Facultad de Ciencias Económicas y Empresariales
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Market Efficiency Testing of Lithium

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ABSTRACT

This study employs the Multifractal Detrended Fluctuation Analysis (MFDFA) methodology and the multifractal spectrum width (MSW) to assess the efficiency of the Lithium market. Lithium's critical role in electric vehicle battery production underscores the importance of understanding its market dynamics to assess the suitability of investments in the commodity which reinforce its role in the global energy transition. Analysis of the Lithium market reveals overall market efficiency, with occasional deviations attributed to news-driven trend-reinforcing behaviors that quickly dissipate. The findings suggest market inefficiencies do not impede investment in the commodity, offering reassurance to private and public investors, regulators, and society. Promising results yielded using the multifractal spectrum width have yet to be validated, as the latter presents similar conclusions, albeit with some discrepancies, relative to the Hurst coefficient. Overall, this research pioneers the application of the MFDFA to analyze lithium's long-term memory and introduces the multifractal spectrum width as a potential efficiency measure, contributing to a deeper understanding of lithium market dynamics, the global energy transition, and advancing developments in the field of financial asset efficiency testing.

Key Words: Lithium market, efficiency testing, energy transition, critical minerals, MFDFA, multifractal spectrum width, Efficient Market Hypothesis (EMH), Hurst exponent.

RESUMEN

Este estudio utiliza la metodología del Análisis Multifractal de Fluctuación sin Tendencia (*MFDFA)* y la amplitud del espectro multifractal (*MSW*) para evaluar la eficiencia del mercado del litio. El papel crítico que desempeña el litio en la producción de baterías para vehículos eléctricos, destaca la importancia de comprender las dinámicas de este mercado para así evaluar la viabilidad de inversiones en la materia prima, que refuercen su papel en la transición energética global. El análisis realizado sobre el mercado del litio demuestra en líneas generales, un comportamiento eficiente de este, con desviaciones ocasionales hacia la persistencia, impulsadas estas por noticias cuyo efecto se disipa rápidamente. Los hallazgos sugieren que al no existir ineficiencias latentes en este mercado, estas no suponen un impedimento de cara a posibles inversiones en la materia prima, ofreciendo seguridad a inversores privados y públicos, reguladores y a la sociedad. Los resultados prometedores obtenidos mediante el uso del ancho del espectro multifractal aún deben ser validados, ya que este último presenta conclusiones similares, aunque con algunas discrepancias, a las obtenidas mediante el coeficiente de Hurst. En general, esta investigación es pionera en la aplicación del *MFDFA* para analizar la memoria a largo plazo del litio, e introduce el ancho del espectro multifractal como una medida potencial para la medición de eficiencia, contribuyendo así a una mejor comprensión de las dinámicas subyacentes del mercado del litio, de la transición energética global, y avanzando en el desarrollo de la disciplina del estudio de la eficiencia de los precios de activos financieros.

Palabras Clave: Mercado del litio, medición de eficiencia, transición energética, mineral crítico, Análisis Multifractal de Fluctuación sin Tendencia (*MFDFA*), amplitud del espectro multifractal, hipótesis del mercado eficiente (*EMH*), exponente de Hurst.

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CHAPTER 1. INTRODUCTION

1.1 Motivation and justification of the topic

Lithium is currently considered one of the critical minerals of the climate transition due to its widespread use in electric vehicle (EV) batteries. These batteries are expected to play a significant role in reducing global CO_2 emissions in the future. Several studies suggest that there is a supply gap in the market for this commodity, and this gap is expected to widen as demand for EVs, and consequently, lithium, continues to grow. If the supply cannot keep up with the increasing demand, the price of the commodity may rise, potentially hindering the adoption of EVs and thus endangering the climate transition. To ensure an adequate supply to meet the growing demand, public and private investments in this commodity are necessary.

Market efficiency is a paramount precondition for the efficient allocation of resources (Pagan, 1996). As such, the efficient behavior of the lithium market is a crucial prerequisite for attracting the needed investments in the asset. Studying the compliance of this market with the Efficient Market Hypothesis (EMH) using the Multifractal Detrended Fluctuation Analysis (MFDFA) methodology will thus help identify potential risks to the climate transition and inform potential investors in the commodity.

The MFDFA methodology serves as one of the available approaches to calculate the Hurst exponent (H) for a given time series. The Hurst exponent is a statistical metric that measures the long-term correlation, indicating the level of persistence within time series data. When a time series of financial returns exhibits long-term memory, it implies the potential for achieving above-average returns through the use of technical trading rules, as it enables accurate predictions based on past behavior. The presence of such persistence contradicts the assumptions of the weak-form Efficient Market Hypothesis and designates the series as informationally inefficient. Unlike other methodologies, the MFDFA allows for the accurate identification of non-constant Hurst exponents throughout different segments of a stochastic process. This makes it particularly relevant for the analysis of financial series, which often display distinct behaviors depending on the specific time intervals under study.

