Loss Aversion in Storage Locker Auctions*

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Abstract

I find that loss averse bidders bid noticeably below risk neutral ones in a novel tractable structural model for ascending auctions with both common and private value components as well as heterogenous bidders' characteristics. Moreover, I assess the empirical relevance of the model using data from storage locker auctions in the popular cable TV show *Storage Wars*, documenting for the first time the presence of loss aversion in actual ascending auctions. Additionally, I find that bidders reduce their bids even further when they incorporate the information of those bidders present who decide not to participate after inspecting the item auctioned.

Keywords: Ascending Auctions, Non-Bidding Participants, Prospect Theory, Structural Model.

JEL: D81, D44, C57, D82

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1 Introduction

The standard framework in most of the empirical and theoretical auction literatures has been expected utility, often with risk neutral bidders. However, Kahneman and Tversky (1979) criticized expected utility because they found that individuals derive their utility from gains and losses relative to some reference point, rather than from absolute levels of wealth as perfectly rational agents do under expect utility. They presented a new model of decision making under risk known as "Prospect Theory", whose key feature -*loss aversion*- is that individuals are much more sensitive to reductions than to increases in wealth. Given that bidders suffer both losses and gains in the auctions they participate, it is important to explore whether loss aversion might better reflect their bidding behavior.

In this paper, I propose a novel and tractable structural ascending auction model that replaces risk neutrality with loss aversion in the well-known framework with symmetric bidders in Milgrom and Weber (1982) and its asymmetric extension in Hong and Shum (2003). Like in standard models, the utility of a bidder depends only on the difference between his own valuation of the object auctioned and his bid, but in this new specification, a bidder is more sensitive to reductions in wealth than to increases of the same magnitude (see Kahneman and Tversky (1979)). In addition, my proposed model allows for both common and private value components in the bidder's valuations as well as heterogeneous bidder's characteristics. Importantly, I find that, ceteris paribus, loss averse bidders bid substantially lower than risk neutral ones.

To empirically assess the model, I focus on storage locker auctions, which have gained a lot of popularity in recent years, with 155,000 of them taking place each year in the US alone at an average price of \$425.[1] Specifically, I exploit a unique dataset of 254 actual auctions from the first three seasons of the popular cable TV show *Storage Wars*, which follows a core group of individual bidders who take part in storage locker auctions throughout the State of California. As shown in numerous empirical studies (see List (2006), Post et al (2008), Belot et al (2010) and van Dolder et al (2015) for examples), TV shows are an ideal setting to empirically test economic theories because they provide an environment with substantially larger economic incentives than lab experiments. Therefore, analyzing the behavior of bidders in these auctions seems especially relevant.

An interesting unique feature that storage locker auctions have compared to other ascending auctions, such as those run by eBay, Sotheby's, etc., is that the contents of the locker are unknown to both the auctioneer and potential buyers before and throughout the entire auction. This situation has the ideal characteristics for bidders to exhibit loss aversion, a feature that often

[1] See <https://www.statisticbrain.com/self-storage-industry-statistics/> for more details.

arises when comparing sure outcomes (not participating in the auction) with a risky prospect (participating and making an uncertain positive or negative profit) (see Kahneman and Tversky (1979) for more details).

Empirically, I find that most *Storage Wars* bidders are loss averse in a model in which bidder's characteristics are heterogeneous. However, the behavior of the most professional bidder is in line with risk neutrality. Not surprisingly, he is the bidder who bids most aggressively.

I also consider a more general framework in which bidders incorporate into their strategies the information of those bidders who are present but decide not to participate after inspecting the item put up for auction. This results in bidders reducing the aggressiveness of their bids even further as the number of non-bidding participants increases. Once again, I find that bidders continue to be loss averse in this more general framework. In addition, my results confirm the empirical relevance of taking into account the presence of non-bidding participants in ascending auctions.

Previous papers have provided experimental evidence of loss aversion in sealed-bid auctions (see Lange and Ratan (2010), Banerji and Gupta (2014), Rosato and Tymula (2019) and Eisenhuth and Grunewald (2020) for independent private values and Balzer and Rosato (2020) for interdependent ones). In contrast, there is little work in ascending auctions. An exception is von Wangenheim (2017), who theoretically showed that under independent private values the second-price sealed-bid auction yields strictly higher revenues than the ascending auction when bidders are expectation-based loss averse (see Kőszegi and Rabin (2006) for more details).

In this sense, this paper makes not only a methodological contribution by incorporating loss aversion in a structural ascending auction model with both private and common value components, but also a substantive one by documenting for the first time the presence of loss aversion in actual ascending auctions.[2]

The rest of the paper is organized as follows. Section 2 describes the TV show in greater detail and provides a summary of the dataset. In Section 3, I discuss my proposed structural model of ascending auctions under loss aversion. Then, in Section 4 I discuss the empirical results. Finally, in Section 5, I introduce a framework that incorporates the signals of the bidders present at the auction who decided not to participate. This is followed by the conclusions and several appendices where proofs and additional details can be found.

[2]Some previous papers have looked at other behavioral biases in ascending auctions. Specifically, Dodonova and Khoroshilov (2005, 2009) argue that bidders with independent private values may feel a quasi-endowment effect toward the object for which they are bidding, so that after making an initial bid of $\$x$ followed by a competitor's bid of $\$(x+1)$, they prefer to pay $\$(x+2)$ to keep the object even though they would never buy the auctioned object for this amount when facing a simple buying decision.

2 Storage Wars

The TV show *Storage Wars*, developed by A&E cable network, first aired on December 2010 and soon became the most watched program in the network's history. Each episode starts with potential bidders gathering outside a storage facility in the State of California. These facilities have the right to put up for auction the contents of a locker when its rent is not paid for three consecutive months. Before bidders are allowed into the storage facility to see the lockers, the auctioneer explains the rules. The auctions are cash only sales, with bidders only being able to bid on the entire contents of the locker (not on an item-to-item basis), and the winner the highest cash bidder.

The lock of the locker is then broken and bidders have exactly five minutes to look around without stepping inside or opening any boxes. During that time, bidders effectively receive a private noisy signal of the unknown contents, and therefore of the valuation of the locker put up for auction. After those five minutes, the auctioneer announces a suggested opening bid for the locker on sale and starts accepting increasingly higher bids from the bidders in the auction.[3] Unlike sealed-bid auctions, there exists "information transparency", in the sense that the identity of all the bidders and their bids are known during the entire auction. The highest bidder at any given moment has the standing bid, which can only be displaced by a higher bid from another bidder. Throughout the auction, every bidder is given the opportunity to outbid the standing bid.[4] Failure to do so results in the end of the auction, with the locker being sold to the winner at a price equal to his bid.

After all the auctions of the day are completed, the winning bidders go through their lockers sorting the "valuable" content from the rest. When they encounter an unusual, potentially very valuable item, bidders consult with experts to find out the actual value of the item.

Although the private valuation might differ from bidder to bidder because they may have different interests, such as collectibles or household items, there is also a clear common component. For example, if a locker contained a standard but very valuable item such as a brand new motorcycle, its value would be very much the same across bidders.

For all those reasons, a model which allows for both common and private values seems adequate to capture the behavior of bidders in these auctions.

[3] In storage locker auctions there are no reserve prices, i.e. the lowest price at which the seller is willing to sell the item. So in principle, the locker could be sold for $1.

[4] There is no predetermined ending time as in eBay. As a consequence, the practice of sniping, i.e. bidding in the very last seconds (see Roth and Ockenfels (2002)), is irrelevant in this auction.

2.1 *The main bidders and the auctioneers*

The first three seasons of the show follows four main regular bidders throughout the auctions: Dave Hester (a professional buyer who operates his own auction house), Darrell Sheets (a less experienced storage auction bidder who makes his living by selling in swap meets and through his online store), Jarrod Schulz (an even less experienced storage auction bidder who owns a thrift store) and Barry Weiss (a lifelong antiques collector who had never participated in storage auctions before). Additionally, there are other bidders present at the auctions whose identity are not shown publicly, but whose bids are.[5]

The auctioneers on the show are Dan and Laura Dotson, who have run their own business (American Auctioneers) since 1983. Their retribution scheme comes from a small percentage of the locker sale they receive from the storage unit company. Therefore, it is in their interest that the locker is sold at a high price.

One of their key roles is to engage bidders. To accomplish this, they have to start the auction by announcing a suggested opening bid low enough to be immediately accepted by one of the bidders. The regression results in Table 1 suggest that the opening bid is set taking into account the location and size of the locker, which is not surprising since the value of the locker is unknown to both the auctioneer and bidders before and throughout the entire auction.

(Table 1)

2.2 *Description of the data*

As explained in the introduction, I examine the bidding behavior of *Storage Wars* participants in 254 actual auctions, which covers seasons 1 (59), 2 (103) and 3 (92) of the TV show.[6] The dataset contains the identity of the bidders, including the four main regular ones, the number of main bidders present at the auction, as well as the total number of bidders bidding per auction (ranging from 2 to 7), the location of the auction, the size of the locker, whether the main regular bidders decide in real time to bid or not after visually inspecting the locker, the entire bid sequence and the ex-post value of the locker. I have also collected per capita income data of the municipality where the locker is located as one would expect a priori that richer neighborhoods have more valuable locker contents.

There are three types of lockers: small (10×10 ft.) fitting household items from 3 rooms, medium (10×20 ft.) fitting items from 5 rooms and large (10×30 ft.) fitting items from 7 rooms.

[5] Given that there is no identifying information on those bidders, I treat them as homogeneous when estimating the empirical model in section 4.

[6] Video clips of each episode are widely available on the Internet, for example, through the A&E website <https://www.aetv.com/shows/storage-wars>.

Table 2 offers a basic description of the data.

(Table 2)

For each season, it shows the number of times a small, medium and large locker has been auctioned, the average profit each bidder makes, the average ex-post value of the locker auctioned, the average median household income of the municipalities where the lockers are located, and the total number of bidders participating per auction.

The most frequent auctions involve 3, 4 and 5 active bidders, with 50, 72 and 63 auctions, respectively. Additionally, there are many more small and medium size lockers auctioned than large ones. After running a standard OLS regression, I find that the order in which the lockers are put up for auction each day is independent of the ex-post value of the locker, which is again not surprising because the contents of the locker are unknown.

Table 3 describes the participation rates of the main bidders in *Storage Wars*.

(Table 3)

As can be seen, none of the four main bidders has actually participated in all of the auctions. Jarrod is the bidder who has participated the most, followed by Darrell, Dave and Barry. However, all four of them only coincide 11.42% of the time. Given that the main regular bidders often publicly indicate whether they will participate in the auction after looking at the locker to be auctioned, I assess whether their actual participation is in line with their claims using a standard independence test (see Sentana (2021) for more details). The results show that the null hypothesis of independence between their actual participation and their claims is massively rejected for all the main bidders (p-value of 0), confirming that their participation decisions are coherent with their announcements. This fact motivates the extension of the model in section 5, in which bidders incorporate into their strategies the information of those bidders who are present but decide not to participate after inspecting the item put up for auction.

3 The Model with Loss Aversion

The theoretical auction model studied in this paper resembles the Japanese "button" auction in Milgrom and Weber (1982), in which prices raise continuously, bidders keep pressing a button to remain active, dropout prices are common knowledge, and once a bidder drops out, he cannot reenter the auction at a higher price.[7]

[7] This standard model has been widley used by most of the subsequent literature (see Athey and Haile (2002), Hong and Shum (2003) and Aradillas-Lopez et al (2013) for examples).

More formally, consider an auction of a single item with N potentially heterogeneous bidders, indexed $i = 1, \ldots, N$, for whom the value of the item auctioned is V_i. However, at the beginning of the auction, they only observe a private noisy signal X_i of their own valuation V_i.

The auction proceeds in rounds, indexed $k = 0, \ldots, N-2$, in which active bidders submit bids. A new round starts whenever a bidder drops out and bidders are indexed by the round in which they drop out: bidder N drops out in round 0 at price P_0 and bidder $N-k$ drops out in round k at price P_k, with bidder 1 winning the auction at price P_{N-2}.[8]

In ascending auctions, a Bayesian-Nash equilibrium consists of bid functions $\beta_i^k(X_i; \Omega_k)$ for each bidder i and round k, where Ω_k is the available information set at the beginning of round k containing the previously observed dropout prices. Effectively, the bidding function $\beta_i^k(X_i; \Omega_k)$ determines the price at which bidder i should quit the auction at round k as a function of his signal and the available information set. The collection of bid functions $\beta_i^0(X_i; \Omega_0), \ldots, \beta_i^{N-2}(X_i; \Omega_{N-2})$ are common knowledge, with $\Omega_0 = \emptyset$.

Like Milgrom and Weber (1982), I assume that the utility of bidder i depends only on the difference between his own valuation of the item put up for auction and his bid. More precisely, let $u[V_i - \beta_i^k(X_i; \Omega_k)]$ denote bidder i's utility at round k, where $u(\cdot)$ is continuous, nondecreasing in its argument and satisfies $u(0) = 0$. But instead of an expected utility framework, as in the standard literature, I draw inspiration from the work in Kahneman and Tversky (1979) by assuming the following functional form:

$$u\Big(V_i - \beta_i^k(X_i; \Omega_k)\Big) = \begin{cases} V_i - \beta_i^k(X_i; \Omega_k) & \text{for } V_i > \beta_i^k(X_i; \Omega_k) \\ \lambda_i[V_i - \beta_i^k(X_i; \Omega_k)] & \phantom{\text{for }} V_i \leq \beta_i^k(X_i; \Omega_k) \end{cases}, \tag{1}$$

where $\lambda_i \geq 1$ captures loss aversion, i.e. the tendency of individuals to prefer avoiding reductions in wealth than equivalent gains. This piecewise linear specification, which has a kink at the origin,[9] has been used by many authors in a variety of economic situations (see Barberis et al (2001), Kőszegi and Rabin (2006) and Sprenger (2015) for examples). The reason is that loss aversion at the kink is very relevant for gambles that can lead to both gains and losses, such as in single item auctions, where "gains" and "losses" correspond to the difference between the value of the item auctioned and the final price.[10]

(Figure 1)

[8] Given that continuous bidding does not take place in practice, I assign the dropout price of a bidder to the bid of the next bidder who outbids him.

