ICADE – Facultad de Ciencias Económicas y Empresariales

BAYESIAN OPTIMISATION FOR FAST-FOOD RESTAURANT BUSINESS STRATEGY

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1. Introduction

The foodservice business is fiercely competitive; customers can easily switch from one service provider to another in such a dynamic and fast-growing market (Pai et al., 2016). Companies strive continuously to attract and retain customers while simultaneously maximising their profits. Still, the high rate of business failures in the restaurant industry suggests that it is not easy to translate customer satisfaction into revenue, and that not every business can succeed in this high-growth competitive industry (Cheng et al., 2021). The impact of COVID-19 on the restaurant and hospitality business, including the restrictions and limitations it has entailed, has only emphasised the extent to which restaurants need to optimise their strategy in order to remain competitive. Firms in the hospitality sector have increasingly turned their attention to the consumer, establishing satisfaction maximization and retention as key elements of their new business strategy. In particular, the fast-food industry is a fully customer-oriented industry (Liu et al., 2016). Their satisfaction is affected by many different factors, and the complexity of this problem, coupled with the limited budget of companies, is a burden for most players in the industry.

Traditionally, restaurants have built their strategy on the basis of qualitative analysis, such as market research, which involves marketing tools such as the 4P analysis system and others that comprise what is referred to as business intelligence (Halim et al. 2019). For the most part, restaurants still use traditional strategic frameworks that rely on a descriptive qualitative research approach, such as DAFO or SWOT analysis. Undeniably, disruptive technologies such as Artificial Intelligence have emerged as game changers in the food industry during the past decades, with the ability to leverage big data into powerful insights and deliver automated intelligent processes and predictive models for real-time results (Sharma & Sharma, 2022). The rise of social media has led to a massive generation of consumer data and restaurants have slowly begun to exploit a small part of all the data available to them by employing machine learning to analyse consumer behaviour, using tools such as sentiment analysis of reviews or topic modelling of content generated on social media.

Essentially, the ultimate goal of all these approaches is to try to deduce the function that underlies behind the conflation of customer satisfaction with business profitability. In the real world, this function is often subjective, multi-factorial and contains noise. Moreover, we are not familiar with the underlying pattern, and obtaining evaluations entails a cost in terms of funds, time or other resources. Bayesian optimisation (BO) is precisely an approach that can handle this kind of problems. Brochu et al. (2010) present this approach as a useful answer to problems explained by unknown functions and with high levels of noise. Over the last few years, BO has broadened beyond the field of hyperparametric optimisation, where it has already gained a definitive foothold. In the business domain, however, this tool is rather infrequent. The aim of this paper is to demonstrate the applicability of Bayesian optimisation to the development of the business strategy of restaurant companies. Using fast food restaurants as our case scenario, we propose a simple way in which the entrepreneur can exploit the information available to him or her in order to optimise his resources and develop an affordable, data-driven strategy that can be easily implemented within his or her business.

2. Methodology

Our paper aims to address a Bayesian Optimisation approach to the business strategy of hospitality companies, specifically, those focused on the fast-food industry. In section 3, we conduct a literature review over the work that other authors have carried out in relation to our field of study. We then explain in section 4 the theoretical background of Bayesian optimisation and present a comparison against its more widespread alternative, namely Random Search (RS). Section 5 describes the implementation of the mentioned approaches on Python and in section 6, we employ such models to simulate a real-world case scenario involving strategic optimisation in the fast-food sector. We analyse the results of our simulation model in section 7 along with a discussion of its potential benefits for entrepreneurs and limitations. Finally, in section 8, we draw our conclusions and suggest areas for future research.

3. Literature review

Prior to presenting our proposal, we conducted a thorough review of the existing literature on the different topics we address. Our survey first analyses the tools used in the restaurant sector (section 3.1) and then delves into the fast-food market (section 3.2). Finally, we examine previous studies dealing with the application of Bayesian optimisation to a variety of fields (3.3).

3.1. Previous work on the hospitality sector

Traditionally, the strategy of restaurants has revolved around profit maximisation through operational improvement of the business. Until recent years, the optimisation of their strategy has been based on capacity management, price management and time management (Tyagi and Bolia, 2021). Authors such as Dickson et al. (2005) propose the use of discounting combined with a reservation system capable of managing demands so that customer traffic would coincide with restaurant space while Kimes (1999) and Kimes et al. (1999) suggest using revenue per available seat hour (RevPASH) as a measure to optimise revenue.

Over the past few years, the focus of the strategy of these businesses has begun to consider the consumer as a key element of their activity. The effects that the COVID-19 pandemic has had on the restaurant and hospitality business and how the restrictions and limitations it has caused have led to a change in the marketing and operational strategies of companies (Craescu, 2022). Companies in the hospitality sector have focused their attention on perceived customer demands, customer satisfaction and retention of existing customers beyond classic profit maximisation. Most restaurants base their strategy on market research, using marketing tools such as the 4P analysis system (Halim et al. 2019), the SWOT (Strengths, Weaknesses, Opportunities, Threats and Opportunities) analysis and the QSPM (Quantitative Strategic Planning Matrix) analysis. Certainly, qualitative research and analysis is the most widely used approach in this sector. Cheng et al. (2021) propose the DINESERV questionnaire as a framework for measuring service quality and improving areas of poor service quality. Wedel and Kamakura (2002) support customer segmentation based on demographic characteristics, such as residential location or gender, psychographic characteristics and product benefits, such as environmental aspects or product guarantees. (N., 2022) also employs a qualitative approach, based on managerial interviews and in-depth qualitative assessment of external data sources, such as media, government regulations, reports and statistics from international consulting firms, to better explain the sales funnel optimisation strategies of a major Indian food-ordering app (customer acquisition and retention in a highly competitive market).