As an alternative measure of long-term memory and thus, presence of inefficiency within the series, this study will be, to the best of our knowledge, one of the firsts to consider the multifractal spectrum plot, and width. Given that white-noise processes generally present consistent monofractal behavior, multifractality within the series as evidenced by a large multifractal spectrum width will be taken as an indication of inefficiency. To complement the static estimation of the Hurst Exponent and the multifractal spectrum width, a rolling-window approach will be conducted for both metrics, to further explore the time-varying efficiency dynamics of the series.

We observe evidence of overall weak-form efficiency within the Lithium market, marked by three distinct periods of trend-reinforcing behavior. These three periods generally coincide with the outbreak of events and news, which had a considerable effect on the expected future performance of the commodity and its key players and thus led the market to enter states of panic and/or euphoria, which translated into a persistent behavior.

1.2 Objectives and methodology

As lithium is not a tradable financial asset, in this study, the adherence to the weak form of the EMH of the return series of the Global X Lithium & Battery Tech ETF and the Lithium Carbonate Future (as traded in the Guangzhou Futures Exchange), are tested by applying the MFDFA and multifractal spectrum width (MSW) methodologies. Both financial assets under study represent suitable alternatives for investors looking to gain an exposure to lithium.

The MFDFA and MSW methodologies are predominantly quantitative and as such, the approach to be followed in this investigation is deductive where the initial hypothesis is that the return series of lithium futures and the Lithium ETF are informationally efficient. To the author's knowledge, the market efficiency of lithium and its related assets hasn't yet been tested making this investigation novel in that sense. However, as Corzo, Martin-Bujack, Portela & Saénz-Diez 's (2022) study demonstrates, the field concerning efficiency testing of critical minerals, is starting to gain larger academic awareness. This thesis is also innovative in the use of the proposed methodology. Although several investigations such as Syed Aun R. Rizvi's (2014), have applied MFDFA to test the price efficiency of various financial assets, none have yet considered a rolling-window approach of the multifractal spectrum width in their analysis, as a plausible measure of the series' efficiency.

The relevance of lithium in the climate transition will also be studied through a qualitative and quantitative approach, which will involve a thorough review of the extensive literature on the subject, focusing on the potential applications and limitations of the current and prospective supply of lithium as a mineral for the climate transition.

1.3 Work scheme

The remainder of this paper is organized as follows. Chapter 2 delves into the role of lithium, its significance in the climate transition, and the current as well as expected conditions of its supply and demand. The literature review conducted within this chapter considered academic articles and research papers retrieved from the Scopus database, the dissemination portal Dialnet, and Google Scholar amongst others. Chapter 3 introduces the theoretical foundations of the MFDFA and multifractal spectrum width methodologies, which will be applied later. Additionally, this chapter outlines the two series that will serve as sample data for the subsequent analysis. Chapter 4 applies the MFDFA & multifractal spectrum width methodologies to the data and provides the results of the analysis, and Chapter 5, summarizes the final conclusions drawn from the study.

CHAPTER 2. CONCEPTUAL FRAMEWORK

2.1 Lithium and relevance to climate transition

Lithium is currently one of the crucial raw materials in the production process of electric vehicle (EV) batteries and grid-scale energy storage systems for renewable electricity (Graham, Rupp, & Brungard, 2021). As such, lithium is widely regarded as one of the critical minerals in the upcoming global energy transition (Sprott Asset Management LP, 2023).

The worldwide adoption of EVs is intended to combat the greenhouse gas emissions of one of the largest global contributing sectors, transport. In 2023, the latter accounted for 8.4 Gigatons of global C02 emissions, thus representing 14.1% of the annual total global greenhouse gas emissions (World Data Lab, 2023). As such, the EV's role in driving the energy transition is recognized by the global powers as one of the pillars in the fight against global warming.

In the 28th meeting of the Conference of the Parties (COP28) in November 2023, the member organizations agreed that in order to align with the target of achieving net-zero, there's a need to reduce road transport emissions by one-third by the year 2030 (United Nations Framework Convention on Climate Change, 2023). EVs currently stand as the foremost tool to achieve this planned reduction in C02 emissions. This has been acknowledged by several policymakers who have recently drawn up large new public investments into this technology and its related infrastructure. In the US alone, $7.5 billion in funding have been channeled through the Infrastructure Investment and Jobs Act in November 2021, for the purpose of construction of EV charging stations, to thus achieve the established goal of making half of all new vehicles sold in the U.S. in 2030, zero-emissions vehicles (U.S. Department of Transportation, 2022).

