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Implementing a Novel Pairs Trading Strategy: A Comprehensive Analysis on the Dow Jones Index

Author: Guerrero Gallego, Álvaro

Director: Paraskevopoulos, Yannis

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Abstract

This study investigates a pairs trading strategy using stocks from the Dow Jones Industrial Average (DJIA) index. It entails estimating parameters, optimizing portfolios, and backtesting utilizing daily price data from 2003 to 2023. The technique seeks to profit from stock price mean-reversion by selecting couples with stable correlations.

A continuous cointegration technique and maximum likelihood estimation are used to estimate the model parameters. Stability requirements and cointegration tests are used to choose pairs. Using current portfolio theory, the estimated parameters, such as mean-reverting speeds and spread volatility, are used to design optimal portfolios.

Backtesting is used to analyze the performance of the strategy using metrics such as excess return, volatility, and Sharpe ratio. When stability prerequisites and cointegration constraints are met, the results show that the technique has the potential to provide consistent profits.

This research adds to our understanding of pair trading methods in the context of DJIA equities. The methods given here can be used by practitioners who want to implement comparable strategies. For successful pairs trading methods in practice, it emphasizes the necessity of accurate parameter estimate, portfolio optimization, and rigorous backtesting.

Keywords: Risk aversion, Interacting agents, Stock selection, Stock cointegration, Portfolio theory, Spread options, Backtesting

Resumen

Este estudio investiga una estrategia de negociación por pares utilizando valores del índice Dow Jones Industrial Average (DJIA). Implica la estimación de parámetros, la optimización de carteras y la realización de pruebas retrospectivas utilizando datos de precios diarios desde 2003 hasta 2023. La técnica trata de beneficiarse de la reversión a la media de los precios de las acciones seleccionando parejas con correlaciones estables.

Para estimar los parámetros del modelo se utiliza una técnica de cointegración continua y una estimación de máxima verosimilitud. Para elegir las parejas se utilizan requisitos de estabilidad y pruebas de cointegración. Utilizando la teoría actual de carteras, los parámetros estimados, como las velocidades de reversión a la media y la volatilidad de los diferenciales, se emplean para diseñar carteras óptimas.

El backtesting se utiliza para analizar el rendimiento de la estrategia utilizando métricas como el exceso de rentabilidad, la volatilidad y el ratio de Sharpe. Cuando se cumplen los prerrequisitos de estabilidad y las restricciones de cointegración, los resultados muestran que la técnica tiene potencial para proporcionar beneficios constantes.

Esta investigación contribuye a nuestra comprensión de los métodos de negociación de pares en el contexto de la renta variable del DJIA. Los métodos aquí expuestos pueden ser utilizados por los profesionales que deseen aplicar estrategias comparables. Para que los métodos de negociación por pares tengan éxito en la práctica, se hace hincapié en la necesidad de una estimación precisa de los parámetros, la optimización de la cartera y un backtesting riguroso.

Palabras clave: Aversión al riesgo, Agentes interactuantes, Selección de valores, Cointegración de valores, Teoría de carteras, Opciones spread, Backtesting

1. Introduction

In this bachelor's thesis, we aim to implement a pairs trading strategy in Python based on a research paper by our director Yannis Paraskevopoulos. The original paper presented a trading strategy in which risk-averse agents cooperated to exploit relative pricing errors in asset prices. Our goal is to adapt this strategy to a more flexible and modern code to allow further optimizations and adapting the triggers of our strategy to improve its results. By doing so, we aim to investigate the limits to the effectiveness of this strategy.

The motivation for this research stems from the diverse justifications for trading between risk-averse agents found in the general equilibrium literature. Previous studies, such as Bhamra and Uppal (2014), Xiouros and Zapatero (2010, 2019), and Scheinkman and Xiong (2003), have explored various factors that influence agents' trading behavior, including heterogeneous priors, preferences, information quality, and disagreement over asset valuation. Our approach differs from these previous studies by focusing on the strategic cooperation between two interacting agents who see a positive benefit in trading. Both agents are risk-averse and have an incentive to cooperate to achieve healthy revenues. The investor operates under a continuous time mispricing model, aiming to profit from relative pricing errors in asset prices. The counterparty facilitates the execution of trades, maintaining continuous hedging and mark-to-market processes to eliminate unwanted risk and neutralize the portfolio.

Traditionally, portfolio building relies on selecting stocks from a cloud of possible alternatives in the market. However, the cointegration-based portfolio literature suggests that portfolios of paired assets, subject to optimal investment weights, can maximize the expected utility of the portfolio value over a defined investment horizon. Existing studies, such as Liu and Timmermann (2013) and Lei and Xu (2015), provide insights into solving general portfolio problems with stochastic cointegrated assets. However, these studies have not yet established a common method to derive optimal weights.

Our thesis aims to contribute by proposing a framework in which a spread investor and its counterparty continuously exchange information to reach an equilibrium based on optimal portfolio weights defined in terms of forward-looking market parameters. To achieve this, we solve a dynamic maximization problem that maps the exchange of flows over a specified period. Specifically, we propose a framework in which the risk-averse investor sets the optimal weights equal to the deltas of a spread option, utilizing forward-looking option implied information. Our key idea revolves around the concept of asset convergence to equilibrium levels with individual mean reversion parameters. For this purpose, we draw inspiration from the pairs trading literature based on cointegration, as explored by Gatev et al. (2006), Andrade et al. (2005), Broussard and Vaihekoski (2012), Bowen and Hutchinson (2016), and others. These studies have demonstrated strong and consistent outperformance of pairs trading strategies applied to equity data in the US and international markets.

In our empirical analysis, we test our proposed framework using daily prices of DJIA constituents over a specific period. We compare the performance of our model-implied pairs portfolio against the benchmark Johansen portfolio, commonly used in pairs trading literature. The results highlight the significant outperformance of our model-implied pairs portfolio, attributable to the introduction of novel algorithms for pairs selection, optimal weights derived from spread option deltas, and a forward-looking trigger based on option sensitivities.

Overall, our findings support the notion that our adapted trading strategy based on option-based pairs trading can generate impressive returns and robust risk-adjusted performance, outperforming traditional benchmark strategies. By incorporating forward-looking option prices and integrating aggregate information on future expectations, our framework allows for improved performance and a superior ability to predict exercise opportunities compared to historical-volatility measures. In conclusion, this thesis aims to contribute to the existing literature by implementing and testing an adapted trading strategy based on a research paper by our professor. Through the use of Python, we will investigate the effectiveness of a pairs trading strategy using call and put options on the DJIA index, following the principles outlined in the original paper. The results of our empirical analysis will provide insights into the performance and potential profitability of this strategy in a different market context.

2. Theoretical framework

In this section, we provide an enhanced overview of the theoretical academic groundwork essential for comprehending pairs trading, which serves as the foundation for the trading strategy implemented in this thesis. These concepts have undergone extensive study in the field of quantitative finance and offer valuable insights into the interrelationships among financial assets.

Before delving into pairs trading, it is crucial to understand the rationale behind the development of trading strategies. Traditionally, investors made allocation decisions based on available information about individual companies or relying on their intuition. In such cases, the construction of a systematic framework for trading strategies was unnecessary, as decisions were based on periodic financial reports issued by listed companies. However, the notion of implementing a structured framework to gain an advantage over the market stems from the recognition that market participants cannot perfectly predict market behavior solely based on fundamental information of companies, leading to exploitable inefficiencies. The Efficient Market Hypothesis (EMH), proposed by Fama (1970), suggests that financial markets are efficient, implying that prices fully reflect all available information. According to this hypothesis, it is deemed impossible to consistently outperform the market by employing trading strategies, as any available information is already incorporated into the prices. Nevertheless, empirical studies have revealed deviations from the EMH, indicating the presence of market anomalies and opportunities for profit (Latif et al., 2011).

An alternative perspective, Behavioral Finance, incorporates psychological and cognitive factors into financial decision-making processes (Barberis & Thaler, 2003). It acknowledges that market participants may not always make rational decisions and can be influenced by biases and emotions, which in turn can lead to predictable patterns in asset prices. By understanding and exploiting these behavioral biases, trading strategies can be designed to generate excess returns (Shefrin, 2002). Aligning with Chan (2019), the primary objective of constructing a trading strategy is to generate profits by capitalizing on market inefficiencies. Market inefficiencies arise when the market price of an asset deviates from its intrinsic value, which represents the true worth of the asset based on its fundamental characteristics such as earnings, dividends, and growth prospects. Trading strategies aim to identify and exploit these inefficiencies to generate profits, and this study seeks to achieve precisely that.

Moreover, it is crucial to understand why one would want to code trading strategies in a programming language like Python. While simple trading strategies have been successfully executed by humans since the beginnings of financial markets, modern strategies, including those examined in this study, rely on complex calculations and thorough historical testing to assess their effectiveness. The increasing complexity of novel trading strategies necessitates a programming language that is both comprehensible and reliable, allowing investors to delegate calculations to a computer. When dealing with financial transactions, the reliability of the system is paramount. As emphasized by Géron (2013), coding trading strategies in Python offers numerous advantages, including the ability to backtest strategies using historical data, automate trading processes, and access a vast array of libraries and tools specifically designed for quantitative finance. This, along with the ease of understanding, is what make Python the programming language selected to implement our pairs strategy.

