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MASTER IN FINANCE

ACTIVE VS PASSIVE INVESTMENT. THE OPTIMAL DIVERSIFICATION EFFECT.

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Abstract.

The debate of active and passive investment has been out for a while. The classic investment methodology is based on the active investment. But in this thesis, it will be proved that the behaviour in a long-term investment period, the passive investment will give better results than the active investment. The second part of this thesis will be based on building a portfolio using passive investment instruments called Exchanged Traded Funds (also known as ETFs). These highly diversified instruments will prove wrong to the common belief of using more than 30 instruments to reduce our diversifiable risk at the minimum. Instead of that, only a combination of 4 instruments will be enough to have all the benefits of the diversification.

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1. Introduction

The passive investment is the strategy whose objective is to replicate the evolution of a certain index in the market. Unlike active management, passive investment tries to minimize the frequency of investment aiming to achieve long-term benefits. In the other hand, the active investment strategy is based in the buying and selling of individual securities, and continuously monitoring their performance in order to beat a certain index in the market.

The upcoming tendency of the passive investment is to gain market share to the active investment strategy, as we can already see in the numerous reports that study this relationship.

This is changing the way of investing. People are getting higher returns assuming less risk and in a cheaper way than it was done before. This is because there is no need to buy numerous assets to have your risk diversified and invest a high amount of money for it.

So, the objective of this thesis is to study the performance of both kind of investment, proving the advantages and the positive tendency of the passive investment. Adding up, the second part of this thesis will be focused in the diversifiable risk reduction of Markowitz´s Minimum Variance Portfolio Theory, using passive investment instruments known as Exchanged Traded Funds to build a portfolio. Therefore, we will be responding to the following subjects:

- The kind of investment (passive or active) that have a better performance of the investment and the tendency of the market share over time.
- The number of instruments needed to reduce the risk of a portfolio as much as possible. This will explain the highly effect of diversification by using passive investment instruments instead of active investment instruments.

2. Passive vs Active Investment

In this part of the thesis we will be comparing the performance of the passive and active investment in different periods of time, to determine which one has been more successful in the last years. This will be decisive in order to see, in which type of investments are the investors willing to put their savings and in which direction are the flows of these investments turning.

2.1. Performance Indicators

The passive investment has gained an important reputation and market share in the last years, and it is proved in many researches of reputable companies like Bloomberg, Morgan Stanley, Morningstar and Standard and Poor (S&P Dow Jones Indices), who study the success rate of the active investment.

To consider that the active company has succeed against passive investment companies, there have to be two main conditions:

1. The fund that started the sample period went on to survive at the end of the period.
2. Generate a return in excess over the passive fund return over the period.

It is important to notice that this has to be done in the point of view of the active investment because it doesn´t make sense to do in the opposite way as the objective of the passive investment is to track the index and not surpass it. And even if passive instruments track the market, they will be always be bellow due to the transaction’s costs involved in buying those instruments.

One of the most reliable company doing this research is Morningstar in its Active vs Passive Barometer. This document measures the performance of European domiciled active funds against passive peers in their respective Morningstar Categories. As we can see in the table 1 of the appendix, in the European Large-Cap Blend Market, only 22% of the active funds succeed over their passive peers. In the Spanish Market this percentage was over 50%, but in the long term, less than 30% of the funds could succeed to their passive peers. This percentage is even higher in Global Equity, where, according to Morningstar, more than 80% of the firms, are not able to succeed against passive investors.

The S&P Dow Jones, in its publication of SPIVA (S&P Indices Versus Active Funds) also measures the performance of actively managed equity against the performance of their respective S&P DJI benchmark Indexes over different time horizons. In the short period of time (1-Year comparison), the percentage of European equity funds outperformed by their benchmarks is almost 86% in Europe and 59% in Spain. As longer is the period of studied, this number tends to go higher, reaching at 98,48% in the global equity in a 10 years' time horizon.

In their latest report of S&P, SPIVA Around the World, they analyse globally the percentage of active funds outperformed by their respective benchmarks over different periods of time (1-, 3- and 5-year periods). The following table shows us the most important keys behind this research.

Table 1. Percentage of active funds outperformed by benchmarks 'over 1-, 3-, and 5-year periods.

Country	1-Year	3-Year	5-Year	Benchmark
Australia	87%	86%	80%	S&P/ ASX 200
Brazil	56%	85%	84%	S&P Brazil BMI
Canada	77%	94%	90%	S&P/ TSX Composite
Chile	60%	86%	81%	S&P Chile BMI
Europe	86%	86%	80%	S&P Europe 350
India	92%	91%	58%	S&P BSE 100
Japan	84%	57%	59%	S&P Topix 150
Mexico	42%	89%	86%	S&P/ BMV IRT
South Africa	39%	70%	77%	S&P South Africa DSW Capped
United States	64%	79%	82%	S&P 500

Source: S&P Dow Jones Indices LLC, Morningstar, Fundata, CRSP. Data as of December 31, 2018. Charts and tables are provided for illustrative purposes. Past performance is no guarantee of future results.

The performance of the passive investment has been surpassed by their respective Benchmark in almost all the different countries. In some cases, the percentage of funds that didn´t surpass their benchmark is considerably high even in the first years, which proves that the passive investment is not only made for long-term periods of investments, but that it can also perform better than the active investment in short-term periods of time.

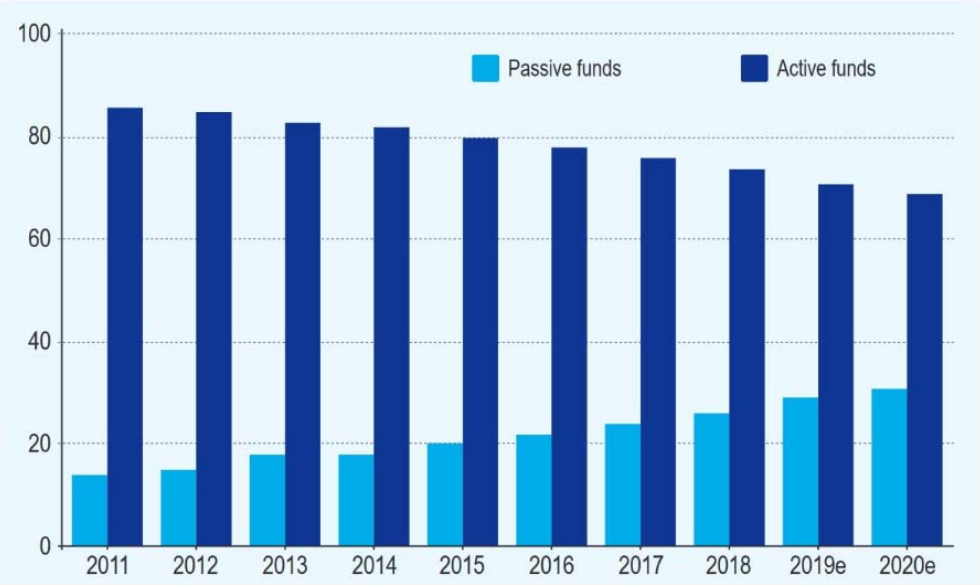
Bloomberg has also done this research (appendix 5), being the latest one available for the year 2017. The period of study is 10 years, and the results are also favourable for the passive investment. The percentage of active funds outperforming passive funds over 1-, 3-, 5- and 10-years have been really low. According to that research, the average equity and the average fixed income who surpassed to their passive peers have been only 25% and 24%, respectively.

2.2. Market Share of the different investment styles over time and future tendency.

The passive investment has been gaining market share over the active investments in all the different categories and the positive results of it is making the investors to move from one investment style to another in a faster path.

The following graph shows us the evolution of the both styles of investment from 2011. Even though the volume of the active investment is considerably high, the percentage of market share of the passive investment has almost doubled. The Active investment has lost almost 20 percent of the market share in the same period.

Figure 1. Distribution of Passive and active Exchanged Traded Funds (ETFs) worldwide 2011 to 2020e.



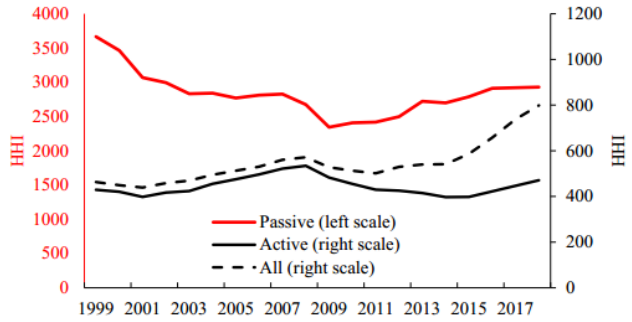
Source: UBS, EFAMA, ETFGI. Data as per 31/12/2018

The net flows of the passive investments have been growing in a stable and consistent path. The active investment has lost an important amount of market share specially in times of uncertainty, as the market situation is not optimal, like in the financial crisis of 2018. In those periods, the flows tend to swift from active to passive investment. This is one of the many reasons that the passive investment has gained higher reputation in the last years.

In concern of that, the Federal Reserve Bank of Boston, wrote an article about the possible effects from changing to one kind of investment to another, called, “The Shift from Active to Passive Investing: Potential Risks to Financial Stability?” In there, they analyse the different kind of possible effects that the swift of the active investment can have in volatility, liquidity and co-movement in assets returns and liquidity. The conclude adding that this changes in investment strategies has a direct effect on the asset management industry. Even though the results don´t prove implication in the financial stability, they around about the future of the investment is going to change as the passive investment has diminished the risk related to liquidity transformation; and the cost are going to be reduced due to the highly competence of the passive investment companies, as they are cheaper and give higher returns compared to the active firms.

The following graph shows us how the market share has been higher in the passive investment in the recent years compared to active investment:

Figure 2. Concentration of the active and passive Mutual Funds and ETFs.



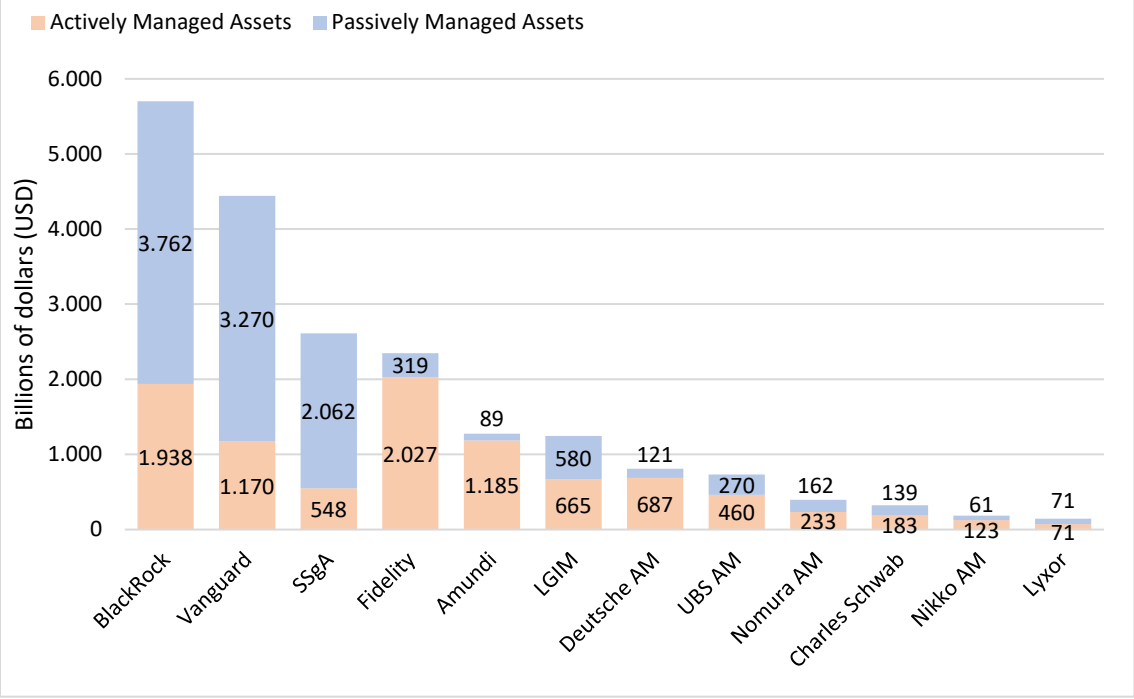
Source: Centre for Research in Securities Prices. Wharton Research Data Services.

