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The Arithmetic of Stepwise Offer Curves

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Abstract

In liberalized electricity markets, aggregated stepwise supply and demand curves are at the core of many relevant processes. Efficient and meaningful representations of the offer curves is an essential procedure for agents participating in those markets. However, there is not a formal framework that allows operating with those offer curves using basic arithmetic operations. In this paper we first formalize the concept of stepwise offer curve by explicitly defining the standard True Offer Curve (TOC). To overcome the inherit difficulties of this non-continuous TOC, we propose the Encoded Offer Curve (EOC), a continuous piecewise version that approximates the steps of the TOC with high accuracy. We present fast and simple specialized algorithms to obtain both TOC and EOC models, as well as a formal framework to deal with elementary mathematical operations involving TOCs and EOCs. The proposed framework has been tested in the Italian electricity market, computing the residual demand curves of the producers in a particular Market Zone; and in the Iberian electricity market, quantifying the differences in the bidding behavior of market agents in different stages of the COVID-19 pandemic.

Keywords: Electricity markets, piecewise linear function, supply function, demand function, residual demand curve.

1. INTRODUCTION

In the last decades many different countries have experienced a deregulation process which has given rise to liberalized markets that allow companies to trade energy in organized auctions [1]. For example, day-ahead electricity markets are generally based on sealed-bid auctions where agents submit their selling and buying offers to the Market Operator (MO) who then determines the market-clearing price and the set of accepted bids for each time period of time [2, 3, 4].

Consider, for the sake of simplicity and without loss of generality, a simple-bid market. In this market each offer is defined by a price p and a quantity q, which refers to the amount of energy the agent is willing to buy or sell at that price p. By sorting the selling (buying) offers in increasing (decreasing) prices, the aggregated supply (demand) function for the agent is built. Once all the agents have submitted their offers, the sum of all the supply curves results in the system supply curve, whereas the sum of all the demand curves results in the system demand curve. Market-clearing price is computed as the intersection of the system aggregated supply and demand curves [5]. In real-world electricity markets these aggregated curves are generally stepwise functions. For simplicity, in this paper we use the term offer curve (OC) to refer both stepwise aggregated supply and demand functions.

These OCs are at the core of many relevant processes in electricity markets. For example, the MO uses these supply and demand functions in order to clear the market [3]. For market agents not only setting their bidding strategies in terms of OCs is critical (see e.g. [2, 6, 7, 8, 9, 10, 11] or [12, 13] for agent-based market equilibrium models), but also analyzing, when possible, the OCs of their competitors is essential to understand the strategic bidding behavior of competitors (see e.g. [14]). For regulators, estimating the slope of the residual demand curves, computed from OCs, also provide useful information regarding the potential market power of the different producers, see e.g. [15]. Additionally, being able to model supply curves is of utmost importance when analyzing transmission-constrained electricity markets [16].

While there is a considerable amount of literature on electricity markets where dealing with OC's is basic, to the best of our knowledge, there is not a formal framework for operating with OCs using basic arithmetic operations. An illustrative example of these needs is the computation of the residual demand curve (RDC) of a given agent, (see Section V for details). RDCs have been extensively used in the strategic bidding literature. For example, [17] proposes a new forecasting approach for the RDC based on functional nonparametric models that require the computation of the historical RDCs. Reference [4] proposes a new method to compute RDCs that cap-

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tures the effect of complex offering conditions and transmission constraints, making extensive use of elementary mathematics with OCs. Both [17] and [4] compute RDCs using a common strategy in practice, based on sampling and lookup tables to manage the operation, but without using any formal framework.

There is some previous work where OCs have been explicitly modeled with different purposes. In [18, 19] the OCs are modeled as continuous piecewise linear functions, obtained using a general-purpose machine learning approach, the so-called Linear Hinges Model (LHM). This model provides a good compromise between the number of pairs defining the piecewise linear function and the obtained accuracy. In [20] this approach is used to create the deterministic component of a probabilistic model of the residual demand of a agent in order to optimize its stepwise offer curve by genetic algorithms.

More recently, in [21] authors propose the X-model for electricity price forecasting based on supply and demand curves. This approach models the OCs using a piecewise linear approximation which allows not only estimating the market clearing price by the intersection of both supply and demand curves, but also calculating the mean supply and demand curves of a time period. Note that although the structure of this piecewise function is equivalent to the LHM of [18], the nature of the algorithm used to obtain the model is very different. The algorithm of [21] is straightforward. In the case of a supply OC, after sorting the offers in ascending order according to the price, for each unique price there is a pair in the set of points defining the piecewise function where, for each unique price, there is a quantity calculated as the sum of all the offer quantities up to this price.

The main contribution of this paper is the definition of a practical framework to deal with OCs, with the corresponding theoretical basis. This goal is achieved by first formalizing the concept of stepwise offer curves, explicitly defining the standard True Offer Curve (TOC). Then, to overcome the inherit difficulties of operating with these non-continuous TOCs, we propose the Encoded Offer Curve (EOC), a very accurate continuous piecewise linear approximation of the original TOC that can be used to combine demand and supply OCs, allowing computing e.g. RDCs. To avoid misunderstandings, fast and simple specialized algorithms to obtain both TOC and EOC models from raw offers are proposed. Furthermore, a formal framework to deal with elementary mathematical operations involving TOCs and EOCs has been developed, including a fast algorithmic implementation of the basic operations. The development of this methodology has a direct application in the analysis and forecasting of supply and demand curves in electricity markets, as it provides a precise and flexible representation method for these bidding curves. One recent example is the application of functional forecasting models [17, 22, 23] to obtain short-term estimations of the aggregated supply curves in day-ahead markets, obtaining point-forecasts of the bidding behavior of all market agents. Additionally, probabilistic functional forecasting models have been recently used to generate scenarios of residual demand curves [24], which can then be used to improve certain stochastic optimization models such as the offering optimization model proposed in [25]. As these functional models use the whole bidding curve as an input, it is of utmost importance that the method selected to define the supply curve is as accurate as possible, in order to obtain an accurate description of each agent's bids. By employing the proposed EOC framework, more accurate forecasts can be obtained, which will result in the development of better decision making models for electricity markets.

The remainder of the paper is organized as follows. Section II defines the TOC. The proposed continuous EOC is presented in Section III. Then the formal framework to deal with elementary mathematics involving TOCs and EOCs is described in Section IV. To illustrate the proposed approach, Section V shows how to compute the residual demand curve of an agent applying basic algebra with TOCs and EOCs. Sections VI and VII present an empirical comparative study of the proposed frameworks in the context of both the Italian and Iberian day-ahead electricity market. Finally, conclusions are drawn in Section VIII.

2. TRUE OFFER CURVES

In a simple-bid market each offer is defined by a price p and a quantity q, which refers to the amount of energy the agent is willing to buy or sell at that price p. The set of N raw (p_i, q_i) offers retains all the basic information required by the MO to clear the market. They are the basic bricks used by the agents to set their bidding strategies (see e.g. the buy and sell offers displayed in the bottom of Fig. 1).