2.1. Pairs trading

Pairs trading is a market-neutral trading strategy that aims to exploit temporary anomalies between closely related securities. The strategy is based on the premise that there exists a long-run equilibrium between the prices of the stocks composing the pair, as articulated by Vidyamurthy (2004). The origins of pairs trading can be traced back to the 1980s at Morgan Stanley, where it was introduced and utilized by a group of quantitative analysts, known as 'quants,' led by Nunzio Tartaglia. Tartaglia, a Jesuit priest turned financier, assembled a team comprising physicists, mathematicians, and computer scientists to develop mathematical models capable of identifying temporary price deviations between pairs of stocks. Subsequently, the team executed trades that sought to profit from the expectation that the prices would ultimately converge. Underpinning their strategy was the principle of mean reversion, which posits that a stock's price tends to move toward its average price over time. Despite initial skepticism within the industry, the success of this strategy led to its widespread adoption by many quantitative trading firms.

The 1990s and early 2000s witnessed significant advancements in computational technologies and the availability of data, enabling the further refinement and expansion of pairs trading strategies. During this period, more sophisticated mathematical models and algorithms were developed to identify potential pairs and execute trades with greater efficiency. Furthermore, the strategy was extended to a wider range of financial instruments, including options, futures, currencies, and stocks.

However, despite its extensive use, pairs trading is not without challenges and criticisms. The effectiveness of the strategy heavily relies on the accurate identification of pairs and the assumption of mean reversion. Failure to meet these conditions can result in significant losses. Additionally, pairs trading has been criticized for potentially contributing to market volatility and its susceptibility to manipulative trading practices. Nevertheless, pairs trading continues to be a prominent strategy in the field of quantitative finance, underscoring the ongoing evolution and innovation in financial markets driven by technological advancements and the ever-growing availability of data.

2.1.1. Cointegration

In the realm of academic research, numerous studies have implemented pair trading strategies, leveraging the concept of cointegration, to identify profitable trading opportunities. Cointegration refers to a statistical relationship between two or more time series variables that exhibits a long-term equilibrium or common trend, despite short-term fluctuations or divergences. The concept of cointegration forms the basis for pairs trading strategies, as it suggests that assets that are cointegrated tend to converge to their equilibrium level over time. This implies that if a pair of stocks exhibits cointegration, deviations from their long-term equilibrium relationship present potential trading opportunities. When the prices of the two stocks temporarily diverge, pairs traders may take positions to profit from the expectation that the prices will eventually revert to their equilibrium.

The pioneering work of Engle and Granger (1987) laid the foundation for cointegration and its application in pairs trading. Their research highlighted the potential for capitalizing on the mean-reverting behavior of pairs of stocks that exhibit cointegration, thus creating profit opportunities. However, Engle and Granger's approach primarily focused on testing the presence of cointegration using the Augmented Dickey-Fuller (ADF) test (1979).

Building upon Engle and Granger's seminal work, Johansen (1991) introduced a more comprehensive methodology known as the Johansen procedure or the Johansen test. This procedure offers a robust framework for estimating the number of cointegrating relationships among a set of variables, making it a valuable tool for selecting cointegrated pairs in pairs trading strategies. The Johansen test is based on vector autoregressive (VAR) models, which capture the joint dynamics of multiple variables. By employing likelihood ratio tests, it assesses the presence of cointegration and provides estimates of the number of cointegrating vectors, also known as the rank of cointegration. The procedure includes the trace test and the maximum eigenvalue test, both of which provide statistical evidence regarding the number of cointegrating relationships in the data. We will delve deeper into the Johansen test as it serves as the benchmark for our strategy's own cointegrated pairs selector.

The practical implementation of the Johansen procedure involves several steps:

- Formulate a VAR model: Construct a VAR model using the selected variables (such as stock prices or returns) that are potentially cointegrated.
- Determine the lag length: Specify an appropriate lag length for the VAR model, considering factors such as information criteria (e.g., Akaike Information Criterion, Bayesian Information Criterion) or econometric judgment.
- Estimate the VAR model: Utilize estimation techniques such as ordinary least squares (OLS) or maximum likelihood to estimate the parameters of the VAR model.
- Perform the Johansen test: Conduct the Johansen test using the estimated VAR model to assess the presence and rank of cointegration. The test involves comparing likelihood ratios against critical values obtained from asymptotic distributions.
- Interpret the results: Analyze the results of the Johansen test to determine the number of cointegrating vectors. Each cointegrating vector represents a long-term equilibrium relationship among the variables.

By employing the Johansen procedure, traders and researchers can systematically identify and select cointegrated pairs of assets for pairs trading strategies. This approach provides a robust and statistically grounded method for identifying pairs with mean-reverting behavior and potential profitability. In our study, the Johansen test serves as an integral component of our framework for selecting cointegrated pairs, thereby enhancing the effectiveness of our pairs trading strategy.

Subsequently, notable studies such as Gatev, Goetzmann, and Rouwenhorst (2006) have extended the work of Johansen by exploring pairs trading in a broader context. Their research examined the effectiveness of the strategy across different asset classes, demonstrating that pairs trading remained viable not only for equity markets but also for other financial instruments such as commodities and currencies. By identifying cointegrated pairs within these diverse asset classes, the study revealed the potential for cross-market pairs trading strategies that exploit deviations from equilibrium relationships.

Furthermore, Avellaneda and Lee (2010) introduced a novel pairs trading approach using statistical arbitrage. Their strategy focused on selecting pairs based on cointegration and implementing a mean-reverting trading algorithm. The research showcased the effectiveness of this approach by generating consistent profits across a variety of financial markets. Additionally, with advancements in machine learning techniques, researchers have explored the integration of artificial intelligence in pairs trading strategies. For instance, Nóbrega and Oliveira (2013) employed a machine learning-based method to identify cointegrated pairs and predict future price movements. By incorporating advanced algorithms such as support vector regression and random forest, their strategy outperformed traditional pairs trading methods, highlighting the potential of machine learning in enhancing pair trading strategies.

In our specific strategy, we employ the Vector Error Correction Model (VECM) methodology. The VECM has been extensively explored in the academic literature, demonstrating its effectiveness in capturing mean-reverting behavior and potential profitability. Studies by Alexander and Dimitriu (2005), Cheung and Ng (1996), Engsted, Tanggaard, and Vinther (2009), and Gregoriou, Kontonikas, and Montagnoli (2009) have successfully employed the VECM to identify cointegrating pairs in various domains, ranging from pairs trading to exchange rates and inflation analysis. These studies have showcased the robustness and reliability of the VECM in capturing long-term equilibrium relationships among variables, providing a strong foundation for the implementation of our strategy.

By incorporating the VECM methodology, we leverage the insights and findings from these studies to identify cointegrated pairs with the potential for mean reversion in our implemented strategy. The VECM's ability to capture the dynamics of cointegration, along with its statistical framework, offers a reliable and well-established approach to selecting pairs that exhibit profitable trading opportunities. Drawing on the success of the VECM in the literature, we anticipate that our utilization of this methodology will enhance the effectiveness and profitability of our pairs trading strategy.

Furthermore, by building upon the existing body of knowledge and leveraging the insights from these studies, we contribute to the broader understanding and development of effective pairs trading techniques. Through a systematic and rigorous approach, we aim to validate the viability and robustness of our strategy and expand the empirical evidence base for the application of the VECM in pairs trading.

2.1.2. Budget allocation

The construction of an optimal portfolio is a critical aspect of pairs trading once the pairs have been selected. It entails determining the budget weights for each pair in the portfolio, as well as the weights for the simultaneous long and short positions within each pair of assets. The optimization of pair weights is a fundamental consideration as it directly influences the expected utility of the portfolio value over the investment horizon.

Numerous studies have proposed innovative methods to address the optimal weights problem in pairs trading strategies. Liu and Timmermann (2013) provided a theoretical justification for market-neutral pairs trading, highlighting the significance of considering the cointegration of stocks in determining optimal weights. They studied optimal investment in a market with two cointegrated stocks and an agent with CRRA utility, emphasizing the importance of incorporating cointegration into the portfolio construction process.

Mudchanatongsuk et al. (2008) employed a singular stochastic control approach to investigate the optimal pairs trading problem with proportional transaction costs. Their work emphasized the necessity of accounting for transaction costs when determining optimal weights. By incorporating these costs into the portfolio optimization problem, traders can obtain more realistic weight allocations and improve the overall effectiveness of their trading strategies.

In a recent study, Hoque et al. (2021) introduced a random weights innovation volatility forecasting (RWIVF) algorithm. This approach extended the Bollinger bands trading strategy and provided a data-driven method to obtain optimal weights based on past observed volatilities. By leveraging this algorithm, traders can dynamically adjust their weights to capture changing market conditions and optimize their portfolio performance.