The actual situation in Spain of the passive investment, is not the best. The proportion of passive funds domiciled in Spain is just a 2%, but we have to consider that this percentage doesn´t include the investments done outside Spanish territory by Spanish investors. But as the tendency of Spain is to follow the tendency of Europe and then, United States, which have a higher weight, we can say that this percentage will increase in the following years considerably

In Europe, the countries where most of the passive investment instruments are domiciled are Ireland and Luxembourg. They include more than 60% of all the passive investment domiciliation.

2.3. Top Active Funds by Assets under Management.

Figure 3. Surveyed Managers with Total AUM Split into Passive and Active Assets.



Source: Asset Manager Data. Data as of 30/06/2017.

As per 31st December of 2018, the total asset managed by passive investment funds is more than 10 trillion US Dollars. The top 3 of those funds manage approximately the 70% of that amount,

which are Blackrock, Vanguard, and State Street. This amount is really significant, and we have to consider that they manage more than any other fund in the world.

This changes a lot in the active investment, as the asset are more distributed and not handled by only 3 companies.

So, with this, it can be said that passive investment not only have a better performance in a long period of time but also that a lot of investors are noticing that and sifting their investment.

3. Portfolio theories

Having reached to the conclusion that in the long term the passive investment has more chances to surpass in performance to the active investment, and it is gaining market share, we are going use passive instruments to create an optimal portfolio for an investment. But before that, the different theories that involve the building of that portfolio are going to be explained.

For that, we are going to express the ideas of Markowitz´s Modern Portfolio Theory, William Sharpe´s Capital Asset Pricing Model and the Efficient Market hypothesis of Fama.

The concepts of this theories come along with the idea of Markowitz that the investors are risk averse. This idea express that they will invest in a portfolio with a lower risk given an amount of return than assume more risk for it. Later on, William Sharpe came out with his theory of Capital Asset Pricing Model (CAPM, A theory of market equilibrium under conditions of Risk, 1964). This theory symbolized the expected return of all the assets of a market as a function of Unsystematic risk, (also known as the "specific risk") and systemic risk (or market risk), calculated by the beta, to show how the market should estimate the price of an individual asset in relation to it peers.

3.1. Markowitz´s Modern Portfolio theory.

In the 1950s, in order to measure and quantify in a specific way the amount of risk, the portfolio Modern Theory was proposed by Harry Markowitz. He derived the expected rate of return for a given portfolio of assets and the measurement of the risk of that portfolio. He proposed a model for dealing with issues concerning choices which involve a great amount of possible financial instruments. According to this paradigm, the goal of the Investor is to select the portfolio of securities that will provide the best distribution of future consumption, given the amount of the investment susceptible to risk. There are mainly two measures that are assumed to be enough for evaluating the desirability of the investor: the expected or mean value and the standard deviation, or the variance, of that value. This approach is often used as a measurement of risk.

Normally, investors use mean-variance analysis to decide about which financial instruments to invest in, based on how much risk they are ready to assume in exchange of different levels of returns.

Markowitz demonstrated that the variance of the expected rate of return of an asset was a good measurement for quantifying the amount of risk under some circumstances. Not only that, but from him was derived the formula that showed the significance of diversifying investments to reduce the risk of a portfolio effectively. His theory was based in the following assumptions of the behaviour of an investor:

1. Investors are rational and they will behave in a way to maximize their utility for one period.
2. All the investors have free access to fair and correct information on the risks and returns of the investments.

3. The markets are efficient, and they process the information quickly and perfectly.
4. Investors are risk averse and they will try to minimise the risk and maximise the return. So, they will choose higher returns instead of lower for a certain amount of risk. Likewise, for a given expected return level, they will prefer less risk to more risk.
5. Investors base decisions by the probability on expected returns and risk (variance or standard deviation) over the holding period.

Considering all these assumptions, we can say that a portfolio or an asset will be considered optimal if there is no other portfolio which can offer higher expected return with a same or lower level of risk, or less risk with the same or higher return.

The portfolio variance for two assets can be calculated by the following formula:

$$\sigma_p^2 = w_A^2 * \sigma^2(R_A) + w_B^2 * \sigma^2(R_B) + 2 * (w_A) * (w_B) * Corr(R_A, R_B) * \sigma(R_a) * \sigma(R_a)$$

The covariance considers the relationship between the different instruments.

If instead of two assets, we have many assets, we can use the following formula:

$$\sigma_p^2 = \sum_{i=1}^{n} \sum_{i=1}^{n} w_i w_j Cov(R_i, R_j)$$

And for the Expected return of the Portfolio:

$$E(R) = w_1 R_1 + w_2 R_2 \dots w_n R_n$$

3.2. Willian Sharpe, Capital Asset Pricing Model.

The Capital Asset Pricing Model (William Sharpe, 1964) result is one of the most famous and known theories in the actual finance, and even arriving for a one period model, it solves the problem of the asset-pricing. For instance, it is well known that a higher risk will have a higher expected return in order to compensate the investor for holding this risk, but the measurement of the risk is a concept which is difficult to calculate precisely. So, with this theory, to see how risky an asset is, it shouldn´t be measured by the returns volatility but by the Beta (β), which is a proportional to its covariance with the market portfolio.

As it was said before, the CAPM theory is derived from the assumptions of Harry Markowitz of the mean-variance analysis (1959). Sharpe (1964) and Lintner (1965) build-up on that model adding two main assumptions. This theory provides a mathematical condition on the asset weights in the mean-variance efficient portfolios. They turn this mathematical statement into a demonstrable prediction about the relationship between the risk and the expected returns by identifying a portfolio that must be efficient.

This theory was extended in a way that allowed investors to evaluate the risk and the return for diversified portfolios and for individual securities. For this, the CAPM model redefines the measure of risk from the total volatility to just stay with the non-diversifiable proportion of that total volatility, which is also known as systematic risk. For this, they introduced the new risk measurements called beta. The beta coefficient calculates the level of the systematic risk of a security compared to the market portfolio. So, using the beta as the measurement risk of the portfolio, the CAPM defines the risk premium that we expect per unit of risk. This takes us to an

expression of the expected return composed by the risk-free rate and the risk premium that we expect.

These are the two assumptions of this theory:

- Investors are risk averse. So, this theory is based that the investors are rational and risk averse. They base their conclusions on the investments normal rate of return, its standard deviation and the covariance between the rate of return of the assets and the other assets composed in the portfolio. So, this investor will select the asset that has the highest expected return rate according to the measurement of the standard deviations, and the assets with the lowest standard deviation, from compared to the expected rates of return.
  Thus, the investor will select the asset that are over on the efficient frontier, and therefore, limit their risk at the same time that the investments returns are being maximized. So, the risk-aversion of the investor will locate its optimal point in the market opportunity line. This line will show a linear function between the rate of return and the level of risk (standard deviation), and at any point of it, the investor will have an optimal point according to his expected level of risk.

- Homogeneity of expectations. So, if all investors come to the previous conclusion, they will have homogeneous expectations, meaning that they all will expect the same distributions for the expected return. When referring to homogeneity of expectations, means that all the investors will have similar beliefs about the probability of their future returns. With this assumption, we are saying that all the financial assets are equally assessible for all the investors. It is assumed then, that the investors will hold identical assets, and so on, means, variance and covariances.

  Some other assumptions for this theory are the following:

  1. The position on the efficient frontier which investors have taken a position will depend on their utility function and the relation between the risk and the expected return according to that risk.
  2. The investors will hold the instruments for the same one period of time.
  3. The investors can buy or sell the portions of their shares for the securities or portfolio that they have in their hold.
  4. The taxes and the cost of transactions are not considered in the selling or in the purchasing of the assets.
  5. It hasn't been considered the inflation or the changes in the interest rates.
  6. The capital markets are considered to be in equilibrium. So, all the investors will find their investments quoted at a fair price. Investors will not be able to affect in the prices.

3.2.1. CML vs SML

As we know, all investors are mean-variance optimizer, which means that they will assume the less risk as possible in relation of the return. So, in order to have a variety of randomly selected portfolio, each of them will hold the same tangency portfolio of risky securities in conjunction with a position in the risk-free asset. So, because of this tangency portfolio is held by all

investors, we can identify this portfolio as the market portfolio. The efficient frontier is then determined by the Capital Market Line (CML).

So basically, the Capital Market Line represents portfolios that combine risk and return optimally. It is a theoretical concept that represents portfolios that combine the risk-free rate of return and the market portfolio of risky assets. This theory says that all investors will choose a position under the CML by borrowing or lending at a risk free, since this maximizes return for a given level of risk.

In the CAPM model, since all the agents have diversified portfolios, they will demand a premium depending on each individual asset. So, the systematic risk and the related profitability, are going to be measured by the beta.

Basically, unlike the CML, the Security Market Line (also known as SML), shows the expected returns of individual assets.

Knowing all that, the return of the security is calculated in the following way:

$$R_e = R_f + \beta(R_M - R_f)$$

Where,

R_e = Expected return of the security,
R_f = Expected return of the risk-free asset,
R_M= Expected return of the Market.

3.2.2. Sharpe Ratio.

William Sharpe also developed the known as The Sharpe ratio (1966), which is used to help investors understand the return of an investment compared to its risk.

The Sharpe Ratio measures the additional return earned per unit of dispersion in an investment asset considered by the variance risk measure. Basically, is telling the investors how well the returns of an asset compensate the risk they are taking for that investment. Is just a way to check the performance of an investment considering the risk involved in it,

It is calculated as:

$$Sharpe\ Ratio = \frac{R_p - R_F}{\sigma_p}$$

Where,

R_P = Expected return of the portfolio,
R_f = Expected return of the risk-free asset,
σ_p = Standard deviation of the portfolio

As a generalization, we should buy assets if the Sharpe ratio is above the Capital market Line (CML) and sell if the Sharpe ratio is below the CML. Considering that, we can deduce that the slope of the CML is the Sharpe ratio of the market portfolio.

3.3. Fama´s, Efficient Market Hypothesis.

The investment theory of Fama is based on the number of assumptions about securities markets and how they work. These assumptions include the idea of the efficient market hypothesis: the belief that all information which is relevant to picking up stocks is freely available among all the investors.

There must be the consideration of "a*ll of the available empirical literature is implicitly or explicitly based on the assumption that the conditions of market equilibrium can be stated in terms of expected returns. This assumption is the basis of the expected returns or "fair game" efficient markets models*".

And as there are always a large number of buying and selling trades in the markets, the movements of the prices always occurs efficiently, so the stocks will always be trading at the current fair market value.

So, if the stocks are always traded at the market fair value, the conclusion of this theory is that it is not possible to buy undervalued stocks nor overvalued. With this information we can conclude that neither stock analysis nor market timing strategies can help us do a better performance over the overall market. So, the only way to generate superior returns is to take higher amount of risk.

3.3.1. Variations of the Efficient Markets Hypothesis.

There three variation or possible scenarios for this hypothesis, the weak form, the semi-strong form and the strong form. Each and every one will represent three different levels of the efficiency of the market.

1. Weak Form. The weak form of the Efficient Market Hypothesis assumes that the prices of the securities reflect all the available information in the market, but they might not reflect the new information publicly available. It also assumes that the future prices are independent form the past information related to the price, the volume and the returns. So, the weak form implies that technical strategies of trading won´t provide excess of returns because the past performances cannot predict future prices. But while it discounts technical analysis, says that the fundamental analysis may provide an outperforming over the market average return.
2. Semi-Strong form. The semi-strong form of the theory discounts the usefulness of the technical and the fundamental analysis, adds to the weak form that the prices are adjusted quickly to any public information; therefore, the future price movements are not predictable using the fundamental analysis.
3. Strong Form. The strong form of the Efficient Markets hypothesis says that the prices are always reflecting the public and the private information. So, if this includes all the information available, historical and new, which includes the insider information, the prices of the company´s current stock will reflect the information which is not publicly available from the investors. So, according to this theory, not even insider information can give knowledge to the investors to general returns and outperform the market.

