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Review

A Review on the Unit Commitment Problem: Approaches, Techniques, and Resolution Methods

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Abstract: Optimizing the schedule of thermal generators is probably the most important task when the operation of power systems is managed. This issue is known as the unit commitment problem in operational research. It has been profoundly studied in the literature, where several techniques have been proposed to address a computationally tractable solution. In turn, the ongoing changes of paradigms in energy markets focus the attention on the unit commitment problem as a powerful tool to handle new trends, such as the high renewable energy sources penetration or widespread use of non-conventional energy-storage technologies. A review on the unit commitment problem is proposed in this paper. The easy understanding of the diverse techniques applied in the literature for new researchers is the main goal of this state-of-art as well as identifying the research gaps that could be susceptible to further developments. Moreover, an overview of the evolution of the Mixed Integer Linear Programming formulation regarding the improvements of commercial solvers is presented, according to its prevailing hegemony when the unit commitment problem is addressed. Finally, an accurate analysis of modeling detail, power system representation, and computational performance of the case studies is presented. This characterization entails a significant development against the conventional reviews, which only offer a broad vision of the modeling scope of their citations at most.

Keywords: unit commitment; optimal thermal generation; numerical optimization; evolutionary optimization; optimization techniques; decomposition techniques; uncertainty management

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1. Introduction

The unit commitment problem (UC) is a traditional optimization problem where the best schedule for a group of thermal units is obtained. Optimizing the electrical generation entails many advantages for market players and final customers. However, that is not an easy task according to the big size of the problem and the computational limitations.

For that reason, there are many works in the literature where different approaches are proposed to find an optimal solution to this problem, constituting an essential target for the advances in operational research. This paper presents a new review of the state-of-art of the unit commitment problem, where the distinctions between optimization techniques, problem formulations, and resolution algorithms are exposed in order to facilitate their understanding.

This section provides a brief description of the main issues that are frequently considered in the unit commitment problem. Several mathematical approaches have been proposed over the years. The principal modeling ideas are gathered in this paper and referenced for more detailed explanations. There are multiple techniques that have been applied to solve the unit commitment problem, being the most popular its presentation as a conventional optimization problem:

$$min(Production\ cost + SU\ cost + SD\ cost + Emission\ cost + Maintenance\ cost) \quad (1)$$

$$\text{subject to:} \qquad \textit{Technical and economical constraints.} \tag{2}$$

The different terms of the objective function (OF) are described below:

- Production cost: This cost is related to the fuel consumption of the thermal units when electricity is generated. Its behavior is usually described through a linear or quadratic function, where there are a fixed term, a linear term concerning the power production, and a quadratic term that multiplies the squared power generation of the unit. The last term can be omitted to work with linear objective functions. Moreover, a piecewise approximation can also be used to linearize the quadratic function. The utilization of integer variables is mandatory for correctly modeling this cost.
- Start-up (SU) cost: This cost is related to the fuel consumption of the starting-up process before a thermal unit is totally committed. It has an exponential behavior according to the number of hours that the unit had been offline. Nevertheless, it is commonly linearized through a stairwise function. Integer variables are also employed for achieving an accurate representation.
- Shut-down (SD) cost: This cost is applied when a thermal unit is shut-down. It is usually modeled as a fixed cost where integer variables are used to define its treatment. Sometimes, this cost is not considered.
- Emission cost: This cost is related to the polluting compounds or the greenhouse gases generated as a consequence of electricity production. It is not linked to the fuel prices, such as those mentioned above, but it is related to fuel consumption and technological efficiency. Its value depends on the local regulation and the emission allowance trading market scheme if it exists. Within the European Union, it relies on CO_2 prices.
- Maintenance cost: This cost represents the increase of the maintenance operations when the thermal unit is running for a longer time. It is modeled as a linear function with respect to power generation, and it is often internalized in the production cost for the sake of simplicity. Integer variables are also associated with this cost.

Fuel consumption is the main cost of the unit commitment problem. The importance of a proper representation of these costs is pointed out in [1], where it is mentioned that saving 0.5% of fuel in electricity generation represents a yearly benefit of millions of euros for a utility. Accordingly, research is focused on improving the modeling detail to increase profitability. In past decades, coal plants were generally the thermal basis of power systems. Meanwhile, combined cycles gas turbines were relegated for high-demand periods, and fast-ramping gas turbines were used to cover demand peaks. This operation was steady over time and did not demand great modeling developments to achieve an efficient asset management.

Nowadays, the trend is changing. The dismantling process of coal plants accounted by many power systems according to greenhouse policies and the loss of their competitiveness due to the implementation of emission-allowance trading markets has brought a paradigm shift. Moreover, the higher renewable-energy source penetration has increased the frequency of the start-up and shut-down processes of the thermal units. Hence, the proper representation of the fuel consumption during these operations is gaining force in current formulations. With regard to the technical constraints used in the unit commitment problem, the following considerations are often assumed:

- Demand constraint: This is a balance equation to assure that the electricity generation meets the load demand for every represented time period. An energy storage term can be added if more accurate management is desired. In turn, it is also possible to introduce a spillage term in the equation to represent situations of production surpluses. It is a linear equation with continuous variables.
- Reserve constraint: This inequality guarantees a technical necessity of power systems, which is the availability of an extra generation capacity reserved for compromising situations, such as a failure in a committed thermal unit, to keep the security of supply. It is a linear inequality that also employs continuous variables.

- Capacity limits: This inequality is used to assure that the electricity generation of each thermal unit respects the minimum and maximum power output according to their technical limits. This inequality is linear and uses integer variables.
- Ramping limits: This inequality assures that the difference between the power generation of a thermal unit during the previous and the current time step does not exceed the ramping rates. It is a linear inequality that uses continuous variables.
- Logic constraint: It establishes logic in the commitment decisions at every time step, indicating the relationships between start-ups, shut-downs, and commitment status along the whole time span. This equation is linear and utilizes integer variables.
- Minimum time up (TU) and time down (TD): This inequality is used to guarantee that the unit is online for a minimum period of time since it is started-up or that it is offline for a minimum time since it is shut-down, in order to accomplish with technical limitations that reduce the risk of failure. It is linear and employs integer variables.
- Operating constraints: This group gathers the constraints that are utilized for a more accurate representation of the operation of the thermal units from a technical point of view. Some examples are situations where some units must run or have a fixed power output, an outage of a unit due to maintenance tasks, or unforeseen problems, etc.
- Emission constraints: They are imposed to bound specific emissions along a time span.
- Network constraints: These constraints are implemented with the aim of representing technical limitations regarding the consideration of the power grid. They increase the accuracy of the unit commitment problem but also the complexity of its resolution. For that reason, the network is frequently disregarded unless a more secure generation schedule is desired. In that case, the capacity of the optimal schedule to overcome an unexpected failure safely is sought. It is known as the Security Constrained Unit Commitment problem (SCUC).

The unit commitment problem is frequently addressed as a minimization problem, where the sum of the total costs constitutes the objective function, and the demand constraint is an equation where the generation meets the power consumption. Sometimes, a non-served energy term is added to the balance equation. If the associated cost is high, there will only be non-served energy situations if the demand exceeds the generation limit. On the contrary, if it is not high enough, there could appear situations in which the electricity production is not profitable.

This representation was suitable before the liberalization of energy markets accomplished in many power systems. Nowadays, the objective function of the unit commitment problem is also represented through the difference between benefits and costs of producing electricity, which is sought to be maximized. In turn, price forecasts are implemented to evaluate profitability, and the demand constraint is transformed into an inequality or omitted. The production of a market player will be based on profitability, which is known as the Price-Based Unit Commitment problem (PBUC). The representation of the competitiveness behavior of market players has also been studied in this problem. Nevertheless, this research topic is out of the scope of this survey.

The following section describes the techniques that can be applied to solve the unit commitment problem further than the optimization problem presentation. However, according to its importance, this methodology is predominantly studied in this paper. The programming options with relation to the formulation are exposed in Section 3.1. In turn, the consideration of uncertainty is addressed in Section 3.2. The decomposition techniques that can be applied to facilitate its resolution are mentioned in Section 3.3. On the other hand, the resolution algorithms that can be applied to find the optimal solution to the optimization problem are exposed in Section 3.4. Section 4.1 describes the modeling trends when the unit commitment problem is formulated as a Mixed Integer Linear Programming (MILP), which is currently the most widespread approach. Meanwhile, Section 4.2 presents an accurate analysis of modeling detail, power system representation, and computational performance of the case studies described in all the references. Finally, Section 5 concludes the article with a brief contribution summary, identifying research gaps for further developments.

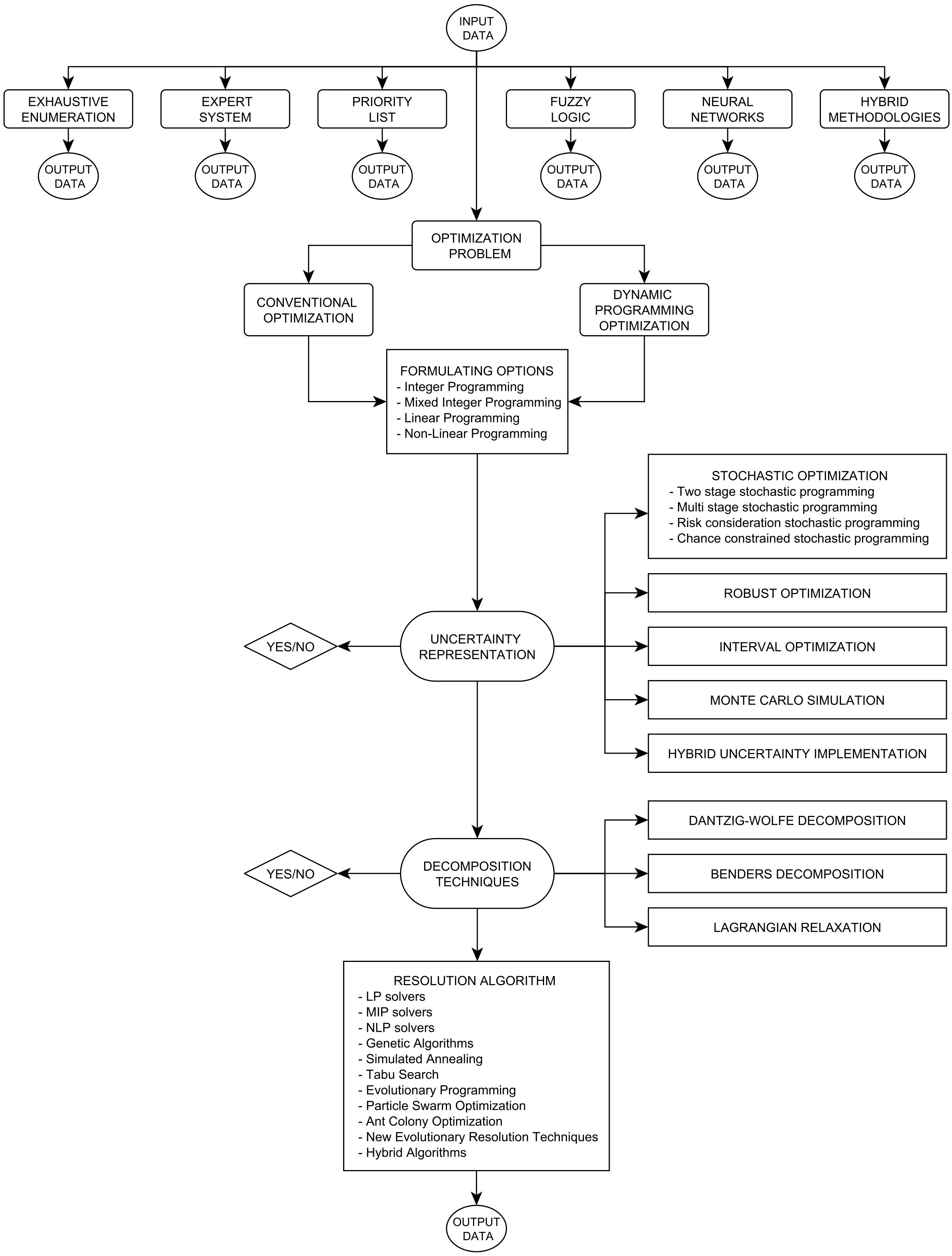

Figure 1. Optimization methodologies applied to the unit commitment problem. At the first level, the optimization techniques are exposed. Later, the most widely used is visually described, underlying the two choices of addressing the optimization problem and its stages. The formulating option and the resolution algorithm that are needed to solve the problem are presented. In turn, the uncertainty representation and the utilization of decomposition techniques are introduced as possible alternatives to enhance the accuracy of the representation of the problem and the performance of its resolution.

2. Optimization Techniques and Unit Commitment

In order to achieve an optimal solution to the unit commitment problem, several optimization techniques have been applied in the literature. These methodologies require a set of input data and return the best thermal schedule obtained after their applications. Figure 1 presents an illustrative diagram where these optimization techniques are classified. In contrast with many other arrangements made in the literature, this layout can help to clarify the differences between optimization techniques, problem formulations, decomposition techniques, and resolution methods, as well as different options of uncertainty management inside the proposed scheme.

Popular optimization methods are described in this section. They constitute different approaches to face the unit commitment problem, whose complexity has often evolved together with the advances in computation. Furthermore, it is also possible to combine them in hybrid methodologies to exploit some of their advantages at once. Regarding the literature, the following classification is made:

- Exhaustive Enumeration (EE): It consists of the evaluation of all the feasible solutions in order to identify the best value as the optimal solution. Exhaustive enumeration is a brute force method that is not computationally affordable. Its scope is very limited [2].
- Expert System (ES): The underlying idea of this method resides in the creation of an algorithm where the good practices and knowledge of proceedings in the resolution of the unit commitment problem are computed. It was employed to save in computational costs [3], but it fell into disuse because of its sub-optimal solutions.
- Priority List (PL): This technique is based on ordering elements of target sets according to their contribution to the objective function. The decisions taken on these target elements during the resolution process will conclude in a better or worse approach to the optimal solution. For that reason, it usually has a mathematical background behind it. Despite returning a sub-optimal solution, it is an attractive optimization technique from a computational perspective due to the obtaining of near-optimal solutions in reasonable run times [4–7].
- Fuzzy Logic (FL): This method allows the application of abstract reasoning into the computational logic to solve a mathematical problem. It utilizes *if–then* rules in order to generalize some input data. This information is treated according to a background (*if* conditions) and output data are obtained consequently (*then* reactions). The utilization of fuzzy logic techniques helps accelerate the resolution of the unit commitment problems but returns a less accurate solution. They are usually applied at a beginning step and combined with other optimization techniques in hybrid proceedings [8–10].
- Neural Networks (NN): This artificial intelligence technique is based on establishing patterns that transform some input data into a near-optimal solution. Its structure consists of a group of interconnected nodes where some mathematical functions are applied to process the information. In order to achieve good results, these processes are trained with a benchmark database. However, it provides sub-optimal solutions, and its implementation and adjustment are quite difficult [11,12]. This machine-learning approach is still being used to solve the unit commitment problem nowadays. For further information about the application of machine learning techniques in the unit commitment, the reader is referred to [13].
- Optimization Problem (OP): The unit commitment problem is frequently addressed as a classical optimization problem, where an objective function is proposed, subject to a set of constraints. This methodology entails the most widespread approach used to solve the unit commitment problem, and it is described in depth in Section 3.
- Hybrid Methodologies (HM): These optimization techniques are sometimes combined in hybrid methodologies in order to improve their performance. Some approaches made in the literature of the unit commitment problem are [14–21].

A comparison of these optimization techniques is presented in Table 1. References are focused on the thermal UC. For representations where the optimization of power systems adds hydro valleys or energy storage facilities, the reader is referred to [22,23], respectively.

Table 1. Comparison of optimization techniques to solve the unit commitment problem.

Technique	Advantages	Disadvantages
EE	Optimal solution	Computationally intractable
ES	Fast resolution Handles a lot of information Combines theoretical–practical knowledge	Non-optimal solution Difficult implementation Problems if schedules are unexpected in database
PL	Fast resolution Mathematical background Generally easy to implement	Non-optimal solution Difficult identification of new cost-saving trends Solution can be very far from the optimum value
FL	Qualitative interpretation Effective solution to complex problems Handles any type of unit characteristic data	Non-optimal solution Fuzzy rules are difficult to implement Results are difficult to analyze in-depth
NN	Handles complex systems efficiently Hidden relationships can be identified Flexible utilization of non linear functions Flexible treatment of noisy data	Non-optimal solution Intricate network structure New additions require a retraining Exponential computation-time/problem-size rate
OP	Optimal solution Generally easy to implement Advances in commercial solvers Much information in the literature	Exponential computation-time/problem-size rate Modeling simplifications are sometimes needed Function linearizations are sometimes needed Unable to work with noisy data

3. Unit Commitment as an Optimization Problem

As mentioned, the resolution of the unit commitment problem is frequently addressed as an optimization problem. Two approaches can be differentiated within this technique. Conventional optimization (CO) exposes the objective function and its constraints as is. On the other hand, dynamic programming (DP) utilizes the principle of optimality proposed by Bellman [24], dividing the problem into overlapping subproblems and optimal substructures [25–27]. However, the curse of dimensionality forces the implementation of heuristic rules to deal with real-size problems, providing near-optimal solutions. Although nowadays, DP is not as relevant as it was before, it is sometimes used to manage uncertainty [28–30]. Furthermore, other distinctions in OP can be made according to their formulation, uncertainty consideration, utilization of decomposition techniques, or the optimization algorithm that is employed to determine an optimal solution.

