COMILLAS
UNIVERSIDAD PONTIFICIA
ICAI

GRADO EN INGENIERÍA EN TECNOLOGÍAS INDUSTRIALES

TRABAJO FIN DE GRADO

Transformación Eléctrica Volkswagen Polo 2009

Autor: Jorge Ojeda López de Uralde
Director: Mike Griffis

Madrid

Declaro, bajo mi responsabilidad, que el Proyecto presentado con el título Transformación Eléctrica Volkswagen Polo 2009 en la ETS de Ingeniería - ICAI de la Universidad Pontificia Comillas en el
curso académico 2021/22 es de mi autoría, original e inédito y
no ha sido presentado con anterioridad a otros efectos.
El Proyecto no es plagio de otro, ni total ni parcialmente y la información que ha sido tomada de otros documentos está debidamente referenciada.

Fdo.: Jorge Ojeda López de Uralde   Fecha: 15/06/2022

Jorge

Autorizada la entrega del proyecto

EL DIRECTOR DEL PROYECTO

Fdo.: Mike Griffis   Fecha: 15/ 06/ 2022

COMILLAS
UNIVERSIDAD PONTIFICIA
ICAI

GRADO EN INGENIERÍA EN TECNOLOGÍAS INDUSTRIALES

TRABAJO FIN DE GRADO

Transformación Eléctrica Volkswagen Polo 2009

Autor: Jorge Ojeda López de Uralde

Director: Mike Griffis

Madrid

TRANSFORMACIÓN ELÉCTRICA VOLKSWAGEN POLO 2009

Autor: Ojeda López de Uralde, Jorge.

Director: Griffis, Mike.

Entidad Colaboradora: ICAI – Universidad Pontificia Comillas.

RESUMEN DEL PROYECTO

La introducción en el mercado de un modelo cien por cien eléctrico, de tamaño comedido y pensado para ciudad, es una gran oportunidad de mercado para Volkswagen. En esta línea, este proyecto presenta una propuesta para la marca basada en la implementación de una unidad de potencia eléctrica en una versión ya existente (Polo 2009) buscando obtener el máximo rendimiento a través de la caja de cambios.

Palabras clave: caja de cambios, transmisión, motor eléctrico, motor de combustión, eficiencia, rendimiento, Volkswagen.

1. Introducción

Debido a las crecientes restricciones para la movilidad de los vehículos de combustión en las grandes ciudades europeas, la introducción de un modelo pequeño y cien por cien eléctrico parece ser una gran oportunidad para el aumento en las cifras de venta de Volkswagen. Con este propósito, se presenta un vehículo con una fuente de energía limpia y un rendimiento atractivo a través del desarrollo de la caga de cambios. Como resultado, dos características atractivas se combinan en el mismo coche.

2. Definición del Proyecto

El objetivo del proyecto es obtener el mejor rendimiento posible del vehículo a través del desarrollo de la caja de cambios. Para ello, el primer paso ha sido la elección de un nuevo motor, una unidad PHI301 con 114 cv procedente de *PhiPower*. La idea es utilizar un motor con un ratio peso-potencia similar a la versión de combustión. Con este propósito, es necesario considerar la diferencia de peso producida por la electrificación. Una vez el vehículo se ha electrificado, se deben considerar diferentes especificaciones del vehículo. Algunas ya están definidas por el modelo base, y otras quedan abiertas para ser establecidas durante el desarrollo del trabajo.

Tomando como base las condiciones de trabajo y la utilización de los diagramas de potencia invertidos, las diferentes marchas se han desarrollado a través de un proceso de iteración. Durante el desarrollo de los mencionados diagramas, es esencial considerar el rango de mayor eficiencia del motor seleccionado. Para el motor en cuestión, el mejor resultado se obtiene entre 4000 y 7000-8000 rpm y, por ello, durante el diseño de la transmisión se ha buscado trabajar dentro de ese intervalo. El objetivo es buscar el cambio de marcha en torno a 7000 rpm cuando se demandan al motor 114 cv y tratar de que el motor se sitúe en 4000 rpm cuando la nueva marcha se engrana.

El siguiente paso ha sido convertir las relaciones de transmisión en engranajes reales. Como resultado, cada marcha introduce dos etapas: la primera, común para todas las marchas, compuesto por los engranajes 9 y 10 y la segunda, independiente para cada marcha, compuesta por los engranajes correspondientes de cada una. Una vez los engranajes están diseñados, es esencial asegurar que son capaces de soportar las condiciones de trabajo a través de la *Lewis Bending Equation*. Esta comprobación también es un proceso iterativo, y durante el estudio, el grosor de los engranajes 1, 2, 5 y 6 ha sido incrementado por tres con respecto al valor de partida para asegurar su correcto funcionamiento.

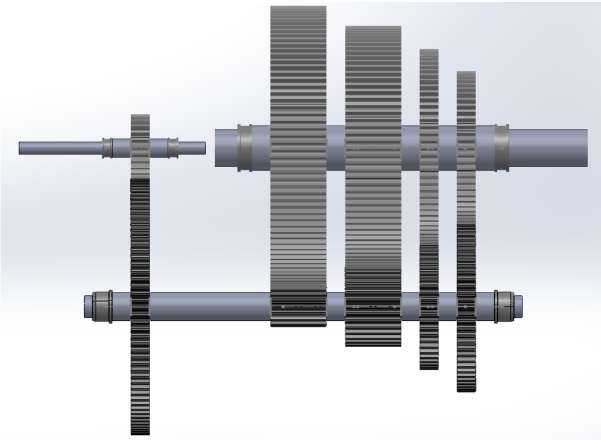
Consecutivamente, los rodamientos y ejes son diseñados. Para ambos elementos, es necesario conocer las reacciones y fuerzas sufridas por el sistema en cada marcha. Con este fin, es necesario aplicar equilibrio de momentos y fuerzas en los diferentes ejes. Después, un estudio completo se realiza para analizar el caso más desfavorable. Si el sistema es capaz de soportar estas condiciones, podrá trabajar bajo cualquier otras. La decisión de diseñar primero los rodamientos y después los ejes es arbitraria, y esta elección hace que los rodamientos puedan cambiar durante el diseño de los ejes como pasa con el OutShaft y CounterShaft. Como paso final de diseño, la carcasa del sistema debe ser elaborada. Es importante recordar que el espacio y el peso son dos factores a tener en cuenta para que el rendimiento no se vea perjudicado. La función principal de esta carcasa es proteger la transmisión y evitar que se introduzcan objetos no deseados.

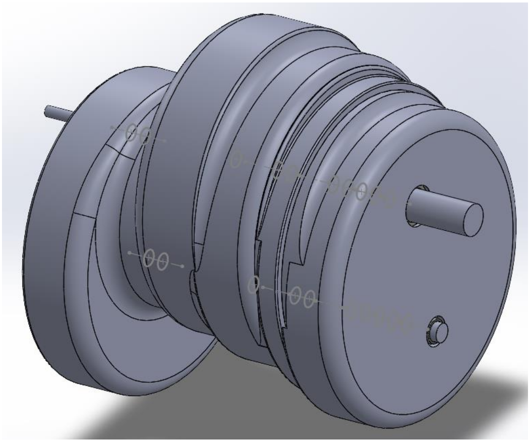
Para terminar, el sistema se simula mediante *SolidWorks* para asegurar su correcto funcionamiento en un vehículo real. Los resultados son comparados con los valores utilizados durante el diseño.

3. Descripción del modelo/sistema/herramienta

El modelo de combustión base es un Volkswagen Polo 2009 con 85 cv y cambio manual de 5 relaciones. Algunas especificaciones del nuevo modelo están definidas por el: tamaño de rueda, peso o el ratio peso-potencia objetivo. Por otro lado, otras están abiertas a ser determinadas durante el desarrollo del proyecto: tracción delantera, cambio manual de 4 marchas o velocidad máxima de 180 $\frac{\mathrm{km}}{\mathrm{h}}$. Con este diseño se consigue que cada marcha trabaje en el rango de mayor eficiencia del motor.

El número de dientes varía entre 15 para el engranaje 5, y 120 para el 1. Todos ellos tienen un grosor de 1 in excepto el 1, 2, 5 y 6 (3 in). Todos los ejes tienen dos cambios de diámetro, lugar en el que se albergan los rodamientos que son, en todos los casos, cilíndricos. Las cargas son más altas en el CounterShaft, donde se ha utilizado un material más resistente para evitar un grosor excesivo que suponga tener que cambiar el tamaño del engranaje 5. Finalmente, el grosor de la carcasa es de 0.5 in para evitar daños, pero suficientemente ligero para no afectar al rendimiento.





. *Figure 2: Vista del sistema final.*

Figure 1: Resultado final

4. Resultados

La Table 1 recoge los resultados de la simulación de 1,5 segundos además del error del resultado en comparación con los valores utilizados para el diseño de la transmisión. El número de datos recogidos aumenta con la velocidad, con el propósito de no perder precisión. Como puede verse, el error alcanzado es muy pequeño y puede considerarse despreciable. Así, la transmisión podría implementarse en un coche rea y funcionaría correctamente.

Gear	Range	# of data collected	Expected value [rpm]	Mean [rpm]	Error [%]
1	First	105	132.5	132.536	0.027
	Second	131	220.83	220.813	0.008
2	First	144	397.49	395.229	0.569
	Second	160	618.32	620.772	0.397
3	First	149	839.15	838.934	0.026
	Second	155	1059.97	1065.66	0.537
4	First	158	1324.97	1323.343	0.123
	Second	145	1589.96	1586.566	0.213

Table 1: Objetivos de diseño vs. Resultados de simulación.

5. Conclusiones

La Table 4 recoge el rendimiento final alcanzado con la electrificación y la implementación de la transmisión diseñada. Como resultado, el coche acelera de 0 a 120 $\frac{\text{km}}{\text{h}}$, incluyendo una pendiente del 5% (valor máximo para autovías en España), en solo 14 segundos. Este tiempo es el mismo que emplea el vehículo original sobre una superficie plana. La fuerza debida a la pendiente es el segundo elemento más grande que se opone al avance del vehículo. Como resultado, la mejora en el rendimiento se puede considerar excelente.

Gear	$a_{Achieved}$
1	0.97g
2	0.355g
3	0.145g
4	0.045g

Table 2: Rendimiento alcanzado.

6. Referencias

Richard G. Budynas and J. Keith Nisbett (2011). Shigley's Mechanical Engineering Design, Ninth Edition.

Carlier, M. (4 Aug 2021) Worldwide deliveries of Volkswagen Group's most popular car model between 2017 and 2020.
https://www.statista.com/statistics/461115/most-popular-models-of-volkswagen-group-by-worldwide-delivery/

Cities with emissions restrictions:
Renault Group (25 March 2020) Renault Group: Low Emission Zones (LEZs) in Europe.
https://www.renaultgroup.com/en/news-on-air/news/low-emission-zones-lezs-in-europe/

Engineering Explained (19 Jul 2017). How Do Electric Cars Produce Instant Maximum Torque? Youtube.
https://www.youtube.com/watch?v=VwFEyL_JJAQ

Gears:
https://www.rushgears.com/

2009 VOLKSWAGEN POLO ELECTRIC TRANSFORMATION

Author: Ojeda López de Uralde, Jorge.

Supervisor: Griffis, Mike.

Collaborating Entity: ICAI – Universidad Pontificia Comillas.

ABSTRACT

The introduction of a hundred percent electrified small city car would be a great market opportunity for Volkswagen. In this line, a proposition to the German brand is made. An electric power engine is introduced in an already existing version (Polo 2009) and the idea is to get the maximum performance from it through the design of the gearbox.

Keywords: gearbox, transmission, electric engine, combustion engine, efficiency, performance, Volkswagen.

1. Introduction

Due to the increasing number of restrictions for combustion vehicles mobility in the European big cities, the introduction of a hundred percent electrified small city car seems to be a great opportunity for Volkswagen to increase their sales figures. To this end, a car with a clean power source and with an attractive performance is presented through the development of a gearbox. As a result, two appealing characteristics are combined in the same car.

2. Methodology

The aim of the project is to get the best possible performance of the car through the development of the gearbox. Therefore, the first step has been the election of the new engine, a PHI301 series unit with 114 Hp from *PhiPower*. The idea is to use one with similar power to weight ratio of the already existing combustion version. To this end, it is necessary to consider the weight difference that the electrification introduces. Once the vehicle has been electrified, there are different specs to consider. Some are already defined by the base model and others are open to be established.

Based on the working conditions and using flipped power flow diagrams, the different gears have been developed through an iteration process. Over the development of the mentioned power flow diagrams, it is essential to consider the most efficient range of the chosen engine, so the performance of the car is the highest. In this particular case, the best result is given between 4000 and 7000-8000 rpm, and, therefore, during the design of the transmission this is the range that has been pursued. The objective is to look for a shift change at around 7000 rpm when 114 Hp are demanded to the engine and go to 4000 rpm in the new meshed gear.

The next step has been to convert the desired gear ratios based on the working conditions and the defined objectives into real gears. As a result, each gear introduces two stages in the system: the first spur, which is common for all the gears, compounded by gears 9 and 10 and the last spur, independent for each gear, compounded by the corresponding gears of each one. Once the gears are designed, it is essential to ensure they withstand their working conditions through the Lewis Bending Equation. This is also an iteration process, and during the study, the face width of gears 1, 2, 5 and 6 has been increased by 3 to ensure the system meets the requirements.

Consecutively, the bearings and shafts are designed. For both elements the reactions and forces suffered by the system on each gear needed to be calculated. To this end, equilibrium of momentum and forces is applied on the different shafts. Then, a complete study is made of the worst possible case to ensure the system is able to withstand every condition under which is going to be working. The decision of designing the bearings before the shafts is arbitrary, but their election is subjected to change as it happened for the bearings in the OutShaft and CounterShaft. As a final design step, a case that covers the entire system is designed. It is important to bear in mind that space and weight are a concern, so the performance doesn't get damaged. Its main function is to avoid the transmission gets harmed or undesired elements get into it.

To sum up with, the system has been simulated using *SolidWorks* to ensure the design is suitable for a real car. Results are compared to the expected value used to design the model.

3. Model description

The base combustion model is a Volkswagen Polo 2009 with 85 Hp and a 5-gear manual transmission. Some specs of the new model are set by it: the tires size, the initial weight or the objective power to weight ratio. Even though, others are open to change and have been established during the development of the project: front wheel drive, 4-gear manual transmission or a top speed of 180 $\frac{\mathrm{km}}{\mathrm{h}}$. The decision of making a 4-gear transmission was made to ensure the full power of the engine was squeeze. With such design, each gear is made to work in the rpm range in which the engine is most efficient.

The teeth number varies from 15 in gear 5, to 120 in gear 1. All gears have a face width of 1 in except from gears 1, 2, 5 and 6 (3 in). All shafts have two shoulders to accommodate the corresponding bearings that are, in all cases, cylindrical roller bearings. The shafts diameters are different depending on the forces they suffer. The loads are higher in the CounterShaft but a harder material has been used so the shaft can be introduced inside the smaller gear (gear 5). Finally, the case has a thickness of 0.5 in, in order to avoid damages, but light enough so it doesn't affect the car performance.

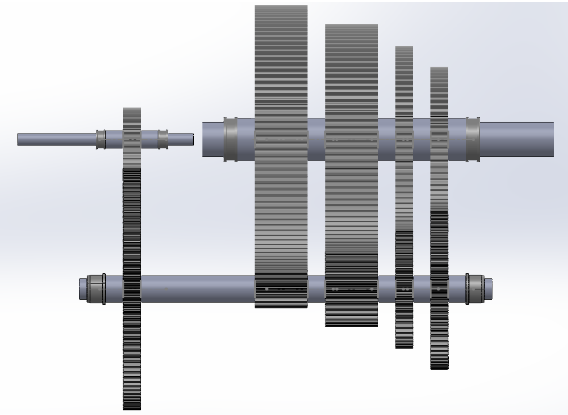


Figure 3: Final system view.

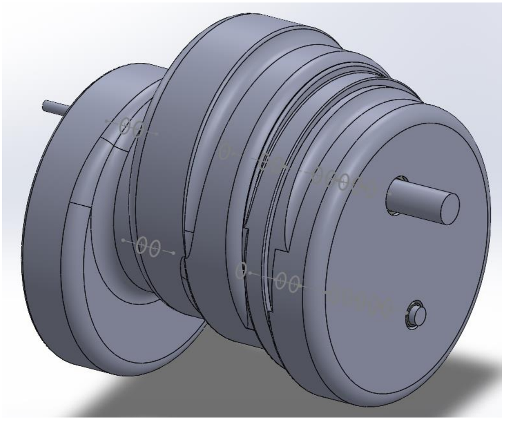


Figure 4: Final result.

4. Results

Table 3 collects the result from a simulation of 1,5 seconds and the error compared to the expected values used for the gearbox design. The number of data collected increases with the speed, so no accuracy is lost. As it can be seen, the error achieved in every gear is small and can be considered negligible. As a result, the transmission might be implemented in a real car that would work successfully.

Gear	Range	# of data collected	Expected value [rpm]	Mean [rpm]	Error [%]
1	First	105	132.5	132.536	0.027
	Second	131	220.83	220.813	0.008
2	First	144	397.49	395.229	0.569
	Second	160	618.32	620.772	0.397
3	First	149	839.15	838.934	0.026
	Second	155	1059.97	1065.66	0.537
4	First	158	1324.97	1323.343	0.123
	Second	145	1589.96	1586.566	0.213

Table 3: Design objective vs. Simulation results.

5. Conclusion

Table 4 collects the final performance achieved by the electrification of the car and the implementation of the transmission designed. As a result, the car gets a 0 to 120 $\frac{\mathrm{km}}{\mathrm{h}}$ with a 5% of slope (maximum value for the Spanish highways) in 14 second. This is the same time the original car would have spent in a flat surface. Because the uphill force is the second bigger load against the vehicle progress, considering the engine has a very similar power to weight ratio the improvement is excellent.

Gear	$a_{Achieved}$
1	0.97g
2	0.355g
3	0.145g
4	0.045g

Table 4: Performance achieved.

6. References

Richard G. Budynas and J. Keith Nisbett (2011). Shigley's Mechanical Engineering Design, Ninth Edition.

Carlier, M. (4 Aug 2021) Worldwide deliveries of Volkswagen Group's most popular car model between 2017 and 2020.
https://www.statista.com/statistics/461115/most-popular-models-of-volkswagen-group-by-worldwide-delivery/

Cities with emissions restrictions:
Renault Group (25 March 2020) Renault Group: Low Emission Zones (LEZs) in Europe.
https://www.renaultgroup.com/en/news-on-air/news/low-emission-zones-lezs-in-europe/

Engineering Explained (19 Jul 2017). How Do Electric Cars Produce Instant Maximum Torque? Youtube.
https://www.youtube.com/watch?v=VwFEyL_JJAQ

Gears:
https://www.rushgears.com/

Index

Figures index

Tables index

Chapter 1. INTRODUCTION

Electric cars have arrived to stay. The transformation that the car sector is undergoing is clearly evidenced by the market closing numbers for the year 2020. The iD3 model, a Volkswagen car similar in size to the well-known Volkswagen Golf, was the second most bought car in Europe in December of the previously mentioned year.

Even though, there are not many models that offer this electric configuration. The German brand on its own, only offers 6 models with a hundred percent electric configuration:

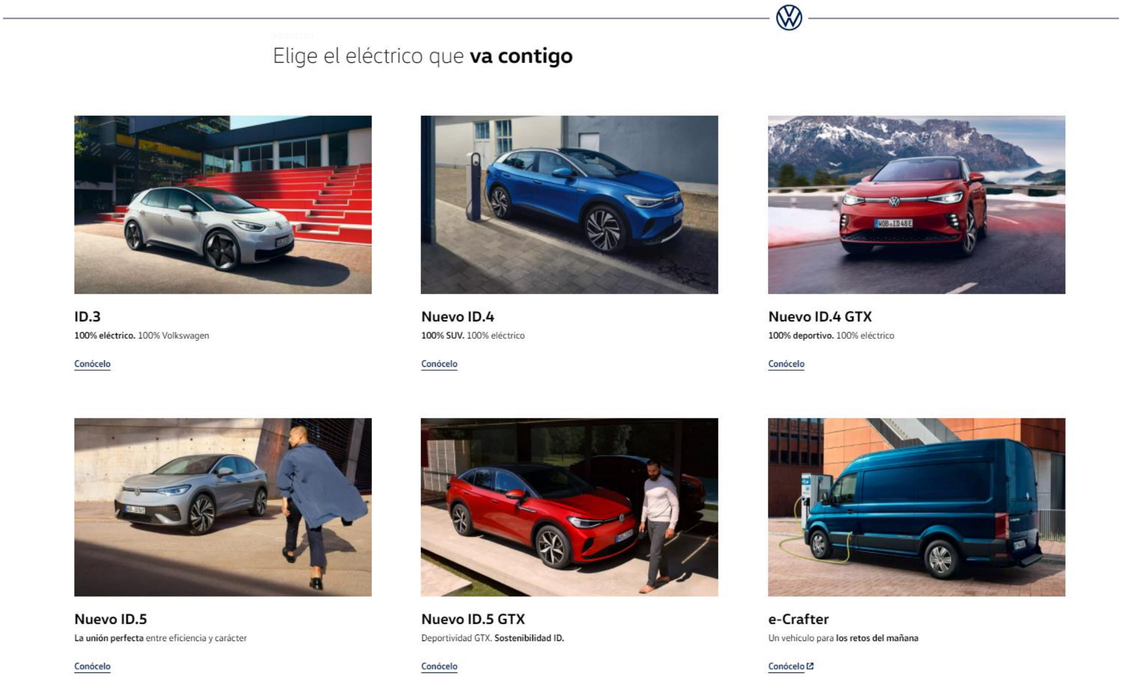


Figure 5: Volkswagen actual offer of 100% electric vehicles.

From all the vehicles exposed in Figure 5, none of them cover the lowest size part of the market (Table 5). As a result, the idea of trying to fulfil this market space seem like a good beginning for the project. Project that is going to be aimed at the design of the gear transmission of an electrified Volkswagen version to get the highest performance possible. The model chosen is a Volkswagen Polo 2009 85 hp and 5 gear manual transmission.

Model	Length [mm]	Width [mm]
ID.3	4261	1809
ID.4	4584	1852
ID.5	4599	1852
Polo (2009 version)	4064	1682

Table 5: Electric versions vs. Polo measures.

Why don't they offer an electric vehicle of similar size characteristics to the Volkswagen Polo? There are many reasons to do it:

- Volkswagen Polo´s combustion version sales in the market. As a first approach, considering the combustion model size in the market was essential. If the traditional model did not cover an important part of it, there was not a reason to produce an electrified version. This could have been the reason for the absence of a small Volkswagen 100 % electrified.

  As a conclusion of the information collected in both Figure 6 and Table 6, the Volkswagen Polo takes up a big part in the market niche. In the last three years, it has achieved better sales numbers than his big brother, the Volkswagen Golf, which has been converted into hybrid in both the GTE and the eHybrid versions. In 2019 there was a general decrease in the number of cars sold. Polo's decrease can be due to the introduction of the 2018 version.

	Polo annual sales [thousands]	Golf annual sales [thousands]
2017	716	974
2018	835	832
2019	724	702
2020	488	481

Table 6: Volkswagen Polo last years sales. Volkswagen Golf annual sales are also included to support that 2020 big decrease is due to the pandemic.

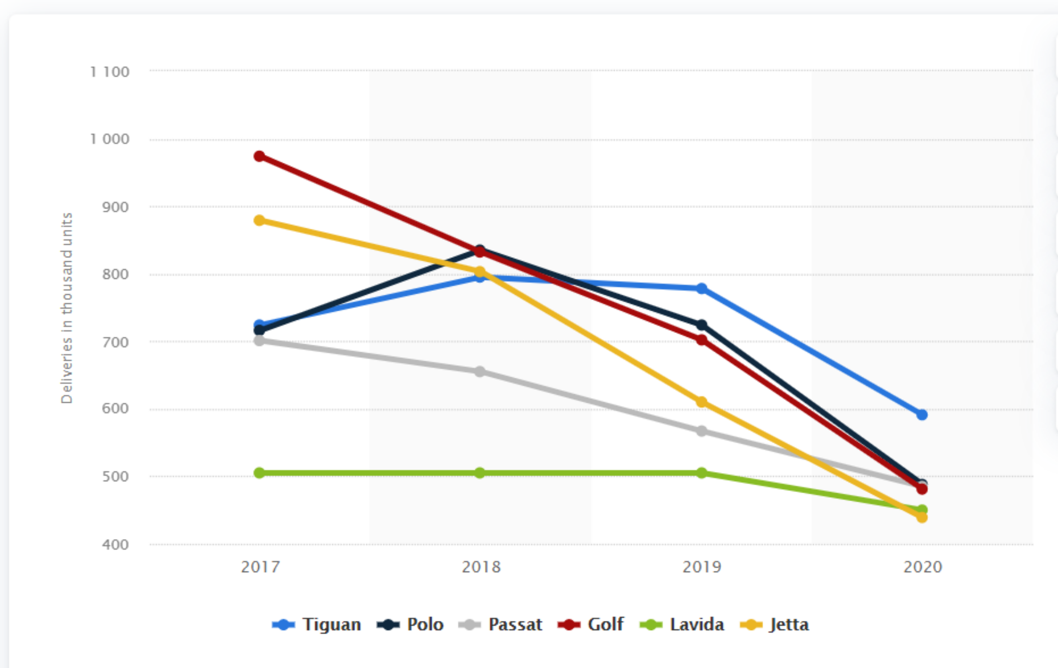


Figure 6: Volkswagen last years sales.

- Clean. As it has just been exposed, the demand of electric cars has been growing in the past recent years. There are many reasons to believe that this model would perfectly fit in the demand curve. An important one is the implementation of new emission restrictions[1] in different European cities[2]. These restrictions are now starting to be implemented, but further measures are to come in the next years. As the Volkswagen polo is a measured size car, perfect for the city, it´s demand could get bigger than that of the combustion version.

[1] Euro 7: "*This initiative, which is part of the European Green Deal, will develop stricter emissions standards (Euro 7) for all petrol and diesel cars, vans, lorries and buses. To ensure vehicles on EU roads are clean over their lifetime, the proposed rules will consider new vehicle technologies and ensure emissions are measured in real-time.*"

[2] "*In Low Emission Zones (LEZs), only the least polluting vehicles are allowed on the road. These zones aim to reduce air pollution in cities. The concept is winning over an increasing number of local authorities: ten different European countries so far have enforced this type of "greener" space.*"

- Efficiency. Electric vehicles are well ahead in the efficiency race. "EVs convert over 77 per cent of the electrical energy from the grid to power at the wheels. Conventional gasoline vehicles only convert about 12 per cent – 30 per cent of the energy stored in gasoline to power at the wheels." According to the US Department of Energy.

- Sporty. Electric motors give great performance results. The main difference between a traditional combustion model and an electric one is the output torque.
  While combustion cars achieve their peak torque at a certain number of rpm, electric cars have instant maximum torque from 0 rpm. At the same time, electric cars give higher output torque. As a result, there is an improvement in acceleration at medium and high speeds.

	Volkswagen Golf GTE	Volkswagen Golf GTI
Power [PS]	245	245
Transmission	DSG	DSG
Weight [kg]	1624	1463
Torque [Nm]	400	370
0-100 km/h [s]	6.7	6.2

Table 7: Comparison between Volkswagen Golf GTE (electric) and Volkswagen Golf GTI (combustion).

The GTE and the GTI have the same power (Table 7). As previously discussed, the electric version output torque is higher than the petrol combustion version. Even though, the 0-100 is slightly lower in the traditional version. This does not contradict the previous argument; it is due to an increase in the weight of 161 kg.

The last two bullet points main topics are important concerns for young people. Bearing in mind that Volkswagen Polo is popular among this group of age, the electrified version could attract even more interest in the market.

Chapter 2. STATE OF ART

Volkswagen has already the capacity to produce electric cars. As a result, there is no need to make big investments for development and machinery. To support this fact, I am going to develop what are considered to be three main pillars in the world of electric cars:

- Platform. Volkswagen has already developed MEB platform, which stands for Modular Electric Drive Matrix. It constitutes the basis for the entire ID family, and it is considered to be very adaptable. It can be used for compacts, SUVs and even for vans.
  Its main characteristic is the large amplitude due to a larger space between shafts. As a result, cars present wide interiors and there is plenty of space for the batteries.

Figure 7: MEB platform.

- Battery. It can be considered the most important element of an electric car. Volkswagen has developed ion-lithium batteries for the iD family. There are different types depending on the power. The higher the power, the greater number of modules the battery is made of.

In addition, the German brand has developed exceptionally efficient batteries. They can be recharged with the kinetic energy from breaking. Urban driving involves a lot of breaking and it is going to be the main environment in which a Volkswagen Polo is going to be driven. Therefore, these batteries seem to be designed for a model like this.

- Engine. Electric motors are made of less components than traditional combustion units. As a result, they are more reliable and need less visits to the garage.
  This project is going to be focussed on this bullet point. Starting from an already existing electric engine, the idea is to develop the best gear transmission to extract the best performance possible.

In addition, and due to the situation given by the Ukrainian war, a small comparison between power sources prices (gasoline and electricity) is going to be made:

- Electricity: 0.21658 $\frac{€}{\text{kWh}}$.
  To get a range of 400 km in a 77 kWh battery will cost 16,68 €
- Fuel (95 gasoline): 2.189 $\frac{€}{\text{l}}$.
  To get a range of 400 km in a car that needs 6,5 l every 100 km will cost 56,91 €

In conclusion, not only because of the increasing restrictions, but also because of the limited petrol access, electric cars demand will keep growing in the incoming years. Therefore, this is another reason why Volkswagen should increase their 100% electric cars offer (June 2022 values).

Chapter 3. DEFINICIÓN DEL TRABAJO

OBJECTIVES

1. Demonstrate that an electrified version of the Volkswagen Polo could be a good market option for the German brand. With the data and graphics included at the beginning of this document, and the performance achieved after the development of the gearbox, support the business opportunity.
2. Establish the situation under which the car is going to work. In order to pick a correct motor, it is essential to recognize the environment in which the car is going to be driven and analyse the conditions. This will set the output power required. Furthermore, analyse the benefits of converting the engine into a clean power source.
3. Once it is demonstrated that introducing this electrified version is a good market strategy, and the motor selected meets the requirements established, develop the transmission to achieve the best engine performance possible. Design it considering the number of gears, the acceleration and speed range desired for each one, and the torque required to reach it. Analyse the optimal design.
4. Simulate the designed transmission. Get a completed analysis using SolidWorks motion study. Use the obtained results to compare the "real" situation with the values on paper. Analyse the efficiency of the transmission designed.

METHODOLOGY

1. Pick an electric motor and thus set vehicle power.

   For the purpose of picking the motor, it is necessary to identify the conditions under which the car is going to work. Different aspects need to be considered, as that a car goes uphill or downhill, the torque required when accelerating to overtake or the weight carried inside the car.

   The goal is to achieve the best performance possible. Even though, it is important to keep in mind that this Volkswagen model is a small car, and hence it is mostly going to be used in the city. Therefore, bearing in mind the high-performance objective, the usual work conditions and being sure it meets the requirements, a similar engine (85 Hp) is the objective in the power source research. Once the power source is selected, a torque vs. rotational speed plot is established.

2. Select the vehicle layout specs.

   Size, front or rear wheel drive, vehicle gross weight, manual transmission, tire diameter, turning radius.

   Some of them are already established, because most of the 2009 Polo version remains the same: size, manual transmission, or tire diameter. Others are open to change, as the turning radius, the weight, or where is the power going to be transmitted.

3. Set some specs.

   Number of gears (1st, 2nd, 3rd, etc.), vehicle acceleration in each gear, vehicle speed range in each gear. All of them depend on the engine chosen. Once it is selected, and taking into consideration the entire speed range, the number of gears is going to be selected. The next step is to design the transmission with the purpose of reaching the best performance possible. To this effect, each gear is going to be assigned a range of speed to work to get the maximum torque available and, therefore, the best acceleration in a specific rpm range.

4. Design a transmission.

   The transmission allows the selection of gear (1st, 2nd, 3rd, etc). It's design needs to consider for what speed range it is going to be used and how much torque is required on each one. To this end, all the gears of the transmission would be designed with a specific dimension.

   The next stage is to optimize the gearbox to give the vehicle the desired acceleration per the electric motor selected and to optimize the differential to allow the vehicle to go straight or turn tightly. The following step in this extensive design is to analyse the proposed system. Therefore, making a technical deep dive to establish that the car meets requirements, and an overall vehicle view is required.

   As a last step, the design and analysis of many components (bearings, shaft, structure, etc.) must be done. This part requires a deep study, in which it is necessary to consider the function of each element.

5. Simulate (Solid Works Motion Study).

   The first step would be to create a Solid CAD model of the system. Once it is done, it would be simulated using the motion study of the program to compare the simulation results with the theory (or expected) results. Evaluate the difference between both.

Chapter 4. GEARBOX DEVELOPMENT

SYSTEM ANALYSIS

As a first approach, it is necessary to recognize the car specs that are going to change, those that are going to remain the same and the working conditions. Furthermore, during the development of the project, it is essential to remember what the aim of the project is: achieving the best possible car performance through the gearbox design.

<u>Car weight</u>

The first parameter that is going to be revised is the car weight. Electric cars are known to be heavy, due to the weight of batteries and the engine electrification. Thus, this is such an important parameter to consider.