4. The risk in a portfolio and the effect of the diversification.

Once we know that the passive investment is the investment style we are going to select in order to choose the assets, and what are the assumption of the theories needed for making a portfolio, we are going to see an example to clarify the process for building the portfolio.

4.1. Risk measurement in a portfolio.

In order to make a portfolio, there are two main risk to be consider, the diversifiable risk and the non-diversifiable risk.

- Diversifiable risk, also known as unsystematic risk, which represents the risk inherent to the portfolios specific asset and is associated with random causes. This kind of risk can be eliminated with the diversification. Unsystematic risk is caused due to factors specific of an industry or a company.
- The non-diversifiable risk, also known as systematic risk, it represents the proportion of risk attributable to the market factors that affect all the companies which associated to that that market. This kind of risk cannot be eliminated through diversification.
  The systematic risk is the one which affects to the entire market, not just to a particular stock or industry. It is almost impossible to completely avoid this type of risk. But in order to reduce the unsystematic risk as much as possible, one of the main variables is the diversification. Adding a variety of assets in a portfolio, we can reduce significantly its risk, as long as their coefficient of correlation is less than 1.0. When the correlation between the assets is 1.0, it means that any change in one asset is equally reflected in the change in the other asset.

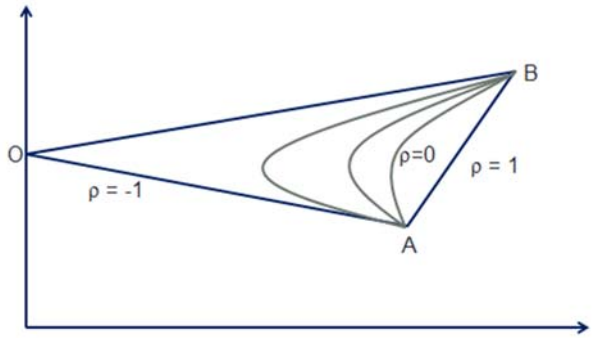
4.2. The effect of the risk in the diversification.

The figure 5, shows us how the efficient frontier will be depending on the level of correlation that we have. When the correlation between the assets is 1.0, as mentioned before, any change in one asset is equally reflected in the change in the other asset. So, considering that, there will be any diversification effect, because, for instance, if one asset goes down 5%, the other asset will also go down 5% as well. As we can see, there is no diversification benefit. In such a case any portfolio derived from a combination perfectly correlated instruments, will fall on the straight line, reflecting the risk of the both assets.

In any case, as the correlation between two or more assets starts reducing, the portfolio will be getting the benefit of the diversification. If we look at the image of the efficient frontier, this will show that it will start being bent to the left, as the correlation starts getting low. So, the lesser the correlation between the two assets, the more the diversification benefit and more it will bend towards the left. Two assets will reach at their maximum level of correlation when they are perfectly negatively correlated, this means, that the coefficient of correlation is -1.0. This is also the point where the risker is at the lowest. As we can see in the previous chart, when the correlation is -1.0., the risk is 0. So, investing in these two assets, will give us return without assuming any kind of risk. In the reality, it is not really possible to find assets that are perfectly negatively correlated. For this reason, the efficient frontier will usually be a curved line bending towards the left, signifying some levels of diversification.

So, in other words, we can reduce significantly its risk, as long as their coefficient of correlation is less than 1.0. When the correlation between the assets is 1.0, it means that any change in one asset is equally reflected in the change in the other asset.

Figure 4. Correlation Curve.



Source: Based on the different concepts explained in this thesis.

While perfectly positively correlated assets do exist, in most of the cases, the coefficient of these assets is less than 1.0. The implications of this fact for risk are main to understand the effect of the diversification. So, we can say that an investor can reduce the risk of the portfolio by having combinations of financial instruments that are not perfectly positively correlated.

Let´s suppose the following example of diversification with a portfolio of three different assets:

	Asset A	Asset B	Asset C
Return	5%	10%	15%
Standard Deviation	7%	10%	15%

Correlation between A-B	0,50%
Correlation between B-C	0,20%
Correlation between A-C	0,30%

As we can see, we have three different assets with different returns and different standard deviation (risk). We will randomly weight each and every asset generating a combination of 20 possible portfolios to invest in.

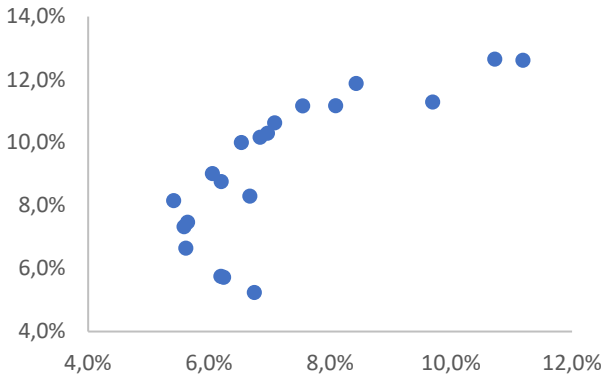
As in the actual situation of Europe, we will consider the Risk-free rates are 0 (this will be useful to calculate the Sharpe ratio, and so, determining the best possible combination in relation of its return and risk).

The following table show us the data used to weight each asset:

Table 2. Example

Combination	Weight A	Weight B	Weight C	Total	Deviation	Return	Sharpe Ratio
1	68,17%	14,29%	17,54%	100,00%	5,6%	7,5%	1,32
2	27,18%	33,16%	39,66%	100,00%	7,1%	10,6%	1,50
3	11,27%	54,18%	34,54%	100,00%	7,6%	11,2%	1,48
4	55,60%	13,65%	30,76%	100,00%	6,2%	8,8%	1,41
5	12,45%	37,65%	49,89%	100,00%	8,4%	11,9%	1,41
6	34,51%	50,78%	14,70%	100,00%	6,1%	9,0%	1,49
7	53,21%	30,50%	16,29%	100,00%	5,4%	8,2%	1,51
8	36,65%	1,05%	62,30%	100,00%	9,7%	11,3%	1,16
9	19,67%	57,40%	22,93%	100,00%	6,8%	10,2%	1,48
10	34,15%	25,89%	39,96%	100,00%	7,0%	10,3%	1,48
11	21,61%	4,57%	73,83%	100,00%	11,2%	12,6%	1,13
12	86,82%	11,44%	1,74%	100,00%	6,2%	5,7%	0,93
13	17,66%	11,79%	70,55%	100,00%	10,7%	12,6%	1,18
14	24,43%	51,19%	24,39%	100,00%	6,5%	10,0%	1,53
15	3,12%	70,43%	26,44%	100,00%	8,1%	11,2%	1,38
16	87,79%	10,11%	2,11%	100,00%	6,2%	5,7%	0,92
17	36,30%	61,51%	2,19%	100,00%	6,7%	8,3%	1,24
18	57,97%	37,60%	4,43%	100,00%	5,6%	7,3%	1,31
19	73,42%	20,33%	6,25%	100,00%	5,6%	6,6%	1,18
20	96,30%	2,67%	1,03%	100,00%	6,7%	5,2%	0,78

Figure 5. Example. Efficient frontier Curve



Source: Own elaboration based on the data of the table 2.

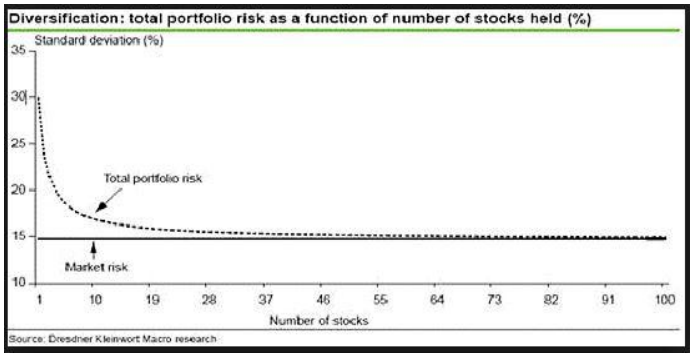
The combination of these three assets, as is shown in the table, will reduce considerable our risk. And being the best combination the number 14, the one with the highest Sharpe Ratio, which will give us a return of 10% and a risk of 6.5% (less than any three of the assets if looked individually).

With this example, we can prove that the diversification really has an effect of reducing the risk if combined with different asset with correlation less than 1. But how many assets do we need to reduce the diversifiable risk at its minimum?

5. Exchanged Traded Funds (ETFs)

The second subject is going to be answered in this part of the thesis, which was the number of instruments needed to reduce the risk of a portfolio as much as possible. This proves the diversification effect that have the combination of certain kind of financial instruments. It was commonly believed by investors that the optimal number of stocks needed to be fully diversified is over 30. But where did this number come from?

Figure 6. Diversification: total portfolio risk as a function of number of stocks held (%).



Source: Dresdner Kleinwort Macro Research.

Lawrence Fisher and James H. Lorie released in 1970 “Some Studies of Variability of Returns on Investments in Common Stocks”, which was published by The Journal of Business. They proved that a randomly created portfolio of 32 stocks can reduce the distribution by 95%, comparing this portfolio to a portfolio of the NYSE. The myth of the 95% of “the benefits of the diversification is captures with a 30 stocks portfolio” came from this paper.

In the other side, Benjamin Graham, in The Intelligent Investor, suggested that the optimal diversification can be acquired with 10 to 30 instruments.

In a section of the Journal of Finance in 1968, Evans and Archer found out that the portfolios with as few as 10 securities had a standard deviation (risk) nearly identical to the market risk. There are also been a "15-stock diversification solution" over the decades, which has become preserved in various texts, the most know was in A Random Walk Down Wall Street:

“By the time the portfolio contains close to 20 equal-sized and well-diversified issues, the total risk (standard deviation of returns) of the portfolio is reduced by 70 percent. Further increase in the number of holdings does not produce any significant further risk reduction.”

At the end, all these studies and papers come to the conclusion that the diversification is a best way to reduce the diversifiable risk as much as possible in a portfolio. But managing, for instance, 32 different kind of assets can be complicated for many reasons. One of them is the time invested in monitoring and balancing these assets, and if we consider the transaction cost involved in these activities, we can lose all the benefits we had in the diversification, handling these stocks. Is not the same having a few numbers of assets than dozens of them to obtain an optimal risk reduction.

Fortunately, all of these concerns are easily overcome by only using low cost, passive institutional funds or exchange traded funds (ETFs).

The ETF´s are securities that are created to behave the same way as a specific index or a collection of different securities. This means that they will track or intend with the securities they are designed to follow. There is a lot of types of ETF´s focusing on specific indexes, market segments and even in entire countries. They give the possibility to investors to diversify over a broad market sector with a single investment. They differ from mutual funds because they trade like common stock in an exchange.

5.1. Advantages and disadvantages of ETFs.

Using ETF´s have a lot of advantages as:

- Diversification. As it was already mentioned previously, diversification is main and most important advantage. Buying shares of an ETF "purchases" an entire stock index, so, for instance, if you decide to buy a kind of ETF that replicates the Spanish stock exchange, IBEX 35, you are buying a share which is composed by shares of 35 companies at once, obtaining a spectacular diversification with a single purchase.
  And that's not the only advantage, ETF´s also allow investors to invest in unknown or emerging countries in an easier way, so they can also diversify geographically as well.
- Low commissions. Despite the concept is very similar to investment funds, ETFs charge very low commissions compared to buy all the stock individually. So, the diversification effect is much faster and cheaper. Rather than monitoring and buying and selling all the stocks constantly.
- They are very easy to acquire and there is a lot of different types. Nowadays most of them can be bought from easily from any online broker, such as Renta4 or ING. Obviously, each and every broker have their own commission which should be consider before making any move.
- There is no minimum amount of investment. Some funds require a minimum amount of money to invest, not like ETFs, which are bought and sold like shares. The lowest amount invested in ETF´s, the higher the effect of the commission, because there are commissions involved in the purchases.