In recent years, some authors have applied artificial intelligence to this field. Decision tree analysis is widely used in the fields of hospitality and tourism (Lee & Kim, 2021 and Yeo & Grant, 2018). Several studies in hospitality and tourism have used decision tree modelling to predict customer behaviour patterns in hotel choice and guest preferences (Min et al., 2002), restaurant preference and restaurant selection (Hwang et al., 2012), tourist consumption behaviour and destination selection (Lopez et al., 2019), and for information technology and online user behaviour (Xu et al., 2019). In addition, the rise of social media has led to the massive generation of consumer data. Oh et al. (2021) propose a model based on semantic networks and ML algorithms to explore the sentiments implicit in Cantonese coffee shop reviews in Hong Kong. Zhang et al. (2011) employ Naive Bayes and SVM to automatically classify a restaurant's customer reviews into positive or negative (Zhang et al., 2011). Kwon et al. (2020) proposed a methodological framework to explore the numerical evaluation of customer feedback based on non-structured online reviews, which is used to better understand customers and improve service operations. Several streams of research focus on measuring and improving customer engagement and satisfaction in foodservice. A growing number of studies examine the effects of consumer rating systems and restaurant brand communications on social media, where consumers comment on the quality, experience and price of restaurant services (Luca & Zervas, 2016). AI is also used to forecast visitor traffic, food orders and sufficient catalogue to meet demand for a given period or date (Wilson & Daugherty, 2018). Analysis of customer shopping patterns has helped businesses improve the shopping experience. Joshi et al. (2022) develop an algorithm called FoodMatch, which maps the vehicle allocation problem to the minimum weight perfect matching problem in a bipartite graph. Other researchers examine how the analysis of worker knowledge and performance data can improve staffing efficiency (Kawaguchi, 2020 and Smirnov & Huchzermeier, 2020), labour allocation, scheduling and scaling (Kamalahmadi et al., 2021 and Tan & Staats, 2020).

Simulation-based approaches, on the other hand, are gaining increasing popularity as a key approach to optimising operations. Whitenack and Mahabir (2022) develop a simulation-based tool to identify inefficiencies in existing drive-thru services. The tool allows a range of scenarios to be tested for both employees and customer service agents, providing important situational awareness to restaurant owners, via a programmable multi-agent modelling environment (Cheng et al., 2021). Previous studies have developed such models to analyse drive-thru performance in restaurants (Brann and Kulick, 2002). These models offer better opportunities to understand not only what may be causing service delays, but also to compare different options that can be customised according to business possibilities.

3.2. Previous work on the fast-food sector

Within the catering field, the fast-food industry is a deeply customer-oriented industry (Liu et al., 2016). Howard and Sheth (1969) explain that customer satisfaction is a psychological conditioning that is based on the perceived value of purchasing a product or service. Hence, the key for companies in this sector to maintain their competitive advantage is their ability to acquire and manage information about their customers and transform them into value. However, customer relationship management is difficult to carry out, as the customer, either individually or as a group, has different preferences and expectations. In the context of the fast-food industry, it is necessary to understand the determinants that drive customer satisfaction in order to develop appropriate strategies (Tama, 2015).

To this end, multiple authors have tried to build different models to explain the determinants of customer satisfaction. Liu et al. (2016) selected 197 customer samples from leading fast-food franchises in Taiwan using clients' views to design a business model that enhances quality of service as well as improving customer satisfaction and loyalty by employing Sobel's test. Most of the existing literature is based on the application of descriptive statistics on questionnaires to try to explain customer patterns. Esmaeilpour et al. (2016) studied the effect of the dimensions of service quality of SERVQUAL model (tangible factors of services, reliability, responsiveness, assurance and empathy) on the brand equity of fast-food industry in Boushehr. Hanaysha (2016) investigated the impact of fair pricing, environment and meal quality upon consumer experience in the fast-food restaurant sector in the Malaysian market. Arora and Singer (2006), Baek et al. (2006), Gupta et al. (2007), Park (2004) and Jang et al. (2021), among others, have investigated attributes related to customer satisfaction using regression analysis, conjoint analysis, descriptive statistics, correlation analysis and factor analysis and structural equation modelling.

The fast-food sector relies heavily on marketing as a strategy to attract customers. Thus, several fast-food restaurants are already contributing to study, analyse and develop marketing campaigns to capture the largest market share of customers and to achieve customer retention and loyalty in order to increase the financial efficiency of the organisation. Anees et al. (2020) use exploratory and quantitative research to determine the impact of CRM strategy on customer retention. Taking a more qualitative approach, Oe & Weeks (2022) explore customer perceptions of McDonald's fast food business strategies in a foreign market in terms of divergence from the traditional local menu, indigenous management and alternative employment opportunities, environmental friendliness and corporate governance..

The intersection of data mining and machine learning in the field of customer relationship management (CRM) has been widely employed as tools for discovering the relationship between data attributes (Tama, 2015). In customer-related research, decision trees and neural networks have been used considerably in closed-loop CRM dimensions such as customer attraction, customer identification, customer development and customer retention (Ngai et al., 2009). One example is the DT and rule extraction algorithm, called NeuroRule (Setiono & Huan Liu, 1996) used for predicting consumer preferences of fast-food franchises. Tama (2015) uses these same tools for the identification of determinants related to customer satisfaction in fast food restaurants. In this way, classification models are developed using decision trees and neural networks to determine the underlying attributes of customer satisfaction. The rules generated are beneficial for management and practical application in the fast-food sector.

3.3. Previous work on Bayesian Optimization and simulation tools

In the real world, the objective function for explaining any kind of phenomena is typically subjective, meaning that it contains noise. In order to infer the underlying function of an unknown pattern, one needs observations of the model, providing some insight into how our function behaves on some situations. Nevertheless, the collection of evaluations invariably entails some sort of related cost. Bayesian optimisation offers precisely the solution to problems that fall within these features. Brochu et al. (2010) present this approach as a useful answer to problems with the characteristics described above. Brochu et al. (2010) present this approach as a helpful solution to such problems. As explained by Garrido-Merchán and Albarca-Molina (2018), BO is based on modelling the output surface using a probabilistic model, most commonly a Gaussian Process (GP), which results in a method which is easy to test and yields predictions for the entire input domain that take uncertainty into account. As an effective algorithm for the optimisation of computationally demanding black-box functions, the widespread popularity of Bayesian optimisation has risen over the past few years along with the boom in machine learning due to its role as the major algorithm for hyperparameter optimisation, applied to genetic algorithm, colony optimization, gradient descent, Newton-Quasi method and L-BFGS-B (Thuan & Logofatu, 2020).

Even to a lesser extent, BO has been adopted in other fields outside of hyperparameter optimization. In the business domain, though, this tool is rather uncommon. Park et al. (2008) used a Bayesian framework to analyse the individual user behaviour and combined it with an analytic hierarchy process method to generate group recommendations, based on the type of restaurant, price, mood, and distance. In the culinary field, it has begun to be employed for recipe optimisation. Solnik et al. (2017) were one of the pioneers, using Google Vizier BO-based platform to obtain the flawless chocolate chip cookie recipe. The approach was a blended initiative process involving human chefs, human raters and a machine optimiser in 144 experiments. To obtain results tailored to the user's preferences, a lot of data was needed. Garrido-Merchán and Albarca-Molina (2018) suggested an approach to solve this challenge by avoiding to cook for each iteration of the BO process,, but just prior at the start of the process. Accordingly, BO would only be applied to a Dataset based on a fitted ML model that has approximated the cooking evaluation by having learned it from previous data. Although this study proves the superiority of BO compared to a RS or an Expert Criterion in terms of optimization, the proposed model may be too simplistic and quantitative for areas such as business strategy.

