 CMES, vol.25, no.2, pp.81-97, 2008

Coupled Electromechanical Optimization of Power Transmission Lines

J.R. Jimenez-Octavio[1], O. Lopez-Garcia[2], E. Pilo[1] and A. Carnicero[2]

Abstract: This paper presents a multidisciplinary design and optimization method of power transmission lines. This optimization method solves both mechanical and electrical problems by a new strongly coupled method that also optimizes the potential designs using a genetic algorithm. A multi-objective function is formulated to simplify a constrained typical optimization problem into an unconstrained one. The scope of this work is the sizing and configuration optimization problem with fixed topology. The method is applied to a railway overhead transmission line. The genetic algorithm is applied to mechanical, electrical and electromechanical optimization problems obtaining good results. Finally, the solution of the electromechanical optimization problem is a trade-off between the multidisciplinary nature of the problem and the sort of optimization, sizing and configuration.

Keyword: multidisciplinary optimization, multi-objective analysis, structural design, electrical design, genetic algorithm.

1 Introduction

Nowadays, multidisciplinary design and optimization play a fundamental role in the current concept of engineering. To afford multidisciplinary designs implies dealing with completely different physical problems. Moreover, optimization of these multi-decision designs should take into account not only multi-objective criteria, but also the coupling of both design variables and constraints.

One of the first applications of multi-objective optimization concepts appeared in Krokosky (1968). In this early paper, the author adopted a random search technique to find the best trade-off, correlating the different objectives in terms of the a priori chosen design parameters. Since then, multi-objective optimization has attracted a lot of attention among engineers, and several surveys are available in the literature. Coello (1999), Van Veldhuizen and Lamont (1998) and Coello, Pulido and Montes (2005) have reviewed evolutionary multi-objective optimization and other topics related to constraint-handling techniques. Some applications of multi-objective optimization to particular engineering designs have been carried out, among others, by Di Barba, Farina and Savini (2001) to electrical devices, Liu (2003) to seismic design of steel frame structures, Morino, Bernardini and Mastroddi (2006) to innovative aircraft designs, and Yoon, Jung, Hyun, Hong and Kim (1999) have tackled the optimization of electromechanical devices.

Deb and Gulati (2001) revealed that optimal design has always been an active area of research in the field of search and optimization. From the design point of view, electromechanical optimization has been widely studied in the literature. Most of these papers are focused on electromechanical devices or materials with a strong electromechanical constitutive behavior. Silva and Kikuchi (1999a, 1999b), using sequential linear programming, tackled topology optimization of piezoelectric transducers. Bai and Chinghong (2003) optimized the overall configuration of piezoelectric radiators by means of a genetic algorithm. Topology optimization of microelectromechanical actuators using adjoint sensitivity analysis was studied in Sigmund (2001a) for one

[1] Institute of Research in Technology, Department of Electrical Engineering, Universidad Pontificia Comillas de Madrid, Santa Cruz de Marcenado, 26, 28015, Madrid, Spain.
[2] Institute of Research in Technology, Department of Mechanical Engineering, Universidad Pontificia Comillas de Madrid, Alberto Aguilera, 23, 28015, Madrid, Spain.

material and in Sigmund (2001b) for two materials. Shape optimization of solenoid actuators by means of sequential quadratic programming was carried out by Yoon, Hur, Chun and Hyun (1997). Branch and Bound algorithm was used to optimize the design of electromechanical actuators taking into account multi-objective criteria in Messine, Nogarede and Lagouanelle (1998). However, to the authors' knowledge optimization of power transmission lines has received little attention. One of the aims of this paper is to revitalize the multipurpose design of power transmission lines by incorporating to this field new methods and tools. However, one important requirement of optimal solutions is to obtain feasible designs in the sense of conventional or expected real-world applications.

Most of the aforementioned techniques can be classified into three main categories: (i) sizing, (ii) configuration, and (iii) topology optimization. In the sizing optimization, cross-sectional areas of members are considered as design variables and the coordinates of the nodes and connectivity among various members are considered to be fixed, see for instance Goldberg and Samtani (1986). The resulting optimization problem is a nonlinear programming (NLP) problem. The sizing optimization problem is extended and made useful by restricting the member cross-sectional areas to take only certain pre-specified discrete values, see Rajeev and Krishnamoorthy (1992). In the configuration optimization, for instance Imai and Schmit (1981), nodal coordinates are treated as design variables. In most studies, simultaneous optimization of sizing and configuration has been used. The resulting problem is also a NLP problem with member area and change in nodal coordinates as variables. A recent publication for these kind of optimizations has been presented by Lamberti and Pappalettere (2007), about a weight optimization of frame structures using multi-point simulated annealing. In the topology optimization, developed by Krish (1989) and Ringertz (1985), the connectivity of members has to be determined. Classical optimization methods present difficulties in topology optimization, simply because they lack efficient ways to represent connectivity of members. However, evolutionary strategies have show promising success tackling the topology optimization. For example Kaveh and Kalatjari (2003b), using graph theory and genetic algorithms, have considered the topology optimization of structures while Wang and Wang (2006) present a coupled structural shape and topology optimization by means of a genetic algorithm. Another interesting branch of application related to structural topology optimization is the material structural one, being recently dealt by Shiwei and Wang (2006) or Michael Yu and Shiwei (2006).

The main contribution of this paper is to present a new multidisciplinary framework to optimize power transmission lines. As it will be seen latter, the proposed method is sufficiently general to easily include thermal, magnetic, electrical and mechanical criteria. Without losing generality, this paper is particularly concerned with the multi-objective optimization of power transmission lines using electrical and mechanical design variables. From a practical point of view, the considered scope of this work is the sizing and configuration optimization problem with fixed topology. One aim of the methodology proposed herein is the exploration of wide regions of the space of design variables and the determination of the global optimum. Thus, a genetic algorithm to optimize cross sections and geometry configuration simultaneously becomes necessary.

2 Problem formulation

Design of power transmission lines, see Figure 1, is a complex engineering problem which involves the fulfillment of different requirements. These requirements usually refer to certain electrical, magnetic, thermal or mechanical objectives, among others. Very often, design variables are coupled by these goals. Therefore, the design of power transmission lines can be posed as an optimization problem in which complex design variables should be considered together. The main goal of this paper is to present an innovative design method of power transmission lines. In what follows, and without losing generality, the method will be focused on the coupling between electrical and mechanical variables and constraints using multi-objective optimization. In this case there are several variables belonging to different problems though which are really coupled. For example the location of the physical conductors, which are related to the electrical impedance problem but also influence the structural design. As far as the authors' knowledge is concerned, this particular methodology establishes an innovative contribution which can be very useful to explore new configurations of power transmission lines.

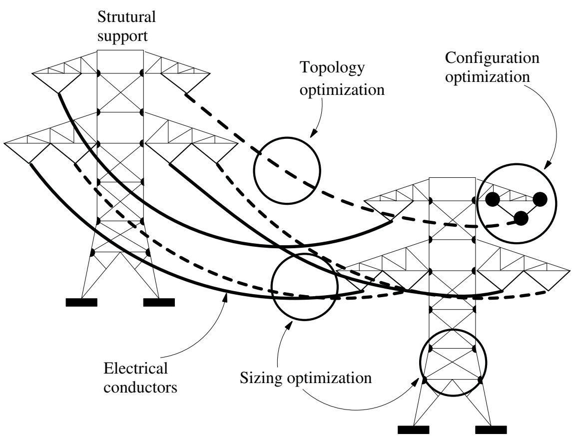

Figure 1: Power transmission lines for optimization

Furthermore, Figure 2 schematizes the coupling problem in a general flowchart. This figure shows the relationships between electrical and mechanical designs through the multi-objective function and the optimization algorithm. These relationships are the main ingredients of the method proposed herein,

Multi-objective optimization can also be called vector optimization given that Osyczka (1978) defines it as "a vector of decision variables which satisfies constraints and optimizes a vector function whose elements represent the objective functions". Therefore, describing variables and constraints of power transmission lines is the first step to establishing the optimization problem, thus: (i) sizing and (ii) configuration of the structural supports, (iii) geometric location and (iv) type of electric conductors are the design variables, while the constraints are: (i) maximum allowable stress,

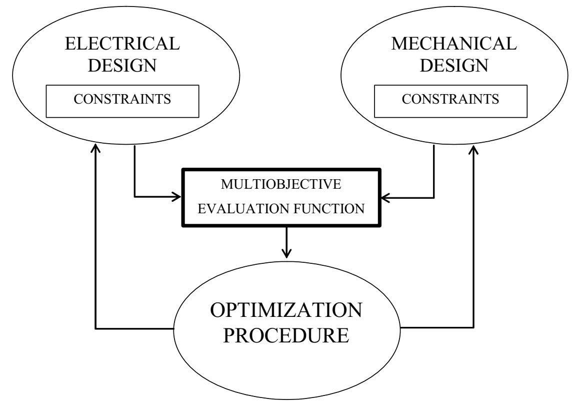

Figure 2: General coupling problem

and (ii) structural static stability, (iii) geometric constraints for the electrical design and (iv) maximum allowable current of each conductor.