The combustion version has an unladen weight of 1104 kg. On the one hand, getting to know the exact weight added due to the electrification of the car is not possible, because it would require getting specific batteries, to previously know the weight of the gear transmission that is going to be designed, and so on with many other components. On the other hand, getting to know the exact weight lost, would require weighing each component that is going to be substituted, including the combustion engine.

With the purpose of making the best possible approach to the electric section, two already existing cars are going to be considered. The data and conclusions are going to be adapted to this specific model.

- Volkswagen Golf GTE Mk8 (8^{th} generation). It is a hybrid version with two engines: 150 hp / 110 kW for the combustion engine and 109 hp / 80 kW for the electric engine. This hybrid version weights 1624 kg.
- Volkswagen Gol Mk8 (8^{th} generation). It is a combustion traditional version with 150 hp / 110 kW. This version weights 1353 kg.

This means that the entire electrification of a 109 hp engine, weights around 271 kg. (1624 kg from the combined version) – (1353 kg of the traditional version).

In order to make the best possible approach to the combustion engine weight question, Volkswagen was contacted. Despite my attempts, I couldn't get the information required. To find accurate information about Volkswagen engines weight, the internet has been used, with no result. The best result found, is a recent post on a Spanish website that gives do-it-yourself advice. It says that a 4 cylinders combustion engine weight between 90-160 kg. Because the engine of the car in this project is small, only 85 hp, an approximate loss weight of 100 kg is going to be considered.

All together:

- Volkswagen Polo 2009 weight = 1104 kg
- Weight gain due to electrification = 271 kg
- Phi301 weight = 29 kg included in the 271 kg gained
- Weight lost due to the remove of the old engine = 100 kg

$$Electrified\ version\ weight = 1104 + 271 - 100 = 1275\ kg$$

Equation 1: Electrified version weight.

$$Increase = 1275 - 1104 = 171\ kg$$

Equation 2: Weight increment.

This means the car has raised its weight a 15.5% compared with its initial weight. An important increment considering it is a small city car, and therefore it shouldn't be heavy. As a result, for the purpose of not having a decrease on the performance, an increment in the engine output power is going to be implemented.

Electric engine

Once the new unladen weight of the car is known, the electric engine can be selected bearing in mind the weight increment previously discussed. The engine is going to be chosen from *Phi-Power*, a Swiss company specialized on engines for both racing and heavy goods vehicles. The main advantage of *Phi-Power* motors is that they can offer significantly more power and torque in the same space, which seems to be a great benefit for a small car.

The power source could have been selected from any other electric motor supplier but focusing on the objective of achieving the best performance possible on a small car, the advantages of the Swiss company seem to be ideal.

Phi-Power offer is divided in two groups:

A) Phe-Motor Series, which includes Ph381 and Ph382 motors. Two engines with high power and torque, but low speed. Therefore, engines specialized for heavy goods machines.
B) Phi-Power Motor Series, which includes Phi27S (45 kW), Phi301 (85 kW) and Phi271 (90 kW). Smaller engines with lower torques and higher speeds. Engines with a much more racing perspective.
   Even though the torques are lower than those of the Phe-Series, they are still higher than what will be offered by a combustion engine of this power.

For a given power (H), if the torque (T) increases, the speed (w) decreases and vice versa as it is reflected in Equation 3.

$$H = T * w$$

Equation 3: Relationship between power, torque and angular speed.

Because this project is centred on a car, and focus on reaching the best performance possible, the engine is going to be selected form the Phi-Motor Series.

The previous weight-power output ratio was 12.988 kg/hp (Equation 4). Taking into consideration the 15.5% (Equation 2) weight increase due to electrification, the new power source needs to be more powerful:

$$\frac{m}{H} = 12.988\ \frac{kg}{hp}$$

Equation 4: Weight-Power ratio.

$$H = \frac{m}{12.988\ \frac{kg}{hp}} = \frac{1275\ kg}{12.988\ \frac{kg}{hp}} = 98.2\ Hp\ = 72.2\ kW$$

Equation 5: Equivalent Power for the electric engine.

There is not an engine with exactly 98.2 Hp (Equation 5). Taking into consideration the objective of reaching the best performance possible, the Phi301 with 85 kW (Table 8) is going to be chosen. It is similar enough to the previous combustion version, but it has 17 Hp extra from the weight—power output ratio to achieve better results and therefore be more attractive.

PHI301 SERIES

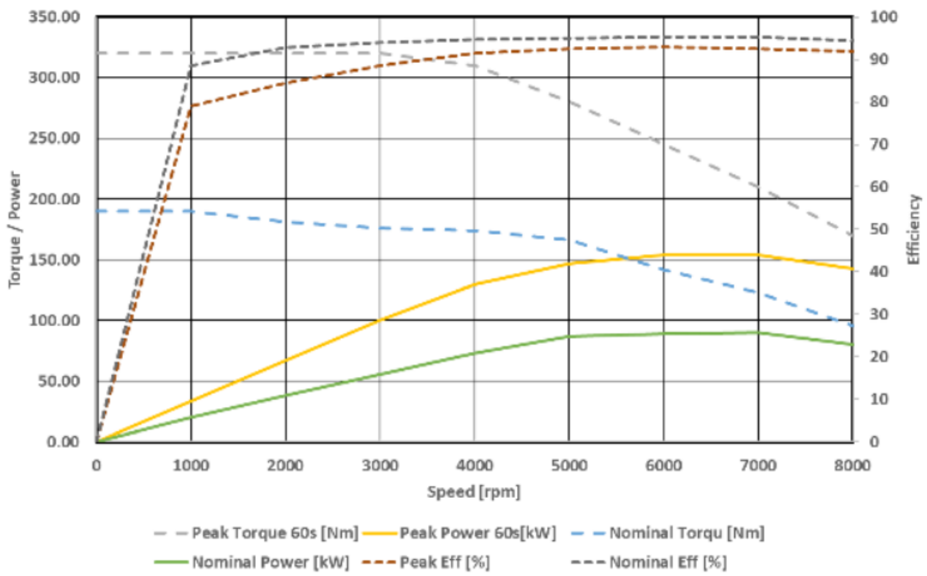


Figure 8: PHI301 Performance.

PARAMETERS	Combustion motor	Phi301 motor
Specifications		
Fuel	Gasoline	Electricity
Power location	Front wheel drive	Front wheel drive
Cylinders	4 cylinders in line	-
Transmission	5 speed manual	To be determined
Engine size [cm^3]	1390	-
Engine weight [kg]	100 kg	29 kg
Performance		
Power [hp/kW] (rpm)	85/63 (5000)	*Nominal:* 114/85
Torque [Nm] (rpm)	132 (3800)	Nominal: 190

Table 8: Combustion vs. Electric engine specifications and performance.

<u>Working conditions:</u>

Having selected the new electric engine, it is time to see if it is enough to satisfy the requirements. Requirements that need to be determined considering the working conditions under which the car is expected to work.

The first idea to bear in mind is that the Polo is a small car, and therefore its main environment of operation is going to be the city. Even though, its operating location is not going to be limited to the urban roads. Thus, different factors of the driving conditions are going to be considered.

A) Slopes. The most adverse road condition while driving is given by the slope of the road. To ensure the design is done with real case situations, the highest slope percentages allowed for road construction in Spain are collected in Table 9 and Table 10. Both tables are taken from the 'BOE'.

Project Speed [km/h]	Maximum Slope [%]
140, 130, 120, 110 and 100	4
90 and 80	5

Table 9: Maximum slope allowed in highways in Spain according to the BOE (Boletín Oficial del Estado).

Project Speed [km/h]	Maximum Slope [%]	Exceptional Slope [%]
100	4	5
90 and 80	5	7
70 and 60	6	8
50 and 40	7	10

Table 10: Maximum slope allowed in conventional roads and multilane roads in Spain according to the BOE (Boletín Oficial del Estado).

B) Transportation. Initially, despite the main objective of this car is not transporting people or a great amount of baggage, a load consisted in five people with a total weight of 380 kg and luggage and personal items with a weight of 80 kg was considered.

$$\text{Weight}_{\text{Total}} = \text{Weight}_{\text{Car}}\text{ (Equation 2)} + \text{Weight}_{\text{Transported}} = 1275\ kg + 460\ kg = 1735\ kg$$

Even though, when the flipped power flow diagrams were developed to set the different accelerations on each corresponding gear, the weight seem to be too high. As a result, the acceleration values were too low. Therefore, the car is going to be considered to be unladen for the 0 to top speed scenario on which the gearbox is going to be developed.

C) Aerodynamic forces.

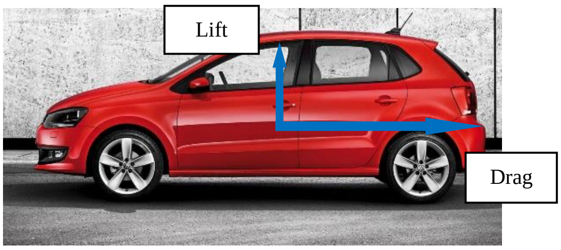


Figure 9: Drag and Lift forces.

When a body is moving in the presence of a fluid as air, it suffers two different forces (Figure 9):

- Drag force. It has a huge effect on fuel consumption and vehicle performance.
- Lift force. Because the movement of the car is in the horizontal direction, and it is not going to achieve extremely high speed, it is going to be omitted.

Drag force is the resultant of two different stresses acting together on the surface of the body, in this case, the car:

Figure 10: Shear and Pressure stress.

- Shear stress (Figure 10). Caused by frictional forces due to fluid viscosity. τ
- Pressure stress (Figure 10). Caused by how pressure is distributed around the car. P

If the precise values of τ and P were known, the exact value of the drag force could be calculated. To do that, a wind tunnel would be necessary.

Instead, Bernoulli equation (Equation 6) is going to be used:

$$Drag = \frac{1}{2}\, C\, \rho\, A\, v^2$$

Equation 6: Bernoulli Equation for drag force.

- Drag coefficient, C.
  Value for a typical car is between 0.25 and 0.35. C=0.3 is going to be used.
- Fluid density, ρ.

As every fluid property, it depends on the location conditions. According to the International Standard Atmosphere (ISA), for sea level with 15ºC and 14.7 psi, ρ = $1.225\ kg/m^3$.

- Cross sectional area, A.

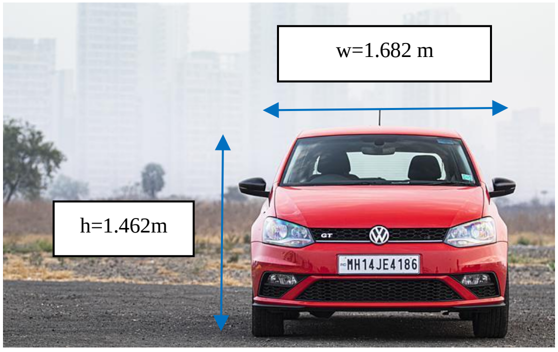


Figure 11: Cross sectional area measures.

The cross-sectional area of the car (**¡Error! No se encuentra el origen de la referencia.**) can be calculated as shown in **¡Error! No se encuentra el origen de la referencia.**.

Equation 7: Cross sectional area of a rectangle.

- Speed, v.

Different speeds are going to be considered depending on the scenario. On each of them, the highest speed achieved is the one considered for the drag force.

D) Tire rolling resistance.

On an ideal situation, where the contact between the tire and the road is just a line, the rolling resistance wouldn't be a concern. However, at an actual driving situation, due to the car weight, the contact is an area and not a line, and rolling resistance needs to be considered (Equation 8).

While the car is moving, the tires are constantly being deformed with the rotation. As a result, there is energy lost. This is due to the hysteresis on the compression process suffered by the tire when the car is moving.

$$Rolling\ resistance = c\ m\ g$$

Equation 8: Rolling Resistance Equation.

- Rolling resistance coefficient, c.

  For ordinary car tires on asphalt, c=0.01.

E) Rolling (Equation 10) vs. Sliding (Equation 9). If the surface is slippery or the acceleration of the car is too high, the wheel won't be able to transform the power of the engine into movement.
Once the driving force is known for the different proposed scenarios, it will be essential to ensure that the wheel is not sliding. For this purpose, it is considered that the total weight of the car is equally distributed between the four wheels.

Rolling (Figure 13)

$$driving\ force < \mu\ \frac{m\ g}{4}$$

Equation 10: Rolling.

Sliding (Figure 12)

$$driving\ force > \mu\ \frac{m\ g}{4}$$

Equation 9: Sliding.

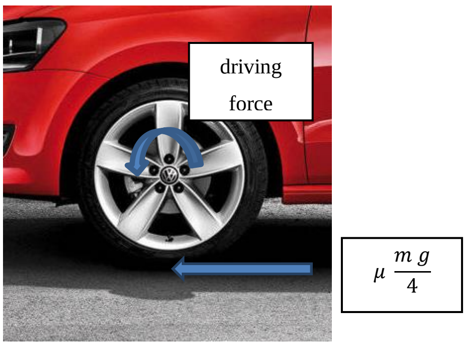


Figure 13: Driving force on a rolling situation.

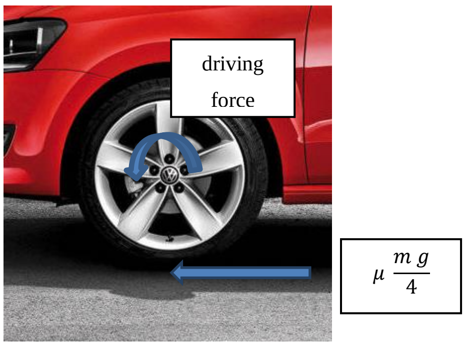


Figure 12: Driving force on a sliding situation.

F) Friction. To determine the friction coefficient between the tires and the road I have used *HPWizard.com*. Setting the tire type and the road type the friction coefficient given. The different values are collected in Table 11.

μ	Road type					
Tyre type	Dry asphalt	Wet asphalt	⟹	Asphalt average	Sand	Ice
Tourism	0.9	0.6		0.75	0.6	0.1
Hi-Performance	1	0.7		0.85	0.6	0.08

Table 11: μ values according to different tyre and road conditions.

These values seem high compared to those used during my studies. In order to ensure the values are accurate, I have decided to make a comparison with other resources. Comparing them to those in *The Friction of Automobile Tires from Jones & Childers, Contemporary College Physics, 3rd ed, 2001*, I realized the values were correct, but it is common to use and average value between the dry conditions and the wet conditions.

The value used is going to be the average between dry and wet asphalt for a tourism tyre type. These combination of tyre and road condition seems to be the most accurate for the case of study.

G) Internal Friction.

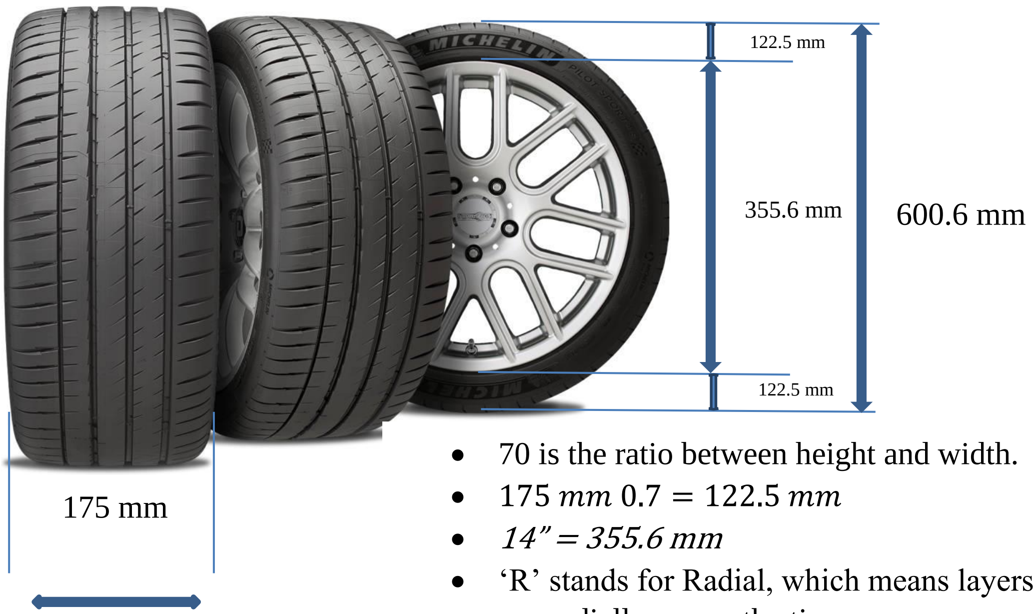
3 stages with a 80 % efficiency as a system.

- 92 % on each spur stage.
- 95 % for the wheel stage.

Car specs

1. Front wheel drive.
2. Tires.

   The case of study version is equipped with the standard 175/70R14 tires.



- 70 is the ratio between height and width.
- $175\ mm\ 0.7 = 122.5\ mm$
- *14" = 355.6 mm*
- 'R' stands for Radial, which means layers run radially across the tire.

Figure 14: Volkswagen Polo tire sizes.

3. Top speed 180 km/h.
4. 4 gears.

   Two different accelerations are going to be considered on each gear. With those two values, and average acceleration is going to be set for the 4 different gears, with the highest value for the first gear and the lowest value for the fourth gear.

   Speed range on each gear:

   - First gear: $(0-25)\frac{\text{km}}{\text{h}}$
   - Second gear: $(25-70)\frac{\text{km}}{\text{h}}$
   - Third gear: $(70-120)\frac{\text{km}}{\text{h}}$
   - Fourth gear: $(120-180)\frac{\text{km}}{\text{h}}$

     Highest interval to get low fuel consumption in the highway.

DESIGN

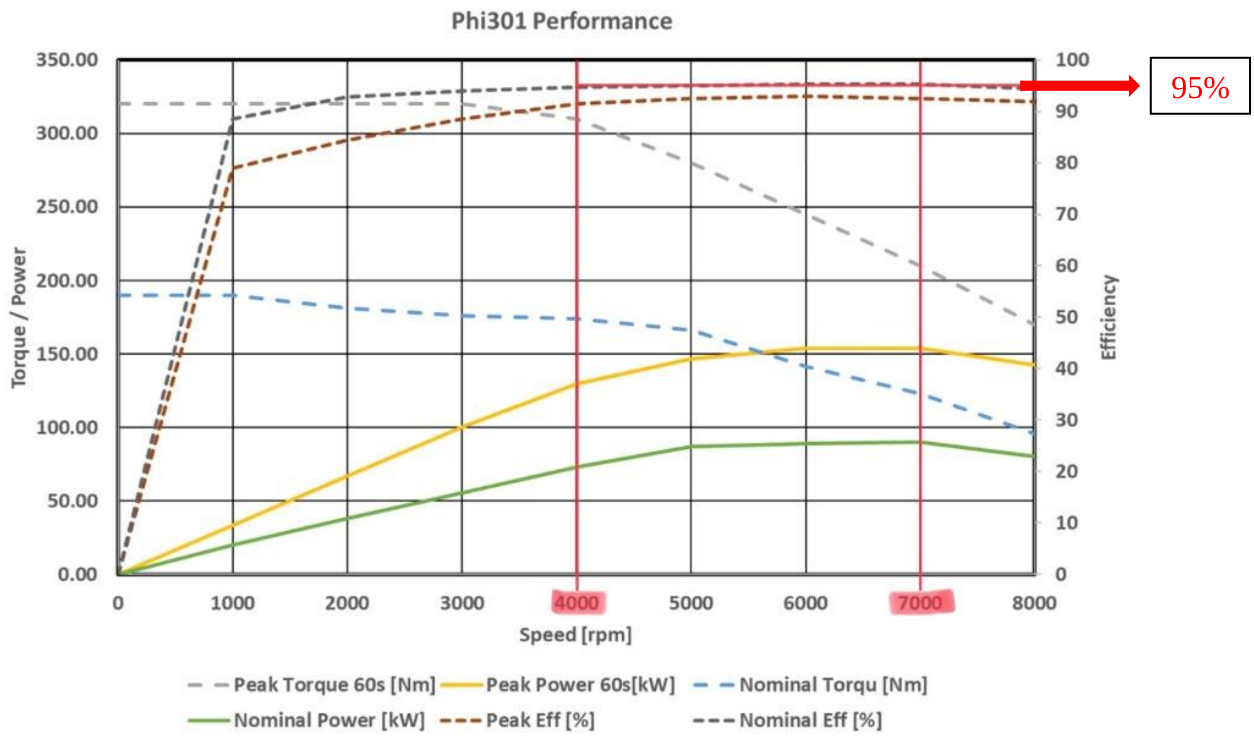


Figure 15: Phi301 best performance range.

Looking for highest efficiency:

- 4000 - 8000 rpm (Figure 15).
- Shift change at around 8000 rpm.

Because the aim of the project is to design the gear transmission to get the highest efficiency of the car, the development is going to be done on that strip as long as it is possible.

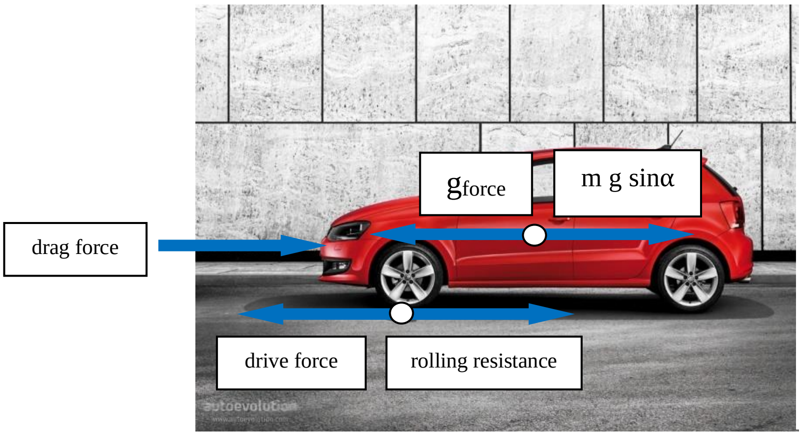


Figure 16: Free body diagram of the car.

$$\sum F = m\,a;$$

$$drive\ force - rolling\ resistance - drag - uphill = m\,a$$

$$drive\ force = m\,a + m\,g\,sin\alpha + c\,m\,g\,cos\alpha + \frac{1}{2}\,cd\,\rho\,A\,v^2$$

Equation 11: Force equilibrium for the study case.

Once the system is defined by Figure 16 and Equation 11, it is time to start the design. With this purpose, the scenario selected is going to be a 0 to top speed situation.

<u>Assumptions:</u>

- Engine efficiency: *η =94 %*.

  The highest efficiency is achieved between 4000 and 8000 rpm. Therefore, the aim is going to keep the motor working on that range. Bearing this idea in mind, the shift change is going to be at 8000 rpm and it has to reach around 4000 rpm when the highest gear is interlocked.

  Between 0 and 1000 rpm, the engine efficiency is low. From 1000 to 4000 rpm is better, but it doesn't reach the top efficiency of 95 %.

  Because it is not possible to start the engine at 4000 rpm to immediately reach the peak efficiency, this low rpm range must be considered. As a result, an efficiency of 94 % is going to be considered for the engine.

- Each gear is going to work on the speed range previously established. At the same time, each range is going to be divided in two subranges: low rpm and high rpm. On each of the subdivisions constant acceleration is going to be assumed. Because the aim of the project is to get the highest performance possible, the acceleration of a Ford Mustang is going to be considered as a reference. Trying to achieve similar values, the gear transmission is going to be develop.

Gear	$a_{Mustang}$
1	1.26g
2	0.82g
3	0.55g
4	0.45g

Table 12: Ford Mustang acceleration on each gear.

- A slope of 5% is going to be implemented (Table 9 and Table 10).
  Using simple trigonometric conversion: $5\% = 2.862^{0}$.
- Engine working under nominal conditions: $\tau\ max = 190\ Nm$ (Figure 15 and Table 8).
- Wheel radius: $r = 0.3003\ m$ (Figure 14)

Statements:

- "Gear ratio is the ratio of the number of rotations of a driver gear to the number of rotations of a driven gear".

$$G = \frac{wi}{wo}$$

Equation 12: Gear ratio in terms of angular velocity.

$$G = \frac{wi}{wo} = \frac{\frac{2v}{di}}{\frac{2v}{do}} = \frac{do}{di} = \frac{No}{Ni}$$

Equation 13: Gear ratio in terms of teeth numbers.

- "Mechanical advantage is the ratio of output force to input force in a system, used to analyse the forces in machines".

$$m = \frac{To}{Ti}$$

Equation 14: Mechanical advantage.

- Efficiency:

$$\eta = \frac{output\ power}{input\ power} = \frac{To\ wo}{Ti\ wi} = \frac{m}{G}$$

Equation 15: Efficiency in terms of mechanical advantage and gear ratio.

- Gears:

-Face Width, F. The face width is going to be established in 1 in = 25.4 mm to start with. In case the Lewis Bending equation (Equation 17) shows this value is not enough, the gear width will be increased.

-Diametral Pitch, P. “Diametral Pitch is the ratio between the number of teeth in a gear and the pitch diameter [in]. Teeth per inch”. It must be the same for all the gears in an assembly.

$$P = \frac{N}{D}$$

Equation 16: Diametral pitch.

-Bore diameter. The bore diameter is the hole created in the gear to introduce the shaft.

-Bending stress in gear teeth. It is a concern to estimate the stress in the gear teeth to ensure the design meets the requirements. With this purpose, The *Lewis Bending Equation* is going to be implemented.

In Equation 17 a dynamic factor is included to take into account the load increases due to high speed of the gears in contact.

$$\tau = Kv \frac{Wt\,P}{F\,Y} = Kv \frac{2\,T\,P^2}{N\,F\,Y}$$

Equation 17: Lewis Bending Equation.

$$\tau \leq \frac{Sy}{nd}$$

Equation 19: Stress limit.

$$P \leq \sqrt{\frac{N\,F\,Y\,Sy}{2\,nd\,Kv\,T}}$$

Equation 18: Lewis Bending equation combined with stress limit.

- Dynamic factor, K_v. It depends on the speed the gear is turning. For cut or milled profile:

$$Kv = \frac{6.1 + v\,[\frac{m}{s}]}{6.1}$$

Equation 20: Dynamic factor for cut or milled profile in SI units.

- Torque, T.
- Diametral Pitch, P.
- Number of teeth, N.
- Face width, F.
- Yield strength of the material, S_y.
- Safety factor, n_d. n_d=2
- Lewis form factor, Y. It is tabulated and depends on the number of teeth.

Number of Teeth	Y	Number of Teeth	Y
12	0.245	28	0.353
13	0.261	30	0.359
14	0.277	34	0.371
15	0.290	38	0.384
16	0.296	43	0.397
17	0.303	50	0.409
18	0.309	60	0.422
19	0.314	75	0.435
20	0.322	100	0.447
21	0.328	150	0.460
22	0.331	300	0.472
24	0.337	400	0.480
26	0.346	Rack	0.485

Table 13: Lewis form factor (Y) for each number of teeth.

Development

Once the engine has been chosen and the car has been electrified, the aim of the project and the work conditions have been established and some concepts have been defined, it is time to start developing the transmission. Always remember that this project is looking for the best possible performance of the car by squeezing to the limit the tiny electric engine selected.

Because there are different unknowns and many parameters to bear in mind, a flipped process is going to be followed. In particular, three different flipped power flow diagrams are going to be made for each gear:

- The first one is made to ensure that when a gear change is made, the higher gear starts working at around 4000 rpm.
- The second one, at around half of the gear speed range, to reach a power requirement of the engine of around 114 Hp by establishing the highest acceleration possible. Nominal torque and power of the engine and a limit of 8000 rpm must be considered.
- The last one, considering the same parameters as in the step before, is made to ensure that the shift change occurs at around 7000-8000 rpm. The acceleration is necessary lower than for the previous step.

Each power flow diagram has, at the same time, three stages as it is shown in Figure 17.

CAR		STAGE 3: WHEEL				STAGE 2: Last Spur				STAGE 1: First Spur		MOTOR	
Vo(kph)	15,00			o1(rad/s)	13,875			o5(rad/s)	111,000			o10(rad/s)	444,000
Vo(m/s)	4,17	G3(rad/m)	3,3300	n1(rpm)	132,50	G2(-)	8,00	n5(rpm)	1059,97	G1(-)	4,00	n10(rpm)	4239,89
Fo(N)	16543,5991	eta4(-)	0,95	T1(Nm)	5229,52	eta3(-)	0,92	T5(Nm)	709,03	eta2(-)	0,92	T10(Nm)	192,26
Ho(kW)	68,932			H(kW)	72,560			H(kW)	78,702			Hin(kW)	85,364
Ho(Hp)	92,40			H(Hp)	97,265			H(Hp)	105,499			Hin(Hp)	114,429

Figure 17: Gear 1, speed range 1, flipped power flow diagram with considerations.

(orange)	Values limited by the engine parameters or car specs
(green)	Values to be determined depending on the conditions
(yellow)	Variable
(blue)	Established

Figure 18: Legend of the flipped power flow diagrams.

The first step is to estimate the drive force using the free body diagram of the model. Because the rest of the parameters had already been set (working conditions), the drive force only depends on the speed and acceleration of the car.

- The speed of the Bernoulli equation in the drag force is going to changed depending on the gear (maximum value of the range is used).
- The acceleration value is going to be adjusted to reach the limits of the engine. In other words, the acceleration value is increased until input power (H_{in}) reaches 114 Hp, which is the maximum power the engine can deliver.

  By using the relationship between output speed (V_o) and output force (F_o), output power (H_o) can be calculated. Now, introducing the concept of efficiencies as shown in Figure 17, the relationship between input and output is also established. When input power (H_{in}) reaches the mentioned value of 114 Hp the objective is achieved.

  This process has been repeated until the objective value was achieved.

At this point, the output required is known and the input limits too. That is why it is called flipped power flow diagram. The next step is to find the gear ratios, so the engine can meet the requirements. The first spur is common for the four gears and the gear ratio is named as *G1*. The last spur is independent for each gear, its value is going to decrease as the gear increases and is named as *G2*. By plotting the different power flows for each gear at the same time to have a whole vision of the design and by making a big number of iterations the transmission design is done.

It is necessary to mention that there are thousands of possible solutions. The one presented in the following pages looks for the highest efficiency. Therefore, makes the engine to work between 4000 and 7000- 8000 rpm and tries to always demand 114 Hp from the engine. Furthermore, the gear ratios shouldn't be extremely complicated values as they are going to be transformed into teeth numbers.

GEAR 1

The first gear is the most demanding torque gear.

Speed range 1

Forces:

slope	5/0,02	º/rad
a	12	m/s
gforce	16140	N
uphill	263,836	N
rolling	131,918	N
drag	7,845	N
dforce	16543,6	N

Table 14: Gear 1, speed range 1 free body diagram values.

- $V_{max} = 15\frac{km}{h} = 4.17\frac{m}{s}$
- $a = 12\frac{m}{s}$

Equation 11:

$$drive\ force = 16543.6\ N\ (\text{ANEX I})$$

CAR		STAGE 3: WHEEL				STAGE 2: Last Spur				STAGE 1: First Spur			MOTOR
Vo(kph)	15,00			o1(rad/s)	13,875			o5(rad/s)	111,000			o10(rad/s)	444,000
Vo(m/s)	4,17	G3(rad/m)	3,3300	n1(rpm)	132,50	G2(-)	8,00	n5(rpm)	1059,97	G1(-)	4,00	n10(rpm)	4239,89
Fo(N)	16543,5991	eta4(-)	0,95	T1(Nm)	5229,52	eta3(-)	0,92	T5(Nm)	709,03	eta2(-)	0,92	T10(Nm)	192,26
Ho(kW)	68,932			H1(kW)	72,560			H5(kW)	78,702			Hin(kW)	85,364
Ho(Hp)	92,40			H1(Hp)	97,265			H5(Hp)	105,499			Hin(Hp)	114,429

Figure 19: Gear 1, speed range 1 flipped power flow diagram.

Speed range 2

Force:

slope	5/0,02	º/rad
a	7,1	m/s
gforce	9549,5	N
uphill	263,836	N
rolling	131,918	N
drag	21,791	N
dforce	9967,045	N

Table 15: Gear 1, speed range 2, free body diagram values.

- $V_{max} = 25\frac{km}{h} = 6.94\frac{m}{s}$
- $a = 7.1\frac{m}{s}$

Equation 11:

$$drive\ force = 9967.045\ N\ \text{(ANEX I)}$$

CAR		STAGE 3: WHEEL				STAGE 2: Last Spur				STAGE 1: First Spur		MOTOR	
Vo(kph)	25,00			o1(rad/s)	23,125			o5(rad/s)	185,000			o10(rad/s)	740,001
Vo(m/s)	6,94	G3(rad/m)	3,3300	n1(rpm)	220,83	G2(-)	8,00	n5(rpm)	1766,62	G1(-)	4,00	n10(rpm)	7066,49
Fo(N)	9967,04529	eta4(-)	0,95	T1(Nm)	3150,64	eta3(-)	0,92	T5(Nm)	427,17	eta2(-)	0,92	T10(Nm)	115,83
Ho(kW)	69,216			H1(kW)	72,859			H5(kW)	79,026			Hin(kW)	85,716
Ho(Hp)	92,78			H1(Hp)	97,666			H5(Hp)	105,933			Hin(Hp)	114,901

Figure 20: Gear 1, speed range 2, flipped power flow diagram.

GEAR 2

Shift change

Force:

slope	5/0,02	º/rad
a	4,5	m/s
gforce	6052,5	N
uphill	263,836	N
rolling	131,918	N
drag	21,791	N
dforce	6470,045	N

Table 16: Gear 2, shift change, free body diagram values.