Before considering buying ETF´s we should be aware of the possible disadvantages that these financial instruments might have:

- The effect of diversification that we want has a price. It is not the same investing in companies chosen by us individually than having to buy all the index.
  This means that, if you buy stocks one by one, you can choose exactly where to put your money, while if you buy an ETF you can only choose the index, and in the process, you will be affected by some companies which may mean obtaining less or losing money.
- Tax like stocks. For instance, if we put our money in Investment funds, they have a great fiscal advantage, and that is that they can be rotated without paying taxes. ETFs, on the other hand, have the same treatment as stocks, so if you sell you will have to pay taxes on capital gains that you generate.
- It has purchase and sale commissions. Same as any other financial assets, the exchanged traded funds have to deal with these kinds of commissions.

After knowing all the advantages and disadvantages of the Exchanged Traded Funds, this thesis will be focused in the diversifiable risk reduction of Markowitz´s Minimum Variance Portfolio Theory and, for this purpose, we will be using Exchange Traded Funds and we will determine the

number of ETF´s necessary to have a perfectly diversified portfolio. The second step for this thesis will be knowing if it's better to make a passive investment or an active investment in order to have better results in a long-term investment.

Although ETFs trade like stocks, investors need to stop treating the two as if they're the same. The construction and the negotiation of these instruments is quite different. And in order to understand exactly what we are buying; we will proceed to give an overview about the creation of these instruments.

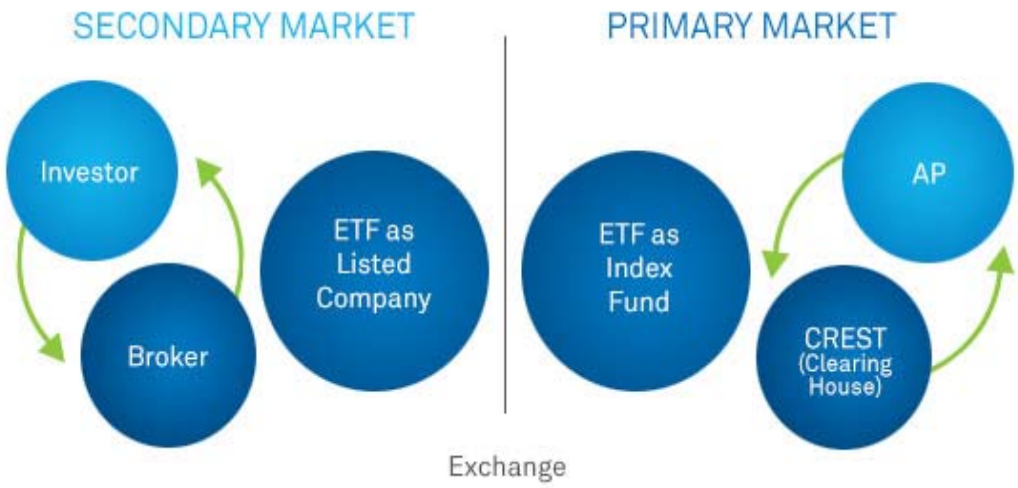
5.2. The creation of ETFs

The creation of Exchanged Traded Funds shares is done by a process called creation and redemption. This happens on a level of fund in the primary market. It allows to the authorised participants, as market makers and institutional trading desk, to exchange cash and securities for ETF Shares.

The capability of creating and redeeming shares keeps the price of the ETF in line according to the underlying net asset value and makes sure that the liquidity derives from the underlying securities held in the fund.

The process of creation of the ETFs is explained in the following graph:

Figure 7. Process of creation of ETFs securities



Source: iShares by BlackRock, Creation and redemption process

The Exchange traded funds can have different structures. These are open-ended collective investments that tracks the performances on a benchmark.

Now a days, ETFs are one of the most common ETPs (Exchanged traded products) available in the markets. One of the biggest advantages of the ETFs is that, they offer to the investors transparent, liquid, flexible and a cheaper way to access to any asset or asset class. They can track the indexes by holding the underlying securities (physical based ETFs), or alternatively by holding a derivative (Swap-Based ETFs).

- Physical-based ETFs. These kinds of ETFs generally buy the security of the underlying index and hold them as fund asset. When it is not possible to buy all the securities in the index, the optimisation process is used, which means, buying a portion of the security and using it to track the performance of the index.
- Swap- based ETFs, use the total return index swaps to replicate an index´s performance. This means that, when you are buying a swap-based ETF, you are buying the performance of the Index, not the securities it contains.

6. The number of ETFs needed for an optimal level of diversification.

6.1. Selection of Data

In order to select the data, we will be using the information from the database of Morningstar Direct. Even though investors can get now a days the information from a lot of different brokers easily, we are going to use the database of a reliable source that is used by a lot of multinational companies for their researches. With Morningstar is possible to get the information for more than 10,000 national and international ETFs, quoted on the Spanish Stock Exchange and on the main European and Global Stock Exchanges. The number is that high because there are a lot of securities that track the same index, issued by different companies. Thanks to that variety of stocks, we can select between a high range of assets needed for the simulation and reach to an optimal conclusion explaining the optimal effect of the diversification.

Now a days, the variety of ETFs is huge and in order to select the best ETF, we don´t consider just the cost, but also all the different characteristics that they have:

1. Type of Asset. These can be fixed income, equities, commodities, etc.
2. Subtype. It is possible to purchase ETFs tracking public debt, corporations, European shares, etc.
3. Geographical area. The ETF can track an entire geographical are as Europe, eurozone, USA, Japan, etc.
4. Underlying index, as MSCI Europe Index, DJ Eurostoxx 50, etc
5. Quoted currency, like EUR, USD, CHF, etc.

As it was mentioned before, the data from Morningstar let us find information for more than 10,000 different kind of ETFs in the market. Not all of them are available. Approximately 3,000 ETFs are obsolete as of 31st December of 2018. So even after that, more than 7,000 different instruments are still available in the sample. But, as we know, most of them track similar variables, being different just the entities by whom they were offered, or the domicile of the same.

The next step was to find a group of ETFs for our purpose, which was, proving the diversification effect of these instruments. In order to do that, 13 ETF´s were selected as a sample for our study, considering different industries, geographies and assets classes, and see how these different variables affect to our portfolio.

We are avoiding using ETF issued by other currency than Euro to try to avoid the currency risk in our investment, as this is not the purpose of the thesis.

So, after a consideration of the different sectors, the selection of ETFs picket for our study was the following:

Table 3. Selection of assets

	ISSUER	Name	Ticker	Domicile	Fund Size EUR
SECTORS					
UTILITIES	Lyxor International Asset Management S.A.S.	Lyxor Stoxx Europe 600 Utilities ETF	UTI	Luxembourg	37.200.587,00
TECHNOLOGY	BlackRock Asset Management (DEU) AG	iShares STOXX Europe 600 Technology (DE)	EXV3	Germany	108.144.349,00
BANKING	Lyxor International Asset Management S.A.S.	Lyxor Stoxx Europe 600 Banks ETF Acc	BNK	Luxembourg	458.455.212,00
OIL & GAS	BlackRock Asset Management (DEU) AG	iShares STOXX Europe 600 Oil & Gas (DE)	EXH1	Germany	543.643.635,00
HEALTH CARE	BlackRock Asset Management (DEU) AG	iShares STOXX Europe 600 HealthCare (DE)	EXV4	Germany	465.980.879,00
REAL ESTATE	BlackRock Asset Management (DEU) AG	iShares STOXX Europe 600 Real Est (DE)	EXI5	Germany	75.258.882,00
GEOGRAPHIC REGIONS					
EUROPE	BlackRock Asset Management (DEU) AG	iShares STOXX Europe 600 (DE)	EXSA	Germany	5.408.277.767,00
EMERGING MARKETS	Amundi Luxembourg S.A.	Amundi IS MSCI Emerging Markets ETF-C €	AEEM	Luxembourg	5.938.005.149,00
ASIA	Lyxor International Asset Management S.A.S.	Lyxor MSCI AC AsiaPac Ex Jpn ETF Acc EUR	AEJ	Luxembourg	383.709.593,00
UNITED STATES	Amundi Luxembourg S.A.	Amundi IS MSCI USA ETF-C EUR	CU2	Luxembourg	1.015.601.973,00
FIXED INCOME					
HIGH YIELD	DWS Investment S.A. (ETF)	X II EUR High Yield Corporate Bd ETF 1D	XHYG	Luxembourg	168.123.144,00
CORPORATE BONDS	BlackRock Asset Management Ireland - ETF	iShares € Corp Bond Lg Cp ETF EUR Dist	IBCX	Ireland	3.789.996.405,00
GOVERNMENT BONDS	DWS Investment S.A. (ETF)	X II Eurozone Government Bond ETF 1C	DBXN	Luxembourg	2.662.877.301,00

Source: Morningstar Direct. Data as of 31st December of 2018.

After picking up all these instruments, the following steps and assumptions are going to be followed:

1st Step. The data we have consider goes from the year 2009 to the year 2018 (10 years). The information that we got from the database of Morningstar, was the annualized return and the annualized standard deviation of each and every ETF we have considered. After doing that, the covariance matrix was calculated for the given returns, which was used to calculate the relationship of the returns of the portfolio.

2nd Step. Once we had all the previous data, the combinations of each and every asset have been created. For instance:

- For a portfolio of one asset, the only possible combination will be:

- ETF 1
- ETF 2
- …
- ETF 13

- For a portfolio of two assets, the combinations are made in the following order:

- ETF 1 & ETF 2
- ETF 1 & ETF 3
- …
- ETF 13 & ETF 12

And so on, until the last portfolio of all the assets available. In this last combination (portfolio of 13 assets), only one combination was possible.

3rd Step. Once we have this, for each portfolio category, which have a number of ETFs, and for each combination of assets obtained in the previous step, we will calculate the minimum portfolio variance.

The minimum portfolio variance is a well-diversified portfolio that consists of picking individually risky assets, which are traded together, in a way that the resulting risk is the lowest possible for an expected rate of return. This process was calculated using the covariance matrix mentioned in the step 1. We had to use the function of Excel optimizer to get the weights for the given portfolio combination. The formula was used considering the covariances of the ETFs of our portfolio and adding a restriction of getting a total of 100% of weight for our total portfolio, without the possibility of short selling (so, no negative weight was allowed).

The given result gave us the optimal weights for the minimum risk. The process was quite intensive to do for all the possible combination (We have to consider that there are more than 8,000 possible combination of the number of ETFs we have selected), so a Macro was created to find the optimal portfolio combination to minimize the portfolio variance.

The 4th and last step was to rank the portfolio combinations within each category so as to find the minimum risk that a combination can achieve with a specific number of ETFs.

6.2. Evolution of the results according to the number of instruments used for an optimal diversification.

The minimum variance portfolio will indicate us an optimal weight of the diversified portfolio considering our risk. So, each investment in a minimum variance portfolio will be riskier if they are traded individually, buy together, in the portfolio, the risk will be hedged at a minimum point.

The following table shows us the annualized return of the selected instruments according to Morningstar Asset Management. These instruments have been used to calculate the covariance matrix for the final results.

Table 4. Annualized returns of the selected instruments.