Feng-Hui Yu et al. (2022) explored optimal pairs trading strategies in terms of both static and dynamic optimality under the mean-variance criterion. They addressed the optimal weights problem using a constrained optimal control problem and the Lagrange multiplier technique. Their approach considered the trade-off between risk and return, providing insights into determining optimal weights for pairs trading strategies.

Li and Tourin (2022) proposed a monotone finite difference scheme that approximates the viscosity solution of the Hamilton-Jacobi-Bellman equation in pairs trading with transaction costs. Their mathematical framework offered insights into optimal trading strategies and the impact of transaction costs on determining optimal weights. By considering the full dynamics of the trading process, their approach provided a comprehensive framework for portfolio construction.

Despite the various methods proposed to address the optimal weights problem in pairs trading strategies, a universally accepted approach that consistently delivers optimal results remains elusive. The ongoing research in this field reflects the continuous efforts to enhance portfolio construction and improve the performance of pairs trading strategies.

To address this literature gap, our implemented strategy aims to contribute by presenting a novel framework that establishes an equilibrium between a spread investor and its counterparty through continuous information exchange. Our approach is based on the determination of optimal portfolio weights using forward-looking market parameters and incorporates forward-looking option implied information to outperform historical benchmarks. By solving the expected utility maximization problem for long-short strategies over the time span (t, T), we demonstrate that risk-averse investors will set the optimal weights equal to the deltas of a spread option.

Our innovative approach takes into account the dynamic nature of the market and leverages forward-looking information to achieve more accurate and effective portfolio construction in pairs trading strategies. By incorporating option implied information, we can capture market expectations and incorporate them into the determination of optimal weights. This enables our strategy to adapt to changing market conditions and more effectively exploit potential profit opportunities.

The process of determining optimal weights involves solving a dynamic maximization problem that maps the exchange of flows within a specified period. Through continuous information exchange and the consideration of forward-looking market parameters, our objective is to achieve the highest expected utility of the portfolio value. By setting the optimal weights equal to the deltas of a spread option, our strategy aligns with the risk preferences of the investor, striking a balance between risk and potential returns.

By incorporating this framework into our pairs trading strategy, we anticipate improved portfolio performance and enhanced risk management. The utilization of forward-looking option implied information provides us with a more accurate assessment of market expectations, allowing for more effective portfolio positioning. Through the dynamic optimization of weights, we aim to capture profit opportunities while minimizing risk exposure.

It is noteworthy that our proposed framework represents a departure from traditional approaches in pairs trading strategies. By incorporating forward-looking option implied information and solving the dynamic maximization problem, our strategy aims to advance the field and contribute to the existing body of knowledge. We believe that this innovative approach holds significant potential for improving the performance of pairs trading strategies and generating consistent profits in dynamic market environments. In the subsequent sections, we will present empirical results and evaluations to validate the effectiveness of our approach and its impact on portfolio performance in pairs trading.

2.2. Strategy definition

After conducting an extensive review of the relevant literature on pairs trading strategies, we now proceed to define the key elements of our approach, taking into account the insights gained from the existing body of knowledge. To establish a robust and comprehensive pairs trading strategy, it is essential to specify several components, including pairs selection, position sizing, entry and exit criteria (triggers), and risk management. These elements play a crucial role in ensuring a systematic and disciplined execution of the strategy. In this section, we will examine the existing literature pertaining to each element and introduce the specific approach adopted in our implemented strategy. Additionally, we will present additional concepts that are vital for a comprehensive understanding of the rationale behind our enhanced strategy.

At a high level, our approach involves the simultaneous purchase and sale of two cointegrated assets to exploit the mean reversion of the price spread between them. Mean reversion refers to the tendency of a value that deviates significantly from its historical mean to eventually revert back to it over the long term. We will apply this strategy to multiple pairs of assets from the same index simultaneously, allocating an equal investment to each pair. To facilitate this, we will describe a model within a continuous-time environment, aiming to capture the fundamental dynamics of the market and the underlying factors influencing asset prices, in order to capitalize on potential trading opportunities.

The VECM model utilized in our strategy builds upon the work of Figuerola-Ferretti et al. (2018), who extensively explored this framework in a previous study. In our proposed model, we consider a continuous time setting where trading occurs without any transaction costs or market frictions. Within this framework, our pair portfolio consists of three securities: a risk-free cash account and two risky assets. Here is the portfolio for a single pair:

$$\Pi_t = \varphi_1 y_t + \varphi_2 x_t + \varphi_3 B_t$$

The cash account, denoted as “Bt”, offers a constant return rate “r”, which will be denoted by the risk free rate. The two risky assets represented by "yt" and "xt," are also tradeable in the market. The dynamics of their prices involve various factors, including potential data dependencies and an error correction mechanism. They can be defined under a dynamic VECM in continuous time with a stationary price spread under the following equations:

$$\frac{dy_t}{y_t} = \mu_y dt - \lambda_1 z_t dt + \sigma_y dW_{y,t}$$

$$\frac{dx_t}{x_t} = \mu_x dt + \lambda_2 z_t dt + \sigma_x dW_{x,t}$$

$$z_t = lny_t - lnx_t$$

In these equations, "μy," "μx," "λ1," "λ2," "σy," and "σx" denote constant parameters. The terms "Wy,t" and "Wx,t" represent standard Brownian motions with zero drift rate and unit variance rate, capturing the inherent stochastic nature of the assets' price movements. The variable "zt" corresponds to the price spread between the logarithm of the two asset prices, indicating temporary mispricing. The parameters "λ1" and "λ2" determine the speed at which the prices of "yt" and "xt" revert to their equilibrium levels, respectively. By incorporating the error correction terms "-λ1zt" and "λ2zt" within the continuous Vector Error Correction Model (VECM) framework, we account for the forces that drive asset prices back towards their long-term equilibrium. The presence of non-zero "-λ1zt" in the equation suggests that "yt" is overpriced, and its price is expected to decrease to restore equilibrium. Conversely, if "λ2zt" is positive, it implies that "xt" is underpriced, and its price is expected to increase. This mean reversion process presents an opportunity for pairs trading, where profits can be made by exploiting the mispricings arising from limits to arbitrage and market uncertainty.

Once the VECM model is defined, the next step involves estimating its parameters. The estimation process entails gathering a dataset that includes the relevant variables, such as the stock prices for all potential pairs. Maximum Likelihood Estimation (MLE) is employed to estimate the parameters of the VECM by maximizing the likelihood function. This involves identifying the parameter set that maximizes the probability of observing the given data under the VECM. The estimated VECM parameters capture the long-term equilibrium relationship and the short-term dynamics between the variables. They serve as the foundation for subsequent analysis and implementation of pairs trading strategies based on the VECM framework. The detailed mathematical procedures for parameter estimation under MLE are fully described in the original paper of the strategy. In our implementation, we utilize the resulting equations to estimate each parameter:

$$\hat{\mu_y} = \frac{1}{n}\sum_{i=1}^{n}\left(\frac{Y(t_i) - Y(t_{i-1})}{\Delta t} + \hat{\lambda_1}(Y(t_{i-1}) - X(t_{i-1})) + \frac{1}{2}\hat{\sigma}_y^2\right)$$

$$\hat{\mu_x} = \frac{1}{n}\sum_{i=1}^{n}\left(\frac{X(t_i) - X(t_{i-1})}{\Delta t} - \hat{\lambda_2}(Y(t_{i-1}) - X(t_{i-1})) + \frac{1}{2}\hat{\sigma}_x^2\right)$$

$$\hat{\lambda_1} = \frac{2\sum_{i=1}^{n}\left(A(t_i) - \frac{1}{n}\sum_{i=1}^{n}A(t_i)\right)\left(z(t_{i-1}) - \frac{1}{n}\sum_{i=1}^{n}z(t_{i-1})\right)}{\Delta t \cdot \sum_{i=1}^{n}\left(z(t_{i-1}) - \frac{1}{n}\sum_{i=1}^{n}z(t_{i-1})\right)^2}$$

$$\hat{\lambda_2} = \frac{2\sum_{i=1}^{n}\left(B(t_i) - \frac{1}{n}\sum_{i=1}^{n}B(t_i)\right)\left(z(t_{i-1}) - \frac{1}{n}\sum_{i=1}^{n}z(t_{i-1})\right)}{\Delta t \cdot \sum_{i=1}^{n}\left(z(t_{i-1}) - \frac{1}{n}\sum_{i=1}^{n}z(t_{i-1})\right)^2}$$

$$\hat{\sigma}_y = \sqrt{\frac{1}{n\Delta t} \cdot \sum_{i=1}^{n}\left(\left(A(t_i) - \frac{1}{n}\sum_{i=1}^{n}A(t_i)\right) - \hat{\lambda_1}\Delta t\left(z(t_{i-1}) - \frac{1}{n}\sum_{i=1}^{n}z(t_{i-1})\right)\right)^2}$$

$$\hat{\sigma}_x = \sqrt{\frac{1}{n\Delta t} \cdot \sum_{i=1}^{n} \left(\left(B(t_i) - \frac{1}{n} \sum_{i=1}^{n} B(t_i) \right) - \hat{\lambda}_2 \Delta t \left(z(t_{i-1}) - \frac{1}{n} \sum_{i=1}^{n} z(t_{i-1}) \right) \right)^2}$$

$$Z_y = \frac{Y(t_i) - Y(t_{i-1}) - \left(\mu_y - \lambda_1 (Y(t_{i-1}) - X(t_{i-1})) - \frac{1}{2}\sigma_y^2\right) \Delta t}{\sigma_y \sqrt{\Delta t}} \sim N(0,1)$$

$$Z_x = \frac{X(t_i) - X(t_{i-1}) - \left(\mu_x + \lambda_2 (Y(t_{i-1}) - X(t_{i-1})) - \frac{1}{2}\sigma_x^2\right) \Delta t}{\sigma_x \sqrt{\Delta t}} \sim N(0,1)$$

$$\hat{\rho_{xy}} = \frac{1}{n} \sum_{i=1}^{n} (Z_y Z_x)$$

where $A(t_i) = Y(t_i) - Y(t_{i-1}), \quad B(t_i) = X(t_i) - X(t_{i-1}), \quad z(t_{i-1}) = Y(t_{i-1}) - X(t_{i-1}).$