3.1. Formulating Options

There are different classifications depending on how an OP can be formulated. According to the variables, a distinction can be made between discrete or continuous variables, as well as the utilization of a mix of them. In contrast, the nature of the objective function and constraints can be discriminated into linear, quadratic, or non-linear functions. Combinations of these programming techniques lead to the classification made in Appendix A. In the unit commitment problem, the most popular formulating option is Mixed Integer Linear Programming. In turn, it can also be faced by Mixed Integer Quadratic Programming (MIQP), Mixed Integer Quadratically Constrained Prog. (MIQCP), Mixed Integer Second-Order Cone Programming (MISOCP), and Mixed Integer Non-Linear Programming (MINLP).

3.2. Uncertainty Representation

The operation of real power systems is implicitly subject to uncertainty. The demand forecasts are exposed to inaccuracy or sudden changes according to unexpected situations. Furthermore, the penetration of renewable energy sources in current power markets increases the uncertainty. Wind and solar generation entail the risk of altering the thermal demand at any time. For those reasons, the consideration of uncertainty in the unit commitment problem enhances the reliability of the optimal schedule that is obtained. If the election of representing uncertainty is made, there are several alternatives to transform a deterministic problem into a probabilistic one. The most popular techniques applied to the unit commitment problem are briefly described below and compared in Table 2.

- Stochastic Optimization (SO): This methodology manages the representation of uncertainty through the utilization of probability distributions that are connected to risk variables. These distributions can be directly included in constraints that require some statistical parameters. Nonetheless, the most common practice is to consider different scenarios. These scenarios are obtained through probability-distribution discretizations. Each scenario has an associated weight according to its frequency of occurrence.
  - Two-Stage (TS) Stochastic Programming: This technique is based on dividing the problem into two steps, distributing decision variables and constraints in these stages. When the first step is accomplished, the first-stage choices are made. Later, these decisions are considered fixed, and the second stage is solved. The two-stage stochastic programming utilizes scenarios to consider uncertainty. When all the scenarios are solved, a solution to the problem is calculated according to the weight of each scenario. It has been widely applied in the UC literature [31–39]. It is widespread to decide which thermal units will be online along the time span in the first stage. Thereafter, the optimal schedule is set in the second stage. Each dispatch obtained in this stage corresponds to a scenario.
  - Multi-Stage (MS) Stochastic Programming: This technique uses a combinatorial tree where every combination of scenarios is represented. The tree is divided into successive nodes that are linked. A branch represents the path between the initial node and a final solution. The weights of scenarios are set in the linking connection between nodes. Thus, it is easy to determine the probability of each solution obtained when each branch is solved, which corresponds to a scenario. Some of its applications to the unit commitment problem are [40–42]. Robust solutions are provided, since decisions are taken dynamically. However, the associated computational burden requires an excellent scenario sampling and reduction to reach acceptable run times.
  - Risk Consideration (RC) Stochastic Programming: This method is based on the addition of some constraints in order to respect the risk exposure of some decisions when the problem is solved. These equations require statistical information of the probability distributions as input and the significant value desired to respect. Thus, a solution is obtained according to the confidence interval that is introduced in the problem. This technique has been applied to the unit commitment problem in order to represent situations such as the expected non served load, loss of load probability, or the variance of the total profit [43–45].
  - Chance Constrained (CCO) Stochastic Programming: It is considered a particularization of two-stage stochastic programming. This technique allows the solution to violate a set of constraints according to a predefined confidence level. As with the risk consideration stochastic programming, it works with probability distributions instead of scenarios. However, the solution has a probabilistic touch. It will be the optimal solution with a confidence interval, not according to a confidence interval associated with risk variables [46–48].
- Robust Optimization (RO): The underlying idea of this methodology is to reach an optimal solution avoiding the worst possible combination of circumstances that can happen according to the uncertainty associated with the presence of risk variables. Robust optimization does not work with probability distributions. It employs a bounded range from which a risk variable can take its value. The bounds imposed on the risk variables are applied in two ways. They are directly used as upper and lower bounds in an inequality that is defined for each dimension in which the variables are formulated and indirectly through the establishment of an uncertainty set. This

uncertainty budget is linked to the deviation of the forecasted value associated with each risk variable from their bounds along the evaluated horizon. It can not exceed a predefined value. The more restrictive the uncertainty set is, the more robust the obtained solution. The load demand is used as the target of RO in the unit commitment problem [49–51].

- Interval Optimization (IO): This technique handles the uncertainty representation through the creation of bounds according to a predefined confidence interval. Firstly, a forecasted value is provided for each considered risk variable. Later, the confidence interval is used to generate an envelope around the expected value. The higher the value of the confidence interval, the tighter the bounding to the risk variable. Afterwards, the problem is optimized at the expected central value of the interval, being also capable of providing a feasible solution for those deviations from the forecast that are contained inside the interval [52–54].
- Monte Carlo Simulation (MCS): The Monte Carlo methodology is frequently employed to achieve an accurate sampling from a set of probabilistic distributions. However, it can be extended to manage a complete uncertainty representation. In that case, the obtained scenarios are optimized as deterministic problems. Later, the output data are processed, and a probabilistic distribution is associated with each result [55–57].
- Hybrid Uncertainty Implementation (HUI): These techniques can be combined to improve their performances. Some examples in the unit commitment literature are [58–60].

Table 2. Comparison of uncertainty representation techniques used in the unit commitment problem.

Technique	Advantages	Disadvantages
SO-TS	Less conservative solution Generally easy to implement Working with discretized scenarios Easy weight-assignment to each solution	Static assumption of uncertainty Necessity of scenario-generation techniques Necessity of scenario-reduction techniques Information of probability distributions
SO-MS	Dynamic assumption of uncertainty Accurate decision-making modeling Working with discretized scenarios Easy-traceable paths in the scenario tree Easy weight-assignment to each solution	Curse of dimmensionality Necessity of scenario-generation techniques Necessity of scenario-reduction techniques Information of probability distributions Difficult construction of scenario trees
SO-RC	Flexible interval confidence for input data Risk variables define robustness Generally easy to implement	Necessity of statistical information Working with probabilistic distributions Solution according to a interval confidence
SO-CCO	Less conservative solution Solution with a probabilistic touch Flexible setting of violable constraints	Solution is not valid in some situations Working with probabilistic distributions Modeling demands extra binary variables
RO	Optimization over the whole horizon Do not need probabilistic distributions Flexible protection from adverse situations Useful if forecasting deviations are bounded	Over-conservative solutions Tight optimality/uncertainty set relation Expertise to construct the uncertainty set Difficult to incorporate uncertainty dynamics
IO	Easier to incorporate uncertainty dynamics Do not need probabilistic distributions Flexible protection from adverse situations Useful if forecasting deviations are bounded	Over-conservative solutions Tight optimality/forecasting bounds relation Expertise to establish forecasting deviations Optimization over particular central values
MCS	Computational efficiency High-efficient sampling process Working with discretized scenarios Resolution process can be parallelized Results have a probabilistic distribution	Implementation can be difficult Necessity of scenario-generation techniques Necessity of scenario-reduction techniques Information of probability distributions Solutions are not valid in some situations

3.3. Decomposition Techniques

Once the formulation of the unit commitment problem is chosen and the decision of considering uncertainty or not is taken, it is time to evaluate if the application of decomposition techniques to the problem is desirable. These techniques can be a powerful tool to solve the unit commitment problem with reasonable run times. Although they were frequently employed when the performance of numerical optimization solvers was computationally limited, particularly the Lagrangian Relaxation, their use has drastically decreased currently because of the advances in commercial solvers and disadvantages such as its difficult implementation and convergence toward an optimal solution. However, the application of decomposition techniques in the unit commitment problem is sometimes addressed nowadays:

- Dantzig-Wolfe Decomposition (DWD): [33,40,61].
- Benders Decomposition (BD): [35,39,48].
- Lagrangian Relaxation (LR): [31,37,62].

3.4. Optimization Algorithms

Finally, when an optimization problem is completely defined, optimization algorithms are applied to find an optimal or near-optimal solution to the problem. The resolution techniques can be classified into numerical methods or meta-heuristic methods.

Numerical optimization (NO) is preferably used to solve the unit commitment problem. In a first step, the improvements in these iterative algorithms were implemented directly by researchers looking to enhance the resolution processes. Currently, commercial solvers have advanced notoriously, and the research is focused on exploiting their properties as best as possible. In order to solve MILP, MIQP, and MIQCP problems, two of the most renowned commercial solvers are Gurobi [63] and CPLEX [64]. In addition, for MISOCP or MINLP problems, MOSEK [65] and BARON [66] respectively offer one of the best performances. For further information about NO, the reader is referred to [67].

On the other hand, evolutionary optimization (EO) is also applied to solve the unit commitment problem. Although these methodologies can not guarantee finding an optimal solution, they can handle more complex formulations without the necessity of utilizing too many simplifications. These meta-heuristic algorithms are based on simulating behaviors that are observed in nature to map the feasible region of an optimization problem looking for the optimal solution. Many of these approaches have been employed to solve the unit commitment problem:

- Genetic Algorithm (GA): [68].
- Simulated Annealing (SA): [69].
- Tabu Search Algorithm (TSA): [70].
- Evolutionary Programming (EP): [71].
- Particle Swarm Optimization (PSO): [72].
- Ant Colony Optimization (ACO): [73].
- Ant Lion Optimizer (ALO): [74].
- Artificial Bee Colony Algorithm (ABCA): [75].
- Artificial Fish Swarm Algorithm (AFSA): [76].
- Artificial Immune System Algorithm (AISA): [77].
- Artificial Sheep Algorithm (ASA): [78].
- Bacterial Foraging Algorithm (BFA): [79].
- Cuckoo Search Algorithm (CSA): [80].
- Differential Evolution Algorithm (DEA): [81].
- Exchange Market Algorithm (EMA): [82].
- Firefly Algorithm (FFA): [83].
- Fireworks Algorithm (FWA): [84].
- Gravitational Search Algorithm (GSA): [85].
- Grey Wolf Optimizer (GWO): [86].

- Imperialist Competitive Algorithm (ICA): [87].
- Intensify Harris Hawks Optimizer (IHHO): [88].
- Memory Management Algorithm (MMA): [89].
- Quantum-Inspired Evolutionary Algorithm (QIEA): [90].
- Quasi-Oppositional Teaching Learning Based Algorithm (QOTLBA): [91].
- Shuffled Frog Leaping Algorithm (SFLA): [92].
- Sine–Cosine Algorithm (SCA): [93].
- Whale Optimization Algorithm (WOA): [94].

In turn, these algorithms can be combined with each other to achieve a better performance, or even with commercial solvers if the problem is divided into multiple resolution stages. Some examples in the unit commitment literature are in [95–99]. Furthermore, an excellent state-of-art on EO is presented in [100].

4. Trends in Computational Efficiency to Solve the Unit Commitment Problem

4.1. Evolution of Unit Commitment Modeling Trends and Current Situation

In spite of the application of advanced evolutionary algorithms to the resolution of the unit commitment problem, the mainstream research trend is the utilization of numerical optimization algorithms to find the global optimum. As it was mentioned in Section 3, the implicit computational burden of this problem limited the scope of numerical optimization.

In the beginning, research was mainly focused on improving optimization algorithms. In turn, decomposition techniques were also deeply studied in order to accelerate the resolution processes. Despite their computational advantages, the utilization of decomposition techniques introduces complexity in the problem, such as difficulties in the implementation or the oscillations in the iterative convergence to the optimal solution.

For that reason, when commercial solvers gained competitiveness, the research in the UC problem started to change its direction. The advances in computation and the huge developments in numerical optimization promoted an exponential enhancement of MIP solvers [101]. Commercial solvers were able to address the unit commitment problem in reasonable run times even if decomposition techniques were not utilized. Then, the research began to focus on the formulation, instead of the decomposition, in order to exploit the properties of the solvers efficiently.

Suddenly, it was possible to think about improving the detail representation in the unit commitment problem. In turn, these advances needed to be in accordance with the solver trends. The performance of MILPs was more developed than MIQPs or MIQCPs and much more than MINLPs. For that reason, even quadratic formulations that were able to be solved as is were linearized in order to achieve a faster resolution.

The foundations of the UC modeling were laid. The detailed representation of the quadratic production costs was assumed through a piecewise linearization. Moreover, the dependence between start-up costs and the offline time steps accounted by the thermal units was modeled by a stairwise function [102]. The importance of convexity was manifested, both for thermal units and hydro generators [103]. Regarding the reserves [104] and power trajectories, efficient formulations were proposed to deal with them [105]. In these formulations, it was usual to employ three binary variables for each thermal unit and time step: one to define the commitment status and the others to represent a start-up or shutdown process. Despite that, binary variables were also utilized for piecewise linearizations, whose performance has been studied as well [106], and for stairwise representations [107].

Nevertheless, despite the improvements accomplished by the commercial solvers, the UC remains a strongly NP-hard problem [108]. For that reason, if greater horizons, thermal portfolios, or technical representations are desired, a trade-off between the modeling detail and the size of the corresponding problem must be chosen.

Modeling simplifications are mandatory when bidding processes are included in the representation [109], even more, if congestion in transmission lines is also desired to be included [110]. In turn, disaggregation methods are studied as an alternative for extending the scope of the representation. Ref. [111] breaks the horizons and solves shorter time spans sequentially. Ref. [112] decomposes the problem according to power market stages to increase the detail and links them heuristically later. Ref. [113] links longer-term with day-ahead decisions. Ref. [114] also uses multiple stages to simulate the competitive behavior of the market players, and in [115], a sequential resolution is proposed, too.

Additionally, the efficiency of the formulations is continuously studied. The optimization of the number of constraints and variables, especially binary variables, to model the same operation is addressed in depth in order to gain accuracy without a high computational cost. Ref. [116] points out the importance of reducing the number of binary variables that are employed per thermal unit and time step. Meanwhile, Ref. [117] analyzes the response of the same thermal portfolio when the optimization is addressed as a cost minimization problem and as a PBUC. On the other hand, Ref. [118] studies how to increase the flexibility in the operation efficiently, especially regarding fuel consumption.

According to the apparent complications introduced by binary variables, Ref. [119] proposed to work with them as relaxed variables and penalize the non-integrality in the OF. On the other hand, Ref. [120] uses a formulation with only one bin and imposes constraints to achieve their integrality behavior after a more efficient resolution. This approach reduces the number of branches that are created by the resolution algorithm because of the lower binary variables defined. This philosophy is also adopted in [121], where bidding processes are considered. Despite the defense of their utilization, the results of [121] show that the conventional three-bin formulations achieve practically the same performances.

This issue was further studied in [122,123] where the tightness of a UC formulation was defined as the proximity of the relaxed solution and the integer solution. These solutions are desired to be close in order to enhance the performance of the solver. For that reason, some additional constraints are proposed to tighten the feasible region when the problem is relaxed in the iterations of the Branch & Cut. Moreover, Ref. [123] concludes that the advances in commercial solvers cause the obsolescence of the formulation proposed in [120], since the solver can manage the branch creation properly and utilizes integer variables to generate cuts as well. If the integrability is not defined, the cuts will ignore these continuous variables that should be integer ones, hindering the performance of the solver. The Tight formulation proposed in [123] is quite renowned in the literature.

Tight formulations improve the performance of the solver by approximating the gap in the resolution process. However, it is also important for the tightened relaxed problem to be easy to solve. In that way, if many constraints are added, the computational cost of this step will be notably increased. Ref. [124] defines the concept of compactness according to the necessity of the lower number of variables, constraints, and non-zero elements in the problem matrix. For that reason, it is mandatory to evaluate a trade-off between Tightness and Compactness. Ref. [124] proposes a Tight and Compact (T&C) formulation where the importance of power trajectories is exposed, as it is reiterated later in [125]. In [126], the T&C formulation is extended to a thermal portfolio and, in [127], it is analyzed in depth and improved. The T&C formulation with power trajectories is exploited to achieve a proper evaluation of the reserve management in real operations of real power systems, Ref. [128] and their properties are deeply studied in [129]. T&C remains a powerful and renowned formulation nowadays.

Since that moment, UC formulations have tried to be as tight and compact as possible. For this reason, the proposition made in [130] to apply perspective functions in the UC problem was retaken. Perspective reformulations can be utilized to replace quadratic constraints by linear or second-order cone programming [131], enhancing the resolution process. The computational advances of [131] are exploited in [132] to develop an efficient tight formulation. Moreover, this technique is evolved in [133–135] towards the proposition of a two-bin formulation. Ref. [135] is a renowned formulation that projects the thermal production in continuous variables that are bounded in [0, 1], and it does not need the utilization of the shut-down binary variables.