	UTILITIES	TECHN	BANKING	OIL & GAS	HEALTH-CARE	REAL ESTATE	EUROPE	EM MARKETS	ASIA	USA	HIGH YIELD	CORP BONDS	GOVN BONDS
Ret 2018	2,28%	-9,98%	-25,39%	-0,79%	-0,74%	-11,18%	-10,97%	-10,66%	-10,40%	-0,09%	-3,65%	-1,19%	0,57%
Ret 2017	9,76%	20,48%	11,70%	1,95%	4,01%	10,37%	10,78%	20,20%	19,27%	6,67%	4,64%	1,50%	-0,07%
Ret 2016	-4,76%	4,98%	-2,83%	28,61%	-8,51%	-5,65%	1,56%	14,22%	8,96%	14,29%	7,73%	4,27%	3,35%
Ret 2015	0,20%	16,05%	-0,50%	-3,01%	17,81%	15,88%	10,80%	-5,59%	0,04%	12,11%	0,00%	-0,90%	1,48%
Ret 2014	18,39%	8,60%	-0,15%	-11,07%	20,80%	24,23%	7,42%	10,72%	16,06%	28,28%	0,00%	8,02%	12,89%
Ret 2013	13,56%	28,16%	22,54%	7,71%	23,66%	6,79%	20,95%	-7,35%	-1,84%	26,04%	0,00%	1,50%	2,06%
Ret 2012	5,07%	23,68%	27,31%	-1,05%	14,62%	29,69%	17,75%	15,07%	19,34%	13,41%	0,00%	10,75%	10,83%
Ret 2011	-12,29%	-11,57%	-32,23%	3,63%	13,87%	-8,94%	-8,75%	-16,31%	-16,36%	4,46%	0,00%	3,07%	3,22%
Ret 2010	-4,20%	19,10%	-11,28%	3,77%	9,76%	16,25%	12,25%	0,00%	23,21%	22,40%	0,00%	4,31%	1,01%
Ret 2009	6,41%	22,56%	47,47%	29,88%	17,45%	29,58%	32,68%	0,00%	64,85%	21,99%	0,00%	12,42%	4,19%

Source: Morningstar Direct. Data as of 31st December of 2018.

The covariance matrix of our selected instruments is the following:

Table 5. Covariance matrix of the selected assets.

	UTILITIES	TECHN	BANK	OIL & GAS	HEALTH-CARE	REAL ESTATE	EUROPE	EM MARKETS	ASIA	USA	HIGH YIELD	CORP BONDS	GOVN BONDS
UTILITIES	0,0085	0,0067	0,0126	-0,0033	0,0041	0,0073	0,0058	0,0045	0,0065	0,0046	-0,0003	0,0010	0,0017
TECHN	0,0067	0,0194	0,0281	0,0024	0,0063	0,0159	0,0167	0,0076	0,0187	0,0083	0,0007	0,0023	0,0006
BANKING	0,0126	0,0281	0,0592	0,0128	0,0097	0,0277	0,0300	0,0129	0,0427	0,0120	0,0010	0,0072	0,0028
OIL & GAS	-0,0033	0,0024	0,0128	0,0176	-0,0046	-0,0017	0,0059	0,0011	0,0147	0,0014	0,0020	0,0020	-0,0016
HEALTH-CARE	0,0041	0,0063	0,0097	-0,0046	0,0104	0,0092	0,0071	-0,0035	0,0036	0,0056	-0,0016	0,0014	0,0017
REAL ESTATE	0,0073	0,0159	0,0277	-0,0017	0,0092	0,0234	0,0164	0,0083	0,0260	0,0091	-0,0005	0,0051	0,0037
EUROPE	0,0058	0,0167	0,0300	0,0059	0,0071	0,0164	0,0174	0,0049	0,0238	0,0084	0,0002	0,0036	0,0010
EM MARKETS	0,0045	0,0076	0,0129	0,0011	-0,0035	0,0083	0,0049	0,0153	0,0125	0,0023	0,0025	0,0023	0,0021
ASIA	0,0065	0,0187	0,0427	0,0147	0,0036	0,0260	0,0238	0,0125	0,0521	0,0106	0,0010	0,0081	0,0021
USA	0,0046	0,0083	0,0120	0,0014	0,0056	0,0091	0,0084	0,0023	0,0106	0,0091	0,0001	0,0022	0,0018
HIGH YIELD	-0,0003	0,0007	0,0010	0,0020	-0,0016	-0,0005	0,0002	0,0025	0,0010	0,0001	0,0010	0,0001	-0,0001
CORP BONDS	0,0010	0,0023	0,0072	0,0020	0,0014	0,0051	0,0036	0,0023	0,0081	0,0022	0,0001	0,0022	0,0014
GOVN BONDS	0,0017	0,0006	0,0028	-0,0016	0,0017	0,0037	0,0010	0,0021	0,0021	0,0018	-0,0001	0,0014	0,0019

Source: Own elaboration based on the data of Morningstar Direct. Data as of 31st December 2018.

After following all the steps, and having the objective of this thesis in mind, which is proving the effect of diversification using passive instruments securities, we have calculated three different variances for our portfolio:

- The minimum variance portfolio. Obtaining a well-diversified portfolio with the minimum risk using an optimal weight for the combination of assets
- The maximum variance portfolio. With this we will obtain the worst possible combination using different securities. This is done to show that even if we use the maximum variance, the risk is still reduced in a way that we don´t need that much amount of securities to reach to the minimum.
- The average variance portfolio. The effect of the risk reduction using an average variance in our combination of portfolios.

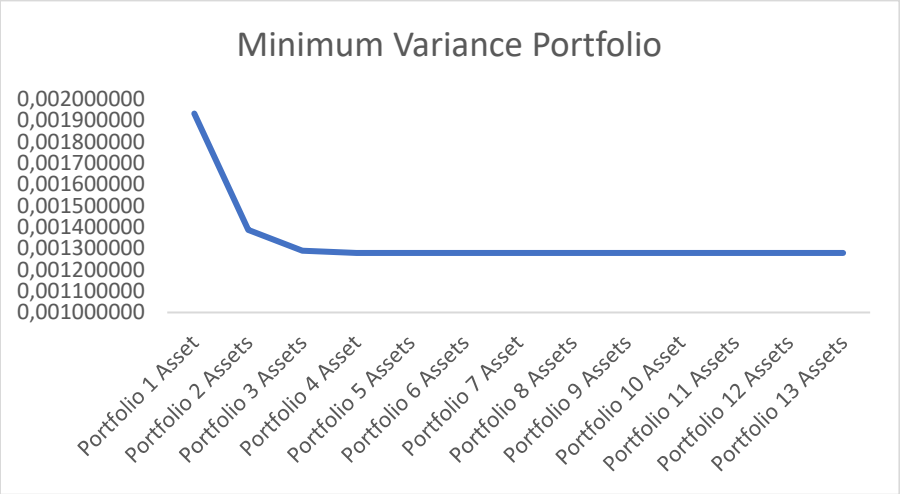
The following tables shows us the minimum, the maximum and the average variance for the select asset for the period from January 2009 to December 2018 (10 years). It also shows the decrease in variance for the three situations year by year, reaching a time where no decrease in variance is possible by adding more assets. We can see that clearly in the graph that is below the table.

Table 6. Minimum Variance Portfolio Results

	Minimum Variance Portfolio	Decrease in Variance	Decrease in Variance (%)
Portfolio 1 Asset	0,001933061		
Portfolio 2 Assets	0,001387271	-0,000545790	-28,2%
Portfolio 3 Assets	0,001289723	-0,000097549	-7,0%
Portfolio 4 Asset	0,001279389	-0,000010334	-0,8%
Portfolio 5 Assets	0,001279389	0,000000000	0,0%
Portfolio 6 Assets	0,001279389	0,000000000	0,0%
Portfolio 7 Asset	0,001279389	0,000000000	0,0%
Portfolio 8 Assets	0,001279389	0,000000000	0,0%
Portfolio 9 Assets	0,001279389	0,000000000	0,0%
Portfolio 10 Asset	0,001279389	0,000000000	0,0%
Portfolio 11 Assets	0,001279389	0,000000000	0,0%
Portfolio 12 Assets	0,001279389	0,000000000	0,0%
Portfolio 13 Assets	0,001279389	0,000000000	0,0%

Source: Own elaboration based on the data of Morningstar Direct. Data as of 31st December 2018.

Figure 8. Minimum Variance Portfolio Graph



Source: Own elaboration based on the data of Morningstar Direct. Data as of 31st December 2018.

As we can observe in the minimum variance graph, with only two ETFs our risk is significantly reduced, and with the amount of four ETFs, considering the optimal weight, our risk is reduced at the maximum level possible, achieving the maximum benefits of the diversification. So, as it was previously mentioned, the average of stock needed to make an optimal diversification, is highly reduced with only four assets needed and not the thirty as it was done previous ETFs securities. And not only that, but also the reduction of cost implied in buying just four assets and not the thirty stocks to reach to the same results.

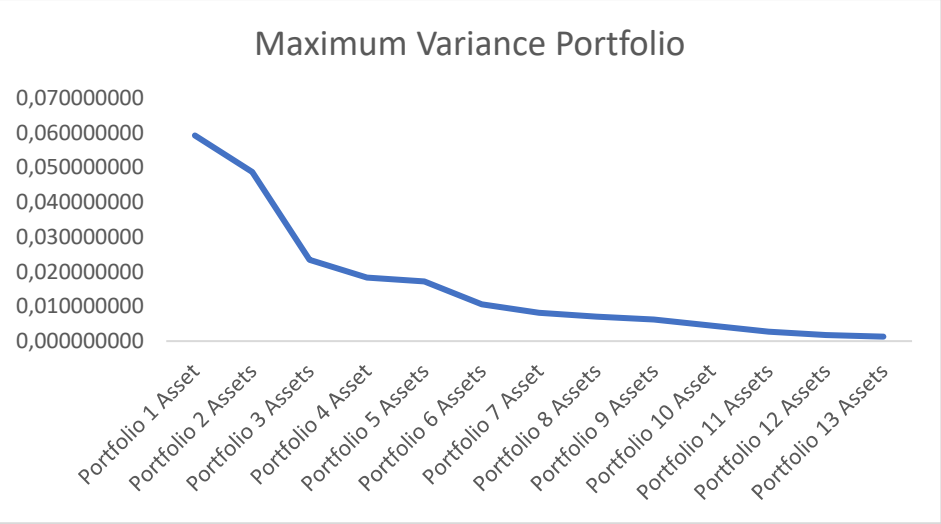
The following table shows us the results taking the maximum variance portfolio.

Table 7. Maximum Variance Portfolio Results

	Maximum Variance Portfolio	Decrease in Variance	Decrease in Variance (%)
Portfolio 1 Asset	0,059212453		
Portfolio 2 Assets	0,048686634	-0,010525820	-17,8%
Portfolio 3 Assets	0,023388640	-0,025297993	-52,0%
Portfolio 4 Asset	0,018276287	-0,005112353	-21,9%
Portfolio 5 Assets	0,017152853	-0,001123434	-6,1%
Portfolio 6 Assets	0,010572579	-0,006580275	-38,4%
Portfolio 7 Asset	0,008099008	-0,002473571	-23,4%
Portfolio 8 Assets	0,007022762	-0,001076245	-13,3%
Portfolio 9 Assets	0,006184646	-0,000838116	-11,9%
Portfolio 10 Asset	0,004447495	-0,001737151	-28,1%
Portfolio 11 Assets	0,002681357	-0,001766138	-39,7%
Portfolio 12 Assets	0,001695518	-0,000985839	-36,8%
Portfolio 13 Assets	0,001279389	-0,000416129	-24,5%

Source: Own elaboration based on the data of Morningstar Direct. Data as of 31st December 2018.

Figure 9. Maximum Variance Portfolio Graph



Source: Own elaboration based on the data of Morningstar Direct. Data as of 31st December 2018.

The maximum variance portfolio was created just to reflect how the effect of the diversification of the ETFs instruments affect to our portfolio. Even taking the portfolio of the maximum variance for the combinations we have, if we add a new ETF, the risk is reduced considerably. When used all the thirteen ETFs selected, our risk reaches to the minimum calculated previously.

So, even using the worst combination possible in our selection of asset, the amount of instrument needed to reduce the risk at the lowest point, is still significantly low.