To effectively implement our pairs trading strategy, it is crucial to estimate the Vector Error Correction Model (VECM) parameters for each possible pair within the assets of the market index. This estimation process involves gathering historical close price data for each pair, transforming them into their logarithms (represented by capital letters X and Y), and applying Maximum Likelihood Estimation (MLE) using the formulas described above to determine the parameter values. By estimating the parameters for each pair, we gain insights into the specific dynamics and equilibrium relationships between the assets. This allows us to identify potential mispricings and exploit profitable trading opportunities. The estimation of these parameters serves as the foundation for constructing a well-informed and robust pairs trading strategy that can be implemented across various pairs within the market index.

2.2.1. Cointegrated pairs selection

Once the parameters of the Vector Error Correction Model (VECM) have been estimated for each possible pair of assets, it is necessary to identify the pairs that exhibit a long-term equilibrium or common trend while allowing for short-term fluctuations or divergences. The selection of appropriate pairs for our trading strategy is contingent upon the data meeting the requirements of the VECM, which include stationarity and cointegration among the variables. Stationarity ensures the stability of the error term, while cointegration indicates a long-term equilibrium relationship that can be exploited for trading opportunities. Therefore, it is crucial to consider both the cointegration and stability conditions.

In our model framework, the stability condition is defined by the sum of the mean-reverting speeds represented by $\lambda 1$ and $\lambda 2$, which must be greater than zero. This condition, consistent with the findings of Lei and Xu (2015), ensures the stationarity of the error term. Cointegration, on the other hand, requires that at least one of the λ coefficients is non-zero. Furthermore, if both coefficients are non-zero, they must exhibit opposite signs and different values in the mean-reverting speed. These conditions serve as a guideline for our empirical exercise and provide a one-step procedure for selecting cointegrated pairs. Therefore for each possible pair both of these conditions must be checked to be determined cointegrated and well modeled under our VECM.

To further validate the cointegration properties of the selected pairs, we employ the Johansen (1991) three-step procedure as a benchmark. Although our model is less stringent than the Johansen test, it is essential to ensure that our selected pairs encompass those identified as cointegrated by Johansen's test. This additional assessment provides a robust validation of the cointegration properties of the pairs, strengthening the foundation of our pairs trading strategy and enhancing its potential for generating profitable trading opportunities. By adhering to these selection criteria and validation procedures, we ensure the reliability and effectiveness of our pairs trading strategy in capturing cointegrated pairs within the selected market index.

2.2.2. Optimal position sizing

To determine the ideal position sizes within each pair of assets, we leverage the hedging strategy employed by the counterparty. The counterparty utilizes spread option contracts to hedge the risk associated with their short position in our portfolio. This idea is consistent with the idea of skilled traders operating in the options markets, as highlighted by DeMiguel et al. (2009). By decomposing the spread options into simpler Black-Scholes option calls and puts on the individual stocks involved in the spread, which was proven possible by Schroder (1999), the counterparty achieves an effective replication of the error correction strategy employed by the spread investor.

Within this hedging framework, the counterparty follows a delta hedging strategy, which involves adjusting their positions in the underlying assets based on the delta values of the options. Delta measures the sensitivity of the option value to changes in the underlying asset price. By holding a long position in the underlying asset (represented by Δy) and a short position in the other underlying asset (represented by Δx), the counterparty aims to achieve an optimal hedge for their portfolio.

The delta values obtained from the Black-Scholes model guide the counterparty in determining the appropriate position sizes. These values reflect the proportionate changes in the option value relative to changes in the underlying asset price. The counterparty adjusts their positions based on the relative proportions of the assets in the spread, ensuring a balanced and effective hedge.

Through this hedging strategy, the investor gains insight into the ideal position sizes within each pair of assets. The delta values serve as a guide, indicating the optimal allocation of capital between the long and short positions. By aligning the position sizes with the delta values, the investor can effectively manage risk and maximize the potential for profit. Hence, the optimal size of the investment in each asset pair can be expressed as follows:

$$\begin{aligned} V(t, \Pi, y, x) &= y_t \Phi(d_1) - x_t \Phi(d_2) \\ &= \Delta_{y,t} \cdot y_t - \Delta_{x,t} \cdot x_t \end{aligned}$$

An important aspect of this approach is that the deltas used as position sizes are calculated using the Black-Scholes formulas and implied volatilities of the ATM options, rather than relying solely on historical data. An ATM (at the money) option refers to an option whose strike price is the same as the underlying asset value. This departure from traditional methods, which utilize in-sample and historical observations, stems from recent research (e.g., Bailey and Lopez de Prado, 2014; Harvey and Liu, 2014) highlighting the pitfalls of calibration based on back-testing, which often leads to overfitting and underperformance of the portfolio. Therefore, we employ implied volatilities, as they contain forward-looking information instead of relying solely on historical data.

In summary, the hedging strategy, involving the use of spread options and delta hedging, empowers the investor to make informed decisions regarding the ideal position sizes within each pair of assets. By following the delta values derived from the Black-Scholes model and utilizing ATM implied volatilities, the investor can achieve a well-balanced and risk-managed portfolio, effectively optimizing their trading strategy for the selected pairs.

2.2.3. Forward looking trigger

In this section, we introduce the trading trigger mechanism that underlies our strategy, which is designed to capture maximum spread levels. The trigger acts as a signal for initiating trades when the price difference between the two assets in a pair reaches its highest point. The trigger point is determined by analyzing the relationship between the gammas of the options associated with the assets, with the gammas calculated using ATM implied volatilities derived from the Black-Scholes formulas, similar to the calculation of deltas. The implementation of the trigger involves several steps, including the calculation of gammas from implied volatilities, comparison of the gamma values, and identification of the trigger point based on the ratio of asset prices.

In this strategy two trigger conditions are used, so we will start explaining the entry trigger condition, where the used trigger is the convergence of the pairs' speeds. The use of convergence speeds as the opening trigger in the pairs trading strategy is based on the underlying principle of mean reversion. Mean reversion suggests that, over time, the prices of two cointegrated assets tend to move back towards their average relationship when they deviate from it.

The convergence speeds, lambda_1 and lambda_2, represent the estimated rates at which the pair of stocks converge towards their average relationship. These speeds capture the tendency of the prices to revert to their mean and indicate how quickly this reversion occurs. By subtracting these convergence speeds from one, we obtain a measure of the remaining deviation from the mean relationship.

To determine whether a position should be opened, this remaining deviation is multiplied by the volatility of the spread at each point in time. The spread volatility represents the level of fluctuation or uncertainty in the spread of the pair's prices. By considering both the remaining deviation and the spread volatility, we can assess the potential profitability and risk associated with opening a position.

Comparing this result with the logarithmic spread of the pair allows us to determine if the spread exceeds the threshold defined by the entry trigger. If the spread is larger than the entry trigger, it indicates that the pair's prices have deviated sufficiently from their mean relationship, presenting a potential trading opportunity. This trigger serves as a signal to initiate a position in the pair, expecting that the prices will eventually revert back towards their mean relationship, resulting in a profitable trade.

The use of convergence speeds as the opening trigger is grounded in the concept of exploiting short-term price inefficiencies and capturing profit opportunities when the pair's prices exhibit deviations from their mean relationship. By incorporating these speeds into the trigger calculation, the strategy is able to identify favorable entry points based on the estimated rates of convergence. This approach allows the strategy to take advantage of mean reversion dynamics and potentially generate profitable trades.