The True Offer Curve (TOC) is the stepwise aggregated offer curve obtained by sorting the selling (buying) offers in increasing (decreasing) prices. In order to differentiate buy and sell curves, the following notation will be used: selling curves will be denoted by TOC+ (monotonically increasing TOC), and buying curves will be referred to as TOC- (monotonically decreasing TOC). Hence, the mathematical expression of each type of TOC will be different. For selling offers (asks) the expression is:

$$f^{TOC+}(x \mid C^{TOC}) = \begin{cases} 0 & x < a_1 \\ b_{j-1} & a_{j-1} \leq x < a_j \\ b_M & x \geq a_M \end{cases} \tag{1}$$

Likewise, for buying offers (bids) the TOC- is defined as:

$$f^{TOC-}(x \mid C^{TOC}) = \begin{cases} b_1 & x \leq a_1 \\ b_{j-1} & a_{j-1} < x \leq a_j \\ 0 & x \geq a_M \end{cases} \tag{2}$$

where M is the number of unique (a_j, b_j) steps sorted in increasing order according to the relation $a_{j-1} < a_j$, and

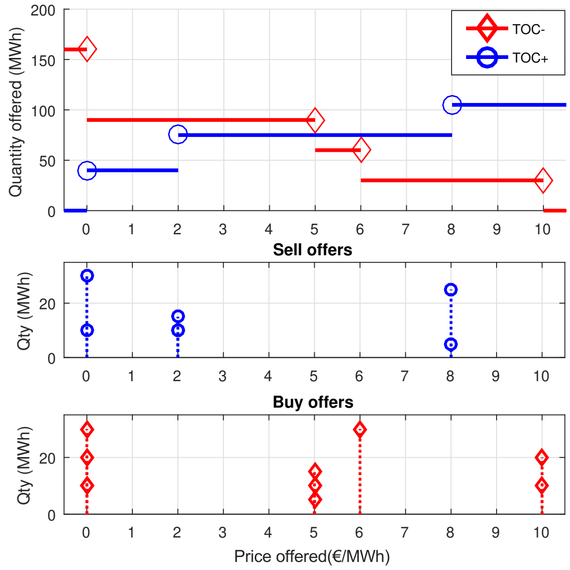

Figure 1: TOC+ and TOC- obtained from the raw buying and selling offers.

$j = 2, \cdots, M$. Thus, the TOC is completely defined by the set of M pairs $C^{TOC} = \{(a_j, b_j)\}_{j=1}^{M}$, being $M \leq N$. Note that a_j are related to the prices of the offers, whereas b_j are related to their quantities.

Fig. 1 illustrates these concepts: the top graph shows the TOC+ and TOC- obtained from the buy and sell offers displayed in the bottom. Note that for a given price there can be several quantities offered that will be stacked in the TOC representation. In order to display the steps of the TOCs this representation includes a marker that signals the points where the curve changes its value. For example, the quantity offered by the TOC+ at price 2 is 75 MWh, due to TOC+ being composed of right half-open intervals.

2.1. Stepwise computing Algorithm

Algorithm 1 is an efficient implementation for computing stepwise aggregated offer curves. In the first step it sorts the offers by price, accumulating those of equal price. Then, the cumulative stepwise function is obtained simply from these stacked and sorted offers. Note that the option for TOC- (not included due to the lack of space), is a simple modification of the detailed TOC+ version, allowing quick reverse calculations without needing a flip or reflection of the set of pairs. Concerning the running time of Algorithm 1, in the worst case it is $O(N \log M) + O(M) \approx O(N \log N)$.

3. ENCODED OFFER CURVES

In this paper we use the idea of encoding in order to convert an original stepwise TOC into a new piecewise linear approximation that can be used to encode in a compact form its main characteristics. Basically, this new representation replaces the non-continuous TOC by a continuous version conserving the steps of the original TOC.

Algorithm 1: TOC Creation Algorithm

Input: Raw offers defined by the set of pairs $\{(p_i, q_i)\}_{i=1}^{N}$ and type of curve $T \in \{\text{TOC+}, \text{TOC-}\}$

Output: TOC f^T of type $T \in \{\text{TOC+}, \text{TOC-}\}$ defined by the set of pairs $C^{TOC} = \{(a_i, b_i)\}_{i=1}^{M}$

initialize $C^{TOC} = \emptyset$ and $M = 0$
if *T is TOC+* **then**
  Step 1: ***Sorting and stacking*:**
  for $i \in \{1, \ldots, N\}$ **do**
    initialize $l = 1$, $r = M - 1$ and *continue* $= 1$
    while *$l \leq r$ and continue* $= 1$ **do**
      compute $m = \lfloor \frac{l+r}{2} \rfloor$
      if $p_i < a_m$ **then**
        actualize $r = m - 1$
      else
        if $p_i > a_m$ **then**
          actualize $l = m + 1$
        else
          increment quantity $b_m = b_m + q_i$
          set *continue* $= 0$
    if *continue* $= 1$ **then**
      add (p_i, q_i) to the end of C^{TOC}
  set M equals to the number of pairs in C^{TOC}
  Step 2: ***Compute the cummulative sum*:**
  for $i \in \{2, \ldots, M\}$ **do**
    compute $b_i = b_i + b_{i-1}$
else
  reverse version for TOC-
return *f^T defined by $C^{TOC} = \{(a_i, b_i)\}_{i=1}^{M}$*

The proposed Encoded Offer Curve (EOC) is a continuous piecewise linear approximation defined by a set of K knots $C^{EOC} = \{(k_j, h_j)\}_{j=1}^{K}$ sorted in increasing order according to the relation $k_{j-1} < k_j$, as shown in Fig. 2. Note that this model is valid for positive and negative TOCs. The mathematical expression of the EOC is

$$f^{EOC}(x \,|\, C^{EOC}) = \begin{cases} h_1, & x < k_1 \\ h_{j-1} + s_j \cdot (x - k_{j-1}), & k_{j-1} \leq x < k_j \\ h_K, & x \geq k_K \end{cases} \tag{3}$$

where $j = 2, \cdots, K$ and $s_j = (h_j - h_{j-1})/(k_j - k_{j-1})$ is the slope of the linear piece between knots j and $j - 1$.

Note that the complexity K of the EOC is twice that of the complexity M of the TOC. For example, if there is only one selling offer (p, q), then the associated TOC+ is defined by the same pair (p, q), but the EOC consists of two hinges $(p - \varepsilon, 0)$ and (p, q).