- ($V_{max} = 25\frac{\mathrm{km}}{\mathrm{h}} = 6.94\frac{m}{s}$)
- $a = 4.5\frac{m}{s}$

Equation 11:

$$drive\ force = 6470.045\ N\ \text{(ANEX I)}$$

CAR		STAGE 3: WHEEL				STAGE 2: Last Spur				STAGE 1: First Spur		MOTOR	
Vo(kph)	25,00			o2(rad/s)	23,125			o6(rad/s)	74,000			o10(rad/s)	296,000
Vo(m/s)	6,94	G3(rad/m)	3,3300	n2(rpm)	220,83	G2(-)	3,20	n6(rpm)	706,65	G1(-)	4,00	n10(rpm)	2826,59
Fo(N)	6470,04529	eta4(-)	0,95	T2(Nm)	2045,22	eta3(-)	0,92	T6(Nm)	693,23	eta2(-)	0,92	T10(Nm)	187,98
Ho(kW)	44,931			H2(kW)	47,296			H6(kW)	51,299			Hin(kW)	55,642
Ho(Hp)	60,23			H2(Hp)	63,399			H6(Hp)	68,766			Hin(Hp)	74,587

Figure 21: Gear 2, shift change, flipped power low diagram.

<u>Speed range 1</u>

Force:

slope	5/0,02	º/rad
a	3,75	m/s
gforce	5043,75	N
uphill	263,836	N
rolling	131,918	N
drag	70,603	N
dforce	5510,107	N

Table 17: Gear 2, speed range 2, free body diagram values.

- (V_{max}= $45\,\frac{\text{km}}{\text{h}} = 12.5\,\frac{m}{s}$)
- $a = 3.75\,\frac{m}{s}$

Equation 11:

$$drive\ force = 5510.107\ N\ \text{(ANEX I)}$$

CAR		STAGE 3: WHEEL				STAGE 2: Last Spur				STAGE 1: First Spur			MOTOR
Vo(kph)	45,00			o2(rad/s)	41,625			o6(rad/s)	133,200			o10(rad/s)	532,801
Vo(m/s)	12,50	G3(rad/m)	3,3300	n2(rpm)	397,49	G2(-)	3,20	n6(rpm)	1271,97	G1(-)	4,00	n10(rpm)	5087,87
Fo(N)	5510,10696	eta4(-)	0,95	T2(Nm)	1741,77	eta3(-)	0,92	T6(Nm)	590,38	eta2(-)	0,92	T10(Nm)	160,09
Ho(kW)	68,876			H2(kW)	72,501			H6(kW)	78,639			Hin(kW)	85,296
Ho(Hp)	92,33			H2(Hp)	97,187			H6(Hp)	105,414			Hin(Hp)	114,337

Figure 22: Gear 2, speed range 2, flipped power flow diagram.

<u>Speed range 2</u>

Force:

slope	5/0,02	º/rad
a	2,2	m/s
gforce	2959	N
uphill	263,836	N
rolling	131,918	N
drag	170,841	N
dforce	3525,595	N

Table 18: Gear 2, speed range 2, free body diagram values.

- (V_{max}= $70\,\frac{\text{km}}{\text{h}} = 19.44\,\frac{m}{s}$)
- $a = 2.2\,\frac{m}{s}$

Equation 11:

$$drive\ force = 3525.595\ N\ \text{(ANEX I)}$$

CAR		STAGE 3: WHEEL				STAGE 2: Last Spur				STAGE 1: First Spur			MOTOR
Vo(kph)	70,00			o2(rad/s)	64,750			o6(rad/s)	207,200			o10(rad/s)	828,801
Vo(m/s)	19,44	G3(rad/m)	3,3300	n2(rpm)	618,32	G2(-)	3,20	n6(rpm)	1978,62	G1(-)	4,00	n10(rpm)	7914,46
Fo(N)	3525,59523	eta4(-)	0,95	T2(Nm)	1114,46	eta3(-)	0,92	T6(Nm)	377,75	eta2(-)	0,92	T10(Nm)	102,43
Ho(kW)	68,553			H2(kW)	72,161			H6(kW)	78,270			Hin(kW)	84,896
Ho(Hp)	91,89			H2(Hp)	96,731			H6(Hp)	104,919			Hin(Hp)	113,801

Figure 23: Gear 2, speed range 2, flipped power flow diagram.

GEAR 3

Shift change

Force:

slope	5/0,02	º/rad
a	2	m/s
gforce	2690	N
uphill	263,836	N
rolling	131,918	N
drag	170,841	N
dforce	3256,595	N

Table 19: Gear 3, shift change, free body diagram values.

- $V_{max} = 70\,\frac{\text{km}}{\text{h}} = 19.44\,\frac{m}{s}$
- $a = 2\,\frac{m}{s}$

Equation 11:

$$drive\ force = 3256.595\ N\ (\text{ANEX I})$$

CAR		STAGE 3: WHEEL				STAGE 2: Last Spur				STAGE 1: First Spur		MOTOR	
Vo(kph)	70,00			o3(rad/s)	64,750			o7(rad/s)	103,600			o10(rad/s)	414,400
Vo(m/s)	19,44	G3(rad/m)	3,3300	n3(rpm)	618,32	G2(-)	1,60	n7(rpm)	989,31	G1(-)	4,00	n10(rpm)	3957,23
Fo(N)	3256,59523	eta4(-)	0,95	T3(Nm)	1029,43	eta3(-)	0,92	T7(Nm)	697,86	eta2(-)	0,92	T10(Nm)	189,23
Ho(kW)	63,323			H3(kW)	66,655			H7(kW)	72,298			Hin(kW)	78,418
Ho(Hp)	84,88			H3(Hp)	89,350			H7(Hp)	96,914			Hin(Hp)	105,118

Figure 24: Gear 3, shift change, flipped power flow diagram.

Speed range 1

Force:

slope	5/0,02	º/rad
a	1,4	m/s
gforce	1883	N
uphill	263,836	N
rolling	131,918	N
drag	314,661	N
dforce	2593,415	N

Table 20: Gear 3, speed range 1, free body diagram values.

- $V_{max} = 95\,\frac{\text{km}}{\text{h}} = 26.39\,\frac{m}{s}$
- $a = 1.4\,\frac{m}{s}$

Equation 11:

$$drive\ force = 2593.415\ N\ (\text{ANEX I})$$

CAR		STAGE 3: WHEEL				STAGE 2: Last Spur				STAGE 1: First Spur		MOTOR	
Vo(kph)	95,00			o3(rad/s)	87,875			o7(rad/s)	140,600			o10(rad/s)	562,401
Vo(m/s)	26,39	G3(rad/m)	3,3300	n3(rpm)	839,15	G2(-)	1,60	n7(rpm)	1342,63	G1(-)	4,00	n10(rpm)	5370,53
Fo(N)	2593,41536	eta4(-)	0,95	T3(Nm)	819,79	eta3(-)	0,92	T7(Nm)	555,74	eta2(-)	0,92	T10(Nm)	150,70
Ho(kW)	68,437			H3(kW)	72,039			H7(kW)	78,138			Hin(kW)	84,752
Ho(Hp)	91,74			H3(Hp)	96,567			H7(Hp)	104,742			Hin(Hp)	113,609

Figure 25: Gear 3, speed range 1, flipped power flow diagram.

Speed range 2

Force:

slope	5/0,02	º/rad
a	0,86	m/s
gforce	1156,7	N
uphill	263,836	N
rolling	131,918	N
drag	502,063	N
dforce	2054,517	N

Table 21: Gear 3, speed range 2, free body diagram values.

- $V_{max} = 120\,\frac{\text{km}}{\text{h}} = 33.33\,\frac{m}{s}$
- $a = 0.86\,\frac{m}{s}$

Equation 11:

$$drive\ force = 2054.517\ N\ (\text{ANEX I})$$

CAR		STAGE 3: WHEEL			STAGE 2: Last Spur				STAGE 1: First Spur				MOTOR
Vo(kph)	120,00			o3(rad/s)	111,000			o7(rad/s)	177,600			o10(rad/s)	710,401
Vo(m/s)	33,33	G3(rad/m)	3,3300	n3(rpm)	1059,97	G2(-)	1,60	n7(rpm)	1695,96	G1(-)	4,00	n10(rpm)	6783,83
Fo(N)	2054,51734	eta4(-)	0,95	T3(Nm)	649,44	eta3(-)	0,92	T7(Nm)	440,26	eta2(-)	0,92	T10(Nm)	119,38
Ho(kW)	68,484			H3(kW)	72,088			H7(kW)	78,191			Hin(kW)	84,810
Ho(Hp)	91,80			H3(Hp)	96,633			H7(Hp)	104,813			Hin(Hp)	113,686

Figure 26: Gear 3, speed range 2, flipped power flow diagram.

GEAR 4

Shift change

Force:

slope	5/0,02	º/rad
a	0,7	m/s
gforce	941,5	N
uphill	263,836	N
rolling	131,918	N
drag	502,063	N
dforce	1839,317	N

Table 22: Gear 4, shift change, free body diagram values.

- $V_{max} = 120\,\frac{\text{km}}{\text{h}} = 33.33\,\frac{m}{s}$
- $a = 0.7\,\frac{m}{s}$

Equation 11:

$$drive\ force = 1839.317\ N\ (\text{ANEX I})$$

CAR		STAGE 3: WHEEL			STAGE 2: Last Spur				STAGE 1: First Spur				MOTOR
Vo(kph)	120,00			o4(rad/s)	111,000			o8(rad/s)	99,900			o10(rad/s)	399,600
Vo(m/s)	33,33	G3(rad/m)	3,3300	n4(rpm)	1059,97	G2(-)	0,90	n8(rpm)	953,98	G1(-)	4,00	n10(rpm)	3815,90
Fo(N)	1839,31734	eta4(-)	0,95	T4(Nm)	581,42	eta3(-)	0,92	T8(Nm)	700,71	eta2(-)	0,92	T10(Nm)	190,01
Ho(kW)	61,311			H4(kW)	64,537			H8(kW)	70,001			Hin(kW)	75,926
Ho(Hp)	82,19			H4(Hp)	86,511			H8(Hp)	93,835			Hin(Hp)	101,778

Figure 27: Gear 4, shift change, flipped power flow diagram.

Speed range 1

Force:

slope	5/0,02	º/rad
a	0,54	m/s
gforce	726,3	N
uphill	263,836	N
rolling	131,945	N
drag	784,473	N
dforce	1642,718	N

Table 23: Gear 4, speed range 1, free body diagram values.

- $V_{max} = 150 \frac{km}{h} = 41.67 \frac{m}{s}$
- $a = 0.54 \frac{m}{s}$

Equation 11:

$$drive\ force = 1642.718\ N\ (\text{ANEX I})$$

CAR	STAGE 3: WHEEL				STAGE 2: Last Spur				STAGE 1: First Spur				MOTOR
Vo(kph)	150,00			o4(rad/s)	138,750			o8(rad/s)	124,875			o10(rad/s)	499,500
Vo(m/s)	41,67	G3(rad/m)	3,3300	n4(rpm)	1324,97	G2(-)	0,90	n8(rpm)	1192,47	G1(-)	4,00	n10(rpm)	4769,88
Fo(N)	1642,71791	eta4(-)	0,95	T4(Nm)	519,27	eta3(-)	0,92	T8(Nm)	625,81	eta2(-)	0,92	T10(Nm)	169,70
Ho(kW)	68,447			H4(kW)	72,049			H8(kW)	78,148			Hin(kW)	84,764
Ho(Hp)	91,75			H4(Hp)	96,580			H8(Hp)	104,756			Hin(Hp)	113,624

Figure 28: Gear 4, speed range 1, flipped power flow diagram.

Speed range 2

Force:

slope	5/0,02	º/rad
a	0,085	m/s
gforce	114,325	N
uphill	0	N
rolling	131,945	N
drag	1129,642	N
dforce	1375,911	N

Table 24: Gear 4, speed range 2, free body diagram values.

- $V_{max} = 180 \frac{km}{h} = 50 \frac{m}{s}$
- $a = 0.085 \frac{m}{s}$

Equation 11:

$$drive\ force = 1375.911\ N\ (\text{ANEX I})$$

CAR	STAGE 3: WHEEL				STAGE 2: Last Spur				STAGE 1: First Spur				MOTOR
Vo(kph)	180,00			o4(rad/s)	166,500			o8(rad/s)	149,850			o10(rad/s)	599,401
Vo(m/s)	50,00	G3(rad/m)	3,3300	n4(rpm)	1589,96	G2(-)	0,90	n8(rpm)	1430,96	G1(-)	4,00	n10(rpm)	5723,85
Fo(N)	1375,91121	eta4(-)	0,95	T4(Nm)	434,93	eta3(-)	0,92	T8(Nm)	524,17	eta2(-)	0,92	T10(Nm)	142,13
Ho(kW)	68,796			H4(kW)	72,416			H8(kW)	78,547			Hin(kW)	85,196
Ho(Hp)	92,22			H4(Hp)	97,073			H8(Hp)	105,290			Hin(Hp)	114,203

Figure 29: Gear 4, speed range 2, flipped power flow diagram.

For this last gear and speed range, no slope has been considered. When the first iteration was made, with the same slope as the rest of the scenarios, the drive force demanded to the engine was too high. Because the aim of the project is to get the best possible performance, instead of suppressing the acceleration and therefore the g force, the slope was removed.

The results are collected in Table 25. The limits or objective values are also included to visualize the good results achieved in terms of performance for the electric engine:

- The objective start point of each gear is at 4000 rpm, where the engine efficiency gets its higher value (95%). Gear 1 necessarily starts below and gears 3 and 4 reach the objective value.
  As it is show in the second gear row of Table 25, the achieved value is not as good as the rest. The way to fix it would have been to increase first gear speed range and reduce the acceleration. Because the aim of the project is what it is, and the efficiency at the achieved point is only about 2% less, the result presented gives the best possible performance of the engine and, therefore, of the car.
- The objective shift change is between 7000 and 8000 rpm. The limit is at the last-mentioned value because the engine cannot work above it. As it is shown, the values achieved for gears 1, 2 and 3 reach the objective values.
  Gear 4 achieved value seem to be below the objective. It is not because the transmission doesn't squeeze enough the engine, but because the car top speed is limited to 180 km/h.
- The power demand is always demanding the best of the Phi301 at around 114 Hp.

Gear	Start Point		Shift Change		Power Demand	
	Achieved [rpm]	Objective [rpm]	Achieved [rpm]	Limit [rpm]	Achieved [Hp] (1^{st} range/2^{nd} range)	Objective [Hp]
1	-	-	7066.49	8000	114.429/114.901	114
2	2826.59	4000	7914.46	8000	114.337/113.801	114
3	3957.23	4000	6783.83	8000	113.609/113.686	114
4	3815.90	4000	5723.85	8000	113.624/114.203	114

Table 25: Achieved values in the iteration process vs. objective or limit values for the four gears.

EXECUTION

Now that the gear ratios to achieve the desired result are known, it is time to convert those numbers into a real design. To this end, the number of teeth on each gear and the disposition of the different elements in the transmission is going to be developed in this point.

All the distances and measures mentioned in this section are in the U.S System (inches) while the rest of the project is in the Metric System. This is due to the utilization of the *Shigley's Mechanical Engineering Design Textbook* for this part as well as several equations exposed on it.

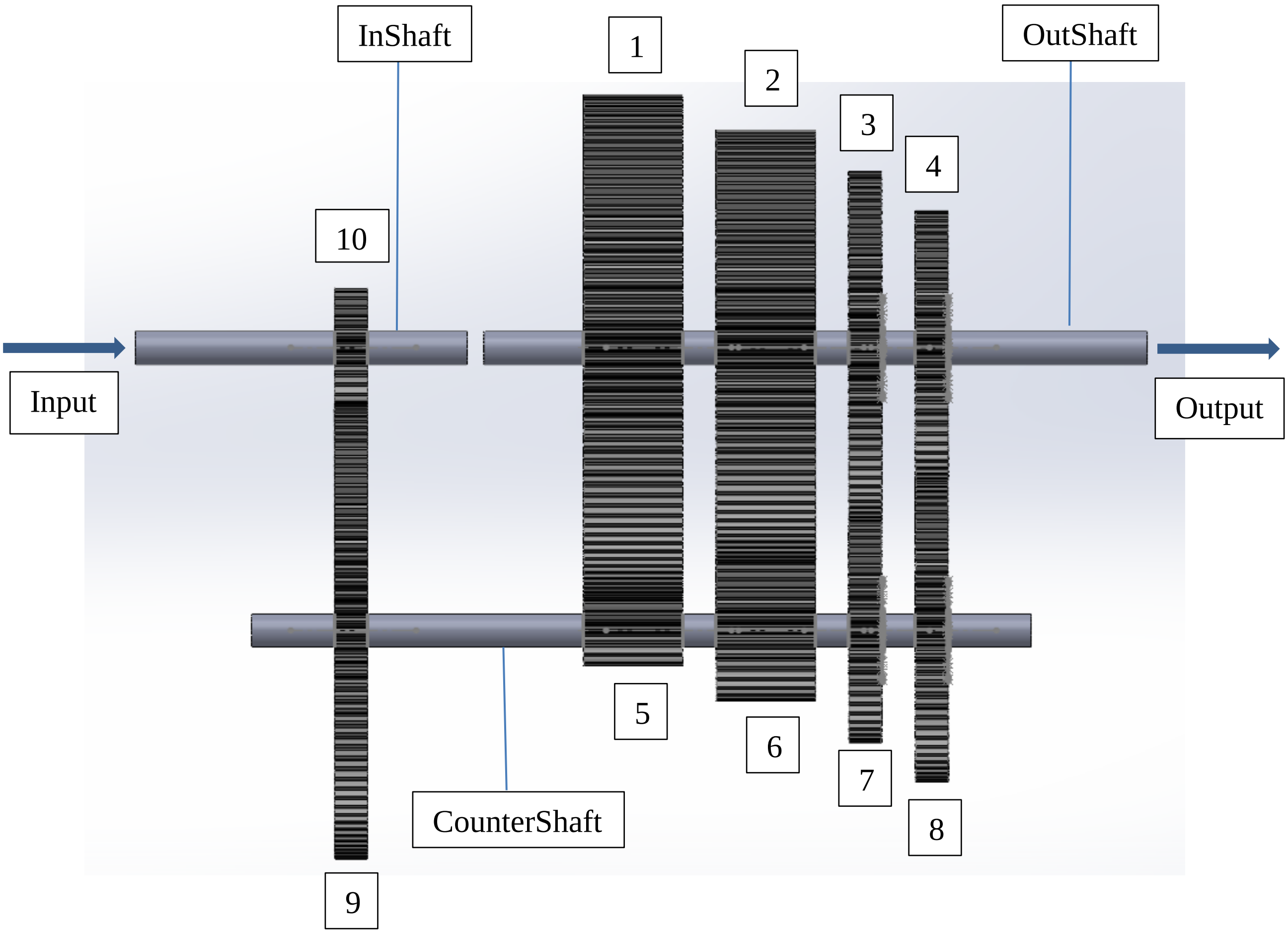


Figure 30: Transmission design with shafts and gears. Labels with names included.

The position shown in Figure 30 corresponds to gear 1 working. Despite it looks like all the gears are meshed, only gear 1 is connected by a square key to the OutShaft. As a result, even though all the gears are in there corresponding position, only gear 1 moves the Outshaft introducing its gear ratio between the Inshaft and the OutShaft.

The initial design consisted in a moving OutShaft as it is shown in Figure 31. Right after it was designed, it was seen that with this conection, the wheels shaft would have been moving, which doesn't make any sense. Because the aim of the project is to make a real transmision that could be introduced in a street car, the key solution was implemented. Instead of moving the entire OutShaft, the little square key connects the selected gear with the Outhaft.

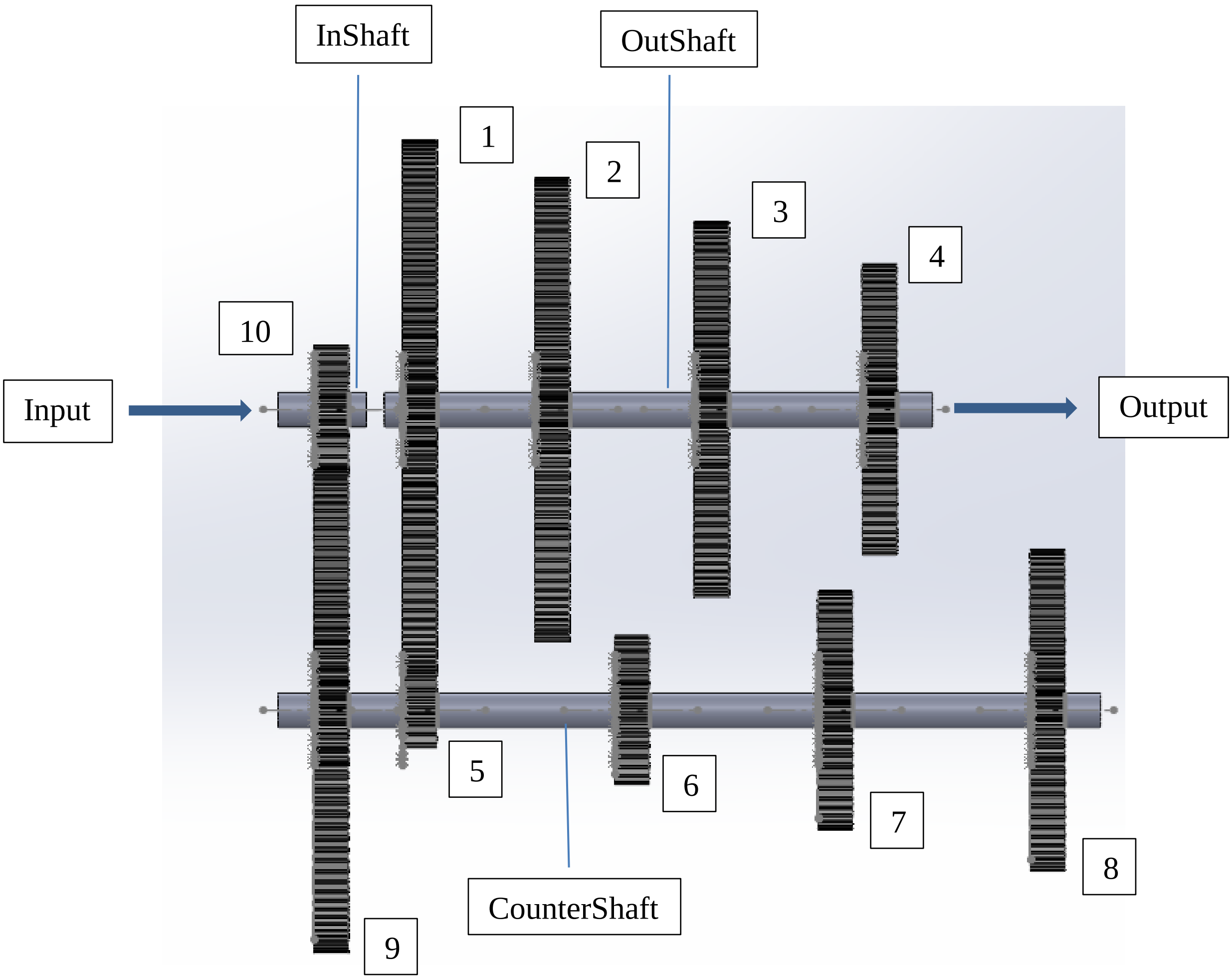


Figure 31: Initial transmission design with shafts and gears. Labels with names included.

Teeth numbers and gear sizes

To get the different gears size and number of teeth, the first consideration is going to be that the distance between the InShaft and OutShaft with the CounterShaft is established and doesn't change (Equation 21).

$$\frac{D10}{2} + \frac{D9}{2} = \frac{D1}{2} + \frac{D5}{2} \rightarrow D10 + D9 = D1 + D5$$

Equation 21: Distance between InShaft and OutShaft and the CounterShaft.

The introduction of Equation 16 into Equation 22 gives the first equation to get the teeth numbers of the different gears (Equation 22).

$$\frac{N10}{P} + \frac{N9}{P} = \frac{N1}{P} + \frac{N5}{P} \rightarrow N10 + N9 = N1 + N5$$

Equation 22: Distance between InShaft and OutShaft and the CounterShaft in teeth number terms.

The second equation used to get the teeth numbers has already been exposed (Equation 13). Even though, the transmission designed has two stages. As a result, the gear ratio is compounded by four terms instead of by two as shown in Equation 23. For gear 1:

$$\left.\begin{array}{l} First\ stage: G1 = \frac{No}{Ni} = \frac{N9}{N10} \\ Second\ stage: G2 = \frac{No}{Ni} = \frac{N1}{N5} \end{array}\right\}$$

$$Gtotal = \frac{N9 * N1}{N10 * N5}$$

Equation 23: Gear ratio for first gear in terms of teeth numbers.

Both Equation 22 and Equation 23 are used to get the teeth number of each gear. In the next few lines the steps to get N1, N5, N9 and N10 is shown. The process is exactly the same for the rest of the gears, and it is developed in ANEX I.

Gear 1 (Table 26):

Equation 23: $G = \frac{N9 * N1}{N10 * N5} = 8 * 4$ $\}$

- $N1 = 8 * N5$
- $N9 = 4 * N10$

Introducing those relationships into Equation 22: $N10 = \frac{9}{5} * N5$

The last step is to give a value to N5. Because N1, N9 and N10 are set as proportional values to it, all the teeth numbers of the assembly are known. Eventhough, not every number is an option, because it is impossible to have decimal teeth number values.

P=8	N	D [in]	D [mm]
1	120	15	381
5	15	1,875	47,625
9	108	13,5	342,9
10	27	3,375	85,725

Table 26: Gear 1 teeth numbers and diameters.

For the rest of the gears, the only two considerations to bear in mind is that N1 and N5 change for the terms of the corresponding gears, and N9 and N10 are common terms for the four gears. Furthermore, once the first gear of the transmission was designed, Equation 22 was set to 135. This value corresponds to the sum of N9 and N10.

P=8	N	d [in]	d [mm]
1	120	15	381
5	15	1,875	47,625
2	103	12,875	327,025
6	32	4	101,6
3	83	10,375	263,525
7	52	6,5	165,1
4	64	8	203,2
8	71	8,875	225,425
9	108	13,5	342,9
10	27	3,375	85,725

Table 27: Teeth number and diameter of all the gears in the transmission.

As it can be seen in Table 27 and Figure 32, the teeth number values are wide. N5 equals 15, which is below the recommended lowest value (16). This value hasn't been fixed because adding an extra tooth wasn't enough. It is complex to reach precise values of teeth that meet the requirements. Because 15 is not extremely lower than the recommended lower value it has been considered good enough.

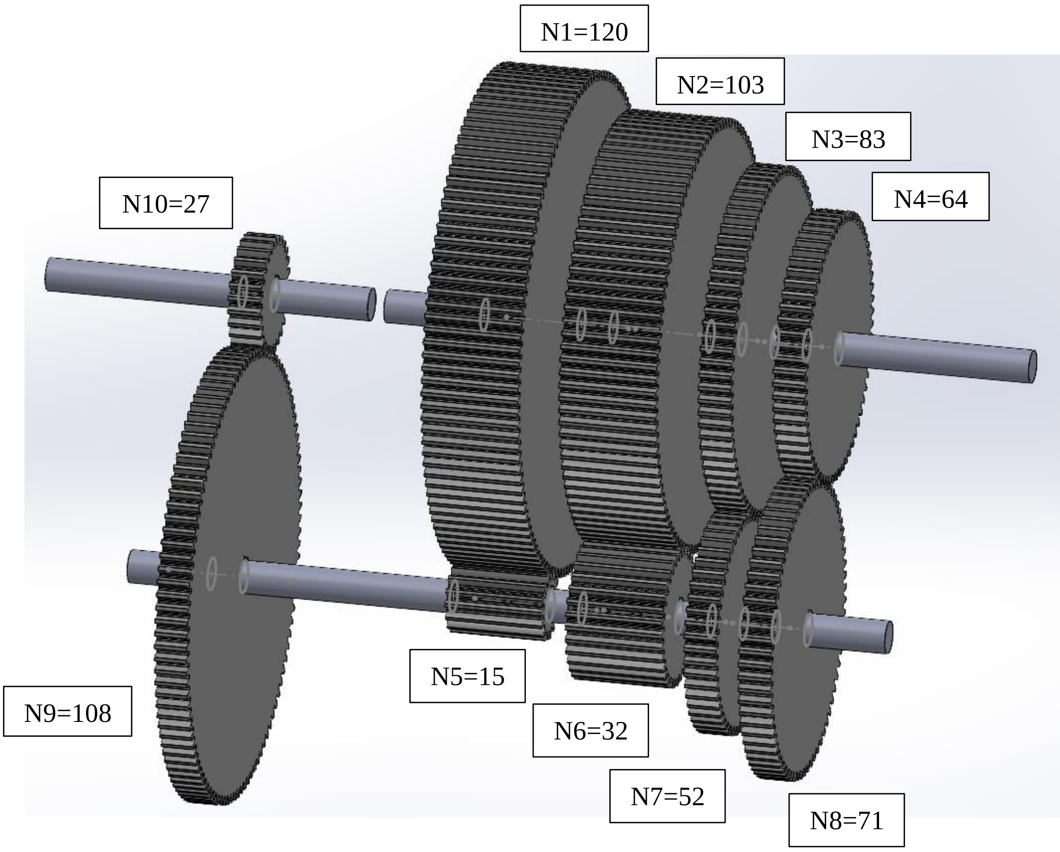


Figure 32: Teeth numbers.

It is necessary to mention that the gap between the gears has been designed with the purpose of having enough space to avoid contacts. Even though, space is a concern. Therefore, the distances have been carefully selected. The result is shown in Figure 33.

Moreover, the distance between the InShaft and OutShaft with the CounterShaft is fixed. This value is given by the sum of the radius as Equation 21 shows. Even though, the real distance must be a bit bigger so the gears can rotate. If the precise value of 8.4375 in is set, the gears would get locked. The final distance is also shown in Figure 33.

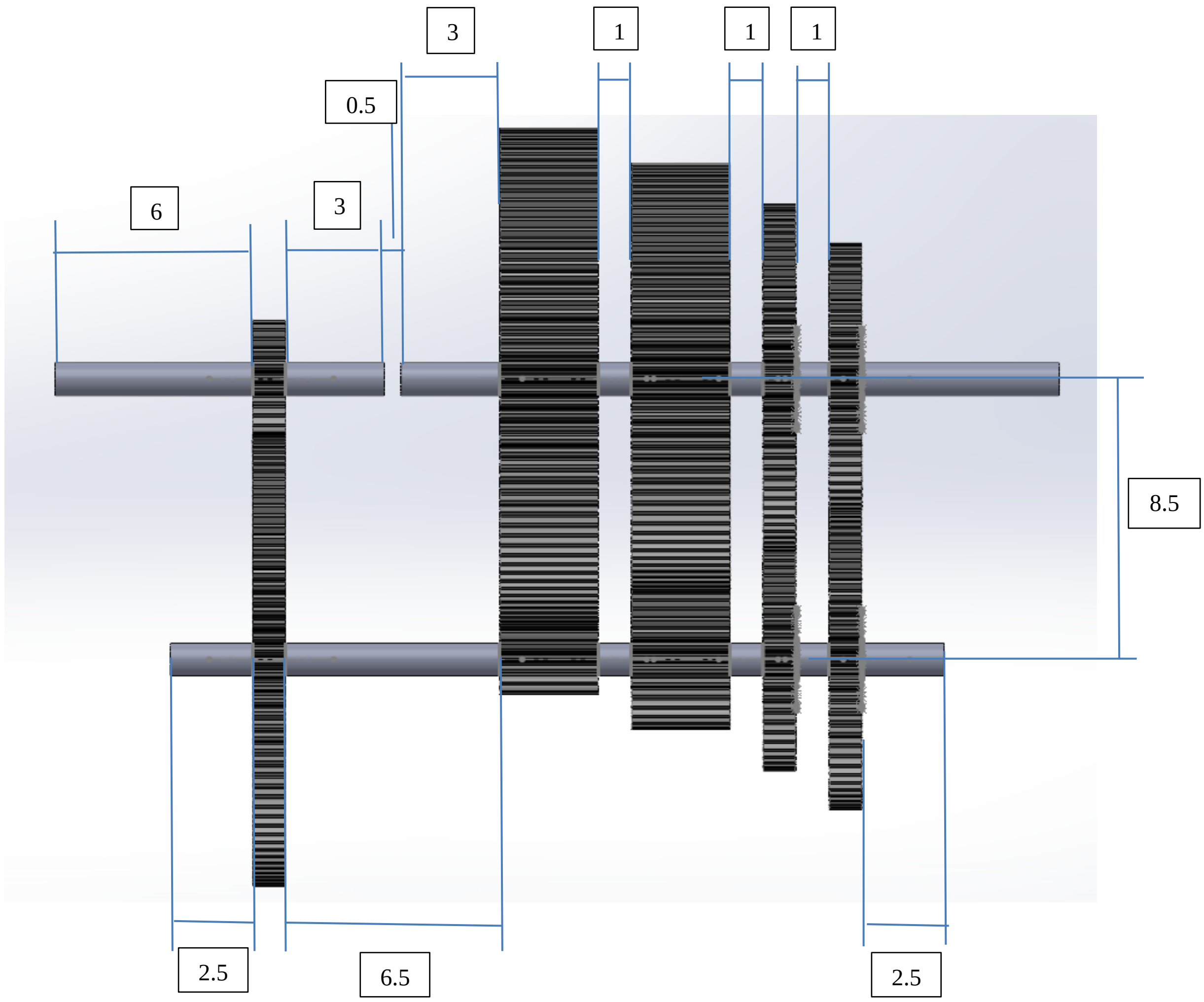


Figure 33: Distances between the different gears in the transmission. Measures in inches.