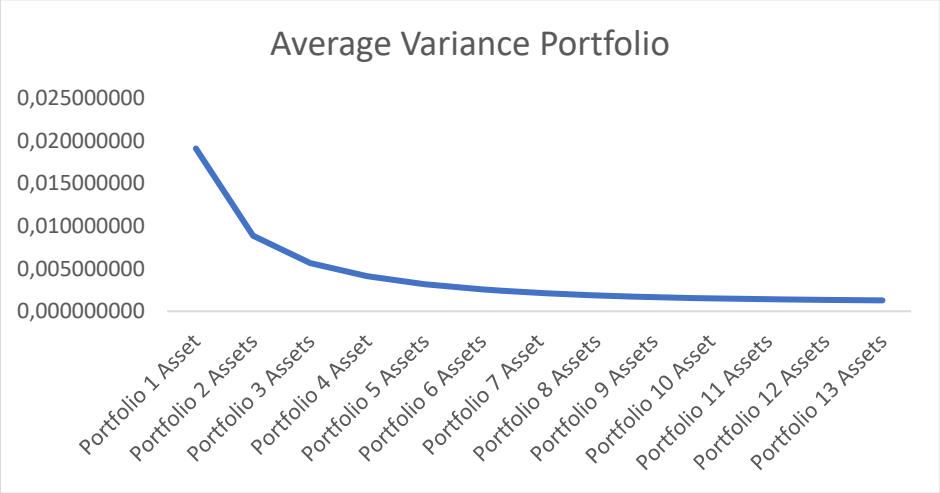
Finally, the following table shows us the result using the average variance portfolio:

Table 8. Average Variance Portfolio Results

	Average Variance Portfolio	Decrease in Variance	Decrease in Variance (%)
Portfolio 1 Asset	0,019113068		
Portfolio 2 Assets	0,008867834	-0,010245235	-53,6%
Portfolio 3 Assets	0,005656804	-0,003211029	-36,2%
Portfolio 4 Asset	0,004106259	-0,001550546	-27,4%
Portfolio 5 Assets	0,003173745	-0,000932513	-22,7%
Portfolio 6 Assets	0,002563393	-0,000610353	-19,2%
Portfolio 7 Asset	0,002147160	-0,000416233	-16,2%
Portfolio 8 Assets	0,001857272	-0,000289888	-13,5%
Portfolio 9 Assets	0,001653866	-0,000203406	-11,0%
Portfolio 10 Asset	0,001510839	-0,000143027	-8,6%
Portfolio 11 Assets	0,001409563	-0,000101275	-6,7%
Portfolio 12 Assets	0,001335991	-0,000073572	-5,2%
Portfolio 13 Assets	0,001279389	-0,000056602	-4,2%

Source: Own elaboration based on the data of Morningstar Direct. Data as of 31st December 2018.

Figure 10. Average Variance Portfolio Graph



Source: Own elaboration based on the data of Morningstar Direct. Data as of 31st December 2018.

If we take the average variance of all the possible combinations of the portfolio, we can also see how the risk is reduced significantly. The decrease in risk reduces exponentially adding the first three instruments, being the risk reduction almost imperceptible as we continue adding more financial instruments.

Once we got all these results, we saw which one offers us the best possible combination. And so, the following table shows us this best combination possible, which is using just 4 instruments.

The portfolio weight used for that combination of those four ETFs, where the level of risk, as we can observe, cannot be lower adding another asset, is the following:

Table 9. Optimal combination of the selected instruments.

Issuer	Asset	Weight
BlackRock Asset Management (DEU) AG	iShares STOXX Europe 600 Oil & Gas (DE)	15,77%
BlackRock Asset Management (DEU) AG	iShares STOXX Europe 600 HealthCare (DE)	3,84%
Amundi Asset Management	Amundi ETF MSCI EMU High Dividend C	7,49%
DWS Investment S.A. (ETF)	X II Eurozone Government Bond ETF 1C	72,89%

Source: Own elaboration based on the data of Morningstar Direct. Data as of 31st December 2018.

So, as we can see in the previous chart, our portfolio is composed in a 16% of commodities, 4% of Health Care Equity, 7% of Equities and 73% of fixed income. So, surprisingly, the instrument with more weight is the X II Eurozone Government Bond ETF 1C, which means that the portfolio is very conservative.

Transactions Costs. As we mentioned before, the most obvious advantage of having just four instruments, and not an average of more than thirty, is the lower transactions cost implied in buying and rebalancing our portfolio. The monitoring of the just 4 instruments also makes us use less resources and save money.

6.3. The calculation of the Sharpe Ratio for the minimum variance portfolio

So, in this example, only four instruments were needed to obtain the desirable results. In order to proceed and find out the best combination considering also the return, we are going to calculate the Sharpe ratio of each combination.

For calculating the Sharpe ratio, we need to have a portfolio return, a risk-asset and the variance of the portfolio.

$$Sharpe\ Ratio = \frac{R_p - R_F}{\sigma_p}$$

Having the risk of each combination of portfolio (the one of the minimum variances), the following steps have been followed:

1st Step. As all the assets are already selected, using the given annualized returns, it has been calculated the total return in a period of 10 years of all these instruments. This total return will be used in the formula.

2nd Step. Once we have the annualized returns, we will proceed to calculate the Sharpe Ratio with the procedure explained for all the possible combinations.

For instance, we will be selecting the following combination:

- the best portfolio of the combination of one asset,
- the best portfolio of the combination of two assets,
- …

- the best portfolio of the combination thirteen assets.

3rd Step. The result of the Sharpe Ratio will give us the most attractive risk adjusted portfolio.

The following table shows us the annualized returns in 10 years of the selected assets.

Table 10. Annualized return (10 years) of the selected ETFs

Asset	Annualized return
Lyxor Stoxx Europe 600 Utilities ETF	5,02%
iShares STOXX Europe 600 Technology (DE)	12,42%
Lyxor Stoxx Europe 600 Banks ETF Acc	0,52%
iShares STOXX Europe 600 Oil & Gas (DE)	4,97%
iShares STOXX Europe 600 HealthCare (DE)	12,28%
iShares STOXX Europe 600 Real Est (DE)	11,65%
iShares STOXX Europe 600 (DE)	9,59%
Amundi IS MSCI Emerging Markets ETF-C €	2,03%
Lyxor MSCI AC AsiaPac Ex Jpn ETF Acc EUR	8,64%
Amundi IS MSCI USA ETF-C EUR	16,07%
Amundi ETF MSCI EMU High Dividend C	8,68%
iShares € Corp Bond Lg Cp ETF EUR Dist	4,10%
X II Eurozone Government Bond ETF 1C	4,01%

Source: Own elaboration based on the data of Morningstar Direct. Data as of 31st December 2018.

Sharpe Ratio, Risk and Return of the portfolio.

To calculate the Sharpe Ratio, we need to consider a risk-free rate instrument. In a normal situation, the German Bund of 10 Years is considered is Europe to be the risk-free rate, but considering the actual situation, we have considered 0% as the value of the returns of the risk-free asset. A risk-free asset is normally considered because it has a future return for your investment without assuming any kind of risk, because treasuries are backed by the government.

As we can see in the following graph, the risk-free rate is negative in the actual conditions of the market. This situation can mean that the policy makers might be afraid of the European future expectations. Normally, when the economic situation is not very favourable, people try to hold on to their cash waiting for the better improvement of the economy. But this behaviour, can make a lack of spending.

The correct consideration of the risk-free asset is much more complex in the real life. For this example, to do it properly, we should have taken the risk-free rate which was 10 years before instead of the one we have right now.

Figure 11. Risk Free Rate.

Source: Bloomberg Intelligence.

With all these considerations, the best combinations of the portfolios depending on the number of instruments used, are the following:

Table 11. Combination with the highest Sharpe Ratio

	Minimum Variance	Standard Deviation	Return of the portfolio	Sharpe Ratio
Portfolio 1 Asset	0,001933061	4,40%	4,01%	0,91
Portfolio 2 Assets	0,001387271	3,72%	4,16%	1,12
Portfolio 3 Assets	0,001289723	3,59%	4,57%	1,27
Portfolio 4 Asset	0,001279389	3,58%	4,83%	1,35
Portfolio 5 Assets	0,001279389	3,58%	4,83%	1,35
Portfolio 6 Assets	0,001279389	3,58%	4,83%	1,35
Portfolio 7 Asset	0,001279389	3,58%	4,83%	1,35
Portfolio 8 Assets	0,001279389	3,58%	4,83%	1,35
Portfolio 9 Assets	0,001279389	3,58%	4,83%	1,35
Portfolio 10 Asset	0,001279389	3,58%	4,83%	1,35
Portfolio 11 Assets	0,001279389	3,58%	4,83%	1,35
Portfolio 12 Assets	0,001279389	3,58%	4,83%	1,35
Portfolio 13 Assets	0,001279389	3,58%	4,83%	1,35
Risk Free Ratio	0%			

Source: Own elaboration based on the data of Morningstar Direct. Data as of 31st December 2018.

Once we have the highest Sharpe Ratio of every portfolio combination with these 13 assets, we can determine which combination has the best risk-return.

Figure 12. Best Risk-Return Portfolio.

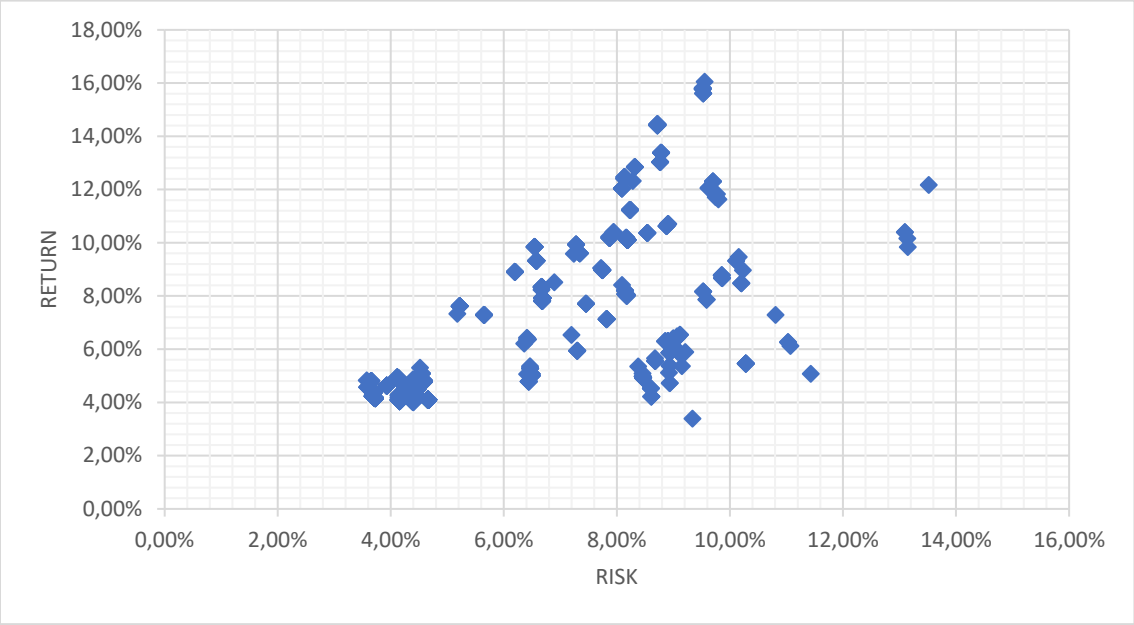
Source: Own elaboration based on the data of Morningstar Direct. Data as of 31st December 2018.

The best combination is the one with 4 instruments, which gives us an annualized return of 4,83% with a standard deviation of just 3,58%.

So, as it was proved with the calculation of the minimum variance portfolio, after the fourth instrument, adding other instruments will not make any difference. If we continue adding instruments in this example, our cost is going to increase in order to monitor and buy and sell those instruments.

But, in the following graph, we can see all the possible combination of risk and return getting a portfolio of just 4 assets.

Figure 13. Risk-return in all the possible combinations of 4 assets.



Source: Own elaboration based on the data of Morningstar Direct. Data as of 31st December 2018.

The selection of 4 different assets is not a guarantee of reducing the risk at the minimum. Even if the select any four-asset portfolio, we need to have the right combination and weights of the asset. Selecting a variety of 13 assets, a portfolio with four assets has more than 700 possible combinations.