Consequently, these functions and constraints will be cast as a mathematical description of performance criteria, although the objectives could be opposed to each other, as follows:

$$\min_{\mathbf{x}} \mathbf{f}(\mathbf{x}) \tag{1}$$

being:

$$\begin{aligned} \mathbf{f} &= [f_1, \ldots, f_{n_k}] \\ \mathbf{x} &= \left[{}^b x_1, \ldots, {}^b x_{n_b}; {}^d x_1, \ldots, {}^d x_{n_d}; {}^s x_1, \ldots, {}^s x_{n_s}\right] \end{aligned} \tag{2}$$

subject to:

$$\begin{aligned} & g_i(\mathbf{x}) \leq 0 \qquad i = 1, ..., n_i \\ & h_j(\mathbf{x}) = 0 \qquad j = 1, ..., n_e \\ & {}^b x_p \in \{0, 1\} \qquad p = 1, ..., n_b \\ & {}^d x_q \in \left\{x_q^1, ..., x_q^{nq}\right\} \qquad q = 1, ..., n_q \\ & {}^s x_r^L \leq {}^s x_r \leq {}^s x_r^U \qquad r = 1, ..., n_s \end{aligned} \tag{3}$$

where $\mathbf{f}(\mathbf{x})$ is a vector of n_k criteria or objectives, $\mathbf{x}$ is the vector of n design variables which define the system, $g(\mathbf{x})$ and $h(\mathbf{x})$ are inequality and equality constraints, respectively. Thus, n_i is the number of inequality constraints, n_e is the number of equality constraints, n_b is the number of Boolean design variables (usually related to topological optimization or member connectivities), n_d is the number of discrete design variables (widely used in sizing) selected from a list of n_q values, and n_s is the number of continuous design variables (typically suitable for configuration optimization).

Nevertheless, multi-objective problems are usually reduced to a single evaluation function. In order to achieve this, the following procedures, which are described in depth by Coello (2002), have been commonly used: (i) the use of a penalty function composed of the various criteria; (ii) the separate solutions of single criterion problems and their trade-off; or (iii) the solution of a single criterion problem, taking the other criteria as constraints.

However, as Coello (2002) has pointed out, the use of penalties is the most common formulation, when evolutionary algorithms are used to solve the optimization problem. The idea of this method, originally proposed by Courant (1943) and later expanded by Carroll and Fiacco (1961) and Fiacco and McCormick (1966), is to add a penalty value to the objective function, whose scale or weight is based on the degree of constraint violation. In this way, a constrained classical optimization problem is changed into an unconstrained one.

In this paper, the formulation of the optimization problem will use exterior dynamic penalties. One aim of the optimization procedure is to explore initially wide regions of the space of design variables. Therefore unfeasible solutions can appear at the first stages of the algorithm. Contrary to interior penalties, exterior penalties can tackle initial unfeasible solutions. Another requirement of the optimization procedure is the determination of a global optimum. Consequently as the search evolves unfeasible solutions should be strongly penalized. Therefore, dynamic penalties should be used. Thus, the formulation of penalty functions developed herein is divided into static and dynamic parts. On one hand, the static part is inspired in the works of Hoffmeister and Sprave (1996) and Morales and Quezada (1998). On the other hand, dynamic penalty uses the approach proposed by Kazarlis and Petridis (1998).

Thus, the general formulation of the optimization problem, equations (1), (2) and (3) using dynamic exterior penalties can be rewritten as:

$$F(\mathbf{x}) = \sum_{k=1}^{n_k} \alpha_k \cdot f_k(\mathbf{x}) + \pi \tag{4}$$

$$\begin{aligned} \pi &= \pi_d \cdot \pi_s \\ \pi_d &= (1 + \beta \cdot (n-1)) \\ \pi_s &= \sum_{i=1}^{n_i} \delta_i \cdot r_i \cdot G_i + \sum_{j=1}^{n_e} \delta_j \cdot c_j \cdot H_j \end{aligned} \tag{5}$$

where $F(\mathbf{x})$ is the new multi-objective function to be optimized; $f_k(\mathbf{x})$ the different criteria, adjusted by its own weight α_k,; and π a vector of penalty functions. In this aspect, π can be split into two parts: a dynamic penalty function π_d, which is considered proportional to a constant β and the number of the current iteration n; and a static one π_s, where G_i and H_j are functions of the degree of violation of the constraints $g_i(\mathbf{x})$ and $h_j(\mathbf{x})$, respectively, and r_i and c_j are positive constants normally called "penalty factors" that are also conditioned by binary factor δ_i which is one if the constraint is active or zero otherwise, mathematically:

$$F(\mathbf{x}) = \begin{cases} \hat{F}(\mathbf{x}), & \text{if } \mathbf{x} \in \Gamma \\ \hat{F}(\mathbf{x}) + \pi, & \text{otherwise} \end{cases} \tag{6}$$

where Γ represents the feasible region.

Regarding power transmission lines two main objectives are usually sought: minimization of weight and impedance. Therefore, the following expression is the most evident multi-objective function:

$$\begin{aligned} \hat{F}(\mathbf{x}) &= \alpha_m \cdot f_m(w_m(\mathbf{x})) \\ &+ \alpha_e \cdot f_e(w_e(\mathbf{x})) + \alpha_z \cdot f_z(z_i(\mathbf{x})) \end{aligned} \tag{7}$$

the first term reflects structural weights, w_m; the second one is also a weight but related to electrical conductors, w_e; and finally the equivalent impedance of the electric system, z_i. Due to the multi-objective nature of the method, other criteria can be easily incorporated into the evaluation function (7). A new way for construction of this kind of fitness functions has been recently suggested by Zhu, Liu, Wang and Yu (2004) for two different multidisciplinary design problems.

3 Optimization procedure

Optimization algorithms can be classified into two main streams: evolutionary strategies and gradient based techniques. The comparison between evolutionary strategies and gradient based optimization has been studied by Papadrakakis, Tsompanakis and Lagaros (1999) and Lagaros, Papadrakakis and Kokossalakis (2002). These works have demonstrated that both evolutionary strategies and genetic algorithms, based on probabilistic searching without needing gradient information, present more computational efficiency in spite of their higher number of analysis until the optimum is reached. Moreover, gradient-based mathematical programming may consume a large part of the total computational effort and could involve major modifications when non linear, discrete or mixed programming techniques are used.

Evolutionary strategies imitate biological evolution in nature and have three characteristics as described Papadrakakis, Lagaros, Thierauf and Jianbo (1998), that differ from any others. Firstly, instead of deterministic operators, they use randomized operators like selection, crossover and mutation. Secondly, in order to avoid convergence to non global minimum these algorithms work simultaneously with the whole population of design points in the space of design variables. Finally, they easily deal with non linear, discrete or mixed programming, making it possible to define a "fitness function" which allows evaluation and adjustment of different variables and penalties in a natural way.