[9] As in Kahneman and Tversky (1979), the primary reference level is the status quo, which in this case is 0, i.e. not participating in the auction.

[10] Kahneman and Tversky (1979) also propose that the utility function should be mildly concave over gains and convex over losses. However, this is most relevant when choosing between prospects that involve only gains or only losses (see Barberis et al (2001) for further discussion of this point).

Figure 1 illustrates the effects of varying the loss aversion parameter λ on the underlying utility function (1). As expected, $\lambda = 1$ implies risk neutrality, i.e. same marginal utility for both gains and losses (the standard model). However, for any other value of $\lambda > 1$, bidders are more sensitive to reductions in wealth than to increases of the same magnitude, preferring not to lose \$10 rather than to gain \$10.

The structure of the Bayesian-Nash equilibrium under loss aversion in increasing bidding strategies (i.e. $\beta_i^k(X_i; \Omega_k)$ is increasing in X_i for $k = 0, \ldots, N-2$) extends the equilibrium described in Milgrom and Weber (1982) and Hong and Shum (2003) as follows. For bidders $i = 1, \ldots, N$ active in round 0, the bid functions are implicitly defined by the equilibrium condition

$$E\{u[V_i - \beta_i^0(X_i; \Omega_0)] | \Upsilon_i^0\} = u(0) = 0,$$

where $\Upsilon_i^0 = \{X_i; X_j = \varphi_j^0[\beta_i^0(X_i; \Omega_0); \Omega_0]\}$ for $j = 1, \ldots, N$ and $j \neq i$, with $\varphi_j^k(.; \Omega_k)$ being the inverse bid function at round $k = 0, \ldots, N-2$ mapping prices into signals, so that $\varphi_i^k[\beta_i^k(X_i; \Omega_k); \Omega_k] = X_i$.

In turn, the analogous condition for bidders $i = 1, \ldots, N-k$ active in round $k = 1, \ldots, N-2$ will be given by

$$E\{u[V_i - \beta_i^k(X_i; \Omega_k)] | \Upsilon_i^k\} = u(0) = 0, \tag{2}$$

where $\Upsilon_i^k = \{X_i; X_j = \varphi_j^k[\beta_i^k(X_i; \Omega_k); \Omega_k], X_h = \varphi_h^{N-h}(P_{N-h}; \Omega_{N-h})\}$, for $j = 1, \ldots, N-k$, $j \neq i$ and $h = N-k+1, \ldots N$, with X_h denoting the signals of the bidders who have dropped out prior to round k. Since the equilibrium bid functions are common knowledge, an active bidder in round k can infer the private information possessed by the previous dropout bidders by inverting their bid functions, so that $X_h = \varphi_h^{N-h}(P_{N-h}; \Omega_{N-h})$.[11]

3.1 *The stochastic setup*

Following Hong and Shum (2003), I use a parametric approach by assuming that bidder's signals and valuations $(X_1, ..., X_N, V_1, ..., V_N)$ are log-normally distributed. This assumption allows me to derive tractable closed-form formulas for the expectations in (2), from which I can then obtain analytic expressions for the equilibrium bid functions $\beta_i^k(X_i; \Omega_k)$, which are exponentially affine.[12]

[11] It is worth mentioning that if several bidders were to quit simultaneously, the equilibrium conditions in (2) would still hold (see Milgrom and Weber (1982) for more details).

[12] Two other empirical studies have previously used Hong and Shum's (2003) auction model. Dionne et al (2009) studied Mauritanian slave auctions in the 19th century, finding evidence of heterogeneity in the quality of the information between bidders, which in turn led to adverse selection. In turn, Koptyug (2016) found that in online car auctions, resellers are better than consumers at appraising the value of the cars they are bidding on.

Let V_i be defined as $V_i = A_i \times V$, where A_i is a bidder-specific private value component and V a common value component to all bidders in the auction. Although A_i and V, or indeed V_i, are not directly observed by the bidders, they are assumed to be independently log-normally distributed so that

$$\ln V = v = m + \epsilon_v \sim N(m, r_0^2),$$

$$\ln A_i = a_i = \bar{a}_i + \epsilon_{a_i} \sim N(\bar{a}_i, t_i^2),$$

and

$$\ln V_i = v_i = \ln V + \ln A_i \sim N(m + \bar{a}_i, r_0^2 + t_i^2).$$

In practice, bidder i only observes a private noisy signal X_i of his own valuation V_i, which will be effectively revealed to the other bidders after he drops out. Given the log-normality assumption,

$$\ln X_i = x_i = v_i + \xi_i \sim N(m + \bar{a}_i, r_0^2 + t_i^2 + s_i^2),$$

where $\xi_i \sim N\left(0, s_i^2\right)$, and s_i^2 captures the amount of information any bidder has about the true value of the item being auctioned (see Dionne et al (2009) and Koptyug (2016) for more details). The common knowledge assumption implies that all the model parameters $\theta \equiv (\bar{a}_i, m, t_i^2, r_0^2, s_i^2)$ are known among the bidders.

In this log-normal setup, the conditional expected value of V_i can be written as:

$$E(V_i|X_1, \ldots, X_N) = \exp[E(v_i|x) + \frac{1}{2}Var(v_i|x)],$$

where $x = (x_1, \ldots, x_N)$, and $E(v_i|x)$ and $Var(v_i|x)$ denote the unconditional mean vector and variance-covariance matrix of (v_i, x) for bidder i, respectively (see Appendix A.2 for further details).

The following proposition, which I prove in Appendix A.1, establishes sufficient conditions to ensure the existence of an equilibrium under loss aversion in this stochastic framework.

Proposition 1 *Let $\eta_i \geq 0$ be the unique solution to*

$$\exp(\eta_i)\left[(1-\lambda_i)\,\mathrm{erf}\left(\frac{\frac{\omega_{v_i|x}}{2} + \eta_i}{\sqrt{2\omega_{v_i|x}}}\right) + (1+\lambda_i)\right] = \left[(\lambda_i - 1)\,\mathrm{erf}\left(\frac{\frac{\omega_{v_i|x}}{2} - \eta_i}{\sqrt{2\omega_{v_i|x}}}\right) + (\lambda_i + 1)\right],$$

where erf*(.) is the error function. Then*

$$\beta_i^k(X_i; \Omega_k) = \exp(-\eta_i)E(V_i|\Upsilon_i^k) \tag{3}$$

is an increasing-strategy Bayesian-Nash equilibrium under loss aversion in the log-normal stochastic setup.

(Figure 2)

Figure 2 compares the equilibrium bidding function under risk neutrality (the standard model) with loss aversion ($\lambda > 1$). This graph shows that, ceteris paribus, loss aversion leads to a substantial reduction in the bids as a function of the signal X_i. As a consequence, the expected seller revenue will decrease relative to risk neutrality. However, the only difference between an equilibrium under risk neutrality and loss aversion is the multiplicative factor $\exp(-\eta_i)$ (see Appendix A.3 for more details).

To define the equilibrium log-bid functions for round k, I use the same notation as Hong and Shum (2003). Let $x_r^k = (x_1, \ldots, x_{N-k})'$ denote the vector of (log) private noisy signals of the bidders active in round k, and $x_d^k = (x_{N-k+1}, \ldots, x_N)'$ the vector of (log) signals of the dropped out bidders before round k. With this notation, the log-bidding function for the bidders active in round k under loss aversion will be

$$b_i^k(x_i; x_d^k) = \log[\beta_i^k(X_i; \Omega_k)] = \frac{1}{\mathcal{A}_i^k}(x_i + \mathcal{D}_i^k x_d^k + \mathcal{C}_i^k) - \eta_i, \; i = 1, \ldots, N-k, \tag{4}$$

where η_i captures the effects of loss aversion in (3) (see Hong and Shum (2003) and Appendix A.4 for detailed expressions for $\mathcal{A}^k$, $\mathcal{C}^k$ and $\mathcal{D}^k$). Note that (4) depends on a bidder's own private signal x_i, as well as on the signals of those bidders who have dropped out prior to round k (x_d^k), except for round 0, where $\mathcal{D}^0$ and x_d^0 are obviously undefined (see again Appendix A.4).

Intuitively, by observing the dropout prices in previous rounds, the remaining active bidders can make inferences about the private information possessed by the bidders who have dropped out. In other words, they can obtain an unbiased estimate of bidder j's (log) valuation from observing his private signal x_j. In common value auctions in which there is correlation across bidders' valuations (V_i), this information allows the remaining active bidders to update their beliefs about their own valuation, causing the prices at which bidders intend to exit to change as the auction progresses. In contrast, Vickrey (1961) showed that in private value auctions this updating does not occur, and each bidder has a weakly dominant strategy which is to bid up to his valuation (see Athey and Haile (2002) and Online Appendix B.1 for a more detailed discussion on the special cases of pure common value and independent private value auctions).

(Figure 3)

Figure 3 plots the equilibrium log-bid functions of a representative bidder in an auction with 5 loss averse bidders. The log-signal x_i is plotted on the x axis, while the log-bid functions in (4) for each round $k = 0, \ldots, 3$ are plotted on the y axis. As depicted in the figure, the slope of the log-bid function decreases for subsequent rounds, implying that, for a given realization of x_i, the targeted dropout price of the representative bidder decreases as the auction progresses. This occurs because bidders can update their bidding functions accordingly after incorporating the private noisy signal of the bidders who have previously drop out, thereby mitigating the chances of suffering the so-called winner's curse (see Online Appendix B.2 for more details).

3.2 *Econometric methodology*

Even though the model parameters $\theta \equiv (\bar{a}_i, m, t_i^2, r_0^2, s_i^2)$ are assumed to be known by the bidders, their values are unknown from an econometrician's point of view. To estimate θ, Hong and Shum (2003), Dionne et al (2009) and Koptyug (2016) employ the simulated non-linear least squares (SNLS) estimator of Laffont et al (1995), but with an independent probit kernel-smoother as in McFadden (1996). In contrast, I use Maximum Likelihood (ML) because when the structural auction model is correctly specified, ML is more efficient than SNLS, while when it is misspecified, SNLS is not more robust than ML given that one must draw prices from the assumed model (see Dridi et al (2007) for more details). For example, suppose one estimates an independent private value model when in fact the true model is a pure common value one. In that case, both the log-bidding functions and the simulated drop out prices will be incorrect, which affects both ML and SNLS. In addition, SNLS does not always identify the parameters of the model with a small number of bidders, while ML identifies all the parameters even when there are only two bidders.

In each auction, an econometrician only observes the vector of dropout prices for bidders $2, \ldots, N$, the order in which bidders drop out and their identities. In this respect, Hong and Shum (2003) make clear that one must condition on the observed dropout sequence to derive the log-likelihood function. In practice, this means that the underlying log-signals $(x_1, \ldots, x_N)$ must be constrained to some region $\mathcal{T}_1(\theta) \subset R^N$, which I describe in Appendix A.5. Furthermore, they also show that since the winner's dropout bid is never observed in ascending auctions, the winner's log-signal x_1 is constrained to some other region $\mathcal{T}_2(x_2, \ldots, x_N|\theta) \subset R^1$, which is consistent with bidder 1 winning the auction. Therefore, if $\mathcal{P} = (p_0, \ldots, p_{N-2})'$ denotes the vector of log-dropout bids, the log-likelihood function for a given auction must be computed as:

$$\mathcal{L}(\mathcal{P}|\theta) = \log f(\mathcal{P}|\theta) + \log \Pr[\mathcal{T}_2(x_2, \ldots, x_N|\theta)] - \log \Pr[\mathcal{T}_1(\theta)|\theta], \quad (5)$$

which resembles the log-likelihood function of a truncated and censored multivariate normal, with $f(\mathcal{P}|\theta)$ reflecting the continuous component corresponding to the likelihood of the observed drop out prices, $\Pr[\mathcal{T}_2(x_2, \ldots, x_N|\theta)]$ the conditional probability associated to the censored winning bid, and $\Pr[\mathcal{T}_1(\theta)|\theta]$ the truncation probability that reflects the order in which the different bidders drop out (see Hong and Shum (2003) and Appendix A.5 for more details).[13]

[13] Crucially, Hong and Shum (2003) prove that the support of x does not depend on θ in the log-normal stochastic setup in section 3.1, so the usual ML regularity conditions hold and standard asymptotic theory applies.

Since the auctions take place independently, the sample log-likelihood function is simply the sum of the log-likelihood function of each auction. Thus, it is straightforward to combine auctions with different number of bidders.

From the practical point of view, the main difficulty in computing the log-likelihood function (5) is the multivariate integral $\Pr[\mathcal{T}_1(\theta)|\theta]$ (see again Appendix A.5 for details). Nevertheless, this is certainly feasible with up to 7 active bidders, although it slows down the numerical optimization. Still, given that the likelihood function is highly non-linear, it is convenient to consider multiple initial values.

4 Empirical Application

Figure 4 displays the boxplot of the profit/losses in *Storage Wars* auctions without a few extreme outliers.

(Figure 4)

The central mark in the box indicates the median profit ($890), and the bottom and top edges indicate the 25th ($-47.5) and 75th ($2,412.5) percentiles, respectively, confirming the presence of losses in these auctions. As can be seen, the profit/losses values involved in these auctions are relatively small. Therefore, the smooth utility functions with moderate risk aversion commonly considered in the literature under expected utility imply that bidders would be close to risk neutral when facing such modest stakes. In contrast, loss aversion may be present in these auctions because the utility in (1) captures the well documented fact that over modest gambles, individuals are noticeably more averse to losses relative to the status quo than they are attracted by gains (see Barberis et al (2001) for more details).