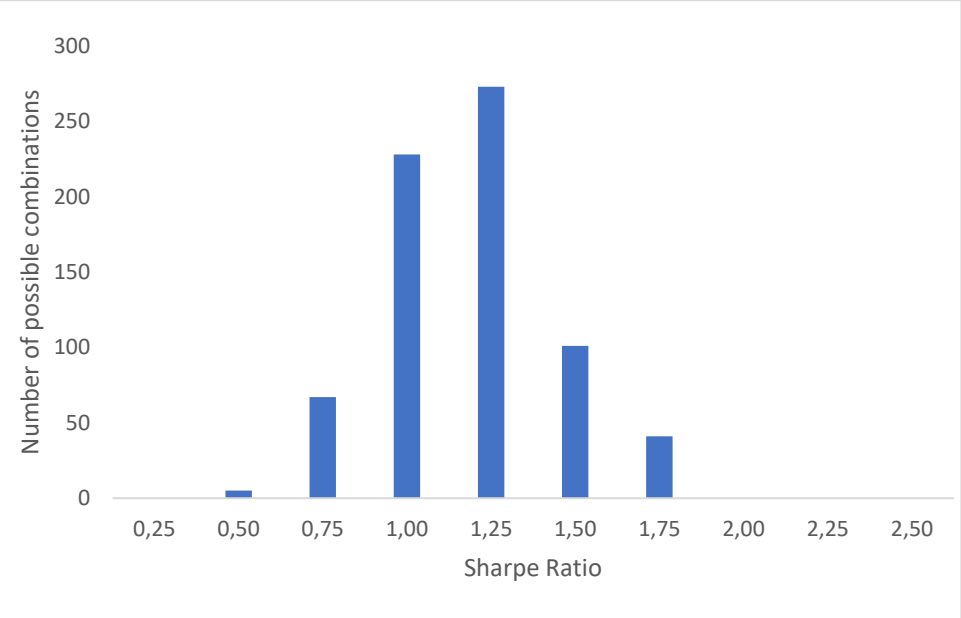
Let's suppose the actual risk of the portfolio and not the minimum, we would have the following distribution:

Table 12. Distribution of the combination of 4 instruments by Sharpe Ratio (data).

Sharpe Ratio	*Frequency*	Relative Frequency	Cumulative Frequency
0,2500	0	0%	0%
0,5000	5	1%	1%
0,7500	67	9%	10%
1,0000	228	32%	42%
1,2500	273	38%	80%
1,5000	101	14%	94%
1,7500	41	6%	100%
2,0000	0	0%	100%
2,2500	0	0%	100%
2,5000	0	0%	100%

Source: Own elaboration based on the data of Morningstar Direct. Data as of 31st December 2018.

Figure 14. Distribution of the combination of a 4 instruments portfolio by Sharpe Ratio (graph).



Source: Own elaboration based on the data of Morningstar Direct. Data as of 31st December 2018.

Even though, the one with the minimum variance has a Sharpe Ratio of 1,35, in the portfolio of a combination of four different asset, most of the combinations have a ratio between 1,00 and 1,25. So basically, they are riskier and offer worst return. We can achieve combinations with

higher Sharpe Ratio as well, but with less possibilities. And the main objective of this thesis was to prove with how many ETFs we can reduce our risk at the minimum point, so this is out of our objective. But having all these combinations, it proves that in order to have a good return and risk in your portfolio, the most important thing is not the number of assets, but how these assets are distributed.

We have to understand that there are a high number of ETFs available in the market, and this result can vary in the combination of each of them. Most of them track the same index. In order to prove the effect of the diversification using ETFs securities, a highly range of ETFs tracking different sectors and markets were selected.

7. Conclusion

The first subject we wanted to solve in this paper was rather to choose an active investment style or a passive investment style. We searched many different resources to see the performance of both kind of investments and it was clear that the passive investment has shown its potential in the last reports of many highly reputed companies. In the long-term period, the active investment is not reaching at the desirable results that the investors seek. More than 80% of the active funds, on average of all the different categories, are not able to survive and beat to their passive investment peers. So, basically, they are charging significantly higher fees and performing worse than passive investment funds. This is having a direct impact in the investors.

Many investors are shifting from active style investment to passive style due to its positive performance in long term periods. Active style of investment has shown to be more costly and less efficient in time, so investors are seeking for a kind of strategy that can solve that issue without risking all their savings

Due to this, Exchanged Traded Funds (ETFs) were created. These instruments are able to replicate an entire market or market sector and investors can purchase and negotiate these instruments as stocks, even though the process involved in the creation of them is completely different than an ordinary stock. This not only offers a highly diversified instrument, but also an affordable for all kind of investors. It's not the same buying one instrument which can replicate an entire market rather than buying each and every stock of those individually. So, for instance, if some investors wanted to buy the IBEX 35, only with one instrument they could replicate it instead of buying all the 35 instruments individually, which will increase also the monitoring and the transactions costs.

The high demand of these kind of instruments, will change the traditional way of investing. The passive investment has diminished the risk related to liquidity transformation; and the cost are going to be reduced due to the highly competence of these companies.

So, thanks to the Exchanged Traded Funds, we reached to the second part of the thesis, which consisted in proving the effect of risk reduction that passive investment instruments have.

It was believed that there were needed more than 30 securities to achieve a proper diversification and reduce the risk of our portfolio at the minimum point possible. But with these instruments, the way of investing has changed completely. So, in order to prove that, we built a portfolio seeking the number of ETFs needed to have in a completely diversified portfolio, where the risk was at the minimum. We achieved this using the Markowitz´s Minimum Variance Portfolio Theory.

For this, we made an assumption of picking up 13 securities, which represent different markets and different sectors to prove how highly diversified these instruments are. Once we made the portfolio with the minimum variance, we could reduce the assets to 4 securities. We also calculated the average variance and the maximum variance in order to prove that the diversification effect of using these kinds of instruments are effective even if we don´t select the right proportion of weights in our portfolio, compare to the active investment instruments.

So, using only 4 instruments instead of 30 instruments, have a few advantages. In one hand, we can make a remarkable saving in costs due to the less transaction needed to buy and rebalance the portfolio. In the other hand, we can also save the cost of monitoring all the instruments individually, as the ETFs by itself, tracks.

In order to continue the research of this thesis, we could have answered to the following question: What if we select all the possible ETF randomly and see, in an average, how many of them are necessary to decrease our risk at the minimum point?

Finally, it´s worth to add that passive investment has not only proved itself by performing better in the last years, but it has also changed the way of investing, making this industry highly competent and efficient. Due to this, active investors have to change their way of investing if they don´t want to lose a considerable amount of market share in the upcoming years.

8. Appendix

Appendix 1: Percentage of European Equity Funds Outperformed by Benchmarks

FUND CATEGORY	COMPARISON INDEX	1-YEAR	3-YEAR	5-YEAR	10-YEAR
EURO-DENOMINATED FUNDS (EUR)					
Europe Equity	S&P Europe 350	85,65	86,01	80,21	86,96
Eurozone Equity	S&P Eurozone BMI	77,17	90,46	89,35	91,32
Nordic Equity	S&P Nordic BMI	86,96	93,10	86,67	96,43
Global Equity	S&P Global 1200	87,00	94,43	96,16	98,47
Emerging Markets Equity	S&P/IFCI Composite	79,33	89,62	97,18	97,95
U.S. Equity	S&P 500	82,68	92,08	95,64	97,73
France Equity	S&P France BMI	98,19	94,55	84,96	87,50
Germany Equity	S&P Germany BMI	74,44	77,17	81,91	82,30
Italy Equity	S&P Italy BMI	74,58	60,00	63,27	65,38
Spain Equity	S&P Spain BMI	59,09	68,97	72,37	78,33
Netherlands Equity	S&P Netherlands BMI	100,00	100,00	90,91	93,10
POUND STERLING-DENOMINATED FUNDS (GBP)					
Europe Equity	S&P Europe 350	80,58	76,24	74,47	78,00
Europe Ex-UK Equity	S&P Europe Ex-UK BMI	81,48	81,40	82,79	79,87
UK Equity	S&P United Kingdom BMI	73,41	80,28	69,09	73,47
UK Large-/Mid-Cap Equity	S&P United Kingdom Large-Mid Cap	78,21	95,32	69,10	73,04
UK Small-Cap Equity	S&P United Kingdom SmallCap	57,14	37,84	50,65	71,26
Global Equity	S&P Global 1200	76,73	89,97	88,99	92,28
Emerging Markets Equity	S&P/IFCI Composite	76,97	81,71	88,89	89,33
U.S. Equity	S&P 500	76,32	87,73	93,09	91,89
FUNDS DENOMINATED IN OTHER EUROPEAN LOCAL CURRENCIES					
Denmark Equity	S&P Denmark BMI	83,78	31,43	60,61	85,29
Poland Equity	S&P Poland BMI	93,18	97,73	93,62	91,43
Switzerland Equity	S&P Switzerland BMI	88,38	52,57	62,72	73,37
Sweden Equity	S&P Sweden BMI	61,74	64,10	54,87	78,99

Source: S&P Dow Jones Indices LLC, Morningstar. Data for periods ending Dec. 31, 2018. Outperformance is based on equal-weighted fund counts. Index performance based on total return. Past performance is no guarantee of future results. Table is provided for illustrative purposes.

Appendix 2: Survivorship Consistency of European Equity Funds		
FUND CATEGORY	**NO. OF FUNDS AT START**	**SURVIVORSHIP (%)**
1 -YEAR (EUR)		
Europe Equity	1124	94,4
Eurozone Equity	574	95,82
Nordic Equity	24	87,5
Global Equity	1316	94
Emerging Markets Equity	340	95,29
U,S, Equity	397	92,7
France Equity	222	95,95
Germany Equity	91	96,7
Italy Equity	59	98,31
Spain Equity	89	97,75
Netherlands Equity	7	71,43
1 -YEAR (OTHER EUROPEAN LOCAL CURRENCIES)		
Denmark Equity	37	97,3
Poland Equity	62	95,16
Switzerland Equity	203	94,58
Sweden Equity	115	93,04
3-YEAR (EUR)		
Europe Equity	1078	85,71
Eurozone Equity	572	88,29
Nordic Equity	31	67,74
Global Equity	1232	83,12
Emerging Markets Equity	333	84,98
U.S. Equity	380	82,63
France Equity	222	89,64
Germany Equity	93	88,17
Italy Equity	50	90
Spain Equity	87	88,51
Netherlands Equity	8	62,5
3-YEAR (OTHER EUROPEAN LOCAL CURRENCIES)		
Denmark Equity	35	91,43
Poland Equity	62	95,16
Switzerland Equity	179	87,15
Sweden Equity	117	78,63
5-YEAR (EUR)		
Europe Equity	1056	75,09
Eurozone Equity	589	77,93
Nordic Equity	32	50
Global Equity	1192	70,47
Emerging Markets Equity	333	71,47
U.S. Equity	365	73,42
France Equity	228	79,39
Germany Equity	94	79,79
Italy Equity	49	79,59

Spain Equity	78	83,33
Netherlands Equity	11	45,45
5-YEAR (OTHER EUROPEAN LOCAL CURRENCIES)		
Denmark Equity	34	91,18
Poland Equity	65	90,77
Switzerland Equity	172	79,07
Sweden Equity	113	72,57
10-YEAR (EUR)		
Europe Equity	1402	45,22
Eurozone Equity	781	49,04
Nordic Equity	30	46,67
Global Equity	1425	43,79
Emerging Markets Equity	250	57,2
U.S. Equity	502	40,24
France Equity	290	52,41
Germany Equity	113	55,75
Italy Equity	78	43,59
Spain Equity	121	47,93
Netherlands Equity	29	17,24
10-YEAR (OTHER EUROPEAN LOCAL CURRENCIES)		
Denmark Equity	35	71,43
Poland Equity	48	81,25
Switzerland Equity	170	61,76
Sweden Equity	139	48,2

Source: S&P Dow Jones Indices LLC, Morningstar. Data for periods ending Dec. 31, 2018. Past performance is no guarantee of future results. Table is provided for illustrative purposes.