It is now evident that the isolated application of BO requires cooking or conducting experiments as many times as the number of iterations we want to undertake. Generating such a large number of different prototype recipes or situations, depending on the problem to be studied, can be costly and can increase our infrastructure, human resources, time and financial constraints. Therefore, in order to be adopted in the field of business strategy, it is necessary to find a cheap and fast alternative to generate these results. In the 1960s, colleges and universities began incorporating Business Game Simulators into their courses to offer an interactive learning experience to their students (Faisal et al., 2022). Since then, BGS have enjoyed increasing popularity among higher education institutions. García et al. (2012) deployed SIMBA (SIMulator for Business Administration),a powerful simulator that is now being used as a web-based platform for business education in different institutions. This robust application is based on the application of reinforcement learning (RL) for the development of smart players that can manage virtual enterprises. Beyond the educational arena, the use of simulators is beginning to spread to the corporate world. Zhao (2022) proposed an experimental financial management simulation system. This model, built on a random forest algorithm, offers support for audit services and, when applied to business cases, provides new insights into the modern audit workflow and provides a rich and lively open business simulation environment for the financial management experiment.

4. Theoretical background

As we previously mentioned, non-linear processes are the principal explanation for most of the behaviour of the real world. Their maximisation has been the field of study of numerous authors over the years. This problem is simpler when the function to be optimised follows a recognised pattern, is convex or, in some way, is cheap to evaluate, giving rise to the possibility of finding its global maximum by means of obtaining infinite simulations. However, typically the derivatives and convexity pattern are unknown, and evaluating the objective function is expensive or perhaps impossible. For example, we may only be able to estimate the objective function by simulating events that will occur in the future for the majority of real-world, serial decision-making problems. The procedure is always expensive, whether using basic Monte Carlo simulation or more sophisticated techniques.Also, in some scenarios, obtaining observations from the function corresponds to pricey procedures like financial investments, destructive testing, or drug testing. In active consumer modelling, for instance, x might stand in for the attributes of a user enquiry, and $f(x)$, for a human response, thus this cost is not always purely economic. In order to avoid upsetting the user, interviewers should ask the appropriate questions and refrain from asking too many.

Bayesian optimisation can be seen as a flawless approach to find the extremes of target functions that are costly to test. It is therefore well suited in an environment where a fixed formal expression for the target function is not known but where (potentially noisy) observations of the function may be derived from sampled values. Bayesian optimisation methods rank amongst the most efficient when it comes to the number of evaluations needed. According to Brochu et al. (2010), Bayesian optimisation's capacity to embed prior assumptions about the problem to help guide sampling and the use of an acquisition function to assess the trade-off between exploration and exploitation of the search space account for a large portion of this efficiency. As a general background, the term Bayesian is used to refer to Bayes' famous theorem and determine the posterior probability of a model in the light of prior evidence. In Bayesian optimisation, the probability a priori reflects our belief about the possible space for the target functions or their behaviour. In the field of business strategy, for example, these prior ideas can come from either competitor analysis or examination of one's own previous or simulated experiences.

The foundation of Bayesian optimization is a set of earlier observations known as priors. The probability function and prior distribution are blended when we gather new observations. In essence, we are questioning the probability of the facts we have observed, given what we think we know about the a priori distribution. By combining both functions, we obtain the posterior probability of the function, which is renewed each time we obtain a further observation, i.e., this posterior distribution reflects our updated assumptions about the unknown target function. This step of Bayesian optimisation can also be interpreted as estimating the objective function via a surrogate function. Part of the efficiency of the model depends on the observations we incorporate into that model. To sample efficiently, Bayesian optimisation uses an acquisition function to determine the next location $x_t + 1 \in A$ to sample. This decision involves an explicit trade-off between exploration and exploitation, i.e., it must decide whether to prioritise obtaining observations in areas where the objective function is very uncertain or in areas where the function values are expected to be high or even maximal. The advantage of this optimization strategy is that it seeks to reduce the number of evaluations of the goal function. Furthermore, it is probably effective even when the goal function contains several local maxima.

The Bayesian model offers an appropriate mechanism for informative priors for characterizing aspects of the objective function, such as consistency or the most likely locations of the maximum, in nature, even in the absence of knowledge about the function that is being sought itself. This approach adheres to the maximizing of expected value or minimization of expected risk, in contrast to many of its alternatives. The basis for the subsequent sample choice process is the choosing of a utility function and a method for optimizing the expectation of this utility with respect to the posterior distribution of the objective function. This secondary optimisation problem is often simpler to solve since the utility is typically chosen in a way that is easy to evaluate, even if the utility is still non-convex. The acquisition function is used as a metaphor for this additional issue. The acquisition function's goal is to guide the hunt for the perfect answer. According to how acquisition functions are frequently stated, a high acquisition typically corresponds to potentially high values of the goal function, either because the prediction is high, the uncertainty is large, or both. The next position at which the function will be evaluated is chosen using the acquisition function's maximization. Given the improvement function's expectation with relation to the Gaussian process' predictive distribution, we may balance the trade-off between exploitation and exploration that we previously outlined. When exploring, we should choose areas with a high surrogate variance, and when exploiting, we should choose areas with a high surrogate mean.