4.1 *Model specification*

Given that the common public information bidders have during the auction are the locker characteristics and the municipality in which they are located, I have regressed the (log) ex-post value of the locker on its size and the per capita income of the municipality. The results are presented in Table 4.[14]

(Table 4)

Not surprisingly, the statistical significance of the results confirm that richer neighborhoods and larger lockers have more valuable locker contents. Consequently, I specify the mean of the

[14] A more flexible non-linear specification that allows for different coefficients for each of the three locker sizes does not offer any statistically significant gains in fit, which is not surprinsing given that the sequence of locker sizes corresponds to 3, 5 and 7 rooms (see section 2.2 for more details).

common value component for a given auction as

$$m = \beta_0 + \beta_1 SIZE + \beta_2 HHI,$$

where $SIZE$ is a variable that measures the size of the locker (small (1), medium (2) or large (3)) and HHI captures the median household income of the municipality where the locker is located in the State of California.

In contrast, private valuations are usually associated with differences in interests across bidders, for which I do not observe any proxies. For that reason, I flexibly define the mean of the private value component of the four main bidders (Barry "Ba", Darrell "Dr", Dave "Dv" and Jarrod "Jr"), as well as of the other active bidders whose identity is not shown publicly, as

$$\bar{a} = \begin{pmatrix} \alpha_0 + \alpha_1 Ba & \alpha_0 + \alpha_2 Dr & \alpha_0 + \alpha_3 Dv & \alpha_0 + \alpha_4 Jr & \alpha_0 & \cdots & \alpha_0 \end{pmatrix},$$

where Ba, Dr, Dv and Jr are mutually exclusive dummy variables. For example, Ba takes the value 1 if Barry is an active bidder in the auction and 0 otherwise. Note that α_0 is the mean of the private value component of those bidders whose identity is unknown.[15]

Furthermore, to guarantee positivity, the variance of the common value component, which is obviously the same across bidders, is modelled as $r_0^2 = \exp(\delta_0)$, while the variance of the noise for each of the bidder's signals is flexibly defined as

$$s^2 = \exp\begin{pmatrix} \gamma_0 + \gamma_1 Ba & \gamma_0 + \gamma_2 Dr & \gamma_0 + \gamma_3 Dv & \gamma_0 + \gamma_4 Jr & \gamma_0 & \cdots & \gamma_0 \end{pmatrix},$$

so that γ_0 is the baseline variance of the anonymous bidders.

I also allow for unrestricted heterogeneity in the variance of the private value component as follows

$$t^2 = \exp\begin{pmatrix} \tau_0 + \tau_1 Ba & \tau_0 + \tau_2 Dr & \tau_0 + \tau_3 Dv & \tau_0 + \tau_4 Jr & \tau_0 & \cdots & \tau_0 \end{pmatrix}.$$

Finally, I set the loss aversion parameter λ to 2.25, a value initially proposed by Tversky and Kahneman (1992) on the basis of experimental evidence which has been used by most of the subsequent literature (see for example Barberis et al (2001), Barberis and Huang (2008) and Post et al (2008)).

4.2 *Results*

The first thing I do is check whether *Storage Wars* bidders exhibit loss aversion. To do so, I fit the model with $\lambda = 2.25$ for all the bidders and compare it to a specification with

[15] Given that in all the formulas all that matters is $m + \bar{a}_i$ (see Appendix A.2 for further details), I set $\beta_0 = 0$ without loss of generality because the constant terms of $\bar{a}$ and m are not separately identified.

risk neutrality ($\lambda = 1$). Surprisingly, the likelihood is actually worse. However, given that the model in section 3 explicitly allows for heterogeneous bidders' characteristics, this two extreme specifications are not the only ones that one could consider. In fact, when I set $\lambda = 1$ for Dave and $\lambda = 2.25$ for all the other bidders, I find that the difference between the log-likelihoods of the risk neutral model and this alternative specification is 9.41, thus confirming the empirical relevance of loss aversion in ascending auctions. Reassuringly, this is the combination of $\lambda = 1$ and $\lambda = 2.25$ across bidders that provides the best log-likelihood fit.[16]

As I explained in section 2.1, Dave is the most professional bidder in *Storage Wars*. Therefore, my finding is not entirely surprising in view of the results in List (2004), who found that professional traders did not exhibit loss aversion.[17] In this respect, it is worth mentioning that Dave suffers the smallest median loss when he losses and enjoys the largest median profit when he wins, regardless of the size of the locker.

The maximum likelihood estimates of the model parameters for this specification are shown in Table 5.

(Table 5)

The results indicate that the coefficients of the size of the locker (β_1) and the per capita income of the municipality (β_2) are both positive and statistically significant, which agrees with the findings in Table 4 regarding the specification of the mean of the common value component (m). Additionally, there is strong evidence of asymmetry in terms of the mean of the private value components (p-value of 0 for LR test of $H_0 : \alpha_1 = \alpha_2 = \alpha_3 = \alpha_4 = 0$) and weaker evidence of heterogeneity in the accuracy of the signals (p-value of 0.07 for LR test of $H_0 : \gamma_1 = \gamma_2 = \gamma_3 = \gamma_4 = 0$). However, there is no evidence of heterogeneity in the importance of the private value component when t^2 is heterogeneously modelled as in section 5.1 (p-value of 0.43 for LR test of $H_0 : \tau_1 = \tau_2 = \tau_3 = \tau_4 = 0$).

The variance of the common value component (r_0^2) explains 74% of the variance of the valuation V_i (see section 3.1 and Appendix A.2), which reflects that the model is neither a pure common value nor an independent private value model, but a mixture of both. To confirm this claim, I formally compare the estimated model to those two extreme versions:

	LR Test	p-value
Independent Private Value	404.23	0
Pure Common Value	77.24	0

Although the pure common value model provides a better match of the results in Table 5, it is

[16] In this respect, setting λ to either 1 or 2.25 substantially decreases the number of parameters to estimate. Nevertheless, allowing λ to vary freely in this range yields similar results.

[17] In contrast, Pope and Schweitzer (2011) found that even the best golfers seem loss averse in the non-pecuniary context of golf putts.

still rejected by a long margin.

(Figure 5)

Figure 5 plots the estimated equilibrium log-bidding functions at round 0 of *Storage Wars* bidders, all of whom are loss averse except for Dave, who is risk neutral. As can be seen, Dave, whose marginal utility is the same for both gains and losses, is the most aggressive bidder for most signal values, although his bidding function has the lowest slope. At the opposite extreme, Barry is the least aggressive bidder. As an illustration, suppose both of them had the same log-signal $x_i = 8.5$ ($4,914), which is approximately the average ex-post value of all the lockers in *Storage Wars*. Then, we can read off the graph that Dave's targeted log-dropout price in round 0 would be 8.48 ($4,821), while for Barry it would be 6.91 ($1,007).

5 The Information from Active Non-Bidding Participants

5.1 *The model*

A standard assumption in auction theory is that the bidders present at the auction coincide with all the potential bidders willing to participate (see Paarsch (1997), Krasnokutskaya and Seim (2011), Athey et al (2011) and Gentry and Li (2014) for examples).[18] Nevertheless, not all the bidders who are present in an ascending auction end up participating after inspecting the item put up for auction. In fact, some bidders decide not to participate when the auctioneer announces the opening bid. Therefore, it is important to distinguish between active bidders and active non-bidding participants in the following sense: active bidders are the ones who bid in the auction and either win or dropout at some point; in contrast, active non-bidding participants are the ones who are present in the auction but effectively drop out at the opening bid.

In storage locker auctions, all potential bidders observe each other when they assess the valuation of the item before the auction starts. Therefore, it seems reasonable to assume that at the beginning of the auction, active bidders can recover the private information active non-bidding participants possess and update their bidding functions accordingly. As I mentioned before, the main bidders in *Storage Wars* publicly indicate their willigness to participate after oberving the locker to be auctioned.

To define the equilibrium log-bid functions in this more general framework, let q denote the number of active non-bidding participants and N the number of active bidders, with $N+q$ being the total number of potential bidders. With this notation, a result analogous to Proposition 1

[18] This assumption is not plausible in eBay auctions, as shown in Song (2004).

shows that the log-bidding function in round -1, i.e. before the auction starts, will be given by

$$b_i^{-1}(x_i) = \log[\beta_i^{-1}(X_i; \dot{\Omega}_{-1})] = \frac{1}{\dot{\mathcal{A}}_i^{-1}}(x_i + \dot{\mathcal{C}}_i^{-1}) - \eta_i,\ i = 1, \ldots, N + q, \tag{6}$$

which again reflects loss aversion, captured by η_i in expression (3), and depends only on bidder's i own private signal x_i (see Appendix A.6 for detailed expressions for $\dot{\mathcal{A}}^{-1}$ and $\dot{\mathcal{C}}^{-1}$).[19] In this context, the active non-bidding participants will be the ones who on the basis of this log-bidding function decide not to participate when the auctioneer announces the opening bid P_{-1}.

Since the equilibrium bid functions are common knowledge, at round 0 active bidders can infer the private signals the q active non-bidding participants by inverting their log-bid functions. Thus,

$$x_q = \ln X_q = b_q^{-1}(x_q)\dot{\mathcal{A}}_q^{-1} - \dot{\mathcal{C}}_q^{-1}.$$

To understand the role of active non-bidding participants in the ascending auction game, suppose that there are only 4 bidders present. Without loss of generality, imagine bidder 4 drops out at the opening bid, so at round 0, only bidders 1, 2 and 3 remain active. These active bidders can now infer the private signal of bidder 4, and update their bidding functions accordingly at the beginning of the auction.

More generally, let $\ddot{x}_r^0 = (x_1, \ldots, x_N)'$ denote the vector of private noisy signals of the active bidders in round 0, and $\ddot{x}_d^0 = (x_{N+1}, \ldots, x_{N+q})'$ the vector of signals of the active non-bidding participants who effectively dropped out in round -1. With this notation, the log-bidding function for the active bidders in round 0 under loss aversion will be

$$b_i^0(x_i; \ddot{x}_d^0) = \log[\beta_i^0(X_i; \ddot{\Omega}_0)] = \frac{1}{\ddot{\mathcal{A}}_i^0}(x_i + \ddot{\mathcal{D}}_i^0 \ddot{x}_d^0 + \ddot{\mathcal{C}}_i^0) - \eta_i,\ i = 1, \ldots, N. \tag{7}$$

(see again Appendix A.6 for detailed expressions for $\ddot{\mathcal{A}}^0$, $\ddot{\mathcal{C}}^0$and $\ddot{\mathcal{D}}^0$). Compared to equation (4) for $k = 0$, which only depends on a bidder's own signal x_i, now $b_i^0(x_i; \ddot{x}_d^0)$ is also a function of the signals of the active non-bidding participants $\ddot{x}_d^0$.

For any subsequent round $k = 1, ..., N-2$, the log-bidding function for the bidders active in round k are entirely analogous to (4), and therefore depends on a bidder's own private signal x_i, as well as the signals of those bidders who have dropped out prior to round k, including the active non-bidding participants, i.e.

$$x_d^k = (\underbrace{x_{N-k+1}, \ldots, x_N}_{N-k}, \underbrace{x_{N+1}, \ldots, x_{N+q}}_{q})'.$$

(Figure 6)

[19] Notice that $b_i^{-1}(x_i)$ is analogous to the log-bidding functions in round 0 without active non-bidding participants in (4).

Figure 6 compares the equilibrium log-bidding function for a loss averse active bidder in round 0 with 0, 1 and 2 active non-bidding participants. As expected, bidders reduce the aggressiveness of their bids even further as the number of active non-bidding participants increases, substantially reducing the chances of suffering the winner's curse. Intuitively, this occurs because at round 0 active bidders recover the private information active non-bidding participants possess, and they update their log-bidding functions accordingly.[20]

5.2 *Econometric methodology*

The structure of the log-likelihood function is similar to the one in section 3.2 once we condition on the vector of active non-bidding participants' dropout bids. In particular, the log-likelihood function for a given auction can be written as:

$$\mathcal{L}(\mathcal{P}|\theta, \dot{\Upsilon}^{-1}) = \log f(\mathcal{P}|\theta, \dot{\Upsilon}^{-1}) + \log \Pr[\mathcal{T}_2(x_d^{N-2}|\theta, \dot{\Upsilon}^{-1})] - \log \Pr[\mathcal{T}_1(\theta)|\theta, \dot{\Upsilon}^{-1}],$$

where $\dot{\Upsilon}^{-1} = [X_l = \varphi_l^{-1}(P_{-1}; \Omega_{-1})]$ for $l = N+1, \ldots, N+q$, with $\varphi_l^{-1}(.; \Omega_{-1})$ being the vector of inverse bid functions at round -1, X_l denoting the signals of the q active non-bidding participants, $f(\mathcal{P}|\theta, \dot{\Upsilon}^{-1})$ reflecting the (conditional) continuous likelihood of the observed drop out prices, $\Pr[\mathcal{T}_2(x_d^{N-2}|\theta, \dot{\Upsilon}^{-1})]$ the (conditional) probability associated to the censored winning bid, and $\Pr[\mathcal{T}_1(\theta)|\theta, \dot{\Upsilon}^{-1}]$ the (conditional) truncation probability that reflects the order in which the different bidders drop out (see Appendix A.7 for more details).

5.3 *Empirical results*

Based on the evidence in section 4.2, I continue to set $\lambda = 1$ for Dave (the most professional bidder in the sample) and $\lambda = 2.25$ for all the other bidders. In this case, the improvement in the log-likelihood function relative to the risk neutral model is 12.73, which is even greater than in section 4.2. Therefore, loss aversion is again empirically relevant in this more general framework.

The maximum likelihood estimates of the model parameters for this specification are shown in Table 6.

(Table 6)

The values of β_1 and β_2 (coefficients for the size of the locker and the per capita income of the municipality) are again statistically significant. Additionally, I find that there is strong evidence of heterogeneity in the precision of the signals (p-value of 0 for LR test of $H_0 : \gamma_1 = \gamma_2 = \gamma_3 = \gamma_4 = 0$) and of asymmetry in the mean of the private value component (p-value of

[20] The log-bidding functions in Figure 6 are equivalent to the log-bidding functions in round 0, 1 and 2 without active non-bidding participants in Figure 3.

