is limited to the expressions for aerodynamic forces and the equations stating the eigenvalue problem. When considering other details of practical flutter analysis, further substantiation of the paper's findings is found. A review of finite-element aeroelastic matrices presented in real notation (Namini 1991) and in complex notation (Starossek 1991, 1993), for instance, reveals the advantages of the complex-number approach, particularly in terms of format compactness and simplicity. In fact, finite-element matrices in real notation grow so voluminous and irregular that even routine tasks such as typesetting and programming become a challenge.

2. The discussers state that "the choice of using a real or complex number format should depend on whether the flutter derivatives or complex coefficients are more conveniently measured or predicted" and that this "depends on the available testing facilities and the expertise of the research team."

   The relationship between real flutter derivatives and complex aerodynamic coefficients is given by (7). Even if the flutter derivatives have been measured in real-number format, the corresponding complex coefficients are readily obtained from these simple equations. Thus, different testing facilities and special expertise are not required for determining complex coefficients.

   In case "available testing facilities" and "expertise of the research team" refers to the difference between the free-oscillation method and the driven-oscillation method for measuring unsteady aerodynamic coefficients, it should be noted that both the real-number derivatives and the complex-number aerodynamic coefficients are usually understood to refer to the critical state of constant-amplitude vibration. In both cases, therefore, only a constant-amplitude procedure such as the driven-oscillation method can, in principle, give accurate results, and both sets of coefficients are related by (7).

3. Finally, the discussers state that "from an educational viewpoint, civil engineers in the United States are indeed more familiar with real than complex numbers."

   Real notation in bridge flutter analysis has become customary in the United States. However, the term "educational standpoint" is used in the paper in quite a different sense: not to justify what has become customary but to illustrate the usefulness of complex flutter analysis in terms of simplicity and ease of understanding.

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SIMPLIFIED MODEL OF LOW CYCLE FATIGUE FOR RC FRAMES[a]

Discussion by Alberto Carnicero,[4] Ricardo Perera,[5] and Enrique Alarcón[6]

To model strength degradation due to low cycle fatigue, at least three different approaches can be considered. One possibility is based on the formulation of a new free energy function and damage energy release rate, as was proposed by Ju (1989).

The second approach uses the notion of bounding surface introduced in cyclic plasticity by Dafalias and Popov (1975). From this concept, some models have been proposed to quantify damage in concrete or RC (Suaris et al. 1990). The model proposed by the author to include fatigue effects is based essentially in Marigo (1985) and can be included in this approach. In the formulation of the fatigue law, the loading-unloading irreversibility concept is employed. However, in this model, an additional constant α is introduced. The physical meaning of this constant is not clear, and neither is its evaluation.

In the approach developed by the discussers, the classical concepts of yield and damage domains are used and the traditional notion of damage energy release rate as a variable related to the elastic strain energy is kept.

For it, the dissipative potentials proposed by the author (1995) are slightly modified to introduce cumulative effects. The discussers propose the following function:

$$g = Y - [Y_{cr} + Z(D)\xi(\omega)] \leq 0 \tag{5}$$

for the damage dissipative potential. In order to keep the coupling between the damage and the plastic domain, a modification of the plastic dissipative potential is also required:

$$f = |M - X(D)| - M_y - R(D)\sqrt{\xi(\omega)} \leq 0 \tag{6}$$

where ω is a cumulative parameter.

The fatigue function, $\xi(\omega)$, must satisfy two restrictions:

$$\begin{aligned} \xi(\omega) = 1 \Leftrightarrow \omega \leq \omega_{min} \\ \xi(\omega) = 0 \Leftrightarrow \omega = \omega_{max} \end{aligned} \tag{7}$$

This new term introduces a softening effect in the two functions in order to include the fatigue effects.