To define the closing trigger point, we rely on the determinant of the Jacobi matrix, denoted as |J|. This determinant serves as a measure of second-order sufficiency conditions for maximizing the expected utility of the portfolio. Specifically, we require the determinant of the Jacobi matrix to be positive semi-definite, ensuring that |J| > 0. The Jacobi matrix encompasses four elements, which represent the second derivatives of the portfolio value with respect to the underlying assets. The trigger condition is met when the determinant of the Jacobi matrix is positive, indicating |J| > 0. Mathematically, the determinant of the Jacobi matrix is expressed as:

$$\det J = \begin{vmatrix} V_{\Pi_y \Pi_y} & V_{\Pi_y y} \\ V_{\Pi_y x} & V_{\Pi_x \Pi_x} \end{vmatrix} = \begin{vmatrix} \Gamma_y - \Gamma_x & \Gamma_y - \frac{x_t}{y_t}\Gamma_x \\ \Gamma_x - \frac{y_t}{x_t}\Gamma_y & \Gamma_x - \Gamma_y \end{vmatrix} = \frac{\Gamma_x^2}{\Gamma_y^2} - \frac{y_t}{x_t} > 0$$

This condition guarantees that the trading trigger is activated at the maximum spread level. In other words, the trigger point is reached when the logarithmic ratio of asset prices, "Yt / Xt," is equal to the squared ratio of the gammas, "(Γx)^2 / (Γy)^2." This optimal spread level serves as an indication for closing a trade in our strategy.

To implement the trigger in practical terms, we rely on implied volatilities, which reflect market-derived expectations of future price fluctuations. This forward-looking nature of implied volatilities is essential for our trigger mechanism. The gammas derived from these implied volatilities provide insights into the sensitivity of option values to changes in the underlying asset prices. After calculating the gammas, we compare their values to determine the trigger point. If the gamma of the long asset, denoted as Γy, is smaller than the gamma of the short asset, denoted as Γx, it suggests that the price movement of the long asset is relatively more significant than that of the short asset. This observation signals a favorable trading opportunity.

By utilizing this trigger mechanism, we can effectively identify optimal spread levels and initiate trades precisely when the spread reaches its maximum point. This approach empowers us to make informed trading decisions based on forward-looking information derived from market-implied volatilities and the interplay between the gammas of the associated options.

2.2.4. Risk management

Effective risk management is essential to protect capital and minimize potential losses in pairs trading. While stop-loss orders and maximum loss thresholds were mentioned earlier as common risk management techniques, additional approaches have been proposed in the literature.

Our pairs trading strategy is designed as a market-neutral strategy, aiming to generate returns regardless of the overall direction of the market. Market-neutral strategies, as defined by Alexander and Dimitriu (2002), involve establishing a portfolio that is insulated from systematic risk factors, such as broad market movements, by taking offsetting positions in correlated assets. By exploiting relative mispricing between the paired assets, our strategy aims to profit from the convergence of their prices while minimizing exposure to systematic risk.

To evaluate the performance of our pairs trading strategy, we employ several metrics that provide insights into its risk and return characteristics, while taking into account the opportunity cost associated with alternative investments. These metrics include annual returns, annual excess returns, and the Sharpe ratio.

Annual returns measure the percentage change in the value of the portfolio over a one-year period, considering the gains or losses generated. They reflect the overall performance of the strategy in terms of its ability to generate profits.

Annual excess returns capture the returns earned by the strategy in excess of a benchmark or risk-free rate, accounting for the opportunity cost of alternative investments. To assess the strategy's ability to outperform a passive investment alternative, we compare the returns to the risk-free rate. In our case, we use the 6-month US Treasury bill rate as the risk-free rate since our trading period is 6 months and we operate in US indexes. This allows us to account for the return that could have been earned by investing in a risk-free instrument over the same period.

The Sharpe ratio, introduced by Sharpe (1998), is a widely used risk-adjusted performance measure. It calculates the ratio of the excess returns of the strategy to its volatility or standard deviation, taking into consideration the opportunity cost of alternative risk-free investments. The Sharpe ratio provides an indication of the risk-adjusted returns generated by the strategy, considering both the average return and the risk involved.

By considering these metrics, we can assess the risk management and performance of our pairs trading strategy, while taking into account the opportunity cost associated with alternative investments. The market-neutral nature of the strategy aims to reduce exposure to systematic risk factors, focusing instead on capturing relative mispricing opportunities. The annual returns, annual excess returns, and Sharpe ratio serve as quantitative measures to evaluate the strategy's success in achieving its objectives.

3. Model implementation

In this section, we present the implementation details of our pairs trading strategy, which follows a systematic methodology encompassing two distinct timeframes: pair formation and trading periods. The pair formation period spans a duration of 3 years, during which we diligently identify and establish pairs based on predetermined criteria. Subsequently, the trading period commences, lasting for 6 months, during which we execute trades based on the pairs formed in the previous stage.

To ensure a comprehensive evaluation of our strategy, we have adopted a moving window approach spanning from January 1st, 2003 to the present day. This time frame allows us to generate a total of 34 samples for assessing the viability and effectiveness of our trading approach. The selection of a 3-year pair formation period and a 6-month trading period was initially chosen based on practical considerations. These time horizons have remained consistent throughout our research, enabling a standardized evaluation of the strategy's performance across multiple iterations.

By maintaining consistent time horizons and utilizing a moving window approach, our implementation approach facilitates robust comparisons and assessments of the pairs trading strategy. This methodology ensures that our findings are not overly influenced by specific time periods or market conditions, enhancing the reliability and generalizability of our results.

3.1. Data origin and retrieval

In the implementation of our pairs trading strategy, the first step involves retrieving the necessary data. As described in the strategy definition section, the required data consist of the spot prices of stocks within the Dow Jones Industrial Average, implied volatilities of their corresponding ATM options, and the daily 6-month treasury bill market rate. The collection of spot price data commenced on January 1st, 2003, marking the onset of the pair formation period. Implied volatilities and the treasury bill market rate data were obtained starting from January 1st, 2006, which corresponds to the first trading day.

To ensure the reliability and precision of the data, utmost care was taken in selecting appropriate data sources. Accurate spot prices are paramount, as even slight deviations can significantly influence the interpretation of trading strategy results. To access the relevant financial data, we utilized the FactSet API—an esteemed and trusted database renowned for providing comprehensive and reliable financial information. The FactSet API facilitated seamless querying of the constituent companies within the two US indexes and automated data retrieval.

For the sake of convenience and efficient data management, we opted to store the retrieved data in a parquet file format. Parquet offers advantages in terms of storage efficiency and query performance compared to traditional text-based file formats such as CSV or Excel. Its optimized binary format enhances data processing speed and facilitates seamless data manipulation. This proved particularly useful when working with the DOW JONES INDUSTRIAL AVERAGE, as retrieving data from 500 companies would have resulted in a text file that would be too large for any computer to process effectively.

The 6-month treasury bill market rate data was directly obtained from the website of the US Federal Reserve System, ensuring the accuracy and up-to-datedness of the information. By leveraging reliable data sources and harnessing the convenience of the parquet file format, we established a robust foundation for implementing our pairs trading strategy. The retrieved data serves as the fundamental basis for subsequent analysis is and evaluation of our trading approach.

The result of the data retrieval were 2 parquet files with the spot prices and implied volatilities data of the companies for each index, and a csv file with the daily rates for the specified dates.

3.2. Pairs selection

In the implementation of our pairs trading strategy, the selection of suitable pairs is a crucial step that involves identifying stock pairs exhibiting cointegration, indicating a potential long-term relationship between their prices. To accomplish this, we employ two distinct methods: Maximum Likelihood Estimation (MLE) and the Johansen test, which serves as a benchmark for comparison.

To initiate the pairs selection process, we retrieve the necessary data from the parquet file, covering the time period from 2003 to 2020, corresponding to the pair formation periods. To ensure the integrity of our analysis, we meticulously filter the data, excluding any missing values that could compromise the reliability and accuracy of our methodology. It is important to note that the utilization of the FactSet API introduces a limitation, as it only provides data for the current components of the index. Consequently, certain companies in our dataset may lack historical price data during the initial years due to their later establishment. To address this limitation, these companies are excluded from the analysis, which introduces the potential for survivorship bias.

Survivorship bias arises from the exclusion of companies that have failed to survive, leading to a skewed representation of the dataset. This bias can influence the evaluation of the pairs trading strategy by overestimating its effectiveness, as underperforming or unsuccessful pairs are omitted from the analysis. Therefore, it is imperative to interpret the results with caution, considering the inherent limitations associated with survivorship bias.

Once the data loading and survivorship bias considerations have been addressed, we proceed to the pairs selection phase by employing the MLE method and the Johansen test. Systematic and efficient implementation is ensured through a structured coding approach for both methods.

For the MLE method, we develop a function that estimates the necessary parameters to assess the integration of a pair of stocks. This function calculates and analyzes the differences and spread between the prices of the selected pair, enabling the estimation of mean returns, price sensitivities, and volatilities. By implementing this function using a rolling window approach, we assess the integration of each pair over time.

Similarly, for the Johansen test, a predefined function from the *statsmodels* library is utilized to perform the test on each pair of stocks. This function evaluates the presence of cointegration by examining the rank of a matrix and comparing it with critical values. The Johansen test is applied within a rolling window framework to assess the cointegration of each pair over different time periods.

To enhance computational efficiency, a critical consideration when analyzing large datasets, we incorporate parallel processing techniques. Specifically, we develop a function for each method that parallelizes the calculation of cointegration for each pair, utilizing multiple CPU cores. This parallelization significantly speeds up the computation time, enabling us to process a large number of pairs more efficiently.