0 for LR test of $H_0 : \alpha_1 = \alpha_2 = \alpha_3 = \alpha_4 = 0$). However, once more I find no evidence of heterogeneity in the importance of the private value component t^2 (p-value of 0.77 for LR test of $H_0 : \tau_1 = \tau_2 = \tau_3 = \tau_4 = 0$), as in Table 5.

(Figure 7)

Figure 7 illustrates the estimated equilibrium log-bidding functions of *Storage Wars* bidders in round 0 with 1 active non-bidding participant, in this case Darrell. This graph shows that the slope of the log-bidding functions for the remaining bidders decreases substantially compared to their round 0 log-bidding functions in Figure 5. Intuitively, this reflects the fact that bidders effectively take into account the private information of the bidder who decided not to participate after inspecting the locker put up for auction, thereby confirming the empirical relevance of active non-bidding participants in ascending auctions.

6 Conclusions

In this paper I propose a novel and tractable structural model with both private and common value components for ascending auctions in which heterogeneous bidders exhibit loss aversion. Importantly, I find that, ceteris paribus, the bidding functions of a loss averse bidder are significantly lower than those of a risk neutral one.

To asses the empirical relevance of the model, I use data from the popular cable TV show *Storage Wars*, which follows a core group of bidders who take part in storage locker auctions throughout the State of California.

Empirically, I find that the behavior of most bidders is consistent with loss aversion in a model in which there are heterogeneous bidders characteristics, thus documenting for the first time the presence of loss aversion in actual ascending auctions. At the same time, I find that the most professional bidder seems to be risk neutral.

Additionally, I consider a more general framework in which bidders incorporate into their strategies the information of those bidders who are present but decide not to participate after inspecting the item put up for auction. In this respect, bidders reduce the aggressiveness of their bids even further as the number of non-bidding participants increases. Loss aversion also persists in this general framework. Moreover, my findings confirm the empirical relevance of taking into account the presence of non-bidding participants in ascending auctions.

Although the empirical analysis of this paper provides reliable evidence of the importance of loss aversion in ascending auctions, there is still much to learn about the behavioral biases that arise in auctions from the field, lab and real life situations.

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Appendix

A Proofs and Auxiliary Results

A.1 *Equilibrium proof*

For notational simplicity, I suppress the arguments of the bid functions so that from now on $\beta_i^k(X_i;\Omega_k) = \beta_i^k(\cdot)$.

Following the discussion in (2), for any round k,

$$E\{u[V_i - \beta_i^k(\cdot)]|\Upsilon_i^k\} = \int_0^{\beta_i^k(\cdot)} \{\lambda_i[V_i - \beta_i^k(\cdot)] \times f[V_i - \beta_i^k(\cdot)|\Upsilon_i^k]\} d[V_i - \beta_i^k(\cdot)] + \int_{\beta_i^k(\cdot)}^{+\infty} \{[V_i - \beta_i^k(\cdot)] \times f[V_i - \beta_i^k(\cdot)|\Upsilon_i^k]\} d[V_i - \beta_i^k(\cdot)].$$

Since $f[V_i - \beta_i^k(\cdot)|\Upsilon_i^k] = f(V_i|\Upsilon_i^k)$, then $E\{u[V_i - \beta_i^k(\cdot)]|\Upsilon_i^k\}$ can be written as

$$E\{u[V_i - \beta_i^k(\cdot)]|\Upsilon_i^k\} = \int_0^{\beta_i^k(\cdot)} \{\lambda_i[V_i - \beta_i^k(\cdot)] \times f(V_i|\Upsilon_i^k)\} dV_i + \int_{\beta_i^k(\cdot)}^{+\infty} \{[V_i - \beta_i^k(\cdot)] \times f(V_i|\Upsilon_i^k)\} dV_i.$$

Given that $V_i = \exp(v_i)$, where $v_i \sim N[E(v_i), Var(v_i)]$, the density of V_i is

$$f(V_i|\Upsilon_i^k) = \frac{1}{V_i\sqrt{2\pi Var(v_i)}} \exp\left[-\frac{[\log(V_i) - E(v_i)]^2}{2Var(v_i)}\right].$$

Moreover,

$$\int_0^{\beta_i^k(\cdot)} \{\lambda_i[V_i - \beta_i^k(\cdot)] \times f(V_i|\Upsilon_i^k)\} dV_i = \int_0^{\beta_i^k(\cdot)} \frac{\Pr[0 < V_i < \beta_i^k(\cdot)]}{\Pr[0 < V_i < \beta_i^k(\cdot)]} \{\lambda_i[V_i - \beta_i^k(\cdot)] \times f(V_i|\Upsilon_i^k)\} dV_i$$

or equivalently

$$\int_0^{\beta_i^k(\cdot)} \{\lambda_i[V_i - \beta_i^k(\cdot)] \times f(V_i|\Upsilon_i^k)\} dV_i = \{\lambda_i[\Pr 0 < V_i < \beta_i^k(\cdot)\} \times \left[\int_0^{\beta_i^k(\cdot)} \frac{[V_i \times f(V_i|\Upsilon_i^k)]}{\Pr[0 < V_i < \beta_i^k(\cdot)]} dV_i - \beta_i^k(\cdot) \int_0^{\beta_i^k(\cdot)} \frac{f(V_i|\Upsilon_i^k)}{\Pr[0 < V_i < \beta_i^k(\cdot)]} dV_i\right],$$

with $\int_0^{\beta_i^k(\cdot)} \{f(V_i|\Upsilon_i^k)/\Pr[0 < V_i < \beta_i^k(\cdot)]\} dV_i = 1$.

Note that $\Pr[0 < V_i < \beta_i^k(\cdot)] = \Pr[\ln(0) < v_i < \ln(\beta_i^k(\cdot)]$, so

$$\Pr[0 < V_i < \beta_i^k(\cdot)] = \Phi\left(\frac{\ln[\beta_i^k(\cdot)] - \varsigma_{v_i|x}}{\sqrt{\omega_{v_i|x}}}\right) = \frac{1}{2}\left(\text{erf}\left\{\frac{\ln[\beta_i^k(\cdot)] - \varsigma_{v_i|x}}{\sqrt{2\omega_{v_i|x}}}\right\} + 1\right)$$

Therefore,

$$\int_0^{\beta_i^k(\cdot)} \{\lambda_i[V_i - \beta_i^k(\cdot)] \times f(V_i|\Upsilon_i^k)\} dV_i = \{\lambda_i \Pr[0 < V_i < \beta_i^k(\cdot)]\} \left\{\int_0^{\beta_i^k(\cdot)} \frac{[V_i \times f(V_i|\Upsilon_i^k)]}{\Pr[0 < V_i < \beta_i^k(\cdot)]} dV_i\right\}.$$

Following Zaninetti (2017),

$$\int_0^{\beta_i^k(\cdot)} \frac{\left[V_i \times f(V_i|\Upsilon_i^k)\right]}{\Pr[0 < V_i < \beta_i^k(\cdot)]} dV_i = E_{T1},$$

where

$$E_{T1} = \frac{\exp[\frac{1}{2}Var(v_i)]\exp[E(v_i)]\,[\mathrm{erf}(a_1) + \mathrm{erf}(a_2)]}{[\mathrm{erf}(a_3) + \mathrm{erf}(a_4)]},$$

with $a_1 = [-\infty - \omega_{v_i|x} - \varsigma_{v_i|x}]/\sqrt{2\omega_{v_i|x}}$, $a_2 = \{\omega_{v_i|x} + \varsigma_{v_i|x} - \ln[\beta_i^k(\cdot)]\}/\sqrt{2\omega_{v_i|x}}$, $a_3 = [-\infty - \varsigma_{v_i|x}]/\sqrt{2\omega_{v_i|x}}$ and $a_4 = \{\varsigma_{v_i|x} - \ln[\beta_i^k(\cdot)]\}/\sqrt{2\omega_{v_i|x}}$.

Hence,

$$E_{T1} = \frac{\exp\left(\frac{1}{2}\omega_{v_i|x}\right)\exp(\varsigma_{v_i|x})\,[\mathrm{erf}(a_2) - 1]}{[\mathrm{erf}(a_4) - 1]},$$

because $\mathrm{erf}(a_1) = \mathrm{erf}(a_3) = -1$.

Therefore,

$$\int_0^{\beta_i^k(\cdot)} \{\lambda_i[V_i - \beta_i^k(\cdot)] \times f(V_i|\Upsilon_i^k)\}dV_i = \{\lambda_i[\Pr 0 < V_i < \beta_i^k(\cdot)]\}[E_{T1} - \beta_i^k(\cdot)].$$

Similarly,

$$\int_{\beta_i^k(\cdot)}^{+\infty} \{[V_i - \beta_i^k(\cdot)] \times f(V_i|\Upsilon_i^k)\}dV_i = \Pr[\beta_i^k(\cdot) < V_i < +\infty]$$
$$\times \left\{ \int_{\beta_i^k(\cdot)}^{+\infty} \frac{\left[V_i \times f(V_i|\Upsilon_i^k)\right]}{\Pr[0 < V_i < \beta_i^k(\cdot)]} dV_i - \beta_i^k(\cdot) \int_0^{\beta_i^k(\cdot)} \frac{f(V_i|\Upsilon_i^k)}{\Pr[0 < V_i < \beta_i^k(\cdot)]} dV_i \right\}.$$

But since $\Pr[\beta_i^k(\cdot) < V_i < +\infty] = \Pr\{\ln[\beta_i^k(\cdot)] < v_i < +\infty\}$, then

$$\Pr[\beta_i^k(\cdot) < V_i < +\infty] = \frac{1}{2}\left(1 - \left\{\mathrm{erf}\,\frac{\ln[\beta_i^k(\cdot)] - \varsigma_{v_i|x}}{\sqrt{2\omega_{v_i|x}}}\right\}\right).$$

Hence,

$$\int_{\beta_i^k(\cdot)}^{+\infty} \{[V_i - \beta_i^k(\cdot)] \times f(V_i|\Upsilon_i^k)\}dV_i = \Pr[\beta_i^k(\cdot) < V_i < +\infty]\left[\int_{\beta_i^k(\cdot)}^{+\infty} \frac{\left[V_i \times f(V_i|\Upsilon_i^k)\right]}{\Pr[0 < V_i < \beta_i^k(\cdot)]} dV_i\right].$$

Again, following Zaninetti (2017),

$$\int_{\beta_i^k(\cdot)}^{+\infty} \frac{\left[V_i \times f(V_i|\Upsilon_i^k)\right]}{\Pr[0 < V_i < \beta_i^k(\cdot)]} dV_i = E_{T2},$$

where

$$E_{T2} = \frac{\exp[\frac{1}{2}Var(v_i)]\exp[E(v_i)]\,[\mathrm{erf}(b_1) + \mathrm{erf}(b_2)]}{[\mathrm{erf}(b_3) + \mathrm{erf}(b_4)]},$$

with $b_1 = -a_2$, $b_2 = [\omega_{v_i|x} + \varsigma_{v_i|x} - \infty]/\sqrt{2\omega_{v_i|x}}$, $b_3 = -a_4$ and $b_4 = [\varsigma_{v_i|x} - \infty]/\sqrt{2\omega_{v_i|x}}$.

Hence,

$$E_{T2} = \frac{\exp\left(\frac{1}{2}\omega_{v_i|x}\right)\exp(\varsigma_{v_i|x})\,[-\mathrm{erf}(a_2) - 1]}{[-\mathrm{erf}(a_4) - 1]},$$

because $\text{erf}(b_2) = \text{erf}(b_4) = -1$.

Therefore,

$$\int_{\beta_i^k(\cdot)}^{+\infty} \{[V_i - \beta_i^k(\cdot)] \times f(V_i|\Upsilon_i^k)\} dV_i = \Pr[\beta_i^k(\cdot) < V_i < +\infty][E_{T2} - \beta_i^k(\cdot)],$$

so $E\{u[V_i - \beta_i^k(\cdot)]|\Upsilon_i^k\}$ is then

$$E\{u[V_i - \beta_i^k(\cdot)]|\Upsilon_i^k\} = \{\lambda_i \Pr[0 < V_i < \beta_i^k(\cdot)]\}[E_{T1} - \beta_i^k(\cdot)] + \{\Pr[\beta_i^k(\cdot) < V_i < +\infty\}[E_{T2} - \beta_i^k(\cdot)].$$

In equilibrium,

$$\{\lambda_i \Pr[0 < V_i < \beta_i^k(\cdot)]\}[E_{T1} - \beta_i^k(\cdot)] + \{\Pr[\beta_i^k(\cdot) < V_i < +\infty]\}[E_{T2} - \beta_i^k(\cdot)] = 0,$$

which simplifies to

$$\exp\left(\frac{1}{2}\omega_{v_i|x}\right) \exp(\varsigma_{v_i|x}) \left[(1-\lambda_i)\left\{\text{erf}\,\frac{\omega_{v_i|x} + \varsigma_{v_i|x} - \ln[\beta_i^k(\cdot)]}{\sqrt{2\omega_{v_i|x}}}\right\} + (1+\lambda_i)\right] - \exp\{\ln[\beta_i^k(\cdot)]\} \left[(\lambda_i - 1)\,\text{erf}\left[\frac{\ln(P_k) - \varsigma_{v_i|x}}{\sqrt{2\omega_{v_i|x}}}\right] + (\lambda_i + 1)\right] = 0.$$

When $\lambda_i > 1$ and $\lambda_i \neq 1$, by "Guess and Verify", it is clear that the solution is:

$$\varsigma_{v_i|x} = \ln[\beta_i^k(\cdot)] - \frac{1}{2}\omega_{v_i|x} + \eta_i,$$

with η_i solving

$$\exp(\eta_i) \left[(1-\lambda_i)\,\text{erf}\left(\frac{\frac{\omega_{v_i|x}}{2} + \eta_i}{\sqrt{2\omega_{v_i|x}}}\right) + (1+\lambda_i)\right] - \left[(\lambda_i - 1)\,\text{erf}\left(\frac{\frac{\omega_{v_i|x}}{2} - \eta_i}{\sqrt{2\omega_{v_i|x}}}\right) + (\lambda_i + 1)\right] = 0. \tag{A1}$$

If there exists an η_i that solves (A1), then the above solution solves the original system.