On the other hand, the detail representation takes advantage of the computationally advanced formulations, choosing their equations as efficient ways to model technical operations. In turn, they are complemented with aspects that are desired to be submitted to study. That is the case of emission targets. The inclusion of emission targets in the UC problem supposes its transformation to a multi-objective problem, which has to be solved through heuristics [136–138]. However, it can be addressed as an MIP problem by giving a penalty to the emissions in the OF, such as [139]. Moreover, non-convex functions that represent emissions targets can be linearized, and the problem can rely on some emission parameters that are introduced as input data [140]. In turn, modeling transmission constraints is also gaining importance in detailed models [141,142] , as well as considering non-conventional energy-storage options [143].

Meanwhile, important issues for solving the UC problem are also studied, such as the presence of symmetry [144]. This phenomenon occurs when there are several identical generation units. In that case, the resolution process is slower because the solver does not cut branches that offer the same solution with a different combination of scheduled units. Symmetric-breaking constraints are added to avoid these situations [145], and their performances are compared against the developments in commercial solvers to handle with symmetry [146]. There is not a clear consensus. Apparently, current solvers can deal with symmetry properly, although the constraints seem to be useful in small size problems [147]. Furthermore, this symmetry can be exploited; Ref. [148] solves aggregated units and later decomposes the results after the optimization.

In turn, the current trends of renewable-energy-sources penetration in modern power systems point out an increment in the amount of start-up and shut-down processes accounted by thermal generators [149]. These ongoing changes highlight the necessity of extending the horizon of the UC problem [150] for considering medium-term technical constraints, such as maintenance schedule [151]. This could be achieved through clustering techniques such as [152]. The reduction of variables due to the aggregation of identical generators can be harmonized with an extension of the time span, keeping the run times assumed before the clustering. According to the research gap for enhancing the start-up costs [153], a more accurate stairwise function representation is presented in [154], and a new efficient formulation is proposed in [155]. Furthermore, more detailed generation schedules can be obtained by considering the possible turbine configurations of a power plant. With this purpose, a tight and compact formulation to represent this accurate operation is presented in [156].

The more efficient modeling techniques are translated into more accurate models with a higher scope. Ref. [157] is focused on a more detailed representation of thermal units, hydro generators, and pumped storage with a clear differentiation between turbine-production and pumping processes, as well as upper and lower reservoirs with their respective inflows. Ref. [158] models these issues and a simplified transmission network, too. The transmission representation was enhanced in [159] and later complemented with a simplified natural gas network in [160], where fluctuating natural gas flows in the pipelines toward the thermal unit that consume this fuel are considered.

Finally, it is important to highlight that new efficient formulations have been proposed during the last years. Ref. [161] is specially designed for fast-ramping thermal units. In [162], the commitment status binary variables are replaced by transition state variables, achieving great computational results. These renowned formulations [126,135,162] are compared in [163], manifesting the good performance of [126] despite being proposed a few years earlier than the others. Another comparison and new formulations are exposed in [164], pointing out the importance of this research field nowadays. Additionally, a standardized method to test formulations is employed, and an excellent explanation of the unit commitment evolution since a formulating point of view is described.

4.2. Precise Description of the Modeling Detail Adopted in the Literature

As it was manifested in [164], it is essential to establish a standardized benchmark to compare the efficiency of the unit commitment formulations. Despite traditional thermal portfolios [165] and power systems such as the IEEE 118-bus [166,167], or the IEEE RTS-96

[168] are frequently employed in the literature, it is also desirable to set other relevant issues. These concerns are different modeling details, algorithm performances, solver options, and computational resources to handle the optimization process.

The unit commitment is a widely studied optimization problem, and, as a consequence, there are several good reviews on this topic. Recent publications are [100,169–175]. There, comparisons about optimization techniques [169,170], uncertainty representation [171,172], and resolution techniques [100,173,174] are presented. However, advantages and disadvantages are usually exposed from a general perspective, as shown in Tables 1 and 2.

The literature reviews proposed in [100,173,174] gathered hardware specifications (if given), case study systems, run times, and a general explanation about the UC constraints that were employed in their references. Nevertheless, a more exhaustive analysis on modeling detail is necessary if a clear vision of the scope of the citations is required. On the other hand, an intensive description of UC modeling is made in [175], but a literature benchmark with the modeling properties of a set of different case studies and methodologies is not presented.

For that reason, bringing out a structured presentation of the modeling detail achieved through the different techniques referenced in this paper satisfies one of the most urgent requirements of the literature reviews. A general description of the technical and economic aspects represented in the unit commitment problem is made in Section 1. Although the theoretical scope of the representation is practically unlimited, reality demands a tight trade-off between modeling detail, system size, and computational resources to find an optimal or near-optimal solution. Considering that, Table 3 exposes a meticulous characterization of these features.

This summary provides the reader with a precise vision of the trade-off decisions assumed by the different authors when presenting their methodologies. An exact idea about the accuracy of the represented power system (detail, size, and horizon) and the corresponding computational tractability (run time and optimality of the solution) is given. In this way, the reader can discern between the scopes of the optimization proceedings presented in this paper, keeping in mind that it is a small collection of the most representative techniques applied to solve the unit commitment problem.

According to the information gathered in Table 3, the following clarifications are done:

- The number of segments in piecewise linearizations is specified when they are reported. If they are not given in the table, it means that quadratic coefficients were presented in the paper, and the author just said that the function was linearized.
- Hourly granularity is considered unless another specification is shown in the time span column. Additionally, time period chronology is also supposed to be respected except if a disaggregation technique is mentioned in that column.
- The symbol (r) means replication. It is shown when the number of units that compose a generation portfolio is repeated to deal with bigger systems in the case study or when the data presented for a shorter time span (typically a day) are imitated to make it longer. It entails computational conditions such as symmetry or identifiable patterns.
- The information presented in the demand constraint column denotes the elements that participate in the balance equation. If it is described as *equal* or *greater*, the unit commitment problem is addressed as a cost minimization problem where the load demand has to be matched or exceeded, respectively. In the case of *PBUC*, it is assumed that the maximization has no generation limits unless a *(lower)* term is added to show that total generation must not exceed the load demand.
- The information presented in the reserve constraints column is not always consistent. Some systems include spinning reserve in primary, secondary, or tertiary reserves. However, other systems establish different distinctions. The reserve dependence on the regulatory framework is out of the scope of this review. For that reason, reserves are shown as they are specified in each paper.

- The operative constraints column is sometimes used to show miscellaneous information, such as hydro representation, market specifications, etc., for the sake of clarity. Thus, the whole case study information is presented in the same row and table, despite space limitations.
- The optimal column determines the capability of the proposed methodology to assure finding the global optimum. In turn, when the paper reports the value of the relative optimality criterion, which is imposed on MIP solvers, it is also exposed in this column. This optimality gap is the quotient of the difference between primal and dual bound, in absolute value, and the maximum of both. This feature can be specified before solving an MIP problem. It should not be confused with the dual gap (defined when decomposition techniques are applied) or the integrality gap (calculated when the optimization ends). It clarifies the scope of the methodologies, providing a significant idea about their efficiency and the trade-off between modeling detail, run time, and computational performance.
- Executions are made in regular computing machines up to the date of the publications to whom any researcher could have access. If they are run in high efficient clusters, whose affordability is limited to generation companies or universities, it will be specified in the run time column.
- The data used in each case study are supposed to be given or properly cited. If some technical or economic aspect is modeled, but any input data are provided, it is specified with a (*). Moreover, if these data are apparently given, but the link is offline by the date that this review is sent to the publisher or the referenced article does not provide the information, it is specified with a (**).
- The generation limits of the thermal units are always considered to be given, except if the rest of the information is also missing.
- If there is at least one start-up cost represented in the methodology, the logic constraint that establishes commitments, start-ups, and shut-downs is formulated.

Table 3. Technical and economic modeling detail accomplished in the literature to represent power systems and computational performance of the methodologies.

Ref	Method	Production Cost	Start-up Cost	Shut-down Cost	Emission Cost	Mainten. Cost	Demand Constraint	Reserve Constraints	Ramp Limits	TU/TD Constraints	Emission Constraints	Operative Constraints	Network Constraints	Optimal	Problem Size	Time Span	CPU Time
[2]	EE	quadratic	1 SU	-	-	yes	equal NSE OE	-	-	-	-	-	-	near-opt.	10 units	1 year through 6 time steps	-
[3]	ES	linear	1 SU	-	-	-	base level mid. level peak level	-	-	-	-	-	-	near-opt.	26 units	24 h	∼20 min
[4]	PL	quadratic	2 SU	-	-	-	equal	primary	-	yes	-	-	-	near-opt.	10 units 100 units (r) 100 units (r)	24 h 24 h 120 h	0.01 s 0.06 s <1 s
[5]	PL	quadratic	2 SU	-	-	-	equal	spinning	yes	yes	-	-	-	near-opt.	10 units 100 units (r)	24 h	2 s 174 s
[6]	PL	quadratic	2 SU	-	-	-	equal	spinning	yes *	yes	-	-	-	near-opt.	10 units 100 units (r)	24 h	0.154 s 0.275 s
[7]	PL	quadratic	2 SU	-	-	-	equal ND-RES EVs	spinning	-	yes	-	battery capacity limits charge/discharge limits charge/discharge efficiency	-	near-opt.	10 units 1 wind gen. 1 solar gen. 50000 PEVs	24 h	-
[8]	FL OP(CO) PSO	quadratic	2 SU	-	-	-	equal	spinning	-	yes	-	-	-	near-opt.	10 units 100 units (r)	24 h	18.34 s 1734.67 s
[9]	FL OP(CO) heur. alg.	quadratic	1 SU	-	-	-	PBUC	-	yes	yes	-	-	-	near-opt.	5 units 36 units	24 h	-
[10]	FL OP(CO) PSO	quadratic	2 SU	-	-	-	equal dem. response ND-RES ND-RES-C EVs	spinning	-	yes	calculated after resolution	demand response—curtailment demand response—shifting battery capacity limits charge/discharge limits quadratic charging cost quadratic discharging cost	-	near-opt.	10 units 1 wind gen. EVs	24 h	-
[11]	NN	quadratic *	-	-	-	-	equal	-	-	-	-	-	-	near-opt.	26 units 11 units	24 h 168 h	-
[12]	NN	quadratic	2 SU	-	-	-	equal	spinning *	yes *	yes	-	prohibited operating zones	-	near-opt.	10 units	24 h	<0.02 s
[14]	HM ES & OP(DP)	quadratic	exponential	-	-	-	PBUC lower	-	-	yes	-	-	-	near-opt.	110 units	24 h	15 min
[15]	HM NN & OP(DP)	quadratic	exponential	-	-	-	equal	spinning	-	yes	-	must run units prohibited operating zones quadratic generation losses	-	near-opt.	7 units	24 h	4.5 s
[16]	HM NN & PL & OP(CO)	quadratic	exponential 1 SU	-	-	-	equal	spinning	yes	yes	-	-	-	near-opt.	26 units 45 units	24 h	2.17 s 1.84 s

Table 3. *Cont.*

Ref	Method	Production Cost	Start-up Cost	Shut-down Cost	Emission Cost	Mainten. Cost	Demand Constraint	Reserve Constraints	Ramp Limits	TU/TD Constraints	Emission Constraints	Operative Constraints	Network Constraints	Optimal	Problem Size	Time Span	CPU Time
[17]	HM PL & OP(CO)	quadratic	2 SU	-	-	-	equal	spinning	-	yes	-	-	-	near-opt.	10 units 100 units (r)	24 h	10 s 73 s
[18]	HM PL & FL & OP(DP) SA	quadratic	2 SU	-	-	-	equal	spinning	yes	yes	-	-	-	near-opt.	10 units 100 units (r)	24 h	-
[19]	HM PL & OP(CO) ACO	quadratic	2 SU	-	-	-	equal	spinning	-	yes	-	-	-	near-opt.	10 units 40 units (r)	24 h	252 s 2578 s
[20]	HM ES & OP(CO) PSO	quadratic	2 SU	-	-	-	equal	spinning	yes	yes	-	must run units*	-	near-opt.	10 units 100 units (r) 40 units	24 h 24 h 168 h	3 s 143 s 1661 s
[21]	HM PL & OP(CO) PSO	quadratic	2 SU 2 SU -	-	-	-	equal	spinning	- - yes	yes	-	-	-	near-opt.	10 units 100 units (r) 26 units	24 h	20 s 1700 s 906 s
[25]	OP(DP) heuristics heur. alg.	quadratic	1 SU	-	-	-	equal	-	yes *	yes	-	-	-	near-opt.	1 unit	24 h 96 h 168 h	<1 s
[26]	OP(DP) heuristics heur. alg.	quadratic **	1 SU **	-	-	-	equal **	-	yes **	yes **	-	-	-	near-opt.	200 unit (r)	24 h	134 s
[27]	OP(DP) OP(DP) heuristics OP(DP) heuristics	quadratic	2 SU	-	-	-	equal	spinning *	-	yes	-	-	-	yes near-opt. near-opt.	10 unit	24 h	458 s 12 s 63 s
[28]	OP(DP) heuristics SO-MS MILP solver	linear	1 SU	1 SD	-	-	equal	-	yes	yes	-	-	flow limits	near-opt.	24 buses 38 lines 33 units	24 h	- 225 scen.
[29]	OP(DP) heuristics SO-MS MIP solver	non specified function	1 SU	1 SD	-	-	equal NSE OE TL	spinning	yes	yes	-	SU/SD capacity limits	flow limits shift factor	near-opt.	118 buses 181 lines 54 units	24 h	~1.25 h 50 scen. 16 cores

Table 3. *Cont.*

Ref	Method	Production Cost	Start-up Cost	Shut-down Cost	Emission Cost	Mainten. Cost	Demand Constraint	Reserve Constraints	Ramp Limits	TU/TD Constraints	Emission Constraints	Operative Constraints	Network Constraints	Optimal	Problem Size	Time Span	CPU Time
[30]	OP(DP) heuristics SO-MS MIP solver	quadratic **	1 SU **	1 SD **	-	-	equal ** ND-RES ** ND-RES-C ** TL **	spinning ** operative **	yes **	yes **	-	SU/SD capacity limits ** CCGT configuration modes capacity limits per mode ramp rates per mode ramp rates per edge TU/TD per mode SU/SD cost per mode mode transition costs gas fuel rates for gens. pipelines—cap. limits gas sources—cap. limits compressors—comp. ratios	flow limits ** shift factor **	near-opt.	24 buses 38 lines 33 units 6 CCGTs 2 wind gen.	24 h by 30 min time steps	~28.3 min 200 scen.
[31]	OP(CO) SO-TS MILP solver	linear **	1 SU **	-	-	-	equal ND-RES TL	-	-	yes **	-	-	flow limits ** reactances ** phase angles **	yes	225 buses 375 lines 130 units wind gen.	24 h	~5.7 h 10 scen. 10 cores ~7.4 h 50 scen. 50 cores
[32]	OP(CO) SO-TS MILP solver	stepwise marginal cost function	1 SU	-	-	-	equal NSE ND-RES ND-RES-C	non-spinning	yes *	yes *	-	interval ramp capabilities * SU/SD durations * SU/SD power outputs *	-	yes 0.1% gap	29 thermal 17 hydro 1 wind gen.	1 year by 15 min time steps and 36 h look ahead horizons	177 h 9 scen.
[33]	OP(CO) SO-TS MILP solver	linear **	1 SU **	-	-	-	equal ND-RES ESS	spinning **	yes **	yes **	-	reservoir energy limits ** turb. and pump. limits ** pumping efficiency **	flow limits **	yes 0.1% gap	27 lines 130 units 16 PSUs	24 h	~375 min 50 scen.
[34]	OP(CO) SO-TS MILP solver	marginal cost	1 SU	-	-	-	equal NSE TL	reserve	yes	-	hourly max. gen-emiss. (2-block piecewise)	max. reserve per unit	flow limits voltage limits reactances phase angles	yes 0.0001% gap	118 bus 186 lines 25 coal plants	24 h	1.7 h 30 scen.
[35]	OP(CO) SO-TS MILP solver	linear	1 SU	1 SD	-	-	equal NSE HS ND-RES ND-RES-C TL	spinning 1&2 tertiary spin. non-spinning	yes	yes	-	spinning 1&2 r. cost tertiary spin. r. cost non-spinning r. cost reservoir inflows reservoir spillage reservoir volume limits reservoir discharge limits	flow limits shift factor	yes	9 bus 9 lines 3 thermal 1 hydro 2 wind gen. 118 bus 179 lines 54 thermal 8 hydro	168 h	10 min 5 scen. 1 h 55 min 1 scen.
[36]	OP(CO) SO-TS heur. alg.	piecewise **	stairwise **	-	-	-	equal ND-RES ND-RES-C	spinning **	yes **	yes **	-	-	-	near-opt.	240 bus 448 lines 85 thermal wind gen.	24 h	~200 s 50 scen. ~800 s 100 scen.