3.1 Genetic algorithm

Over the years, much work has been done in engineering optimization and the current tendency is to deal with real-life applications which are multi-objective by nature. Hence, metaheuristic techniques inspired by the mechanics of natural selection suggested by Darwin (1929), initially developed and formalized for genetic algorithms by Holland (1975) and applied in engineering by Goldberg (1989), seem suitable to solve multi-objective optimization problems. Moreover, genetic algorithms are also well-recognized as a powerful optimization method for a wide range of problems. For example, an unusual application, about the determination of non linear polarization curves of buried slender structures through potential measurements at the soil free surface, has been successfully dealt and presented by de Lacerda and da Silva (2006).

In this aspect, Kaveh and Kalatjari (2003b) indicate that optimization by genetic algorithms can be performed by the following steps: (i) a random generation of a set of primary designs (initial population) in which each one, consisting of a string of characters, is called individual. At the same time, each string contains some substrings representing the different design variables. (ii) The real value of each design variable is evaluated by decoding. After that, (iii) penalty functions, i.e. equations (5), are defined representing the violation of constraints, and combining them with the objective function, equation (7), result in a modified objective function called "fitness function", equation (4). Finally, (iv) the next population is produced by the known genetic operators: selection, crossover and mutation, whose own probabilities are related to the exploration and exploitation of the searching space.

After performing these operations, a new allegedly better population, is produced. Repeating steps (ii)-(iv), other populations are obtained, finishing this process when a fixed number of generations are computed or the termination criterion is satisfied.

Therefore, a standard genetic algorithm has been applied to the electromechanical optimization, although some specific tools and functions have been tailored too. As Kaveh and Kalatjari (2003a, 2003b), have pointed out, sizing and configuration optimization problems could present a large number of design variables, both continuous or discontinuous, consisting of cross sectional areas and nodal coordinates, thus resulting in the development of modified genetic algorithms. Knowing each chromosome's fitness, the usual selection process takes place to choose the individuals that will be the parents of the following generation. Nevertheless, in this work a new ranking selection scheme that could be named "pseudo-stochastic remainder selection" (PSMER) has been developed. This is an original contribution of this work, and from the authors' experience, fast and robust convergence is obtained. Other selection processes were used (roulette-wheel, tournament and stochastic universal sampling selections), and algorithm convergence was compromised. In the PSMER scheme the population is scaled and sorted from best to worst. Then each individual is copied as many times as it can, depending on its fitness scale and according to a non-increasing assignment function, and finally a roulette-wheel selection is performed according to that assignment, as Coello and Christiansen (1998) indicates. The aim of this method is to properly mix some selection concepts like scale, elitism and proportionate reproduction without increasing population.

3.2 Coupling criteria

Scientific literature tackles the coupling between variables, constraints and objectives with different methods. For instance, Missoum and Gürdal (2002) consider a weak coupled model. They propose a method based on a nested two-level approach. The inner level solves sizing optimization problem. While the outer level tackles the configuration optimization problem. On the contrary, Regnier, Sareni and Roboam (2005) use a strongly coupled model. This method is able to optimize electrical and mechanical variables using the same fitness function. More particularly, these authors apply the second version of the well known multi-objective technique, based on non–dominated sorting genetic algorithm (NSGA-II) developed by Deb, Pratap, Agarwal and Meyarivan (2002). In this paper the electromechanical coupling can be found at different levels. On one hand, both electrical and mechanical design variables define an individual. On the other hand, a multi-objective electromechanical function has been presented. Finally, sizing and configuration problems are treated as the same problem by expanding design variables to include not only physical parameters but also nodal locations.

Figure 3 shows a detailed flowchart of the proposed optimization problem, reflecting two key aspects of the coupling method: on one hand the relation between mechanical and electrical problems; and in the other hand the relation between both problems and the genetic algorithm.

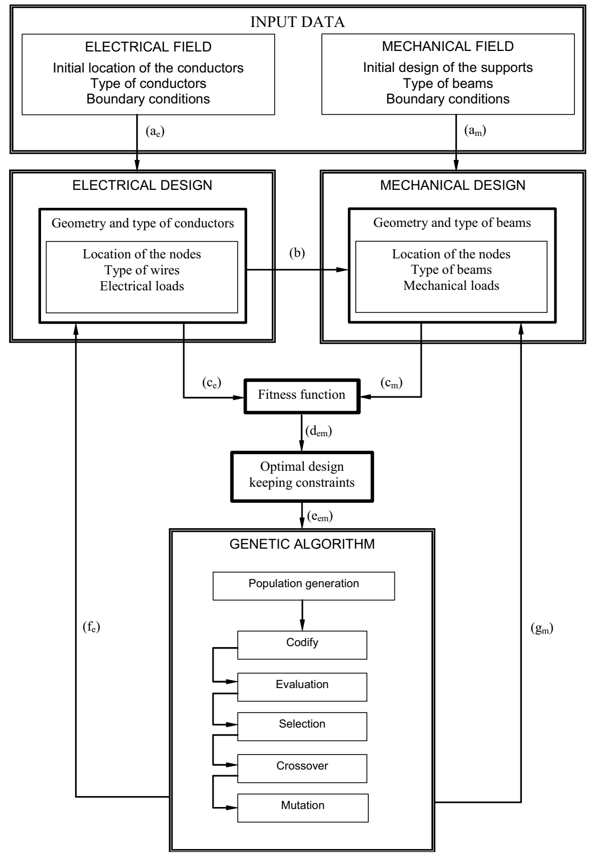

Figure 3: Optimization problem flowchart

First of all, taking into account the initial data and the constraints of the electrical (a_e) and mechanical (a_m) problems respectively, an initial population of potential designs is generated. After considering the effect of the type and location of the electrical conductors on the loads that are going to support the mechanical structure (b), the coupling of both problems takes place by defining the multi-objective fitness function of the electrical (c_e) and mechanical (c_m) variables and boundary constraints, whose result (d_{em}) is the population of potential electromechanical designs. Finally, these designs are evaluated and reproduced by the genetic algorithm (e_{em}), which feedbacks its previous blocks (f_e) and (f_m) until the optimization process finishes fulfilling the imposed termination criteria.

4 Application of the Optimization Method to High Speed Railways Overheads

As a typical application of power transmission lines, a coupled electromechanical optimization of high speed railway overheads is tackled in this section. A more detailed study can be found in Jimenez-Octavio, Pilo, Lopez-Garcia and Carnicero (2006).

Railway overheads design, composed of overhead supports and electric lines, should take into consideration, among other things, different criteria like ease of construction, use of standards, maximum safety or reliability, minimum cost, etc. Without losing generality the electromechanical design proposed herein will be driven by minimization of weight, structural safety requirements and power supply quality.

The proposed method couples electrical and mechanical variables. In addition, from an algorithmic point of view, individual genotypes used by the genetic algorithm are fully coupled. Thus, their genes couple different variables of the mechanical and the electrical problem storing some consecutive binary bit-strings as follows: (a) type of structural beams or electrical conductors with the discrete position in a standard system specification; (b) mechanical beam's node or physical conductor's location. Moreover, it is important to remark that this codification includes different horizontal and vertical locations in nodal coordinates, and the possibility of relating displacements of different nodes.

The numeric parameters of the genetic algorithm used in the numerical simulations correspond to standard values of randomized operators and another related to penalty functions as shown in Table 1.

The proposed method is applied to the design of high speed railway overhead. A typical configuration of this power transmission line is shown in Figure 4. This AC railway overhead, called EAC-350 and used in the new line Madrid-Barcelona-French Border, has been analyzed considering a bitension or dual electrical system 2×25[kV], although the proposed optimization method is also reliable for monotension systems 1×25[kV]. The line contains several physical conductors that can be grouped into three categories: positive, negative and ground or neutral wires.

Table 1: Genetic algorithm parameters

Property	***Value***
Static penalty, r	10^6
Dynamic penalty β	0.2
Crossover probability	0.5
Mutation probability	0.02
Population size	800

The positive wires are the positive phase feeder, the sustainer or messenger wire and the contact wire. There is usually only one negative wire called negative phase feeder. The neutral wires are the rails, the collector wire and the return wire. On the other hand a simplified mechanical support (Figure 5) for the two railways has been considered.