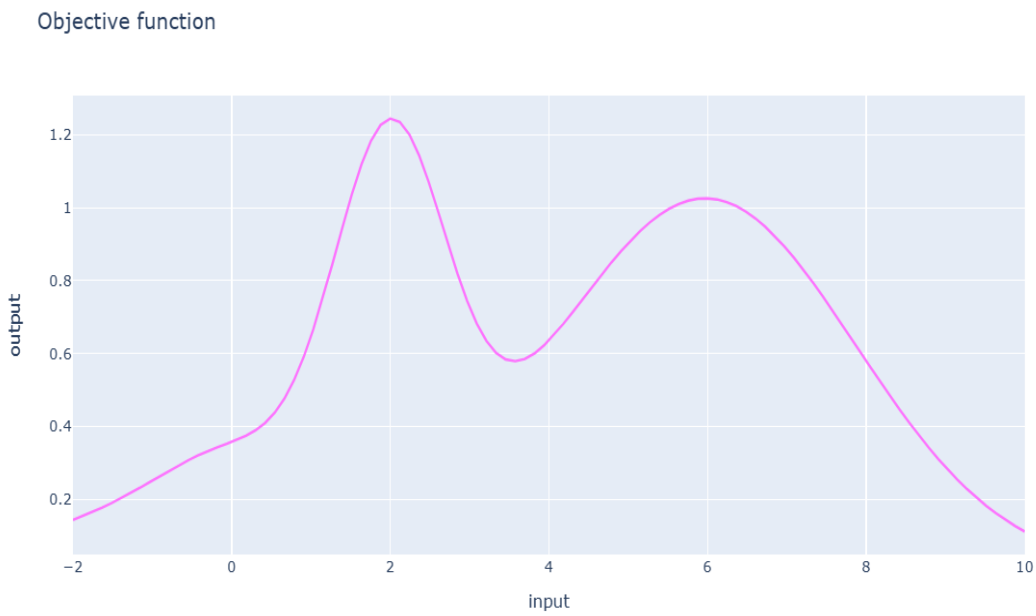
The optimization procedure actually begins with a small number of samples. Each iteration chooses the next sample point by maximising the acquisition function, which represents the sampling utility by taking into account the mean and variance of the predictions in the space. The Gaussian process is then updated, the objective is sampled at the acquisition function's argmax, and the procedure is repeated. It is important to note that in this method, the target is assumed to be Lipschitz-continuous, i.e., there exists some constant C, such that for all $x_1,\ x_2\ \in\ A$: $f(x_1)\ -\ f(x_2)\ \leq\ Cx_1\ -\ x_2$ although C can and usually is unknown. Furthermore, we cannot assume that the negative goal function is convex in our optimization problems. Although we are just aware of a few observations, we may not be aware of the pattern that our function tracks before beginning. In global optimization, it is typical for the objective function to be a "black box" function, meaning that neither its expression nor its derivatives are known to us. The only way to measure the function is to conduct a search at a point x and receive a typically noisy response.

This kind of global optimization can be approached in a wide variety of ways, all of which have been well studied in the literature (Zhigljavsky and Zilinskas, 2008). Branch and bound techniques and slotted optimization are both used in deterministic procedures. In machine learning contexts, the stochastic approach is a well-liked concept for optimizing uncertain objective functions (Kushner and Yin, 1997). Unfortunately, because they need a lot of samples, which are expensive to gather in the active user modelling domain, these methods are frequently inappropriate for our field. This can make purchasing insurance against improbable events exceedingly expensive. As a result, it makes sense to weaken the protections against the worst-case pathological circumstances. Instead, the objective is to maximize the posterior using evidence and previous knowledge at each stage, reducing the gap between the actual global maximum and the maximum predicted by the model with each subsequent evaluation. Bayesian optimization employs a priori variables and trials to define a posterior distribution in function space, providing an effective yet affordable method for real-life optimization problems, encompassing corporate strategy.

5. Implementation of Bayesian Optimization in Python

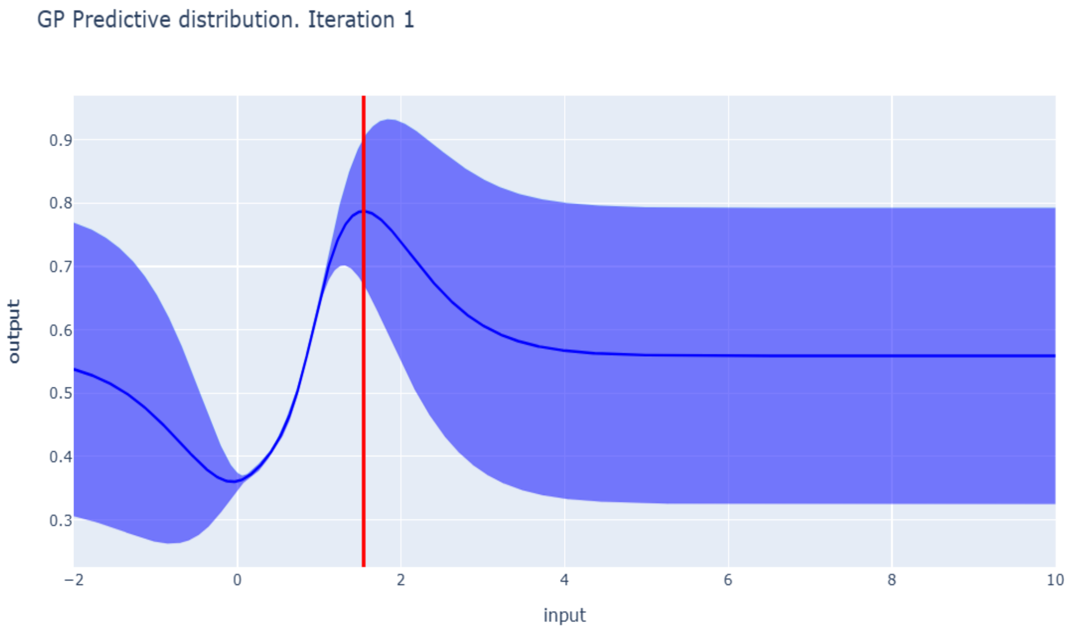
Our Bayesian Optimization model was built in Python using the BoTorch module. BoTorch is a multipurpose programming framework developed in PyTorch for Monte Carlo-based BO research (Saikai, 2022). BoTorch facilitates the setup of GPs and acquisition functions, computation of the GP posterior, marginal likelihood optimization, and optimization of acquisition functions, and was specially designed to efficiently handle the latter operation. Other popular BO libraries include Spearmint, GPyOpt, Cornell-MOE, RoBO, Emukit, and Dragonfly. However, BoTorch is remarkable for the fact that all module operations are highly parallelizable on modern hardware and end-to-end differentiable, allowing efficient optimization of acquisition functions. In essence, this library provides implementations of many state-of-the-art algorithms for vanilla BO, multi-target BO and multi-fidelity BO. Given that the goal of our proposal is to provide a support tool for entrepreneurs in the restaurant industry, we have chosen BoTorch for its ability to implement BO in a simple and efficient manner. The framework is open source and available at https://github.com/pytorch/botorch.

We have mentioned above that the objective function to be optimized follows an unknown pattern of which we only know occasional observations or individual points. Based on these priors, our model will be able to modulate this function and forecast the global maximum of the curve. For the purpose of illustrating how it really works and to demonstrate its accuracy, we apply our model to a function for which we do know the exact behaviour, yet our method is used to shape and optimize it. Consider that the profitability of our business can be explained by the function shown below (picture 1). For the sake of simplicity, the function in this case will depend on a single independent variable, which could be the number of hours open to the public.



