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PROBLEM SOLVING IN PRIMARY EDUCATION MATHEMATICAL TEXTBOOKS IN SPAIN

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This study analyzes the approach to problem solving in nine mathematical textbooks from three of the leading publishers in Spain. It shows that a heuristic approach is not applied to problem solving and that over 60% of the problems posed to students are mechanical activities concealed behind a statement.

INTRODUCTION

This study explores a problem that can be divided into three key lines of argument. Firstly, the fact that many students struggle with mathematics, as highlighted in the PISA (Organisation for Economic Co-operation and Development [OCDE], 2013) and TIMSS (Mullis, Martin, Ruddoch, O'Sullivan & Preuschoff, 2012) international reports on education.

Secondly, the heavy influence of mathematical textbooks on teaching and learning processes around the world (Erbaç, Alacaci & Bulut, 2012; Pepin & Haggarty, 2001; Vincent & Stacey, 2008). However, despite this fact, studies on how mathematical textbooks are used in the classroom prove that academic failure in this subject could be due in part to the difference between the methodological approach of mathematical textbooks, and the knowledge developed by students from these textbooks (Ewing, 2006; Gómez Chacón, 2002; Pehkonen, 2004). In Spain, a report by the Ministry of Education's Institute of Evaluation (IEME, 2009) showed that textbooks were used by 99.1% of all students and that their use is on the rise, to the detriment of group work.

Thirdly, we take a closer look at problem solving as one of the key elements of mathematical education that could contribute to the general development of student capacities (National Council of Teachers of mathematics [NCTM], 2003). In the Spanish education system, the latest education laws (Ministry of Education & Science [MEC], 2006, 2013) state that problem solving should form the backbone of all mathematical activity in the classroom.

The abovementioned problem points to the need to analyze how problem solving is dealt with in mathematical textbooks as these are the main teaching resource used in primary education. In fact, the body of work consulted for this study shows that problem solving has been included as an organizer or indicator in various studies on both textbooks in general and specific mathematical textbooks (Abrate, Delgado & Pochulu, 2006; Erbaç et al., 2012; Monterrubio & Ortega, 2011; Pepin & Haggarty, 2001; Serradó & Azcárate, 2003; Vincent & Stacey, 2008).

METHOD

A textbook analysis tool was developed from a design that mainly focuses on descriptive methods; namely, content analysis.

As our theoretical framework, we have used the model on textbook use proposed by Rezat (2006) which is built around the contradictory characteristics inherent to textbooks. When defining the

methods associated with the data collection tool, we have taken into account the cross referencing of categories and criteria provided by the work of the American Association for the Advancement of Science (AAAS, 2000), González and Sierra (2004), Martínez Bonafé (1992), Monterrubio and Ortega (2011) or Serradó and Azcárate (2003).

The practical approach is based on an *a priori* exploratory and descriptive analysis (Van Dormolen, 1986) of how problem solving is dealt with in nine Primary Education mathematical textbooks produced by three of the leading publishers of educational material in Spain, which were used during the 2011-2012 academic year.

The data collection tool was piloted before the study through direct application and was approved in testing by expert judges.

In order to show how problem solving is dealt with in these mathematical textbooks, we will consider if problem solving is used as a central vehicle for presenting the corresponding content or if its use is limited to certain specific sections. We will also analyze if there is a genuinely heuristic approach to problem solving or if the term "problem" merely serves as a heading to introduce mechanical exercises that only allow for a single, closed solution.

A textbook will be considered to have a heuristic approach to problem solving when it presents open problems and activities that are sufficiently unstructured that they require students to put into practice prior knowledge, to come up with their own problem-solving strategies or to use systematic estimation, in order to test out and collect data that could support future decisions. We will also consider if the textbook asks students to invent or edit the heading or question used to introduce the problem.

In specific problem-solving sections, we will analyze if detailed protocols are followed, if they are characterized by strategies or if students are asked to follow models from previously solved problems.

FINDINGS AND DISCUSSION

Despite the fact that none of the textbooks used problem solving as a central vehicle for presenting the content, they did all have a space dedicated solely to problem solving within a specific section of each teaching unit. However, problems only accounted for 27.44% of the 7889 activities presented in the nine textbooks in total.

As has been the case with other studies (Jiménez & Verschaffel, 2014; Rohrer & Taylor, 2006; Vincent & Stacey, 2008), we have found that the mathematical textbooks do not take a heuristic approach to problem solving. As such, they encourage a superficial or shallow type of teaching characterized by very repetitive problems with a low level of procedural complexity and a lack of mathematical reasoning.

In general, problem solving in these textbooks consists of examples or models used to repeat or imitate, placing a higher value on the end result than on the problem-solving process and insisting on the verification of the obtained results. However, although they do encourage students to reflex on the solution, they give little or no importance to prior estimation as a source for comparing and contrasting hypotheses. Even still, although the percentage of open or "thinking" problems is not very high (17% on average), it is much higher in many cases than the percentage of problems that, in order

to be solved, require the use of detailed protocols or that are characterized by the need to follow logical steps (12.4% and 11.5%, respectively).

In general, there is one section dedicated to testing student competencies that poses problems obtained from their own experiences. These problems require the students to put into practice their prior knowledge, to invent their own problem-solving strategies, to use trial-and-error strategies or systematic estimation, and to collect data that can be used to argue, justify or explain their decisions. However, there is a marked absence of problems that encourage students to analyze potential alternatives before attempting to solve the problem. In addition, these textbooks neither ask students to propose, invent or edit the headings of problems or questions to given headings, nor encourage problem solving in groups.

Furthermore, this study has proven that textbooks from the same publisher share a similar lineal layout that affects not only the quantity and the variety of activities but also the way in which problem solving is approached. In line with Serradó and Azcárate (2003), we would argue that it is not so much the difficulty of the content that determines the chosen activities but rather the physical space left over after the theory has been presented.

Given that experts in mathematical education (Guzmán, 2007) recommend problem solving as *the* method for putting into practice the main objective of active learning, the findings of this study show that both publishers and authors should revise their approach and adopt the necessary measures in order to align mathematical textbooks with approaches that will ensure quality mathematical education.

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