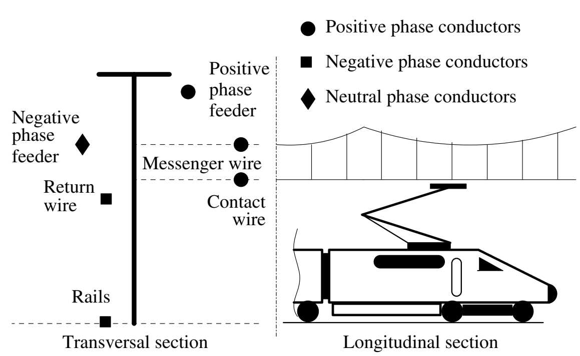

Figure 4: Typical railway overhead configuration

In the following subsections mechanical, electrical and finally electromechanical optimization will be shown.

4.1 Mechanical Optimization

The optimization of overhead supports could be included into the optimal design of truss structures topic. This problem has always been an active area of research in the field of search and optimization.

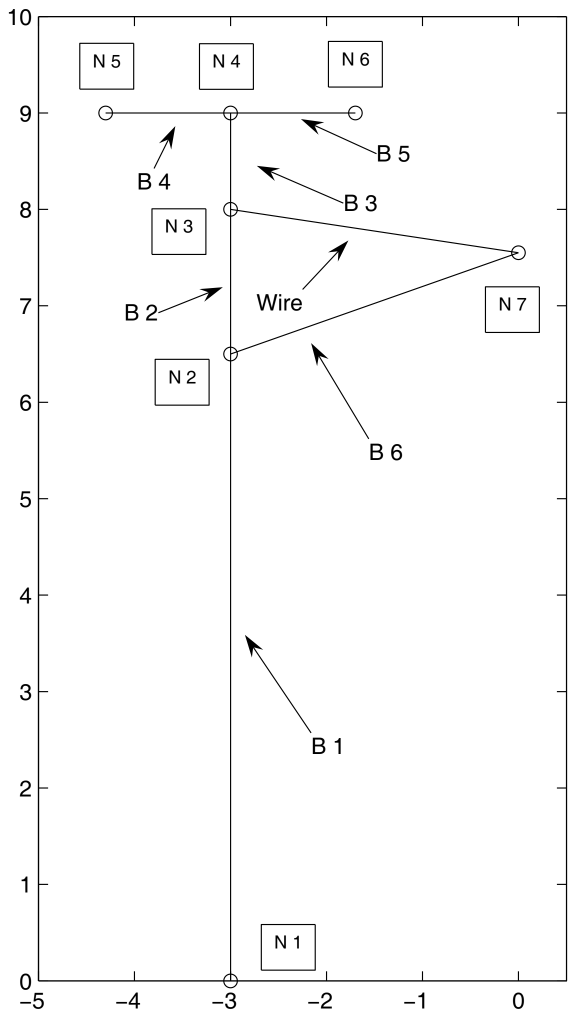

Figure 5: Simplified mechanical model

In the sizing optimization, cross-sectional areas of members are considered as design variables, restricting them to take only certain pre-specified discrete values from a UPN[1] beams standard system specification. These are the first genes built on the population's chromosomes string. Configuration optimization considers nodal coordinates as design variables too.

The material properties of the overhead supports, corresponding to common steel, are shown in the following Table 2:

Table 2: Material properties

Property	***Value***
Elastic modulus	$2.1\cdot10^{11}$ [N/m^2]
Poisson coefficient	0.3
Maximum allowable stress	$2.6\cdot10^{6}$ [N/m^2]
Density	7800 [Kg/m^3]

The following variables have been taken into account in the simplified mechanical model (Figure 5), which represents one of the two symmetric overhead railway supports. The first variable is the cross-section of all the members (from B1 to B6). In order to reduce the number of variables, a column composed of B1, B2 and B3 in the same profile set has been considered. Subsequently, they will have the same type of beam profile after optimization. This can directly improve computational costs. On the other hand, the change in nodal coordinates is possible: vertically at nodes N2, N3 and N4; vertically and horizontally at nodes N5 and N6; whereas nodes N1 and N7 are fixed. The first one for being attached to earth, and the second one for the structure gauge limited by the height of the train.

The aim of this optimization is to minimize the weight of the truss structure, whose influence is obviously related to final building costs. For this reason the evaluation function in this case only tries to minimize the global structure weight (8).

$$F(\mathbf{x}) = \begin{cases} W_m(\mathbf{x}), & \text{if } \mathbf{x} \in \Gamma \\ W_m(\mathbf{x}) + \pi, & \text{otherwise} \end{cases} \tag{8}$$

Thus, the fitness function is penalized adding a term which takes into account the structural integrity. Based on the equation (5), the penalty function can be expressed by:

$$\pi = \pi_d \cdot r \cdot G \tag{9}$$

$$G := \sum_{i=1}^{n_i} \delta_i \cdot |S_i - \sigma_{cr,i}(\sigma_i, \lambda_i)| \tag{10}$$

where the inequality penalty functions G_i reflect the difference between the axial stress S_i and the critical allowable stress $\sigma_{cr,i}$ of each member i. The critical allowable stress represents the maximum stress to avoid buckling or collapse of the

[1] It corresponds to the Spanish regulation NBE-EA-95. This algorithm also allows the use of another standard structural shapes.

member. This value can be obtained from structural standards and usually depends on maximum allowable stress or yield stress σ_i and the slenderness ratio λ_i.

In Figure 6 the optimized geometry together with the initial geometry of the structural support are shown. In Figure 7 evolution and convergence of the global weight W_m along generations are presented. In this figure, the optimal solution is marked with a circle, although the algorithm continues until the termination criterion is fulfilled. A good convergence tendency is shown. In Table 3 optimal design values are also presented.

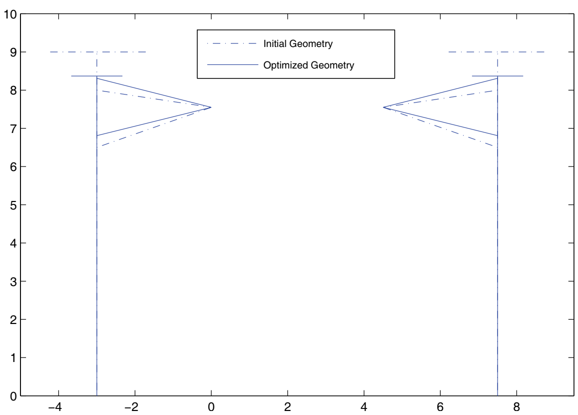

Figure 6: Initial and optimized mechanical configuration

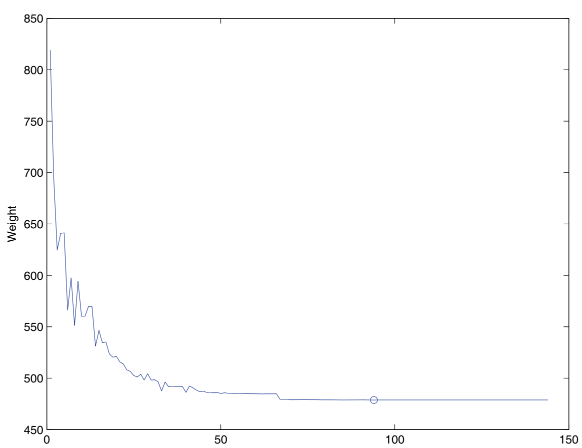

Figure 7: Global weight evolution

Table 3: Optimal weight, generation and types of beam profiles

Property	***Value***
Optimal weight	467.6 [kg]
Optimal population	94
Beam B1	UPN 180
Beam B2	UPN 180
Beam B3	UPN 180
Beam B4	UPN 80
Beam B5	UPN 80
Beam B6	UPN 80

4.2 Electrical Optimization

Electrical optimization considers two main design variables, namely: (i) type of electric conductors and (ii) their geometric location. In the first one, physical conductors are restricted to take only certain pre-specified discrete values from a conductor's standard system specification collected in Table 4.