The discussers propose as fatigue function

$$\xi(\tilde{\theta}, \theta_t) = 1 - \left[\frac{\tilde{\theta}}{N_f(\theta_t)\cdot\theta_t}\right]^{1/\mu} = 1 - \left[\frac{n}{N_f(\theta_t)}\right]^{1/\mu} \tag{8}$$

where $\tilde{\theta}$ and θ_t represent the total cumulative rotation and the total rotation (semiamplitude loop), respectively; and $N_f(\theta_t)$ = number of cycles to failure at θ_t semiamplitude. Therefore, the relation between $\tilde{\theta}$ and θ_t is a measure of the number of cycles, n. The most important influence of the higher cycles over the lower ones is considered across the ductility μ.

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[a]September 1998, Vol. 124, No. 9, by Edward Thomson, América Bendito, and Julio Flórez-López (Technical Note 15092).

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Apparently, this model introduces a new parameter, N_f. However, this parameter can be identified physically following Mander et al. (1994) or Koh and Stephen (1991); both of them gave relationships between N_f and the deformation of the longitudinal reinforcement, but whereas the first one uses the plastic amplitude, the second uses the total amplitude of the loop. Writing these two expressions in terms of plastic rotation, we have

$$N_f = 2\left(\frac{0.16}{\phi_p}\frac{l_p}{d}\right)^2 \tag{9}$$

for the relationship given by Mander et al. and

$$N_f = \frac{1}{2}\left(\frac{0.16}{d\left(\frac{M_y}{EI} + \frac{\theta_p}{l_p}\right)}\right)^3 \tag{10}$$

for the relationship given by Koh and Stephen. In both cases, the plastic hinge length, l_p, must be calculated. In the previous equations θ_p = plastic rotation; d = distance between the reinforcement; M_y = yield moment; E = elastic modulus; and I = moment of inertia. The correlation between numerical and experimental results is, on balance, better using the second expression.

Two examples are presented to show the performance of this approach. Fig. 5 represents the experimental results of a circular cross-section reinforced concrete column tested by Kunnath et al. (1997), which is subjected to a constant axial load of 806 kN and a cyclic lateral displacement of constant amplitude. The numerical simulation, shown in Fig. 6, was performed with the following parameters: $EI/L = 2.51E + 7$ Nm; $M_{cr}^+ = M_{cr}^- = 27.420$ kNm; $M_p^+ = 87.808$ kNm; $M_u^+ = M_u^- = 98.784$ kNm; $\theta_{pu}^+ = \theta_{pu}^- = 0.029$; $\alpha^+ = \alpha^- = 1$.

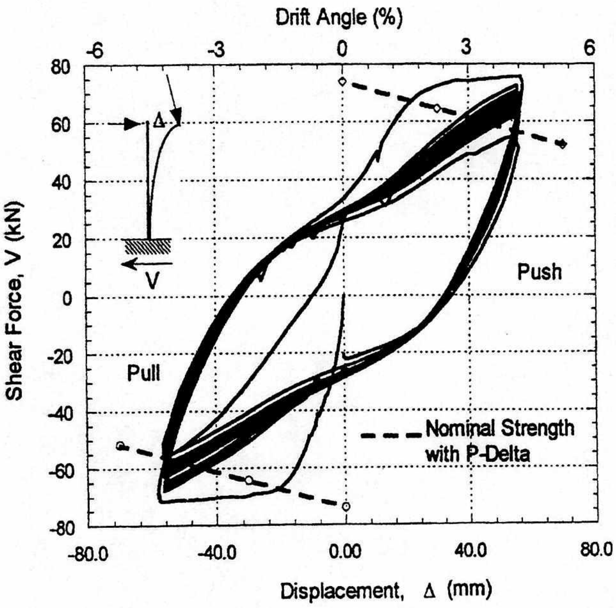

FIG. 5. Test Results Reported in Kunnath et al. (1997)