Table 3. *Cont.*

Ref	Method	Production Cost	Start-up Cost	Shut-down Cost	Emission Cost	Mainten. Cost	Demand Constraint	Reserve Constraints	Ramp Limits	TU/TD Constraints	Emission Constraints	Operative Constraints	Network Constraints	Optimal	Problem Size	Time Span	CPU Time
[37]	OP(CO) SO-TS MILP solver	linear	1 SU	-	-	-	equal HS TL	spinning	yes	yes	-	reservoir inflows reservoir spillage reservoir final volume reservoir volume limits reservoir discharge limits	flow limits shift factor	yes	83 bus 143 lines 15 thermal 28 hydro	24 h	15 min 4 scen. 69 min 9 scen. 149 min 15 scen. 326 min 25 scen.
[38]	OP(CO) SO-TS MILP solver	linear **	1 SU **	-	-	-	equal NSE	reserve **	yes **	yes **	-	flexible line rating recourse (off, normal, high) line min. hours normal ** line max. hours high **	flow limits ** reactances ** phase angles **	yes	364 bus 594 lines 113 slow units 26 fast units	24 h	18 h 10 scen.
[39]	OP(CO) SO-TS MIP solver	piecewise **	1 SU **	-	-	-	equal NSE ND-RES ND-RES-C ESS TL	-	yes **	yes **	-	expected unserved energy ESS charge ramp rates ESS discharge ramp rates	flow limits ** shift factor **	yes	39 bus 46 lines 10 units 1 wind gen. 1 ESS	24 h	551 s 4 scen.
[40]	OP(DP) SO-MS MIP solver	quadratic	1 SU	-	-	-	greater	-	-	yes	-	-	-	yes	10 units 20 units (r) 10 units 20 units (r)	24 h 24 h 48 h (r) 48 h (r)	79 s 2 scen. 379 s 2 scen. 7117 s 4 scen. 8962 s 4 scen.
[41]	OP(DP) SO-MS MILP solver	linear **	-	-	-	yes **	piecewise residual demand and revenue curves 10-20 blocks **	-	yes **	yes **	-	generation company offers self-consumption in units reservoir energy inflows reservoir energy limits reservoir energy splits turb. and pump. limits pumping efficiency	-	yes	Spanish market company with 20 units	168 h with 24 h look ahead capacity	∼1500 s 8 scen.
[42]	OP(DP) SO-MS MILP solver	linear	1 SU	-	-	-	equal	-	yes	yes	-	spinning reserve costs payments for possible being committed for a particular hour	-	yes 5% gap	32 units	24 h (discrete) 24 h 5 min data continuous cubic spline	40 min 24 scen. 67 h 24 scen.
[43]	OP(CO) SO-RC MILP solver	piecewise (3 blocks)	1 SU	-	-	-	equal NSE TL	spinning	yes	yes	-	expected energy not served loss of load probability unit outage probability	flow limits reactances phase angles	yes	73 bus 147 lines 96 units (r)	24	<1 h
[44]	OP(CO) SO-RC MILP solver	piecewise (3 blocks)	1 SU	-	-	-	equal NSE	spinning	yes	yes	-	expected energy not served loss of load probability unit outage scenarios	-	yes 0.5% gap	96 units (r)	24	<1200 s

Table 3. *Cont.*

Ref	Method	Production Cost	Start-up Cost	Shut-down Cost	Emission Cost	Mainten. Cost	Demand Constraint	Reserve Constraints	Ramp Limits	TU/TD Constraints	Emission Constraints	Operative Constraints	Network Constraints	Optimal	Problem Size	Time Span	CPU Time
[45]	OP(CO) SO-RC MILP solver	linear	1 SU	-	linear	-	equal NSE ND-RES ND-RES-C ESS	spinning	yes	yes	-	expected energy not served loss of load probability reservoir energy limits turb. and pump. limits pumping efficiency	-	yes 0.5% gap	71 slow units 35 fast units 1 PSU	24 h by 15 min time steps	5.5 h to 6.9 h
[46]	OP(CO) SO-CCO MIP solver	quadratic	1 SU	-	-	-	equal ND-RES TL	spinning non-spinning	yes	yes	-	reserve ramp limits spinning reserve costs non-spin. reserve costs wind farm outages	flow limits shift factor	yes	30 bus 41 lines 6 units 2 wind gen.	24 h	tens of minutes
[47]	OP(CO) heuristics SO-CCO MILP solver	linear	1 SU	-	-	-	equal NSE ND-RES ND-RES-C TL	spinning	yes	yes	-	reserve costs loss of wind probability loss of load probability trans. line overloading trans. line overload. prob.	flow limits shift factor	near-opt.	118 bus 186 lines 15 fast units 3 wind gen.	24 h	432.6 s
[48]	OP(CO) SO-CCO MILP solver	linear	2 SU	1 SD	-	-	equal NSE ND-RES ND-RES-C TL	spinning	yes	yes	-	spinning reserve costs SU/SD capacity limits	flow limits shift factor	yes 0.5% gap	118 bus 186 lines 54 units 10 wind gen.	24 h	1092 s 500 scen.
[49]	OP(CO) RO MILP solver	piecewise (3 blocks)	2 SU	-	-	-	equal ND-RES ND-RES-C TL	reserve	yes	yes	-	1 EIE is composed by · Electricity generators · EFs (batch operation) · EAs (always on)	flow limits shift factor	yes	24 bus 40 lines 32 units (r) 1 wind gen. 1 EIE = 4 gen. units 12 EFs 1 EA	24 h	175.74 s
[50]	OP(CO) RO MILP solver	linear	2 SU	1 SD	-	-	equal NSE ND-RES ND-RES-C TL	spinning	yes	yes	-	power trajectories SU/SD durations SU/SD power trajectories wind generation bids flow-limit violation penalties	flow limits shift factor	yes	118 bus 186 lines 54 units 3 wind gen.	24 h	13.8 s
[51]	OP(CO) RO MILP solver	piecewise	1 SU	-	-	-	greater	-	yes	yes	-	prohibited operating zones SU/SD capacity limits dynamic ramp rates reservoir inflows reservoir spillage cascade reservoirs reservoir volume limits reservoir discharge limits	-	yes	54 thermal 8 hydro	24 h	-
[52]	OP(CO) IO MILP solver	piecewise	2 SU	-	-	-	equal ND-RES ND-RES-C TL	spinning operative	yes	yes	horizon max. gen-emiss. ** SU emiss. ** SD emiss. **	-	flow limits shift factor	yes	118 bus 186 lines 54 units 3 wind gen.	24 h	73 s
[53]	OP(CO) IO MILP solver	piecewise	1 SU	-	-	-	equal ND-RES ND-RES-C TL	spinning	yes	yes	-	max. spin. reserve per unit	flow limits shift factor	yes	118 bus 186 lines 54 units 4 wind gen.	24 h	246.39 s

Table 3. *Cont.*

Ref	Method	Production Cost	Start-up Cost	Shut-down Cost	Emission Cost	Mainten. Cost	Demand Constraint	Reserve Constraints	Ramp Limits	TU/TD Constraints	Emission Constraints	Operative Constraints	Network Constraints	Optimal	Problem Size	Time Span	CPU Time
[54]	OP(CO) IO MILP solver	piecewise	1 SU	-	-	-	equal ND-RES ND-RES-C TL	-	yes	yes	-	-	flow limits reactances phase angles	yes 1% gap	73 bus 147 lines 96 units (r) 4 wind gen.	24 h	<2500 s
[55]	OP(CO) MCS MIP solver	non specified function	1 SU	-	-	-	equal ND-RES TL	spinning operative	yes	yes	-	-	flow limits reactances phase angles	yes	118 bus 186 lines 76 units 1 wind gen.	24 h	<30 min 10 scen.
[56]	OP(CO) MCS rMIP solver	linear *	1 SU *	1 SD *	linear *	yes *	equal * NSE * OE * HS * ND-RES * ND-RES-C * ESS * imports exports	spinning *	yes *	yes *	-	competitive gen. companies * unplanned unavailabilities * programmed mainten. * hydro condition profiles * reservoir energy inflows * reservoir energy limits * turb. and pump. limits * pumping efficiency * cascade reservoirs * run-of-river generation * wind generation profiles * solar generation profiles * regulatory framework *	flow limits at system interconn. facilities *	yes	Spanish system 800,000 variables 350,000 const. after presolve?	1–2 month through ≤40 time steps	<2 min (relaxed) per scen.
[57]	OP(CO) MCS rMIP solver	linear *	1 SU *	1 SD *	linear *	yes *	equal * NSE * OE * HS * ND-RES * ND-RES-C * ESS * imports exports	spinning *	yes *	yes *	-	competitive gen. companies * unplanned unavailabilities * programmed mainten. * hydro condition profiles * reservoir energy inflows * reservoir energy limits * turb. and pump. limits * pumping efficiency * cascade reservoirs * run-of-river generation * wind generation profiles * solar generation profiles * grid-stability commitments * regulatory framework *	flow limits at system interconn. facilities *	yes	Spanish system French system Portuguese system 1120000 variables 720000 const. after presolve	3 year through 940 time steps	∼2000 s (relaxed) per scen.
[58]	OP(CO) HUI SO-TS & SO-RC MILP solver	piecewise	1 SU	-	-	-	equal NSE ND-RES ESS TL	spinning *	yes	yes	-	electricity price elasticity max. spin. reserve per unit ESS generation capacities ESS storage capacities loss of load probability	flow limits reactances phase angles	yes	118 bus 186 lines 54 units 1 wind gen. 4 ESSs	24 h	∼60 min 100 scen.

Table 3. *Cont.*

Ref	Method	Production Cost	Start-up Cost	Shut-down Cost	Emission Cost	Mainten. Cost	Demand Constraint	Reserve Constraints	Ramp Limits	TU/TD Constraints	Emission Constraints	Operative Constraints	Network Constraints	Optimal	Problem Size	Time Span	CPU Time
[59]	OP(CO) HUI SO-TS & IO MILP solver	piecewise (3 blocks)	2 SU	-	-	-	equal NSE ND-RES ND-RES-C TL	-	yes	yes	-	-	flow limits reactances phase angles	yes 0.1% gap	24 bus 40 lines 32 units (r) 1 win gen.	24 h	∼800 s 10 scen. hour 1 switch SO→IO ∼15,000 s 10 scen. hour 24 switch SO→IO
[60]	OP(CO) HUI SO-TS & RO MILP solver	linear	1 SU	-	-	-	equal NSE ND-RES ND-RES-C TL	-	yes	yes	-	SU/SD capacity limits **	flow limits reactances phase angles	yes 0.03% gap	73 bus 147 lines 96 units (r) 15 win gen.	24 h	1322.7 s 50 scen.
[61]	OP(DP) heuristics MIP solver	quadratic	1 SU	-	-	-	greater	-	-	yes	-	-	-	near-opt.	20 units (r)	24 h	137 s
[62]	OP(CO) heur. alg.	quadratic	2 SU	-	-	-	equal	spinning	-	yes	-	-	-	near-opt.	29 units (r)	24 h	∼37 s
[68]	OP(CO) GA	quadratic	2 SU	-	-	-	equal	spinning	yes *	yes	-	-	-	near-opt.	10 units 100 units (r)	24 h	3.4 s 21.6 s
[69]	OP(CO) SA	quadratic	exponential	-	-	-	equal	spinning	-	yes	-	-	-	near-opt.	12 units	24 h	2 h 25 min
[70]	OP(CO) TSA	quadratic	1 SU	-	-	-	greater	spinning	-	yes	-	bounded spinning reserve	-	near-opt.	40 units	24 h	187 s
[71]	OP(CO) EP	quadratic	2 SU	-	-	-	equal	spinning	-	yes	-	-	-	near-opt.	100 units (r)	24 h	1 h 42 min
[72]	OP(CO) PSO	quadratic	2 SU	-	-	-	equal	spinning	yes	yes	-	-	-	near-opt.	10 units	24 h	∼9 s
[73]	OP(CO) ACO	quadratic	2 SU	-	-	-	equal	spinning	yes *	yes	-	-	-	near-opt.	10 units 100 units (r)	24 h	39 s 1535 s
[74]	OP(CO) ALO	quadratic	2 SU	-	-	-	equal	spinning	-	yes	-	-	-	near-opt.	10 units	24 h	∼4 min

Table 3. *Cont.*

Ref	Method	Production Cost	Start-up Cost	Shut-down Cost	Emission Cost	Mainten. Cost	Demand Constraint	Reserve Constraints	Ramp Limits	TU/TD Constraints	Emission Constraints	Operative Constraints	Network Constraints	Optimal	Problem Size	Time Span	CPU Time
[75]	OP(CO) ABCA	quadratic	2 SU	-	-	-	PBUC lower	-	-	yes	-	-	-	near-opt.	10 units 100 units (r) 1000 units (r) 10 units 100 units (r) 1000 units (r)	24 h	135.23 s (1 core) 1216.08 s (1 core) 8456.52 s (1 core) 14.87 s (20 cores) 109.16 s (20 cores) 457.24 s (20 cores)
[76]	OP(CO) AFSA	quadratic	1 SU	-	-	-	PBUC lower	spinning	yes	yes	-	-	-	near-opt.	10 units 100 units (r)	24 h	∼13.25 s ∼664.52 s
[77]	OP(CO) AISA	quadratic	2 SU	-	-	-	PBUC lower	spinning *	-	yes	-	-	-	near-opt.	10 units	24 h	81 s
[78]	OP(CO) ASA	quadratic	2 SU	-	-	-	equal ND-RES ESS	spinning	-	yes	-	reservoir energy limits turb. and pump. limits	-	near-opt.	10 units 1 PSU 1 wind gen. 1 solar gen.	24 h	-
[79]	OP(CO) BFA	quadratic	2 SU	-	-	-	equal	spinning *	yes *	yes	-	-	-	near-opt.	10 units 100 units (r)	24 h	110 s 5800 s
[80]	OP(CO) CSA	quadratic	2 SU	-	-	-	equal	spinning	-	yes	-	-	-	near-opt.	4 units	8 h	1.91 s
[81]	OP(CO) DEA	quadratic	2 SU	-	-	-	equal	spinning	-	yes	-	-	-	near-opt.	10 units 100 units (r)	24 h	27.4 s 663.9 s
[82]	OP(CO) EMA	quadratic	-	-	-	-	equal	-	yes	-	-	prohibited operating zones generation losses	-	near-opt.	15 units	1 time step	<1 s
[83]	OP(CO) FFA	quadratic	2 SU	-	-	-	equal	spinning	yes *	yes	-	-	-	near-opt.	100 units (r)	24 h	113 s
[84]	OP(CO) FWA	quadratic	2 SU	-	-	-	equal PBUC lower	spinning	yes	yes	-	-	-	near-opt.	10 units 100 units (r) 10 units 100 units (r)	24 h	61.57 s (equal) 1039.80 s (equal) 58.77 s (PBUC) 1124.87 s (PBUC)
[85]	OP(CO) GSA	quadratic	-	-	-	-	equal	-	-	-	-	fuel cost of valve points (sinusoidal function)	-	near-opt.	13 units	1 time step	150.32 s
[86]	OP(CO) GWO	quadratic	2 SU	-	-	-	equal ND-RES ND-RES-C	-	yes	yes	-	-	-	near-opt.	10 units 1 wind gen.	24 h	-

Table 3. *Cont.*

Ref	Method	Production Cost	Start-up Cost	Shut-down Cost	Emission Cost	Mainten. Cost	Demand Constraint	Reserve Constraints	Ramp Limits	TU/TD Constraints	Emission Constraints	Operative Constraints	Network Constraints	Optimal	Problem Size	Time Span	CPU Time
[87]	OP(CO) ICA	quadratic	2 SU	-	-	-	equal TL	-	yes	yes	horizon max. gen-emiss. (quadratic)	reactive generation limits	flow limits voltage limits	near-opt.	118 bus 186 lines 54 units	24 h	71.2 s
[88]	OP(CO) IHHO	quadratic *	1 SU *	-	-	-	equal *	spinning *	-	yes *	-	-	-	near-opt.	10 units	24 h	-
[89]	OP(CO) MMA	quadratic	1 SU	-	-	-	PBUC lower	spinning	-	-	-	-	-	near-opt.	10 units	24 h	-
[90]	OP(CO) QEIA	quadratic	2 SU	-	-	-	equal	spinning	-	yes	-	-	-	near-opt.	10 units 100 units (r)	24 h	∼30 s ∼300 s
[91]	OP(CO) QOTLBA	quadratic	2 SU	-	-	-	equal	spinning	yes *	yes	-	-	-	near-opt.	100 units (r)	24 h	61.4 s
[92]	OP(CO) SFLA	quadratic	2 SU	-	-	-	equal	spinning	-	yes	-	-	-	near-opt.	10 units	24 h	-
[93]	OP(CO) SCA	quadratic	2 SU	-	-	-	PBUC lower	spinning	yes *	yes	-	-	-	near-opt.	10 units	24 h	34.86 s
[94]	OP(CO) WOA	quadratic	2 SU	-	-	-	PBUC lower	spinning	yes *	yes	-	-	-	near-opt.	10 units	24 h	37.86 s
[95]	OP(CO) BFA & GA	quadratic	1 SU 2 SU 2 SU	-	-	-	equal	spinning	yes	yes	-	-	-	near-opt.	38 units 10 units 100 units (r)	24 h	459 s 41 s 4503 s
[96]	OP(CO) EP & PSO	quadratic	2 SU	-	-	-	PBUC	spinning	yes	yes	-	bilateral contracts	-	near-opt.	10 units	24 h	-
[97]	OP(CO) DE & SM	quadratic	1 SU	-	-	-	PBUC lower	spinning	-	yes	-	-	-	near-opt.	10 units	24 h	116 s
[98]	OP(CO) DE & PSO	quadratic	1 SU	1 SD	-	-	equal TL	-	yes	yes	-	TSO-DSO coordination 1 DSs bus ∼5 generators	flow limits reactances phase angles	near-opt.	118 bus 186 lines 54 units 2 DSs 118 bus 186 lines 54 units 32 DSs	24 h	2.53 s 279.13 s
[99]	OP(CO) SA & PSO	quadratic	2 SU	-	-	-	equal	spinning	-	yes	-	-	-	near-opt.	10 units	24 h	-
[102]	OP(CO) MILP solver	piecewise (3 blocks)	11 SU	1 SD	-	-	PBUC	spinning	yes	yes	-	SU/SD capacity limits	-	yes	1 unit	24 h	slight