To confirm this claim, let

$$Y(\eta_i) \equiv \exp(\eta_i) \left[(1-\lambda_i)\,\text{erf}\left(\frac{\frac{\omega_{v_i|x}}{2} + \eta_i}{\sqrt{2\omega_{v_i|x}}}\right) + (1+\lambda_i)\right] - \left[(\lambda_i - 1)\,\text{erf}\left(\frac{\frac{\omega_{v_i|x}}{2} - \eta_i}{\sqrt{2\omega_{v_i|x}}}\right) + (\lambda_i + 1)\right] = 0.$$

To check whether η_i is a solution to $Y(\eta_i) = 0$, one can exploit the fact that

$$\left.\begin{array}{l} 1)\ \lim_{\eta_i \to -\infty} Y(\eta_i) < 0 \\ 2)\ \lim_{\eta_i \to +\infty} Y(\eta_i) > 0 \end{array}\right\}$$

Specifically, given that $0 < \omega_{v_i|x} < \infty$ and $\lambda_i > 1$, then

$$\lim_{\eta_i \to -\infty} Y(\eta_i) = -2\lambda_i < 0 \quad \text{and} \quad \lim_{\eta_i \to +\infty} Y(\eta_i) = +\infty > 0.$$

As a special case,

$$\lim_{\eta_i \to 0} Y(\eta_i) = 2(1-\lambda_i)\,\mathrm{erf}\left(\frac{\frac{\omega_{v_i|x}}{2}}{\sqrt{2\omega_{v_i|x}}}\right) \leq 0.$$

The continuity of $Y(\eta_i)$ guarantees that there exists an η_i that solves $Y(\eta_i) = 0$.

If in addition $\partial Y(\eta_i)/\partial \eta_i > 0$ for any $-\infty < \eta_i < \infty$, the solution will be unique. In particular,

$$\partial Y(\eta_i)/\partial \eta_i = \exp(\eta_i)\left\{(1-\lambda_i)\,\mathrm{erf}\left(\frac{\frac{\omega_{v_i|x}}{2}+\eta_i}{\sqrt{2\omega_{v_i|x}}}\right) + (1+\lambda_i) + \frac{2(1-\lambda_i)}{\sqrt{2\pi\omega_{v_i|x}}}\exp\left[-\left(\frac{\frac{\omega_{v_i|x}}{2}+\eta_i}{\sqrt{2\omega_{v_i|x}}}\right)^2\right]\right\}$$
$$+\left\{\frac{2(\lambda_i-1)}{\sqrt{2\pi\omega_{v_i|x}}}\exp\left[-\left(\frac{\frac{\omega_{v_i|x}}{2}-\eta_i}{\sqrt{2\omega_{v_i|x}}}\right)^2\right]\right\} > 0,$$

or equivalently

$$\frac{(1+\lambda_i)}{(\lambda_i - 1)} - \mathrm{erf}\left(\frac{\frac{\omega_{v_i|x}}{2}+\eta_i}{\sqrt{2\omega_{v_i|x}}}\right) > 0,$$

which is true for any value of η_i and $0 < \omega_{v_i|x} < \infty$ because $[(1+\lambda_i)/(\lambda_i - 1)] > 1$ for $1 < \lambda_i < \infty$ and $\mathrm{erf}[(\frac{1}{2}\omega_{v_i|x} + \eta_i)/\sqrt{2\omega_{v_i|x}}] \in [-1, 1]$. As an aside, it is worth mentioning that η_i, despite being heterogeneous, does not depend on the round of the auction or on the bidder's own private signal.

Therefore, given the existence and uniqueness of η_i,

$$\varsigma_{v_i|x} = (v_i - \sigma'_{v_i x}\Sigma^{*-1}\Psi) + \sigma'_{v_i x}\Sigma^{*-1}_{k,1}x^k_r + \sigma'_{v_i x}\Sigma^{*-1}_{k,2}x^k_d,$$

where $\Sigma^{*-1} = (\Sigma^{*-1}_{k,1}, \Sigma^{*-1}_{k,2})$. Solving for x^k_r yields:

$$x^k_r = (\sigma'_{v_i x}\Sigma^{*-1}_{k,1})^{-1}[\varsigma_{v_i|x} + (\sigma'_{v_i x}\Sigma^{*-1}\Psi - v_i) - \sigma'_{v_i x}\Sigma^{*-1}_{k,2}x^k_d].$$

Given that in the log-normal setup $\omega_{v_i|x}$ is constant and $\varsigma_{v_i|x}$ is linear in the log of x_i, then $E(V_i|X_1, \ldots, X_N)$ is monotonically increasing in X_i.

For a proof of the existence of an increasing-strategy Bayesian-Nash equilibrium see Milgrom and Weber (1982) theorem 10 and Hong and Shum (2003, pp. 352). Specifically, they show that if all bidders $j \neq i$ follow their equilibrium strategies $\beta^k_j(\cdot)$, bidder i's best response is to play $\beta^k_i(\cdot)$ because this guarantees that bidder i will win the auction if and only if his expected net payoff is positive conditional on winning.

A.2 *Mean, variances and covariances of values and signals*

As stated in section 3.1, the conditional expected value of V_i is given by

$$E(V_i|X_1, \ldots, X_N) = \exp[E(v_i|x) + \frac{1}{2}Var(v_i|x)],$$

where $E(v_i|x) \equiv \varsigma_{v_i|x} = \upsilon_i + \sigma'_{v_ix}\Sigma^{*-1}(x - \Psi)$ and $Var(v_i|x) \equiv \omega_{v_i|x} = \sigma^2_{v_i} - \sigma'_{v_ix}\Sigma^{*-1}\sigma_{v_ix}$ with

$$\mu_i = \begin{pmatrix} \upsilon_i & \Psi \end{pmatrix} \text{ and } \Sigma_i = \begin{pmatrix} \sigma^2_{v_i} & \sigma'_{v_ix} \\ \sigma_{v_ix} & \Sigma^* \end{pmatrix}$$

Noting that $\upsilon_i = E(v_i) = E(a_i + v) = m + \bar{a}_i$, then $\upsilon = (\upsilon_1, \ldots, \upsilon_N)' = (m + \bar{a}_1, \ldots, m + \bar{a}_N)'$. Similarly, $E(x_i) = E(v_i + s_i\xi_i) = E(v_i) = m + \bar{a}_i$, so $\Psi = E(x) = (m + \bar{a}_1, \ldots, m + \bar{a}_N)'$. Also,

$$Var(v_i) = Var(a_i + v) = Var(a_i) + Var(v) + 2Cov(a_i, v) = r_0^2 + t_i^2$$

and

$$Cov(v_i, v_j) = E(v_iv_j) - E(v_i)E(v_j) = E(v^2) - [E(v)]^2 = Var(v) = r_0^2$$

for all $i, j \in N$ and $i \neq j$, so

$$\sigma^2_v = \begin{pmatrix} r_0^2 + t_1^2 & \cdots & r_0^2 \\ \vdots & \ddots & \vdots \\ r_0^2 & \cdots & r_0^2 + t_N^2 \end{pmatrix}.$$

In addition,

$$Cov(v_i, x_i) = E[v_i(v_i + s_i\xi_i)] - E(v_i)E(x_i) = E(v_i^2) - [E(v_i)]^2 = Var(v_i)$$

and

$$Cov(v_i, x_j) = E[v_i(v_j + s_j\xi_j)] - E(v_i)E(x_j) = E(v_iv_j) - E(v_i)E(v_j) = Cov(v_i, v_j)$$

for all $i, j \in N$ and $i \neq j$. As a consequence,

$$\sigma_{v_i\Psi} = \begin{pmatrix} r_0^2 + t_1^2 & \cdots & r_0^2 \\ \vdots & \ddots & \vdots \\ r_0^2 & \cdots & r_0^2 + t_N^2 \end{pmatrix}.$$

Finally, since

$$Var(x_i) = Var(v_i + s_i\xi_i) = Var(v_i) + s_i^2Var(\xi_i) + 2s_iCov(v_i, \xi_i) = r_0^2 + t_i^2 + s_i^2$$

and

$$Cov(x_i, x_j) = E(x_ix_j) - E(x_i)E(x_j) = E(v_iv_j) - E(v_i)E(v_j) = Cov(v_i, v_j)$$

for all $i, j \in N$ and $i \neq j$, we have that

$$\Sigma^* = \begin{pmatrix} r_0^2 + t_1^2 + s_1^2 & \cdots & r_0^2 \\ \vdots & \ddots & \vdots \\ r_0^2 & \cdots & r_0^2 + t_N^2 + s_N^2 \end{pmatrix}.$$

A.3 *Difference between loss aversion and expected utility*

When $\lambda_i = 1$, one gets the standard risk neutral case (see Hong and Shum (2003)), so

$$\exp[\frac{1}{2}Var(v_i^k|\Upsilon_i^k)]\exp[E(v_i^k|\Upsilon_i^k)] = \exp[\ln(\beta_i^k(\cdot)]$$

or equivalently

$$E(v_i^k|\Upsilon_i^k) = \ln[\beta_i^k(\cdot)] - \frac{1}{2}Var(v_i^k|\Upsilon_i^k),$$

with $x_r^k = (\sigma'_{\upsilon_i x}\Sigma_{k,1}^{*-1})^{-1}[E(v_i^k|\Upsilon_i^k) + (\sigma'_{\upsilon_i x}\Sigma^{*-1}\Psi - \upsilon_i) - \sigma'_{\upsilon_i x}\Sigma_{k,2}^{*-1}x_d^k]$.

Hence, the difference between loss aversion (LA) and expected utility (EU) in this ascending model is simply:

$$E^{LA}(v_i^k|\Upsilon_i^k) - E^{EU}(v_i^k|\Upsilon_i^k) = \eta_i.$$

Moreover, since $x_r^k = (\sigma'_{\upsilon_i x}\Sigma_{k,1}^{*-1})^{-1}[E(v_i^k|\Upsilon_i^k) + (\sigma'_{\upsilon_i x}\Sigma^{*-1}\Psi - \upsilon_i) - \sigma'_{\upsilon_i x}\Sigma_{k,2}^{*-1}x_d^k]$, then at round 0,

$$x_r^{0,PR} - x_r^{0,EU} = (\sigma'_{\upsilon_i x}\Sigma_{k,1}^{*-1})^{-1}\eta_i,$$

while in subsequent rounds,

$$x_r^{k,PR} - x_r^{k,EU} = \left(\sigma'_{\upsilon_i x}\Sigma_{k,1}^{*-1}\right)^{-1}(\eta_i - \sigma'_{\upsilon_i x}\Sigma_{k,2}^{*-1}x_d^{k,PR} + \sigma'_{\upsilon_i x}\Sigma_{k,2}^{*-1}x_d^{k,EU}).$$

A.4 *Log-bidding functions*

To define the log-bid functions first partition the inverse of the variance-covariance matrix of the private noisy signals as

$$\Sigma^{*-1} = \begin{pmatrix} \Sigma_{k,1}^{*-1} & \Sigma_{k,2}^{*-1} \end{pmatrix},$$

where $\Sigma_{k,1}^{*-1}$ is a $(N-k)\times N$ matrix corresponding to the remaining active bidders in round k, and $\Sigma_{k,2}^{*-1}$ is a $k \times N$ matrix corresponding to the bidders who have dropped out prior to round k.

Moreover, let

$$\begin{aligned}
\Gamma_k &= \begin{pmatrix} \sigma_{\upsilon_1}^2 & \cdots & \sigma_{\upsilon_{N-k}}^2 \end{pmatrix}', \\
\Lambda_k &= \begin{pmatrix} \sigma_{\upsilon_1 x} & \cdots & \sigma_{\upsilon_{N-k}x} \end{pmatrix}', \\
\mu_k &= \begin{pmatrix} \upsilon_1 & \cdots & \upsilon_{N-k} \end{pmatrix}',
\end{aligned}$$

and ℓ_k a $(N-k)\times 1$ vector of ones.

Additionally, let $\mathcal{A}^k$ and $\mathcal{C}^k$ be two $(N-k)\times 1$ vectors, and $\mathcal{D}^k$ a $(N-k)\times k$ matrix, with

$$\mathcal{A}^k = (\Lambda_k\Sigma_{k,1}^{*-1\prime})^{-1}\ell_k, \quad \text{(A2)}$$

$$\mathcal{C}^k = \frac{1}{2}(\Lambda_k\Sigma_{k,1}^{*-1\prime})^{-1}[\Gamma_k - diag(\Lambda_k\Sigma^{*-1}\Lambda'_k) + 2\mu_k - 2(\Lambda_k\Sigma^{*-1}\Psi)], \quad \text{(A3)}$$

$$\mathcal{D}^k = (\Lambda_k\Sigma_{k,1}^{*-1\prime})^{-1}(\Lambda_k\Sigma_{k,2}^{*-1\prime}), \quad \text{(A4)}$$

where $diag\,(.)$ is a matrix whose entries outside the main diagonal are all zero.

Therefore, $\mathcal{A}_i^k$ and $\mathcal{C}_i^k$ in (4) denote the ith element of the vectors (A2) and (A3), and $\mathcal{D}_i^k$ denotes the ith row of matrix (A4).