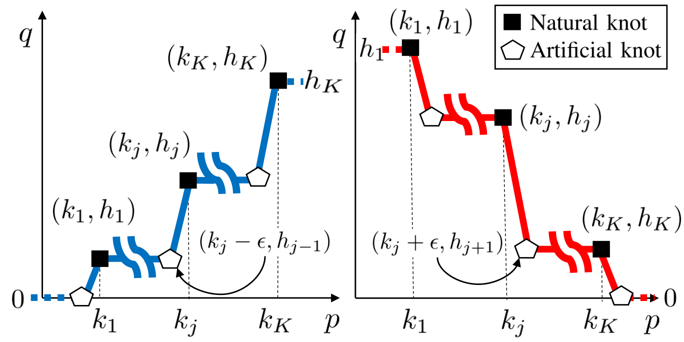

Figure 2: Encoded Offer Curve (EOC) consisting of natural and artificial knots, valid for supply function curves (left) and demand function curves (right).

3.1. Piecewise Encoding Algorithm

Exact details of the proposed approach to obtain the set of pairs C^{EOC} defining the EOC can be found in Algorithm 2 but, briefly speaking, the idea consists in creating the EOC by completing the set of *natural* knots C^{TOC} defining the original offer curve with *artificial* knots. In particular, for each natural knot representing a step, a new artificial knot is created to obtain a good approximation of this step with the piecewise function. Depending on the type of TOC, the relative position of the artificial knots is different (see Fig. 2). Although the artificial knots are under the natural ones, for TOC+s they are horizontally shifted by ε to the left, whereas for TOC-s the ε shift is to the right.

Finally, concerning the running time of Algorithm 2, it is extremely fast, in the worst case is linear on the complexity of the TOC, i.e. $O(M)$.

4. ARITHMETIC WITH OFFER CURVES

This paper proposes a formal framework to deal with elementary arithmetic operations involving OCs. This is carried out by defining the operations of addition and scalar multiplication of OCs, as well as by providing an efficient algorithmic implementation of both operations. Using these two standard operations it is possible to compute mathematical expressions involving OCs. This arithmetic is valid for both TOCs types (TOC+ and TOC-) as well as EOC.

This approach fits with the mathematical concept of vector spaces (see e.g. [26, 27]). Indeed, the three sets of demand TOCs, supply TOCs and EOCs are different vector spaces. The proposed operations of addition and scalar multiplication satisfy the axioms listed in Table 1 for each set. These axioms hold for all offer curves φ, ϕ and ψ and for all scalars c and d. Proving these axioms for the particular case of OCs, although being straightforward, is out of scope of this paper. Let us just remark that the set of TOCs (i.e. ignoring the difference between positive and negative types) is not a vector space because the closure property is not guaranteed. Fig. 3 illustrates this point

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Algorithm 2: EOC Creation Algorithm

Input: TOC f^T of type $T \in \{\text{TOC+}, \text{TOC-}\}$ defined by the set of pairs $C^{TOC} = \{(a_i, b_i)\}_{i=1}^{M}$
Output: EOC f^{EOC} defined by the set of pairs $C^{EOC} = \{(k_j, h_j)\}_{j=1}^{K}$

```
set ε to a small value but larger than the machine
  epsilon to avoid rounding errors in the floating point
  arithmetic (e.g. 100 times the machine epsilon)
initialize set C^EOC = ∅
if T is TOC- then
    for i ∈ {1, ..., M − 1} do
        add natural knot (a_i, b_i) to the end of C^EOC
        add artificial knot (a_i + ε, b_{i+1}) to C^EOC
    add natural knot (a_M, b_M) to the end of C^EOC
    add artificial knot (a_i + ε, b_{i+1}) to C^EOC
else
    add artificial knot (a_1 − ε, 0) to the end of C^EOC
    add natural knot (a_1, b_1) to the end of C^EOC
    for i ∈ {2, ..., M} do
        add artificial knot (a_i − ε, b_{i−1}) to C^EOC
        add natural knot (a_i, b_i) to the end of C^EOC
set K equals to the number of pairs in C^EOC
return f^EOC defined by C^EOC = {(k_j, h_j)}_{j=1}^K
```

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by a simple counterexample where two offer curves f^{TOC-} and g^{TOC+} are combined by computing $f^{TOC-} - g^{TOC+}$. The resultant function is neither a TOC- nor a TOC+, i.e. the set of TOCs has no closure under this operation because it does not produce a TOC of any type.

Table 1: Axioms satisfied by TOC-, TOC+ and EOC frameworks.

Property	*Axiom*
Closure	$\varphi + \phi$ and $c\varphi$ are offer curves
Associativity	$\varphi + (\phi + \psi) = (\varphi + \phi) + \psi$, $c(d\varphi) = (cd)\varphi$
Commutativity	$\varphi + \phi = \phi + \varphi$
Distributivity	$c(\varphi + \phi) = c\phi + c\varphi$, $(c + d)\varphi = c\varphi + d\varphi$
Identity	$\varphi + 0 = \varphi$, $1(\varphi) = \varphi$
Inverse	$\varphi + (-\varphi) = 0$

4.1. Addition

Offer Curve addition is defined for two OCs of the same type (i.e. TOC+, TOC- or EOC). The sum of two OCs ϕ and ψ, denoted by $\phi + \psi$, is again an OC of the same type:

$$\varphi = \phi + \psi, \tag{4}$$

where φ, ϕ and ψ are either TOCs or EOCs, as appropriate.

In order to implement this addition operation, we propose the *stacking law*, i.e. an efficient rule for addition of two or more OCs.

Consider ϕ is defined by the M^ϕ pairs $\{(u_i^\phi, v_i^\phi)\}_{i=1}^{M^\phi}$, whereas ψ is given by M^ψ pairs $\{(u_i^\psi, v_i^\psi)\}_{i=1}^{M^\psi}$. Then $\varphi = \phi + \psi$ is defined by a new set of M^φ pairs $\{(u_i^\varphi, v_i^\varphi)\}_{i=1}^{M^\varphi}$.

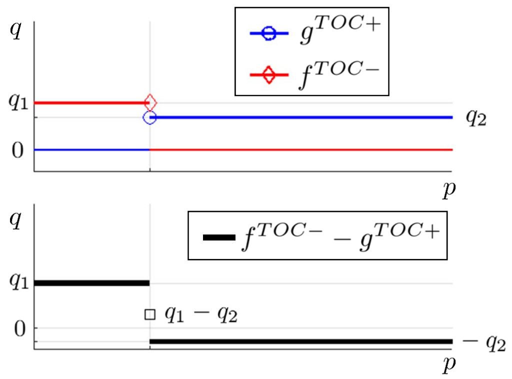

Figure 3: Example of non-closure property when combining TOC- and TOC+.