As more countries commit to reaching net-zero targets in their carbon emissions to combat global warming, the pivotal role EVs will play in the future economy becomes clearer. The Energy Information Administration (EIA) forecasted in 2023, that EV sales will account for between 29% and 54% of global new vehicle sales by 2050 (Energy Information Administration, 2023). This entails an approximate threefold growth from the 2022 levels where electric vehicles accounted for 14% of new vehicle sales (International Energy Agency, 2023). In terms of the global light-duty vehicle fleet, the EIA projected that the latter will increase from 1.31 billion in 2020 to 2.21 billion vehicles in 2050, and that the percentage of the fleet represented by electric vehicles will grow from 0.7% to 31% thus anticipating the incorporation of 672 million new EVs into the current fleet (Energy Information Administration, 2021). This widespread adoption of EVs as a mode of transportation is not however, without its limitations.

One of the key challenges that EVs and the consequent energy transition face is related to supply shortages in the sourcing of raw materials for the production process of the batteries. Although there are several available battery types, each dependent on different minerals, lithium-ion (Li-ion) batteries are the prevalent kind out of them all due to the performance advantages they offer over other alternatives (Egbue & Long, 2012). As its name suggests, Li-ion batteries are highly reliant on lithium as a raw material of the cathode of the latter, thus making it one of the denominated critical minerals of the energy transition.

To accommodate for the forecasted increased adoption of EVs, necessary to reach the established net-zero emissions targets, the sourcing capacity of lithium, and other critical minerals, will have to grow (Energy Tranistions Commission, 2023). According to Industry experts consulted by The Economist, under current conditions lithium will "foresee a shortfall of … 50,000-100,000 tons, [which represents] a 2-4% deficit by 2030" (The Economist, 2023). To address this supply crunch, and thus avoid the destruction of demand of EVs that would ensue, which would potentially endanger the energy transition, public and private investments in current and new mining sources are imperative (Sprott Asset Management LP, 2023). Under these conditions, the study of financial barriers and specifically, the determination of efficiency in the lithium market becomes a valuable source of information for potential investors and public policymakers.

Throughout history, several authors have explored the potential risks and benefits associated with investments in lithium. For example, firms like AuAg and DeGiro have facilitated exposure to this commodity through various funds, advocating for a bullish stance on the asset (AuAg, 2024; Degiro, 2024). They highlight its promising future due to increasing demand driven by the energy transition, societal electrification, and its diversification capabilities with respect to other financial securities. Most existing studies related to investments in this commodity, have however, predominantly focused on explaining and predicting price movements in the latter (Wulandari, 2022; Sanin, 2022; Restrepo, Uribe, & Guillen, 2023).

Moreover, despite the growing interest in lithium as an investment, the market efficiency and adherence to the Efficient Market Hypothesis (EMH) of the commodity hasn't yet been tested. Thus, our study aims to fill this gap by employing the Multifractal Detrended Fluctuation Analysis (MFDFA) and multifractal spectrum width methodologies, whereby studying the Hurst exponent and multifractal spectrum plots and widths on two prominent lithium-related financial series, we seek to shed light on the efficiency of the lithium market and identify potential financial barriers hindering investment in this asset.

CHAPTER 3. METHODOLOGY REVIEW & SAMPLE DESCRIPTION

3.1 MFDFA & MSW Methodology

In his seminal paper "Random Walks in Stock-Market Prices", Eugene Fama developed the idea of weak-form market efficiency. A market is said to be weak-form informationally efficient if successive price changes of each individual security traded in it are independent from one another. As such, these price changes exhibit no memory and therefore, the historical performance of the series is not indicative of its future behavior in any meaningful manner (Fama, 1965) .

This idea stems from the belief that in efficient markets, future developments and the information conveyed by them are random and instantaneously reflected in the prices of securities, which thus make price changes of the latter behave in a 'random-walk' like manner. Weak-form efficient markets, therefore, don't allow mechanical trading rules applied to them to attain above-average, consistent returns which thus show that long-term memory dependence is inconsistent with the weak-form of the efficient market hypothesis (Sadique & Silvapulle, 2001).

In 1951, Harold Hurst introduced a statistical measure called the Hurst Exponent (H) which he developed through the study of the flow of the river Nile (Hurst, 1951). The latter attempts to quantify in a single measure the long-term memory of a time series. Although there are several estimators available to calculate the H parameter (Mielniczuk & Wojdyllo, 2007), the rescaled range statistic (R/S) method continues to be the traditional one.