A.5 *Calculating the likelihood of baseline model*

A.5.1 *Continuous component*

Using the log-bidding function in section 3.2, the bid functions of bidders dropping out in round k will be given by (4). Let

$$F = \begin{pmatrix} \frac{C_N^0}{A_N^0} - \eta_N & \cdots & \frac{C_2^{N-2}}{A_2^{N-2}} - \eta_2 \end{pmatrix}$$

be an $(N-1) \times 1$ vector,

$$\mathcal{G}_i = \begin{pmatrix} \underbrace{0,\ldots,0}_{N-i-2} & 1/\mathcal{A}_{N-i}^i & \mathcal{D}_{N-i}^i/\mathcal{A}_{N-i}^i \end{pmatrix}$$

a $1 \times (N-1)$ vector and

$$\mathcal{G} = \begin{pmatrix} \mathcal{G}_0' & \cdots & \mathcal{G}_{N-2}' \end{pmatrix}$$

an $(N-1) \times (N-1)$ matrix. Thus, the vector of dropout bids can be written as

$$\mathcal{P} = \mathcal{G}\,(x_2, \ldots, x_N)' + \mathcal{F}. \tag{A5}$$

Let $\psi_2(\theta)$ be the $N-1$ subvector of Ψ and $\Sigma_2^*(\theta)$ the $(N-1) \times (N-1)$ submatrix of Σ^* corresponding to bidders $2, \ldots, N$. Then, equation (A5) implies that the mean and variance of the vector of dropout bids will be

$$\left.\begin{array}{l} \mu_p(\theta) = \mathcal{F}(\theta) + \mathcal{G}(\theta)\psi_2(\theta) \\ \Sigma_p(\theta) = \mathcal{G}(\theta)\Sigma_2^*(\theta)\mathcal{G}(\theta)' \end{array}\right\}.$$

Therefore, the continuos part of the $(N-1)$-variate normal log-likelihood function for a given auction is

$$\log f(\mathcal{P};\theta) = -\frac{1}{2}(N-1)\log(2\pi) - \frac{1}{2}\log(|\Sigma_p(\theta)|) - \frac{1}{2}\left\{[\mathcal{P} - \mu_p(\theta)]'\Sigma_p(\theta)^{-1}[\mathcal{P} - \mu_p(\theta)]\right\}.$$

A.5.2 *Characterization of* $T_2(\theta)$ *and its probability*

In an ascending auction, one does not observe the winner's dropout bid, only the price at which the second highest bidder stops. As a result, the signal of the winning bidder is constrained to a region $\mathcal{T}_2(x_2,\ldots,x_N;\theta) \subset R^1$. Hong and Shum (2003) show that the set $\mathcal{T}_2[\mathcal{G}^{-1}(\mathcal{P} - \mathcal{F});\theta]$ consist of the following conditions:

$$\{x_1 : b_1^l(x_1; x_d^l, \theta) \geq p_l, \text{ for all } l = 0, \ldots, N-2\}.$$

This implies that for any dropout order, the winning bidder will never regret having remained active in all prior rounds. However, given the ascending nature of the auction, the only binding constraint will be

$$b_1^{N-2}(x_1; x_d^{N-2}, \theta) \geq p_{N-2}. \tag{A6}$$

To illustrate the calculation, consider an auction with $N = 3$ bidders. Without loss of generality, suppose bidder 3 had the lowest bid in round 0, so at round 1 only bidders 1 and 2 remain active. Then,

$$b_1^1(x_1; x_3, \theta) \geq p_1,$$

which can then be simplified to

$$x_1 \geq \mathcal{A}_1^1 p_1 - \mathcal{C}_1^1 - \mathcal{D}_1^1 x_3 + \mathcal{A}_1^1 \eta_1.$$

Therefore,

$$\Pr\{\mathcal{T}_2[\mathcal{G}^{-1}(\mathcal{P} - \mathcal{F})|\theta]\} = \Pr\left[\frac{x_1 - E(x_1|x_2, x_3)}{\sqrt{Var(x_1|x_2, x_3)}} \geq \frac{\mathcal{A}_1^1 p_1 - \mathcal{C}_1^1 - \mathcal{D}_1^1 x_3 + \mathcal{A}_1^1 \eta_1 - E(x_1|x_2, x_3)}{\sqrt{Var(x_1|x_2, x_3)}}\right]$$

or equivalently

$$\Pr\{\mathcal{T}_2[\mathcal{G}^{-1}(\mathcal{P} - \mathcal{F})|\theta]\} = \Phi\left[\frac{E(x_1|x_2, x_3) + \mathcal{C}_1^1 + \mathcal{D}_1^1 x_3 - \mathcal{A}_1^1 p_1 - \mathcal{A}_1^1 \eta_1}{\sqrt{Var(x_1|x_2, x_3)}}\right].$$

Unfortunately, there is a mistake in the expression for the probability of $\mathcal{T}_2$ after the formula (24) that Hong and Shum (2003) provide. Specifically, they seem to have used unconditional moments when they should have used conditional ones instead because $\Pr\{\mathcal{T}_2[\mathcal{G}^{-1}(\mathcal{P} - \mathcal{F}); \theta]\}$ denotes the probability that $x_1 \in \mathcal{T}_2(\theta)$ conditional on $\mathcal{P}$.

To obtain $E(x_1|x_d) = \bar{\Psi}$ and $Var(x_1|x_d) = \bar{\Sigma}^*$, first partition the vector x as

$$x = [x_1 \ \underbrace{(x_2, x_3)}_{N-1}]',$$

and then partition Ψ and Σ^* accordingly:

$$\Psi = [\Psi_1 \ \underbrace{\Psi_d}_{N-1}]' \ \text{ and } \ \Sigma^* = \begin{pmatrix} \Sigma_{11}^* & \overbrace{\Sigma_{1d}^*}^{1\times(N-1)} \\ \underbrace{\Sigma_{d1}^*}_{(N-1)\times 1} & \underbrace{\Sigma_{dd}^*}_{(N-1)\times(N-1)} \end{pmatrix}.$$

Then, the distribution of x_1 conditional on (x_2, x_3) is multivariate normal $x_1|x_2, x_3 \sim N(\bar{\Psi}, \bar{\Sigma}^*)$, where $\bar{\Psi} = \Psi_1 + \Sigma_{1d}^* (\Sigma_{dd}^*)^{-1} [(x_2, \ldots, x_{N+q}) - \Psi_d]$ and $\bar{\Sigma}^* = \Sigma_{11}^* - \Sigma_{1d}^* (\Sigma_{dd}^*)^{-1} \Sigma_{d1}^*$.

A.5.3 *Characterization of* $Pr[T_1(\theta);\theta]$

For the dropout to occur in the correct order (CO), it must be the case that

$$b_i^k(x_i; x_d^k, \theta) \geq b_{N-k}^k(x_{N-k}; x_d^k, \theta) = p_k, \text{ for all } k \text{ and } i = 0, \ldots, N-k-1.$$

The truncation region $\mathcal{T}_1(\theta)$ for a given value of θ is defined as the values of the log-signals such that CO is satisfied. More formally,

$$\mathcal{T}_1(\theta) = \{x_1, \ldots, x_N : CO \text{ is satisfied}|\theta\}.$$

Given the ascending nature of the auction and that the log-bidding functions for rounds k and $k-1$ intersect when they are equal, Hong and Shum (2003) show that the CO condition can be simplified to the following $N-1$ inequalities

$$b_{N-k-1}^k(x_{N-k-1}; x_d^k, \theta) \geq b_{N-k}^k(x_{N-k}; x_d^k, \theta), \text{ for all } k = 0, \ldots, N-2,$$

which implies that the log-bidding functions of the bidders remaining in round k have to be greater than the log-bidding functions of all the ones who have dropped out.

To illustrate the calculations for $\Pr[T_1(\theta);\theta]$, suppose that, for example, $N = 3$. The only binding constraints are:

$$\left.\begin{array}{l} b_2^0(x_2;\theta) \geq b_3^0(x_3;\theta) \\ b_1^1(x_1, x_3;\theta) \geq b_2^1(x_2, x_3;\theta) \end{array}\right\}$$

which can be written in matrix form as

$$\underbrace{\begin{pmatrix} 0 \\ 0 \end{pmatrix}}_{Z} \geq \underbrace{\begin{bmatrix} \frac{C_3^0}{A_3^0} - \frac{C_2^0}{A_2^0} + (\eta_2 - \eta_3) \\ \frac{C_2^1}{A_2^1} - \frac{C_1^1}{A_1^1} + (\eta_1 - \eta_2) \end{bmatrix}}_{h} + \underbrace{\begin{bmatrix} 0 & -\frac{1}{A_2^0} & \frac{1}{A_3^0} \\ -\frac{1}{A_1^1} & \frac{1}{A_2^1} & \left(\frac{D_2^1}{A_2^1} - \frac{D_1^1}{A_1^1}\right) \end{bmatrix}}_{H} \underbrace{\begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix}}_{x},$$

The probability that $Z \leq 0$ is simply a multivariate normal cdf with $E(Z) = h + H\Psi$ and $V(Z) = H\Sigma^* H'$ because $x \sim N(\Psi, \Sigma^*)$. To calculate this multivariate normal cdf, I use a numerical quadrature procedure for bivariate and trivariate distributions, and a quasi-Monte Carlo integration algorithm for four or more dimensions (see Matlab (2019) *mvncdf* entry).

As an aside, it is worth mentioning that the CO condition in $\Pr[\mathcal{T}_1(\theta)|\theta]$ in the fully homogeneous case (see section Online Appendix B.1.4) implies that the log-signal of the winner has to be greater than the log-signal of the second highest bidder, and similarly the log-signal of the third highest bidder, etc. For example, when there are only three bidders,

$$\Pr[\mathcal{T}_1(\theta)|\theta] = \Pr(x_1 \geq x_2; x_2 \geq x_3|\theta) = \Pr(z_1 \geq 0; z_2 \geq 0|\theta),$$

where $z_1 = x_1 - x_2$ and $z_2 = x_2 - x_3$. But notice that this is the probability that a bivariate normal with zero means, unit variances and some correlation coefficient between z_1 and z_2 ($\rho_{z_1 z_2}$)

lies in the first quadrant. In this case, it is easy to prove that

$$\Pr[\mathcal{T}_1(\theta)|\theta] = 1/N!$$

because there are $N!$ possible orderings, which are all equally likely in the fully homogeneous case. Consequently, $\Pr[\mathcal{T}_1(\theta)|\theta]$ does not depend on the model parameters.

A.6 *Log-bidding functions with active non-bidding participants*

To define the log-bid functions in this more general framework ,let

$$\begin{aligned}
\dot{\Gamma}_{-1} &= (\ \sigma^2_{\upsilon_1} \ \cdots \ \sigma^2_{\upsilon_{N+q}} \ )', \\
\dot{\Lambda}_{-1} &= (\ \sigma_{\upsilon_1 x} \ \cdots \ \sigma_{\upsilon_{N+q} x} \ )', \\
\dot{\mu}_{-1} &= (\ \upsilon_1 \ \cdots \ \upsilon_{N+q} \ )',
\end{aligned}$$

and $\dot{\ell}_{-1}$ a $(N+q) \times 1$ vector of ones.

Additionally, let $\dot{\mathcal{A}}^{-1}$ and $\dot{\mathcal{C}}^{-1}$ be two $(N+q) \times 1$ vectors, with

$$\begin{aligned}
\dot{\mathcal{A}}^{-1} &= (\dot{\Lambda}_{-1}\Sigma^{*-1\prime})^{-1}\dot{\ell}_{-1}, \\
\dot{\mathcal{C}}^{-1} &= \frac{1}{2}(\dot{\Lambda}_{-1}\Sigma^{*-1\prime})^{-1}[\dot{\Gamma}_{-1} - diag(\dot{\Lambda}_{-1}\Sigma^{*-1}\dot{\Lambda}'_{-1}) + 2\dot{\mu}_{-1} - 2(\dot{\Lambda}_{-1}\Sigma^{*-1}\Psi)].
\end{aligned}$$

With this notation, the log-bidding functions are given by (6).

At round 0, partition the inverse of the variance-covariance matrix of the private noisy signals as

$$\Sigma^{*-1} = (\ \ddot{\Sigma}^{*-1}_{0,1} \ \ \ddot{\Sigma}^{*-1}_{0,2} \ ),$$

where $\ddot{\Sigma}^{*-1}_{0,1}$ is a $N \times (N+q)$ matrix corresponding to the active bidders in round 0, and $\ddot{\Sigma}^{*-1}_{0,2}$ is a $q \times (N+q)$ matrix corresponding to the active non-bidding participants who dropped out in round -1. Moreover, let

$$\begin{aligned}
\ddot{\Gamma}_0 &= (\ \sigma^2_{\upsilon_1} \ \cdots \ \sigma^2_{\upsilon_N} \ )', \\
\ddot{\Lambda}_0 &= (\ \sigma_{\upsilon_1 x} \ \cdots \ \sigma_{\upsilon_N x} \ )', \\
\ddot{\mu}_0 &= (\ \upsilon_1 \ \cdots \ \upsilon_N \ )',
\end{aligned}$$

and $\ddot{\ell}_0$ a $N \times 1$ vector of ones.

Finally, let $\ddot{\mathcal{A}}^0$ and $\ddot{\mathcal{C}}^0$ be two $N \times 1$ vectors, and $\ddot{\mathcal{D}}^0$ a $N \times k$ matrix, with

$$\begin{aligned}
\ddot{\mathcal{A}}^0 &= (\ddot{\Lambda}_0\ddot{\Sigma}^{*-1\prime}_{0,1})^{-1}\ddot{\ell}_0, \\
\ddot{\mathcal{C}}^0 &= \frac{1}{2}(\ddot{\Lambda}_0\ddot{\Sigma}^{*-1\prime}_{0,1})^{-1}[\ddot{\Gamma}_0 - diag(\ddot{\Lambda}_0\Sigma^{*-1}\ddot{\Lambda}'_0) + 2\ddot{\mu}_0 - 2(\ddot{\Lambda}_0\Sigma^{*-1}\Psi)], \\
\ddot{\mathcal{D}}^0 &= (\ddot{\Lambda}_0\ddot{\Sigma}^{*-1\prime}_{0,1})^{-1}(\ddot{\Lambda}_0\ddot{\Sigma}^{*-1\prime}_{0,2}).
\end{aligned}$$

With this notation, the log-bidding functions are given by (7).