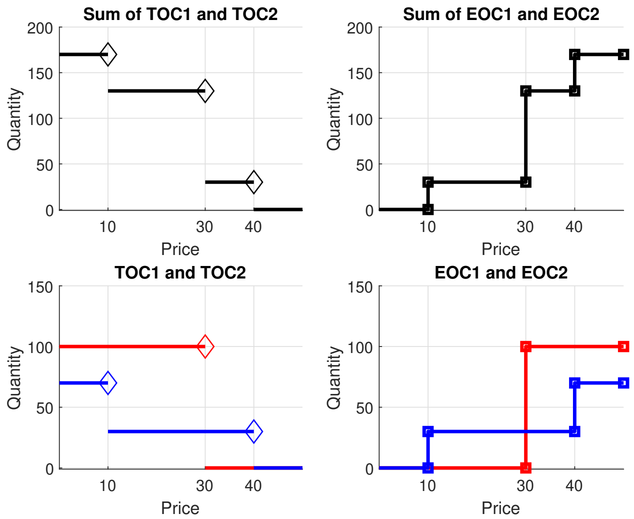

Figure 4: Addition of two TOCs (left) and two EOCs (right).

In the same way the result of adding two polynomials is a polynomial whose algebraic expression can be computed analytically from the original polynomials, the stacking law states that the coefficients that define the sum $\phi + \psi$ can be obtained directly by combining the coefficients defining ϕ and ψ. Basically, the resultant φ is obtained by vertically stacking ϕ and ψ only in the horizontal coordinates given by the unique set $\{u_i^\phi\}_{i=1}^{M^\phi} \cup \{u_i^\psi\}_{i=1}^{M^\psi}$, i.e. without repetitions, (see examples in Fig. 4).

Because of the sets of pairs describing ϕ and ψ are sorted in increasing order of the first coordinate according to the relation $u_{i-1} < u_i$, it is possible to design an efficient algorithm to implement the proposed stacking law of addition. Algorithm 3 exploits first the idea of merging the set of ordered coordinates u^ϕ and u^ψ, avoiding repetitions, to obtain the ordered and unique set u^φ in linear time $O(M^\phi + M^\psi) \approx O(M^\varphi)$. Then, in a second stage, the value of each vertical coordinate v_k^φ is computed by adding the corresponding values of ϕ and ψ in u_k^φ

$$v_k^\varphi = \phi(u_k^\varphi \,|\, C^\phi) + \psi(u_k^\varphi \,|\, C^\psi), \tag{5}$$

where $\phi(\cdot)$ and $\psi(\cdot)$ are computed using Eqs. (1), (2) or (3) depending on their type (TOC+, TOC- or EOC), as appropriate. Note that because evaluating these functions can be computed using binary search, the worst-case time-complexity of the second step of the algorithm is $O(M^\varphi(\log(M^\phi) + \log(M^\psi))) \approx O(M^\varphi \log(M^\varphi))$. Thus, the running time of Algorithm 3 in the worst-case is $O(M^\varphi) + O(M^\varphi \log(M^\varphi)) \approx O(M^\varphi \log(M^\varphi))$.

Algorithm 3: Addition for TOCs and EOCs

Input: Two curves ϕ and ψ of the same type $T \in \{\text{TOC+}, \text{TOC-}, \text{EOC}\}$ defined by the set of parameters $C^\phi = \{(u_i^\phi, v_i^\phi)\}_{i=1}^{M^\phi}$ and $C^\psi = \{(u_i^\psi, v_i^\psi)\}_{i=1}^{M^\psi}$, respectively.
Output: Sum curve $\varphi = \phi + \psi$ of type T, defined by the set of parameters $C^\varphi = \{(u_i^\varphi, v_i^\varphi)\}_{i=1}^{M^\varphi}$

```
Step 1: Compute u^φ = {u_i^φ}_{i=1}^{M^φ}:
    initialize set u^φ = ∅; i = 1; j = 1
    while i ≤ M^ϕ & j ≤ M^ψ do
        if u_i^ϕ < u_j^ψ then
            if u_i^ϕ ∉ u^φ then
                add u_i^ϕ to the end of set u^φ
                i = i + 1
        else
            if u_j^ψ ∉ u^φ then
                add u_j^ψ to the end of set u^φ
                j = j + 1
    while i ≤ M^ϕ do
        if u_i^ϕ ∉ u^φ then
            add u_i^ϕ to the end of set u^φ
            i = i + 1
    while j ≤ M^ψ do
        if u_j^ψ ∉ u^φ then
            add u_j^ψ to the end of set u^φ
            j = j + 1
Step 2: Compute v^φ = {v_i^φ}_{i=1}^{M^φ}:
    initialize set v^φ = ∅
    set M^φ equals to the size of u^φ
    for k ∈ {1, ..., M^φ} do
        compute v_k^φ using Eq. (5)
return φ defined by {(u_i^φ, v_i^φ)}_{i=1}^{M^φ}
```

4.2. Scalar multiplication

Scalar multiplication allows scaling a given OC of any type (TOC+, TOC- or EOC) by a constant factor. The result is again an OC of the same type.

In particular, the multiplication $\varphi = c \cdot \phi$ of a scalar c by a OC ϕ defined by the set of M^ϕ pairs $\{(u_i^\phi, v_i^\phi)\}_{i=1}^{M^\phi}$ produces a new OC φ defined by the pairs $\{(u_i^\varphi, v_i^\varphi)\}_{i=1}^{M^\varphi}$, where $M^\varphi = M^\phi$, $u_i^\varphi = u_i^\phi$ and $v_i^\varphi = c \cdot v_i^\phi$.

Scalar multiplication obeys the rules of vector spaces, e.g. multiplying by 1 does not change ϕ, whereas multiplying by 0 gives the zero OC. Multiplying by -1 gives the additive inverse

$$-\phi = (-1)\phi, \tag{6}$$

which allows the definition of the subtraction of two OCs as

$$\psi - \phi = \psi + (-\phi). \tag{7}$$

This operation is very useful in practice because it is the base for computing the residual demand curve of an agent. However, it is important to recall that the subtraction operation is only valid for OCs of the same type (TOC+, TOC- or EOC). This is discussed in the following section.

5. COMPUTATION OF RESIDUAL DEMAND CURVES

This section presents an illustrative example where the proposed methodology is applied to the computation of the Residual Demand Curve (RDC) of a particular agent of the market. The RDC is a function that gives the maximum energy quantity that the company can sell in the market at a given price (see e.g. [28]). Note that RDCs are used by utilities and other institutions in order to help the analysis of participants' behavior and the decision making process in the electricity market. For example, estimating the slope of the RDC of a particular agent provides useful information regarding its capability to influence market prices. Thus, the analysis of historical data allows regulators to compare the potential market power of the different companies, (see [4] for further details).

According to [4], assuming that the agent i is a pure generation company, the RDC that agent faces can be calculated as

$$R_i = D - S_{-i}, \tag{8}$$

where D is the system demand function and S_{-i} is the supply function of the firm's competitors. The sum of the demand functions of the firms D_i results in the system demand function

$$D = \sum_{j=1,A} D_j, \tag{9}$$

whereas the supply function of the firm's competitors is easily calculated as

$$S_{-i} = S - S_i = \sum_{\substack{j=1,A \\ j \neq i}} S_j, \tag{10}$$

where S is the system supply function given by the sum of the supply functions of each firm and S_i is the supply function submitted by the agent i.