Table 3. *Cont.*

Ref	Method	Production Cost	Start-up Cost	Shut-down Cost	Emission Cost	Mainten. Cost	Demand Constraint	Reserve Constraints	Ramp Limits	TU/TD Constraints	Emission Constraints	Operative Constraints	Network Constraints	Optimal	Problem Size	Time Span	CPU Time
[103]	OP(CO) MIP solver	piecewise * (3 blocks)	1 SU	1 SD	-	-	equal HS imports exports	spinning *	-	yes	-	spinning reserve bounds reservoir inflows * reservoir spillage * cascade reservoirs * reservoir volume limits * reservoir discharge limits HR function (piecewise) * reservoir SU/SD costs reservoir TU/TD import/export limits import loss coefficient * export loss coefficient * system loss coefficient *	-	yes	13 thermal 16 hydro 1 intercon.	24 h	∼69 s
[104]	OP(CO) MILP solver	piecewise (10 blocks)	11 SU	1 SD	-	-	PBUC	10 min spinning 10 min non-spin. 30 min operative	yes	-	-	power trajectories SU/SD capacity limits AGC representation	-	yes	1 unit	24 h	51.51 s
[105]	OP(CO) MILP solver	linear	1 SU	1 SD	-	-	PBUC	-	yes	yes	-	power trajectories SU/SD durations SU/SD power trajectories	-	yes	1 unit	24 h 48 h (r) 96 h (r) 168 h (r)	0.66 s 4.10 s 14.35 s 153.01 s
[107]	OP(DP) SO-MS MILP solver	piecewise *	stairwise *	-	-	-	greater ESS	spinning *	-	yes *	-	reservoir energy limits turb. and pump. limits pumping efficiency	-	yes	25 thermal 7 PSUs	192 h	97.61 s 3 scen. 305.43 s 9 scen. 569.18 s 15 scen. 794.93 s 22 scen.
[109]	OP(CO) MILP solver	piecewise (4 blocks)	1 SU	-	-	-	residual demand and price-quota curves	-	yes	yes	-	-	-	yes	40 units in portfolio 120 units in market	24 h	∼15 min
[110]	OP(CO) heur. alg.	quadratic	-	-	-	-	piecewise supply curve TL	-	-	-	-	-	flow limits reactances phase angles	near-opt.	8 bus 11 lines 6 units	24 h	-
[111]	OP(CO) heur. alg.	quadratic	exponential	-	-	-	equal	primary secondary	yes	yes	-	-	-	near-opt.	10 units	24 h	19 s
[112]	OP(CO) heuristics MILP solver	linear *	2 SU *	1 SD *	-	-	equal * NSE HS ESS	spinning *	yes *	-	-	reservoir inflows reservoir spillage reservoir final volume reservoir volume limits reservoir discharge limits HR func. (3-block piecewise) pumping capacity limits pumping efficiency	-	near-opt.	44 thermal 102 hydro 6 PSUs	168 h by ≤4 h time steps	∼40 min

Table 3. *Cont.*

Ref	Method	Production Cost	Start-up Cost	Shut-down Cost	Emission Cost	Mainten. Cost	Demand Constraint	Reserve Constraints	Ramp Limits	TU/TD Constraints	Emission Constraints	Operative Constraints	Network Constraints	Optimal	Problem Size	Time Span	CPU Time
[113]	OP(CO) MILP solver	linear *	-	-	-	-	equal * NSE	spinning *	yes *	-	-	dispatch-deviation penalties reserve-deviation penalties non-smooth power profile pen. power trajectories AGC representation	-	yes	20 thermal 6 hydro 5 PSUs	28 h by 20 min time steps	<10 s
[114]	OP(CO) heur. alg.	quadratic	1 SU	-	-	-	PBUC lower	spinning	-	yes	-	-	-	near-opt.	10 units 100 units (r)	24 h	6 s 27 s
[115]	OP(CO) heuristics MILP solver	piecewise (10 blocks)	1 SU	-	-	-	equal	spinning	yes	yes	-	-	-	near-opt.	54 units	24 h	6.57 s
[116]	OP(CO) MILP solver	piecewise	1 SU	-	-	-	PBUC	spinning non-spinning	yes	yes	-	bilateral contracts CCGT configuration modes mode transition fuel	-	yes	54 thermal 12 CCGTs	24 h	∼90 s
[117]	OP(CO) MILP solver	piecewise * (2 blocks)	1 SU *	1 SD *	-	-	PBUC * equal *	spinning *	yes *	yes *	-	-	-	yes	27 units	24 h	-
[118]	OP(DP) heur. alg.	quadratic	1 SU	-	-	-	equal ESS TL	spinning operative	yes	yes	horizon max. ho. unit max. gen-emiss. (linear) SU emiss. (1 step)	reactive generation limits max. sustained ramp rate CCGT configuration modes TU per configuration fuel blending operation fuel switching operation unit fuel consump. limits horizon fuel consump. limits	flow limits voltage limits	near-opt.	118 bus 186 lines 54 thermal 12 CCGTs 2 FBUs 2 FSUs 7 hydro 3 PSUs	24 h	∼100 s
[119]	OP(CO) heuristics rMIP solver	quadratic *	1 SU *	-	-	-	equal HS ESS imports	operative	-	yes	-	reservoir energy inflows reservoir energy limits reservoir energy splits turb. and pump. limits pumping efficiency import facilities max. LNG consumption	-	near-opt.	2 bus 2 lines 21 thermal 8 gas-units 4 hydro 2 PSUs 3 intercon.	24 h	-
[120]	OP(CO) MILP solver	piecewise (4 blocks)	2 SU	-	-	-	equal	spinning	yes *	yes	-	SU/SD capacity limits *	-	yes 0.5% gap	10 units 100 units (r)	24 h	1 s 123 s
[121]	OP(CO) MILP solver	piecewise (3 blocks)	10 SU	1 SD	-	-	PBUC	-	yes	yes	-	bilateral contracts SU/SD capacity limits	-	yes 0.0001% gap	15 units	24 h	∼300 s
[122]	OP(CO) MILP solver	piecewise	-	-	-	-	PBUC	-	yes	yes	-	SU/SD capacity limits	-	yes	1 unit (8 type of units)	24 h	∼20 s
[123]	OP(CO) MILP solver	piecewise (2 blocks)	2 SU	-	-	-	greater	spinning	yes	yes	-	SU/SD capacity limits	-	yes 1% gap	187 units (r)	24 h	3556.6 s
[124]	OP(CO) MILP solver	linear	5 SU	1 SD	-	-	PBUC	-	yes	yes	-	power trajectories SU/SD durations SU/SD power trajectories	-	yes 0.0001% gap	1 unit	96 h (r) 6144 h (r)	0.109 s 53.181 s

Table 3. *Cont.*

Ref	Method	Production Cost	Start-up Cost	Shut-down Cost	Emission Cost	Mainten. Cost	Demand Constraint	Reserve Constraints	Ramp Limits	TU/TD Constraints	Emission Constraints	Operative Constraints	Network Constraints	Optimal	Problem Size	Time Span	CPU Time
[125]	OP(CO) SO-TS MILP solver	linear	2 SU	1 SD	-	-	equal ** NSE ND-RES ** ND-RES-C TL	reserve	yes	yes	-	power trajectories SU/SD durations SU/SD power trajectories flow-limit violation penalties	flow limits shift factor	yes 0.05% gap	118 bus 186 lines 54 slow units 10 fast units ** 3 wind gen.	24 h by 5 min time steps	<2 h 20 scen.
[126]	OP(CO) MILP solver	linear	2 SU	1 SD	-	-	equal NSE	spinning	yes	yes	-	SU/SD capacity limits	-	yes	280–1870 units (r) 28–187 units (r)	24 h 168 h (r)	1% gap or 10 h 0.1% gap or 1 h
[127]	OP(CO) MILP solver	linear	1 SU	-	-	-	PBUC	-	-	yes	-	SU/SD capacity limits	-	yes 0.0001% gap	10 units	1536 h (r) 12288 h (r)	0.57 s 8.08 s
[128]	OP(CO) MILP solver	linear	3 SU	-	-	-	equal	secondary tertiary offline tertiary	yes	yes	-	power trajectories SU/SD durations SU/SD power trajectories interval ramp capabilities	-	yes 0.0001% gap	10 units 100 units (r)	24 h	9.1 s 1001 s
[129]	OP(CO) MILP solver	linear	2 SU	1 SD	-	-	equal ND-RES ND-RES-C TL	reserve	yes	yes	-	power trajectories SU/SD capacity limits SU/SD durations SU/SD power trajectories	flow limits shift factor	yes 0.0001% gap	118 bus 186 lines 54 units 3 wind gen.	24 h 60 h	27.61 s 267.23 s
[130]	OP(CO) MILP solver	piecewise	1 SU	-	-	-	equal HS	-	yes **	yes	-	reservoir inflows ** reservoir spillage ** cascade reservoirs ** reservoir volume limits ** reservoir discharge limits **	-	yes 0.5% gap	200 thermal (r) 100 hydro (r)	24 h	314 s
[131]	OP(CO) MILP solver	quadratic **	1 SU **	-	-	-	equal **	reserve **	yes **	yes **	-	SU/SD capacity limits **	-	yes 0.5% gap yes 0.01% gap	200 units (r) 50 units (r)	24 h	1243 s 4877 s
[132]	OP(CO) heuristics rMISOCP solver	quadratic	2 SU	-	-	-	equal	spinning	yes	yes	-	SU/SD capacity limits	-	near-opt.	10 units 100 units (r)	24 h	1.58 s 29.09 s
[133]	OP(CO) MILP solver	quadratic **	2 SU **	-	-	-	equal **	spinning **	yes **	yes **	-	SU/SD capacity limits **	-	yes 0.1% gap yes 0.1% gap yes 0.01% gap yes 0.01% gap	10 units 200 units (r) 10 units 200 units (r)	24 h	4.25 s 104.87 s 11.12 s 3553.05 s
[134]	OP(CO) heuristics rMISOCP solver	quadratic **	2 SU **	-	-	-	equal **	spinning **	yes **	yes **	-	SU/SD capacity limits **	-	near-opt.	10 units 200 units (r)	24 h	1.05 s 21.33 s

Table 3. *Cont.*

Ref	Method	Production Cost	Start-up Cost	Shut-down Cost	Emission Cost	Mainten. Cost	Demand Constraint	Reserve Constraints	Ramp Limits	TU/TD Constraints	Emission Constraints	Operative Constraints	Network Constraints	Optimal	Problem Size	Time Span	CPU Time
[135]	OP(CO) MILP solver	quadratic	2 SU	-	-	-	equal	spinning	yes	yes	-	SU/SD capacity limits	-	yes 0.5% gap yes 0.5% gap yes 0.1% gap yes 0.1% gap yes 0.5% gap yes 0.5% gap	10 units 200 units (r) 10 units 200 units (r) 28 units (r) 1080 units (r)	24 h	0.21 s 19.28 s 0.84 s 51.37 s 1.87 s 3775.95 s
[136]	OP(CO) Pareto optimz.	quadratic	1 SU	-	-	-	PBUC lower	-	yes *	yes	horizon max. prod-emiss (quadratic & exponential)	trade-off curve (201 runs)	-	near-opt.	6 units	168 h	0.05 s per run
[137]	OP(CO) Pareto optimz.	quadratic	2 SU	-	-	-	PBUC lower	-	yes	yes	horizon max. prod-emiss (quadratic & exponential)	trade-off curve (201 runs)	-	near-opt.	15 units	168 h	0.12 s per run
[138]	OP(CO) heur. alg.	quadratic	2 SU	-	quadratic	-	equal TL	spinning	yes	yes	-	-	flow limits reactances phase angles	near-opt.	118 bus 186 lines 54 units	24 h	208 s
[139]	OP(CO) heuristics MILP solver	piecewise (5 blocks)	1 SU	1 SD	linear	-	PBUC HS	spinning	yes	yes	-	bilateral contracts SU/SD capacity limits max. sustained ramp rate reservoir inflows reservoir spillage cascade reservoirs reservoir volume limits reservoir discharge limits HR func. (3-block piecewise) reservoir SU/SD costs reservoir TU/TD	-	near-opt. 5% gap	13 thermal 5 hydro	24 h	reasonab.
[140]	OP(CO) MILP solver	piecewise (5 blocks)	1 SU	-	piecewise	-	PBUC lower	-	yes	yes	-	-	-	yes	20 units (r)	24 h 168 h (r)	9.06 s 652.78 s
[141]	OP(CO) MILP solver	piecewise	1 SU	-	-	-	equal HS TL	spinning * operative *	yes	yes	-	prohibited operating zones max. sustained ramp rate reservoir inflows reservoir spillage reservoir volume limits reservoir discharge limits HR function (piecewise) reservoir SU cost reservoir TU/TD	flow limits voltage limits reactances phase angles	yes	8 bus 10 lines 5 thermal 1 hydro	24 h	-

Table 3. *Cont.*

Ref	Method	Production Cost	Start-up Cost	Shut-down Cost	Emission Cost	Mainten. Cost	Demand Constraint	Reserve Constraints	Ramp Limits	TU/TD Constraints	Emission Constraints	Operative Constraints	Network Constraints	Optimal	Problem Size	Time Span	CPU Time
[142]	OP(CO) MIQCP solver	quadratic *	2 SU *	-	-	-	equal * NSE TL	spinning *	yes *	yes *	-	fuel consumption limits * gas consumption limits * load losses limits *	flow limits * shift factor *	yes	194 thermal 9 CCGTs 1 HCCP 4 IPPs 2 gas sector Mexican transmission network	24 h	15.12 min
[143]	OP(CO) MILP solver	piecewise	1 SU	-	-	-	equal ND-RES ND-RES-C ESS TL	spinning non-spinning	yes	yes	horizon max. gen-emiss. (piecewise)	fuel consump. limits reactive generation limits CAES volume limits CAES inject limits CAES release limits inject efficiency release efficiency	flow limits voltage limits	yes	118 bus 186 lines 54 units 1 CAES 3 wind gen.	24 h	-
[144]	OP(CO) MILP solver	-	-	-	-	-	-	-	-	-	-	branching symmetry exploitting	-	yes 0.2% gap	118 bus 186 lines (9 parallel) 54 units	4 h	875 s
[145]	OP(CO) MILP solver	piecewise	2 SU	-	-	-	equal	spinning	yes	yes	-	symmetry breaking constraints	-	yes 0.0% gap	100 units (r)	24 h	115.64 s
[146]	OP(CO) MILP solver	piecewise (2 blocks)	2 SU	-	-	-	greater	spinning	yes	yes	-	SU/SD capacity limits	-	yes	46-72 units (r)	24 h	∼73.09 s
[147]	OP(CO) MILP solver	piecewise	2 SU	-	-	-	equal	spinning	yes *	yes	-	symmetry breaking constraints SU/SD capacity limits *	-	yes <0.03% gap	100 units (r)	24 h	∼7200 s
[148]	OP(CO) MILP solver	piecewise (2 blocks)	2 SU	-	-	-	equal	spinning	yes	yes	-	SU/SD capacity limits	-	yes 0.01% gap	≤8 agg. units (28–187 units r)	24 h	∼38.03 s
[149]	OP(CO) MILP solver	piecewise (3 blocks)	3 SU	1 SD	-	-	equal NSE ND-RES	secondary tertiary spinning tertiary non-spin.	yes	yes	calculated after resolution	AGC representation ramp rates under AGC	-	yes	45 units (r) 1 wind gen. 1 solar gen.	336 h daily runs with 38 h look ahead capacity	∼1 h
[151]	OP(CO) MILP solver	linear	1 SU	-	linear	yes	equal NSE ND-RES ND-RES-C	-	yes	yes	-	must run units mayor overhaul costs relation with increases of SU in slow&fast units	-	yes 0.1% gap	301 units (r)	336 h	-
[152]	OP(CO) MILP solver	linear	1 SU	-	-	-	equal NSE ND-RES ND-RES-C TL	secondary	yes	yes	-	SU/SD capacity limits	flow limits shift factor	yes 0.01% gap	118 bus 186 lines 54 agg. units (540 units r) 3 wind gen.	24 h	406 s