Lewis Bending Stress requirements:

Once the gear ratios are established and the gears are designed, it is necessary to ensure that the system meets the requirements and properly works under the circumstances the car is going to be driven.

With the purpose of ensuring the proper performance of the transmission, it is essential that each element can withstand the hardest conditions under which is going to be subjected. Each gear is going to be carefully reviewed using Lewis Bending Equation.
To this end, the data from the different flipped power flow diagrams, Table 27 and Figure 33 are going to be used.

Common values for all the gears on its corresponding scenarios:

- P: $P = 8\frac{teeth}{in} = 314.961\frac{teeth}{m}$
- S_y: Q&T Steel 43440 - $Sy = 1470\ MPa$
- n_d: $nd = 2$ (It is consider the typical value in the automotive industry).

Parameters that vary depending on the gear and scenario:

- N: Table 27.
- Y: Table 13.
- K_v: the speed is calculated from the data on the corresponding flipped power flow diagram.
- T: Corresponding flipped power flow diagram.
- F: was initially established at a value of $F = 25.4\ mm$ but it might need to change if the gear cannot withstand the conditions.

The results obtained are collected in Table 47 (ANEX I). It is essential to mention that not all the gears met the requirements with the starting parameters selection. With those values previously exposed, gears 1, 5 and 6 did not give a result bigger than $P = 314.961\frac{teeth}{m}$ and, therefore, were not prepared to resist the maximum bending stress they were supposed to. To repair this issue, there was more than just one possibility. The pitch diameter could have been decreased, but then the gears would have been too big to fit under the car, and the transmission would have been too heavy. As a result, the performance of the car would have been resentful. A material with a higher yield strength could have been chosen, but the price would have increase.

The solution chosen has been to select a material with a yield strength high enough, and to make a growth in the face width of those gears just mentioned. The final decision implies changing the starting design shown in Figure 31 and making a new *SolidWorks* model. The shafts must be redesigned to accommodate the bigger gears and the face width of gears 1, 2, 5 and 6 must be changed. Bear in mind that the increment of gear 6 made necessary to change also gear 2 so both gears could mesh. The final F value is three times the starting point (F=0.076 mm).

The equilibrium reached between weight and price seems to be the best possible solution. The car doesn't get too heavy, so the energy consumption doesn't get too high, and the performance is not compromised. Furthermore, the price does not increase in excess due to excessively treated materials.

<u>Bearings</u>

At this point, the gears are designed with their corresponding teeth numbers to achieve the desired gear ratios. In addition, the distribution of the different elements in the transmission is established. The next step is to select the bearings that will allow the shafts to move inside the case.

To select the bearings for the design, the forces that they are going to suffer must be calculated. Depending on the working conditions, these reactions are going to be different. Even though, it is enough to study the most critical scenario for each gear. If the bearing can resist it, it will also be able to work under the rest of the circumstances.

Most critical scenario on each shaft:

- InShaft. The most critical scenario for this shaft where gear 10 is located is when 190 Nm are demanded to the engine. This value corresponds to the maximum torque the engine can produce.

- OutShaft. The most critical scenario for the shaft were gears 1, 2, 3 and 4 are located corresponds to the shift change for the last three gears and to the starting point for gear 1. At this point, the speed is in the lowest value and therefore the torque is in the maximum point.

  The four mentioned situations are going to be studied and compared. The scenario under which the reactions are higher will be the one used to select the bearings for the transmission.
- CounterShaft. The maximum torque transmitted from the InShaft to the CounterShaft is in the shift change. For all the four gears is around 190 Nm, which corresponds to the study case of the InShaft. Due to Newton's Third Law, the reactions between gears 9 and 10 are already known.

  At the same time, the maximum torque transmitted from the CounterShaft to the OutShaft corresponds to the same scenario. Applying the same law, the reactions between gears can be taken from the OutShaft study.

For the three different shafts studies, the starting point is going to be the equilibrium of both forces and momentums. There are no reactions on the x axis, so the four unknown reactions of each shaft are going to be solved with the four equations established by the equilibriums in y and z axis. The position where the bearings are going to be set regarding the gears is reflected on the corresponding figures (Figure 33).

It is also important to mention that only the gear selected is considered on the study. The rest of the gears are not transferring any force or torque. As an example, when studying the OutShaft in gear one, gears 2, 3 and 4 are omitted.

Furthermore, not all the calculations, equations and figures are going to be included in this section, only those that have resulted to be the most critical. The rest of them can be found in ANEX I.

INSHAFT:

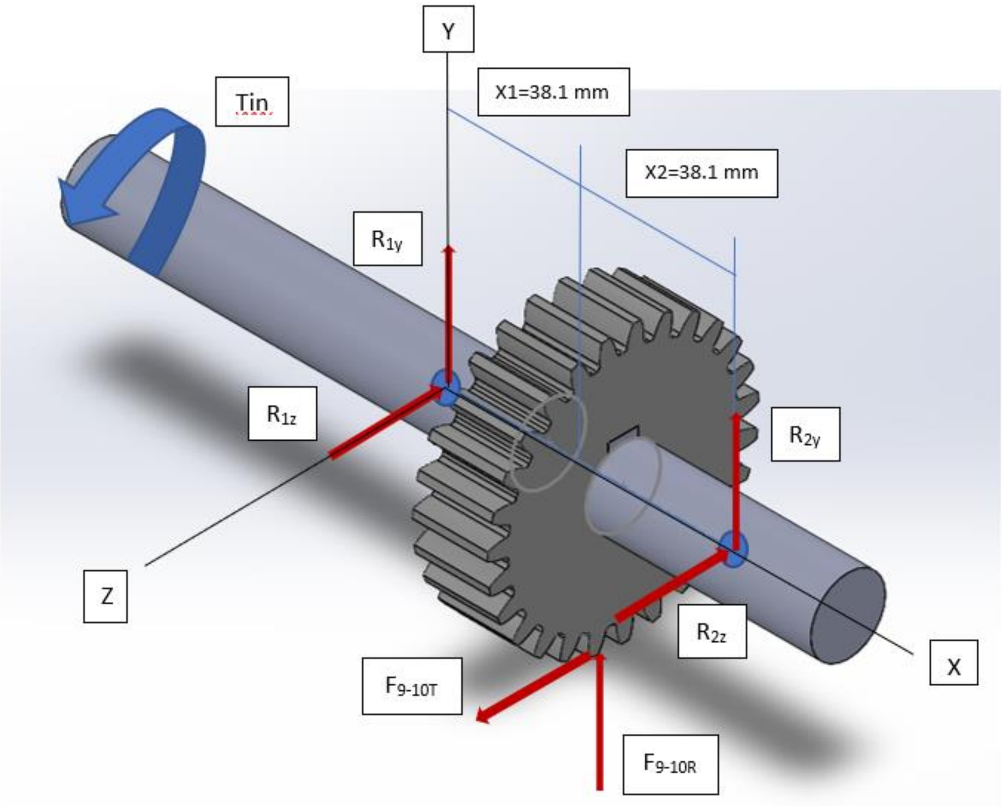


Figure 34: InShaft bearings study.

- $T_{in\ max}$=190 Nm
- r_{10}=0.043 m

From the equilibrium of momentum in the x axis, the tangential force on the gear is known:

$$\sum Mx = 0\,;$$

$$Tin - F910T * r10 = 0 \rightarrow F910T = 4432.779\ N$$

Equation 24: InShaft, x axis momentum equilibrium.

From Equation 24 and the definition of pressure angle (ϕ=14.5º), the resultant force and the radial force are known:

$$F910T = F910 * \cos(14.5)\,; \rightarrow F910 = \frac{F910T}{\cos(14.5)} = \frac{4432.779}{\cos(14.5)} = 4578.619\ N$$

Equation 25: Resultant force between gears 9 and 10.

$$F910R = F910 * \sin(14.5) = 4578.619 * \sin(14.5) = 1146.395\ N$$

Equation 26: Radial force between gears 9 and 10.

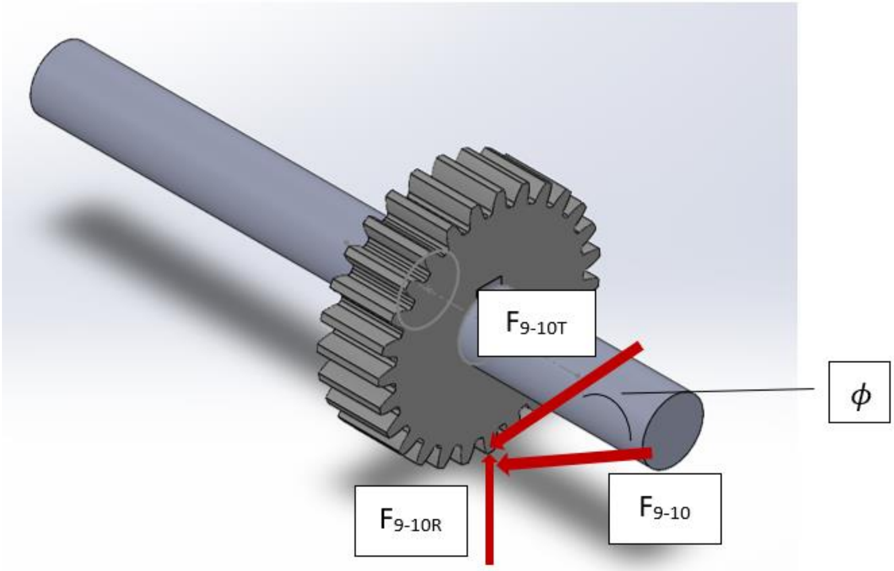


Figure 35: Tangential, Radial and Resultant forces and pressure angle in gear 10.

Once the force transmitted between gears is known, it is necessary to know the reactions the bearings are going to suffer. To this end, force and momentum equilibrium around the two missing axis is going to be applied.

$\sum F = 0\,;$

(i) y: $R1y + R2y + F910R = 0$

Equation 27: InShaft, y axis force equilibrium.

(ii) z: $-R1z - R2z + F910T = 0$

Equation 28: InShaft, z axis force equilibrium.

$\sum M = 0$; (Momentum around bearing 1)

(iii) y: $+R2z * (X1 + X2) - F910T * X1 = 0$

Equation 29: InShaft, y axis momentum equilibrium.

(iv) z: $+R1y * (X1 + X2) + F910R * X1 = 0$

Equation 30: InShaft, z axis momentum equilibrium.

R_{1Y}	-573.197 N (Equation 30)
R_{1Z}	2216.39 N (Equation 28)
R_1	2289.31 N (Equation 31)
R_{2Y}	-573.197 N (Equation 27)
R_{2Z}	2216.39 N (Equation 29)
R_2	2289.31 N (Equation 31)

Table 28: InShaft Bearing reactions.

$$R = (RY^2 + RZ^2)^{\frac{1}{2}}$$

Equation 31: Resultant force from tangential and radial force.

For the selection of the pair of bearings the greater value between R1 and R2 must be chosen. In this particular case, both reactions are the same as it is a symmetric situation.

OUTSHAFT:

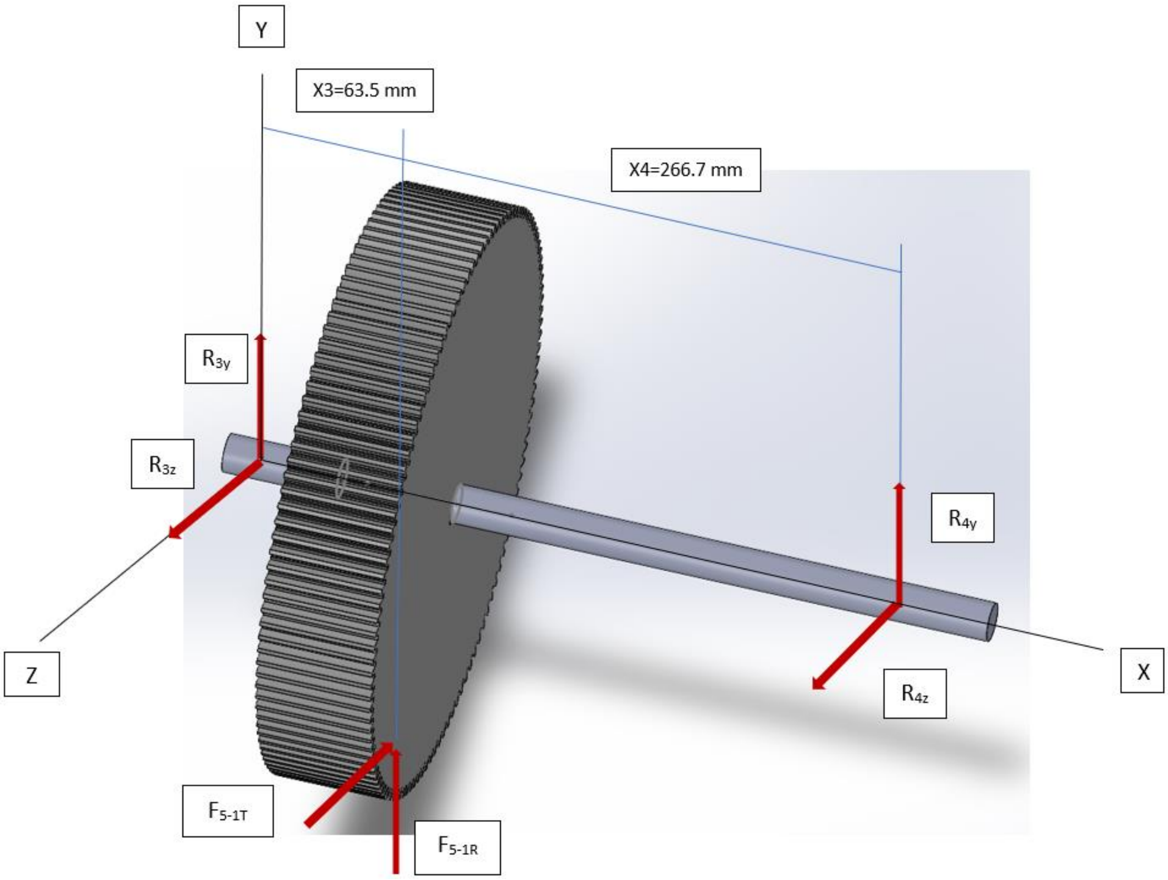


Figure 36: OutShaft bearing study.

- T_{1_max}=5229.52 Nm
- r_1=0.191 m

From the equilibrium of momentum in the x axis, the tangential force on the gear is known:

$$\sum Mx = 0\,;$$

$$T1 - F51T * r1 = 0 \rightarrow F51T = 27451.549\ N$$

Equation 32: OutShaft, x axis momentum equilibrium.

From Equation 32 and the definition of pressure angle (ϕ=14.5º), the resultant force and the radial force are known:

$$F51T = F51 * \cos(14.5)\,; \rightarrow F51 = \frac{F51T}{\cos(14.5)} = \frac{27451.549}{\cos(14.5)} = 28354.713\ N$$

Equation 33: Resultant force between gears 1 and 5.

$$F51R = F51 * \sin(14.5) = 28354.713 * \sin(14.5) = 7099.453\ N$$

Equation 34: Radial force between gears 1 and 5.

Once again, the force transmitted between gears is known. Now it is necessary to get the reactions the bearings are going to suffer. To this end, force and momentum equilibrium around the two missing axis is going to be applied.

$\sum F = 0$;

(v) y: $R3y + R4y + F51R = 0$

Equation 35: OutShaft, y axis force equilibrium.

(vi) z: $-R3z - R4z + F51T = 0$

Equation 36: OutShaft, z axis force equilibrium.

$\sum M = 0$; (Momentum around bearing 3)

(vii) y: $+R4z * (X3 + X4) - F51T * X3 = 0$

Equation 37: OutShaft, y axis momentum equilibrium.

(viii) z: $+R3y * (X3 + X4) + F51R * X3 = 0$

Equation 38: OutShaft, z axis momentum equilibrium.

R_{3Y}	-5734.174 N (Equation 38)
R_{3Z}	22172.405 N (Equation 36)
R_3	22901.884 N (Equation 31)
R_{4Y}	-1365.279 N (Equation 35)
R_{4Z}	5279.144 N (Equation 37)
R_4	5452.829 N (Equation 31)

Table 29: OutShaft Bearing reactions.

For the selection of the pair of bearings the greater value between R3 and R4 must be chosen. In this case, R3=22901.884 N is going to be the value chosen for selecting the bearing for the OutShaft.

COUNTERSHAFT:

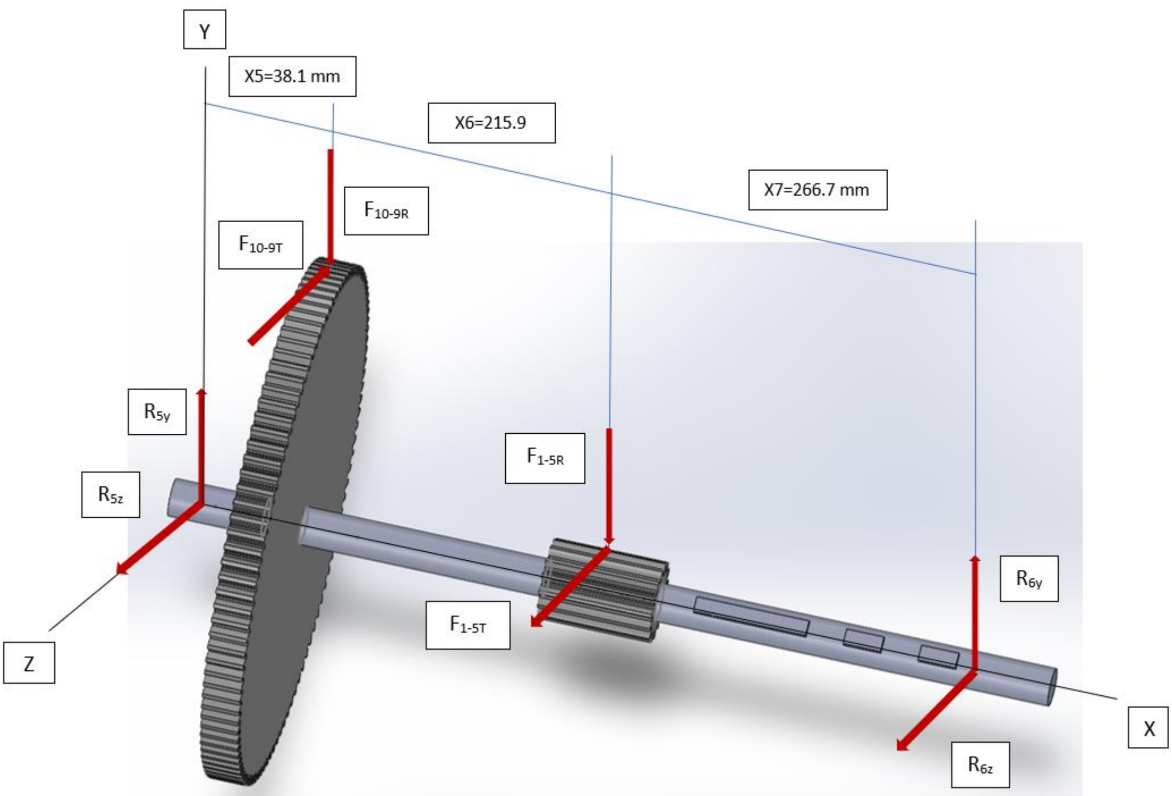


Figure 37: CounterShaft bearings study.

By the application of Newton's third law, the forces applied on gears 9 and 5 of Figure 37 are already know. These values are collected in Equation 25 and Equation 33. By simply applying the forces and momentum equilibriums already explained, the reactions on the different bearings are known.

$\sum F = 0$;

(i) y: $R5y + R6y - F15R - F109R = 0$

Equation 39: CounterShaft, y axis force equilibrium.

(ii) z: $+R5z + R6z + F15T - F109T = 0$

Equation 40: CounterShaft, z axis force equilibrium.

$\sum M = 0$; (Momentum around bearing 5)

(iii) y: $-R6z * (X5 + X6 + x7) - F15T * (X1 + X2) + F109T * X1 = 0$

Equation 41: CounterShaft, y axis momentum equilibrium.

(iv) z: $+R6y * (X5 + X6 + X7) - F15R * (X5 + X6) - F109R * X5 = 0$

Equation 42: CounterShaft, z axis momentum equilibrium.

R_{5Y}	4698.817 N (Equation 39)
R_{5Z}	-9952,12 N (Equation 40)
R_5	11005,615 N (Equation 31)
R_{6Y}	3547.03 N (Equation 42)
R_{6Z}	-13066,65 N (Equation 41)
R_6	13539,526 N (Equation 31)

Table 30: OutShaft Bearing reactions.

For the selection of the pair of bearings the greater value between R5 and R6 must be chosen. In this case, R6=13539,526 N is going to be the value chosen for selecting the bearing for the CounterShaft.

Shaft	Radial Load [N]	Speed [rpm]
InShaft	2289.31	4239.89
OutShaft	22901.884	132.5
CounterShaft	13539,526	1059.97

Table 31: Radial load and Speed of each shaft.

As it was mentioned before, there is not any force applied in the shafts in the x direction. As a result, there is not axial load to bear in mind. In this situation, ball bearings or roller bearings are used, but not sleeve or flat bearings.
To determine the life of the bearings according to the work conditions, Equation 43 is going to be used. It considers both radial loads and the rotational speed. Several new parameters are introduced:

- Catalog Load Rating, c_{10}. Constant radial load that causes 10% of a group of bearings to fail at the bearing manufacturer's rating life.
- Radial load applied, Fr.
- Desired life, L_d [hours]. This parameter is defined by the designer based on how long he wants the bearing to work. For this case, the next assumption is going to be done:

  $$\text{Life in distance } 200000 \text{ km}$$

  $$\text{Average car speed along its life } 40\,\frac{\text{km}}{\text{h}}$$

  $$Ld = \frac{200000\ km}{40\,\frac{km}{h}} = 5000\ hours$$

- Rotational speed, n_d [rpm].

- Rating Life, L_{10}. Life required for 10% of sample to fail.

It depends on the manufacturer. For this project, *SKF* bearings are going to be used. The company rates its bearings in 1 million revolutions.

- Constant, a. The value depends on the type of bearing: a $= 3$ for ball bearings and a $= \frac{10}{3}$ for roller bearings.

$$c10 = Fr * \left(\frac{Ld\ nd\ 60}{L10}\right)^{\frac{1}{a}}$$

Equation 43: Catalog Load Rating formula.

With the data collected in Table 31 and Equation 43, the different c_{10} required values are set. Initially, ball bearings (a=3) were going to be used in the InShaft, where the load is lower. Finally, because space is a concern and cylindrical roller bearings (a $= \frac{10}{3}$) can withstand higher loads, roller bearings are going to be implemented in all the shafts. The selected bearing must have a c_{10} value above of the c_{10} required so it is ensured that it last working under the conditions established. The results and bearings chosen are collected in Table 32.

Shaft	C10 required [kN]	SKF bearing chosen	C10 chosen [kN]
InShaft	19,546	N 303 ECP	28,5
OutShaft	69,131	NU 2306 ECML	83
CounterShaft	76,265	NJ 2208 ECJ	81.5

Table 32: Catalog Load Rating required and bearings chosen.

The technical specification of each bearing is attached in ANEX I. It includes an image of the bearings, sizes and calculation data.

Shaft

Before adding the case that is going to protect the gearbox, which is going to be the final stage, the shafts are going to be designed. The steps that are going to be followed are presented in *Shigley's Mechanical Engineering Design Textbook*, which has already been mentioned.

The first step is to identify the most critical location of each shaft. To this end, the bearing reactions and the forces transmitted between gears that were found in the bearings section are going to be used. As it happened there, it is not necessary to make the process for each gear and speed. If it is guaranteed that the system works under the most critical conditions, then it will also make it in any other.

For a better understanding of the result and identification of the most critical locations, shaft shear, bending and torque diagrams have been made.

Once the most critical point is found, the *DE-Goodman Equation* (Equation 48) is going to be used to analyse the stresses in the shafts. Because the term of study (diameter) is inside the equation, it is going to be an iteration process. The equation is compounded by the following parameters:

- Design factor, n_d. $n_d=2$
- Endurance limit, S_e.

$$Se = Ka\ \ Kb\ \frac{Sut}{2}$$

Equation 44: Endurance limit.

Marin Surface Modification Factor, K_a: $Ka = a\ Sut^b$

Equation 45: Marin Surface Modification equation.

a and b values taken from Figure 71

S_{ut} depends on the material chosen

Marin size factor, K_b: $Kb = 0.9$ as a first estimation

Figure 72 for the rest of iterations.

- Ultimate Tensile Strength, Sut. Depends on the material chosen.
- Alternating bending momentum, M_a. Given by the diagrams.
- Mid-range bending momentum, M_m.

  For a rotating shaft with constant bending, the bending stress is completely reversed and it can be simplified so M_m=0.
- Alternating torsional momentum, T_a.

  For a rotating shaft with constant torsion, the torsion stress is steady and it can be simplified so T_a=0.
- Mid-range torsional momentum, T_m. Given by the diagrams
- Stress Concentration Factors, K_f and K_{fs}.

$K_f=1+q\ (K_t-1)$ Figure 74 and Figure 76.

Equation 46: Stress Concentration Factor for bending.

$K_{fs}=1+q_s\ (K_{ts}-1)$ Figure 75 and Figure 77.

Equation 47: Stress Concentration Factor for torsion.

  - First Estimation Stress Concentration Factors, K_t and K_{ts}. Figure 73.

    $K_f=K_t$ and $K_{fs}=K_{ts}$
  - Notch Sensitivity, q and q_s. Figure 76 snd Figure 77.

$$d=\left\{\frac{16\,n}{\pi}\left[\frac{1}{Se}\left(4\,(kf\ Ma)^2+3\,(Kfs\ Ta)^2\right)^{\frac{1}{2}}+\frac{1}{Sut}\left(4\,(Kf\ Mm)^2+3\,(Kfs\ Tm)^2\right)^{\frac{1}{2}}\right]\right\}^{\frac{1}{3}}$$

Equation 48: DE-Goodman Equation.

$$d=\left\{\frac{16\,n}{\pi}\left[\frac{1}{Se}\left(4\,(kf\ Ma)^2\right)^{\frac{1}{2}}+\frac{1}{Sut}\left(3\,(Kfs\ Tm)^2\right)^{\frac{1}{2}}\right]\right\}^{\frac{1}{3}}$$

Equation 49: DE-Goodman Equation simplified.

INSHAFT

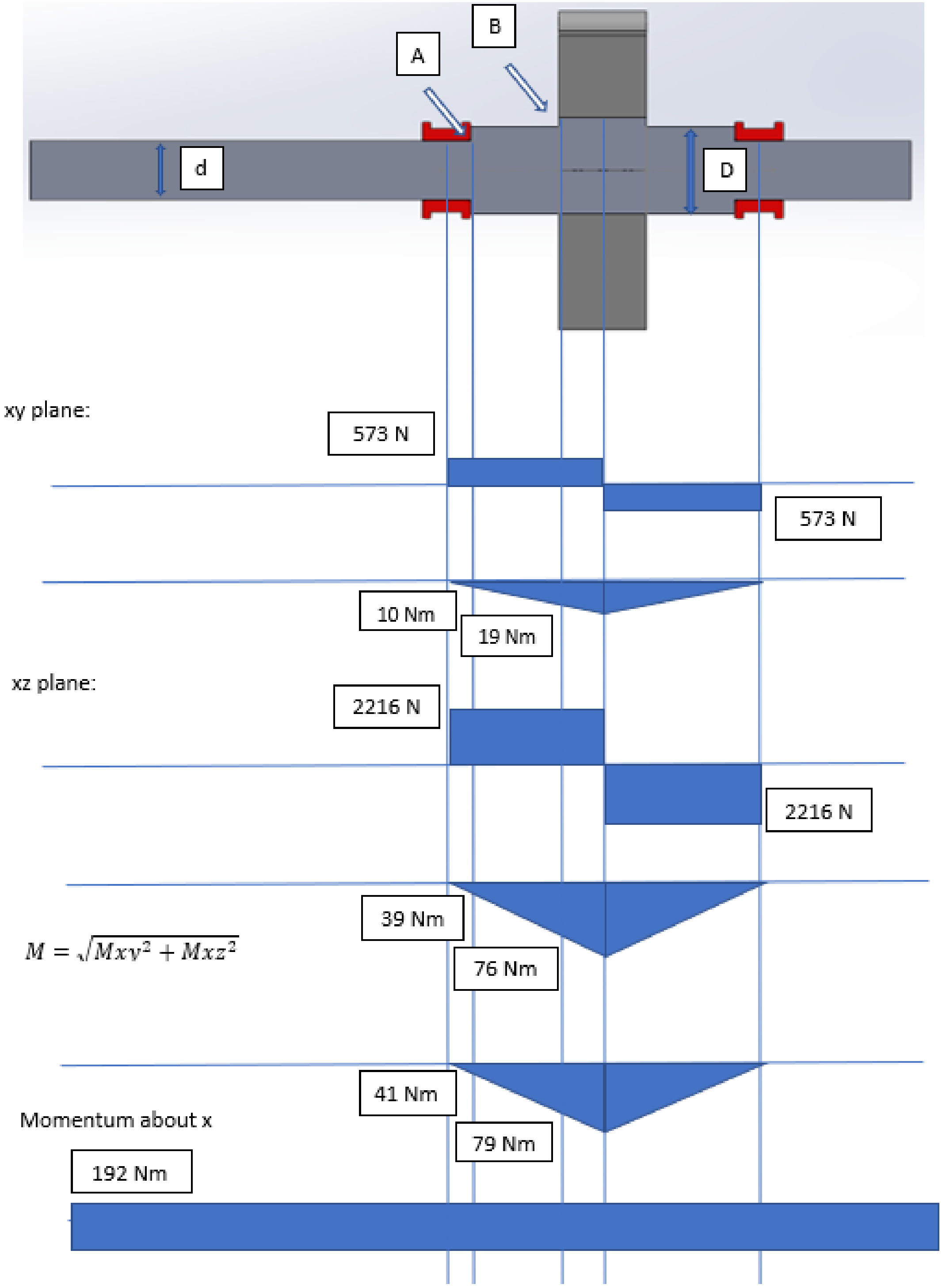


Figure 38: InShaft diagrams.

Once the diagrams have been developed, two different point are going to be considered: A and B (Figure 38). Those two points have been chosen because of the loads they are suffering and due to their condition of shoulders where the diameter changes. If the requirements are satisfied in those two critical locations, the shaft designed would be able to work under the conditions established. The shaft study is going to start from the values just mentioned for the first iteration, and a deeper study will be developed afterwards. For the first two iteration, the material considered is a 1050 Cold-Drawn (CD), with a tensile strength of 620 MPa. Results are collected in Table 33.

Before moving to the next iteration, two more parameters must be defined: the relationship between the diameters in the shoulders and the radius of the shoulder fillets. Both values are to be set, and there are not specific requirements to establish them. Standard values are going to be taken from *Shigley's Mechanical Engineering Design Textbook*:

$$\frac{D}{d} = 1.2 \qquad r = \frac{3}{32}\ in$$

Frist Iteration			Second Iteration		
	A	B		A	B
n	2	2	d [in]	1	1
a	4,51	4,51	D [in]	1,2	1,2
b	-0,265	-0,265	n	2	2
Sut [MPa]	620	620	a	4,51	4,51
Ka	0,821	0,821	b	-0,265	-0,265
Kb	0,9	0,9	Sut [Pa]	620	620
Se [MPa]	22,898	22,898	Ka	0,821	0,821
Kt	1,7	1,7	Kb	0,788	0,788
Kts	1,5	1,5	Se	200,502	200,502
Kf	1,7	1,7	Kt	1,6	1,6
Kfs	1,7	1,7	Kts	1,35	1,35
Ma [Nm]	40,7	78,5	q	0,85	0,85
Tm [Nm]	192,26	192,26	qs	0,9	0,9
d [m]	0,025	0,0278	Kf	1,51	1,51
d [in]	0,981	1,089	Kfs	1,315	1,315
			Ma [Nm]	40,7	78,5
			Tm [Nm]	192,26	192,26
			d [m]	0,023	0,026
			d [in]	0,916	1,025

Table 33: InShaft first iterations with first material.

As it can be seen, these values do not meet with the bearings bore diameters and neither the first designed shafts with a 1 in diameter. There are two options at this point:

- The first one would be to increase the shafts diameters and, therefore, making a new selection of bearings, ensuring the new ones also satisfy the conditions.
- Option B would be changing the material to a stronger one with a higher tensile strength.

Option B is chosen, and the second material selected is a 1050 Quenched and Tempered (Q&T). Results are presented in Table 34. Once again, the results do not meet the design measures. Even though, the values on this last iteration are close to the objective. Therefore, a third material is going to be tried: 4140 Q&T. Results are presented in Table 35.