Some important electric properties and specially the conductor's voltage are shown in Table 5.

Thus, the variables taken into account in this optimization are: on one hand, the type of electric conductor used for the positive and negative phase feeders and the return wire. The type of physical conductors used for all the others are initially fixed as: LA-180 for the contact wire; LA-180 for the messenger wire too; and UIC for the rails. On the other hand, the change of nodal coordinates is considered for the same first set of conductors, assuming the contact and messenger wires are motionless and obviously the rails too. In this aspect, return wire displacement has been restricted, having less action space than the others.

The aim of this optimization is two fold: firstly, to minimize the weight of the electrical conductors, whose influence is obviously related to final costs. And, secondly, the equivalent impedance minimization is considered. This second objective is related to the improvement of the efficiency of the electrical system.

From the electrical point of view, power transmission lines are usually modeled by the concen-

Table 4: Standard system specification of conductors

Standard name	Radius [m]	Internal resistance [Ω/m]	Internal reactance [Ω/m]
LA-30	$3.57{\cdot}10^{-3}$	$1.18{\cdot}10^{-3}$	$1.48{\cdot}10^{-5}$
LA-56	$4.72{\cdot}10^{-3}$	$6.74{\cdot}10^{-4}$	$1.48{\cdot}10^{-5}$
LA-78	$5.67{\cdot}10^{-3}$	$4.68{\cdot}10^{-4}$	$1.48{\cdot}10^{-5}$
LA-110	$7.00{\cdot}10^{-3}$	$3.37{\cdot}10^{-4}$	$1.45{\cdot}10^{-5}$
LA-145	$7.87{\cdot}10^{-3}$	$2.66{\cdot}10^{-4}$	$1.45{\cdot}10^{-5}$
LA-180	$8.75{\cdot}10^{-3}$	$2.16{\cdot}10^{-4}$	$1.45{\cdot}10^{-5}$
LA-280	$1.09{\cdot}10^{-2}$	$1.31{\cdot}10^{-4}$	$1.47{\cdot}10^{-5}$
LA-380	$1.27{\cdot}10^{-2}$	$9.42{\cdot}10^{-5}$	$1.48{\cdot}10^{-5}$
LA-455	$1.39{\cdot}10^{-2}$	$7.89{\cdot}10^{-5}$	$1.48{\cdot}10^{-5}$
LA-545	$1.52{\cdot}10^{-2}$	$6.55{\cdot}10^{-5}$	$1.48{\cdot}10^{-5}$
LA-635	$1.64{\cdot}10^{-2}$	$5.61{\cdot}10^{-5}$	$1.48{\cdot}10^{-5}$
100Cu	$5.64{\cdot}10^{-3}$	$1.93{\cdot}10^{-4}$	$1.50{\cdot}10^{-5}$
150Cu60%	$6.91{\cdot}10^{-3}$	$2.15{\cdot}10^{-4}$	$1.50{\cdot}10^{-5}$
225Cu	$8.46{\cdot}10^{-3}$	$5.59{\cdot}10^{-5}$	$1.50{\cdot}10^{-5}$
UIC-60	$4.95{\cdot}10^{-2}$	$2.47{\cdot}10^{-5}$	$1.50{\cdot}10^{-4}$

Table 5: Conductor's voltage

Property	***Value***
Frequency	50 [Hz]
Electric demand	20[MVA], per railway
Bay length	60[m]
Contact wire	25 [kV]
Messenger wire	25 [kV]
Positive feeder	25 [kV]
Negative feeder	-25 [kV]
Return wire	0 [kV]
Rails	0 [kV]

trated parameter π-model, see for instance Dommel (1986). This model reduces power transmission lines to serial impedance and shunt admittance. The serial impedance matrix represents the effect of resistance, self-inductance and magnetic coupling between conductors. While the shunt admittance matrix represents capacitances and the leakage resistances between conductors (or between conductors and earth). Moreover, due to the fact that sectors of high speed railways are about 50 km long, shunt effects are commonly neglected.

Physical conductor models represent each conductor of the railway overhead separately (Figure 4). In such models, serial impedance matrices can be calculated from Carson formulas that take into account the effect of earth currents.

Grounded conductor voltages can be assumed to be zero, and the equivalent ground conductor can be eliminated. Consequently, dual system catenaries are usually modeled as two mutually coupled conductors.

In the equivalent conductors model used in the electrical optimization, voltage drops can be expressed as:

$$Z \cdot \begin{bmatrix} i_p \\ i_n \end{bmatrix} = \begin{bmatrix} z_{pp} & z_{pn} \\ z_{np} & z_{nn} \end{bmatrix} \cdot \begin{bmatrix} i_p \\ i_n \end{bmatrix} = \begin{bmatrix} v_p \\ v_n \end{bmatrix} \tag{11}$$

where Z is the serial impedance matrix of the section of the railway overhead, and their subindexes represents the positive and negative conductors respectively, being impedance expressed per length unit [Ω/m].

Finally, the equivalent impedance used in this optimization can be written following Pilo (2003) and Pilo, Rouco and Fernández (2003), as follows:

$$Z_{eq} = \left| \frac{z_{pp} \cdot z_{nn} - z_{pn} \cdot z_{np}}{z_{pp} + z_{nn} + z_{pn} + z_{np}} \right| \tag{12}$$

Thus, the evaluation function considers the two objectives, that is electrical weight and equivalent impedance together with a penalty term to include constraint violation and is written as:

$$F(\mathbf{x}) = \begin{cases} W_e(\mathbf{x}) + \alpha \cdot Z_{eq}(\mathbf{x}), & \text{if } \mathbf{x} \in \Im \\ W_e(\mathbf{x}) + \alpha \cdot Z_{eq}(\mathbf{x}) + \pi, & \text{otherwise} \end{cases} \quad (13)$$

where the parameter α=2·10^5 is necessary to adjust equivalent impedance to the same order of magnitude of the weight. Moreover, the penalty function can be expressed as:

$$\pi = \pi_d \cdot \sum_{j=1}^{5} r_j \cdot G_j \quad (14)$$

Being G_j different violations of the electrical constrains that can be broken down into the following terms:

$$G_1 := \sum_{i=1}^{ni} \delta_i \cdot |I_i - I_{\max,i}|$$

$$\text{being } \delta_i \text{ active if } I_i > I_{\max,i} \quad (15)$$

$$G_2 := \sum_{i=1}^{n\hat{i}} \delta_{\hat{i}} \cdot \left| I_{\hat{i}} - I_{dem,\hat{i}} \right|$$

$$\text{being } \delta_{\hat{i}} \text{ active if } I_{\hat{i}} < I_{dem,\hat{i}} \quad (16)$$

$$G_3 := \sum_{i=1}^{ni} \delta_i \cdot \left| D_{i,f} - d_{i,f} \right|$$

$$\text{being } \delta_i \text{ active if } D_{i,f} > d_{i,f} \quad (17)$$

$$G_4 := \sum_{j=1}^{ni} \sum_{i>j}^{ni} \delta_{ij} \cdot \left| D_{i,j} - d_{i,j} \right|$$

$$\text{being } \delta_{ij} \text{ active if } D_{i,j} > d_{i,j} \quad (18)$$

$$G_5 := \sum_{i=1}^{ni} \delta_i \cdot |D_{i,s} - d_{i,s}|$$

$$\text{being } \delta_i \text{ active if } D_{i,s} > d_{i,s} \quad (19)$$

These equations reflect electric current and geometric constraints. On one hand, equation (15) penalizes over current in each conductor i, while (16) tries to ensure electric demand satisfaction of each set of electrical conductors with equal voltage $\hat{i}$. On the other hand, equations (17), (18) and (19) respond to the structure gauge to limit the position of physical conductors. Upper case D's denote minimum distances limited by electrical standards while lower case d's are the actual distances. More particularly, the distance violation between each conductor i and the field f is described by equation (17); between conductors i and j is defined by equation (18); and between each conductor i and the structural supports s is shown in equation (19). The numerical values of D's have been defined by the Spanish regulation of the Ministerio de Industria y Energía (1989). More detailed information can be found in Jimenez-Octavio, Pilo, Lopez-Garcia and Carnicero (2006).