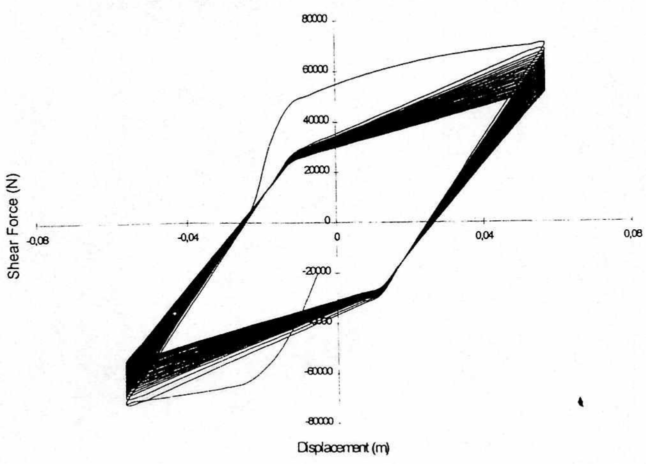

FIG. 6. Numerical Simulation of Test Reported in Kunnath et al. (1997)

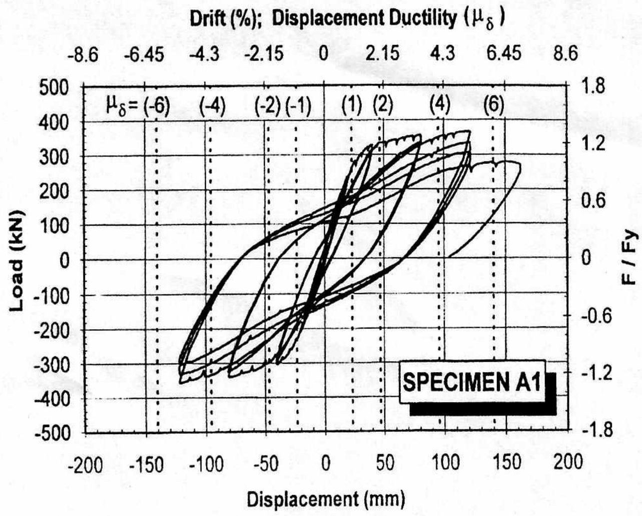

FIG. 7. Test Results Reported in Wehbe et al. (1994)

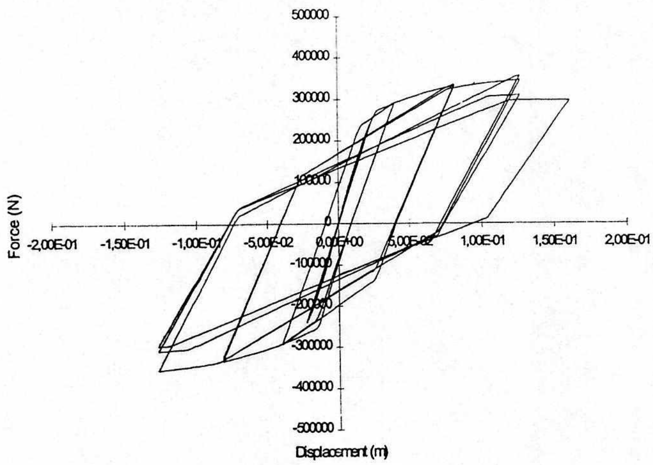

FIG. 8. Numerical Simulation of Test Reported in Wehbe et al. (1994)

Figs. 7 and 8 show experimental and numerical results of a rectangular cross-section reinforced concrete column with moderate confinement tested by Wehbe et al. (1994). As in the previous case, the column is subjected to a constant axial load of 641 kN and the lateral displacement is controlled. The numerical simulation has been done using the following parameters: $EI/L = 2.21E + 7$ Nm; $M_{cr}^+ = M_{cr}^- = 210$ kNm; $M_p^+ = M_p^- = 643$ kNm; $M_u^+ = M_u^- = 850$ kNm; $\theta_{pu}^+ = \theta_{pu}^- = 0.05$; $\alpha^+ = \alpha^- = 1$.