Iteration 1			Iteration 2			Iteration 3		
	A	B		A	B		A	B
n	2	2	d [in]	1	1	d [in]	1	1
a	4,51	4,51	D [in]	1,2	1,2	D [in]	1,2	1,2
b	-0,265	-0,265	n	2	2	n	2	2
Sut [MPa]	1090	1090	a	4,51	4,51	a	4,51	4,51
Ka	0,707	0,707	b	-0,265	-0,265	b	-0,265	-0,265
Kb	0,9	0,9	Sut [MPa]	1090	1090	Sut [MPa]	1090	1090
Se [MPa]	346,655	346,655	Ka	0,707	0,707	Ka	0,707	0,707
Kt	1,7	1,7	Kb	0,91	0,91	Kb	0,91	0,91
Kts	1,5	1,5	Se [MPa]	350,507	350,507	Se [MPa]	350,507	350,507
Kf	1,7	1,7	Kt	1,6	1,6	Kt	1,6	1,6
Kfs	1,7	1,7	Kts	1,35	1,35	Kts	1,35	1,35
Ma [Nm]	40,7	78,5	q	0,9	0,9	q	0,9	0,9
Tm [Nm]	192,26	192,26	qs	0,92	0,92	qs	0,92	0,92
d [m]	0,021	0,024	Kf	1,54	1,54	Kf	1,54	1,54
d [in]	0,83	0,929	Kfs	1,322	1,322	Kfs	1,322	1,322
			Ma [Nm]	40,7	78,5	Ma [Nm]	40,7	78,5
			Tm [Nm]	192,26	192,26	Tm [Nm]	192,26	192,26
			d [m]	0,02	0,022	d [m]	0,02	0,022
			d [in]	0,779	0,879	d [in]	0,779	0,879

Table 34: InShaft iterations for second material.

Iteration 1			Iteration 2			Iteration 3		
	A	B		A	B		A	B
n	2	2	d [in]	0,75	0,75	d [in]	0,669	0,669
a	4,51	4,51	D [in]	0,9	0,9	D [in]	0,803	0,803
b	-0,265	-0,265	n	2	2	n	2	2
Sut [MPa]	1770	1770	a	4,51	4,51	a	4,51	4,51
Ka	0,707	0,707	b	-0,265	-0,265	b	-0,265	-0,265
Kb	0,9	0,9	Sut [MPa]	1770	1770	Sut [MPa]	1770	1770
Se [MPa]	562,917	562,917	Ka	0,707	0,707	Ka	0,707	0,707
Kt	1,7	1,7	Kb	0,91	0,91	Kb	0,969	0,969
Kts	1,5	1,5	Se [MPa]	569,172	569,172	Se [MPa]	606,249	606,249
Kf	1,7	1,7	Kt	1,6	1,6	Kt	1,6	1,6
Kfs	1,7	1,7	Kts	1,35	1,35	Kts	1,35	1,35
Ma [Nm]	40,7	78,5	q	0,9	0,9	q	0,9	0,9
Tm [Nm]	192,26	192,26	qs	0,92	0,92	qs	0,92	0,92
d [m]	0,018	0,02	Kf	1,54	1,54	Kf	1,54	1,54
d [in]	0,706	0,79	Kfs	1,322	1,322	Kfs	1,322	1,322
			Ma [Nm]	40,7	78,5	Ma [Nm]	40,7	78,5
			Tm [Nm]	192,26	192,26	Tm [Nm]	192,26	192,26
			d [m]	0,017	0,019	d [m]	0,017	0,019
			d [in]	0,663	0,748	d [in]	0,657	0,738

Table 35: InShaft iterations for third material.

To sum up with, the selection of the strongest material (4140 Q&T) is enough to meet the established parameters. The smaller diameter is going to be set to 0,669 in, so the chosen bearing can fit in it. For the wider part of the InShaft, instead of 0,75 in, a diameter of 1 in is going to be set. If the size chosen is bigger than the limit it wouldn’t be a problem.

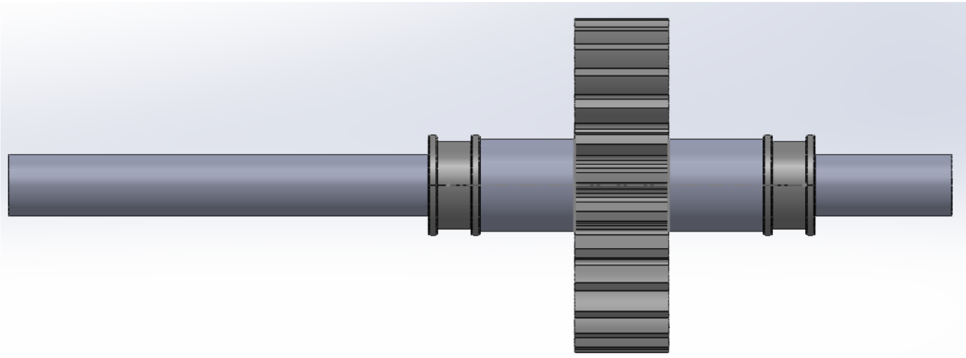


Figure 39: InShaft final result.

OUTSHAFT:

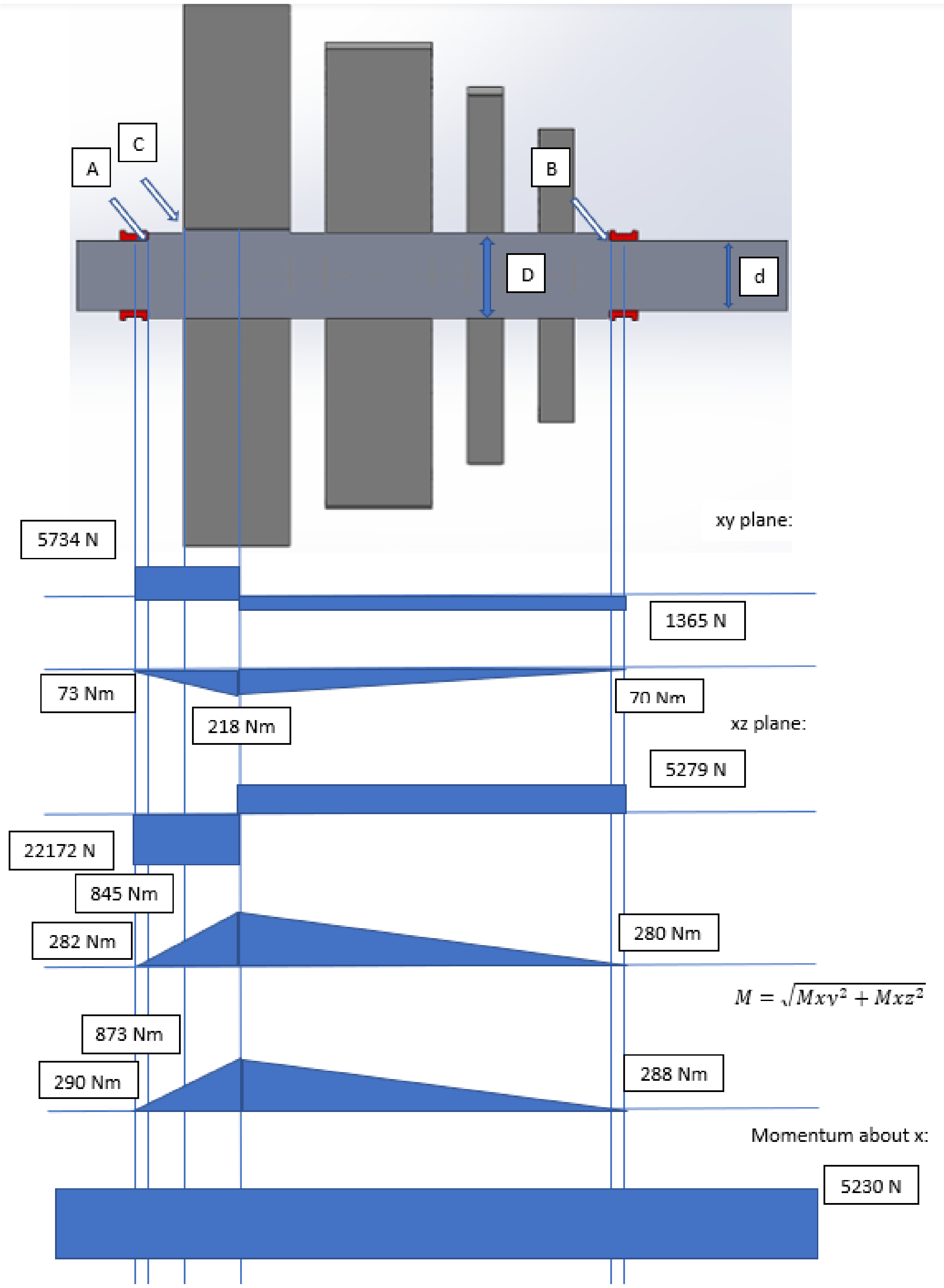


Figure 40: OutShaft diagrams.

The OutShaft suffers bigger reactions and must deal with greater loads as it is shown in Figure 40. Therefore, this shaft study is going to start with a stronger material from the beginning. The material chosen for the first iteration is going to be 4140 Q&T, the third iteration material of the InShaft.

For the last shaft (CounterShaft), the material chosen will be even more important. It is the shaft with higher loads and has the smallest gear (gear 5), with a diameter of only 1.875 in. As a result, it cannot be a very thick shaft.

The same process as in the InShaft is followed. Firstly, the diagrams are done to identify the critical locations. Points A and B are very similar, with just a small difference that makes point A more critical. Therefore, point B is omitted in the study and only points A and C are revised. Once the critical locations are established, it is time to start the iterations following the steps previously exposed. Lastly, the bearings must be revised if the bore diameter changes. Results for the iterations are collected in Table 36.

Iteration 1			Iteration 2			Iteration 3		
	A	C		A	C		A	C
n	2	2	d [in]	2	2	d [in]	2	2
a	4,51	4,51	D [in]	2,4	2,4	D [in]	2,4	2,4
b	-0,265	-0,265	n	2	2	n	2	2
Sut [MPa]	1770	1770	a	4,51	4,51	a	4,51	4,51
Ka	0,622	0,622	b	-0,265	-0,265	b	-0,265	-0,265
Kb	0,9	0,9	Sut [MPa]	1770	1770	Sut [MPa]	1770	1770
Se [MPa]	495,05	495,05	Ka	0,622	0,622	Ka	0,622	0,622
Kt	1,7	1,7	Kb	0,816	0,816	Kb	0,816	0,816
Kts	1,5	1,5	Se [MPa]	448,938	448,938	Se [MPa]	448,938	448,938
Kf	1,7	1,7	Kt	1,92	1,92	Kt	1,92	1,92
Kfs	1,7	1,7	Kts	1,6	1,6	Kts	1,6	1,6
Ma [Nm]	290	873	q	0,92	0,92	q	0,92	0,92
Tm [Nm]	5230	5230	qs	0,96	0,96	qs	0,96	0,96
d [m]	0,048	0,053	Kf	1,846	1,846	Kf	1,846	1,846
d [in]	1,88	2,09	Kfs	1,576	1,576	Kfs	1,576	1,576
			Ma [Nm]	290	873	Ma [Nm]	290	873
			Tm [Nm]	5230	5230	Tm [Nm]	5230	5230
			d [m]	0,047	0,054	d [m]	0,047	0,054
			d [in]	1,866	2,116	d [in]	1,866	2,116

Table 36: OutShaft iterations.

As it can be seen in Table 36, results are far away from the expected values. There is not a material that will let the OutShaft diameter to be 1 in. Therefore, the smaller diameter (bearings part) is going to be set to a minimum of 1.866 in and the wider part of the OutShaft diameter is going to be set to 2.4 in.

Once the minimum diameter size has been set, it is necessary to look for the new bearings. The reactions and the Catalog Load Rating (c_{10}) stays unchanged. Therefore, the aim is to look for a bearing with at least 1.866 in bore diameter with an acceptable c_{10} value (greater than 69.131 kN). The model chosen is a SKF bearing *N 210 ECP* (Figure 65, Figure 66 and Table 53). With c_{10}=73.5 kN, it has enough margin for the dynamic load rating. The bore diameter is 1.969 in, the closest higher value to the one founded with Equation 49.

To sum up with the OutShaft study, the smaller diameter is going to be set to 1.969 in, so it fits with the bearings, and the bigger diameter is going to be se to to 2.4 in.

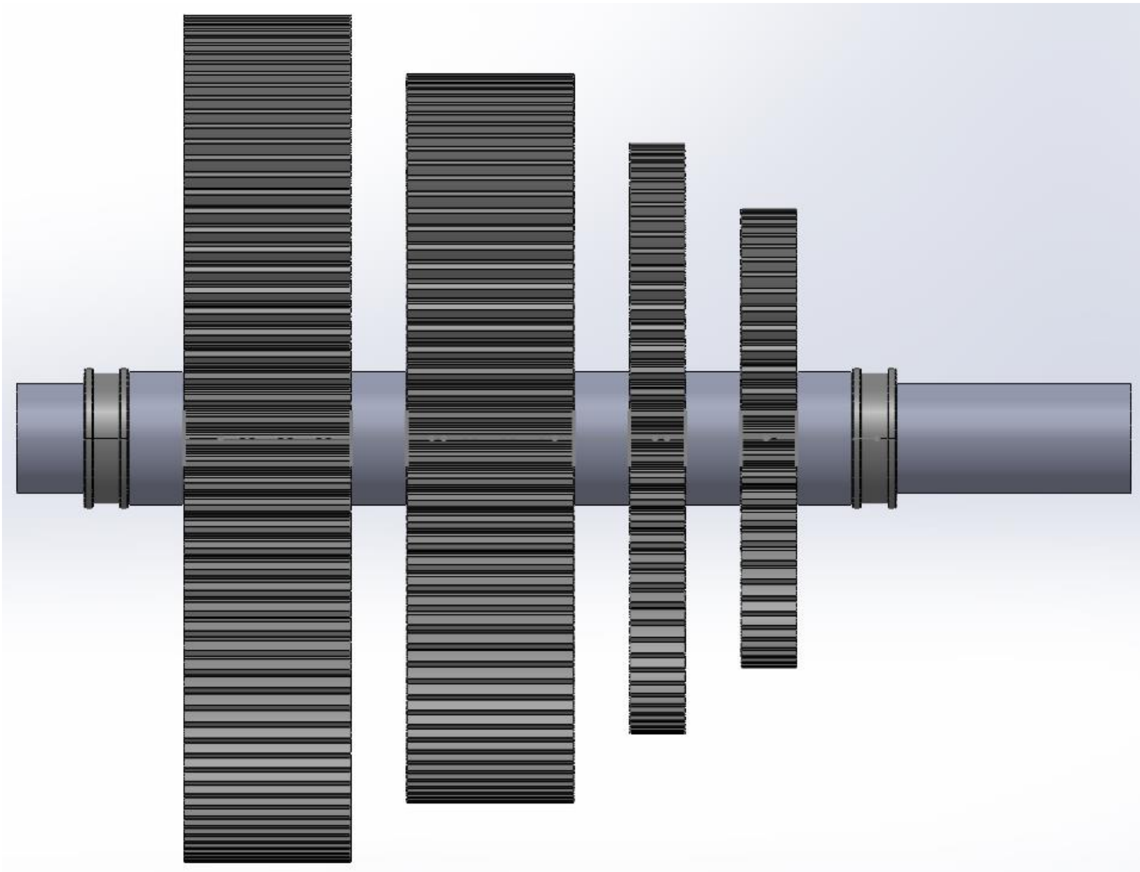


Figure 41: OutShaft final result.

In Figure 42 it is shown how gear 1 is meshed with the OutShaft. The square key lets the shaft rotate at the same time as the mentioned gear. When the gear is changed, the key joins the selected new gear with the shaft, introducing the corresponding gear ratio to the system.

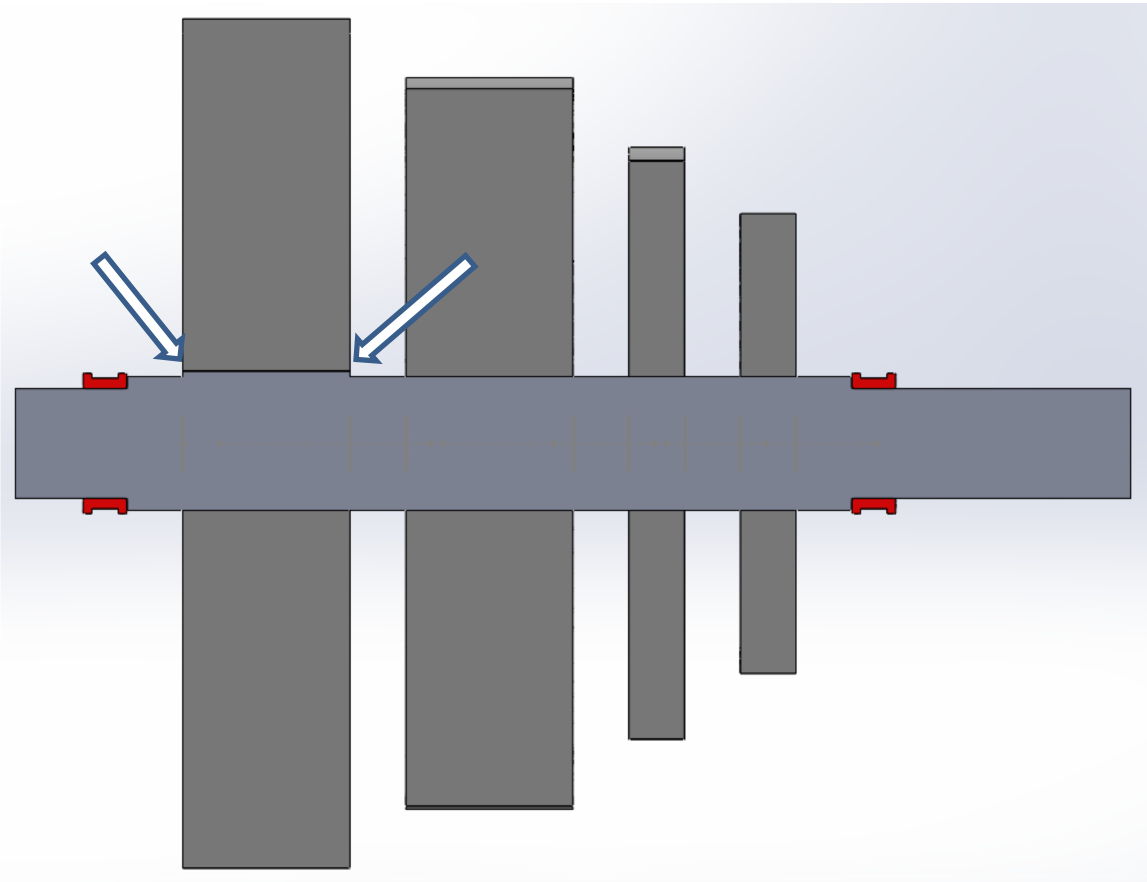


Figure 42: OutShaft final result side view.

COUNTERSHAFT:

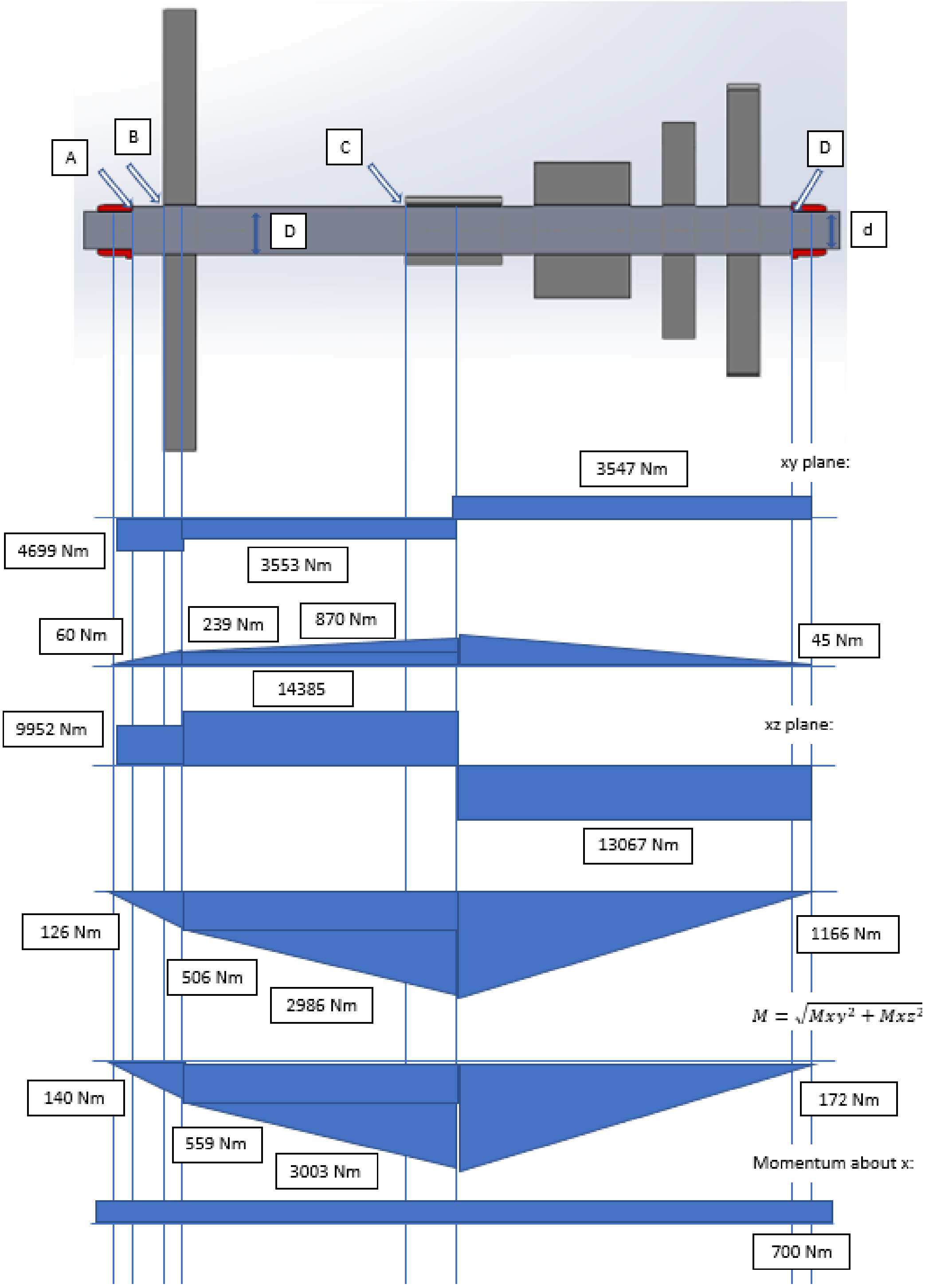


Figure 43: CounterShaft diagrams.

As it happened with the OutShaft, the CounterShaft suffers bigger reactions and must deal with greater loads than the InShaft as it is shown in Figure 43. Therefore, this shaft study is going to start with a stronger material from the beginning. The material chosen for the first iteration is going to be 4140 Q&T, the third iteration material of the InShaft. It is important to bear in mind that gear 5 is small (15 teeth), so the diameter of this shaft cannot be too big. This is going to be the major concern in this shaft design.

The same process as in the InShaft and OutShaft is followed. Firstly, the diagrams are done to identify the critical locations. In this case, there are 4 critical locations: A, B, C and D. Points A and D, and B and C are very similar. Therefore, the aim is going to be to identify the most critical point in both pairs for the study. As it can be inferred from Figure 43 and Table 37, C and D are the critical locations that are going to set the CounterShaft diameters. Once the critical locations are established, it is time to start the iterations following the steps previously exposed. Lastly, the bearings must be revised if the bore diameter changes.

Iteration 1				
	A	B	C	D
n	2	2	2	2
a	4,51	4,51	4,51	4,51
b	-0,265	-0,265	-0,265	-0,265
Sut [MPa]	1770	1770	1770	1770
Ka	0,622	0,622	0,622	0,622
Kb	0,9	0,9	0,9	0,9
Se [MPa]	495,05	495,05	495,05	495,05
Kt	1,7	1,7	1,7	1,7
Kts	1,5	1,5	1,5	1,5
Kf	1,7	1,7	1,7	1,7
Kfs	1,5	1,5	1,5	1,5
Ma [Nm]	140	559	3003	172
Tm [Nm]	700	700	700	700
d [m]	0,027	0,037	0,06	0,028
d [in]	1,073	1,446	2,379	1,111

Table 37: CounterShaft first iteration.

Once the two critical locations are identified, the study is going to be centred on them. As it can be seen in Table 38, the material chosen leaves the results away from the objectives, with big bore diameters. If gear 5 wouldn't have been so small, the diameters could have been increased adapting the bearings to it as it has been done in the OutShaft. However, with N=5 teeth and a pitch diameter of 8 $\frac{\text{teeth}}{\text{in}}$, resulting on a diameter of 1.875 in, these are not executable values.

Iteration 2			Iteration 3		
	D	C		D	C
d [in]	1,25	1,25	d [in]	1,25	1,25
D [in]	2,5	2,5	D [in]	2,5	2,5
n	2	2	n	2	2
a	4,51	4,51	a	4,51	4,51
b	-0,265	-0,265	b	-0,265	-0,265
Sut [MPa]	1770	1770	Sut [MPa]	1770	1770
Ka	0,622	0,622	Ka	0,622	0,622
Kb	0,879	0,879	Kb	0,879	0,879
Se [MPa]	483,318	483,318	Se [MPa]	483,318	483,318
Kt	1,84	1,84	Kt	1,84	1,84
Kts	1,6	1,6	Kts	1,6	1,6
q	0,9	0,9	q	0,9	0,9
qs	0,92	0,92	qs	0,92	0,92
Kf	1,756	1,756	Kf	1,756	1,756
Kfs	1,552	1,552	Kfs	1,552	1,552
Ma [Nm]	140	3003	Ma [Nm]	140	3003
Tm [Nm]	700	700	Tm [Nm]	700	700
d [m]	0,028	0,062	d [m]	0,028	0,062
d [in]	1,089	2,423	d [in]	1,08947129	2,423

Table 38: CounterShaft iterations 2 and 3 of the critical locations.

The first solution, presented in iteration 4 in Table 38, was to change the material to the strongest material found (*ASM Metals Reference Book, 2nd ed, American Society for Metals, Metals Park, Ohio, 1983, p217*): H-11 Ausformed (S_{ut}=2585 MPa). Furthermore, the safety factor was reduced to a value of 1,5 and the concentration factors were carefully adjusted on each iteration. Even though, this wasn't enough too. Therefore, a further step was necessary.

For iteration 6, the safety factor was suppressed (n=1), $\frac{D}{d}$ parameter was reduced to the minimum possible value and the concentration factors were, once again, carefully revised. The material selected was the same as for the previous two iterations.

As it is shown in the last iteration of Table 39, the bore diameter of the wider part of the CounterShaft meets the requirements. However, it is necessary to mention that having a safety factor with a value of just 1 is not the best scenario. Even though, it is not for the entire shaft but only for the bigger diameter zone. If the gearbox study was started again, a different N for gear 5 would have been selected, letting a bigger margin for the bore diameter and the CounterShaft.

D/d		2	D/d		2,4	D/d		2
r/d		0,15	r/d		0,15	r/d		0,1875
Iteration 4			Iteration 5			Iteration 6		
	D	C		D	C		D	C
d [in]	1,25	1,25	d [in]	1,25	1,25	d [in]	1	1
D [in]	2,5	2,5	D [in]	3	3	D [in]	2	2
n	2	1,5	n	2	1,5	n	2	1
a	4,51	4,51	a	4,51	4,51	a	4,51	4,51
b	-0,265	-0,265	b	-0,265	-0,265	b	-0,265	-0,265
Sut [MPa]	2585	2585	Sut [MPa]	2585	2585	Sut [MPa]	2585	2585
Ka	0,562	0,562	Ka	0,562	0,562	Ka	0,562	0,562
Kb	0,325	0,325	Kb	0,879	0,879	Kb	0,879	0,91
Se [MPa]	236,41	236,41	Se [MPa]	638,457	638,457	Se [MPa]	638,696	661,221
Kt	1,4	1,4	Kt	1,5	1,5	Kt	1,3	1,3
Kts	1,35	1,35	Kts	1,45	1,45	Kts	1,2	1,2
q	0,9	0,9	q	0,9	0,9	q	0,9	0,9
qs	0,9	0,9	qs	0,92	0,92	qs	0,9	0,9
Kf	1,36	1,36	Kf	1,45	1,45	Kf	1,27	1,27
Kfs	1,315	1,315	Kfs	1,414	1,414	Kfs	1,18	1,18
Ma [Nm]	140	3003	Ma [Nm]	140	3003	Ma [Nm]	140	3003
Tm [Nm]	700	700	Tm [Nm]	700	700	Tm [Nm]	700	700
d [m]	0,028	0,065	d [m]	0,024	0,048	d [m]	0,022	0,038
d [in]	1,115	2,54	d [in]	0,931	1,882	d [in]	0,884	1,496

Table 39: CounterShaft final study.

As it can be seen in Table 39, differ a bit from the expected values. There is not a material that will let the CounterShaft diameter to be 1 in. Therefore, the smaller diameter (bearings part) is going to be set to a minimum of 0.884 in and the wider part of the shaft diameter is going to be set to 1.5 in.

Once the minimum diameter size has been set, it is necessary to look for the new bearings. The reactions and the Catalog Load Rating (c_{10}) stays unchanged. Therefore, the aim is to look for a bearing with at least 0.884 in bore diameter with an acceptable c_{10} value (greater than 76,265 kN). The model chosen is a SKF bearing *NJ 2306 ECP* (Figure 63, *Figure 70* and Table 55). With c_{10}=83 kN, it has enough margin for the dynamic load rating. The bore diameter is 1.181 in, the closest higher value to the one founded with Equation 49, and the smaller value that could resist the dynamic load.

To sum up with the CounterShaft study, the smaller diameter is going to be set to 1.181 in, so it fits with the bearings, and the bigger diameter is going to be se to to 1.5 in.

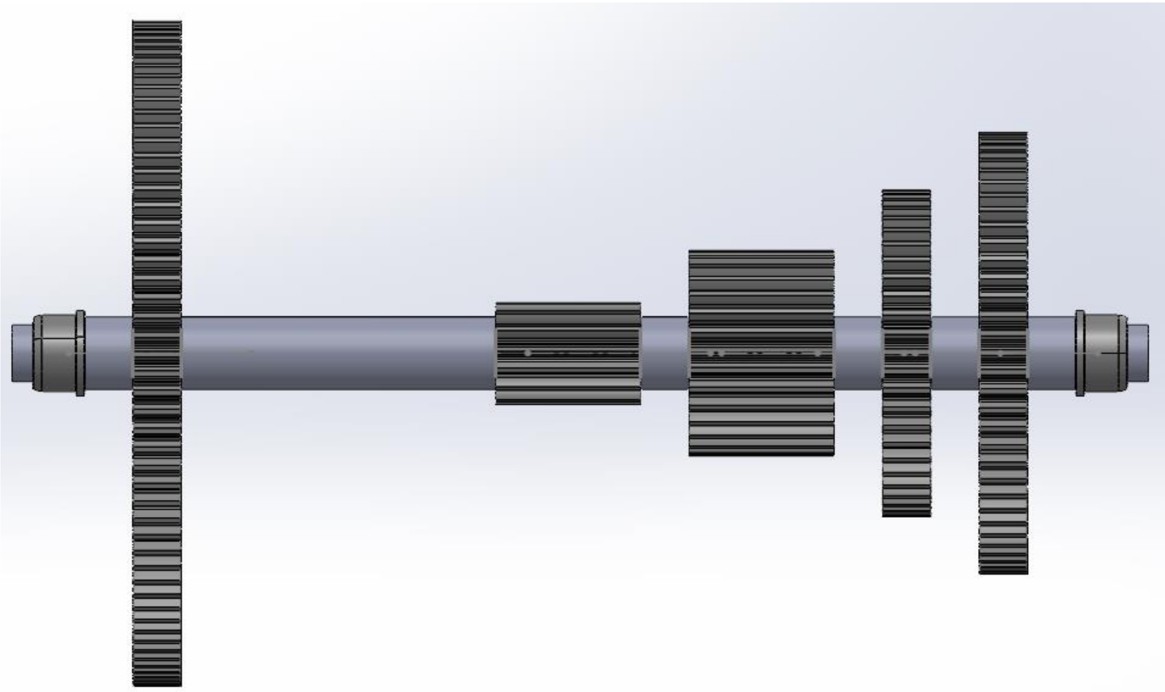


Figure 44: CounterShaft final result.