By employing both the MLE method and the Johansen test, we obtain two sets of pairs that exhibit potential cointegration. These pairs form the foundation of our pairs trading strategy, representing stocks with long-term relationships and potential profit opportunities. The use of complementary methods allows us to evaluate and compare their effectiveness in identifying suitable trading pairs. The pairs selected using each method will be explored further in the results and discussion section.

3.3. Portfolio construction calculations

The portfolio construction calculations represent a pivotal stage in evaluating the performance of our pairs trading strategy. This phase encompasses the computation of various parameters that shed light on the behavior of the selected assets during the trading period. Additionally, it involves the derivation of key indicators essential for the strategy, including opening and closing triggers and position sizes. To facilitate these calculations, we implement the function "process_pair_oos" in a parallelized manner, as has been consistently employed throughout this study. This function operates on six-month batches of cointegrated pairs' stock prices, along with their corresponding implied volatilities and interest rates, enabling the iteration over the rows to extract the pertinent data for each pair.

In the calculation process, we first determine the log spread of the pair, a vital metric used to assess the relationship between the two stocks. Subsequently, we compute the volatilities for the stock spread by utilizing the implied volatilities and the correlation coefficient obtained during the pairs selection phase. The determination of appropriate position sizes is achieved through the calculation of deltas for each stock in the pair. These deltas represent the sensitivity of the option price to changes in the underlying stock price. They are derived from the cumulative distribution function (CDF) of the standardized log spread. Moreover, we compute the gammas, which provide insights into the curvature of the option price concerning variations in the underlying stock price. The gammas are obtained using the probability density function (PDF) of the standardized log spread.

The most critical aspect of the portfolio construction calculations revolves around the trigger calculation. In this context, the entry trigger is computed by subtracting the estimated convergence speeds, represented by lambda_1 and lambda_2, from one. This result is then multiplied by the volatility of the spread at each point in time and compared with the logarithmic spread of the pair. A position is opened when the spread exceeds this threshold. Conversely, the closing trigger is derived from the Jacobian matrix using the lambdas and is determined by the moment when the expression (gamma_x^2 / gamma_y^2) - (price_y / price_x) equals zero. In practice, handling this condition becomes slightly intricate due to working with daily data, which can lead to zeros between consecutive days. To address this issue, we examine the sign change of the aforementioned equation and consider a zero to be present at that point. As previously stated, the closing trigger indicates the attainment of the maximum spread level, signifying that no further profits can be derived from maintaining a position on that particular pair. If none of these conditions are met, a NaN value is assigned to the trigger for that day, signifying that no changes to the position are required.

To ensure the efficient execution of portfolio construction, we incorporate parallel processing techniques into our calculations. This approach facilitates the simultaneous computation of cointegration parameters and key indicators for multiple pairs, leveraging the computational power offered by multiple CPU cores. By employing parallelization, we substantially reduce processing time and enhance the efficiency of analyzing a large number of pairs.

Furthermore, the code encompasses a crucial component that involves the computation of position sizes and returns based on the deltas and triggers. This component plays a vital role in determining the optimal allocation of capital and evaluating the performance of the trading strategy. To facilitate the determination of position sizes, we define the function "position_sizes," which accepts the deltas and triggers as inputs and computes the position sizes for the two stocks in the pair. Positions are opened when the trigger changes to 1 and closed when the trigger becomes 0. If the trigger value is NaN, the position is maintained without any changes. The calculation of position sizes enables us to assess the allocation of capital and monitor the state of the positions held during the trading period.

3.4. Returns

Once the portfolio has been constructed and the position sizes determined, the next step is to calculate the returns of the pairs trading strategy and analyze its performance. This involves assessing the returns generated over time and evaluating the risk-adjusted performance of the strategy.

To calculate the returns, the position sizes determined during the portfolio construction phase are applied to the price data. The returns are computed by taking the differences between consecutive prices and normalizing them by the product of the previous prices and the corresponding position sizes. This calculation effectively captures the changes in stock prices and reflects the impact of the portfolio positions. By aggregating the returns of both stocks in each pair, an overall measure of the strategy's performance on a given trading day is obtained.

To evaluate the performance of the pairs trading strategy, the dataset is divided into trading batches, with each batch covering a 6-month period. Within each batch, the returns are calculated using a function that considers the differences in returns between consecutive dates. The returns for each date are then aggregated to capture the cumulative returns and determine the count of pairs contributing to those returns. This information is stored and tracked, enabling further analysis of the performance.

To facilitate a comprehensive analysis, the calculated returns, average daily returns, and other relevant performance metrics are consolidated into a combined dataframe. This consolidated dataframe serves as a foundation for assessing the strategy's performance across different time periods.

The annual performance of the strategy is evaluated by grouping the data in the combined dataframe by year. The annual returns are computed by taking the product of the cumulative returns for each year and subtracting 1. This approach enables an assessment of the strategy's performance over longer time horizons and provides insights into its consistency and stability.

In addition to the annual returns, the excess returns are calculated by adjusting the average returns for the risk-free rate. This adjustment provides a measure of the strategy's performance relative to a baseline return and accounts for the opportunity cost of holding risk-free assets. The annualized excess returns, along with the corresponding Sharpe ratios, are computed to gauge the risk-adjusted performance of the strategy, taking into account its volatility and the risk-free rate of return.

Further insights into the strategy's characteristics are obtained through the calculation of additional statistics. The skewness of the daily excess returns is computed to assess the symmetry or skewness of the distribution. This analysis helps identify potential deviations from a normal distribution and provides insights into the strategy's risk profile. Additionally, the minimum and maximum daily returns are determined to understand the range of return fluctuations and the potential downside and upside risks associated with the strategy.

Visualizations play a crucial role in conveying the performance of the pairs trading strategy. Cumulative returns over time are plotted to illustrate the growth trajectory of the portfolio throughout the trading period. These plots provide a visual representation of the strategy's ability to generate consistent returns. Furthermore, cumulative plots of excess returns adjusted for the risk-free rate offer insights into the strategy's capacity to outperform risk-free investments.

To analyze the performance on a shorter time scale and capture more granular trends, the data can be grouped into custom periods, such as 6-month intervals. This grouping allows for an evaluation of the strategy's performance within specific timeframes, offering a deeper understanding of its dynamics and potential variations across different market conditions. Semiannual returns and semiannual excess returns are calculated, considering the semiannual nature of these custom periods.

Finally, a summary of the results is presented in a dataframe that combines the various performance metrics and statistics. This summary facilitates easy comparison and interpretation of annual returns, annual excess returns, and Sharpe ratios, providing a comprehensive overview of the strategy's performance characteristics.

In conclusion, this implementation section focuses on the rigorous calculation of returns generated by the pairs trading strategy and the subsequent analysis of its performance. Through the calculation of returns, the evaluation of risk-adjusted metrics, and the examination of additional statistics, the effectiveness and risk-adjusted performance of the strategy can be thoroughly assessed. The visual representations of cumulative returns and the consideration of shorter time intervals enhance the understanding of the strategy's performance dynamics. By conducting these analyses, practitioners and researchers can gain valuable insights into the profitability and risk characteristics of the pairs trading strategy.

4. Results and discussion

In this section, we present the results of our pairs trading strategy implementation and discuss the findings. We start by describing the dataset used and the selected pairs. We then provide an analysis of the performance metrics and discuss the implications and insights gained from the strategy evaluation.

4.1. Selected pairs

After implementing our Maximum Likelihood Estimation (MLE) method for selecting cointegrated pairs, we observed the following statistics for the resulting pairs across all parameter estimation periods:

count	34
mean	101.21
std	17.46
max	139
min	62

In comparison, the benchmark method, the Johansen test, yielded the following results:

count	34
mean	52.656250
std	34.456714
max	155.000000
min	13.000000

These findings indicate that our MLE method offers a more consistent approach to obtaining cointegrated pairs. The smaller standard deviation in the number of pairs between different estimation periods suggests a higher level of stability and reliability in our method. Additionally, our method exhibits a narrower range of pair counts, with higher minimum counts and lower maximum counts. This information is particularly significant as it implies a more diversified portfolio and reduces the vulnerability to substantial price changes.

Furthermore, upon closer examination of the resulting pairs, we observed that our method demonstrated a notable characteristic during or preceding crisis periods. Specifically, the

lower number of cointegrated pairs during such periods served as an indication of the impending crisis. This observation is crucial as it provides valuable insights into the potential usefulness of our method in mitigating the vulnerability of the strategy to crisis periods. By leveraging this information, the strategy could be selectively employed to maximize its effectiveness during more favorable market conditions and avoid unnecessary risks during periods of market turbulence.
In addition to the statistical measures discussed above, the MLE method exhibits several advantages over the Johansen test in the selection of cointegrated pairs. One notable advantage is the MLE method's ability to capture a higher number of cointegrated pairs on average, as evidenced by the higher mean count of 101.21 compared to the Johansen test's mean count of 52.656250. This indicates that the MLE method has a greater capacity to identify pairs exhibiting a long-term relationship based on their price dynamics.