To calculate the H coefficient using the R/S method, we follow the steps detailed in Weron (2002):

Step 1. Given a time series of returns Z_t, of length L, we divide the latter into d subseries of length n.

Step 2. For each subseries, $m = 1,\dots,d$, calculate the mean E_m and standard deviation S_m.

Step 3. Normalize the data by subtracting the sample mean $X_{t,m} = Z_{t.m} – E_m$ ($m = 1, \dots , d$; $t = 1, \dots , n$)

Step 4. Calculate the cumulative time series $Y_{t,m} = \sum_{j=1}^{t} X_{j,m}$ for $j = 1,2,\dots n$

Step 5. For each subseries m, compute the rescaled range the following way:

$$\left(\frac{R}{S}\right)_m = \frac{\max(X_{1,m} \dots X_{n,m}) - \min(X_{1,m} \dots X_{n,m})}{S_m}$$

Step 6. Calculate the mean value of the rescaled range for all subseries of length n, which will thus give us the R/S(n) statistic.

Step 7. The R/S(n) statistic follows the asymptotical relationship R/S(n) ≈ Cn^H (Mandelbrot B. B., 1975). As such, the value of H can be obtained by fitting a linear regression, under ordinary least squares and taking log (n) as our independent variable and log (R/S(n)) as our dependent variable. The slope of the resulting fitted line corresponds with the Hurst exponent (H) of the observed time series.

The Hurst coefficient (H) measures the long-term correlation and persistence of stochastic processes. The range of H is from 0 to 1. A time series with a Hurst coefficient of 0.5, displays random and uncorrelated behavior, proper of a white noise process (Cannon, Percival, Caccia, Raymond, & Bassingthwaighte, 1997). A Hurst coefficient greater than 0.5 denotes a time series with persistent, correlated, trend-reinforcing behavior where the values in the process increase or decrease throughout time to an extent which would be impossible to attain for a random walk. A Hurst coefficient smaller than 0.5 describes a time series with anti-persistent, negatively correlated behavior, and as such describes series with mean-reverting characteristics. The strength of the trend-reinforcing and mean-reverting behaviors of the latter series, increases as the value of the Hurst exponent approaches 1 and 0 respectively (Corzo Santamaría, Martin-Bujack, Portela, & Sáenz-Diez, 2022).

The Hurst exponent calculated trough the rescaled range statistic (R/S) method has been used in a variety of fields ranging from geophysics (Mandelbrot & Wallis, 1969), to healthcare (Díaz M & Córdova, 2022) and even ecology (Wang, Yu-Zhi; Li, Bo; Wang, Ren-Qing; Su, Jing; Rong, Xiao-Xia, 2011). In finance its use has been widespread being the statistic applied to different assets such as FOREX (Galluccio, Caldarelli, Marsili, & Zhang, 1997), equities (Palágyi & Mantegna, 1999), commodities (Turvey, 2007; Kristoufek & Vosvrda, 2014) and futures (Scalas, 1998) , mostly for financial modelling and predictive purposes (Qian & Rasheed, 2005).

Compared to other methods available for calculating the H parameter (namely Detrended Fluctuation Analysis (DFA) and Variance Time Plot), several studies show that the R/S statistic has the smallest Mean Square Error for different sample sizes which makes it the most efficient estimator of them all (Ceballos & Largo, 2017). However, the latter was found by Anis and Lloyd (1976) to be biased for small values of *n*, which thus led to the creation of the Adjusted Rescaled Range Statistic (R/S-AL) which takes this limitation into account.

Furthermore, the R/S statistic assumes that the persistence of the time series under study can be represented using a single power law exponent and as such, that the scale invariance of the process is independent on the passage of time. Stochastic processes which behave in the previous manner are referred to as possessing a monofractal structure. However, some series do see their scale invariant structure affected by temporal variations (multifractal structures). These processes call for a more robust analysis that can consider these changes in the power law exponent across the series (Ihlen, 2012). The method thus chosen in this study to calculate the time-varying Hurst Exponent of the stochastic processes is the Multifractal Detrended Fluctuation Analysis (MFDFA).

To calculate the H coefficients using the MFDFA procedure, we follow the steps detailed in Ihlen (2012):

Step 1. Given a time series of returns $Z_{t,,}$ of length *L*, convert the latter into a white noise process X_t by subtracting the mean value E_m and integrating the time series.

Step 2. Divide the series into equal-sized non-overlapping segments $X_{t,d}$ of length *n*. Do so for different values of *n*.

Step 3. For each value of *n*, compute the Root-Mean-Square (RMS) variation of each segment $X_{t,d}$ where $RMS_d = \sqrt{\frac{1}{n}\sum_{t=1}^{n}(X_{t,d} - E_{m,d})^2}$.

