3.9923	31.1299	1.0912	33.8276	18.9908
	4	41.443	6.6935	31.0574	1.082	34.0458	23.2806
	3	38.1652	10.3227	30.816	1.0295	34.6366	23.0847
	2	33.2763	13.0767	30.6392	1.2317	35.6315	21.469
	1	34.2763	14.6454	30.5978	2.1219	36.6152	22.5857
	0	35.2763	16.7163	30.8752	4.5636	36.865	23.477
Medium	**INF**	59.0243	4.2518	59.0243	4.2518	59.0243	4.2518
	15	59.0243	4.2518	59.0243	4.2518	59.022	4.2255
	12	59.0243	4.2518	59.0243	4.2518	58.9663	3.4414
	11	59.032	4.2418	59.0243	4.2518	59.0269	3.0454
	10	60.0461	4.346	59.0243	4.2518	58.1282	3.1846
	9	62.1493	4.5137	59.0243	4.2518	56.9702	2.6326
	8	61.9645	4.6903	59.0243	4.2518	56.0114	2.7493
	7	61.7789	5.23	59.0243	4.2518	56.1122	3.6946
	6	61.2808	6.1872	58.8254	4.2014	51.123	4.456
	5	58.4089	6.996	57.3849	3.928	47.6202	6.0547
	4	55.1872	8.199	54.4868	3.1491	45.473	9.9904
	3	51.2586	10.2529	51.4645	2.2092	46.1681	11.9421
	2	46.8106	13.8749	48.8118	1.6399	43.8067	13.6072
	1	39.876	17.8479	47.2972	2.1847	41.4307	16.2724
	0	36.4517	19.1575	60.0227	2.8896	41.9671	16.9989
High	**INF**	71.612	6.3356	71.612	6.3356	71.612	6.3356
	15	71.612	6.3356	71.612	6.3356	71.5753	6.1624
	12	71.6028	6.2763	71.612	6.3356	71.5058	6.9898
	11	71.656	6.1494	71.612	6.3356	70.3487	7.5692
	10	73.4755	5.9148	71.612	6.3356	69.0386	8.5014
	9	73.3085	5.3546	71.612	6.3356	67.5127	9.7101
	8	72.288	4.7726	71.6749	6.3555	65.7508	11.2073
	7	71.3176	4.1196	71.2763	6.1104	62.8051	13.2065
	6	69.0799	3.2929	69.6946	5.9249	59.0792	15.2376
	5	65.9099	2.4355	66.5712	5.9237	55.8585	18.1859
	4	61.661	1.5267	62.0929	6.1305	52.7792	21.3228
	3	56.7538	1.1723	57.7609	6.6878	52.2874	24.2257
	2	49.8127	1.6941	53.249	8.1693	48.3037	28.0082
	1	44.7342	2.7223	56.2586	8.9668	50.2797	31.6618
	0	36.9787	3.8434	86.7824	10.0755	54.7833	32.0918

Table 0-12 – Academic network. Results for norm-constraint shrinkage with continuous variables. δ represents the maximum allowed distance between the solution and the robust benchmark.

	Minimum-generation benchmark	Expected-generation benchmark	Maximum-generation benchmark
Low Uncertainty	1%	2%	0%
Medium Uncertainty	38%	20%	30%
High Uncertainty	48%	26%	33%

Table 0-13 – Academic network. Best reduction achieved in average cost with norm-constraint shrinkage and continuous variables.

	Minimum-generation benchmark	Expected-generation benchmark	Maximum-generation benchmark
Low Uncertainty	68%	9%	3%
Medium Uncertainty	0%	61%	38%
High Uncertainty	81%	7%	3%

Table 0-14 – Academic network. Best reduction achieved in average squared decision error with norm-constraint shrinkage and continuous variables.