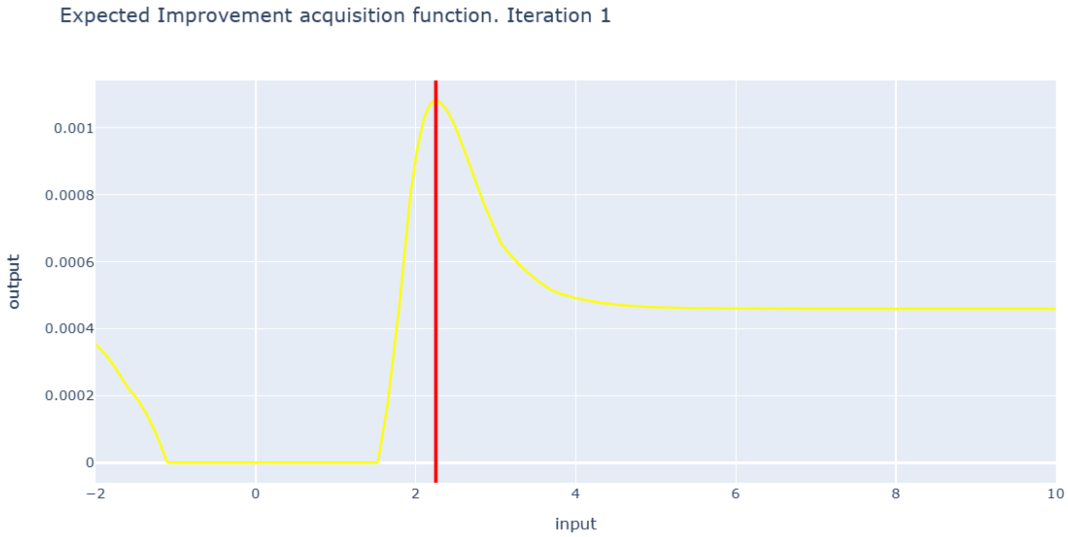
Picture 1: test function for Bayesian Optimization

Now suppose we do not actually know the function above, but we are provided with a set of ten prior observations for which we are aware of the true value our function yields. Those are the priors we will introduce in our process and the first step our process will follow is to estimate the distribution our function would follow based on the data we have, capturing the uncertainty present in each stretch, as shown in picture 2. Thus, our model will predict higher uncertainty in those legs for which we do not possess (many) observations, since the behaviour of our function in those intervals may follow different traces. The model then proceeds to optimise the acquisition function for the given distribution, found in picture 3, with the objective of finding the candidate that maximises the expected improvement, i.e. that decreases the uncertainty the most or maximises our function.



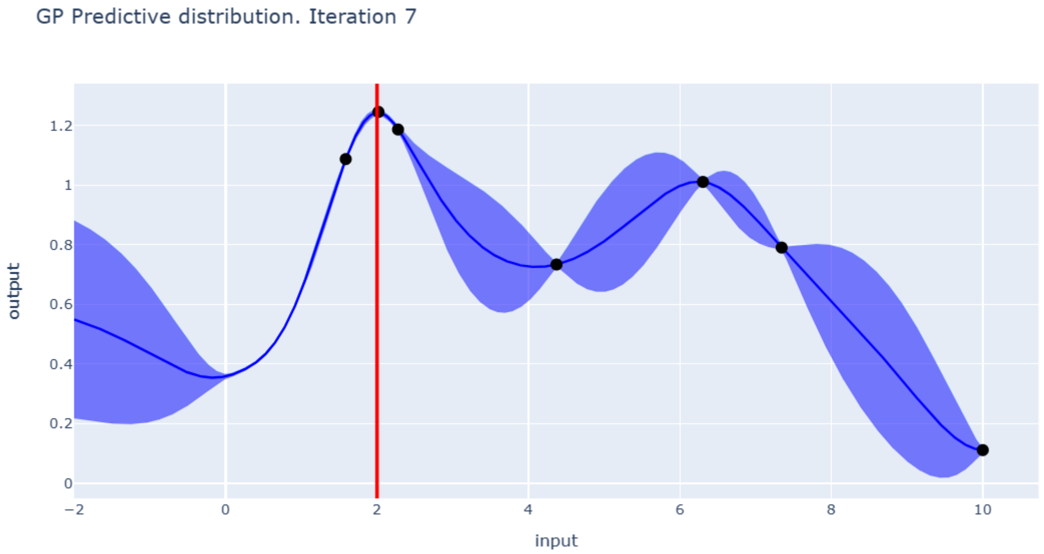
Picture 2: predictive distribution for the test function, showing its mean and standard deviation

As we have observed above, there is a trade-off between exploitation and exploration, since the expected improvement may refer to generating candidates in those areas where we have fewer observations or in those areas where the known observations generate higher values, which may be a sign that we are approaching the peak of the function. Optimising the acquisition function is of critical importance as in reality, our budget will be limited and obtaining each new simulation implies a certain economic cost.



Picture 3: acquisition function computed based on the predictive distribution of the previous Gaussian process

Considering the priors entered and the forecasted distribution, the candidate that maximises the utility of the acquisition function is the one that delivers the highest expected improvement, which is why we proceed to obtain the value of our function for that entry point. The process is iterative, in other words, with each new observation the predicted distribution will be updated and new candidates will be estimated to be implemented. The number of iterations will depend on the budget available to the entrepreneur, although it has been shown that BO is efficient even with less than 20



Picture 4: new predictive distribution of our test function after introducing the candidate that maximizes the expected improvement