Some series display slow varying trends across them which therefore makes detrending of the process necessary to determine the scale invariant structure around these trends. To do so, a polynomial of order m is fitted to each segment $X_{t,d}$, and the RMS variation formula is adjusted so that it calculates the variation between the data points of the series and the fitted polynomial $P_{m,d}$.

$$RMS_d = \sqrt{\frac{1}{n}\sum_{t=1}^{n}(X_{t,d} - P_{m,d})^2}$$

Step 4. In monofractal time series, the Detrended Fluctuation Analysis (DFA) methodology (on which MFDFA is built on) would follow by computing the average fluctuation F(n) of the different segments for each different scale n. This average fluctuation, in monofractal time series, follows the relationship F(n) = Cn^H hence, by fitting a regression line to the (log) average fluctuation (as was done in the R/S methodology) we would be able to determine the value of the Hurst exponent H.

However, multifractal time series display non-constant Hurst exponents throughout different segments of the stochastic process. As such, these series have local fluctuations with both extreme and small magnitudes. Therefore, in the MFDFA methodology, we compute the q-order RMS calculated as $qRMS_d = {RMS_d}^{q(nq)}$ for the different segments d and for a set of different pre-defined q-orders. The q-order will weight segments with large and small fluctuations differently (positive and negative q's will weigh more heavily segments with large and small fluctuations (RMS), respectively).

Step 5. Once the q-order RMS_d has been calculated for all segments with different lengths n, we can calculate the q-order Hurst exponent as the slope of the regression line for each q-order RMS. Multifractal time series, will display q-dependent Hurst exponents whilst monofractal and white noise time series will display a constant q-order coefficient.

The multifractal spectrum plot is a visual tool that allows to differentiate between monofractal & white noise series, and multifractal processes. The spectrum gives an approximation of the different q-order Hurst exponents present in the series under study. Due to monofractal & white noise series having constant q-order Hurst exponents, their multifractal spectrum plot displays the shape of a small arc. Meanwhile, multifractal stochastic processes have q-dependent Hurst coefficients and as such, their multifractal spectrum plot resembles the shape of a large arc.

Given that white noise processes are generally monofractal (Ihlen, 2012), the presence of multifractality in a time series, suggests the presence of non-random behavior in it, which in financial series translates to weak-form inefficiency. The multifractality spectrum width, calculated as the difference between the maximum and minimum q-order Hurst exponents of the series will therefore be considered as a measure of the informational efficiency of the process.

A large width is considered as evidence of multifractality and thus inefficiency in the series whilst a small width will be taken to indicate monofractality or white noise behavior, which will thus call for further analysis though consideration of the value of the traditional Hurst exponent (2^{nd}-order Hurst coefficient). Right and left – side truncations in the spectrum plot also show whether the series is insensitive to local fluctuations of small or large magnitudes respectively, thus giving insights on the scale invariant structure of the process (Ihlen, 2012).

To determine how the long-term dependence of the series has evolved throughout time, we've conducted a rolling-window analysis with a period length of 252 days (number of trading days in a year) of the 2^{nd}-order Hurst exponent and the multifractal spectrum width.

To evaluate the statistical significance of the results, we generated distinct confidence intervals for each assessed metric. This was achieved by computing the 5th or/and 95th percentiles of each metric across 10,000 simulations of a white noise process. The length of the white noise series was adjusted based on the number of observations in the data under examination, as well as whether the metric was calculated over the entire dataset or using a moving window.

The characteristics of the financial series which are considered in this study are now presented.

3.2 Sample Description

Unlike other commodities, lithium isn't listed in any financial exchange. Therefore, to assess the efficiency of the lithium market, this study has considered two financial assets which best serve as proxies for the behavior of the commodity.

The first of these assets is a lithium future quoted in the Guangzhou Futures Exchange (GFE) in China. The underlying asset of this future is Lithium Carbonate 99.5% which corresponds to lithium readily available to be used in electric car batteries and thus makes this price series especially relevant for the study at hand. The data available for analysis corresponds to daily closing spot prices of Lithium Carbonate 99.5% futures, ranging from January 3^{rd}, 2019, (earliest available date of the retrieved series) to February 1^{st}, 2024, and quoted in Chinese Yuan (CNH).

The second of these assets is an Exchange Traded Fund (ETF) which includes the most relevant listed companies dedicated to the lithium industry. As was demonstrated by Corzo et al. (2022) in their investigation, the companies which conform a given industry serve as good proxies of the behavior of the underlying asset of the latter. As such, the consideration of the price series of an ETF for this study was deemed a valid approach towards studying the efficiency of the lithium market.