Uncertainty	δ	Minimum-generation benchmark Avg cost [M€]	Avg sq error decision [p.u.]	Expected-generation benchmark Avg cost [M€]	Avg sq error decision [p.u.]	Maximum-generation benchmark Avg cost [M€]	Avg sq error decision [p.u.]
Low	INF	33.0618	8.362	33.0618	8.362	33.0618	8.362
	15	33.0618	8.362	33.0618	8.362	33.0618	8.362
	12	32.9726	6.3615	33.0618	8.362	33.1033	10.8026
	11	31.8718	2.8806	33.0618	8.362	33.1033	10.8026
	10	31.776	0	33.0773	7.0416	33.2429	13.0433
	9	31.776	0	33.0773	7.0416	33.3746	17.0041
	8	31.776	0	33.0773	7.0416	33.3746	17.0041
	7	34.7045	7.3216	32.5807	6.9215	33.3746	17.0041
	6	34.7044	7.0015	32.5807	6.9215	34.1646	26.0063
	5	35.1043	10.6022	32.5807	6.9215	34.1646	26.0063
	4	34.7761	13.003	31.8718	2.8806	34.1646	26.0063
	3	34.7761	13.003	32.0753	9.0019	36.8934	40.4101
	2	36.7767	17.0052	31.159	9.0019	37.1651	35.0092
	1	36.7767	17.0052	32.0753	9.0019	37.1651	35.0092
	0	36.7767	17.0052	32.0753	9.0019	37.1651	35.0092
Medium	INF	53.1546	10.2423	53.1546	10.2423	53.1546	10.2423
	15	53.1546	10.2423	53.1546	10.2423	53.2331	9.5222
	12	51.2999	11.3226	53.1546	10.2423	46.5679	7.0017
	11	47.9697	10.1223	53.1546	10.2423	45.4526	4.9212
	10	47.9713	12.0027	45.5836	9.9222	46.342	9.1624
	9	50.2142	12.3628	45.3298	9.2021	44.5629	8.4423
	8	43.9182	13.243	45.2737	8.3219	44.2607	10.0827
	7	40.9966	15.8838	43.4946	7.6017	46.5113	14.724
	6	40.896	15.5637	43.2208	7.0416	42.4094	15.0841
	5	41.8873	21.2449	43.3616	5.8013	43.2897	18.6854
	4	37.0565	21.5651	38.7091	5.6813	44.4678	23.4867
	3	37.0565	21.5651	35.7902	0.9603	47.3191	25.2871
	2	37.9521	26.007	35.6896	0.6402	44.5984	24.567
	1	37.9521	26.007	35.5258	0	65.1171	26.0073
	0	37.9521	26.007	35.5258	0	65.1171	26.0073
High	INF	60.7942	9.5976	60.7942	9.5976	60.7942	9.5976
	15	60.7942	9.5976	60.7942	9.5976	60.905	12.0832
	12	60.7268	7.8371	60.7942	9.5976	55.218	16.3244
	11	57.8834	8.3973	60.7942	9.5976	54.3118	16.5244
	10	53.9645	3.6184	55.8859	9.2375	52.0975	19.8053
	9	56.6793	4.2986	55.8505	8.4998	48.8213	20.5255
	8	50.1684	4.5386	55.7438	8.98	45.6179	21.9258
	7	46.2781	3.1606	52.4677	9.7002	47.7874	25.9269
	6	46.1914	2.8406	48.5683	7.762	44.712	26.6471
	5	47.8263	8.4824	48.8465	14.3811	44.9976	31.8888
	4	40.7422	8.1623	42.5819	13.5409	48.3324	35.3698
	3	40.7422	8.1623	39.8935	10.9225	49.7197	37.3303
	2	38.4791	13.0037	39.8068	10.6025	46.2657	36.6101
	1	38.4791	13.0037	43.1322	13.0033	91.4528	39.0106
	0	38.4791	13.0037	43.1322	13.0033	91.4528	39.0106

Table 0-15 – Academic network. Results for norm-constraint shrinkage and discrete variables. δ represents the maximum allowed distance between the solution and the robust benchmark.

	Minimum-generation benchmark	Expected-generation benchmark	Maximum-generation benchmark
Low Uncertainty	4%	6%	0%
Medium Uncertainty	30%	33%	20%
High Uncertainty	37%	35%	26%

Table 0-16 – Academic network. Best reduction achieved in average cost with norm-constraint shrinkage and discrete variables.

	Minimum-generation benchmark	Expected-generation benchmark	Maximum-generation benchmark
Low Uncertainty	100%	66%	0%
Medium Uncertainty	1%	100%	52%
High Uncertainty	70%	19%	0%

Table 0-17 – Academic network. Best reduction achieved in average squared decision error with norm-constraint shrinkage and discrete variables.

δ	Avg cost [M€]	Avg sq error decision [p.u.]
+INF	11,300.00	1.0007
8.9814	11,400.00	1.0072
6.736	10,600.00	0.8799
4.4907	10,700.00	0.8963
3.368	11,300.00	0.9992
2.2453	11,100.00	0.9517
2.0208	11,100.00	0.9517
1.7963	11,200.00	0.9814
1.5717	11,200.00	0.9814
1.3472	11,200.00	0.9814
1.1227	11,200.00	0.9814
0.8981	11,400.00	1.0128
0.6736	11,400.00	1.0128
0.4491	11,400.00	1.0128
0.2245	11,400.00	1.0128

Table 0-18 – Realistic Network. Results for norm-constraint shrinkage and discrete variables.

Avg cost	6.23%
Avg sq error decisión	12.07%

Table 0-19 – Realistic Network. Best reductions achieved in average cost and average squared decision error achieved with norm-constraint shrinkage and discrete variables.

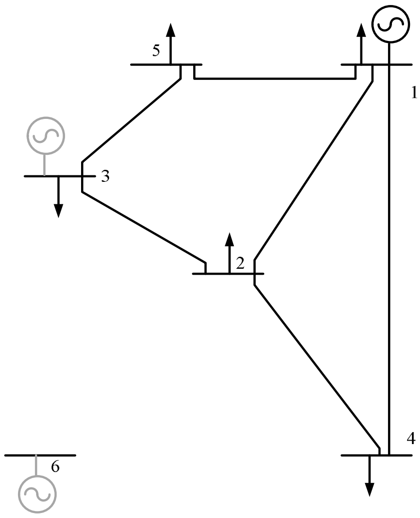


Figure 0-1 – Academic network.

Figure 0-2 – Realistic case study: initial state of the network, import and export node

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