Appendix 3: Average European Equity Fund Performance (Equal Weighted)

CATEGORY	1 -YEAR (%)	3-YEAR (ANNUALIZED %)	5-YEAR (ANNUALIZED %)	10-YEAR (ANNUALIZED %)
EURO-DENOMINATED FUNDS (EUR)				
Europe Equity	-14,35	-1,24	2,84	7,98
S&P Europe 350	-9,9	1,06	3,88	8,8
Eurozone Equity	-16,17	0,77	2,51	6,42
S&P Eurozone BMI	-12,82	1,71	4,44	8,14
Nordic Equity	-9,75	0,09	4,72	11,47
S&P Nordic BMI	-6,97	2	6,2	13,52
Global Equity	-8,36	1,96	5,54	8,52
S&P Global 1200	-3,85	5,5	9,21	12,51
Emerging Markets Equity	-12,9	4,92	3,76	8,14
S&P/IFCI Composite	-9,89	7,59	6,48	11,18
U, S, Equity	-3,97	4,25	8,76	12,16
S&P 500	0,44	7,42	12,62	15,35
France Equity	-19,17	0,13	4,16	7,71
S&P France BMI	-8,45	4,37	6,19	9,18
Germany Equity	-20,92	-1,4	2,24	8,62
S&P Germany BMI	-18,02	0,34	3,06	9,22
Italy Equity	-17,07	-2,71	2,33	4,6
S&P Italy BMI	-14,13	-2	2,66	3,48
Spain Equity	-11,3	0,34	0,93	4,15
S&P Spain BMI	-11,05	0,73	1,2	3,71
Netherlands Equity	-13,12	2,15	5,56	8,99
S&P Netherlands BMI	-8,72	5,63	8,72	12,4

Source: S&P Dow Jones Indices LLC, Morningstar, Data for periods ending Dec, 31, 2018, Index performance based on total return, Funds are equal weighted, but indices are not, Past performance is no guarantee of future results, Table is provided for illustrative purposes.

Appendix 4. Active Funds' Success Rate by Category (%)

Category	1-year	3-year	5-Year	10-Year
Asia-Pacific ex-Japan Equity	44,2	46,5	39,3	11,7
Asia-Pacific including Japan Equity	15,2	26,1	28,4	17,4
China Equity	40,8	44,3	40,7	33,8
Global Emerging Markets Equity	24,8	20,2	31,7	31,6
Europe ex-UK Large-Cap Equity	32,4	55,8	41,9	21,1
Europe Large-Cap Blend Equity	22,1	12,6	17,7	16,6
Europe Large-Cap Value Equity	33	7,8	22,3	17,2
Europe Small-Cap Equity	39,7	54,1	63,2	28,6
Hong Kong Equity	22,2	15,8	17,6	13,3
India Equity	26	59,5	73,5	46,9
Japan Large-Cap Equity	31,2	31,2	25,8	13,7
Japan Small/Mid-Cap Equity	25,7	66,7	62,2	31,7
Korea Equity	26,7	5,9	29,4	12,5
Latin America Equity	37,1	24,7	27,3	27,2
Nordic Equity	28,9	41,1	42,9	30,8
US Large-Cap Blend Equity	33,3	17,8	21,2	9,5
US Large-Cap Growth Equity	33	9,3	2,6	0,7
US Large-Cap Value Equity	27	12,6	19,5	9,2
US Small-Cap Equity	60,3	41,3	42,1	17,2
Norway Equity	23,1	31,4	54,8	44,6
Sweden Equity	24,7	38,7	57,3	24,7
Switzerland Large-Cap Equity	26,8	38,5	45,4	23,1
UK Large-Cap Blend Equity	32,5	21,5	37,2	30,4
Global Large-Cap Blend Equity	22,7	15,8	14,1	9,3
CHF Bond	23,9	35,7	21	22,5
USD Diversified Bond	30	30,8	8,5	15,8
USD Government Bond	36,4	9,4	28,2	14,3
EUR Diversified Bond	36,9	17,4	19,6	18,5
EUR Government Bond	11,8	19,7	23,3	16,7
GBP Corporate Bond	33	60,4	53,4	26
JPY Bond	36,4	50	38,5	0
GBP Government Bond	15,6	30,3	18,6	8,8
EUR Bond - Long Term	42,4	25	17,1	14,6
EUR Inflation-Linked Bond	24,1	8,2	18,8	9,8
Canada Equity	85,7	12,5	18,2	11,1
Emerging Europe ex-Russia Equity	32,3	2,9	17,5	11,7
France Large-Cap Equity	10,1	6,4	13,8	24,8
Germany Large-Cap Equity	33,8	21,7	41	30
Italy Equity	29,3	46,8	53,3	35,9
Netherlands Equity	25	0	27,3	9,1
Russia Equity	14	37	36,5	34
Spain Equity	55,8	45,9	32,9	29,3
Switzerland Small/Mid-Cap Equity	29,8	56,3	40,9	52
Taiwan Large-Cap Equity	22,7	22,7	20,7	15,6
Property - Indirect Asia	50	20,8	16	16
Property - Indirect Europe	57,1	49,2	47,4	22,5
EUR Corporate Bond	20,3	56,8	45,6	59,5
USD Corporate Bond	39,8	38,6	30,8	31,8
UK Mid-Cap Equity	25	26,7	63,6	66,7

Source: Morningstar´s European Active/Passive Barometer Year End 2018

Appendix 5. Percentage of active funds outperforming over 1, 3, 5 and 10 years

Universe	Benchmark	1Y	3Y	5Y	10Y
Europe large caps	MSCI Europe Net Return EUR Index	45%	39%	36%	36%
Eurozone large caps	EURO STOXX 50 Net Return EUR	55%	50%	38%	41%
Europe small caps	MSCI Europe Small Cap Net TR EUR	72%	57%	27%	10%
Germany large caps	Deutsche Boerse AG German Stock Index	61%	27%	24%	14%
France large caps	CAC 40 Total Return Index	38%	13%	13%	13%
UK all caps	FTSE UK Series FTSE All Share TR	30%	20%	40%	26%
Italy large caps	FTSE MIB Net Total Return Index	81%	75%	79%	0%
Spain large caps	IBEX 35 NET RETURN INDEX	32%	46%	42%	38%
Switzerland large caps	Swiss Exchange Swiss Performance Index	37%	52%	34%	23%
US large caps	MSCI USA Net Total Return USD Index	32%	15%	16%	11%
US small caps	Russell 2000 Total Return Index	57%	41%	70%	50%
Japan all caps	Topix Total Return Index JPY	49%	30%	26%	23%
World large caps	MSCI World Net Total Return USD Index	54%	22%	14%	11%
Emerging markets large caps	MSCI Emerging Net Total Return USD Index	42%	34%	41%	24%
China large caps	MSCI China Net Total Return USD Index	24%	38%	44%	37%
Euro govies	FTSE MTS Eurozone Government Bond IG Index (Ex-CNO Etrix)	20%	11%	21%	16%
Euro corporate	Bloomberg Barclays Euro Aggregate Corporate Total Return Index Value Unhedged EU	52%	43%	40%	26%
Euro high yield	ICDE BofAML Euro High Yield Index	16%	37%	20%	0%
Euro inflation linked	Bloomberg Barclays Euro Govt Inflation-Lined Bond All Maturities Total Return I	6%	47%	64%	82%
US corporate	Bloomberg Barclays US Corporate Total Return Value Unhedged USD	57%	43%	67%	0%
US high yield	Bloomberg Barclays US Corporate High Yield Total Return Index Value Unhedged USD	56%	33%	20%	0%
Global bonds - EUR HDG	Bloomberg Barclays Global-Aggregate Total Return Index Value EUR Hedged	67%	48%	37%	50%
Emerging debt	JP Morgan GBI-EM Global Diversified Composite Unhedged EUR	41%	31%	20%	15%
Average Equity		47%	37%	38%	25%
Average Fixed Income		39%	37%	36%	24%
Average 2017		44%	37%	36%	25%
Average 2016		28%	25%	24%	19%

Source: Bloomberg and Morningstar data from 31/12/2007 to 29/12/2017

Appendix 6. RENTABILIDADES MEDIAS ANUALES PONDERADAS DE LOS FONDOS DE INVERSIÓN AL 31/01/2019										Patrimonio
Tipo de Fondo	**Mes**	**2.019**	**1 Año**	**3 Años**	**5 Años**	**10 Años**	**15 Años**	**20 Años**	**25 Años**	**(en miles €)**
MONETARIOS	0,03	0,03	-0,47	-0,28	-0,09	0,63	1,06	1,39	2,24	6.870.914
RENTA FIJA EURO CORTO PLAZO	0,2	0,2	-1,12	-0,23	-0,02	0,7	1,09	1,41	2,23	35.761.317
RENTA FIJA EURO LARGO PLAZO	0,51	0,51	-0,98	0,3	1,21	2,36	2,15	2,29	3,12	15.289.784
RENTA FIJA MIXTA EURO	1,32	1,32	-3,47		0,57	1,91	1,78	1,52	2,76	20.755.422
RENTA VARIABLE MIXTA EURO	3,28	3,28	-6,47	1,16	1,27	3,42	2,47	1,28	3,24	3.418.297
RENTA VARIABLE NACIONAL EURO	6,04	6,04	-9,1	5,67	2,37	5,41	4,5	2,44	5,5	6.548.116
RENTA FIJA INTERNACIONAL	1,01	1,01	-0,65	-0,21	1,72	2,19	1,88	2,1	2,78	6.544.436
RENTA FIJA MIXTA INTERNACIONAL	1,76	1,76	-2,99	0,04	0,33	1,45	0,98	0,74	1,99	18.734.711
RENTA VARIABLE MIXTA INTERNACIONAL	3,12	3,12	-4,19	1,52	1,29	4	2,08	0,77	2,76	20.967.191
RENTA VARIABLE EURO RESTO	6,21	6,21	-12,1	2,91	2,27	7,58	4	1,5	4,5	6.124.155
RENTA VBLE. INTERNACIONAL EUROPA	7,47	7,47	-8,09	2,12	2,97	6,15	3,04	0,76	3,28	6.594.408
RENTA VBLE. INTERNACIONAL EEUU	7,68	7,68	-1,13	9,72	9,65	11,79	5,37	2,57	1,79	2.486.651
RENTA VBLE. INTERNACIONAL JAPÓN	5,13	5,13	-9,7	5	6,53	6,64	2,32	-0,15	-0,89	827.894
RENTA VBLE. INTERNACIONAL EMERGENTES	8,75	8,75	-8,27	9,96	5,22	7,36	6,31	5,55	3,42	1.686.782
RENTA VBLE. INTERNACIONAL RESTO	7,37	7,37	-7,57	5,2	4,97	10,18	4,73	1,23	3,28	13.980.506
GLOBALES	2,7	2,7	-4,3	2,04	2,15	2,93	2,09	1,19	2,83	43.079.073
GARANTIZADOS DE RENDIMIENTO FIJO	0,72	0,72	0,76	0,45	0,69	1,95	2,08	2,19		5.025.740
GARANTIZADOS DE RENDIMIENTO VARIABLE	0,71	0,71	-0,74	0,6	0,95	1,64	1,86	1,79		14.208.305
DE GARANTÍA PARCIAL	0,61	0,61	-2,63	0,58	1,37					586.405
DE GESTIÓN PASIVA	1,7	1,7	-2,27	1,14	1,9					16.126.320
RETORNO ABSOLUTO	1,29	1,29	-4,02	-0,33	0,06					14.357.175
FONDOS DE INVERSIÓN LIBRE (FIL)	5,08	5,08	-4,74	6,14	4,46					2.049.156
FONDOS DE FIL	0,71	0,71	-2,89	-1,51	-0,33					209.366
TOTALES FONDOS:	2,33	2,33	-3,36	0,96	1,09	2,17	2,02	1,5	2,6	262.232.127

Fuente: Inverco

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