There are several ETFs available that provide investors with exposure to lithium and to the complete production chain of electric batteries. These include, Global X Lithium and Battery Tech (LIT), Amplify Lithium and Battery Technology ETF (BATT), WisdomTree Battery Solutions UCITS ETF (CHRG), ARK Autonomous Tech & Robotics ETF (ARKQ) and First Trust Nasdaq Clean Edge Smart Green Energy ETF (QCLN) amongst others. Out of them all, Global X Lithium and Battery Tech has been designated by the author as the most representative ETF for investing in lithium and li-ion batteries. This choice is underpinned by the ETF's global, exclusive focus on lithium and electric battery production, along with it having the most extensive time series data of the available indices (July 2010 to January 2024).

The Global X Lithium and Battery Tech ETF replicates the performance of the Solactive Global Lithium Index (SGLI) (Moreno & Gil-Alana, 2018; Gomes, 2022). The latter is denominated in US$ and "tracks the performance of the largest and most liquid listed companies active in exploration and/or mining of lithium or the production of lithium (Li-Ion) batteries" (Solactive German Index Engineering, 2024, p. 1). The positions forming the index must comply certain rules and guidelines to form part of the latter and are rebalanced on a semi-annual basis.

The index constituents are initially selected based on their adherence to predefined guidelines, thereby establishing the "Index Universe" (Solactive AG, 2023). To be included in this universe, equities must meet various criteria, including significant revenue generation from lithium mining or production of lithium batteries and a free float market capitalization of at least 50 million USD$. Additionally, to be included in this universe, companies not currently part of the index must have a 3-month average traded value of at least 200,000 USD, while those already included in the index should have a minimum average traded value of 100,000 USD.

Following the determination of the index universe, the companies within it are ranked in descending order based on their market capitalization. The largest companies are subsequently selected to constitute the new index. The index is designed to include a minimum of 20 positions and a maximum of 40.

As of February 1st, 2024, the ETF contained 40 positions, shown in Table 1, and had generated a return of 4.03% (p.a.) since 2010. The data used for the analysis of the ETF price series corresponds to the daily closing prices of the Global X Lithium and Battery Tech ETF since July 19th, 2010 (date of ETF inception) to February 1st, 2024, thus considering 3521 observations.

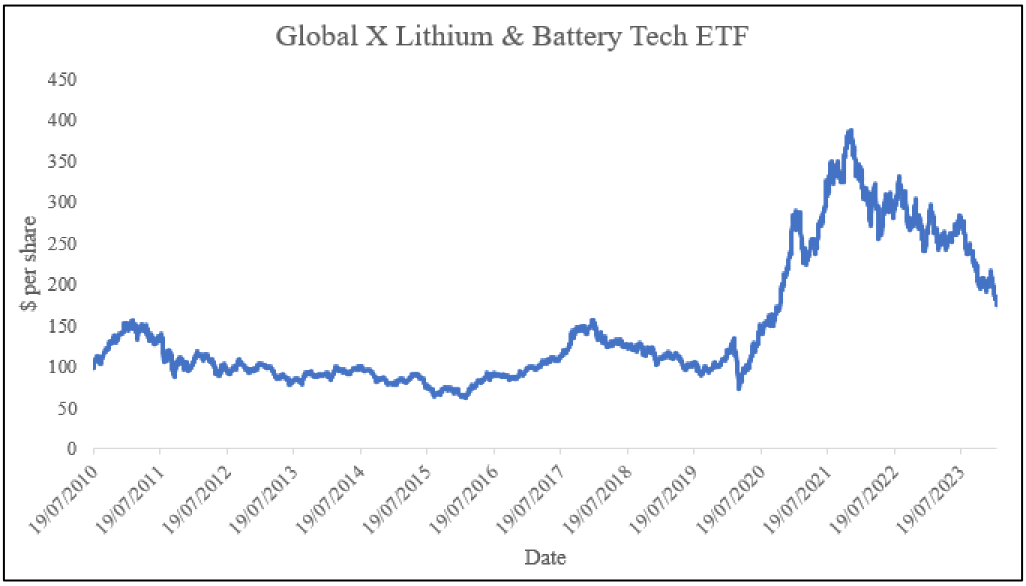
Table 1. Solactive Global Lithium Index composition as of February 1st, 2024