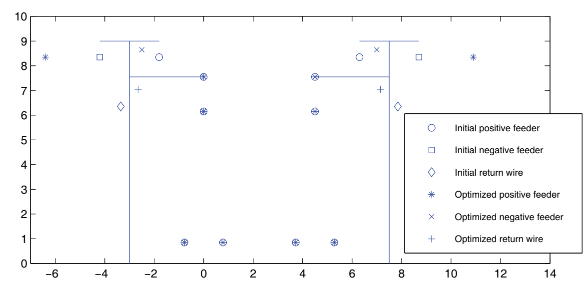

Figure 8: Initial and optimized electrical configuration

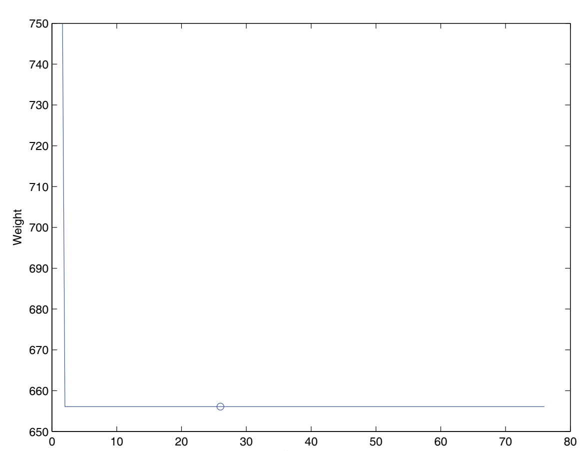

Figure 9: Global weight evolution

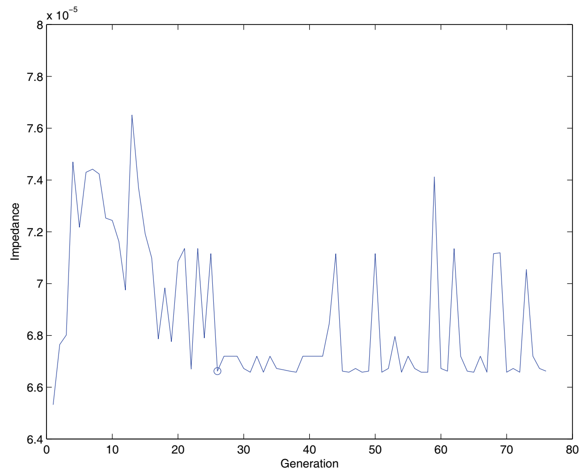

Figure 10: Equivalent impedance evolution

In what follows, electrical optimization results are exposed. In Figure 8 the electrical optimized configuration together with the initial geometry is shown. In Figure 9 and Figure 10 evolution and convergence of global weight and global equivalent impedance are respectively pictured along generations.

Equivalent impedance minimization implies just as can be observed in Figure 8: that the positive phase feeder moves far away from the rest of positive conductors, decreasing mutual series inductance (although at the same time this decreases magnetic field cancellation with the negative phase feeder); while the negative phase feeder locates close to the positive equivalent magnetic center location, trying to reduce global equivalent impedance too. Note that these excessive location displacements are allowed because this model is not yet coupled to the mechanical one. Nevertheless, free location displacements could be expected to move positive and negative phase feeders further down in order to reduce their own series inductance, but geometric penalties (particularly equation (17)) leads the genetic algorithm to search in upper regions.

Sizing optimization, shown in Figure 9, converges very quickly (less than 5 populations), but equivalent impedance evolution of Figure 10 presents an oscillatory tendency. There are two reasons to explain this behavior. Firstly, there are only three kinds of conductors to optimize, thus, with a big population of 800 individuals it is quite easy to reach a fast convergence in this aspect. Secondly, equivalent impedance could modify both kinds of conductors and location. Hence, though minimum impedance is reached earlier, equivalent impedance continues changing with each population without improving its minimum value.

Nevertheless, the behavior of the equivalent impedance as a function of the number of generations is concerned with the agreement between exploration and exploitation mentioned in the problem formulation. Ideally, as suggested by Coello (2002), penalty factors should remain just close to the optimality feasible border. This is a key topic that can be observed from two perspectives. On one hand, if the penalty is too high the optimal solution has a high tendency of nearing the feasible border and, consequently, the algorithm tries to quickly lead solutions inside this region, damaging exploration. On the other hand, if penalty factors are too low, the searching process become slower due to the excessive infeasible solutions that reflect a wide exploration in the whole searching space but a marginal exploitation of the feasible region.

Numerical results are shown in Table 6, which presents optimal weight, impedance and population; and also the selected conductors in the electrical optimization.

Table 6: Optimal weight, impedance, population; and types of electrical conductors

Property	***Value***
Optimal weight	656 [kg]
Optimal Z_{eq}	$6.66{\cdot}10^{-5}$ [Ω/m]
Optimal population	26
Messenger wire	LA-180
Contact wire	LA-180
Positive feeder	100 Cu
Negative feeder	100 Cu
Return wire	100 Cu
Rails	UIC-60

4.3 Electromechanical Optimization

Coupled electromechanical optimization includes both previous electrical and mechanical optimizations as is shown in the evaluation function (20), whose purpose is to add the three optimization objectives with all mechanical and electrical penalties symbolized with the parameter π.

$$F(\mathbf{x}) = \begin{cases} W_m(\mathbf{x}) + W_e(\mathbf{x}) + \alpha \cdot Z_{eq}(\mathbf{x}), & \text{if } \mathbf{x} \in \Gamma \\ W_m(\mathbf{x}) + W_e(\mathbf{x}) + \alpha \cdot Z_{eq}(\mathbf{x}) + \pi, & \text{otherwise} \end{cases} \tag{20}$$

The electromechanical model completely couples both optimization variables and design constraints. On one hand, variables build chromosome strings with the same criterion as before: firstly, sizing variables like cross sections for structural members or type of wires for electrical conductors; and next, geometric location or configuration for all the nodes, both mechanical and electrical, that define the whole geometric problem. On the other hand, constraints considered are: the maximum allowable stress and displacements, and buckling conditions in the mechanical problem (equations (9) and (10)); in addition to the structure gauge to limit the position of physical conductors, minimum distance between conductors (18), between each conductor and the ground (17) and between conductors and overhead supports (19), and finally maximum allowable current of each conductor (15) and electric demand satisfaction (16) in the electrical problem.

Some graphical results of the electromechanical optimization appear below. Figure 11 shows the optimized configuration together with the initial geometry. In Figure 12 and Figure 13 evolution and convergence of electrical and mechanical weights and equivalent impedance are respectively shown as functions of the number of generations.

As previously observed, Figure 11 shows that optimum configuration is lead by weight minimization instead of equivalent impedance optimization.