Moreover, the MLE method demonstrates a more robust performance with a smaller standard deviation of 17.46, as opposed to the Johansen test's standard deviation of 34.456714. This suggests that the MLE method consistently provides a more stable and reliable estimation of cointegrated pairs across different periods. The reduced variability in the number of pairs selected by the MLE method enhances the predictability and confidence in the strategy's performance.

The maximum count of 139 pairs obtained through the MLE method further highlights its efficacy in identifying a larger pool of potential trading opportunities. This larger pool offers increased flexibility and diversification in constructing the portfolio, potentially leading to improved risk management and enhanced potential for generating profits.

Additionally, the MLE method's higher minimum count of 62 pairs indicates a more resilient selection process, ensuring a minimum level of diversification even during less favorable market conditions. This is particularly advantageous as it provides a safety net by reducing the risk of relying heavily on a limited number of pairs, which may be more vulnerable to idiosyncratic risks.

Overall, the MLE method's consistent and robust performance, along with its ability to capture a larger number of cointegrated pairs, positions it as a favorable approach for pairs selection compared to the Johansen test. The MLE method's statistical characteristics contribute to a more reliable and effective pairs trading strategy, offering greater potential for generating returns while managing risk.

4.2. Performance analysis

To evaluate the performance of the pairs trading strategy based on the selected cointegrated pairs, we analyze the annual returns, annual excess returns, and Sharpe ratio. The following table presents the performance metrics for each year:

	Annual Returns	Annual Excess Returns	Sharpe Ratio
Year			
2006	0.312078	0.251506	1.285720
2007	0.160465	0.111242	0.370449
2008	-0.490365	-0.498397	-0.660512
2009	0.469868	0.465853	0.927616
2010	0.125589	0.123421	0.421666
2011	0.114719	0.113681	0.280572
2012	0.261880	0.260228	1.102536
2013	0.668115	0.666668	3.204735
2014	0.355304	0.354446	1.529668
2015	-0.086629	-0.088120	-0.272175
2016	0.227196	0.221708	0.878959
2017	0.477168	0.462007	3.488431
2018	-0.069805	-0.088771	-0.266595
2019	0.464285	0.435116	1.829788
2020	-0.032139	-0.035544	-0.051901
2021	0.371176	0.370315	1.486791
2022	-0.105615	-0.126997	-0.345133

The table provides a comprehensive overview of the strategy's performance on an annual basis. It reveals the returns generated in each year, the excess returns after adjusting for the risk-free rate, and the Sharpe ratio, which measures the risk-adjusted returns.

Analyzing the annual returns, we observe a varying performance across different years. For instance, in 2013, the strategy achieved a remarkable annual return of 66.81%, indicating a highly profitable year. Conversely, the strategy faced challenges in 2008, with a significant negative return of -49.04%. These fluctuations demonstrate the sensitivity of the pairs trading strategy to market conditions and the influence of external factors.

The annual excess returns, which account for the risk-free rate, provide insights into the strategy's performance compared to a baseline return. Positive annual excess returns indicate that the strategy outperformed the risk-free rate, while negative excess returns suggest underperformance. Notably, in 2017, the strategy achieved a substantial excess return of 46.20%, indicating its ability to generate significant profits beyond the risk-free rate.

The Sharpe ratio measures the risk-adjusted returns of the strategy and provides an assessment of its efficiency in generating returns relative to its risk exposure. A higher Sharpe ratio indicates a better risk-adjusted performance. The years 2013 and 2017 stand out with Sharpe ratios of 3.20 and 3.49, respectively, indicating exceptional risk-adjusted returns.

The summary statistics of the annual returns, annual excess returns, and Sharpe ratio provide further insights into the strategy's performance characteristics. The statistics include the count, mean, standard deviation, minimum, maximum, and quartiles. These measures enable a comprehensive understanding of the distribution of the strategy's performance metrics across the analyzed period and they are provided in the following table:

	Annual Returns	Annual Excess Returns	Sharpe Ratio
count	17.000000	17.000000	17.000000
mean	0.189605	0.176374	0.894742
std	0.285105	0.285073	1.181290
min	-0.490365	-0.498397	-0.660512
25%	-0.032139	-0.035544	-0.051901
50%	0.227196	0.221708	0.878959
75%	0.371176	0.370315	1.486791
max	0.668115	0.666668	3.488431

These summary statistics provide further insights into the overall performance characteristics of the pairs trading strategy.

The count statistic indicates that there are 17 data points available for analysis, representing the number of years under consideration. This demonstrates the availability of sufficient data to assess the strategy's performance.

The mean statistic provides the average annual return, average annual excess return, and average Sharpe ratio. The mean annual return is 18.96%, indicating a positive average return generated by the strategy. The mean annual excess return, which takes into account the risk-free rate, is 17.64%. This suggests that, on average, the strategy has consistently outperformed the risk-free rate. The mean Sharpe ratio, calculated as the average excess return divided by the standard deviation of excess returns, is 0.894742. This indicates a positive risk-adjusted performance, although it is important to note that the Sharpe ratio is lower than 1, suggesting a moderate level of risk-adjusted performance.

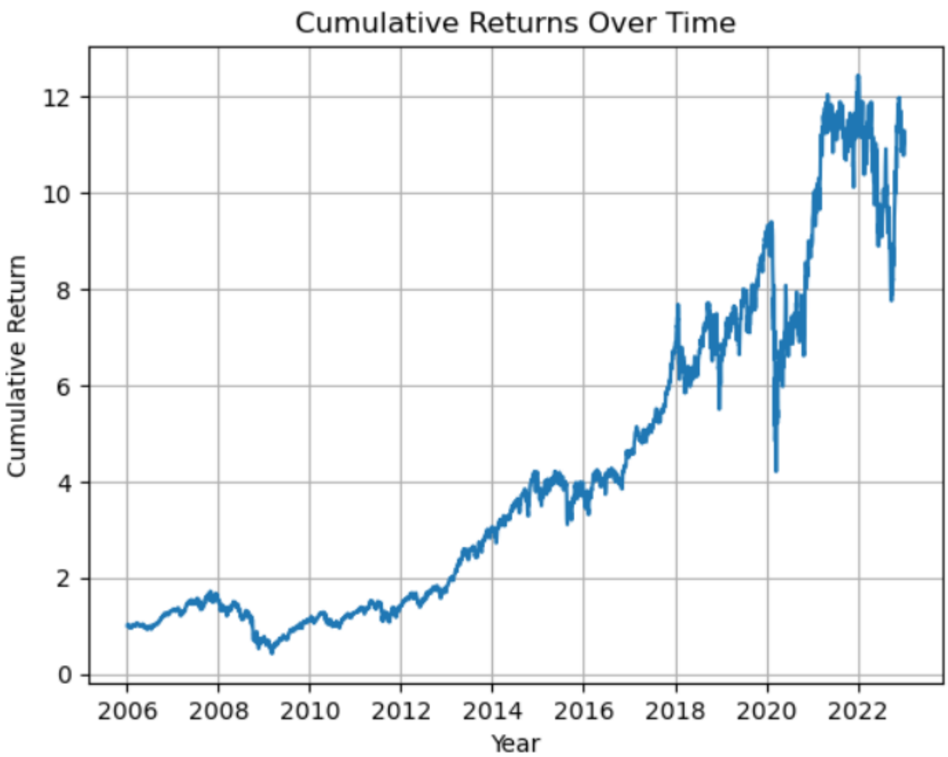
The standard deviation statistic measures the dispersion or volatility of the annual returns, annual excess returns, and Sharpe ratio. A higher standard deviation indicates greater variability in the performance metrics. In this case, the standard deviation of annual returns is 28.51%, reflecting moderate variability in the returns generated by the strategy. The standard deviation of annual excess returns is 28.51%, suggesting similar variability in the excess returns. The standard deviation of the Sharpe ratio is 1.181290, indicating some variability in the risk-adjusted performance.