Table 3. *Cont.*

Ref	Method	Production Cost	Start-up Cost	Shut-down Cost	Emission Cost	Mainten. Cost	Demand Constraint	Reserve Constraints	Ramp Limits	TU/TD Constraints	Emission Constraints	Operative Constraints	Network Constraints	Optimal	Problem Size	Time Span	CPU Time
[155]	OP(CO) MILP solver	piecewise (1 to 10 blocks)	≤2 SU	-	-	-	equal ND-RES ND-RES-C	spinning	yes	yes	-	SU/SD capacity limits	-	yes 0.1% gap	PJM Inteconnection	48 h	136 s 2% wind 185 s 30% wind
[156]	OP(CO) MILP solver	linear **	1 SU **	-	-	-	-	spinning **	yes **	yes **	-	CCGT configuration modes ** capacity limits per mode ** ramp rates per mode ** transition ramp rates ** mode transition costs ** prod. costs per mode ** SU cost per mode ** TU/TD per mode **	-	yes 0.1% gap	4 units (r) 10 units (r) 16 units (r)	24 h	2.4 s 4.8 s 124.9 s
[157]	OP(CO) MILP solver	piecewise **	1 SU **	-	-	-	equal ESS	-	-	-	-	reservoir inflows cascade reservoirs reservoir volume limits reservoir discharge limits HR func. (s-block piecewise) pumping capacity limits pumping efficiency	-	yes 0.5% gap	198 thermal 72 hydro 34 IPPs 10 PSUs	168 h	<10 s 1 HR 3 blocks 4549.14 s 3 HR 3 blocks 39304.84 s 3 HR 6 blocks
[158]	OP(CO) MILP solver	piecewise *	2 SU	-	-	-	greater ESS TL	spinning	yes	yes	-	reservoir inflows cascade reservoirs reservoir volume limits reservoir discharge limits HR function (piecewise) pumping capacity limits pumping efficiency	flow limits reactances phase angles	yes 0.0% gap	31 bus 43 lines 16 thermal 2 PSUs	24 h	116.9 s
[159]	OP(CO) MILP solver	linear	2 SU	-	-	-	greater TL	spinning	yes	yes	-	SU/SD capacity limits *	flow limits reactances phase angles	yes 0.0% gap	31 bus 43 lines 16 units 118 bus 186 lines 54 units	24 h	104.5 s 37.2 s

Table 3. *Cont.*

Ref	Method	Production Cost	Start-up Cost	Shut-down Cost	Emission Cost	Mainten. Cost	Demand Constraint	Reserve Constraints	Ramp Limits	TU/TD Constraints	Emission Constraints	Operative Constraints	Network Constraints	Optimal	Problem Size	Time Span	CPU Time
[160]	OP(CO) MILP solver	linear	2 SU	-	-	-	greater ESS TL	-	yes	yes	-	reservoir inflows cascade reservoirs reservoir volume limits reservoir discharge limits HR function (piecewise) pumping capacity limits pumping efficiency gas-unit consump. piecewise f. gas network representation gas pressure linearizations: · node piecewise function · pipeline piecewise funct. · compressor piecewise funct.	flow limits reactances phase angles	yes 1% gap	31 bus 43 lines 13 no-gas units 3 gas units 7 gas nodes 5 gas loads 5 pipelines 1 compressor 2 supply nodes 2 PSUs	24 h	480.5 s
[161]	OP(CO) MILP solver	linear	1 SU	1 SD	-	-	equal TL	-	yes	yes	-	-	flow limits shift factor	yes 0.01% gap	118 bus 186 lines 54 units	24 h	∼296 s
[162]	OP(CO) MILP solver	piecewise (2 blocks) (2 blocks) (1–10 blo.) (1–10 blo.)	2 SU 2 SU ≤2 SU ≤2 SU	-	-	-	equal	spinning	yes	yes	-	-	-	yes 0.01% gap	280–1870 units (r) 28–187 units (r) PJM Interconnection PJM Interconnection	24 h 168 h 24 h 168 h	∼367.8 s ∼286.3 s ∼34.3 s ∼664.7 s
[163]	OP(CO) MILP solver	linear	1 SU	1 SD	-	-	equal	spinning	yes	yes	-	SU/SD capacity limits	-	yes 0.1% gap	800 units (r)	24 h	∼285 s
[164]	OP(CO) MILP solver	piecewise (1 to 10 blocks)	≤2 SU	-	-	-	equal NSE OE ND-RES ND-RES-C TL	spinning	yes	yes	-	reserve-deviation penalties SU/SD capacity limits	flow limits reactances phase angles	yes 0.01% gap	PJM Interconnection	48 h	∼300 s

* The constraint is formulated, but the corresponding input data for the case study are not provided. ** The constraint is formulated, but the corresponding input data for the case study are missing due to an offline link or the lack of information in a reference.

5. Conclusions

This article presents a literature review on the different available alternatives to set out and solve the unit commitment problem. One of the main goals is establishing a hierarchical classification of the computational techniques that could be applied to each stage of the problem. This proposal differs from other surveys, which mix identification principles and describe several modeling techniques, resolution algorithms, etc., without any more profound distinction.

The exposed layout allows to quickly acquire a general idea of the different options to represent and optimize the operation of power systems. Furthermore, their advantages and disadvantages are elaborately described, allowing an easy recognition of the research gaps for further developments and introducing a solid basis to implement new improvements.

Some examples of research necessities in this field are enhancing the computational efficiency when handling uncertainty; improving modeling detail, such as a more precise characterization of the start-up processes, or going in-depth with the operational flexibility representation; extending the unit commitment horizon to facilitate coordination with markedly medium-term tasks, such as hydro management, fuel purchases, or financial contracting; representing the real demand variability for thermal generators in current and future electricity markets with high penetration of non-dispatchable energy sources; or studying the most efficient procedure for achieving widespread integration of non-conventional energy-storage technologies in modern power systems.

In order to not provide an immeasurable amount of references, the most popular and recent unit commitment approaches have been cited throughout the different sections. To conclude this review, the historical events and ongoing trends in the most widespread technique applied to the unit commitment problem are introduced. Hence, the current state is presented, focusing on the thermal generation of electricity markets which is optimized through commercial solvers. The reader is referred to recent surveys if interested in obtaining a deeper knowledge of specific problems, such as the hydro unit commitment problem, the operational representation of energy-storage technologies, and evolutionary optimization.

Moreover, the scope of the different methodologies described in the paper is clearly identified through a comparison table, where the modeling detail adopted in the references is analyzed in depth. In turn, the problem sizes are also precisely presented, providing the elements that compound the power systems represented in the case studies. Furthermore, the computational performance of each proceeding can be discerned, too, according to the corresponding run times and optimality of the solutions.

This characterization entails a significant development against the conventional reviews on the unit commitment problem, which only offer a broad vision of the modeling scope of their cites at most. Additionally, this accurate description can also be used as a benchmark to look for different options to model a specific technical or economic aspect, choosing the formulation that apparently offers a greater computational efficiency according to the desired methodology.

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Abbreviations

The following abbreviations are used in this manuscript:

ACO	Ant Colony Optimization
AGC	Automatic Generation Control
ALO	Ant Lion Optimizer
ABCA	Artificial Bee Colony Algorithm
AFSA	Artificial Fish Swarm Algorithm
AISA	Artificial Immune System Algorithm
ASA	Artificial Sheep Algorithm
BD	Benders Decomposition
BFA	Bacterial Foraging Algorithm
CAES	Compressed Air Energy Storage
CCGT	Combined Cycle Gas Turbine
CO	Conventional Optimization
CCO	Chance Constrained Optimization
CSA	Cuckoo Search Algorithm
DEA	Differential Evolution Algorithm
DS	Distribution System
DSO	Distribution System Operator
DP	Dynamic Programming
DWD	Dantzig-Wolfe Decomposition
EA	Electrolytic Aluminium series
EE	Exhaustive Enumeration
EF	Electrolytic arc Furnance
EIE	Energy Intensive Enterprise
EMA	Exchange Market Algorithm
EO	Evolutionary Optimization
EP	Evolutionary Programming
ES	Expert System
ESS	Energy Storage System
EV	Electric Vehicle
FBU	Fuel Blending Unit
FFA	Firefly Algorithm
FL	Fuzzy Logic
FSU	Fuel Switching Unit
FWA	Fireworks Algorithm
GA	Genetic Algorithm
GSA	Gravitational Search Algorithm
GWO	Grey Wolf Optimizer
HCCP	Hybrid Combined Cycle Plants
HM	Hybrid Methodologies
HS	Hydro Spillage
HR	Head Range
HUI	Hybrid Uncertainty Implementation
ICA	Imperialist Competitive Algorithm
IHHO	Intensify Harris Hawks Optimizer
IO	Interval Optimization
IP	Integer Programming
IPP	Independent Power Producer
LNG	Liquefied Natural Gas
LP	Linear Programming
LR	Lagrangian Relaxation
MCS	Monte Carlo Simulation
MILP	Mixed Integer Linear Programming
MIQP	Mixed Integer Quadratic Programming
MIQCP	Mixed Integer Quadratically Constrained Programming
MINLP	Mixed Integer Non Linear Programming

MIP	Mixed Integer Programming
MISOCP	Mixed Integer Second Order Cone Programming
MMA	Memory Management Algorithm
MS	Multi Stage
ND-RES	Non-Dispatchable Renewable Energy Sources
ND-RES-C	Non-Dispatchable Renewable Energy Sources Curtailment
NLP	Non Linear Programming
NN	Neural Network
NO	Numerical Optimization
NSE	Non-Served Energy
OE	Oversupplied Energy
OF	Objective Function
OP	Optimization Problem
QIEA	Quantum-Inspired Evolutionary Algorithm
QOTLBA	Quasi-Oppositional Teaching Learning Based Algorithm
PBUC	Price-Based Unit Commitment
PJM	Pennsylvania, New Jersey, and Maryland
PL	Priority List
PSO	Particle Swarm Optimization
PSU	Pumping Storage Unit
QP	Quadratic Programming
QCP	Quadratically Constrained Programming
RC	Risk Consideration
rMILP	Relaxed Mixed Integer Linear Programming
rMIQP	Relaxed Mixed Integer Quadratic Programming
rMIQCP	Relaxed Mixed Integer Quadratically Constrained Programming
rMINLP	Relaxed Mixed Integer Non Linear Programming
rMISOCP	Relaxed Mixed Integer Second Order Cone Programming
RES	Renewable Energy Sources
RO	Robust Optimization
SA	Simulated Annealing
SCA	Sine–Cosine Algorithm
SCUC	Security Constrained Unit Commitment
SD	Shut-Down
SFLA	Shuffled Frog Leaping Algorithm
SO	Stochastic Optimization
SOCP	Second Order Cone Programming
SU	Start-Up
TD	Time Down
TL	Transmission Losses
TS	Two Stage
TSA	Tabu Search Algorithm
TSO	Transmission System Operator
TU	Time Up
UC	Unit Commitment
WOA	Whale Optimization Algorithm

Appendix A. Formulating Options

- Integer Programming (IP): The formulation employs only discrete variables. A discrete variable, that is commonly called an integer variable, accomplishes an integer value after the problem resolution ($x_I \in \mathbb{Z}$). According to the numerical methods utilized by computers to solve the problem, the integrality of these variables is related to a tolerance. Formulating exclusive-integer problems is not common. The habitual practice is to utilize continuous variables as well. For that reason, the integer programming term is usually employed as a reference to the usage of integer variables in the unit commitment problem and energy models.

- Mixed Integer Programming (MIP): The formulation uses a mix of integer and continuous variables ($x_C \in \mathbb{R}$). The MIP has several subgroups attending to the mathematical formulation of the OF and constraints. If the OF and all the constraints are linear, the problem is called MILP. If the constraints are linear but the OF is quadratic, it is called MIQP. On the other hand, if there are some quadratic constraints and a linear OF, the problem is an MIQCP. This quadratic constraints can be represented as second-order cones in a MISOCP problem. Finally, if there is any non-linear constraint or OF, that will be an MINLP problem.
- Linear Programming (LP): The formulation only employs continuous variables. In turn, both their objective function and constraints are linear. LP can be a result of relaxing the discrete variables of a MIP. When the integer variables of MILP are relaxed (considered as continuous in order to facilitate the resolution of the problem), the new problem is known as an rMILP and there is not any difference with an LP.
- Non-Linear Programming (NLP): The formulation utilizes continuous variables; meanwhile, the objective function or some constraint is a non-linear function. As well as in LP, the relaxation of an MINLP turns out into a problem which it is solved as an NLP, an rMINLP. Quadratic Programming (QP), Quadratically Constrained Programming (QCP), and Second-Order Cone Programming (SOCP) are non-linear techniques. Nevertheless, they are not frequently included in this group in the literature because their convexity properties allow them to be solved easier than NLP used to be. Relaxing MIQP, MIQCP, or MISOCP converts the problem in an rMIQP, rMIQCP, or rMISOCP that is also solved such as a QP, QCP, or SOCP problem.

References

1. Padhy, N. Unit commitment-A bibliographical survey. *IEEE Trans. Power Syst.* **2004**, *19*, 1196–1205.
2. Hara, K.; Kimura, M.; Honda, N. A method for planning economic unit commitment and maintenance of thermal power systems. *IEEE Trans. Power App. Syst.* **1966**, *PAS-85*, 427–436.
3. Li, S.; Shahidehpour, S.M.; Wang, C. Promoting the application of expert systems in short-term unit commitment. *IEEE Trans. Power Syst.* **1993**, *3*, 286–292.
4. Delarue, E.; Cattrysse, D.; D'haeseleer, W. Enhanced priority list unit commitment method for power systems with a high share of renewables. *Electr. Power Syst. Res.* **2013**, *105*, 115–123.
5. Moradi, S.; Khanmohammadi, S; Hagh, M.T.; Mohammadiivatloo, B. A semi-analytical non-iterative primary approach based on priority list to solve unit commitment problem. *Energy* **2015**, *88*, 244–259.
6. Elsayed, A.M.; Maklad, A.M.; Farrag, S.M. A New Priority List Unit Commitment Method for Large-Scale Power Systems. In Proceeding of the 2017 Nineteenth International Middle East Power Systems Conference (MEPCON), Cairo, Egypt, 19–21 December 2017.
7. Shahbazitabar, M.; Abdi, H. A novel priority-based stochastic unit commitment considering renewable energy sources and parking lot cooperation. *Energy* **2018**, *161*, 308–324.
8. Wang, B.; Li, Y.; Watada, J. Re-Scheduling the Unit Commitment Problem in Fuzzy Environment. In Proceedings of the 2011 IEEE International Conference on Fuzzy Systems (FUZZ-IEEE 2011), Taipei, Taiwan, 27–30 June 2011; pp. 1090–1095.
9. Leou, R. A Price-Based Unit Commitment Model considering uncertainties with a Fuzzy Regression Model. *Int. J. Energy Sci.* **2012**, *2*, 51–58.
10. Zhang, N.; Hu, Z.; Han, X.; Zhang, J.; Zhou, Y. A fuzzy chance constrained program for unit commitment problem considering demand response, electric vehicle and wind power. *Int. J. Electr. Power Energy Syst.* **2015**, *65*, 201–209.
11. Shanti-Swarup, K.; Simi, P.V. Neural computation using discrete and continuous Hopfield networks for power system economic dispatch and unit commitment. *Neurocomputing* **2006**, *70*, 119-129.
12. Jahromi, M.Z.; Bioki, M.M.H.; Nejad, M.R.; Fadaeinedjad, R. Solution to the unit commitment problem using an artificial neural network. *Turkish J. Electr. Eng. Comput. Sci.* **2013**, *21*, 198-212.
13. Yang, Y.; Wu, L. Machine learning approaches to the unit commitment problem: Current trends, emerging challenges, and new strategies. *Electr. J.* **2021**, *34*, 369–389.
14. Pokharel, B.K.; Shrestha, G.B.; Lie T.T.; Fleten, S.E. Price based unit commitment for Gencos in deregulated markets. In Proceedings of the Power Engineering Society General Meeting, IEEE, San Francisco, CA, USA, 12–16 June 2005; pp. 428–433.
15. Kumar, S.S.; Palanisamy, V. A dynamic programming based fast computation Hopfield neural network for unit commitment and economic dispatch. *Electr. Power Syst. Res.* **2007**, *77*, 917–925.
16. Dieu, V. N.; Ongsakul, W. Ramp rate constrained unit commitment by improved priority list and augmented Lagrange Hopfield network. *Electr. Power Syst. Res.* **2008**, *78*, 291–301.