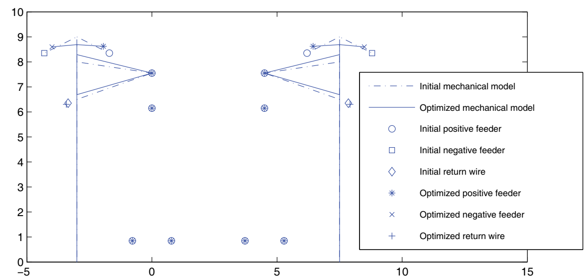

Figure 11: Initial and optimized electromechanical configuration

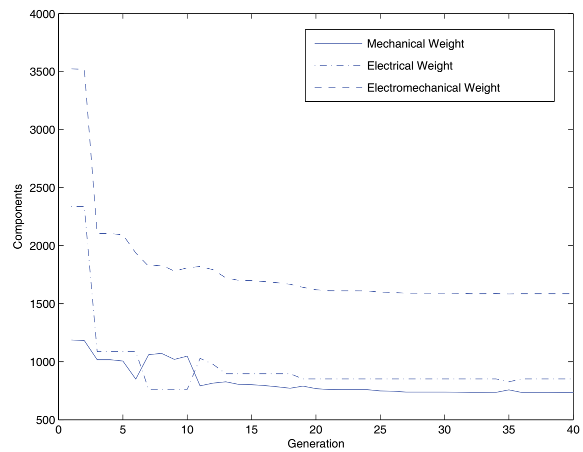

Figure 12: Electrical and mechanical weights evolution

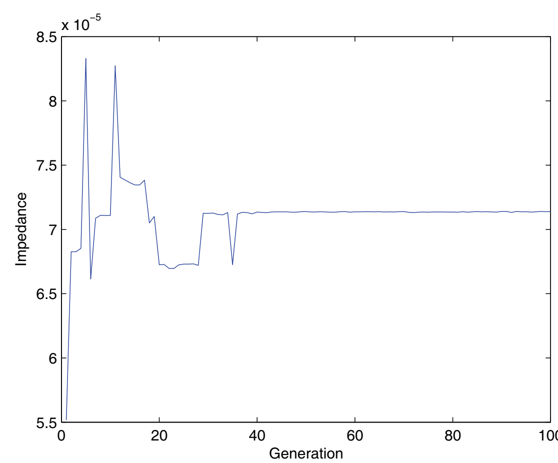

Figure 13: Equivalent impedance evolution

Coupled electromechanical optimization is more constrained than uncoupled ones, electrical or mechanical. Basically, the coupled problem not only includes the mechanical and electrical constraints but also objective function and design variables are fully coupled. Then, the electromechanical optimization algorithm should explore smaller feasible regions of the space of design variables. Therefore, the fact that the electromechanical feasible region is a more restricted environment of configurations inevitably leads to optimized solutions which are relatively close to traditional and somewhat expected configurations.

There are also detailed numerical results in Table 7 which shows optimal weights, impedance and population; the optimal structure members obtained; and finally, the optimal physical conductors.

Table 7: Optimal weights, equivalent impedance, population; types of beam profiles; and types of electrical conductors

Property	***Value***
Optimal mechanical weight	482 [kg]
Optimal electrical weight	656 [kg]
Optimal Z_{eq}	$7.1\cdot10^{-5}$ [Ω/m]
Optimal population	38
Beam B1	UPN 180
Beam B2	UPN 180
Beam B3	UPN 180
Beam B4	UPN 80
Beam B5	UPN 80
Beam B6	UPN 80
Messenger wire	LA-180
Contact wire	LA-180
Positive feeder	100 Cu
Negative feeder	100 Cu
Return wire	100 Cu
Rails	UIC-60

5 General conclusions

A general framework to electromechanically optimize power transmission lines has been presented. Due to its multi-objective nature, complex multidisciplinary designs, that is, both mechanical and electrical, can be optimized. In order to guarantee that the optimum design corresponds to a global optimum an evolutionary strategy has been selected. A genetic algorithm has been developed and implemented for sizing and configuration optimization of electromechanical problems. Furthermore, the ranking selection scheme has been tailored to improve the behavior of the optimization algorithm.

The validity of the methodology proposed herein is independent of the complexity of the models used to evaluate structural and electrical designs. Evidently, the multi-objective nature of the method, which is easily embedded into the genetic algorithm, provides the model's independence of the optimization algorithm.

Regarding the different kind of optimization problems carried out, the genetic algorithm have exhibited an independent behavior on the penalty constants. Moreover, the same set of constants have been found to be good enough to be valid to optimize electrical, mechanical and electromechanical problems.

More particularly, and regarding the application of the method to railway overhead transmission lines the conclusions are numerical efficiency, theoretical consistency and finally a realistic design solution.

From the numerical point of view, an excellent and fast convergence behavior of the genetic algorithm is obtained. Regarding this aspect, the development of the so-called PSMER ranking selection scheme has become one of the most determining factors. The numerical results of the algorithm have exhibited a robust behavior shared by all described optimization problems.

The following theoretical conclusions are worth mentioning. From the comparison between mechanical, electrical and electromechanical optimization, two tendencies have been observed. On one hand, weight is the responsible for the solution of the sizing optimization problem, and on the other hand equivalent impedance determines the solution of the configuration optimization problem. Thus, proven the versatility of the methodology and taking into account the suitability of genetic algorithms, the incorporation of more complex physical couplings, like termomechanic, electromagnetic, wind load effects, etc. can be easily included.

The electromechanical optimization solution represents a trade-off between the multidisciplinary nature of the problem and the sort of optimization, sizing and configuration. As previously stated, one aim of the presented method is the production of affordable optimal designs. Therefore, the combination of both arguments leads to an optimized design close to a conservative one.

Consequently, if more innovative or unexpected designs are required, topological optimization is the next step ahead whose application allows the obtention of more advanced designs and results. The authors are currently working on its implementation.

Acknowledgement: This work has been partially supported by the Spanish *Ministerio de Educación y Ciencia* (Ministry of Education and Science) under the project *Cálculo del desgaste en el sistema de captación de energía de trenes de alta velocidad TRAN2004-06889/TREN* (Wear evaluation in high speed electric overhead systems). This support is gratefully acknowledged.

References

Bai, M. R.; Chinghong, H. (2003): Optimization and implementation of piezoelectric radiators using the genetic algorithm. *Journal of the Acoustical Society of America*, vol. 113, pp. 3197-208.

Carroll, C. W.; Fiacco, A. V. (1961): The created response surface technique for optimizing nonlinear restrained systems. *Operations Research*, vol. 9, pp. 169-85.

Coello, C. A. C. (1999): An updated survey of evolutionary multiobjective optimization techniques: state of the art and future trends. *Proceedings of the 1999 Congress on Evolutionary Computation-CEC99 (Cat. No. 99TH8406)*, vol. 1, pp. 3-13.

Coello, C. A. C. (2002): Theoretical and numerical constraint-handling techniques used with evolutionary algorithms: a survey of the state of the art. *Computer Methods in Applied Mechanics and Engineering*, vol. 191, pp. 1245-87.

Coello, C. A. C.; Christiansen, A. D. (1998): Two New GA-based Methods for Multiobjective Optimization. *Civil Engineering Systems*, vol. 15, pp. 207-43.

Coello, C. A. C.; Pulido, G. T.; Montes, E. M. (2005): Current and Future Research Trends in Evolutionary Multiobjective Optimization. In *Information Processing with Evolutionary Algorithms: From Industrial Applications to Academic Speculations*, pp. 213-31.

Courant, R. (1943): Variational Methods for the Solution of Problems of Equilibrium and Vibrations. *Bulletin of the American Mathematical Society*, vol. 49, pp. 1-23.

Darwin, C. (1929): On the Origin of Species by Means of Natural Selection or the Preservation of Favored Races in the Struggle for Life, *The Book League of America*, 1929. Originally published in 1859.

de Lacerda, L. A.; da Silva, J. M. (2006): A dual BEM genetic algorithm scheme for the identification of polarization curves of buried slender structures. *CMES: Computer Modeling in Engineering & Sciences*, vol. 14, pp. 153.

Deb, K.; Gulati, S. (2001): Design of truss-structures for minimum weight using genetic algorithms. *Finite Elements in Analysis and Design*, vol. 37, pp. 447-65.

Deb, K.; Pratap, A.; Agarwal, S.; Meyarivan, T. (2002): A fast and elitist multiobjective genetic algorithm: NSGA-II. *IEEE Transactions on Evolutionary Computation*, vol. 6, pp. 182-97.

Di Barba, P.; Farina, M.; Savini, A. (2001): Multiobjective design optimization of real-life devices in electrical engineering: a cost-effective evolutionary approach. *Evolutionary Multi-Criterion Optimization. First International Conference, EMO 2001. Proceedings (Lecture Notes in Computer Science Vol.1993)*, vol. 1993, pp. 560-73.

Dommel, H. W. (1986): Electromagnetic transients program reference manual. Vancouver, Canada: Bonneville Power Administration, 1986.