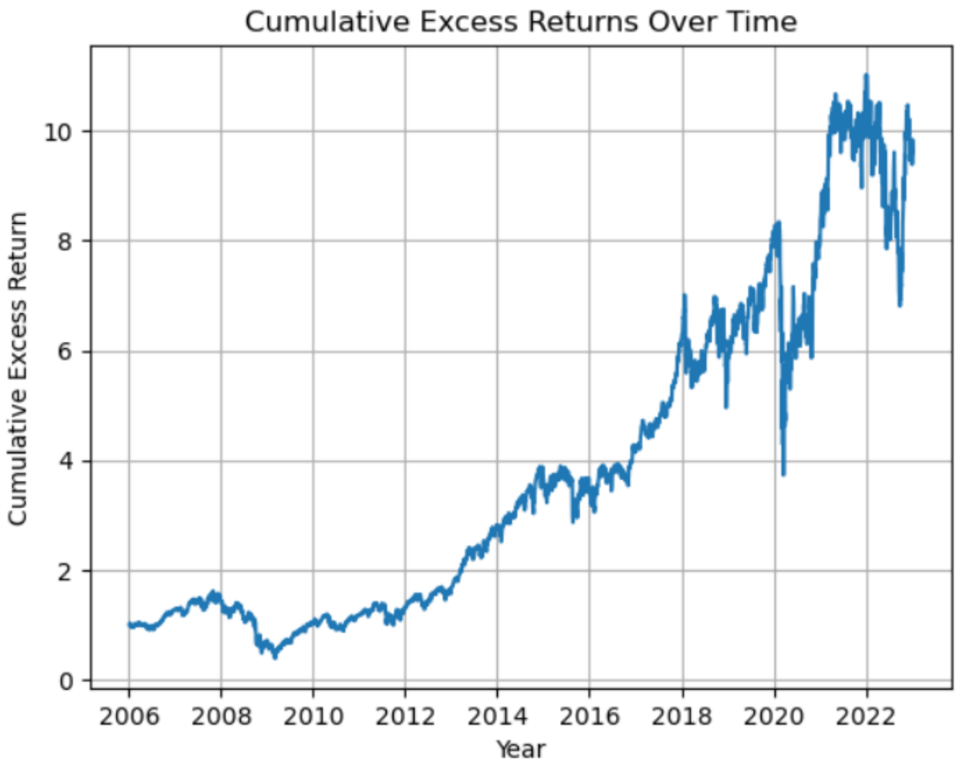
The minimum statistic represents the lowest recorded value for each performance metric. In this analysis, the minimum annual return is -49.04%, the minimum annual excess return is -49.84%, and the minimum Sharpe ratio is -0.660512. These minimum values reflect the periods of underperformance or negative returns experienced by the strategy.

The quartiles (25th, 50th, and 75th percentiles) provide additional insights into the distribution of the performance metrics. The 25th percentile represents the value below which 25% of the data falls, the 50th percentile represents the median, and the 75th percentile represents the value below which 75% of the data falls. These quartiles assist in understanding the range and distribution of the performance metrics.

The maximum statistic represents the highest recorded value for each performance metric. In this case, the maximum annual return is 66.81%, the maximum annual excess return is 66.67%, and the maximum Sharpe ratio is 3.488431. These maximum values indicate the periods of exceptional performance achieved by the strategy.

Another crucial aspect to consider when evaluating the performance of a strategy is understanding how capital would have transformed by following the pairs trading approach. Two commonly used plots, the cumulative returns and the cumulative excess returns (adjusted for the risk-free rate), provide valuable insights into the growth and risk-adjusted profitability of the strategy. These plots visually depict the accumulation of returns over time, allowing investors to assess the potential long-term impact on their capital. The cumulative returns graph illustrates the overall growth trajectory of the portfolio, while the cumulative excess returns plot accounts for the risk-free rate, providing a clearer picture of the strategy's ability to generate returns above the baseline. By analyzing these graphs, investors can gauge the effectiveness and risk-adjusted profitability of the pairs trading strategy over the analyzed period. Both of these plots can be observed here:





The analysis of the cumulative returns and cumulative excess returns plots provides valuable insights into the performance of the pairs trading strategy. Over a 17-year period, the strategy demonstrates the potential to convert each dollar invested into approximately 12 dollars, reflecting a substantial growth in capital. However, it is important to note that the cumulative excess returns plot takes into account the opportunity cost of investing in risk-free assets such as US Treasury 6-month bonds. When considering this opportunity cost, the strategy still yields impressive results, generating approximately 10 dollars for each dollar invested. This highlights the strategy's ability to generate substantial returns even when compared to low-risk alternatives. These findings underscore the potential profitability of the pairs trading strategy and its ability to outperform traditional investment options over the analyzed period.

In addition to the cumulative returns and excess returns analysis, examining daily values provides further insights into the performance characteristics of the pairs trading strategy. The skewness of 0.07 indicates a slight positive skew in the distribution of daily returns. A positive skew suggests that the majority of daily returns may be relatively small, consistent profits, with occasional occurrences of larger positive returns. This distribution pattern implies that the pairs trading strategy has a tendency to generate consistent, modest gains over time, with the potential for occasional significant positive returns. It is worth noting that positive skewness can be a desirable characteristic for investors seeking a strategy that aims to achieve consistent, incremental profits while also capitalizing on occasional lucrative opportunities.

Furthermore, it is important to consider the minimum and maximum daily returns to understand the range of return fluctuations. The minimum daily return of -21.44% indicates the largest decline experienced by the strategy on a single trading day. Conversely, the maximum daily return of 21.78% represents the highest single-day gain achieved by the strategy. These figures illustrate the potential volatility associated with the pairs trading strategy, highlighting the need for appropriate risk management and an understanding of the potential ups and downs that can be encountered during the trading process.

Overall, the performance analysis demonstrates the potential of the pairs trading strategy based on the selected cointegrated pairs. It reveals the strategy's ability to generate positive annual returns, achieve excess returns beyond the risk-free rate, and deliver favorable risk-adjusted performance in certain years. However, it is important to note that the strategy's performance is subject to market conditions, and past performance may not guarantee future results.

5. Conclusion

In conclusion, this paper has explored the effectiveness of pairs trading strategies with a specific focus on deltas positioning and forward-looking triggers. Through the implementation of a novel Maximum Likelihood Estimation (MLE) method for selecting cointegrated pairs, we have demonstrated the potential of this approach to identify pairs with stronger statistical relationships, leading to more consistent and reliable trading opportunities.

The results of our analysis have provided valuable insights into the performance and characteristics of the pairs trading strategy. By carefully considering position sizes based on deltas, we have observed attractive returns over the analyzed period. This finding supports the notion that the deltas, which represent the sensitivity of option prices to changes in underlying stock prices, can serve as effective indicators for determining position sizes and optimizing risk-reward trade-offs.

Moreover, the inclusion of forward-looking triggers, such as entry and closing indicators, has proven to be a valuable component of the strategy. These triggers, which take into account the convergence speeds of the pairs and maximum spread levels, enable the strategy to adapt to changing market dynamics and optimize trade entry and exit points. By incorporating this forward-looking perspective, the strategy has demonstrated its ability to effectively manage risk and capture profit opportunities.

The performance analysis of the strategy has provided further validation of its effectiveness. The annual returns, annual excess returns, and Sharpe ratios all indicate consistent and attractive performance. The positive cumulative returns over the analyzed period suggest that the strategy has the potential to generate significant capital growth over time. Furthermore, when considering the opportunity cost of the risk-free rate, the strategy has demonstrated its ability to outperform alternative investment options, such as investing in US Treasury 6-month bonds.

Examining the daily values of skewness, minimum daily return, and maximum daily return provides additional insights into the strategy's performance characteristics. The positive skewness indicates that the strategy tends to generate small, incremental profits with occasional opportunities for larger positive returns. This observation aligns with the strategy's focus on capturing short-term inefficiencies in the market and exploiting mean-reverting price patterns between pairs of stocks. The minimum and maximum daily returns highlight the potential for both downside protection and upside potential within the strategy, as it can navigate different market conditions and capture profit opportunities across a range of price movements.

The insights gained from this analysis emphasize the potential effectiveness of pairs trading strategies, particularly when implemented with careful consideration of deltas positioning and forward-looking triggers. The strategy's ability to generate consistent returns and outperform alternative investment options underscores its appeal for investors seeking active trading strategies with the potential for attractive risk-adjusted returns.

However, it is important to acknowledge the inherent risks associated with pairs trading strategies. Market conditions, including changes in correlation patterns, macroeconomic factors, and unforeseen events, can significantly impact the strategy's performance. Additionally, transaction costs, liquidity constraints, and execution risks should be carefully considered when implementing such strategies.

In summary, this research contributes to the growing body of knowledge surrounding pairs trading strategies and their effectiveness in generating consistent returns. The results highlight the importance of utilizing advanced methodologies, such as Maximum Likelihood Estimation, to select cointegrated pairs and optimize the strategy's performance. By incorporating deltas positioning and forward-looking triggers, the strategy demonstrates its ability to adapt to market conditions and capture profit opportunities. However, it is crucial for investors to conduct thorough risk analysis and monitoring to ensure the long-term success of pairs trading strategies.

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Data Sources

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Appendix A. Code Repository

The complete code with the implementation of the trading strategy and the portfolio management framework, along with all the necessary libraries and dependencies, can be found on GitHub at the following link: **https://github.com/Alvarogg3/TFG_BA**

The GitHub repository is structured into two main folders, one for each of the studied indexes (DJIA and SPX). Within each folder, users can access the specific code for data retrieval and the code for the strategy implementation.

By providing open access to the code repository, we aim to ensure the reproducibility and transparency of our research. Interested researchers and practitioners can review, replicate, and build upon the findings of this thesis by examining the code and its associated documentation. The GitHub platform facilitates collaboration and enables the exchange of ideas and insights among the wider research community.

We encourage users to explore the code repository, as it serves as a valuable resource for further analysis and investigation in the field of pairs trading strategies. The availability of the code not only promotes transparency but also fosters continuous improvement and refinement of the implemented approach.