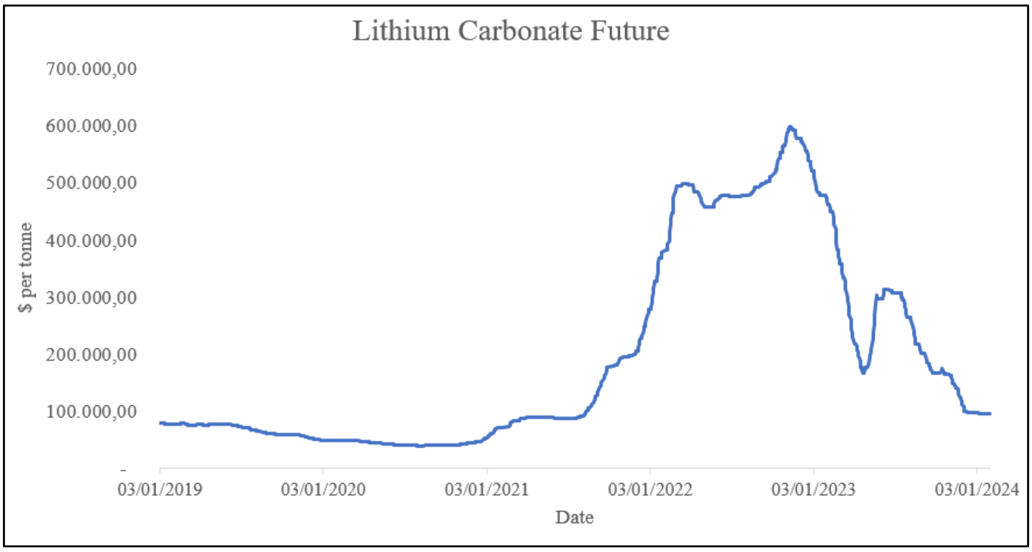
Company Name	Weighting %	Market Price ($)	Shares Held	Market Value ($)
Albemarle Corp	9,64%	122,58	1.261.198,00	154.597.650,84
TDK Corp	6,18%	50,45	1.964.788,00	99.119.347,55
Naura Tech GR-A	5,65%	39,91	2.268.841,00	90.550.058,49
Mineral Resources Ltd	5,32%	46,60	1.828.762,00	85.226.272,55
Pilbara Minerals Ltd	5,11%	2,62	31.279.819,00	81.963.811,43
Contemporary A-A	4,88%	25,93	3.016.609,00	78.210.226,60
Quimica Y-SP ADR	4,58%	47,94	1.530.672,00	73.380.415,68
Tianqi Lithium Corp-A	4,17%	6,73	9.939.141,00	66.914.767,86
Panasonic Holdings Corp	4,14%	9,24	7.176.056,00	66.318.111,51
BYD Co Ltd-h	4,12%	26,82	2.462.374,00	66.040.742,18
Eve Energy co ltd-a	3,93%	5,05	12.460.688,00	62.918.217,35
Ganfeng Lithiu-A	3,75%	5,31	11.319.640,00	60.132.131,07
LG Energy Solution	3,65%	270,11	216.663,00	58.523.506,38
Samsung SDI Co Ltd	3,57%	291,56	196.113,00	57.179.127,49
Tesla INC	3,18%	171,05	298.187,00	51.004.886,35
Guangzhou Tinc-A	2,47%	3,32	11.924.178,00	39.647.157,49
Enersys	2,46%	90,91	434.354,00	39.487.122,14
Igo Ltd	2,44%	4,85	8.052.323,00	39.073.772,32
Wuxi Lead Inte-A	2,03%	3,20	10.169.736,00	32.526.136,36
L&F Co Ltd	1,91%	116,33	263.223,00	30.621.790,82
Sinomine Resou-A	1,77%	5,32	5.321.600,00	28.327.964,44
Yunnan Energy-A	1,72%	5,64	4.883.659,00	27.529.096,33
Arcadium Lithium PLC-C	1,70%	4,30	6.345.982,00	27.262.759,11
Sunwoda Electr-A	1,67%	1,91	14.054.591,00	26.788.886,41
Shanghai Putai-A	1,50%	2,75	8.751.411,20	24.087.668,28
Lucid Group Inc	1,36%	2,49	8.781.185,00	21.865.150,65
Jiangxi Specia-A	1,33%	1,46	14.596.577,00	21.273.240,91
Arcadium Lithium PLC	1,18%	4,11	4.619.040,00	18.984.254,40
Lithium Americas Corp	0,75%	7,08	1.686.063,00	11.937.813,85
Liontown Resources Ltd	0,71%	0,84	13.537.388,00	11.342.467,30
Sigma Lithium Corp	0,54%	14,65	590.056,00	8.644.320,40
AMG Critical Mat	0,52%	26,00	322.566,00	8.387.422,38
Lithium Americas	0,42%	5,18	1.292.451,00	6.689.384,01
Eramet	0,40%	76,65	82.752,00	6.342.598,73
Patriot Battery	0,36%	5,34	1.078.598,00	5.755.031,67
Latin Resources Ltd	0,20%	0,13	23.533.205,00	3.121.316,66
Piedmont Lithium Inc	0,15%	12,68	187.250,00	2.374.330,00
Sayona Mining Ltd	0,14%	0,03	87.509.986,00	2.208.132,41
Core Lithium Ltd	0,13%	0,10	20.650.357,00	2.137.718,12
Standard Lithium Ltd	0,11%	1,13	1.540.449,00	1.746.883,61

Source: Own elaboration

Figure 1 shows a graphical representation of both price series.