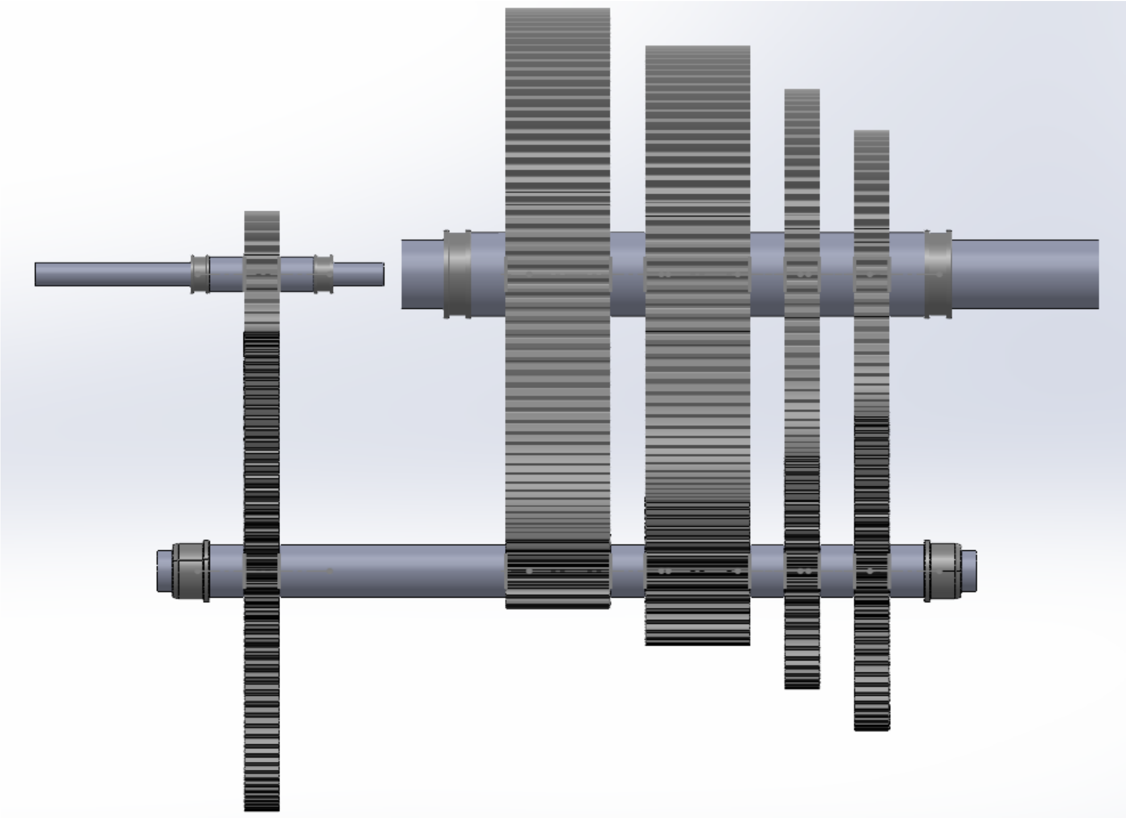


Figure 45: Gearbox result without the case.

Figure 45 shows the gearbox result before designing the case. The final shafts, bearings and gears and their positions can be perfectly seen. The last step would be to design a case that covers the transmission and protects it from being damaged.

Case

As it has just been exposed, the very last step is going to be the case design. There is not any parameter that drives it, and the only concern is the space. Furthermore, it is essential to bear in mind that the gears must not get in contact with the case, and the bearings must fit into the designated spots in the gearbox. Figure 46 shows how this is made for the InShaft and the CounterShaft.

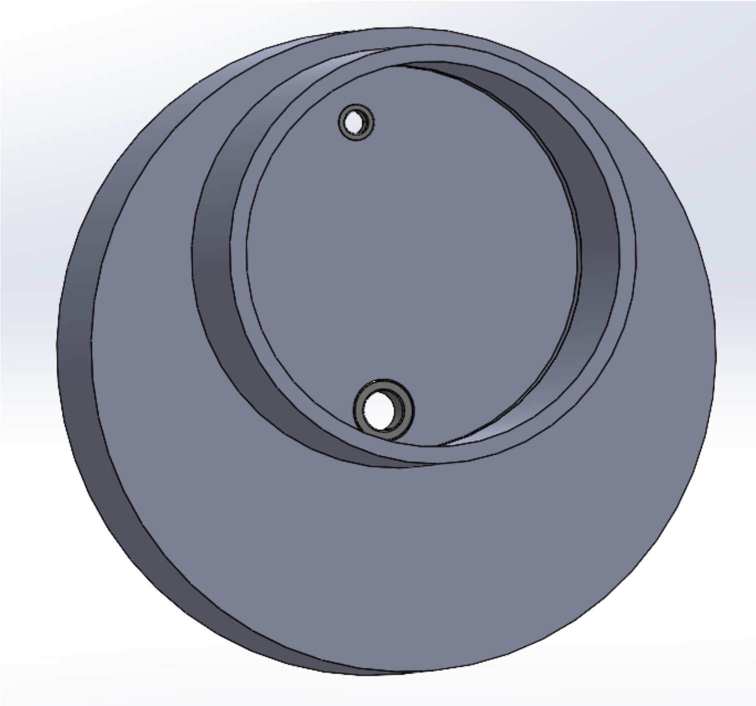


Figure 46: InShaft and CounterShaft bores in the case and the corresponding bearings.

The most important point in the design, is the structure that is going to store the bearings and hold the shafts. Figure 47 shows two bores of 1.05 in a 2.5 in for the bearings and the corresponding shafts. This data is taken from the bearings specifications, and it is found in ANEX I. In addition, Figure 48 shows how the structure perfectly fits not only with the bearings but also with the shafts. More figures are included in ANEX I.

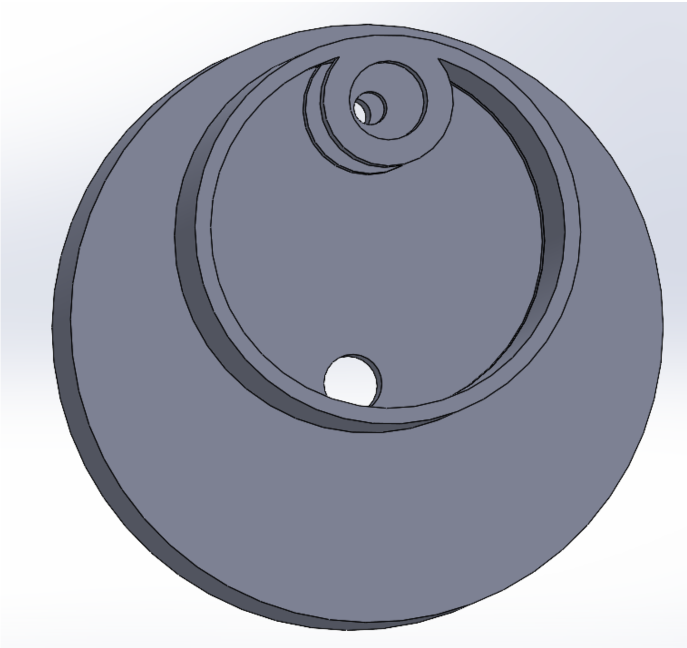


Figure 47: Case structure.

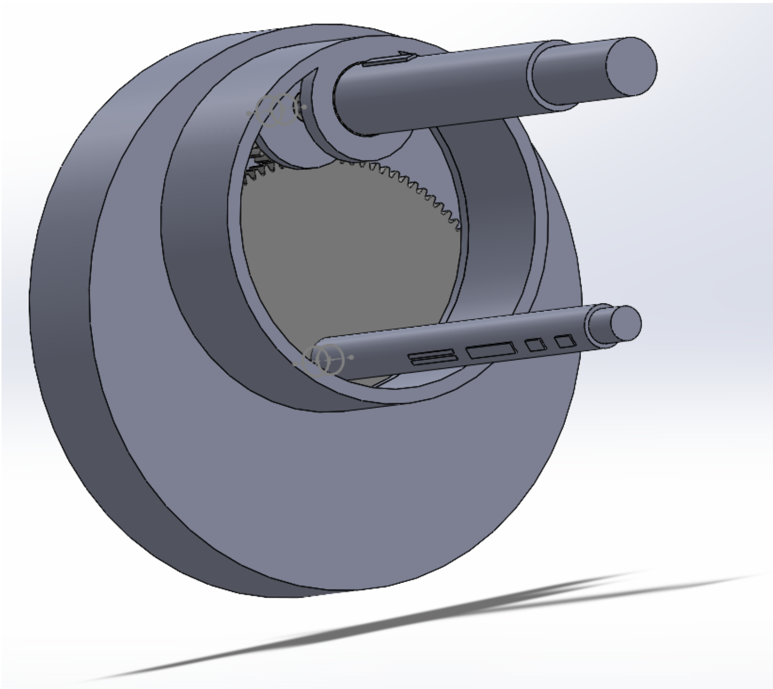


Figure 48: Case structure with shafts, bearings and gears.

Final result is shown in Figure 50. It can be seen how the cover has been cut to the maximum to avoid excessive added weight that might be detrimental in the car performance. Furthermore, the gearbox needs to be as small as possible to fit under the car.

Figure 49 lets the viewer see how the entire assembly is and looks. With a translucent cover it is the best view to see the details. The left and right covers thickness is set by the CounterShaft bearing. While, on the centre structure, two different bores have been designed to fit the two different bearings off the InShaft and the CounterShaft. The entire case is made of a 0.5 in thickness steel

.

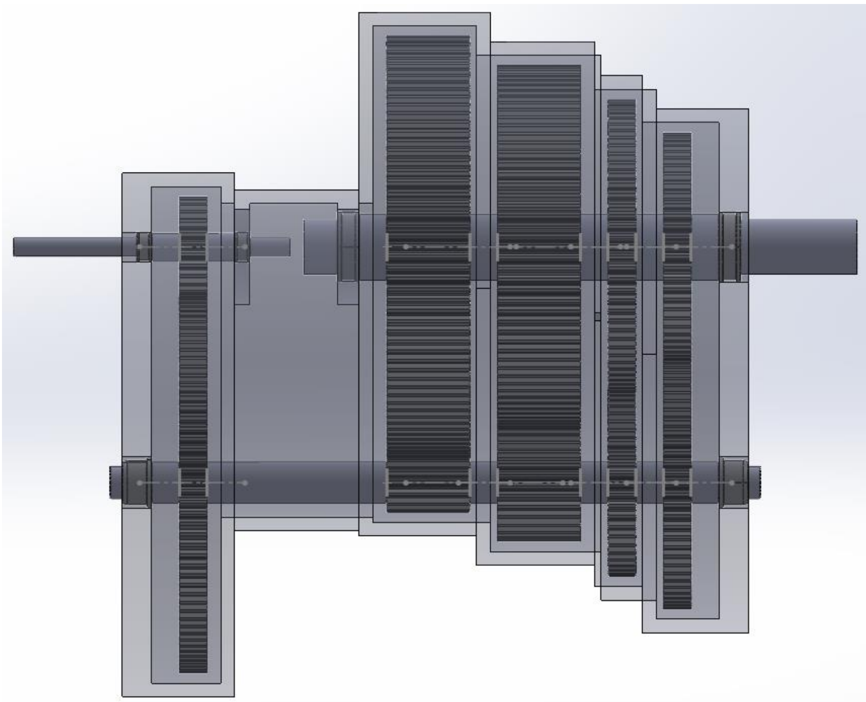


Figure 49: Case side view.

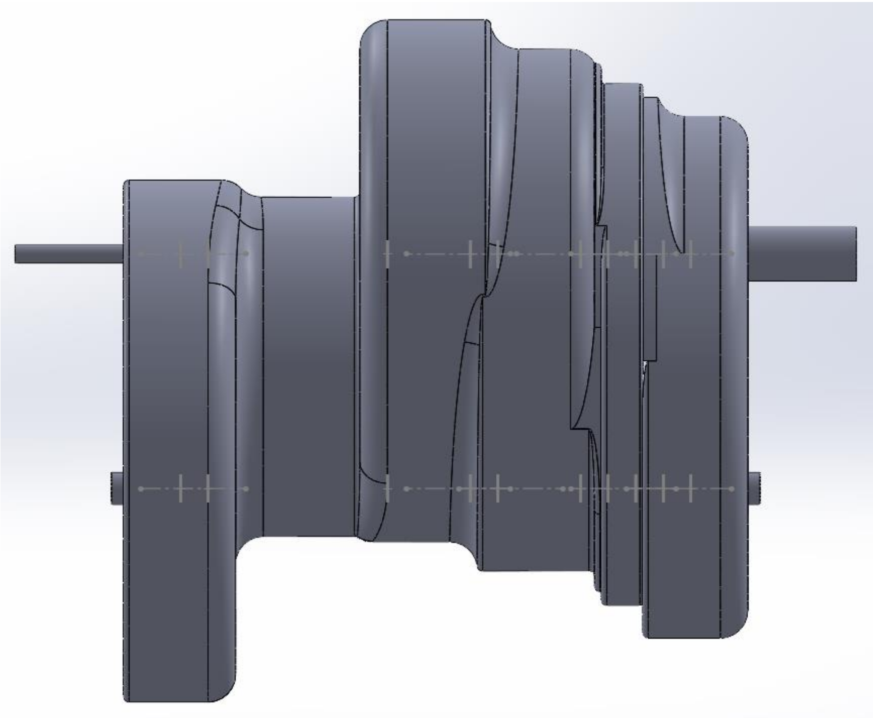


Figure 50: Case final result.

Chapter 5. RESULTS

Once the entire design is finish, it is time to simulate the gearbox with *SolidWorks*. The simulation time is set to 1.5 seconds for all the gears and the different speed ranges with the purpose of avoiding excessive long runs. Once the simulations are finished, the results are going to be compared with the expected values.

GEAR 1

First speed range (15 km/h)

- $\text{Expected value} = 132.5\ \text{rpm}$
- $\text{Mean} = 132.535847\ \text{rpm}$
- $|\text{Error}| = \frac{132.5-132.535847}{132.5} * 100 = 0.027\%$

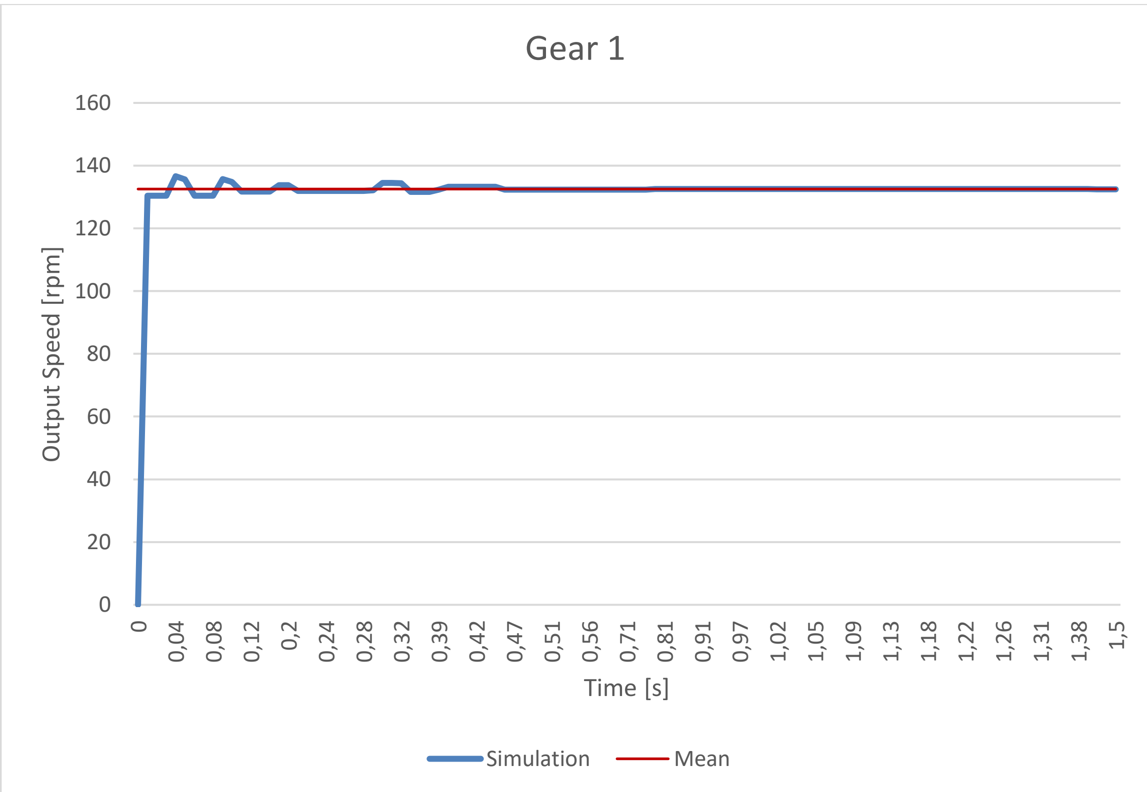


Figure 51: Gear 1, speed range 1, simulation result.

Second speed range (25 km/h)

- Expected value $= \ 220.83$ rpm
- Mean = 220.812812 rpm
- Error $= \frac{220.83-220.812812}{220.83} * 100 = 0.008\%$

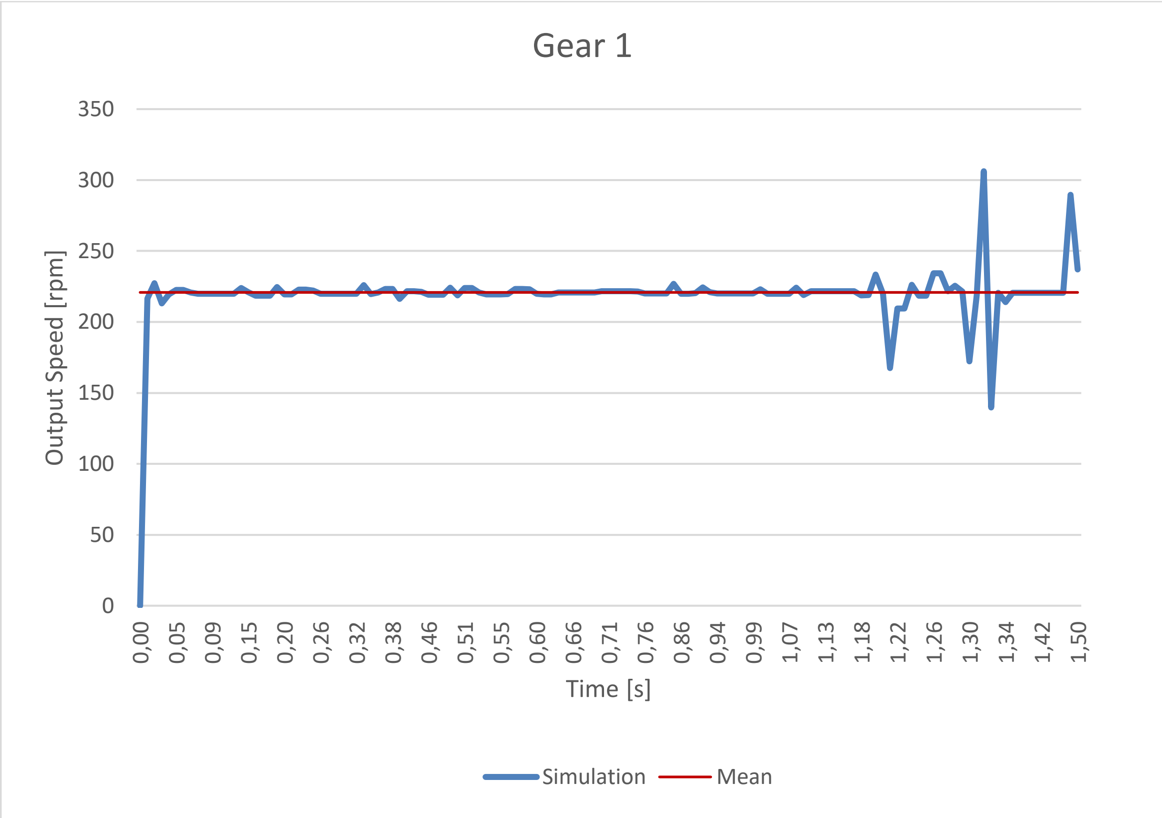


Figure 52: Gear 1, speed range 2, simulation result.

The results are coherent. As Table 40 shows, the differences between the expected values and those obtained by simulation are nearly the same and the errors are small.

Even though, as it happens in Figure 54 and Figure 55, there seem to be some sliding between teeth at the end of the simulation.

GEAR 2

First speed range (45 km/h)

- Expected value = 397.49 rpm
- Mean = 395.228561 rpm
- $|\text{Error}| = \frac{397.49-395.228561}{397.49} * 100 = 0.569\%$

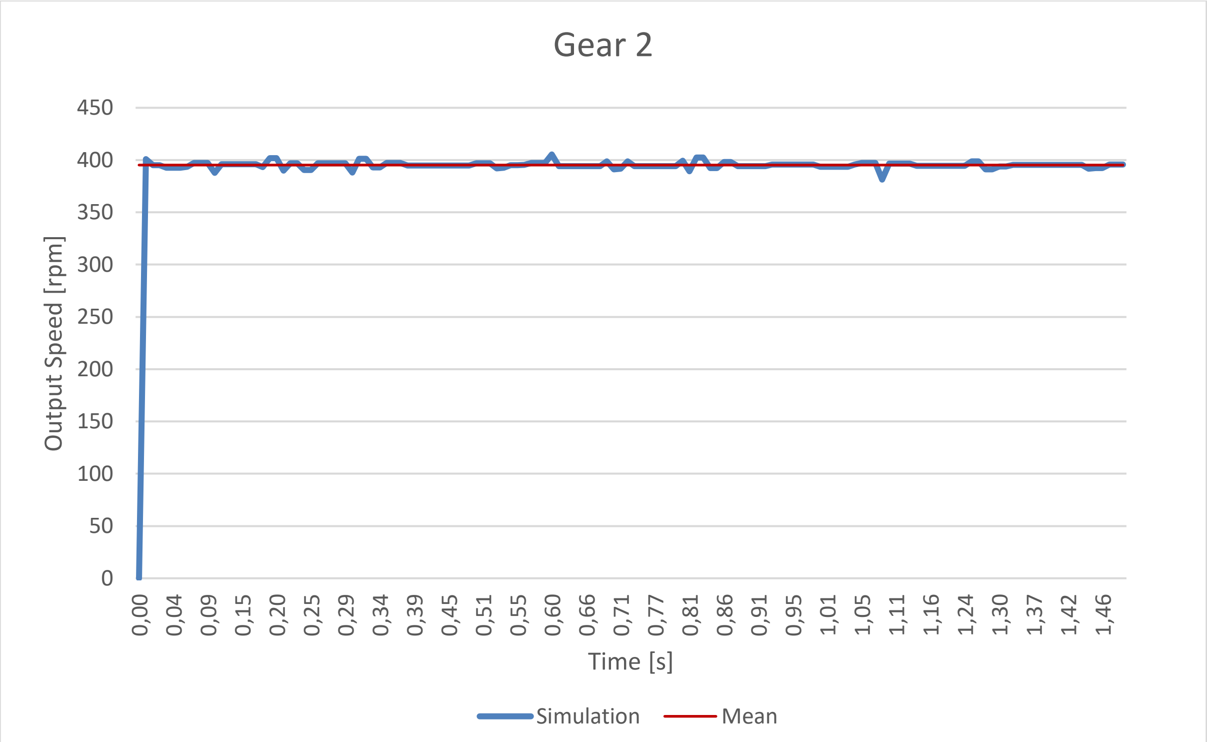


Figure 53: Gear 2, speed range 1, simulation result.

Second speed range (70 km/h)

- Expected value $= 618.32$ rpm
- Mean $= 620.771833$ rpm
- Error $= \frac{618.32-620.771833}{618.32} * 100 = 0.397\%$

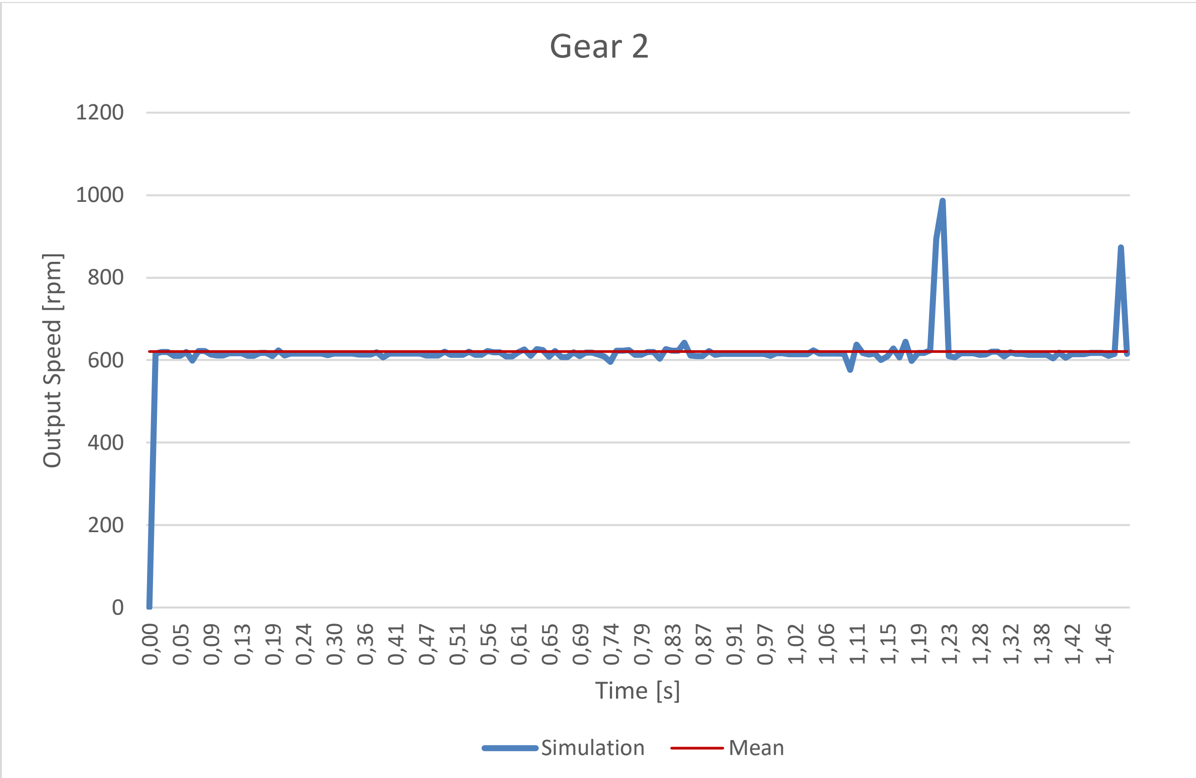


Figure 54: Gear 2, speed range 2, simulation result.

GEAR 3

First speed range (95 km/h)

- Expected value = 839.15 rpm
- Mean = 838.933705 rpm
- $|\text{Error}| = \frac{839.15-838.933705}{839.15} * 100 = 0.026\%$

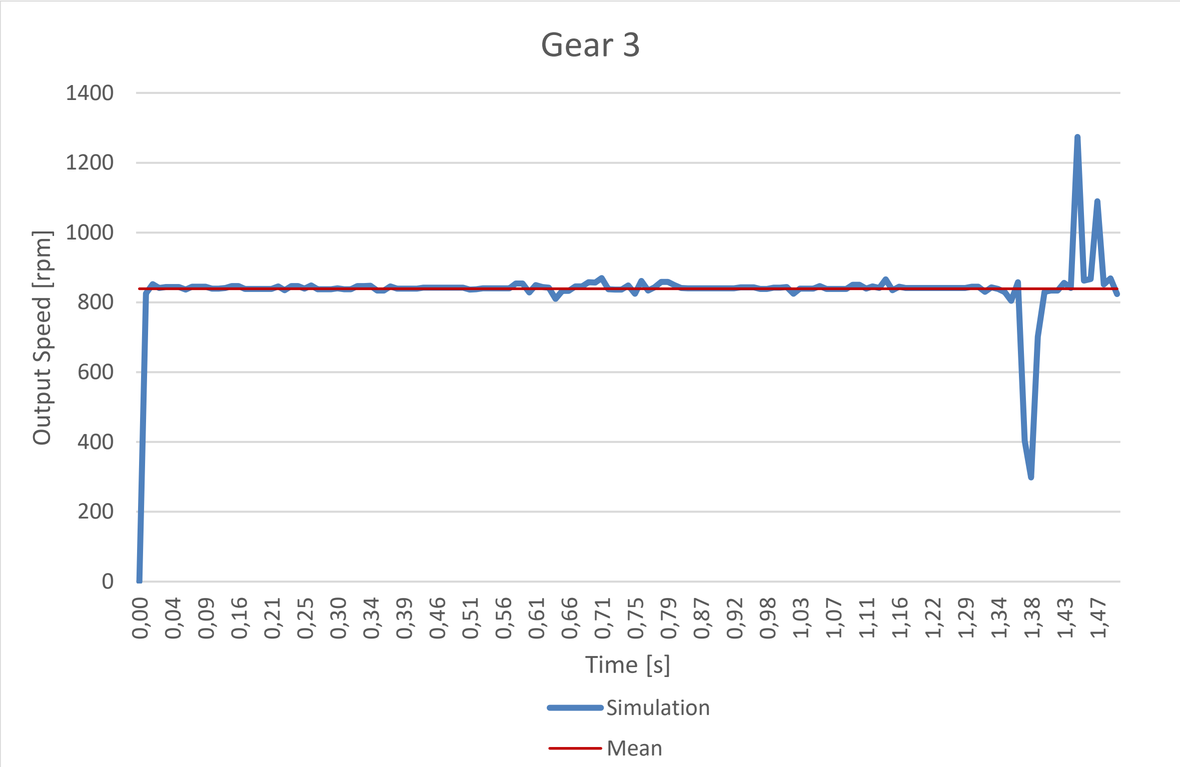


Figure 55: Gear 3, speed range 1, simulation result.

Second speed range (120 km/h)

- Expected value $= 1059.97$ rpm
- Mean $= 1065.65985$ rpm
- $|\text{Error}| = \frac{1059.97 - 1065.65985}{1059.97} * 100 = 0.537\%$

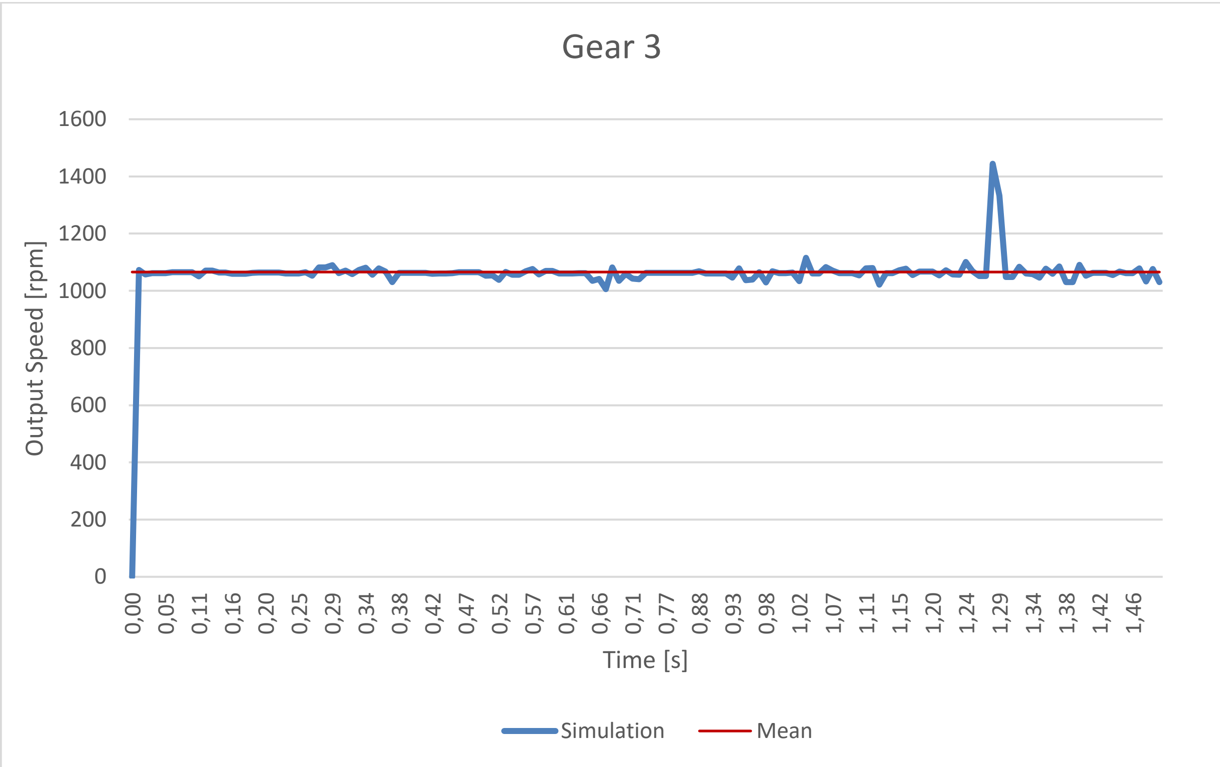


Figure 56: Gear 3, speed range 2, simulation result.

GEAR 4

First speed range (150 km/h)

- Expected value $= 1324.97$ rpm
- Mean $= 1323.34261$ rpm
- $|\text{Error}| = \frac{1324.97 - 1323.34261}{1324.97} * 100 = 0.123\%$

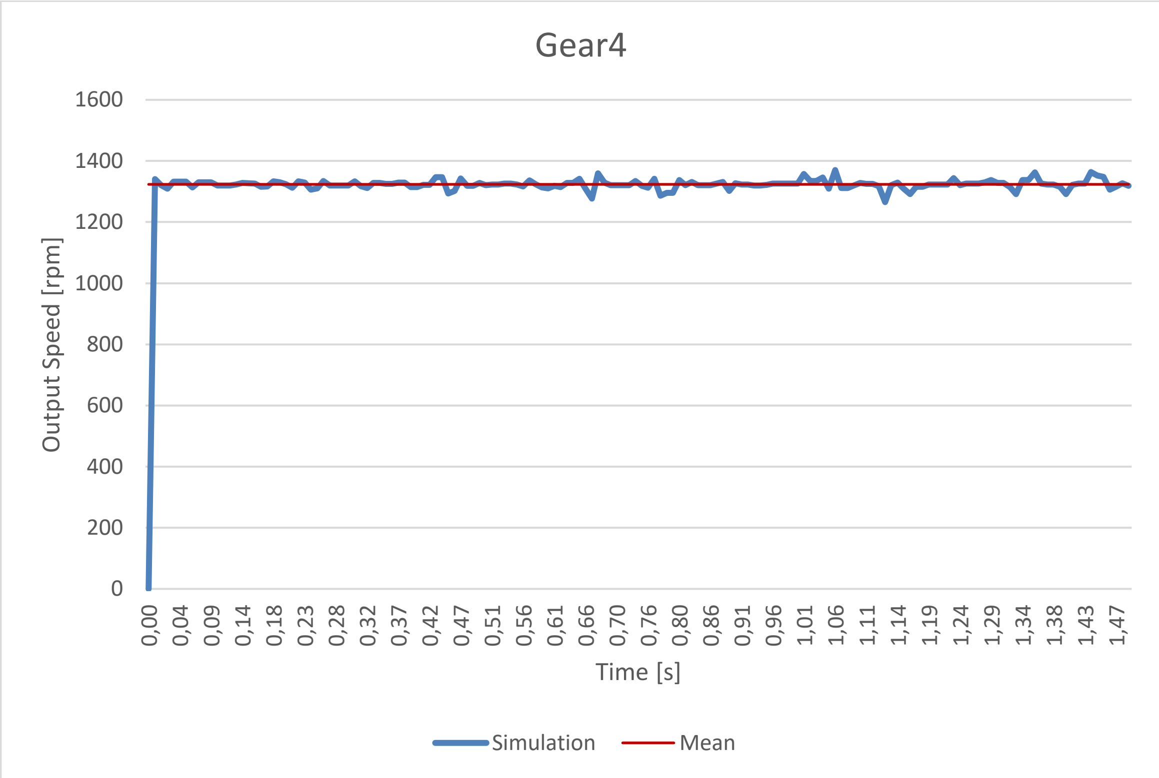


Figure 57: Gear 4, speed range 1, simulation result.