A.7 *Calculating the likelihood with active non-bidding participants*

A.7.1 *Continuous component*

Define

$$\tilde{\mathcal{F}} = \begin{pmatrix} \frac{\mathcal{C}_{N+q}^{-1}}{\mathcal{A}_{N+q}^{-1}} - \eta_{N+q} & \cdots & \frac{\mathcal{C}_{N+1}^{-1}}{\mathcal{A}_{N+1}^{-1}} - \eta_{N+1} & \frac{\mathcal{C}_{N}^{0}}{\mathcal{A}_{N}^{0}} - \eta_{N} & \cdots & \frac{\mathcal{C}_{2}^{N-2}}{\mathcal{A}_{2}^{N-2}} - \eta_{2} \end{pmatrix}$$

as an $(N+q-1)\times 1$ vector, with q being the number of non-bidding participants and N the number of active bidders. Similarly, let

$$\begin{array}{l} \tilde{\mathcal{G}}_j = \begin{pmatrix} \underbrace{0,\ldots,0}_{N-j-2} & 1/\mathcal{A}_{N-j}^{-1} & \underbrace{0,\ldots,0}_{q+j} \end{pmatrix}, \text{ for } j=-q,\ldots,-1 \\ \tilde{\mathcal{G}}_i = \begin{pmatrix} \underbrace{0,\ldots,0}_{N-i-2} & 1/\mathcal{A}_{N-i}^{i} & \underbrace{\mathcal{D}_{N-i}^{i}/\mathcal{A}_{N-i}^{i}}_{q+i} \end{pmatrix}, \text{ for } i=0,\ldots,N-2. \end{array}$$

denote two $1\times(N+q-1)$ vectors and

$$\tilde{\mathcal{G}} = \begin{pmatrix} \tilde{\mathcal{G}}_{-q}' & \cdots & \tilde{\mathcal{G}}_{-1}' & \tilde{\mathcal{G}}_{0}' & \cdots & \tilde{\mathcal{G}}_{N-2}' \end{pmatrix}$$

an $(N+q-1)\times(N+q-1)$ matrix. As before, the vector of dropout bids can be written as

$$\tilde{\mathcal{P}} = \tilde{\mathcal{G}}\,(x_2,\ldots,x_{N+q})' + \tilde{\mathcal{F}}. \tag{A7}$$

This equation describes the mapping from the unobserved log-signals

$$x_{dr} \equiv (x_2,\ldots,x_N, x_{N+1},\ldots,x_{N+q})'$$

to the observed log-bids $\tilde{\mathcal{P}} = (\underbrace{p_{-1},\ldots,p_{-1}}_{q}, p_0,\ldots,p_{N-2})'$.

Let $\tilde{\psi}_2(\theta)$ be the $N+q-1$ subvector of Ψ and $\tilde{\Sigma}_2^*(\theta)$ the $(N+q-1)\times(N+q-1)$ submatrix of Σ^* corresponding to the signals of bidders $2,\ldots,N+q$. Then, equation (A7) implies that the mean and variance of the vector of dropout bids will be

$$\left.\begin{array}{l} \tilde{\mu}_p(\theta) = \tilde{\mathcal{F}}(\theta) + \tilde{\mathcal{G}}(\theta)\tilde{\psi}_2(\theta) \\ \tilde{\Sigma}_p(\theta) = \tilde{\mathcal{G}}(\theta)\tilde{\Sigma}_2^*(\theta)\tilde{\mathcal{G}}(\theta)' \end{array}\right\}.$$

Similarly, partition the price vector $\tilde{\mathcal{P}}$ as:

$$\tilde{\mathcal{P}} = (\underbrace{(p_{-1},\ldots,p_{-1})}_{q}\ \underbrace{(p_0,\ldots,p_{N-2})}_{N-1})'$$

and then partition $\tilde{\mu}_p(\theta)$ and $\tilde{\Sigma}_p(\theta)$ accordingly:

$$\tilde{\mu}_p(\theta) = (\underbrace{\tilde{\mu}_{p,1}}_{q}\ \underbrace{\tilde{\mu}_{p,2}}_{N-1})' \text{ and } \tilde{\Sigma}_p(\theta) = \begin{pmatrix} \overbrace{\tilde{\Sigma}_{p,11}}^{q\times q} & \overbrace{\tilde{\Sigma}_{p,12}}^{q\times(N-1)} \\ \underbrace{\tilde{\Sigma}_{p,21}}_{(N-1)\times q} & \underbrace{\tilde{\Sigma}_{p,22}}_{(N-1)\times(N-1)} \end{pmatrix}.$$

Then, the distribution of $(p_0, \ldots, p_{N-2})$ conditional on $(p_{-1}, \ldots, p_{-1})$ is multivariate normal $[(p_0, \ldots, p_{N-2}) \,|\, (p_{-1}, \ldots, p_{-1})] \sim N\left(\bar{\mu}_p, \bar{\Sigma}_p\right)$, where $\bar{\mu}_p = \tilde{\mu}_{p,2} + \tilde{\Sigma}_{p,21}\tilde{\Sigma}_{p,11}^{-1}\left[(p_{-1}, \ldots, p_{-1}) - \tilde{\mu}_{p,1}\right]$ and $\bar{\Sigma}_p = \tilde{\Sigma}_{p,22} - \tilde{\Sigma}_{p,21}\tilde{\Sigma}_{p,11}^{-1}\tilde{\Sigma}_{p,12}$.

Therefore, the continuos part of the $(N-1-q)$-variate normal log-likelihood function for a given auction conditional on the initial dropout bidders is

$$\log f(\mathcal{P}_a|P_{-1}, \theta) = -\frac{1}{2}(N-1-q)\log(2\pi) - \frac{1}{2}\log(|\bar{\Sigma}_p(\theta)|) - \frac{1}{2}\left\{[\mathcal{P}_a - \bar{\mu}_p(\theta)]'\bar{\Sigma}_p(\theta)^{-1}[\mathcal{P}_a - \bar{\mu}_p(\theta)]\right\},$$

where $\mathcal{P}_a = (p_0, \ldots, p_{N-2})'$.

A.7.2 *Characterization of* $T_2(\theta)$ and $Pr[T_1(\theta); \theta|P_{-1}]$

In this case, the probability of $\mathcal{T}_2$ will be the same as (A6). To illustrate how the $\Pr[T_1(\theta); \theta]$ looks like in this context suppose that, for example, $N = 3$ and $q = 1$. At round -1, bidder 4 drops out at price p_{-1}. Therefore, the only binding constraints will be:

$$\left.\begin{array}{l} b_3^{-1}(x_3|x_4) \geq b_4^{-1}(x_4|x_4) \\ b_2^0(x_2|x_4) \geq b_3^0(x_3|x_4) \\ b_1^1(x_1; x_d|x_4) \geq b_2^1(x_2; x_d|x_4) \end{array}\right\}$$

which can be written in matrix form as

$$\underbrace{\begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}}_{Z} \geq \underbrace{\begin{bmatrix} \frac{\mathcal{C}_4^{-1}}{\mathcal{A}_4^{-1}} - \frac{\mathcal{C}_3^{-1}}{\mathcal{A}_3^{-1}} + (\eta_3 - \eta_4) \\ \frac{\mathcal{C}_3^0}{\mathcal{A}_3^0} - \frac{\mathcal{C}_2^0}{\mathcal{A}_2^0} + (\eta_2 - \eta_3) \\ \frac{\mathcal{C}_2^1}{\mathcal{A}_2^1} - \frac{\mathcal{C}_1^1}{\mathcal{A}_1^1} + (\eta_1 - \eta_2) \end{bmatrix} + \begin{bmatrix} 1/\mathcal{A}_4^{-1} \\ \left(\frac{\mathcal{D}_{3,1}^0}{\mathcal{A}_3^0} - \frac{\mathcal{D}_{2,1}^0}{\mathcal{A}_2^0}\right) \\ \left(\frac{\mathcal{D}_{2,2}^1}{\mathcal{A}_2^1} - \frac{\mathcal{D}_{1,2}^1}{\mathcal{A}_1^1}\right) \end{bmatrix} \underbrace{(x_4)}_{x_b}}_{h}$$

$$+ \underbrace{\begin{bmatrix} 0 & 0 & -1/\mathcal{A}_3^{-1} \\ 0 & -1/\mathcal{A}_2^0 & 1/\mathcal{A}_3^0 \\ -1/\mathcal{A}_1^1 & 1/\mathcal{A}_2^1 & \left(\frac{\mathcal{D}_{2,1}^1}{\mathcal{A}_2^1} - \frac{\mathcal{D}_{1,1}^1}{\mathcal{A}_1^1}\right) \end{bmatrix}}_{H} \underbrace{\begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix}}_{x_a},$$

Then, partition the vector x as

$$x = \left(x_a', x_b'\right)' = [\underbrace{(x_1, \ldots, x_N)}_{N} \; \underbrace{(x_{N+1}, \ldots, x_{N+q})}_{q}]',$$

and then partition Ψ and Σ^* accordingly:

$$\Psi = (\underbrace{\Psi_a}_{N} \; \underbrace{\Psi_b}_{q})' \text{ and } \Sigma^* = \begin{pmatrix} \overbrace{\Sigma_{aa}^*}^{N\times N} & \overbrace{\Sigma_{ab}^*}^{N\times q} \\ \underbrace{\Sigma_{ba}^*}_{q\times N} & \underbrace{\Sigma_{bb}^*}_{q\times q} \end{pmatrix}.$$

The distribution of x_a conditional on x_b is multivariate normal $x_a|x_b \sim N(\hat{\Psi}, \hat{\Sigma}^*)$, where $\hat{\Psi} = \Psi_a + \Sigma_{ab}^*\left(\Sigma_{bb}^*\right)^{-1}[(x_{N+1}, \ldots, x_{N+q}) - \Psi_b]$ and $\hat{\Sigma}^* = \Sigma_{aa}^* - \Sigma_{ab}^*\left(\Sigma_{bb}^*\right)^{-1}\Sigma_{ba}^*$.

The probability that $Z \leq 0$ conditional on x_b is simply a multivariate normal cdf with $E[Z|\,(x_{N+1}, \ldots, x_{N+q})] = h + H\tilde{\Psi}$ and $V[Z|\,(x_{N+1}, \ldots, x_{N+q})] = H\tilde{\Sigma}^*H'$.

Tables

Table 1: Auctioneer Behavior

	Estimate	Std. Error
HHI^{**}	0.015	0.005
$SIZE^{***}$	0.548	0.106
Constant	2.879	0.350

Notes: Multiple regression of (log) opening bid. HHI captures the median household income of the municipality where the locker is located in the State of California, $SIZE$ is a variable that measures the size of the locker (small (1), medium (2) or large (3)). Additionally, * indicates rejection of the null at the 10% significance level, ** 5% level, *** 1% level.

Table 2: Summary Statistics

Variable	#Obs.	Season 1	Season 2	Season 3
Auction characteristics				
Small locker	100	23	37	40
Medium locker	117	27	54	36
Large locker	37	9	12	16
Average HHI	78	57641	61473	58750
Average *Ex-post*	250	4797	3954	6319
Average Profit	250	3949	2282	4821
Number of auctions	254	59	103	92
Number of bidders per auction				
$N = 2$	14	3	6	5
$N = 3$	50	9	13	28
$N = 4$	72	20	29	23
$N = 5$	63	13	34	16
$N = 6$	36	8	16	12
$N = 7$	19	6	5	8

Notes: HHI denotes the median household income of the municipality where the locker is located in the State of California, *Ex-post* denotes the ex-post value of the locker and profit denotes the difference between the ex-post value of the locker and the winner's bid.

Table 3: Bidder's Frequency Participation

	# Obs.	First Bidder	J	Dr	Dv	J-Dr	J-Dv	Dr-Dv	J-Dr-Dv
Barry	139	66	82	86	76	52	44	50	29
	# Obs.	First Bidder	J	B	Dv	J-B	J-Dv	B-Dv	J-B-Dv
Darrell	161	25	96	86	98	52	57	50	29
	# Obs.	First Bidder	J	Dr	B	J-Dr	J-B	Dr-B	J-Dr-B
Dave	151	10	99	98	76	57	44	50	29
	# Obs.	First Bidder	B	Dr	Dv	B-Dr	B-Dv	Dr-Dv	B-Dr-Dv
Jarrod	165	31	82	96	99	52	44	57	29

Notes: The four main bidders are Barry "B", Darrell "Dr", Dave "Dv" and Jarrod "Jr". Additionally, "J-Dv" means that Jarrod and Dave were the only two main bidders out of the four who were active bidding participants, i.e. they participated in the auction.

Table 4: Mean Common Value

	Estimate	Std. Error
HHI^{**}	0.012	0.005
$SIZE^{***}$	0.368	0.123
Constant	6.242	0.408

Notes: Multiple regression of (log) *Ex-post* value. HHI captures the median household income of the municipality where the locker is located in the State of California and $SIZE$ is a variable that measures the size of the locker (small (1), medium (2) or large (3)). Additionally, * indicates rejection of the null at the 10% significance level, ** 5% level, *** 1% level.

Table 5: Maximum Likelihood Estimates Baseline Model

	Estimate	p-value
β_1^{***}	0.381	0
β_2^{**}	0.009	0.01
δ_0	0.014	-
α_0	5.534	-
α_1^{***}	-0.185	0
α_2^{***}	0.198	0
α_3^{*}	0.101	0.06
α_4	0.039	0.51
τ_0	-1.081	-
γ_0	0.371	-
γ_1	-0.059	0.85
γ_2	0.053	0.77
γ_3^{***}	0.923	0
γ_4^{*}	-0.717	0.09

Notes: The mean of the common value component for a given auction is $m = \beta_0 + \beta_1 SIZE + \beta_2 HHI$, where $SIZE$ is a variable that measures the size of the locker (small (1), medium (2) or large (3)) and HHI captures the median household income of the municipality where the locker is located in the State of California. Additionally, the mean of the private value component of the four main bidders (Barry "Ba", Darrell "Dr", Dave "Dv" and Jarrod "Jr"), as well as of the other active bidders whose identity is not shown publicly, is $\bar{a} = (\alpha_0 + \alpha_1 Ba, \alpha_0 + \alpha_2 Dr, \alpha_0 + \alpha_3 Dv, \alpha_0 + \alpha_4 Jr, \alpha_0, \ldots, \alpha_0)$, where Ba, Dr, Dv and Jr are mutually exclusive dummy variables. Furthermore, the variance of the common and private value component is modelled as $r_0^2 = \exp(\delta_0)$ and $t^2 = \exp(\tau_0)$, respectively, while the variance of the noise for each of the bidder's signals is $s^2 = \exp(\gamma_0 + \gamma_1 Ba, \gamma_0 + \gamma_2 Dr, \gamma_0 + \gamma_3 Dv, \gamma_0 + \gamma_4 Jr, \gamma_0, \ldots, \gamma_0)$. p-values correspond to the likelihood ratio. Finally, * indicates rejection of the null at the 10% significance level, ** 5% level, *** 1% level.