Fiacco, A. V.; McCormick, G. P. (1966): Extensions of SUMT for nonlinear programming: equality constraints and extrapolation. *Management and Science*, vol. 12, pp. 816-28.

Goldberg, D. E. (1989): Genetic Algorithms in Search, Optimization and Machine Learning. Massachusetts: Addison-Wesley Publishing Co., 1989.

Goldberg, D. E.; Samtani, M. P. (1986): Engineering optimization via genetic algorithms. *Proceedings of the Ninth Conference on Electronic Computations*, pp. 471-82.

Hoffmeister, F.; Sprave, J. (1996): Problem-independent handling of constraints by use of metric penalty functions. In *Evolutionary Programming V. Proceedings of the Fifth Annual Conference on Evolutionary Programming*, pp. 289-94.

Holland, J. H. (1975): Adaptation in Natural and Artificial Systems. *University of Michigan Press*.

Imai, K.; Schmit, L. A. J. (1981): Configuration Optimization of Trusses. *Journal of the Structural Division*, vol. 107, pp. 745-756.

Jimenez-Octavio, J. R.; Pilo, E.; Lopez-Garcia, O.; Carnicero, A. (2006): Coupled Electromechanical Cost Optimization of High Speed Railway Overhead. *Proceedings of the Joint Rail Conference*, vol. 31, pp. 231-240.

Kaveh, A.; Kalatjari, V. (2003a): Size/geometry optimization of trusses by the force method and genetic algorithm. *ZAMM*, vol. 84, pp. 347-357.

Kaveh, A.; Kalatjari, V. (2003b): Topology Optimization of Trusses using Genetic Algorithm, Force Method and Graph Theory. *International Journal for Numerical Methods in Engineering*, vol. 58, pp. 771-791.

Kazarlis, S.; Petridis, V. (1998): Varying fitness functions in genetic algorithms: studying the rate of increase of the dynamic penalty terms. In *Parallel Problem Solving from Nature - PPSN V. 5th International Conference. Proceedings*, pp. 211-20.

Krish, U. (1989): Optimal topologies of truss structures. *Computer Methods in Applied Mechanics and Engineering*, vol. 72, pp. 15-28.

Krokosky, E. M. (1968): The ideal multifunctional constructural material. *Journal of the Structural Division*, vol. 94, pp. 958-81.

Lagaros, N. D.; Papadrakakis, M.; Kokossalakis, G. (2002): Structural optimization using evolutionary algorithms. *Computers & Structures*, vol. 80, pp. 571-89.

Lamberti, L.; Pappalettere, C. (2007): Weight Optimization of Skeletal Structures with Multi-Point Simulated Annealing. *CMES: Computer Modeling in Engineering & Sciences*, vol. 18, pp. 183.

Liu, M. (2003): Development of Multiobjective Optimization Procedures for Seismic Design of Steel Moment Frame Structures, pp. 259. *Department of Civil an Environmental Engineering*. University of Illinois at Urbana-Champaign, Illinois.

Messine, F.; Nogarede, B.; Lagouanelle, J. L. (1998): Optimal design of electromechanical actuators: a new method based on global optimization. *IEEE Transactions on Magnetics*, vol. 34, pp. 299-308.

Michael Yu, W.; Shiwei, Z. (2006): Phase field: a variational method for structural topology optimization. *CMES: Computer Modeling in Engineering & Sciences*, vol. 6, pp. 547.

Ministerio de Industria y Energía. (1989): Reglamento de Líneas Aéreas de Alta Tensión. Centro de Publicaciones del Ministerio de Industria y Energía. Madrid, Spain.

Missoum, S.; Gürdal, Z. (2002): Displacement-Based Optimization for Truss Structures Subjected to Static and Dynamic Constraints. *AIAA Journal*, vol. 40, pp. 154-161.

Morales, A. K.; Quezada, C. V. (1998): A universal eclectic genetic algorithm for constrained optimization. In *6th European Congress on Intelligent Techniques and Soft Computing. EUFIT '98*, vol. vol.1, pp. 518-22.

Morino, L.; Bernardini, G.; Mastroddi, F. (2006): Multi-Disciplinary Optimization for the Conceptual Design of Innovative Aircraft Configurations. *CMES: Computer Modeling in Engineering & Sciences*, vol. 13, pp. 1-18.

Osyczka, A. (1978): An approach to multicriterion optimization problems for engineering design. *Computer Methods in Applied Mechanics and Engineering*, vol. 15, pp. 309-33.

Papadrakakis, M.; Lagaros, N. D.; Thierauf, G.; Jianbo, C. (1998): Advanced solution methods in structural optimization based on evolution strategies. *Engineering Computations*, vol. 15, pp. 12-34.

Papadrakakis, M.; Tsompanakis, Y.; Lagaros, N. D. (1999): Structural Shape Optimization Using Evolution Strategies. *Engineering optimization*, vol. 31, pp. 515-40.

Pilo, E. (2003): Diseño óptimo de la electrificación de ferrocarriles de alta velocidad, pp. 308. *Departamento de Electrotecnia y Sistemas*. Universidad Pontificia Comillas, Madrid.

Pilo, E.; Rouco, L.; Fernández, A. (2003): A reduced representation of 2x25kV electrical systems for high-speed railways. *Proceedings of the Joint Rail Conference*, pp. 199-205.

Rajeev, S.; Krishnamoorthy, C. S. (1992): Discrete optimization of structures using genetic algorithms. *Journal of Sructural Engineering*, vol. 118, pp. 1233-50.

Regnier, J.; Sareni, B.; Roboam, X. (2005): System optimization by multiobjective genetic algorithms and analysis of the coupling between variables, constraints and objectives. *COMPEL - The International Journal for Computation and Mathematics in Electrical and Electronic Engineering*, vol. 24, pp. 805-20.

Ringertz, U. T. (1985): On topology optimization of trusses. *Engineering optimization*, vol. 9, pp. 209-18.

Shiwei, Z.; Wang, M. Y. (2006): 3D multi-material structural topology optimization with the generalized Cahn-Hilliard equations. *CMES: Computer Modeling in Engineering & Sciences*, vol. 16, pp. 83.

Sigmund, O. (2001a): Design of multiphysics actuators using topology optimization. I. One-material structures. *Computer Methods in Applied Mechanics and Engineering*, vol. 190, pp. 6577-604.

Sigmund, O. (2001b): Design of multiphysics actuators using topology optimization. II. Two-material structures. *Computer Methods in Applied Mechanics and Engineering*, vol. 190, pp. 6605-27.

Silva, E. C. N.; Kikuchi, N. (1999a): Design of piezocomposite materials and piezoelectric transducers using topology optimization - Part III. *Archives of Computational Methods in Engineering*, vol. 6, pp. 305-29.

Silva, E. C. N.; Kikuchi, N. (1999b): Design of piezoelectric transducers using topology optimization. *Smart Materials and Structures*, vol. 8, pp. 350-64.

Van Veldhuizen, D. A.; Lamont, G. B. (1998): Multiobjective Evolutionary Algorithm Research: a History and Analysis. *Air Force Institute of Technology*.

Wang, S. Y.; Wang, M. Y. (2006): Structural Shape and Topology Optimization Using an Implicit Free Boundary Parametrization Method. *CMES: Computer Modeling in Engineering & Sciences*, vol. 13, pp. 119-48.

Yoon, S. B.; Hur, J.; Chun, Y. D.; Hyun, D. S. (1997): Shape optimization of solenoid actuator using the finite element method and numerical optimization technique. *IEEE Transactions on Magnetics*, vol. 35, pp. 4140-4142.

Yoon, S. B.; Jung, I. S.; Hyun, D. S.; Hong, J. P.; Kim, Y. J. (1999): Robust shape optimization of electromechanical devices. *IEEE Transactions on Magnetics*, vol. 35, pp. 1710-13.

Zhu, Z. Q.; Liu, Z.; Wang, X. L.; Yu, R. X. (2004): Construction of integral objective function/fitness function of multi-objective/multi-disciplinary optimization. *CMES: Computer Modeling in Engineering & Sciences*, vol. 6, pp. 567.