Second speed range (180 km/h)

- Expected value = $1589.96\ rpm$
- Mean = $1586.56584\ rpm$
- Error = $\frac{1589.96-1586.56584}{1589.96} * 100 = 0.213\%$

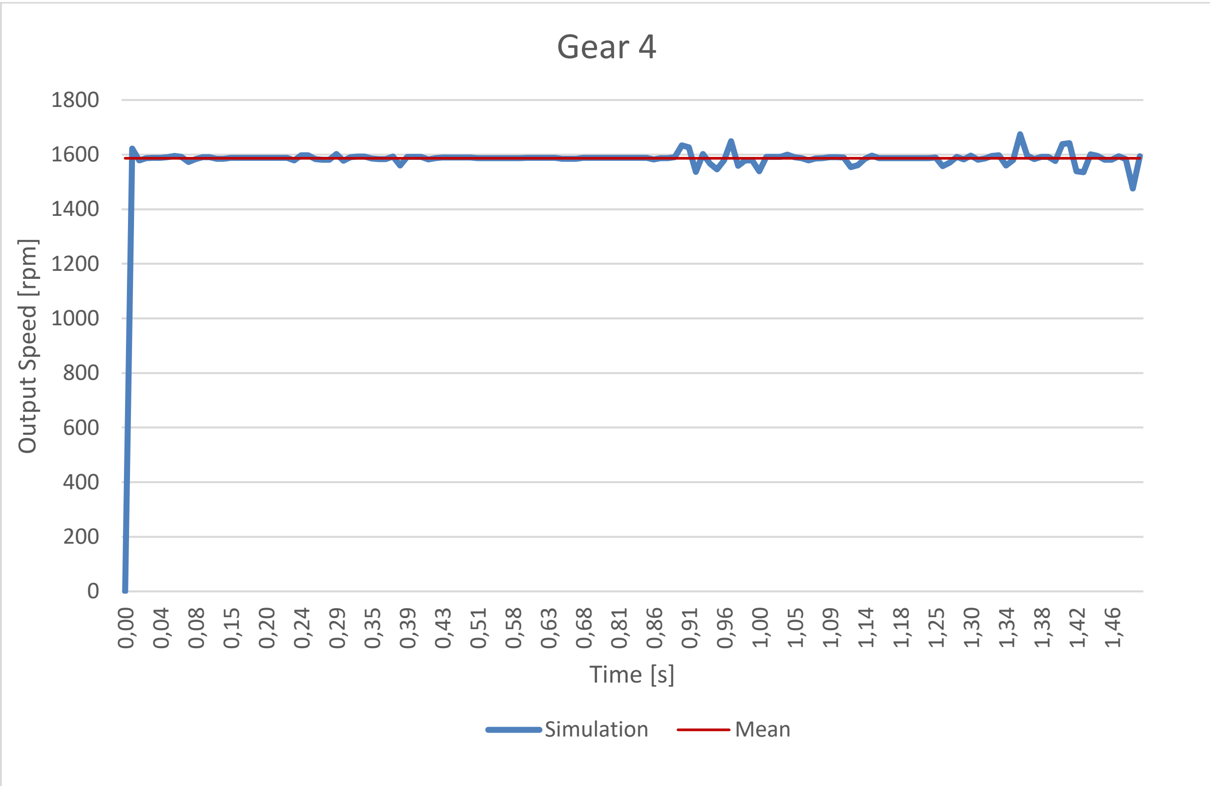


Figure 58: Gear 4, speed range 2, simulation result.

Chapter 6. CONCLUSION AND RESULTS ANALYSIS

The simulation results are collected in Table 40. The error achieved is extremely low, so it can be concluded that the conducted study has worked successfully. The difference between the expected values, or values used for the design, and the result values is negligible.

The number of data values collected on each simulation is also included in Table 40 to show that the numbers achieved have enough credibility. It seems that this value increase as the speed of the car increases. As a result, no accuracy is lost.

Gear	Range	# of data collected	Expected value [rpm]	Mean [rpm]	Error [%]
1	First	105	132.5	132.536	0.027
	Second	131	220.83	220.813	0.008
2	First	144	397.49	395.229	0.569
	Second	160	618.32	620.772	0.397
3	First	149	839.15	838.934	0.026
	Second	155	1059.97	1065.66	0.537
4	First	158	1324.97	1323.343	0.123
	Second	145	1589.96	1586.566	0.213

Table 40: Simulation data collected.

Furthermore, Table 41 includes the acceleration achieved by the developed model and the Ford Mustang acceleration that was used as a reference at the beginning of the project. The objective was not to reach the same acceleration, as the Volkswagen Polo is extremely less powerful than the Mustang. It was used in order to get a sense of how good the results are and how much this model can improve the Volkswagen performance.

Gear	$a_{Mustang}$	$a_{Achieved}$
1	1.26g	0.97g
2	0.82g	0.355g
3	0.55g	0.145g
4	0.45g	0.045g

Table 41: Acceleration achieved compared to the Ford Mustang acceleration.

The developed model gets from 0-120 $\frac{\mathrm{km}}{\mathrm{h}}$ in just 14 seconds. It is important to remember that this result comes from the worst possible conditions. Therefore, a slope of a 5%, which is the highest possible value in the highway as Table 9 reflects, was included. The base model, which is the start point of this project, takes 14 seconds in the 0-120 $\frac{\mathrm{km}}{\mathrm{h}}$ in a flat surface. This means that the original model spends more time under the same conditions and, therefore, the performance of the car has been improved. As a result, it can be concluded that the objective of the project in terms of performance has been achieved.

To sum up with, not only a fastest car has been developed, which was the main objective. As it was exposed in ***¡Error! No se encuentra el origen de la referencia.***, its power source has plenty of benefits due to the increasing number of restrictions for combustion cars. Therefore, after the entire evolution of the project, developing an electrified Volkswagen Polo with high performance results would be an incredible market opportunity for the German brand.

In adition, the development of this project produces the following conclusions of four of the Sustainable Development Goals (ODS):

3. *Health and well-being.*

Cities are responsible of 75% of the entire carbon emissions production and around 70% of the entire energy consumption with today population, which is around 50% of the total Earth population. Furthermore, the urban population is expected to grow up to reach 60% of the entire Earth population by 2030.

Considering that according to the WHO there are 4.2 million deaths due to atmospheric contamination nowadays, if the growth of cities is not measured, it can result in devastating health problems. The application of action is already going on (Euro 7 or Low Emissions Zones).

Even though these restrictions are nowadays a reality, there is a need of technology development and implementation. Electric cars will not only be acceptable by law, but also be benefit for citizens health with its 0 emissions engine. As a result, and despite the cities population growth, the carbon emission production will decrease and therefore health problems and diseases due to atmospheric contamination decrease. On the other hand, the energy consumption will still be high, even increase, but it can be obtained from cleaner sources and not from fossil fuel resources.

7. *Affordable and non-contaminant energy.*

Energy is the key factor of climate change, as it represents 60% of all the worldwide emission of greenhouse gases. In 2012 EIA (Energy Information Administration) reported that only 13.6% of the energy production comes from total non-fossil fuel resources: 7% nuclear, 5% hydro, 0.5 % biomass and waste, and 1.1 % from other sources as solar, wave, wind or geothermal.

Therefore, finding cleaner ways of work production is a lead objective factor for all the industries. Particularly in the automotive industry, in which there have been great advances in the recent years.

In this line, the contribution of this project is to convert an already existing combustion version to a cleaner source. The energy required to recharge the batteries can be obtained from clean renewable sources as the solar or wind mentioned before. This is something already going on, but this project goes further. It presents the idea of implementing developed technology solutions to existing bestsellers.

11. *Sustainable cities and communities.*

The development of a small electric car will not only contribute to the citizens health and the contamination reduction, but also to the city's development and organization.
In the past recent years, cars have been experimenting a clear growth in size as shown in Table 42. Because urban population is expected to suffer a big increment, as explained before, there seem to be a space conflict coming.

Version	Length [m]
2021	4.05
2018	4.05
2014	3.97
2009	3.97
2005	3.92
2002	3.89

Table 42:Volkswagen Polo measures for the different versions.

Therefore, having an electric version of a car that still conserves restrained measures, looks like a good approach to the issue.

13. *Action for the environment.*

As explained in factors 3 and 7, bearing in mind the actual health situation and the energy resources consumptions, the contribution to the health and environment of an electrified car are unquestionable. As unquestionable as the climate emergency and its urgency.

The sector with the highest percentage of greenhouse gas emissions is the transportation sector. As shown in Figure 59 it is responsible of nearly 1/3 of the total emission in the U.S.

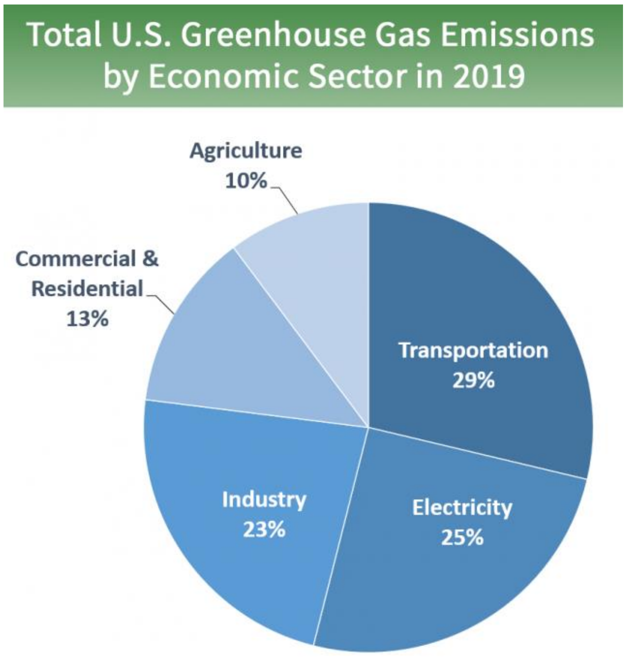


Figure 59: Greenhouse Gas Emissions by Economic Sector in the U.S in 2019.

The development of cleaner and more efficient engines, as well as other mechanical components as transmissions, is essential. This project is focused on the private and personal vehicle, but the conclusions extracted from this project, as well as the process follow, could be extrapolated to lorries, buses, or vans, which are higher originators of the harmful gases.

Starting from cars, the study and work done can be implemented in any way of transportation, from public transportation to freight transportation. Altogether, as a sum, would considerably reduce the emission of greenhouse gases to the atmosphere.

Chapter 7. BIBLIOGRAPHY

Manual gear transmission:
Lesics (4 Mar 2015) Manual Transmission, How it works?
https://www.youtube.com/watch?v=wCu9W9xNwtI

Introduction

1. Measures:
   Volkswagen Polo:
   Km77 // (23 Oct 2012) Volkswagen Polo (2009)
   https://www.km77.com/coches/volkswagen/polo/2009/3-puertas/informacion
   Electric Vehicle Database:
   https://ev-database.org/car/1127/Volkswagen-ID3-Pure-Performance
   https://ev-database.org/car/1273/Volkswagen-ID4-1st
   https://ev-database.org/car/1494/Volkswagen-ID4-GTX
   https://ev-database.org/car/1558/Volkswagen-ID5-GTX
2. Volkswagen Polo Combustion version sales in the market:
   Volkswagen sales:
   Carlier, M. (4 Aug 2021) Worldwide deliveries of Volkswagen Group's most popular car model between 2017 and 2020.
   https://www.statista.com/statistics/461115/most-popular-models-of-volkswagen-group-by-worldwide-delivery/
   Volkswagen Polo versions along the years:
   Km77: Volkswagen.
   https://www.km77.com/coches/volkswagen?market[]=available&market[]=discontinued
3. Clean
   Emisions restriction:
   The AA (11 Dic 2017) Limits to improve air quality and health.
   https://www.theaa.com/driving-advice/fuels-environment/euro-emissions-standards
   Euro 7:
   European Commission. European vehicle emissions standards – Euro 7 for cars, vans, lorries and buses.
   https://ec.europa.eu/info/law/better-regulation/have-your-say/initiatives/12313-European-vehicle-emissions-standards-Euro-7-for-cars-vans-lorries-and-buses_en

Cities with emissions restrictions:
Renault Group (25 March 2020) Renault Group: Low Emission Zones (LEZs) in Europe.
https://www.renaultgroup.com/en/news-on-air/news/low-emission-zones-lezs-in-europe/

4. Efficiency:
   Renault Group (23 Feb 2021). Renault Group: Understanding the power output of an electric motor.
   https://www.renaultgroup.com/en/news-on-air/news/understanding-the-power-output-of-an-electric-motor/
5. Electric cars output torque:
   Engineering Explained (19 Jul 2017). How Do Electric Cars Produce Instant Maximum Torque? Youtube.
   https://www.youtube.com/watch?v=VwFEyL_JJAQ
   GTE vs.GTI:
   Km77// (22 Jan 2021) Volkswagen Golf GTI (2020).
   https://www.km77.com/coches/volkswagen/golf/2020/5-puertas/gti/golf-gti/datos
   Km77//(10 Jun 2021) Volkswagen Golf GTE y Golf eHybrid (2020).
   https://www.km77.com/coches/volkswagen/golf/2020/5-puertas/gte/golf-5p-gte/datos

State of art

1. MEB platform:
   Volkswagen España (2 Dec 2020). Para todos los públicos 1x02 – La plataforma MEB, los cimientos de un eléctrico. Retrieved from
   https://www.youtube.com/watch?v=H2Wnw4vyz-k
   Volkswagen. Modular electric drive matrix (MEB). Retrieved from
   https://www.volkswagen-newsroom.com/en/modular-electric-drive-matrix-meb-3677
2. Batteries:
   Volkswagen España (7 Dec 2020). Para todos los públicos 1x03 – El corazón de los coches eléctricos: la batería. Youtube.
   https://www.youtube.com/watch?v=YJsggQH0yag&list=PLX86Ukgkvm9OmjHoGFU-XSknnUV2l8QtJ&index=3

3. Engine:
Volkswagen España (13 Jan 2021). Para todos los públicos 1x04 – Ventajas del motor eléctrico. Youtube.
https://www.youtube.com/watch?v=0A2rf9dmlnU&list=PLX86Ukgkvm9OmjHoGFU-XSknnUV2l8QtJ&index=4

Work definition
Methodology:
Procedure:
Engineering Explained (29 Jan 2020). Did Ford Ruin Their Manual Transmission? Youtube.
https://www.youtube.com/watch?v=Y25ordkIfgM
How does an electric motor work:
Volkswagen España (13 Jan 2021). Para todos los públicos 1x04 – Ventajas del motor eléctrico. Youtube.
https://www.youtube.com/watch?v=0A2rf9dmlnU&list=PLX86Ukgkvm9OmjHoGFU-XSknnUV2l8QtJ&index=4

Gearbox Development
System Analysis
1. Car Weight:
Wiki Automobile Catalog (n.d) 2009 Volkswagen Polo V 1.4 (85 Hp) DSG Specs
https://www.auto-data.net/en/volkswagen-polo-v-1.4-85hp-dsg-16804
Ventos (n.d) ¿Cuánto pesa el motor de un automóvil? (Motores pequeños o grandes)
https://www.ventos.site/como/cuanto-pesa-el-motor-de-un-automovil-motores-pequenos-o-grandes/#:~:text=El%20tama%C3%B1o%20del%20motor%20es,pesar%20m%C3%A1s%20de%20700%20libras.
GTE:
Km77 (n.d) Volkswagen Golf GTE 1.4 TSI 180 kW (245 CV) DSG 6 vel. (2020) | Precio y ficha técnica
https://www.km77.com/coches/volkswagen/golf/2020/5-puertas/gte/golf-5p-gte/datos
Golf mk8:
Wiki Automobile Catalog (n.d) 2020 Volkswagen Golf VIII 2.0 TDI (150 Hp)
https://www.auto-data.net/en/volkswagen-golf-viii-2.0-tdi-150hp-43584

2. Electric engine:
   Engine:
   phi power ag (2016) Innovation in axial flux motor design and manufacturing
   https://www.phi-power.com/en/home/
   Electric motors:
   Engineering Explained (24 Jul 2013) Electric Motor
   https://www.youtube.com/watch?v=ErV5lGVso1w&list=PL2ir4svMoaYj48N0VWoic25P9LaU2wlbA
   Engineering Explained (25 Sept 2013) Electric Car
   https://www.youtube.com/watch?v=MWcmUgKP1GU&list=PL2ir4svMoaYj48N0VWoic25P9LaU2wlbA&index=5
   Engineering Explained (30 Agu 2017) Could Electric Cars Have A Manual Transmission?
   https://www.youtube.com/watch?v=42MVi6Fel-E
   Combustion motor specs:
   Km77// (n.d) Volkswagen Polo 5p Advance 1.4 85 CV (2010-2012) | Precio y ficha técnica
   https://www.km77.com/coches/volkswagen/polo/2009/5-puertas/advance/polo-5p-advance-14-85-cv2/datos

3. Working conditions:
   Slope:
   Orden FOM/273/2016, Boletín Oficial del Estado de 19 de febrero, por la que se aprueba la Norma 3.1- IC Trazado, de la Instrucción de Carreteras.
   BOE Núm. 55 Viernes 4 de marzo de 2016 Sec. I. Págs. 17714 -17723.
   http://www.carreteros.org/normativa/trazado/31ic_2016/pdfs/5.pdf
   Drag:
   The Efficient Engineer (2020) Understanding Aerodynamic Drag. Youtube.
   https://www.youtube.com/watch?v=GMmNKUlXXDs
   MIT Blossoms. Math & Science Vídeo Lessons. Drag force handout [video]
   https://blossoms.mit.edu/sites/default/files/video/download/Drag-force-Handout.pdf
   Hall, N. (13 May 2021) The Drag Coeficient. National Aeronautics and Space Administration (NASA). Glen Reseach Center.
   https://www.grc.nasa.gov/www/k-12/airplane/dragco.html
   Elert, G. (1998) Aerodynamic Drag. The Physics Hypertextbook
   https://physics.info/drag/

Will, V. (21 Nov 2019) What Is the Density of Air at Standard Temperature and Pressure (STP). MAC Instruments.
https://macinstruments.com/blog/what-is-the-density-of-air-at-stp/

Km77 (2009) Volkswagen Polo 1.4 85 CV (2010-2012) Ficha técnica.
https://www.km77.com/coches/volkswagen/polo/2009/5-puertas/advance/polo-5p-advance-14-85-cv2/datos

Rolling resistance:

Learn Engineering. (2021) Understanding Rolling Resistance. Youtube
https://www.youtube.com/watch?v=_S2lyaMgBQ8

StaticProf (6 Oct 2016) Rolling vs Sliding Friction. Demonstration showing rolling friction calculations vs sliding friction calculations. Youtube.
https://www.youtube.com/watch?v=qo9NO88KTMo

Lesics (2021) The importance of tire slip. Youtube.
https://www.youtube.com/watch?v=tsnYqCRWTbE

The Tire Digest (2020) Ultimate energy tire
https://thetiredigest.michelin.com/michelin-ultimate-energy-tire

Skill-Lync (26 Apr 2020) What is Rolling Resistance. Youtube
https://www.youtube.com/watch?v=PwdG9bFsJyA

Engineering ToolBox, (2008). *Rolling Resistance*. [online]
https://www.engineeringtoolbox.com/rolling-friction-resistance-d_1303.html

Friction:

HP Wizard (n.d) Tire friction and rolling resistance coefficients
http://hpwizard.com/tire-friction-coefficient.html

Jones and Childers (2002) The Friction of Automobile Tires. *Contemporary College Physics*, University of South Carolina
http://boson.physics.sc.edu/~rjones/phys101/tirefriction.html#:~:text=A%20patterned%20tire%20gives%20typical,obtained%20with%20a%20smooth%20tire.

Overtakes:

Ministerio de Transportes, movilidad y agenda urbana ( ) Transporte Terrestre Marco de referencia del sector. Inspección y seguridad en el transporte. Pesos y dimensiones
https://www.mitma.gob.es/transporte-terrestre/inspeccion-y-seguridad-en-el-transporte/pesos-y-dimensiones/longitudes/longitudes-mercancias/esquema-longitud-vehiculos-rigidos

Other polo´s acceleration:

KM77 (2009) Volkswagen Polo 5 puertas. Mediciones.
https://www.km77.com/coches/volkswagen/polo/2009/5-puertas/mediciones-propias/detalle

4. Car specs

Tires:

Llantasneumaticos.com (2022) Volkswagen Polo 2009 Guía con toda la información para la relación Neumático-Llanta
https://llantasneumaticos.com/size/volkswagen/polo/2009/#trim-14-mpi-eudm-84

Goodyear Auto Service (2022) How to read a tire size
https://www.goodyearautoservice.com/en-US/learn/tire-size

Google Search (2022) Wolksvagen golf tire
https://www.google.com/search?q=volkswagen+golf+tire&rlz=1C1CHBD_esES813ES813&sxsrf=APq-WBt8ghKp6oIfFk85XJjBS3EbFy9KbA:1645393342767&source=lnms&tbm=isch&sa=X&ved=2ahUKEwiZ_Nftn4_2AhX8RTABHaRGB9YQ_AUoAXoECAIQAw&biw=1536&bih=754&dpr=1.25#imgrc=-2eoosFONziksM

Design

1. Statements:

SAE International (2022) Developing the Gear Ratio Formula
https://www.sae.org/binaries/content/assets/cm/content/learn/education/motortoycar-samplelessonplan.pdf

J.M.K.C. Donev et al. (2018). Energy Education - Mechanical advantage Available: https://energyeducation.ca/encyclopedia/Mechanical_advantage.
https://energyeducation.ca/encyclopedia/Mechanical_advantage

Redacción (5 Marc 2017) Así se fabrica una caja de cambios. *La Vanguardia*.
https://www.lavanguardia.com/motor/actualidad/20170305/42562053801/caja-de-cambios-seat.html

2. Assumptions:

Richard G. Budynas and J. Keith Nisbett (2011). Shigley's Mechanical Engineering Design, Ninth Edition.

Development

Gears:
https://www.rushgears.com/

Bearings:
https://www.skf.com/es

Conclusion

1. Sustainable Development Goals:

Good health and well-being:

United Nations. Goal 3: Ensure healthy lives and promote well-being for all at all ages.
https://www.un.org/sustainabledevelopment/es/cities/

Affordable and clean energy:

United Nations. Ensure access to affordable, reliable, sustainable and modern energy.
https://www.un.org/sustainabledevelopment/es/energy/

U.S. Energy Information Administration (EIA).
https://www.eia.gov/

Sustainable cities and communities:

United Nations. Goal 11: Make cities inclusive, safe, resilient and sustainable. Retrieved from
https://www.un.org/sustainabledevelopment/cities/

2021 model:

Km77// 29 Jun 2021). Volkswagen Polo (2021).
https://www.km77.com/coches/volkswagen/polo/2021/estandar/informacion

2018 model:

Km77// (17 Jan 2018). Volkswagen Polo (2018).
https://www.km77.com/coches/volkswagen/polo/2018/5-puertas

2014 model:

Km77// (12 Apr 2017). Volkswagen Polo (2014).
https://www.km77.com/coches/volkswagen/polo/2014/5-puertas

2009 model:

Km77// (23 Oct 2012). Volkswagen Polo (2009).
https://www.km77.com/coches/volkswagen/polo/2009/5-puertas

2005 model:

Km77// (19 Jun 2007). Volkswagen Polo (2007).
https://www.km77.com/coches/volkswagen/polo/2005/5-puertas

2002 model:

Fernández, V.M. (30 Oct 2001). Volkswagen Polo (2002).
https://www.km77.com/coches/volkswagen/polo/2002/5-puertas

Climate action:

United Nations. Goal 13: Take urgent action to combat climate change and its impacts.
https://www.un.org/sustainabledevelopment/es/climate-change-2/

United States Environmental Protection Agency.
https://www.epa.gov/climatechange-science/causes-climate-change

United States Environmental Protection Agency. Greenhouse Gas Emissions: Sources of Greenhouse Gas Emissions. Retrieved from https://www.epa.gov/ghgemissions/sources-greenhouse-gas-emissions#:~:text=The%20largest%20source%20of%20greenhouse,electricity%2C%20heat%2C%20and%20transportation.&text=Electricity%20production%20(25%20percent%20of,share%20of%20greenhouse%20gas%20emissions.

ANEX I

Drive force equation development for each gear and speed range

GEAR 1

Speed range 1:

$$drive\ force = m\ a + m\ g\ sin\alpha + \mathrm{c\ m\ g\ cos}\alpha + \frac{1}{2}\ cd\ \rho\ A\ v^2$$

$$drive\ force = 1345 * 12 + 1345 * 9.81 * \sin(2.862) + 0.01 * 1345 * 9.81 * \cos(2.862) + \frac{1}{2} * 0.3 * 1.225 * 2.459 * 4.17^2$$

$$drive\ force = 16543.599\ N$$

Speed range 2:

$$drive\ force = m\ a + m\ g\ sin\alpha + \mathrm{c\ m\ g\ cos}\alpha + \frac{1}{2}\ cd\ \rho\ A\ v^2$$

$$drive\ force = 1345 * 7.1 + 1345 * 9.81 * \sin(2.862) + 0.01 * 1345 * 9.81 * \cos(2.862) + \frac{1}{2} * 0.3 * 1.225 * 2.459 * 6.94^2$$

$$drive\ force = 9967.045\ N$$

GEAR 2

Shift change:

$$drive\ force = m\ a + m\ g\ sin\alpha + \mathrm{c\ m\ g\ cos}\alpha + \frac{1}{2}\ cd\ \rho\ A\ v^2$$

$$drive\ force = 1345 * 4.5 + 1345 * 9.81 * \sin(2.862) + 0.01 * 1345 * 9.81 * \cos(2.862) + \frac{1}{2} * 0.3 * 1.225 * 2.459 * 6.94^2$$

$$drive\ force = 6470.045\ N$$

Speed range 1:

$$drive\ force = m\ a + m\ g\ sin\alpha + \mathrm{c\ m\ g\ cos\alpha} + \frac{1}{2}\ cd\ \rho\ A\ v^2$$

$$drive\ force = 1345 * 3.75 + 1345 * 9.81 * \sin(2.862) + 0.01 * 1345 * 9.81 * \cos(2.862) + \frac{1}{2} * 0.3 * 1.225 * 2.459 * 12.5^2$$

$$drive\ force = 5510.107\ N$$

Speed range 2:

$$drive\ force = m\ a + m\ g\ sin\alpha + \mathrm{c\ m\ g\ cos\alpha} + \frac{1}{2}\ cd\ \rho\ A\ v^2$$

$$drive\ force = 1345 * 2.2 + 1345 * 9.81 * \sin(2.862) + 0.01 * 1345 * 9.81 * \cos(2.862) + \frac{1}{2} * 0.3 * 1.225 * 2.459 * 19.44^2$$

$$drive\ force = 3525.595\ N$$

GEAR 3

Shift change:

$$drive\ force = m\ a + m\ g\ sin\alpha + \mathrm{c\ m\ g\ cos\alpha} + \frac{1}{2}\ cd\ \rho\ A\ v^2$$

$$drive\ force = 1345 * 2 + 1345 * 9.81 * \sin(2.862) + 0.01 * 1345 * 9.81 * \cos(2.862) + \frac{1}{2} * 0.3 * 1.225 * 2.459 * 19.44^2$$

$$drive\ force = 3256.595\ N$$

Speed range 1:

$$drive\ force = m\ a + m\ g\ sin\alpha + \mathrm{c\ m\ g\ cos\alpha} + \frac{1}{2}\ cd\ \rho\ A\ v^2$$

$$drive\ force = 1345 * 1.4 + 1345 * 9.81 * \sin(2.862) + 0.01 * 1345 * 9.81 * \cos(2.862) + \frac{1}{2} * 0.3 * 1.225 * 2.459 * 26.39^2$$

$$drive\ force = 2593.415\ N$$

Speed range 2:

$$drive\ force = m\ a + m\ g\ sin\alpha + \mathrm{c\ m\ g\ cos\alpha} + \frac{1}{2}\ cd\ \rho\ A\ v^2$$

$$drive\ force = 1345 * 0.86 + 1345 * 9.81 * \sin(2.862) + 0.01 * 1345 * 9.81 * \cos(2.862) + \frac{1}{2} * 0.3 * 1.225 * 2.459 * 33.33^2$$

$$drive\ force = 2054.517\ N$$

GEAR 4

Shift change:

$$drive\ force = m\ a + m\ g\ sin\alpha + \mathrm{c\ m\ g\ cos\alpha} + \frac{1}{2}\ cd\ \rho\ A\ v^2$$

$$drive\ force = 1345 * 0.7 + 1345 * 9.81 * \sin(2.862) + 0.01 * 1345 * 9.81 * \cos(2.862) + \frac{1}{2} * 0.3 * 1.225 * 2.459 * 33.33^2$$

$$drive\ force = 1839.317\ N$$

Speed range 1:

$$drive\ force = m\ a + m\ g\ sin\alpha + \mathrm{c\ m\ g\ cos\alpha} + \frac{1}{2}\ cd\ \rho\ A\ v^2$$

$$drive\ force = 1345 * 0.54 + 1345 * 9.81 * \sin(2.862) + 0.01 * 1345 * 9.81 * \cos(2.862) + \frac{1}{2} * 0.3 * 1.225 * 2.459 * 41.67^2$$

$$drive\ force = 1642.718\ N$$

Speed range 2:

$$drive\ force = m\ a + m\ g\ sin\alpha + \mathrm{c\ m\ g\ cos\alpha} + \frac{1}{2}\ cd\ \rho\ A\ v^2$$

$$drive\ force = 1345 * 0.085 + 0 + 0.01 * 1345 * 9.81 + \frac{1}{2} * 0.3 * 1.225 * 2.459 * 50^2$$

$$drive\ force = 1375.911\ N$$

Teeth numbers:

GEAR 1:

Equation 13: $G = \frac{N9 * N1}{N10 * N5} = 8 * 4$

- $N1 = 8 * N5$
- $N9 = 4 * N10$

Introducing those relationships into Equation 21: $N10 = \frac{9}{5} * N5$

P=8	N	D [in]	D [mm]
1	120	15	381
5	15	1,875	47,625
9	108	13,5	342,9
10	27	3,375	85,725

Table 43: Gear 1 teeth numbers and diameters.

GEAR 2:

Equation 13: $G = \frac{N9*N2}{N10*N6} = 3.2 * 4$

- $N2 = 3.2 * N6$
- $N9 + N10 = 135$

Introducing those relationships into Equation 21: $N6 = \frac{135}{4.2}$

P=8	N	D [in]	D [mm]
2	103	12,875	327,025
6	32	4	101,6
9	108	13,5	342,9
10	27	3,375	85,725

Table 44: Gear 2 teeth numbers and diameters.

GEAR 3:

Equation 13: $G = \frac{N9*N3}{N10*N7} = 1.6 * 4$

- $N3 = 1.6 * N7$
- $N9 + N10 = 135$

Introducing those relationships into Equation 21: $N7 = \frac{135}{2.6}$

P=8	N	D [in]	D [mm]
3	83	10,375	263,525
7	52	6,5	165,1
9	108	13,5	342,9
10	27	3,375	85,725

Table 45: Gear 3 teeth numbers and diameters.

GEAR 4:

Equation 13: $G = \frac{N9*N4}{N10*N8} = 0.9 * 4$

- $N4 = 0.9 * N8$
- $N9 + N10 = 135$

Introducing those relationships into Equation 21: $N8 = \frac{135}{0.9}$

P=8	N	D [in]	D [mm]
4	64	8	203,2
8	71	8,875	225,425
9	108	13,5	342,9
10	27	3,375	85,725

Table 46: Gear 4 teeth numbers and diameters.