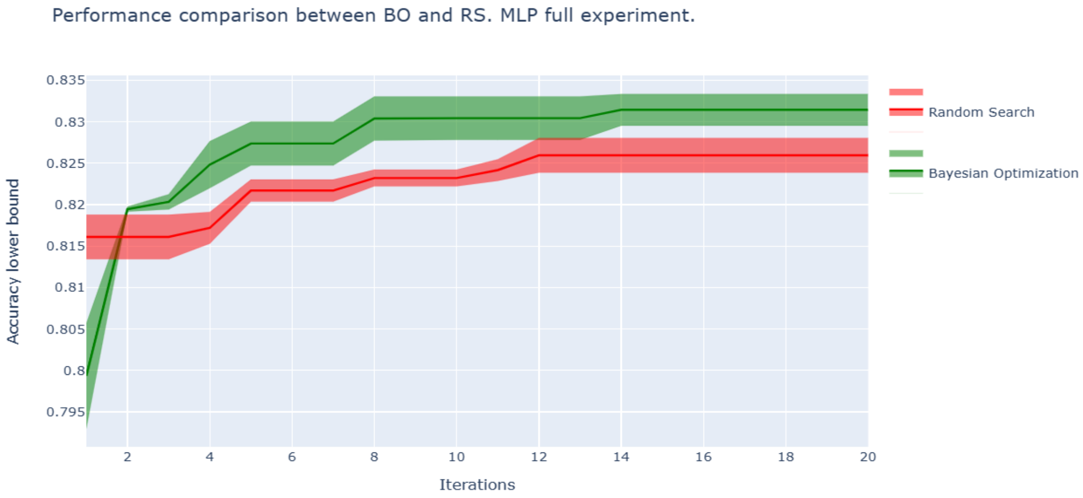
observations. In picture 4 we can see how, with each candidate introduced, our model outlines the pattern that our function follows and effectively reduces its uncertainty.

In essence, our Bayesian model searches for the global optimum of the function by estimating the objective function based on the priors introduced and the candidates chosen. By using the acquisition function, the search of these candidates is optimised and provides a robust approach to finding the true maximum in a noisy function.

Bayes search is not the only way to model a function with these characteristics. The easiest solution is to try a bunch of combinations and figure out which one performs best. The idea of creating a "grid" of parameters and testing all possible permutations is called a grid search. Grid search has traditionally been used to find the optimal hyperparameters of a model that yield the most "accurate" predictions. Evidently, if we try all possible points, we will exactly recreate the objective function. However, this is a heavily computationally intensive method, being sub-optimal in terms of resources used. In the field of restaurant business strategy, the grid search method involves conducting thousands of experiments, with the time and financial burden that this entails. A resource-efficient alternative, which is often used as a comparison to the Bayesian method, is Random Search. Creating and analysing random inputs for the goal function is the procedure behind random search. Because it assumes nothing about the structure of the objective function but instead models it based on random points, it is the most computationally efficient method. Its similarity to Bayes search lies precisely in the inference of the objective function based on mere observations which makes it more suitable for the field of business strategy and for all those cases where each observation incurs a high cost.

However, random search belongs, together with grid search, to the field of naive optimisation. Any approach that relies on no assumptions about the objective function being optimized is categorized as naïve optimization. It is fairly simple to build, and the algorithm's best outcome can be used as a standard against which to compare more complex ones. A more complex algorithm should be abandoned if it can't, on average, outperform a simple algorithm in solving the problem at hand.

For the purpose of proving the superiority of our model using the random search as a benchmark, we carry out three simulations in which we apply both models to the same function, comparing their accuracy. For each simulation, we have start from ten preliminary points, being random observations for the random search model and using BoTorch to generate the priors of the Bayesian model, and perform twenty iterations for each experiment. For the random search model, the next twenty simulations remain random while these candidates are chosen by optimizing the acquisition function for the Bayesian model. As we have indicated before, the accuracy of these approaches improves as we increase the number of iterations, since in both cases it is possible to model the function underneath with less uncertainty. Figure 5 shows the average accuracy of each of them for the different iterations in the three experiments.



Picture 5: comparison of the accuracy of BO and RS for 10 experiments with 20 iterations

6. Experimental implementation of BO & RS

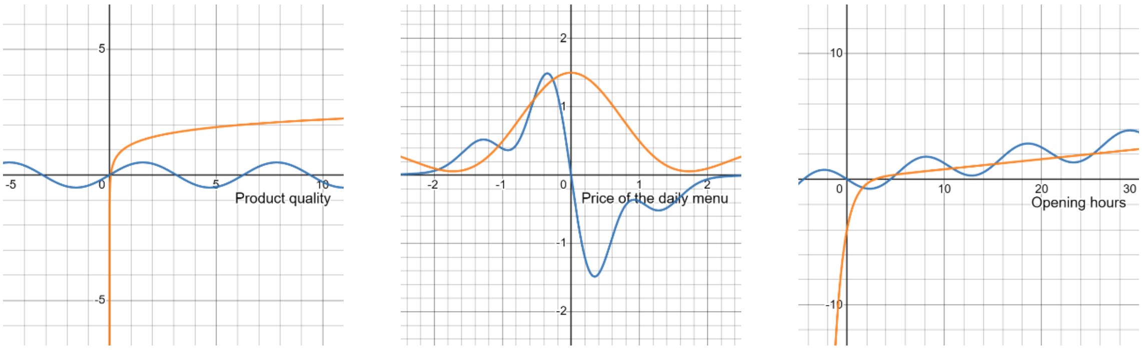
6.1. Formulation of the objective function

In the previous section, we explained the underlying logic of Bayesian optimisation and compared it to random search with a unidimensional test function. Based on this approach, our goal is to develop a business tool that can be used by the entrepreneur and adjustable to his or her specific needs. Traditionally, the goal of an optimisation strategy has been to maximise or minimise a single objective function. In the business world, the situation is not always so straightforward and often more than one objective is pursued simultaneously. In recent years, in the restaurant industry, and more specifically in the fast-food sector, the strategy has focused on customer retention. Ganatra et al. (2021) state that it is an irrefutable fact that the cost of retaining a customer is much lower than the cost of acquiring a new one. Customer satisfaction is undoubtedly one of the most important factors in ensuring customer retention. In the past, several researches have been carried out to identify the factors that influence customer satisfaction and, in turn, lead to customer retention. Convenience is one of the factors that can build customer loyalty (Al Masud et al, 2017).

In addition to serving high-quality cuisine, convenience plays a significant part in retaining customers. According to a 2018 article by Al Masud et al., a good convenience encourages a high level of positive client loyalty. The only component of the marketing mix that directly affects resources is price. Setting and managing prices is thus a crucial problem. In a market that is extremely competitive, the organization must set a price and determine how customers will react to it. Yieh et al. (2007) claim that when buyers believe the price being supplied by the vendor is reasonable and fair, their positive view of the seller would steadily increase and these kinds of feelings will eventually manifest as behavioural intents. According to research by Ferreira and Coelho (2015), consumers who are knowledgeable about the costs of products and locations are more dependable. client demand grows along with the number of businesses, but for the client to remain devoted, the pricing must be fair and reasonable (Al-Tit, 2015). We have therefore assumed that the interests of the entrepreneur will normally be divided between the need to retain customers and the goal of remaining profitable. Our tool will therefore focus on the optimisation of two functions, customer retention and business profitability, offering the entrepreneur the possibility to choose which weight to give to each of them. In order to test the usefulness of our proposal in an environment as close as possible to the real one, we have applied our tool to a situation in which the entrepreneur wishes to develop his strategy according to the factors mentioned above: the price of his menu of the day, the quality of his product and the number of hours open, deciding to give 60% importance to customer retention and 40% to maximising his income.