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Closure by Edward Thomson,[7] América Bendito,[8] and Julio Flórez-López[9]

The writers would like to thank the discussers for their interest in the technical note. The writers learned of the discussers' model only after the technical note had been published. Since then, they have studied the discussers' model with great interest.

It seems clear that the model presented in the note is less ambitious than the discussers' model, but it is also simpler. The note's model only intends to represent concrete cracking during repeated loads. The writers cautioned in the note that other phenomena besides concrete cracking are responsible for failure under repeated loads, mainly the buckling of reinforcement. The model, as mentioned in the paper, does not describe this effect, since it is based on a generalization of the Griffith criterion.

The authors disagree with the discussers' comment that "the physical meaning of this constant [i.e., the parameter α] is not clear, and neither is its evaluation." The parameter α is related to additional cracking under repeated loads as compared with the damage in monotonic loads of the same maximum solicitation. In this sense, this parameter is not any less clear than the term N_f used in the discussers' model, which has a clear physical meaning only in repeated loads of constant amplitude.

The writers think that the main interest of the model presented in the note is that the parameter α can be considered as a constant ($\cong 2$) for RC members. This conclusion was reached after a dozen experimental tests and numerical simulation of some twenty tests.

AXIALLY LOADED CONCRETE-FILLED STEEL TUBES[a]

Discussion by C. N. Srinivasan[2]

The authors have addressed most of the fundamental questions that influence the behavior of concrete-filled steel tubes. The discusser is happy to note that almost all the author's conclusions bear out results of earlier studies by the discusser. A summary of the results of studies on the following aspects carried out by the discusser (Srinivasan 1985, 1991) are reported in this discussion:

1. Confining action in circular and square in-filled circular tubes (ICT)
2. Triaxial effects on the core and biaxial effects on the shell
3. The presence of lateral strain compatibility and complete interaction of the core and the shell, over the entire range of loading
4. The effect of loading method and a study of connections
5. The factors governing the service and the ultimate load behavior
6. The effect of wall thickness, diameter, and concrete mix

The test program was as follows:

- 150 mm diameter in-filled circular and square tubes of 300–1,200 mm lengths were tested to develop a stress-strain relationship for the core and the shell and to study items 1–3 above.
- A comparative study of samples where the bond between the core and the shell has been broken, and where no such steps have been taken, was studied. After the tests, the samples were split longitudinally to study the concrete face, and it was found that the concrete core retained its integrity.
- 6 circular short in-filled specimens were studied to verify

TABLE 4. Test Results of Axially Loaded Specimen to Study Influence of Loading Method

Brief description of building (1)	Failure load (tons) (2)	Observation at failure (3)
Infilled circular tube core only loaded	185	Formation of Luder lines and spalling of mill scales and kink at midheight
Infilled circular tubes both shell and core loaded	200	Luder lines with spalling of mill scales and kink at midheight
Infilled circular tube, bracketed arrangement beam to column type connection shell loaded	110	Bulges at top and local buckling
Hollow circular tube	90	Bulges at top and local buckling
Infilled circular tube, shell only loaded	85	Bulges at top and local buckling
Infilled circular tube flat slab to column connection core and shell loaded	205	Formation of Luder lines, bending with kink at midheight
Square tubes—core only loaded	200	Cracks formation at welds
Square tubes—both core and shell loaded	240	Cracks at welds
Infilled square tube-bracketed arrangement, "beam to column" type connection—shell loaded	120	Bulges at top and local buckling
Hollow square tube	75	Bulges at top and local buckling
Infilled square tube, shell only loaded	88	Bulges at top and local buckling
Infilled square tube—flat slab column type connection—core and shell loaded	250	Formation of Luder lines and bending with kink at midheight

Note: Description: 15 cm nominal diameter or 15 cm^2 × 6 mm wall thickness × 122 cm long tube infilled concrete corresponding to a 15 cm cylinder strength of 0.295 tons/cm^2.

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