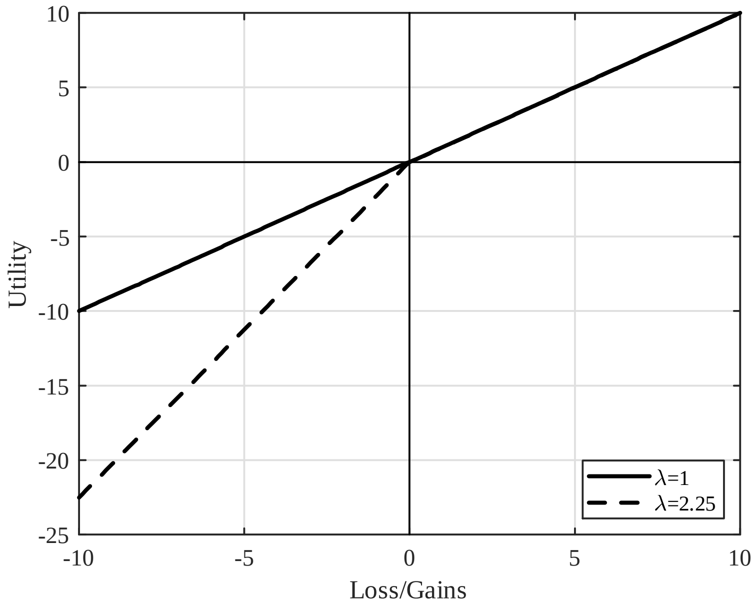
Table 6: Maximum Likelihood Estimates With Active Non-Bidding Participants

	Estimate	p-value
β_1^{***}	0.291	0
β_2^{***}	0.009	0
δ_0	-0.002	-
α_0	5.926	-
α_1^{***}	-0.439	0
α_2	-0.007	0.55
α_3^{***}	-0.126	0
α_4	0.001	0.59
τ_0	0.001	-
γ_0	1.628	-
γ_1^{*}	-0.427	0.08
γ_2	-0.022	0.79
γ_3^{***}	1.024	0
γ_4	0.058	0.76

Notes: The mean of the common value component for a given auction is $m = \beta_0 + \beta_1 SIZE + \beta_2 HHI$, where $SIZE$ is a variable that measures the size of the locker (small (1), medium (2) or large (3)) and HHI captures the median household income of the municipality where the locker is located in the State of California. Additionally, the mean of the private value component of the four main bidders (Barry "Ba", Darrell "Dr", Dave "Dv" and Jarrod "Jr"), as well as of the other active bidders whose identity is not shown publicly, is $\bar{a} = (\alpha_0 + \alpha_1 Ba, \alpha_0 + \alpha_2 Dr, \alpha_0 + \alpha_3 Dv, \alpha_0 + \alpha_4 Jr, \alpha_0, \ldots, \alpha_0)$, where Ba, Dr, Dv and Jr are mutually exclusive dummy variables. Furthermore, the variance of the common and private value component is modelled as $r_0^2 = \exp(\delta_0)$ and $t^2 = \exp(\tau_0)$, respectively, while the variance of the noise for each of the bidder's signals is $s^2 = \exp(\gamma_0 + \gamma_1 Ba, \gamma_0 + \gamma_2 Dr, \gamma_0 + \gamma_3 Dv, \gamma_0 + \gamma_4 Jr, \gamma_0, \ldots, \gamma_0)$. p-values correspond to the likelihood ratio. Finally, * indicates rejection of the null at the 10% significance level, ** 5% level, *** 1% level.

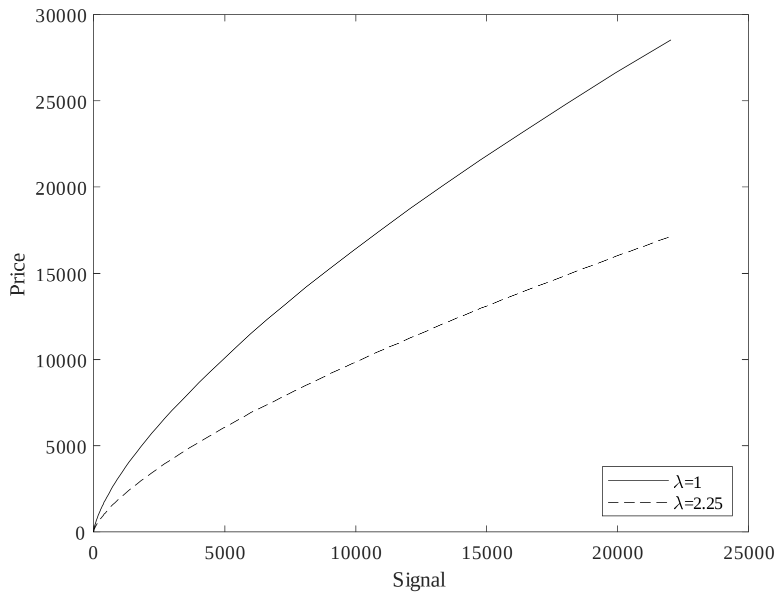
Figures

Figure 1: Loss Aversion Utility Function



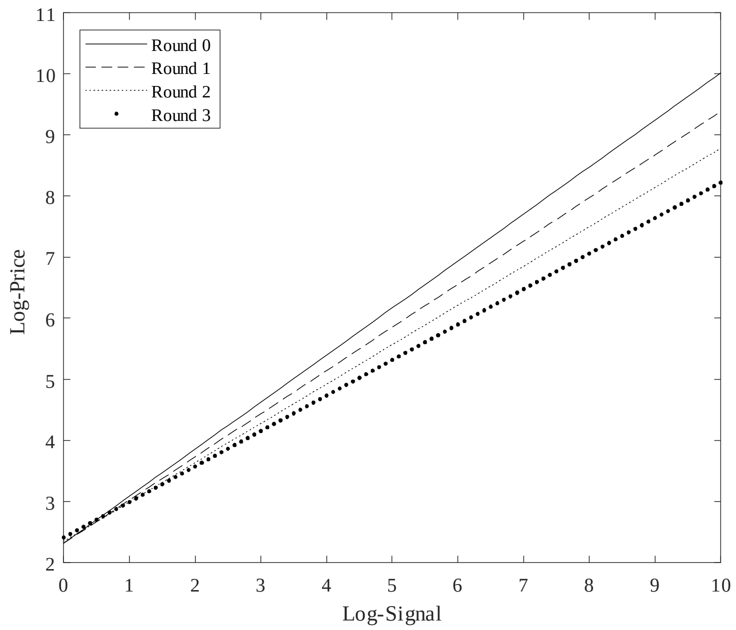
Notes: This graph displays the shape of the utility function (1) plotted against gains and losses for $\lambda = 1$ (risk neutrality) and $\lambda = 2.25$ (loss aversion), with the marginal utility of losses being λ times the marginal utility of gains.

Figure 2: Bidding Functions



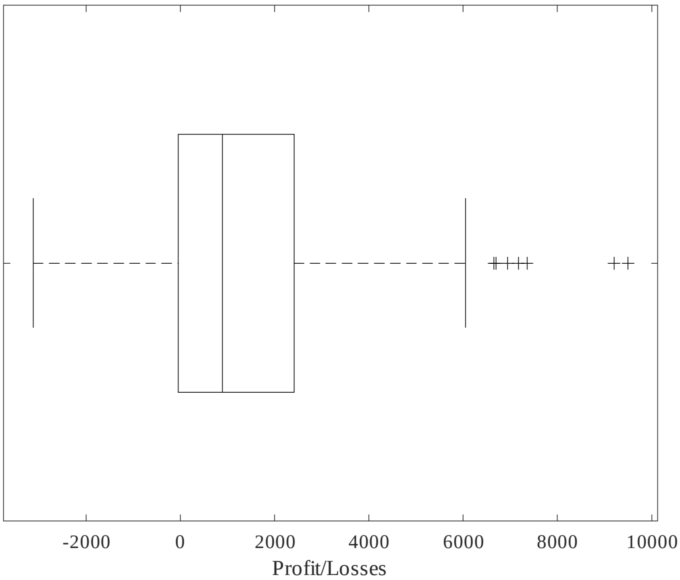
Notes: This graph displays the equilibrium bid functions for $\lambda = 1$ (risk neutrality) and $\lambda = 2.25$ (loss aversion), with bidders bidding substantially lower under loss aversion.

Figure 3: Log-Bid Functions in Multiple Rounds



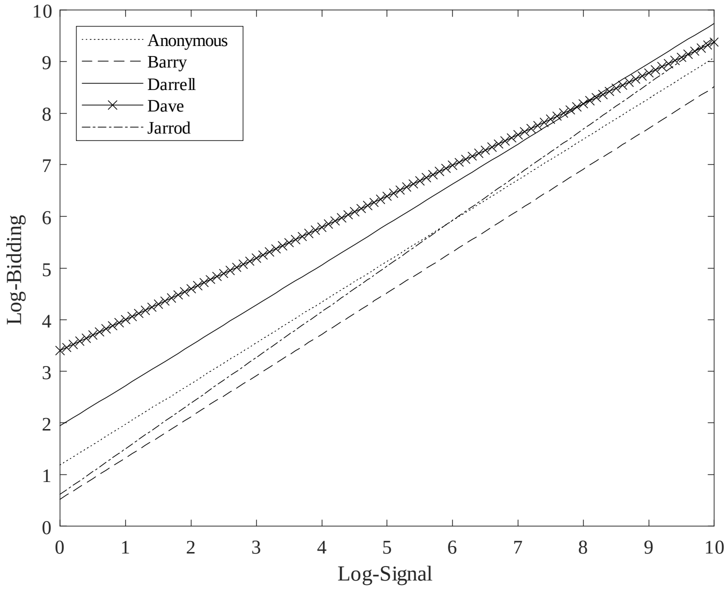
Notes: This graph displays the log-bid functions of a representative bidder for each round in an auction with 5 loss averse bidders.

Figure 4: Distribution of Storage Wars Profit/Losses



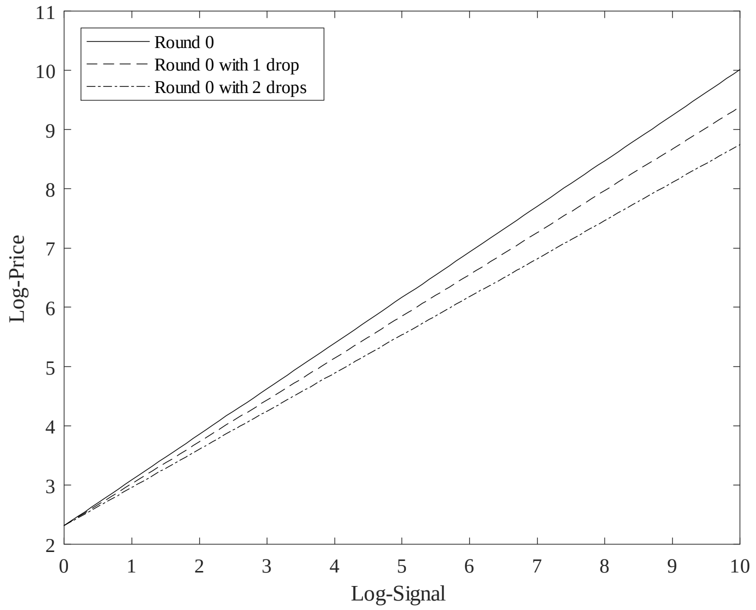
Notes: This graph displays the boxplot of the profit/losses in storage locker auctions, with the outliers being plotted using the + symbol.

Figure 5: Log-Bid Functions of Storage Wars Bidders Baseline Model



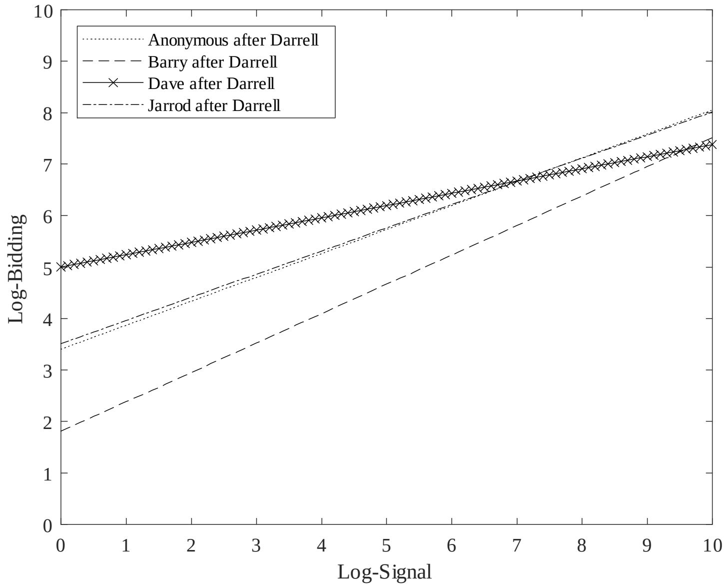
Notes: This graph displays the round 0 log-bid functions of *Storage Wars* bidders under loss aversion, except for Dave who is risk neutral.

Figure 6: Log-Bid Functions with Active Non-Bidding Participants



Notes: This graph displays the log-bid function of a representative loss averse bidder when he takes into account the private information active non-bidding participants have in round 0.

Figure 7: Log-Bid Functions of Storage Wars Bidders With Active Non-Bidding Participants



Notes: This graph displays the log-bid functions of *Storage Wars* bidders in round 0 with 1 active non-bidding participant, which in this case is Darrell.