Initial design:

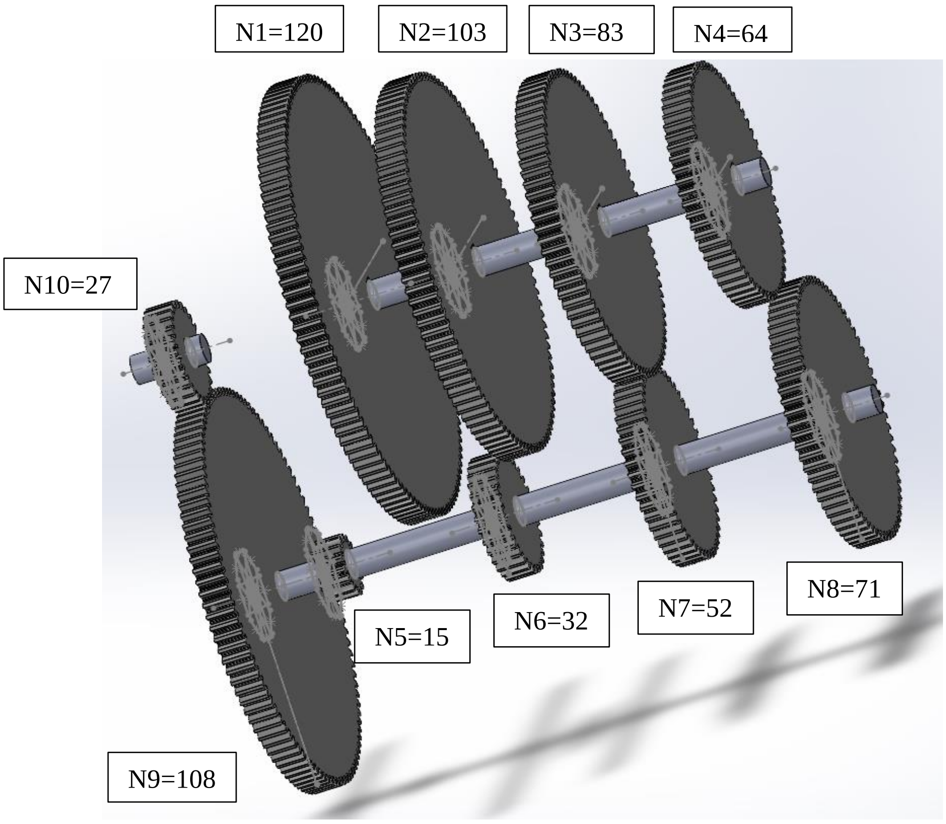


Figure 60: Number of teeth for the initial design.

Figure 60 shows a different vie of the initial OutShaft moving design. As it can be seen, the number of teeth is the same as for the final design. Even though, the face width and the position of the gears change.

Lewis Bending results:

	T	N	v [m/s]	Kv	Y	Sy	F [m]	n	√
1_1_1	5229,519	120	2,643	1,433	0,452	1,47E+09	0,076	2	450,258
1_1_2	3150,635	120	4,405	1,722	0,452	1,47E+09	0,076	2	529,204
2_2_0	2045,215	103	3,781	1,620	0,448	1,47E+09	0,076	2	624,377
2_2_1	1741,774	103	6,806	2,116	0,448	1,47E+09	0,076	2	592,007
2_2_2	1114,459	103	10,587	2,736	0,448	1,47E+09	0,076	2	650,871
3_3_0	1029,427	83	8,532	2,399	0,439	1,47E+09	0,025	2	371,072
3_3_1	819,792	83	11,579	2,898	0,439	1,47E+09	0,025	2	378,291
3_3_2	649,444	83	14,626	3,398	0,439	1,47E+09	0,025	2	392,534
4_4_0	581,418	64	11,278	2,849	0,425	1,47E+09	0,025	2	391,738
4_4_1	519,272	64	14,097	3,311	0,425	1,47E+09	0,025	2	384,498
4_4_2	434,933	64	16,916	3,773	0,425	1,47E+09	0,025	2	393,555
5_1_1	709,026	15	2,643	1,433	0,290	1,47E+09	0,076	2	346,218
5_1_2	427,168	15	4,405	1,722	0,290	1,47E+09	0,076	2	406,923
6_2_0	693,234	32	3,759	1,616	0,365	1,47E+09	0,076	2	540,296
6_2_1	590,381	32	22,837	4,744	0,365	1,47E+09	0,076	2	341,742
6_2_2	377,750	32	8,881	2,456	0,365	1,47E+09	0,076	2	593,770
7_3_0	697,856	52	8,552	2,402	0,412	1,47E+09	0,025	2	345,236
7_3_1	555,743	52	11,607	2,903	0,412	1,47E+09	0,025	2	351,921
7_3_2	440,263	52	14,661	3,403	0,412	1,47E+09	0,025	2	365,150
8_4_0	700,707	71	11,260	2,846	0,432	1,47E+09	0,025	2	378,693
8_4_1	625,810	71	14,075	3,307	0,432	1,47E+09	0,025	2	371,708
8_4_2	524,168	71	16,890	3,769	0,432	1,47E+09	0,025	2	380,475
9_1_1	704,504	108	19,031	4,120	0,449	1,47E+09	0,025	2	394,946
9_1_2	422,702	108	31,718	6,200	0,449	1,47E+09	0,025	2	415,639
9_2_0	1056,756	108	12,687	3,080	0,449	1,47E+09	0,025	2	372,962
9_2_1	587,087	108	22,837	4,744	0,449	1,47E+09	0,025	2	403,186
9_2_2	377,413	108	35,524	6,824	0,449	1,47E+09	0,025	2	419,277
9_3_0	754,826	108	17,762	3,912	0,449	1,47E+09	0,025	2	391,566
9_3_1	556,187	108	24,106	4,952	0,449	1,47E+09	0,025	2	405,441
9_3_2	440,315	108	30,450	5,992	0,449	1,47E+09	0,025	2	414,249
9_4_0	782,782	108	17,128	3,808	0,449	1,47E+09	0,025	2	389,726
9_4_1	626,226	108	21,410	4,510	0,449	1,47E+09	0,025	2	400,382
9_4_2	521,855	108	25,692	5,212	0,449	1,47E+09	0,025	2	407,992
10_1_1	192,262	27	19,031	4,120	0,350	1,47E+09	0,025	2	333,476
10_1_2	115,832	27	31,718	6,200	0,350	1,47E+09	0,025	2	350,228
10_2_0	187,979	27	12,687	3,080	0,350	1,47E+09	0,025	2	390,057
10_2_1	160,090	27	22,837	4,744	0,350	1,47E+09	0,025	2	340,570
10_2_2	102,432	27	35,524	6,824	0,350	1,47E+09	0,025	2	354,996

10_3_0	189,233	27	17,762	3,912	0,350	1,47E+09	0,025	2	344,955
10_3_1	150,697	27	24,106	4,952	0,350	1,47E+09	0,025	2	343,572
10_3_2	119,383	27	30,450	5,992	0,350	1,47E+09	0,025	2	350,916
10_4_0	190,006	27	17,128	3,808	0,350	1,47E+09	0,025	2	348,921
10_4_1	169,697	27	21,410	4,510	0,350	1,47E+09	0,025	2	339,262
10_4_2	142,135	27	25,692	5,212	0,350	1,47E+09	0,025	2	344,832

Table 47: Lewis Bending equation parameters and results.

Even though the naming of the first column seems to be tricky, it is not. It was the easiest way found to explain the scenario in the least possible space:

Gear of study_Gear the engine is working at_Scenario the gear is at the study moment

- Gear of study. Correspond to the gear that is going to be analysed.
- Gear the engine is working at. Corresponds to the gear that is running the motor at that moment.
- Scenario the gear is at the study moment. There are three different scenarios: shift change (0), first speed range (1) and second speed range (2).

<u>Bearings force and reactions study:</u>

InShaft:

Gear	Tmax [Nm]	r	FT [N]	pangle	F[N]	FR[N]	x1[m]	x2[m]
10	190	0,043	4432,779	0,253	4578,619	1146,395	0,038	0,038

R2y	R2z	R1y	R1z	R1	R2
-573,197	2216,390	-573,197	2216,390	2289,310	2289,310

Table 48: InShaft complete bearings study.

OutShaft:

Gear	Tmax [Nm]	r	FT [N]	pangle	F[N]	FR[N]	x3[m]	x4[m]
1	5229,52	0,191	27451,549	0,253	28354,713	7099,453	0,064	0,267
2	2045,22	0,164	12508,035	0,253	12919,553	3234,798	0,165	0,165
3	1029,43	0,132	7812,769	0,253	8069,812	2020,519	0,241	0,089
4	581,42	0,102	5722,638	0,253	5910,914	1479,975	0,292	0,038

R4y	R4z	R3y	R3z	R3	R4
-1365,279	5279,144	-5734,174	22172,405	22901,884	5452,829
-1617,399	6254,017	-1617,399	6254,017	6459,776	6459,776
-1476,039	5707,419	-544,481	2105,351	2174,617	5895,194
-1309,208	5062,333	-170,766	660,304	682,029	5228,886

Table 49: OutShaft complete bearings study.

CounterShaft:

Gear	F9T	F9R	F5T	F5R	x5	x6	x7
1	4432,779	1146,395	27451,549	7099,453	0,038	0,216	0,267
2	4432,779	1146,395	12508,035	3234,798	0,038	0,318	0,165
3	4432,779	1146,395	7812,769	2020,519	0,038	0,394	0,089
4	4432,779	1146,395	5722,638	1479,975	0,038	0,445	0,038

R6y	R6z	R5y	R5z	R5	R6
3547,030	-52395,632	4698,817	29376,863	29750,277	52515,557
2293,013	-23696,543	2088,180	15621,288	15760,239	23807,227
1759,435	-14679,179	1407,479	11299,189	11386,513	14784,245
1455,566	-10665,002	1170,803	9375,143	9447,967	10763,872

Table 50: CounterShaft complete bearings study.

Those values written in bold correspond to the most critical scenario for each shaft. The entire calculation is explained in the bearings section.

The next figures and tables collect the data of the different bearings selected. The calculation data is in IS units while the dimensions are in imperial system. As it has already been explained, this is because some of the formulas used in the design of the gear box are written in this system.

INSHAFT

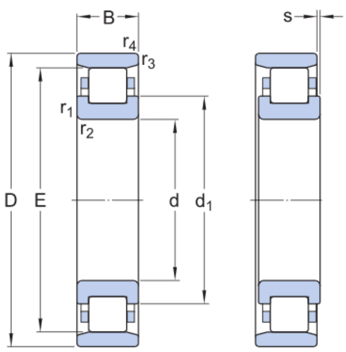
N 303 ECP:

Figure 61: N 303 ECP.

CALCULATION DATA

Basic dynamic load rating	C	28.5 kN
Basic static load rating	C_0	20.4 kN
Fatigue load limit	P_u	2.55 kN
Reference speed		17 000 r/min
Limiting speed		20 000 r/min
Minimum load factor	k_r	0.12
Limiting value	e	0.2
Axial load factor	Y	0.6

Table 51: N 303 ECP calculation data.



DIMENSIONS

d	0.669 in	Bore diameter
D	1.85 in	Outside diameter
B	0.551 in	Width
d_1	≈ 1.091 in	Shoulder diameter of inner ring
E	1.583 in	Raceway diameter of outer ring
$r_{1,2}$	min. 0.039 in	Chamfer dimension
$r_{3,4}$	min. 0.024 in	Chamfer dimension
s	max. 0.039 in	Permissible axial displacement

Figure 62: N 303 ECP dimensions.

OUTSHAFT

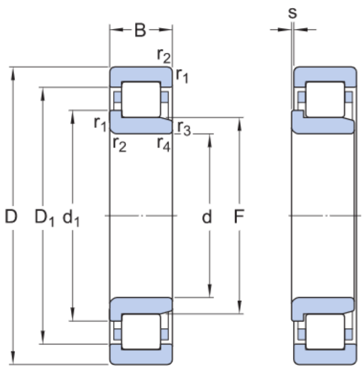
Initial-NJ 2306 ECML:

Figure 63: NJ 2306 ECML.

CALCULATION DATA

Basic dynamic load rating	C	83 kN
Basic static load rating	C_0	75 kN
Fatigue load limit	P_u	9.65 kN
Reference speed		11 000 r/min
Limiting speed		19 000 r/min
Minimum load factor	k_r	0.38
Limiting value	e	0.3
Axial load factor	Y	0.4

Table 52: NJ 2306 ECML calculation data.



DIMENSIONS

d	1.181 in	Bore diameter
D	2.835 in	Outside diameter
B	1.063 in	Width
d_1	≈ 1.772 in	Shoulder diameter of inner ring
D_1	≈ 2.317 in	Shoulder diameter of outer ring
F	1.595 in	Chamfer dimension of loose flange ring
$r_{1,2}$	min. 0.043 in	Chamfer dimension
$r_{3,4}$	min. 0.043 in	Chamfer dimension
s	max. 0.095 in	Permissible axial displacement

Figure 64: NJ 2306 ECML dimensions.

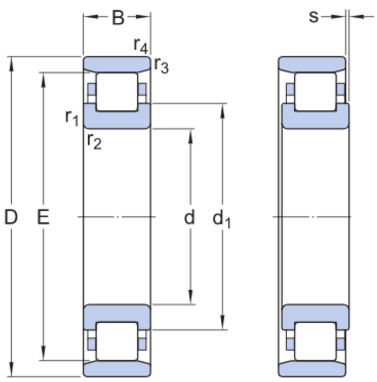
Final-N 210 ECP

Figure 65: N 210 ECP

CALCULATION DATA

Basic dynamic load rating	C	73.5 kN
Basic static load rating	C_0	69.5 kN
Fatigue load limit	P_u	8.8 kN
Reference speed		8 500 r/min
Limiting speed		9 000 r/min
Minimum load factor	k_r	0.12
Limiting value	e	0.2
Axial load factor	Y	0.6

Table 53: N 210 ECP calculation data.



DIMENSIONS

d	1.969 in	Bore diameter
D	3.543 in	Outside diameter
B	0.787 in	Width
d_1	≈ 2.52 in	Shoulder diameter of inner ring
E	3.209 in	Raceway diameter of outer ring
$r_{1,2}$	min. 0.043 in	Chamfer dimension
$r_{3,4}$	min. 0.043 in	Chamfer dimension
s	max. 0.059 in	Permissible axial displacement

Figure 66: N 210 ECP dimensions

COUNTERSHAFT

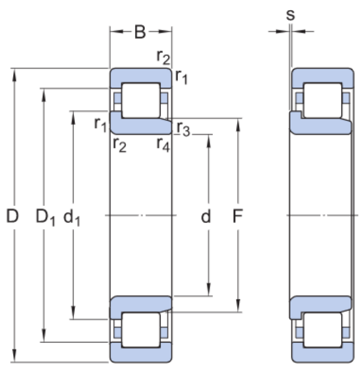
Initial-NJ 2208 ECJ:

Figure 67: NJ 2208 ECJ

CALCULATION DATA

Basic dynamic load rating	C	81.5 kN
Basic static load rating	C_0	75 kN
Fatigue load limit	P_u	9.65 kN
Reference speed		9 500 r/min
Limiting speed		11 000 r/min
Minimum load factor	k_r	0.2
Limiting value	e	0.3
Axial load factor	Y	0.4

Table 54: NJ 2208 ECJ calculation data.



DIMENSIONS

d	1.575 in	Bore diameter
D	3.15 in	Outside diameter
B	0.906 in	Width
d_1	≈ 2.126 in	Shoulder diameter of inner ring
D_1	≈ 2.659 in	Shoulder diameter of outer ring
F	1.949 in	Chamfer dimension of loose flange ring
$r_{1,2}$	min. 0.043 in	Chamfer dimension
$r_{3,4}$	min. 0.043 in	Chamfer dimension
s	max. 0.075 in	Permissible axial displacement

Figure 68: NJ 2208 ECJ dimensions.

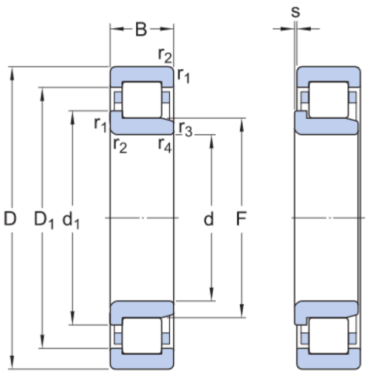
Final-NJ 2306 ECP:

Figure 69: NJ 2306 ECP

CALCULATION DATA

Basic dynamic load rating	C	83 kN
Basic static load rating	C_0	75 kN
Fatigue load limit	P_u	9.65 kN
Reference speed		11 000 r/min
Limiting speed		12 000 r/min
Minimum load factor	k_r	0.25
Limiting value	e	0.3
Axial load factor	Y	0.4

Table 55: NJ 2306 ECP calculation data.



DIMENSIONS

d	1.181 in	Bore diameter
D	2.835 in	Outside diameter
B	1.063 in	Width
d_1	≈ 1.772 in	Shoulder diameter of inner ring
D_1	≈ 2.302 in	Shoulder diameter of outer ring
F	1.595 in	Chamfer dimension of loose flange ring
$r_{1,2}$	min. 0.043 in	Chamfer dimension
$r_{3,4}$	min. 0.043 in	Chamfer dimension
s	max. 0.095 in	Permissible axial displacement

Figure 70: NJ 2306 ECP dimensions.

Shaft:

$$k_a = aS_{ut}^b \tag{6–19}$$

Table 6–2

Parameters for Marin Surface Modification Factor, Eq. (6–19)

Surface Finish	Factor a S_{ut}, kpsi	Factor a S_{ut}, MPa	Exponent b
Ground	1.34	1.58	−0.085
Machined or cold-drawn	2.70	4.51	−0.265
Hot-rolled	14.4	57.7	−0.718
As-forged	39.9	272.	−0.995

From C.J. Noll and C. Lipson, "Allowable Working Stresses," *Society for Experimental Stress Analysis*, vol. 3, no. 2, 1946 p. 29. Reproduced by O.J. Horger (ed.) *Metals Engineering Design ASME Handbook*, McGraw-Hill, New York. Copyright © 1953 by The McGraw-Hill Companies, Inc. Reprinted by permission.

Figure 71: Marin Surface Modification Factor. Source: Shigley's Mechanical Engineering Design Textbook.

$$k_b = \begin{cases} (d/0.3)^{-0.107} = 0.879d^{-0.107} & 0.11 \le d \le 2 \text{ in} \\ 0.91d^{-0.157} & 2 < d \le 10 \text{ in} \\ (d/7.62)^{-0.107} = 1.24d^{-0.107} & 2.79 \le d \le 51 \text{ mm} \\ 1.51d^{-0.157} & 51 < d \le 254 \text{ mm} \end{cases} \tag{6–20}$$

Figure 72: Marin Size Factor. Source: Shigley's Mechanical Engineering Design Textbook.

Table 7–1

First Iteration Estimates for Stress-Concentration Factors K_t and K_{ts}.

Warning: These factors are only estimates for use when actual dimensions are not yet determined. Do *not* use these once actual dimensions are available.

	Bending	Torsional	Axial
Shoulder fillet—sharp (r/d = 0.02)	2.7	2.2	3.0
Shoulder fillet—well rounded (r/d = 0.1)	1.7	1.5	1.9
End-mill keyseat (r/d = 0.02)	2.14	3.0	—
Sled runner keyseat	1.7	—	—
Retaining ring groove	5.0	3.0	5.0

Figure 73: First Iteration Estimates for Stress-Concentration Factors. Source: Shigley's Mechanical Engineering Design Textbook.

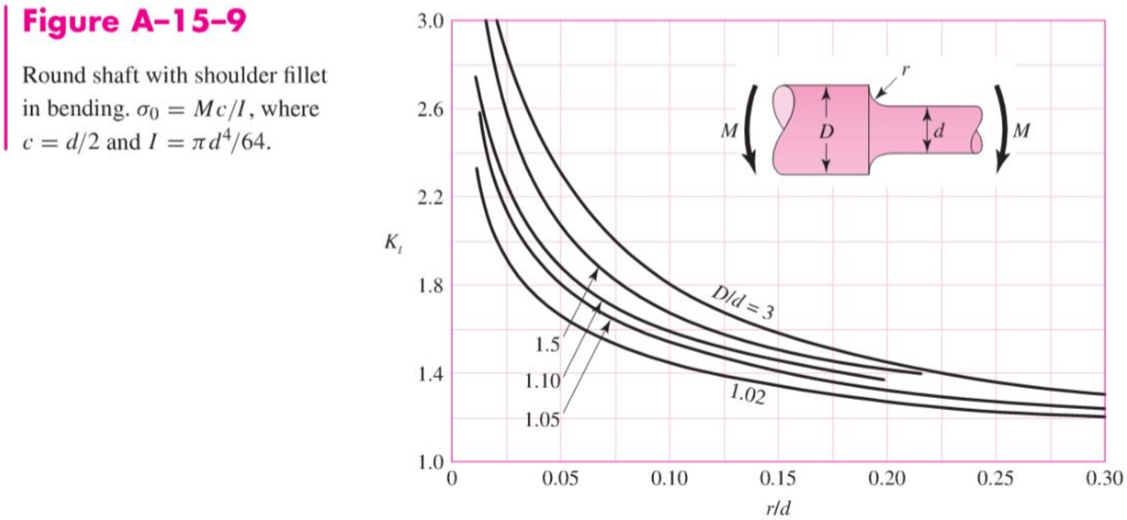


Figure 74: Stress Concentration factors for the rest of the iterations on bending. Source: Shigley's Mechanical Engineering Design Textbook.

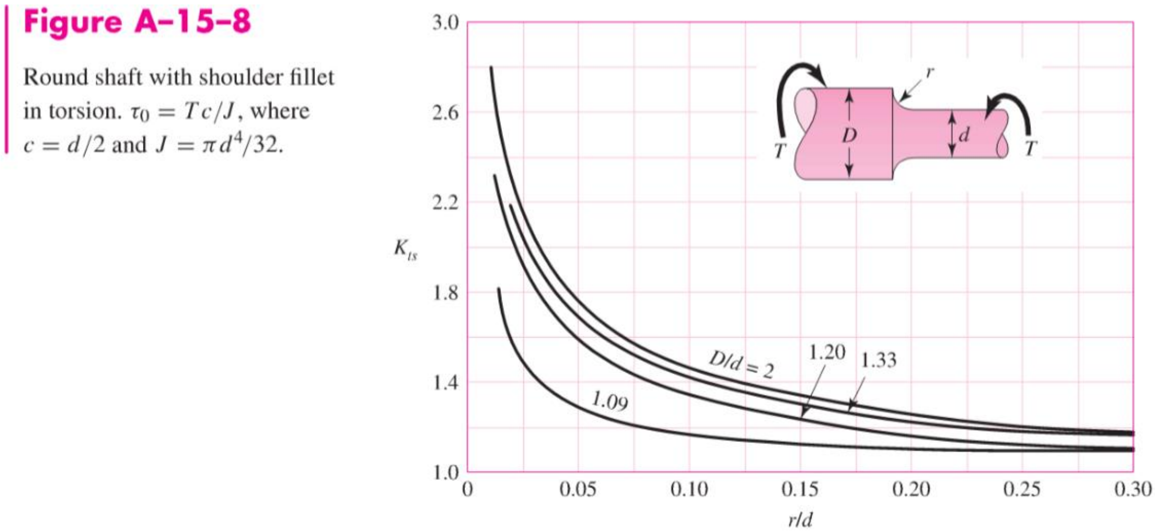


Figure 75: Stress Concentration factors for the rest of the iterations on torsion. Source: Shigley's Mechanical Engineering Design Textbook.

Figure 6–20

Notch-sensitivity charts for steels and UNS A92024-T wrought aluminum alloys subjected to reversed bending or reversed axial loads. For larger notch radii, use the values of q corresponding to the $r = 0.16$-in (4-mm) ordinate. *(From George Sines and J. L. Waisman (eds.),* Metal Fatigue, *McGraw-Hill, New York. Copyright © 1969 by The McGraw-Hill Companies, Inc. Reprinted by permission.)*

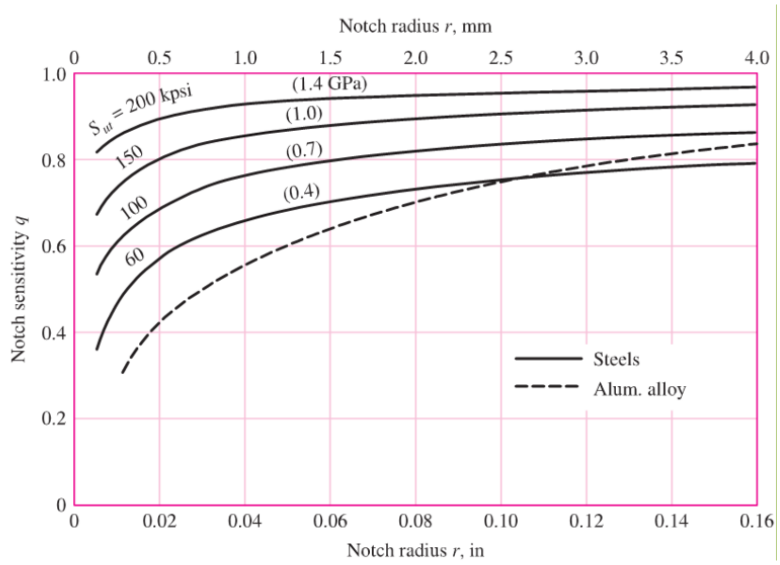


Figure 76: Notch Sensitivity on bending. Source: Shigley's Mechanical Engineering Design Textbook.

Figure 6–21

Notch-sensitivity curves for materials in reversed torsion. For larger notch radii, use the values of q_{shear} corresponding to $r = 0.16$ in (4 mm).

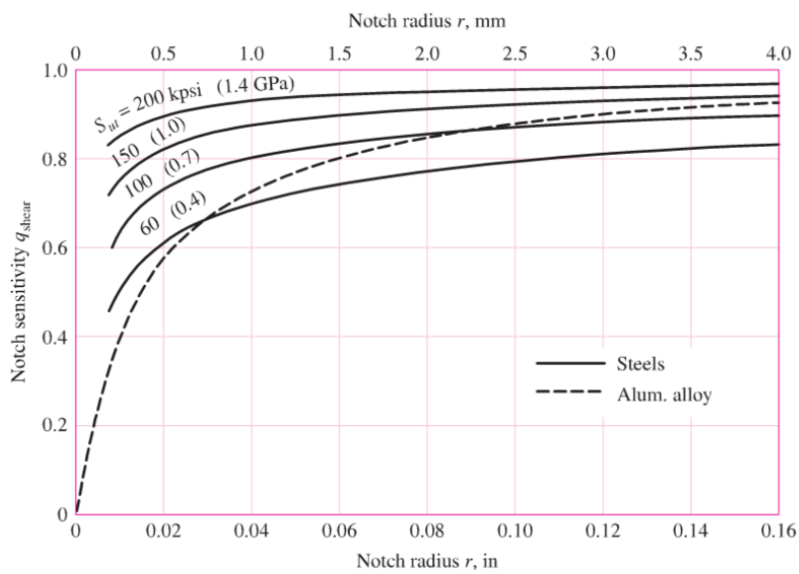


Figure 77: Notch Sensitivity on torsion. Source: Shigley's Mechanical Engineering Design Textbook.

<u>Case</u>

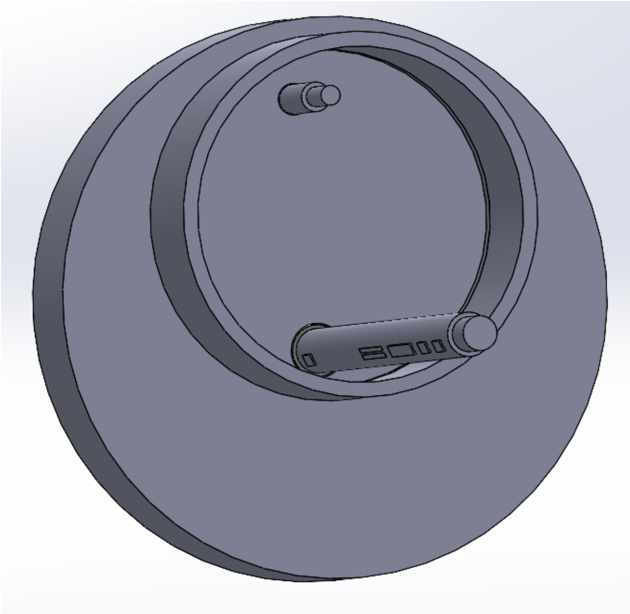


Figure 78: Case with the InShaft and CounterShaft.

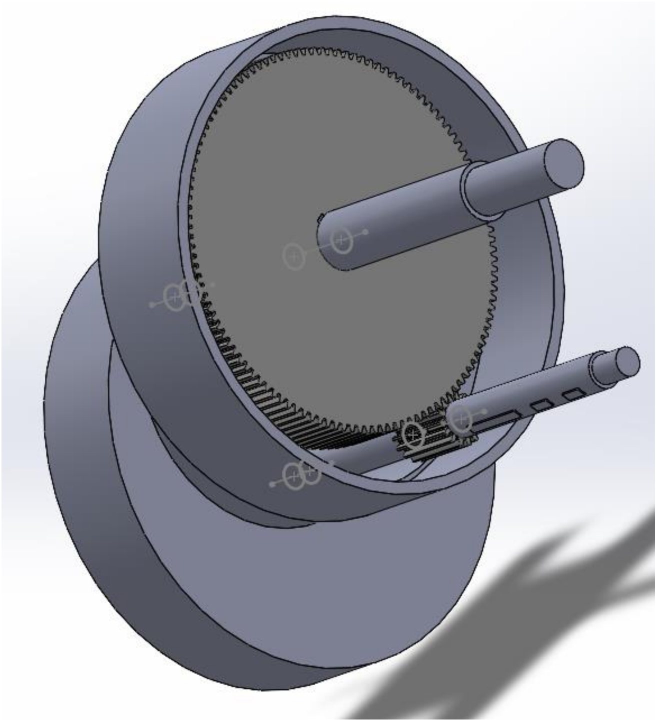


Figure 79: Case with the first gear meshed.

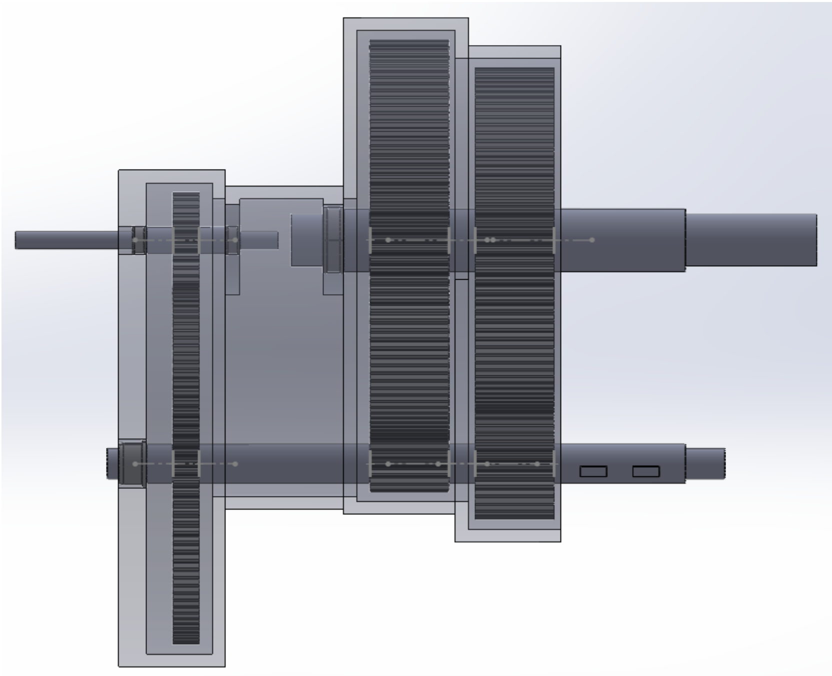


Figure 80: Case development.

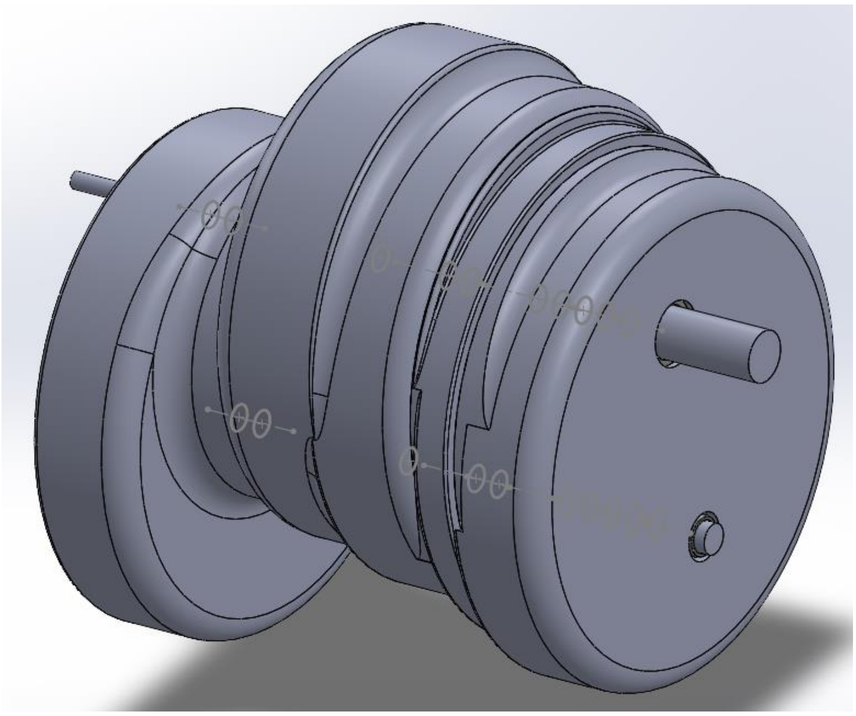


Figure 81: Case final result second view.