Note that according to the TOC mathematical framework, Eqs. (9) and (10) can be calculated using the proposed addition and scalar multiplication. However, Eq. (8) can not be calculated using the TOC framework because it consists of a mixture of demand and supply curves. It requires a new model that support the result, for example a lookup table obtained using a discretization method.

Table 2: Synthetic demand and supply TOCs for example

D_1		D_2		S_1		S_2		S_3	
a	b	a	b	a	b	a	b	a	b
0	300	0	160	10	100	0	25	20	70
15	200	55	50	20	150	5	45	35	90
27	180			30	170	60	150	50	130
				40	200				

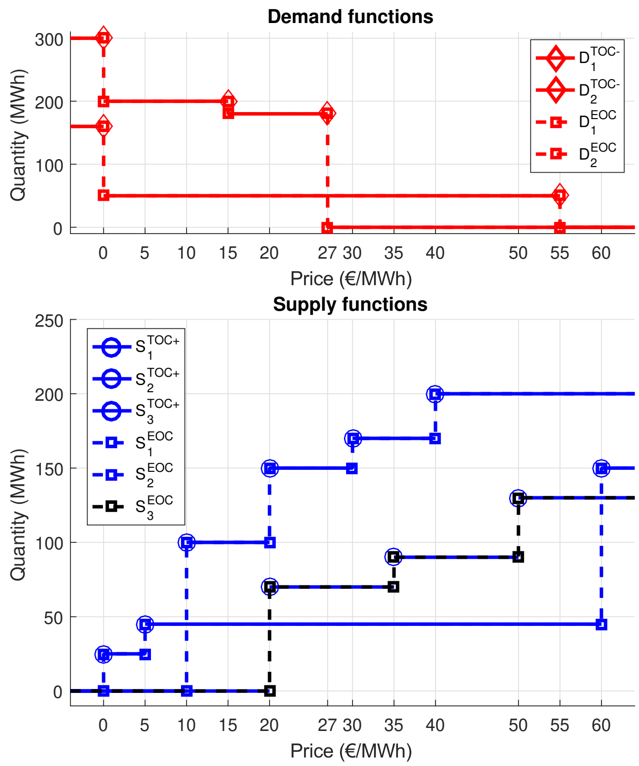

Figure 5: Demand and supply functions for the synthetic example (TOC and EOC versions.)

The EOC framework allows, as opposed as to the TOC one, carrying out all the mathematical operations involved in these equations. Furthermore, using this framework, the RDC of the agent can be directly calculated as

$$R_i = \sum_{j=1,A} D_j - \sum_{j=1,A} S_j + S_i, \tag{11}$$

or even

$$R_i = \sum_{j=1,A} (D_j - S_j) + S_i = \sum_{\substack{j=1,A \\ j \neq i}} (D_j - S_j). \tag{12}$$

Because of the flexibility of the proposed approach, it is also possible to compute Eqs. (9) and (10) using TOC- and TOC+, respectively, and then transform both results to EOCs using Algorithm 2. From those EOCs the arithmetic of Eq. (8) is straightforward, obtaining directly the RDC as an EOC, without using sampling and a lookup table to manage the operation. Therefore, the value of the residual demand at a given price can be obtained using Eq. (3).

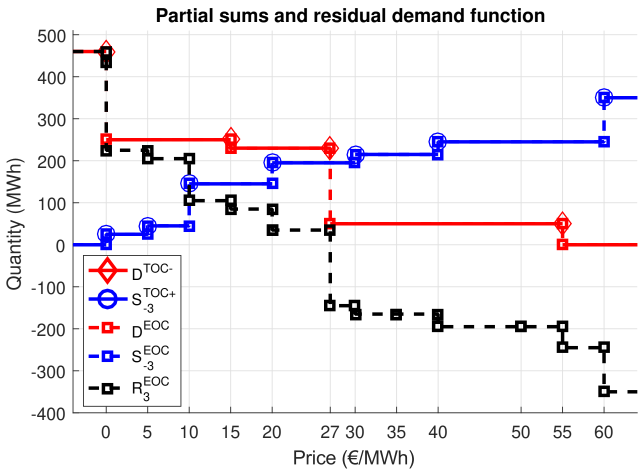

Figure 6: RDC for the third agent of the synthetic example, total demand and supply function of the competitors of this agent.

5.1. *Illustrative example*

This section provides an example in order to illustrate the application of the TOC and EOC frameworks to calculate the residual demand function of an agent. Consider the synthetic case where there are only three agents. According to the TOCs of Table 2, the first two agents have submitted both selling and buying offers, whereas the third one is a pure generation company.

Fig. 5 shows not only the five TOCs of the agents (given in Table 2), but the corresponding five EOCs. Note that there are not practical differences between the EOCs and the original TOCs.

Fig. 6 shows the system demand curve D and the supply function of the competitors of the third agent S_{-3}, obtained using Eqs. (9) and (10), respectively. For these curves both TOC as well as the EOC versions have been included. These versions have been obtained by direct addition of the corresponding Agent's OC of Fig. 5, taking into account their types. For example, $S_{-3}^{TOC+} = S_1^{TOC+} + S_2^{TOC+}$, whereas $S_{-3}^{EOC} = S_1^{EOC} + S_2^{EOC}$. Finally, the residual demand curve R_3 of the third agent is shown, obtained using Eq. (8) under the EOC framework, i.e. applying the subtraction operation of two EOCs $R_3^{EOC} = D^{EOC} - S_{-3}^{EOC}$. Note that the same R_3 is obtained by computing Eqs. (11) or (12).

6. APPLICATION TO THE DAY-AHEAD ITALIAN MARKET

We have applied the proposed TOC and EOC frameworks to compute the residual demand in a real case, the day-ahead Italian market. Italy is one of the largest energy consumers in Europe, after countries such as France and Germany. According to the Italian System Operator (TERNA), in 2017 more than half of the country's installed capacity came from conventional themal plants (54.7%), whereas the remaining production sources were

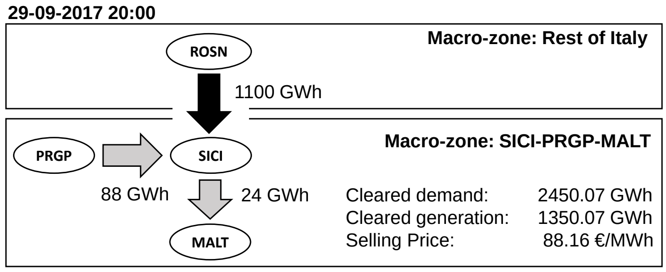

Figure 7: The Market Zone SICI-PRGP-MALT 29-09-2017, hour 20).