As explained above, in a real situation, the entrepreneur will not know in advance what pattern the function of his business strategy follows. The entrepreneur can, however, know how profitable his business is when he sets a certain price, improves or worsens the quality of his product or opens for a certain number of hours. The same will be true for customer retention; he will know the results he achieves on certain occasions. In other words, he will have access to specific observations of his target function but, in order to know how his function behaves for specific values, he will have to change the price of his product, play with the quality of his food or experiment with the opening hours, and obtaining these points involves a cost. Therefore, the objective of our tool will be to maximise this business strategy with as few observations as possible. In our case, as we did not possess this prior knowledge, we simulated these observations. In addition, to prove the actual accuracy of our tool, we have applied it to a function of which we know its behaviour, in order to compare the performance of our Bayesian optimisation with a strategy based on random search. For this purpose, we have designed how our two objective functions would behave for each variable, i.e. for each dimension, and then constructed their aggregate, which would be the complete strategy to be optimised.

Our variable *daily menu price* will range between minus two and two, and will represent the extent to which our price relates to that of our competitors. The *quality of our product* will range between one and ten, while the *opening hours* will vary between two and twenty-four hours. In the following graphs we can see how we have estimated how each of them would behave in relation to customer retention, in orange, and business profitability, in blue.



Picture 6: profitability, in blue, and consumer retention, in orange, based on our three variables: product quality, price of the daily menu and opening hours, respectively

According to our estimates, our two objective functions could be mathematically represented as follows:

$$f(p,c,h) = 0.5 + (1-p^2)e^{-0.5p^2} + \log 16c - 4e^{-h} + 0.08h \qquad (1)$$

$$g(p,c,h) = -(2p + \sin 5p)e^{-p^2} + 0.5\sin c + \sin -0.6h + 0.1h \qquad (2)$$

$$subject\ to$$

$$-2 \leq p \leq 2, \qquad 1 \leq c \leq 10, \quad 2 \leq h \leq 24$$

where $f(p, c, h)$ represents customer retention as a function of p, price of the menu of the day, c, quality of the product and h, number of hours open, and $g(p, c, h)$, the profitability of the business as a function of the same variables. The aggregate of these two functions would shape our business strategy, being different for the different importance that the entrepreneur gives to each of these two functions. Their mathematical representation is as follows:

$$z = a \times f(p, c, h) + b \times g(p, c, h) \qquad (3)$$

$$0 \leq a, b \leq 1 \qquad \text{and} \qquad a + b = 1$$

where a and b represent these weights. In our case, they will be 0.6 and 0.4, respectively, but our tool allows the entrepreneur to adapt it to his specific business.

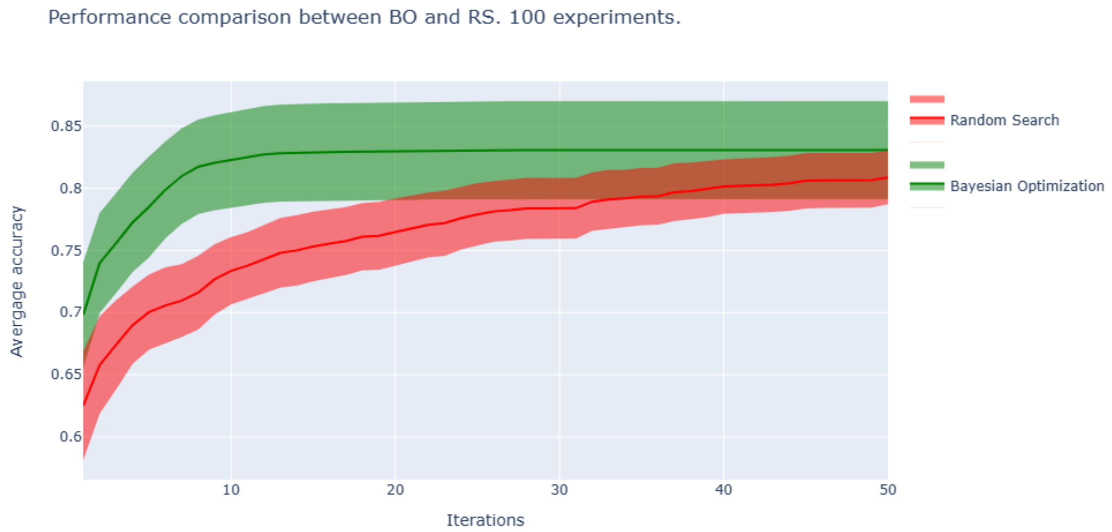
6.2. Experimental set up and Implementation details

Once we have defined the function driving our business strategy, we proceed to implement the optimisation via both our models: Bayesian optimisation and random search. As we have explained above, random search does not rely on any prior knowledge; rather, the search merely retrieves a given number of observations and the result returned will be the maximum observed value among those values. Bayesian optimisation would follow a different path; in this case, we will incorporate prior knowledge, which may be previous experiences that the entrepreneur has already gathered. On the basis of these priors, our model will determine where our global maximum is most likely to be found along with the areas where we find the greatest uncertainty regarding the behaviour of our function. The trade-off between exploitation and exploration will be evaluated by our acquisition function, which, based on the information already known, will determine which future observation will bring us the greatest expected improvement. In order to compare the behaviour of the two models accurately, we conducted a thousand experiments and analysed the accuracy of the lower bound for ten, twenty, thirty, forty and fifty iterations. Each iteration entails the collection of an additional observation and we considered valuable to compare how these models behave in different situations. The basis of the evaluation is the average value of the lower bound accuracy of these one thousand experiments for the five proposed scenarios.

Thus, each experiment will start with the simulation of the equivalent of three previous experiences that would be held by the entrepreneur. To this end, we draw three random observations, different for each experiment, from our objective function, which would serve as our input priors for the Bayesian model. According to these three values, our Bayesian tool would estimate the distribution of the underlying function as well as the uncertainty found within each area, while our acquisition function would evaluate which one among all the candidates, i.e. the rest of the unknown observations, would provide us with the highest expected improvement. The value that maximises the acquisition function becomes our best candidate and its value is introduced in our model, so that both the acquisition function and the expected distribution of the main function are updated with every iteration. As for Random Search, our model would select a blind candidate at each interaction. Finally, both models will choose the highest value obtained and we will assess the accuracy of the estimation of our resultant maxima with respect to the global maximum of the underlying function.