17. Silva, I.C.; Carneiro, S.; de Oliveira, E.J.; Pereira, J.L.R.; Garcia, P.A.N.; Marcato, A.L.M. A lagrangian multiplier based sensitive index to determine the unit commitment of thermal units. *Int. J. Electr. Power Energy Syst.* **2008**, *30*, 504–510.
18. Patra, S.; Goswami, S.K.; Goswami, B. Fuzzy and simulated annealing based dynamic programming for the unit commitment problem. *Expert Syst. Appl.* **2009**, *36*, 5081–5086.
19. Nascimento, F.R.; Silva, I.C.; Oliveira, E.J.; Dias, B.H.; Marcato, A.L.M. Thermal Unit Commitment Using Improved Ant Colony Optimization Algorithm via Lagrange Multipliers. In Proceedings of the 2011 IEEE Trondheim PowerTech, IEEE, Trondheim, Norway, 19–23 June 2011. pp. 1-5.
20. Chen, P. H. Two-level hierarchical approach to unit commitment using expert system and elite PSO. *IEEE Trans. Power Syst.* **2012**, *27*, 780–789.
21. Najafi, A.; Farshad, M.; Falaghi, H. A new heuristic method to solve unit commitment by using a time-variant acceleration coefficients particle swarm optimization algorithm. *Turkish J. Electr. Eng. Comput. Sci.* **2015**, *23*, 354–369.
22. Taktak, R.; D'Ambrosio, C. An overview on mathematical programming approaches for the deterministic unit commitment problem in hydro valleys. *Energy Syst.* **2015**, *7*, 1–23.
23. Howlader, H.; Adewuyi, B.; Hong, Y.Y.; Mandal, P.; Hemeida, A.M.; Senjyu, T. Energy storage system analysis review for optimal unit commitment. *Energies* **2020**, *13*, 158.
24. Bellman, R. *Dynamic Programming*; Princeton University Press: Princeton, NJ, USA, 1957.
25. Frangioni, A.; Gentile, C. Solving nonlinear single-unit commitment problems with ramping constraints. *Oper. Res.* **2006**, *54*, 767–775.
26. Frangioni, A.; Gentile, C.; Lacalandra, F. Solving unit commitment problems with general ramp constraints. *Intl. J. Elec. Power Syst.* **2008**, *30*, 316–326.
27. Singhal, P.K.; Sharma, R.N. Dynamic programming approach for large scale unit commitment problem. In Proceedings of the International Conference on Communication Systems and Network Technologies (CSNT), Katra, India, 3–5 June 2011; pp. 714–717.
28. Analui, B.; Scaglione, A. A dynamic multistage stochastic unit commitment formulation for intraday markets. *IEEE Trans. Power Syst.* **2018**, *33*, 3653–3663.
29. Zou, J.; Ahmed, S.; Sun, X.A. Multistage stochastic unit commitment using stochastic dual dynamic integer programming. *IEEE Trans. Power Syst.* **2018**, *34*, 1814-1823.
30. Xu, Y.; Ding, T.; Qu, M.; Du, P. Adaptive dynamic programming for gas-power network constrained unit commitment to accommodate renewable energy with combined-cycle units. *IEEE Trans. Sustain. Energy* **2019**, *11*, 2028-2039.
31. Papavasiliou, A.; Oren, S.S.; Rountree, B. Applying high performance computing to transmission-constrained stochastic unit commitment for renewable energy integration. *IEEE Trans. Power Syst.* **2015**, *30*, 1109–1120.
32. Bakirtzis, E.A.; Biskas, P.N. Multiple time resolution stochastic scheduling for systems with high renewable penetration. *IEEE Trans. Power Syst.* **2017**, *32*, 1030–1040.
33. Schulze, T.; Grothey, A.; McKinnon, K. A stabilised scenario decomposition algorithm applied to stochastic unit commitment problems. *Eur. J. Oper. Res.* **2017**, *261*, 247–259.
34. Geng, Z.; Conejo, A.J.; Chen, Q. Power generation scheduling considering stochastic emission limits. *Int. J. Electr. Power Energy Syst.* **2018**, *95*, 374–383.
35. Lopez-Salgado, C.J.; Año O.; Ojeda-Esteybar, D.M. Stochastic unit commitment and optimal allocation of reserves: A hybrid decomposition approach. *IEEE Trans. Power Syst.* **2018**, *33*, 5542-5552.
36. Rachunok, B.; Staid, A.; Watson, J.P.; Woodruff, D.L.; Yang, D. Stochastic unit commitment performance considering Monte Carlo wind power scenarios. In Proceedings of the 2018 IEEE International Conference on Probabilistic Methods Applied to Power Systems (PMAPS), Boise, ID, USA, 24–28 June 2018; pp. 1–6.
37. Scuzziato, M.R.; Finardi, E.C.; Frangioni, A. Comparing spatial and scenario decomposition for stochastic hydrothermal unit commitment problems. *IEEE Trans. Sustain. Energy* **2018**, *9*, 1307–1317.
38. Shi, J.; Oren, S.S. Flexible line ratings in stochastic unit commitment for power systems with large-scale renewable generation. *Energy Syst.* **2020**, *11*, 1–19.
39. Vatanpour, M.; Yazdankhah, A.S. The impact of energy storage modeling in coordination with wind farm and thermal units on security and reliability in a stochastic unit commitment. *Energy* **2018**, *162*, 476–490.
40. Shiina, T.; Birge, J.R. Stochastic unit commitment problem. *Int. Trans. Oper. Res.* **2004**, *11*, 19–32.
41. Cerisola, S.; Baíllo, A.; Fernández-López, J.M.; Ramos, A.; Gollmer, R. Stochastic power generation unit commitment in electricity markets: A novel formulation and a comparison of solution methods. *Oper. Res.* **2009**, *57*, 32–46.
42. Hreinsson, K.; Analui, B.; Scaglione, A. Continuous time multi-stage stochastic reserve and unit commitment. In Proceedings of the 2018 Power Systems Computation Conference (PSCC), Dublin, Ireland, 11–15 June 2018; pp. 1–7.
43. Ahmadi-Khatir, A; Bozorg, M.; Cherkaoui, R. Probabilistic spinning reserve provision model in multi-control zone power system. *IEEE Trans. Power Syst.* **2013**, *28*, 2819–2829.
44. Xiong P.; Jirutitijaroen, P. A stochastic optimization formulation of unit commitment with reliability constraints. *IEEE Trans. Smart Grid* **2013**, *4*, 2200–2208.
45. Bruninx, K.; Delarue, E. Endogenous probabilistic reserve sizing and allocation in unit commitment models: Cost-effective, reliable, and fast. *IEEE Trans. Power Syst.* **2017**, *32*, 2593–2603.

46. Hreinsson, K.; Vrakopoulou, M.; Andersson, G. Stochastic security constrained unit commitment and non-spinning reserve allocation with performance guarantees. *Int. J. Electr. Power Energy Syst.* **2015**, *72*, 109–115.
47. Wu, Z.; Zeng, P.; Zhang, X.P.; Zhou, Q. A solution to the chanceconstrained two-stage stochastic program for unit commitment with wind energy integration. *IEEE Trans. Power Syst.* **2016**, *31*, 4185–4196.
48. Zhang, Y.; Wang, J.; Zeng, B.; Hu, Z. Chance-constrained two-stage unit commitment under uncertain load and wind power output using bilinear benders decomposition. *IEEE Trans. Power Syst.* **2017**, *32*, 3637–3647.
49. Jin, H.; Li, Z.; Sun, H.; Guo, Q.; Chen, R.; Wang, B. A robust aggregate model and the two-stage solution method to incorporate energy intensive enterprises in power system unit commitment. *Appl. Energy* **2017**, *206*, 1364–1378.
50. Morales-Espana, G.; Lorca, A.; de Weerdt, M.M. Robust unit commitment with dispatchable wind power. *Electr. Power Syst. Res.* **2018**, *155*, 58–66.
51. Razavi, S.E.; Nezhad, A.E.; Mavalizadeh, H.; Raeisi, F.; Ahmadi, A. Robust hydrothermal unit commitment: A mixed-integer linear framework. *Energy* **2018**, *165*, 593-602.
52. Wu, L.; Shahidehpour, M.; Li, Z. Comparison of scenario-based and interval optimization approaches to stochastic SCUC. *IEEE Trans. Power Syst.* **2012**, *27*, 913–921.
53. Zhou, M.; Xia, S.; Li, G.; Han, X. Interval optimization combined with point estimate method for stochastic security-constrained unit commitment. *Int. J. Electr. Power Energy Syst.* **2014**, *63*, 276–284.
54. Pandzic, H.; Dvorkin, Y.; Qiu, T.; Wang, Y.; Kirschen, D.S. Toward cost efficient and reliable unit commitment under uncertainty. *IEEE Trans. Power Syst.* **2016**, *31*, 970–982.
55. Wang, J.; Shahidehpour, M.; Li, Z. Security-constrained unit commitment with volatile wind power generation. *IEEE Trans. Power Syst.* **2008**, *23*, 1319–1327.
56. Bello, A.; Bunn, D.W.; Reneses J.; Muñoz, A. Medium-term probabilistic forecasting of electricity prices: A hybrid approach *IEEE Trans. Power Syst.* **2017**, *32*, 334-343.
57. Montero, L.; Bello, A.; Reneses, J. A new methodology to obtain a feasible thermal operation in power systems in a medium-term horizon. *Energies* **2020**, *12*, 3056.
58. Huang, Y.; Zheng, Q.P.; Wang, J. Two-stage stochastic unit commitment model including non-generation resources with conditional value-at-risk constraints. *Electr. Power Syst. Res.* **2014**, *116*, 427–438.
59. Dvorkin, Y.; Pandzic, H.; Ortega-Vazquez, M.A.; Kirschen, D.S. A hybrid stochastic/interval approach to transmission-constrained unit commitment. *IEEE Trans. Power Syst.* **2015**, *30*, 621–631.
60. Blanco, I.; Morales, J.M. An efficient robust solution to the two-stage stochastic unit commitment problem. *IEEE Trans. Power Syst.* **2017**, *32*, 4477-4488.
61. Shiina, T.; Yurugi, T.; Morito, S.; Imaizumi, J. Unit commitment by column generation. In *Operations Research Proceedings 2014*; Springer: Cham, Switzerland, February 2016; pp. 559–565.
62. Kuo M.T.; Lu, S.D. Random feasible directions algorithm with a generalized Lagrangian relaxation algorithm for solving unit commitment problem. *J. Chin. Inst. Eng.* **2015**, *38*, 547–561.
63. Gurobi: Gurobi Optimizer. Available online: https://www.gurobi.com/ (accessed on 5 November 2021)
64. IBM: CPLEX Optimizer. Available online: https://www.ibm.com/es-es/analytics/cplex-optimizer (accessed on 5 November 2021)
65. MOSEK: MOSEK Solver. Available online: https://www.mosek.com/ (accessed on 5 November 2021)
66. BARON: BARON Solver. Available online: https://minlp.com/baron-solver (accessed on 5 November 2021)
67. Nocedal, J.; Wright, S.J. *Numerical Optimization*; ser. Springer Series in Operations Research; Springer: New York, NY, USA, 1999.
68. Jo K.H.; Kim, M.K. Improved genetic algorithm-based unit commitment considering uncertainty integration method. *Energies* **2018**, *11*, 1–18.
69. Dudek, G. Adaptive simulated annealing schedule to the unit commitment problem. *Electr. Power Syst. Res.* **2010**, *80*, 465–472.
70. Chang, C.S.; Chen, R.C. Optimal unit commitment decision with risk assessment using tabu search. *J. Inf. Optim. Sci.* **2007**, *28*, 965–984.
71. Juste, K.A.; Kita, H.; Tanaka, E.; Hasegawa, J. An evolutionary programming solution to the unit commitment problem. *IEEE Trans. Power Syst.* **1999**, *14*, 1452–1459.
72. Darvishan, A.; Mollashahi, H.; Ghaffari, V.; Lariche, M.J. Unit commitment-based load uncertainties based on improved particle swarm optimisation. *Int. J. Ambient Energy* **2019**, *40*, 594–599.
73. Jang, S.H.; Roh, J.H.; Kim, W.; Sherpa, T.; Kim, J.H.; Park, J.B. A novel binary ant colony optimization: Application to the unit commitment problem of power systems. *J. Electr. Eng. Technol.* **2011**, *6*, 174–181.
74. Sam'on, I.N.; Yasin, Z.M.; Zakaria, Z. Ant Lion Optimizer for solving unit commitment problem in smart grid system. *Indones. J. Electr. Eng. Comput. Sci.* **2017**, *8*, 129–136.
75. Columbus, C.C.; Simon, S.P. Profit based unit commitment: a parallel ABC approach using a workstation cluster. *J. Compu. Electr. Eng.* **2012**, *38*, 724–745.
76. Singhal, P.K.; Sharma, V.; Naresh, R. Binary fish swarm algorithm for profit-based unit commitment problem in competitive electricity market with ramp rate constraints. *IET Gener. Transmiss. Distrib.* **2015**, *9*, 1697–1707.
77. Lakshmi, K.; Vasantharathna, S. Genco's profit based unit commitment using artificial immune system in day ahead competitive electricity markets. *J. Appl. Sci. Eng.* **2014**, *17*, 275–282.

78. Wang, W.; Li, C.; Liao, X.; Qin, H. Study on unit commitment problem considering pumped storage and renewable energy via a novel binary artificial sheep algorithm. *Appl. Energy* **2017**, *187*, 612–626.
79. Eslamian, M.; Hosseinian, S.H.; Vahidi, B. Bacterial foragingbased solution to the unit-commitment problem. *IEEE Trans. Power Syst.* **2009**, *24*. 1478–1488.
80. Zhao, J.; Liu, S.; Zhou, M.; Guo, X.; Qi, L. Modified cuckoo search algorithm to solve economic power dispatch optimization problems. *IEEE/CAA J. Autom. Sin.* **2018**, *5*, 794–806.
81. Datta D.; Dutta, S. A binary-real-coded differential evolution for unit commitment problem. *Int. J. Electr. Power Energy Syst.* **2012**, *42*, 517–524.
82. Ghorbani, N.; Babaei, E. Exchange market algorithm for economic load dispatch. *Int. J. Electr. Power Energy Syst.* **2016**, *75*, 19–27.
83. Koodalsamy, B.; Veerayan, M.B.; Koodalsamy, C.; Simon, S.P. Firefly algorithm with multiple workers for the power system unit commitment problem. *Turkish J. Electr. Eng. Comput. Sci.* **2016**, *24*, 4773–4789.
84. Reddy, K.S.; Panwar, L.K.; Kumar, R.; Panigrahi, B.K. Binary fireworks algorithm for profit based unit commitment (PBUC) problem. *Int. J. Electr. Power Energy Syst.* **2016**, *83*, 270–282.
85. Swain, R.K.; Sahu, N.C.; Hota, P.K. Gravitational search algorithm for optimal economic dispatch. *Procedia Technol.* **2012**, *6*, 411–419.
86. Sakthi, S.S.; Santhi, R.K.; Krishnan, N.M.; Ganesan, S.; Subramanian, S. Wind integrated thermal unit commitment solution using grey wolf optimizer. *Int. J. Electr. Comput. Eng.* **2017**, *7*, 2088–8708.
87. Aghdam F.H.; Hagh, M.T. Security Constrained Unit Commitment (SCUC) formulation and its solving with Modified Imperialist Competitive Algorithm (MICA). *J. King Saud Univ. Eng. Sci.* **2019**, *31*, 253–261.
88. Nandi, A.; Kamboj, V.K. A New Solution to Profit Based Unit Commitment Problem Considering PEVs/BEVs and Renewable Energy Sources. In *E3S Web of Conferences*; EDP Sciences, Les Ulis, France, 2020; p. 01070.
89. Amudha, A.; Rajan, C.C.A. Effect of Reserve in Profit Based Unit Commitment Using Worst Fit Algorithm. In Proceedings of the 2011 International Conference on Process Automation, Control and Computing, IEEE, Coimbatore, India, 20–22 July 2011; pp. 1–7.
90. Chung, C.Y.; Yu, H.; Wong, K.P. An advanced quantum-inspired evolutionary algorithm for unit commitment. *IEEE Trans. Power Syst.* **2011**, *26*, 847–854.
91. Roy P.K.; Sarkar, R. Solution of unit commitment problem using quasi-oppositional teaching learning based algorithm. *Int. J. Electr. Power Energy Syst.* **2014**, *60*, 96–106.
92. Barati, M.; Farsangi, M.M. Solving unit commitment problem by a binary shuffled frog leaping algorithm. *IET Gener. Transmiss. Distrib.* **2014**, *8*, 1050–1060.
93. Reddy, K.S.; Panwar, L.K.; Panigrahi, B.K.; Kumar, R. A new binary variant of sine–cosine algorithm: Development and application to solve profit-based unit commitment problem. *Arab. J. Sci. Eng.* **2017**, *48*, 4041–4056.
94. Reddy, K.S.; Panwar, L.K.; Panigrahi, B.K.; Kumar, R. Binary whale optimization algorithm: a new metaheuristic approach for profitbased unit commitment problems in competitive electricity markets. *Eng. Optim.* **2019**, *51*, 369-389.
95. Elattar, E.E. Adaptive bacterial foraging and genetic algorithm for unit commitment problem with ramp rate constraint. *Int. Trans. Electr. Energy Syst.* **2016**, *26*, 1555–1569.
96. Bikeri, A.; Kihato, P.; Maina, C. Profit Based Unit Commitment Using Evolutionary Particle Swarm Optimization. In Proceedings of the 2017 IEEE AFRICON, IEEE, Cape Town, South Africa, 18–20 September 2017; pp. 1137–1142.
97. Sudhakar, A.; Karri, C.; Laxmi, A.J. Profit based unit commitment for GENCOs using lagrange relaxation—differential evolution. *Int. J. Eng. Sci. Technol.* **2017**, *20*, 738–747.
98. Nawaz, A.; Wang, H.; Wu, Q.; Ochani, M.K. TSO and DSO with large-scale distributed energy resources: A security constrained unit commitment coordinated solution. *Int. Trans. Electr. Energy Syst.* **2020**, *30*, 1–26.
99. Zhai, Y.; Liao, X.; Mu, N.; Le, J. A two-layer algorithm based on PSO for solving unit commitment problem. *Soft Comput.* **2020**, *24*, 9161–9178.
100. Muralikrishnan, N.; Jebaraj, L. and Rajan, C.C.A. A Comprehensive Review on Evolutionary Optimization Techniques Applied for Unit Commitment Problem. *IEEE Access* **2020**, *8*, 132980–133014.
101. Koch, T.; Achterberg, T.; Andersen, E.; Bastert, O.; Berthold, T.; Bixby, R.E.; Danna, E.; Gamrath, G.; Gleixner, A.M.; Heinz, S.; Lodi, A.; Mittelmann, H.; Ralphs, T.; Salvagnin, D.; Steffy, D.E.; Wolter, K. MIPLIB 2010. *Math. Program. Comput.* **2011**, *3*, 103–163.
102. Arroyo, J.M.; Conejo, A.J. Optimal response of a thermal unit to an electricity spot market. *IEEE Trans. Power Syst.* **2000**, *15*, 1098–1104.
103. Yu, Z.; Sparrow, F.T.; Bowen, B.; Smardo, F.J. On convexity issues of short-term hydrothermal scheduling. *Electr. Power Energy Syst.* **2000**, *22*, 451–457.
104. Arroyo, J.M.; Conejo, A.J. Optimal response of a power generator to energy, AGC, and reserve pool-based markets. *IEEE Trans. Power Syst.* **2002**, *17*, 404–410.
105. Arroyo, J.M.; Conejo, A.J. Modeling of start-up and shut-down power trajectories of thermal units. *IEEE Trans. Power Syst.* **2004**, *19*, 1562–1568.
106. Wu, L. A tighter piecewise linear approximation of quadratic cost curves for unit commitment problems. *IEEE Trans. Power Syst.* **2011**, *26*, 2581–2583.