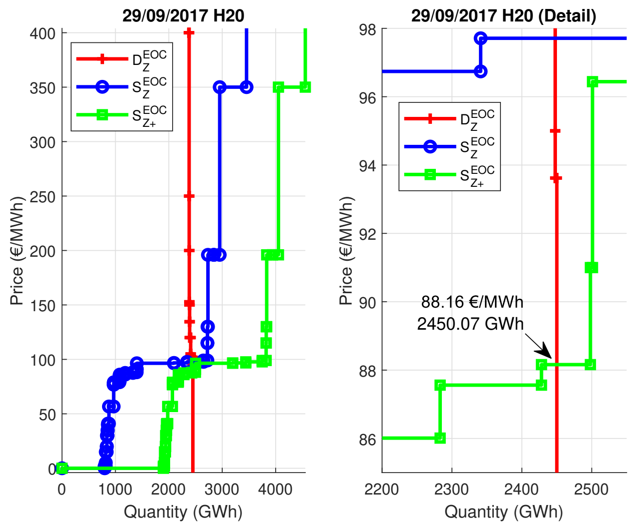

Figure 8: Zonal demand and supply TOCs in Market Zone SICI-PRGP-MALT (Italian market, 29-09-2017, hour 20).

hydro power (19.5%), photovoltaic generation (16.8%), wind power (8.3%) and geothermal (0.7%); providing a maximum generation capacity of 117.1 GW [29]. The Italian power grid is split in several geographical and virtual trading zones. These zones are aggregated on an hourly basis forming Market Zones (i.e. aggregation of geographical and virtual zones such that the flows between the same zones happens without transmission congestion. As such, these are zones with the same zonal price), as a result of the day-ahead European market coupling, solved using the Euphemia algorithm [3]. For example, according to the website of the Italian MO (GME, Gestore dei Mercati Energetici SpA, www.mercatoelettrico.org), at hour 20 of day 29-09-2017 two market Zones were formed (see Fig. 7). The Market Zone SICI-PRGP-MALT, in the South of Italy, consists of the regions Sicily, Priolo G. (a pole of limited production) and Malta (a virtual zone). This Market Zone imported 1100 GWh from ROSN, the maximum capacity of the existing connection.

Using the offer data obtained from the GME's website, we first have calculated the EOCs for the supply and demand of the Market Zone SICI-PRGP-MALT, one of the two aggregated Italian Market zones formed in that hour. Note that in the GME's website both the submitted and

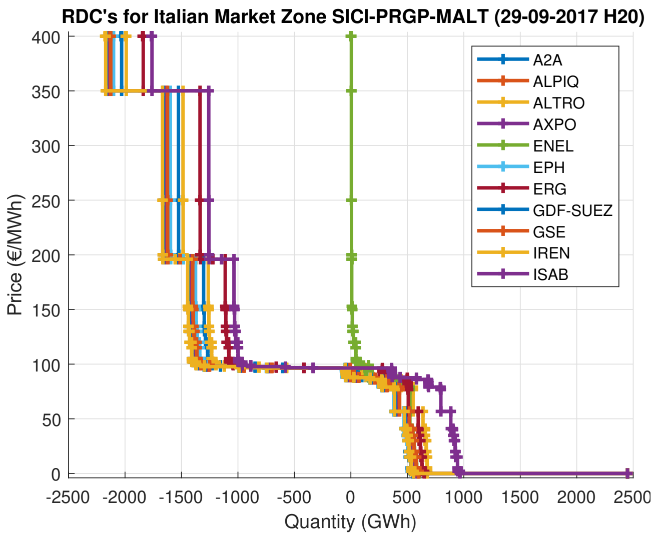

Figure 9: RDCs obtained using the proposed framework for all the generators in Market Zone SICI-PRGP-MALT (Italian market, 29-09-2017, hour 20).

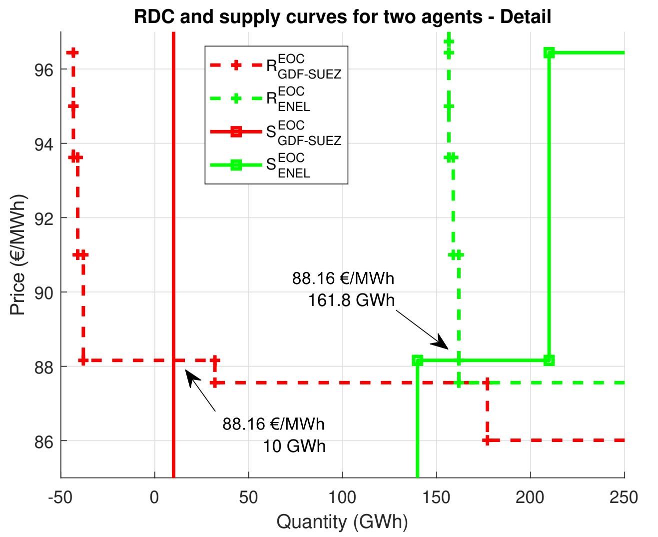

Figure 10: Residual demand and supply curves of GDF-SUEZ and ENEL in the Market Zone SICI-PRGP-MALT (Italian market, 29-09-2017, hour 20).

the accepted offers and bids are available. To compute the standard supply and demand curves as well as the RDC that is used in the literature (see e.g. [4]), cleared functions are used, i.e. those curves obtained by aggregating only accepted offers/bids, but extending the curve to the full range using the no-cleared ones. Therefore, we have used all the available raw offers and bids, except those that were rejected and their offer price is lower or equal than the cleared price. Furthermore, in the case of demand bids, a zero price in the Italian Market expresses the market participant's willingness to buy at any price. Thus, before using our framework we have changed the bid prices equal to zero by 3000 €/MWh.

Figure 8 shows the zonal demand D_Z^{EOC} and supply S_Z^{EOC} functions, as well as the supply function S_{Z+}^{EOC} that includes the 1100 GWh required from ROSN in order to cover the demand of 2450.07 GWh. The zonal equilibrium is obtained from the intersection between S_{Z+}^{EOC} and D_Z^{EOC} at 88.16 €/MWh (see right graph of Fig. 8). These stepwise functions and figures are consistent with the information published on GME's website for that hour. Note that, in order to facilitate this visual comparative, we have flipped the axis in all the figures of this section (now price is on x-axis).

In order to obtain these EOCs, first we have computed the total demand D_Z^{TOC-} and supply S_Z^{TOC+} functions of the Market Zone by summing the TOC- demand functions of the 52 buying agents and the TOC+ supply functions of the 11 selling firms, respectively. Then, we have estimated D_Z^{EOC} and S_Z^{EOC}, the EOC version of these two TOC functions. Obviously, an equivalent result can be obtained by direct computation of D_Z^{TOC-} and S_Z^{TOC+} functions by calculating both TOCs from the raw offers of the Market Zone, without previously computing the TOCs of the agents.