7. Result analysis and discussion

Following the completion of one hundred experiments, we analysed the collected results. As we have discussed above, each trial involves the implementation of both methods to our objective function, conducting fifty iterations per experiment. Each round is essentially equivalent to collecting one additional observation. For the Bayesian model, every run would start with the imputation of a distinct prior while for Random Search, however, every trial would produce a completely random point, free from any background knowledge. In a purely real scenario, the entrepreneur using the Bayesian model could impute any number of available or desired priors, but here, we have opted to use only one. Our rationale behind this approach is to test whether the Bayesian method is truly superior to Random Search in terms of effectiveness for the estimation of future optimal observations in order to maximise the unknown function. If we rely on a multitude of previous observations, the comparison would not be fair, as the Bayesian model would have access to more information than the random search-based model. Consequently, a higher performance could be attributed to such factor rather than to the use of an acquisition function and the feeding of the new observations produced to efficiently decide the next one to be collected. Our tool is intended to be implementable by all types of entrepreneurs in the sector, both new businesses as well as large established chains. Prior experience varies from user to user, therefore we aim to provide a solution that, besides accurate, can be tailored to the food service sector, where, as in many other cases, obtaining an additional observation is a high economic cost. For this reason, we strive for a model that is able to perform reliably while reducing the required number of observations.



Picture 7: Comparison of the results obtained by the Bayesian model and Random Search for 100 experiments of 50 iterations

After completing the hundred experiments, we obtain the average accuracy for each model as well as the standard deviation. In picture 7, we note the evolution of the performance of our models based on the number of iterations conducted. Accordingly, it can be affirmed that the Bayesian model is, on average, superior to Random Search regarding the accuracy of their predictions. The standard deviation is, globally, higher for the Bayesian model and is slightly increased with the number of iterations. In other words, the average accuracy of the Bayesian model fluctuates to a greater extent than in the case of Random Search. Nevertheless, the lower bound accuracy for the Bayesian approach is superior to the Random Search for every scenario, implying that poorer predictions from the Bayesian tool would be closer to the true optimum than those from the Random Search model. Regarding the upper bound accuracy, the value of the Bayesian model is significantly higher compared to the Random Search model. The best predictions of the latter fall below the mean of the former, i.e. the average predictions of the Bayesian model would outperform the most accurate outcomes of the Random Search. The above statements are held to be true for all the different values of iterations between zero and fifty, although these vary as the frequency of iterations increases, as we can observe in figure 8.

	Bayesian model			Random Search model		
Iterations	Lower bound	Mean	Higher bound	Lower bound	Mean	Higher bound
10	0.784	0.823	0.861	0.707	0.734	0.761
20	0.790	0.829	0.869	0.738	0.765	0.790
30	0.791	0.831	0.870	0.759	0.784	0.808
40	0.791	0.831	0.870	0.779	0.801	0.823
50	0.791	0.831	0.870	0.787	0.809	0.830

Picture 8: Comparison of the accuracy of the Bayesian model and Random Search for 100 experiments of 50 iterations

We have mentioned above that the final strategic tool for our case study, the fast-food restaurants, should be able to optimise our resources with as few simulations as possible. An additional observation implies an increase in cost, and we must not forget that there is a limited budget at our disposal. From the figure above, we can notice that the performance of both models tends to converge as we scale up the number of iterations. As the number of repetitions increases, gradually we approach the Grid Search method which, as we previously described in this work, which is based on the reconstruction of the function to be optimised by gathering a wide set of observations. The marginal improvement in the accuracy of our Bayesian model decreases as the number of iterations grows, and remains almost steady from the twentieth iteration onwards. The reason behind it is that, early in the experiment, when our known observations are reduced, the uncertainty of the underlying function is maximal, and retrieving our optimal candidate value leads to a substantial improvement. By obtaining new points, this improvement in the prediction of the objective function is diminished, and, in most cases, the model would continue to incorporate new observations even after the global maximum has already been found. Since the function to be optimised is unknown for our model, it will be unaware of whether or not the global maximum of the function has been found and will continue to add new observations until the allowed quantity of iterations has been reached.

Conversely, the accuracy of the Random Search model increases roughly in a linear fashion as the number of iterations increases. The generation of new candidates is completely arbitrary and does not obey any rule. Therefore, the more observations obtained, the greater the probability of finding the maximum of the function. In the case of unlimited resources, Random Search eventually equals the Bayesian model. However, the superiority of the Bayesian model compared to Random Search is strongly perceived within the first twenty iterations. Briefly, based on our experiment, the Bayesian model is able to predict with a higher accuracy the global maximum of our strategy function for less than twenty repetitions when compared to the Random Search model. We propose to implement this model, employing twenty iterations, which is a realistic threshold for the enterprises. The average accuracy of the Bayesian model for such a scenario is 0.829, compared to the 0.765 obtained by the Random Search. We conclude, therefore, that our proposed model offers a solid basis for decision-making in the business strategy of companies in our sector and offers a clear improvement over the benchmark model, the random search.

8. Final conclusions and further areas for research

Based on the results we have obtained in our work, we believe that a sample of one hundred experiments is a solid demonstration that the Bayesian model has potential advantages for entrepreneurs in the fast food industry and in the business world in general. The optimisation of unknown functions is a widespread issue in all domains. Only occasionally is it possible to determine from the outside the exact function that explains our phenomena. In the majority of real world scenarios, these functions fluctuate and follow a path that is not easily predictable. Optimising a function by gathering evidence assures us that the evaluated points are truly accurate. However, the collection of these points is resource-intensive and difficult to implement in areas where the budget is scarce.

Our model may be regarded as an initial step towards that end, but further research is needed in this field. Our model admits considerably more variables than the ones we have chosen, being three for the sake of simplicity. In addition, the predictive ability of our model strongly relies on the acquisition function employed to optimise the gathering of new inputs required by our process. Multiple functions are available to consider, besides the expected improvement we have proposed. Furthermore, the priors introduced in the first run play an important role in the performance of the Bayesian strategy, and it may be useful to study the effect that different sets of priors may have on the behaviour of our tool. The modelling of human behaviour is a very complex field but also a field of application for most of the tools used traditionally in the most technical fields. We therefore encourage researchers to contribute to bridging the gap between the technical sector and the classical entrepreneur.

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