107. Nowak, M.P.; Römisch, W. Stochastic Lagrangian relaxation applied to power scheduling in a hydro-thermal system under uncertainty. *Ann. Oper. Res.* **2000**, *100*, 251–272.
108. Bendotti, P.; Fouilhoux, P.; Rottner, C. On the complexity of the unit commitment problem. *Ann. Oper. Res.* **2019**, *274*, 119–130.
109. de La Torre, S.; Arroyo, J.M.; Conejo, A.J.; Contreras, J. Price-maker self-scheduling in a pool-based electricity market: A mixed-integer LP approach. *IEEE Trans. Power Syst.* **2002**, *17*, 1037–1042.
110. Li, T.; Shahidehpour, M. Strategic bidding of transmission-constrained GENCOS with incomplete information. *IEEE Trans. Power Syst.* **2005**, *20*, 437–447.
111. Larsen, T.J.; Wangensteen, I.; Gjengedal, T. Sequential Time Step Unit Commitment. In Proceedings of the 2001 IEEE Power Engineering Society Winter Meeting, Columbus, OH, USA, 28 January–1 February 2001; pp. 1524–1529.
112. Parrilla, E.; García-González, J. Improving the B&B search for large-scale hydrothermal weekly scheduling problems. *Int. J. Electr. Power Electr. Syst.* **2006**, *28*, 339–348.
113. García-González, J.; Roque, A.M.S.; Campos, F.A.; Villar, J. Connecting the intraday energy and reserve markets by an optimal redispatch. *IEEE Trans. Power Syst.* **2007**, *22*, 2220–2231.
114. Sharma, D.; Trivedi, A.; Srinivasan, D.; Thillainathan, L. Multiagent modeling for solving profit based unit commitment problem. *Appl. Soft Comput.* **2013**, *13*, 3751–3761.
115. Hosseini, S.H.; Khodaei, A.; Aminifar, F. A novel straightforward unit commitment method for large-scale power systems. *IEEE Trans. Power Syst.* **2007**, *22*, 2134–2143.
116. Li, T.; Shahidehpour, M. Price-based unit commitment: A case of lagrangian relaxation versus mixed integer programming. *IEEE Trans. Power Syst.* **2005**, *20*, 2015–2025.
117. Chang, G.W.; Tsai, Y.D.; Lai, C.Y.; Chung, J.S. A Practical Mixed Integer Linear Programming Based Approach for Unit Commitment. In Proceedings of the IEEE Power Engineering Society General Meeting, Denver, CO, USA, 6–10 June 2004; Volume 221, pp. 221–225.
118. Lu, B.; Shahidehpour, M. Unit commitment with flexible generating units. *IEEE Trans. Power Syst.* **2005**, *20*, 1022–1034.
119. Sawa, T.; Sato, Y.; Tsurugai, M. Security Constrained Integrated Unit Commitment Using Quadratic Programming. In Proceedings of the 2007 IEEE Lausanne Power Tech, Lausanne, Switzerland, 1–5 July 2007; pp. 1858–1863.
120. Carrion, M.; Arroyo, J.M. A computationally efficient mixed-integer linear formulation for the thermal unit commitment problem. *IEEE Trans. Power Syst.* **2006**, *21*, 1371–1378.
121. Bisanovic, S.; Hajro, M.; Dlakic, M. Mixed integer linear programming based thermal unit commitment problem in deregulated environment. *J. Electr. Syst.* **2010**, *6*, 466–479.
122. Ostrowski, J.; Anjos, M.; Vannelli, A. Tight Mixed-Integer Linear Programming Formulations for Generator Self-Scheduling. In Proceedings of the 2010 23rd Canadian Conference Electrical and Computer Engineering (CCECE), Calgary, AB, Canada, 2–5 May 2010; pp. 1–4.
123. Ostrowski, J.; Anjos, M.; Vannelli, A. Tight mixed integer linear programming formulations for the unit commitment problem. *IEEE Trans. Power Syst.* **2012**, *27*, 39–46.
124. Morales-Espana, G.; Latorre, J.M.; Ramos, A. Tight and compact MILP formulation of start-up and shut-down ramping in unit commitment. *IEEE Trans. Power Syst.* **2013**, *28*, 1288–1296.
125. Morales-España, G.; Ramírez-Elizondo, L.; Hobbs, B.F. Hidden power system inflexibilities imposed by traditional unit commitment formulations. *Appl. Energy* **2017**, *191*, 223–238.
126. Morales-Espana, G.; Latorre, J.M.; Ramos, A. Tight and compact MILP formulation for the thermal unit commitment problem. *IEEE Trans. Power Syst.* **2013**, *28*, 4897–4908.
127. Gentile, C.; Morales-Espana, G.; Ramos, A. A tight MIP formulation of the unit commitment problem with start-up and shut-down constraints. *EURO J. Comput. Optim.* **2016**, *5*, 1–25.
128. Morales-Espana, G.; Ramos, A.; Garcia-Gonzalez, J. An MIP formulation for joint market-clearing of energy and reserves based on ramp scheduling. *IEEE Trans. Power Syst.* **2014**, *29*, 476–488.
129. Morales-Espana, Gentile, C.; G.; Ramos, A. Tight MIP formulations of the power-based unit commitment problem. *OR Spectrum* **2015**, *37*, 929–950.
130. Frangioni, A.; Gentile, C.; Lacalandra, F. Tighter approximated MILP formulations for unit commitment problems. *IEEE Trans. Power Syst.* **2009**, *24*, 105–113.
131. Jabr, R.A. Tight polyhedral approximation for mixed-integer linear programming unit commitment formulations. *IET Gener. Transm. Distrib.* **2012**, *6*, 1104–1111.
132. Quan, R.; Jian, J.B.; Mu, Y.D. Tighter relaxation method for unit commitment based on second-order cone programming and valid inequalities. *Int. J. Electr. Power Energy Syst.* **2014**, *55*, 82–90.
133. Yang, L.; Jian, J.; Wang, Y.; Dong, Z. Projected mixed integer programming formulations for unit commitment problem. *Int. J. Electr. Power Energy Syst.* **2015**, *68*, 195–202.
134. Yang, L.; Jian, J.; Zhu, Y.; Dong, Z. Tight relaxation method for unit commitment problem using reformulation and lift-andproject. *IEEE Trans. Power Syst.* **2015**, *30*, 13–23.
135. Yang, L.; Zhang. C.; Jian, J.; Meng, K.; Xu. Y.; Dong, Z. A novel projected two-binary-variable formulation for unit commitment in power systems. *Appl. Energy* **2017**, *187*, 732–745.

136. Catalao, J.P.S.; Mariano, S.J.P.S.; Mendes, V.M.F.; Ferreira, L.A.F.M. A practical approach for profit-based unit commitment with emission limitation. *Electr. Power Energy Syst.* **2010**, *32*, 218–224.
137. Catalao, J.P.S.; Mendes, V.M.F. Influence of environmental constraints on profit-based short-term thermal scheduling. *IEEE Trans. Sustain. Energy* **2011**, *2*, 131–138.
138. Shafie-Khah, M.; Moghaddam, M.P.; Sheikh-El-Eslami, M.K. Unified solution of a non-convex SCUC problem using combination of modified branch-and-bound method with quadratic programming. *Energy Convers. Manage.* **2011**, *52*, 3425–3432.
139. Lotfi, M.M. Short-term price-based unit commitment of hydrothermal gencos: A pre-emptive goal programming approach. *Int. J. Eng.* **2013**, *26*, 1017–1030.
140. Che, P.; Shi, G. An MILP Approach for a Profit-Based Unit Commitment Problem with Emissions Penalty. In Proceedings of the 26th Chinese Control and Decision Conference (2014 CCDC), Changsha, China, 31 May–2 June 2014; pp. 4474–4477.
141. Daneshi, H.; Choobbari, A.L.; Shahidehpour S.M.; Li, Z. Mixed Integer Programming Method to Solve Security Constrained Unit Commitment with Restricted Operating Zone Limits. In Proceedings of the 2008 IEEE International Conference on Electro/Information Technology, IEEE EIT 2008 Conference, Ames, IA, USA, 18–20 May 2008; pp. 187–192.
142. Alvarez-Lopez, J.; Ceciliano-Meza, J.L.; Guillén-Moya, I.; Nieva-Gómez, R. A MIQCP formulation to solve the unit commitment problem for large-scale power systems. *Int. J. Electr. Power Energy Syst.* **2012**, *36*, 68–75.
143. Daneshi, H.; Srivastava, A.K. Security-constrained unit commitment with wind generation and compressed air energy storage. *IET Gener. Transm. Distrib.* **2012**, *6*, 167–175.
144. Ostrowski, J.; Wang, J.; Liu, C. Exploiting symmetry in transmission lines for transmission switching. *IEEE Trans. Power Syst.* **2012**, *27*, 1708–1709.
145. Alemany, J.; Magnago, F.; Moitre, D.; Pinto, H. Symmetry Issues in Mixed Integer Programming based Unit Commitment. *Int. J. Electr. Power Energy Syst.* **2014**, *54*, 86-90.
146. Ostrowski, J.; Anjos, M.F.; Vannelli, A. Modified orbital branching for structured symmetry with an application to unit commitment. *Math. Program.* **2015**, *150*, 99–129.
147. Lima, R.M.; Novais, A.Q. Symmetry breaking in MILP formulations for unit commitment problems. *Comput. Chem. Eng.* **2016**, *85*, 162–176.
148. Knueven, B.; Ostrowski, J. Exploiting Identical Generators in Unit Commitment *IEEE Trans. Power Syst.* **2017**, *33*, 4496–4507.
149. Aliprandi, F.; Stoppato, A.; Mirandola, A. Estimating CO2 emissions reduction from renewable energy use in Italy. *Renew. Energy* **2016**, *96*, 220-232.
150. Keatley, P.; Shibli, A.; Hewitt, N.J. Estimating power plant start costs in cyclic operation. *Appl. Energy* **2013**, *111*, 550–557.
151. Hermans M.; Delarue, E. Impact of Start-Up Mode on Flexible Power Plant Operation and System Cost. In Proceedings of the 2016 13th International Conference on the European Energy Market (EEM), Porto, Portugal, 6–9 June 2016; pp. 1–6.
152. Morales-Espana, G.; Tejada-Arango, D.A. Modeling the hidden flexibility of clustered unit commitment. *IEEE Trans. Power Syst.* **2019**, *34*, 3294–3296.
153. Brandenberg, R.; Huber, M.; Silbernagl, M. The summed start-up costs in a unit commitment problem. *EURO J. Comput. Optim.* **2017**, *5*, 203–238.
154. Brandenberg, R.; Silbernagl, M. Implementing a unit commitment power market model in FICO Xpress-Mosel. *arXiv* 2014, arXiv:1412.4504.
155. Knueven, B.; Ostrowski, J.; Watson, J.P. A novel matching formulation for startup costs in unit commitment. *Math. Program. Comput.* **2017**, *12*, 225–248.
156. Morales-Espana, G.; Correa-Posada, C.M.; Ramos, A. Tight and compact MIP formulation of configuration-based combined-cycle units. *IEEE Trans. Power Syst.* **2016**, *31*, 1350–1359.
157. Chen, C.H.; Chen, N.; Luh, P.B. Head dependence of pump-storageunit model applied to generation scheduling. *IEEE Trans. Power Syst.* **2017**, *32*, 2869–2877.
158. Alvarez, G.E.; Marcovecchio, M.G.; Aguirre, P.A. Security constrained unit commitment problem including thermal and pumped storage units: An MILP formulation by the application of linear approximations techniques. *Electr. Power Syst. Res.* **2018**, *154*, 67–74.
159. Alvarez, G.E.; Marcovecchio, M.G.; Aguirre, P.A. Security constrained unit commitment scheduling: A new MILP formulation for solving transmission constraints. *Comput. Chem. Eng.* **2018**, *115*, 455–473.
160. Alvarez, G.E.; Marcovecchio, M.G.; Aguirre, P.A. Optimization of the integration among traditional fossil fuels, clean energies, renewable sources, and energy storages: An MILP model for the coupled electric power, hydraulic, and natural gas systems. *Comput. Ind. Eng.* **2020**, *139*, 106141.
161. Pan, K.; Guan, Y.; Watson, J.P.; Wang, J. Strengthened MILP formulation for certain gas turbine unit commitment problems. *IEEE Trans. Power Syst.* **2016**, *31*, 1440–1448.
162. Atakan, S.; Lulli, G.; Sen, S. A state transition MIP formulation for the unit commitment problem. *IEEE Trans. Power Syst.* **2017**, *33*, 736–748.
163. Tejada-Arango, D.A.; Lumbreras, S.; Sánchez-Martín, P.; Ramos, A. Which unit-commitment formulation is best? A comparison framework. *IEEE Trans. Power Syst.* **2020**, *35*, 2926–2936.
164. Knueven, B.; Ostrowski, J.; Watson, J.P. On mixed-integer programming formulations for the unit commitment problem. *INFORMS J. Comput.* **2020**, *32*, 857–876.

165. Kazarlis, S.A.; Bakirtzis, A.G.; Petridis, V. A genetic algorithm solution to the unit commitment problem. *IEEE Trans. Power Syst.* **1996**, *11*, 83–92.
166. IEEE 118-Bus: System Data. Available online: http://motor.ece.iit.edu/Data/ (accessed on 5 November 2021)
167. Morales-España, G. Unit Commitment Computational Performance, System Representation and Wind Uncertainty Management. Ph.D. Thesis, Comillas Pontifical University, Madrid, Spain, 2014.
168. Grigg, C.; Wong, P.; Albrecht, P.; Allan, R.; Bhavaraju, M.; Billinton, R.; Chen, Q.; Fong, C.; Haddad, S.; Kuruganty, S.; Li, W.; Mukerji, R.; Patton, D.; Rau, N.; Reppen, D.; Schneider, A.; Shahidehpour, M.; Singh, C. The IEEE Reliability Test System—1996. A report prepared by the Reliability Test System Task Force of the Application of Probability Methods Subcommittee. *IEEE Trans. Power Syst.* **1999**, *14*, 1010–1020.
169. Abujarad, S.Y.; Mustafa, M.; Jamian, J. Recent approaches of unit commitment in the presence of intermittent renewable energy resources: A review. *Renew. Sustain. Energy Rev.* **2017**, *70*, 215–223.
170. Abdou, I.; Tkiouat, M. Unit commitment problem in electrical power system: a literature review. *Int. J. Electr. Comput. Eng.* **2018**, *8*, 1357.
171. Håberg, M. Fundamentals and recent developments in stochastic unit commitment. *Int. J. Electr. Power Energy Syst.* **2019**, *109*, 38–48.
172. Hong, Y.Y.; Apolinario, G.F.D. Uncertainty in Unit Commitment in Power Systems: A Review of Models, Methods, and Applications. *Energies* **2021**, *14*, 6658.
173. Saravanan, B.; Das, S.; Sikri, S.; Kothari, D.P. A solution to the unit commitment problem—A review. *Front. Energy* **2013**, *7*, 223–236.
174. Abdi, H. Profit-based unit commitment problem: A review of models, methods, challenges, and future directions. *Renew. Sustain. Energy Rev.* **2020**, *1*, 1–28.
175. Anjos, M.F.; Conejo, A.J. Unit commitment in electric energy systems. *Found. Trends Electr. Energy Syst.* **2017**, *1*, 220–310.