Concerning S_{Z+}^{EOC}, the most elegant way to obtain it consists in first computing $S_{Z+}^{TOC+} = S_Z^{TOC+} + S_{ROSN}^{TOC+}$, where S_{ROSN}^{TOC+} is defined by the price-quantity pair (3000 €/MWh, 1100 GWh), and then estimate S_{Z+}^{EOC} from S_{Z+}^{TOC+}.

Fig. 9 shows the residual demand curves for the generator agents that submitted bids for hour 20 of day 29-09-2017 in the Market Zone SICI-PRGP-MALT. In this case the RDC of each agent has been computed using the EOC framework as $R_i^{EOC} = D_Z^{EOC} - S_{Z+}^{EOC} + S_i^{EOC}$, using our previous estimations D_Z^{EOC} and S_{Z+}^{EOC}, as well as the supply function S_i^{EOC} estimated for that agent. The view of Fig. 10 shows, for two particular agents, a detail of their residual demand and supply functions focused on the zonal clearing price. Note that each agent has its own RDC, expressing the zonal clearing price and the amount of energy that could have been cleared if the agent had submitted a different supply curve, assuming the agent's competitors do not change their offers. Thus, according to the information provided by these EOCs, the agent GDF-SUEZ is a price taker at that hour as its RDC is quite horizontal, whereas ENEL has the role of price maker as small changes in its offer produces great changes on the zonal clearing price, e.g. increasing the price of its offer at 137 GWh the agent would have increased the zonal clearing price in the same amount. Furthermore, using these EOCs it is also possible to compute easily the amount of energy cleared in this Market Zone at that hour by GDF-SUEZ (10 GWh) and ENEL (161.8 GWh).

Finally, in order to quantify the accuracy of the proposed EOC approach, Table 3 compares EOC with the model obtained using the LHM approach of [18], and the X-model of [21]. It summarizes the Root Mean Squared Error (RMSE) and the Mean Absolute Error (MAE) when modeling the total demand and supply curves of the Market Zone SICI-PRGP-MALT from 1-11-2016 to 30-9-2017

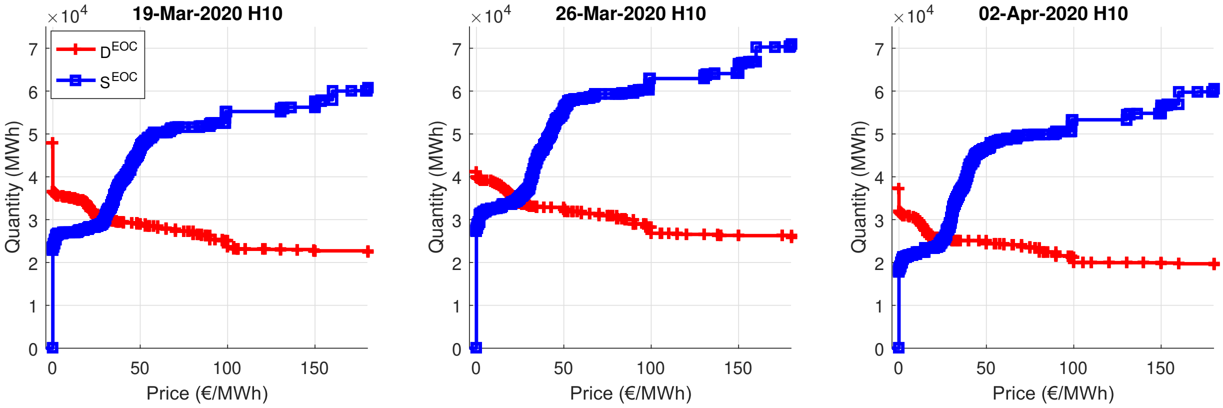

Figure 11: Demand and supply EOCs for three different sessions in the Iberian Day-Ahead Electricity Market.

Table 3: RMSE and MAE (in brackets) for the three models.

Model	min	mean	max
	Demand curve in Market Zone SICI-PRGP-MALT (TOC-)		
LHM	11.34 (3.67)	28.82 (12.53)	77.93 (32.71)
X-Model	17.89 (4.39)	51.91 (24.97)	130.71 (57.99)
EOC	5.17e-13 (3.78e-13)	0.49 (0.00552)	3.66 (0.056)
	Supply curve in Market Zone SICI-PRGP-MALT (TOC+)		
Model	min	mean	max
LHM	0.05 (0.016)	0.53 (0.26)	4.16 (1.42)
X-Model	0.24 (0.07)	2.76 (1.10)	18.12 (6.49)
EOC	2.48e-15 (1.74e-15)	0.0072 (6.26e-05)	0.29 (0.00242)

(8016 hours). According to Table 3, the proposed EOC is clearly more accurate than the other two approaches.

7. APPLICATION TO THE IBERIAN DAY-AHEAD ELECTRICITY MARKET

The previous section was devoted to illustrate how the proposed EOC framework can be used to compute and analyze residual demand curves in electricity markets. The flexibility of the proposed methodology is displayed in this section, whereas the EOC framework is used to quantify the differences between two supply curves in the spot Iberian Electricity Market (MIBEL). This market, which covers the mainland of Portugal and Spain, provides market agents with a venue to submit their selling and takeover bids for electrical energy for next day's 24 hours. Each day at 12:00 CET an auction takes place, which sets the prices and volume of energy being traded for the following 24 hours. Similarly to its Italian counterpart, the price and energy volume for each hour h is obtained as the intersection of the aggregated supply and demand curves. By obtaining the EOC of both the supply and demand curves the differences in the bidding behaviour of the agents in different market sessions can be quantified. This case study will analyze the aggregated curves of the MIBEL spot market of March 2020, in order to measure the impact that the COVID-19 pandemic had in the bidding curves.

In March 2020 the first cases of COVID-19 were detected in Spain, quickly evolving and creating the first wave of the COVID-19 pandemic [30]. Under this unusual situation, the Spanish government adopted different public health measures in order to slow down the spread of the virus. These measures entailed enforced social distancing by means of stay-at-home orders. On March 29, 2020 (Sunday), an enforced complete lockdown with non-essential business closures started, significantly impacting the electricity demand. As such, this study will use the day 26-03-2020 as a baseline, being the last Thursday before the complete lockdown of the country. Both the previous (19-03-2020) and next Thursday (02-04-2020) will be included in this comparison, in order to take into account the weekly periodicity exhibited by these curves [23]. Using the public bidding data obtained from OMIE's website (MIBEL's Market Operator, `www.omie.es`), the EOCs for the supply and demand curves for these dates have been estimated. These have been obtained following the same procedure as the one presented in the previous section: aggregating only accepted offers/bids and then extending the curve to the full price range using non-cleared bids.

Fig. 11 shows the EOC's of both the demand and supply curves for hour 10 of the three aforementioned days, detailing the bidding behaviour of market agents before and after the lockdown. As can be seen, the market clearing price in all three sessions is similar, in the $[20, 26]$ €/MWh range. Nevertheless, for a market agent it is of utmost im-

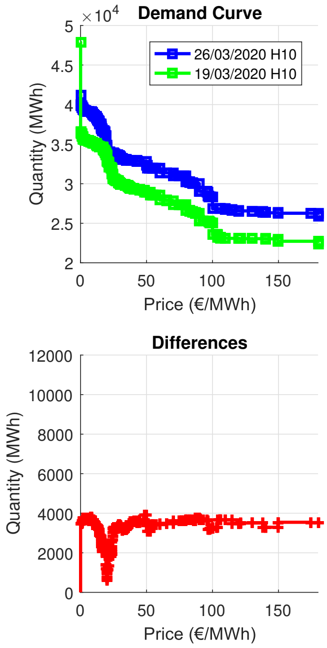

Figure 12: Top: Hourly demand EOCs of the Day-Ahead Iberian Electricity Market for three different dates. Bottom: Difference EOC obtained as the subtraction of the two previous demand EOCs.

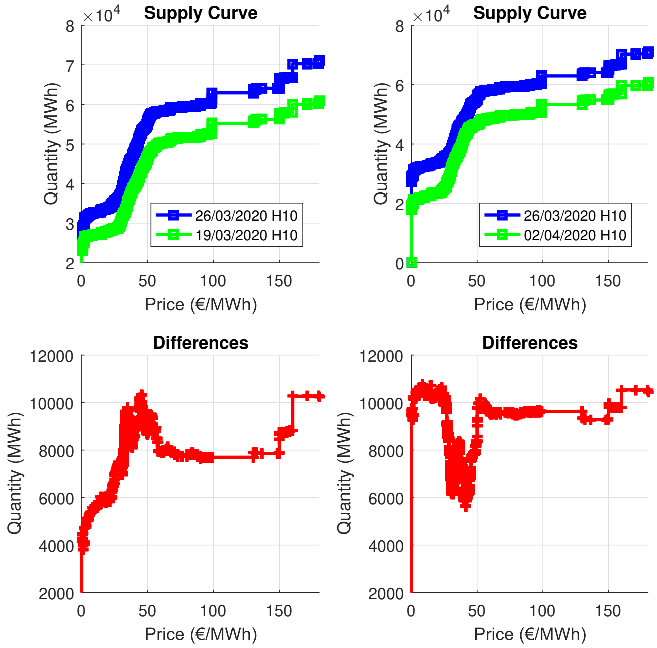

Figure 13: Top: Hourly supply EOCs of the Day-Ahead Iberian Electricity Market for three different dates. Bottom: Difference EOC obtained as the subtraction of the two previous supply EOCs.

portance to measure the changes in the bidding patterns of its competitors in order to adapt its own strategy. An useful tool to quantify these deviations is to obtain the difference between the observed supply (demand) curves in two different hours. This difference can be easily computed using the EOCs that have been previously calculated and the framework proposed in this paper, as the arithmetic that has been developed can be used to define this new curve as

$$Y_i^{EOC} + (-Y_j^{EOC}), \tag{13}$$

where i, j denote the different hours being compared and $-Y_j^{EOC}$ denotes the additive inverse of the EOC (defined in Section 4.2). By analyzing the shape of this curve, relevant information about the changes in the bidding behavior of the agents can be obtained. Figs. 12 and 13 illustrate this concept by obtaining the difference between Fig. 11's consecutive demand and supply curves, respectively. As can be seen in the right panel of Fig. 12, the enforced lockdown shifted the whole demand curve, decreasing the mean level of the curve. The shape of both pre- and post-lockdown demand curves are similar, only exhibiting small shape differences in the $[10, 25]$ €/MWh price range. The left panel of Fig. 12 shows the difference between the demand curves of the two last Thursdays before the lockdown, which can be used to quantify the impact that the lockdown had on these demand curves. As can be seen, the typical level difference between two demand curves was around 4000 MWh in the weeks before the lockdown; whereas the post-lockdown curve exhibited a drawdown of approximately 7000 MWh when compared to the previous week's curve. A totally different situation is observed in the analysis of the supply curves, illustrated in Fig. 13. The left and right panels show the difference between the EOCs of the supply curves before and after the complete lockdown, highlighting the changes in the bidding behavior of the agents. When comparing the two pre-lockdown supply curves, a similar shape is observed except in the price range of $[0, 50]$ €/MWh, where the two curves present a different slope. However, this relationship changes when comparing the post-lockdown curve with the previous week's curve: in this case, the lockdown curve exhibits an average level lower than the reference curve, while showing strong shape discrepancies in the $[25, 50]$ €/MWh price range of the curve. As the market's clearing price usually falls in this price range, these variations could be attributed to sudden changes in the bidding behavior of the market agents. This analysis illustrates how the proposed EOC framework provides flexible tools to analyze sudden changes in aggregated offer curves, which is of utmost importance when analyzing electricity markets.

8. CONCLUSIONS

Liberalized electricity markets allow companies to trade energy by submitting selling and buying offers. These raw offers are aggregated to build stepwise supply and demand curves. That offer curves are at the core of many relevant processes. For example, the market operator uses these supply and demand curves to clear the market. For market agents not only setting their bidding strategies in terms of supply curves is critical, but also analyzing, when possible, the offer curves to understand the bidding behavior of their competitors. For regulators, estimating the slope of the residual demand curves, computed from offer curves, also provide useful information regarding the potential market power of the different producers.

In this paper we first formalize the concept of stepwise offer curves, explicitly defining the True Offer Curve (TOC). Although these TOCs have no error approximation, they do not allow for example computing the residual demand curve of an agent by simple subtraction. To overcome the inherit difficulties of those non-continuous TOCs, we propose the Encoded Offer Curve (EOC), a continuous piecewise version that approximates the steps of the original TOC with high accuracy and that can be used to model and combine both demand and supply offer curves. We also present in this paper fast and simple specialized algorithms to obtain both TOC and EOC models from raw offers. Furthermore, a formal framework to deal with elementary arithmetic operations involving TOCs and EOCs has been developed, including a fast algorithmic implementation of the basic operations. Thanks to the proposed framework, we can operate with offer curves using the rules of algebra. This has been successfully tested in both the Italian electricity market, computing the residual demand curves of the producers in a particular Market Zone; and in the Iberian electricity market, quantifying the differences between supply and demand curves in different stages of the COVID-19 pandemic.

In addition, the proposed framework could also be extended by including the development of a specialized algorithm to estimate the market-clearing price as the intersection of EOCs. Further research would also include the quantification of the accumulation of rounding errors as result of a large number of consecutively performed arithmetic operations with EOCs. Finally, note that machine learning techniques could be used to extend the applications of the proposed approach. For example, forecasting techniques can be used to estimate EOCs for those periods where the submitted offers by the agents are not yet available, possibly due to certain rules of confidentially of the market.

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