Figure 1. Graphical representation of the price series of the Global X Lithium & Battery Tech ETF and the Lithium Carbonate Future





Source: Own elaboration

The MFDFA and multifractal spectrum width methodologies however consider time series of returns as their input. As such, before conducting our analysis, we've computed the daily returns of the Global X Lithium and Battery Tech ETF and Lithium carbonate future price series. The results represent the data that will effectively be used in our analysis. The main statistics of the latter are presented in Table 2.

Table 2. Main statistics of the return series of the Global X Lithium & Battery Tech ETF and the Lithium Carbonate Future

Series	Num. Obs	Mean (%)	Std (%)	Max (%)	Min (%)	Skew	Kurtosis
Global X Lithium & Battery Tech ETF	3520	-0,01%	0,63%	4,89%	-4,65%	0,36	5,66
Lithium Carbonate Future	1216	-0,01%	0,50%	2,15%	-3,13%	-0,45	5,25

Source: Own elaboration

Table 2 shows both series have similar mean and standard deviation metrics. However, the Global X Lithium & Battery Tech ETF displays a broader range of values within the data due to having both a larger maximum and a smaller minimum value.

In terms of skewness, this coefficient serves as a measure of the symmetry within the distribution of returns. When the mean of the latter is close to 0, a positive skewness indicates that the distribution has a higher concentration of the mass of returns located to the right, suggesting more days with positive returns. Conversely, a negative skewness indicates a higher concentration of returns to the left, indicating more days with negative returns. As such, the Global X Lithium & Battery Tech ETF demonstrates a positive skew, indicating more days with positive returns, while the Lithium Carbonate Future exhibits a negative skew, suggesting more days with negative returns.

Additionally, the kurtosis coefficient measures the 'tailedness' of the distribution, and therefore the concentration of data within the latter, relative to the mean. Both series have a similar kurtosis coefficient, greater than three, thus indicating that most of their returns are concentrated around their average value.

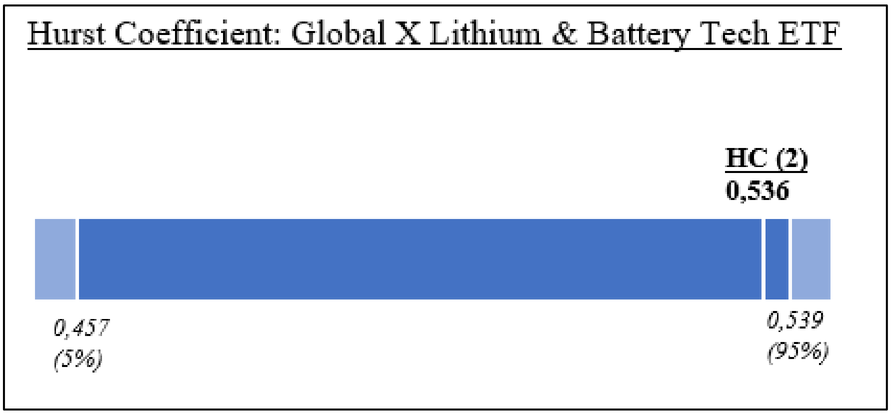
A closer examination of the Lithium future data revealed that, in a considerable number of periods (65%) throughout the series, the asset's price level remained unchanged, resulting in a 0% return between those dates. This established a horizontal trend in the series and thus made the latter unsuitable for potential investments and for the correct application of the multifractal methodology on it. As such, Chapter 4 of the study presents the results of the conducted analysis for the Global X Lithium and Battery Tech ETF, whilst the results of the fractal analysis for the Lithium Carbonate Future are presented in the appendix.

CHAPTER 4. FINANCIAL DATA ANALYSIS

4.1 Fractal Analysis

The second-order Hurst coefficient calculated using the MFDFA methodology, for the Global X Lithium and Battery Tech ETF is displayed in Figure 2. This coefficient represents the Hurst exponent derived from applying DFA to the series, offering insights into the average fractality of the process.

Figure 2. Second-order Hurst coefficient and Confidence Interval (5%,95%) for the Global X Lithium and Battery Tech ETF for the period spanning July 2010- February 2024.



Source: Own elaboration

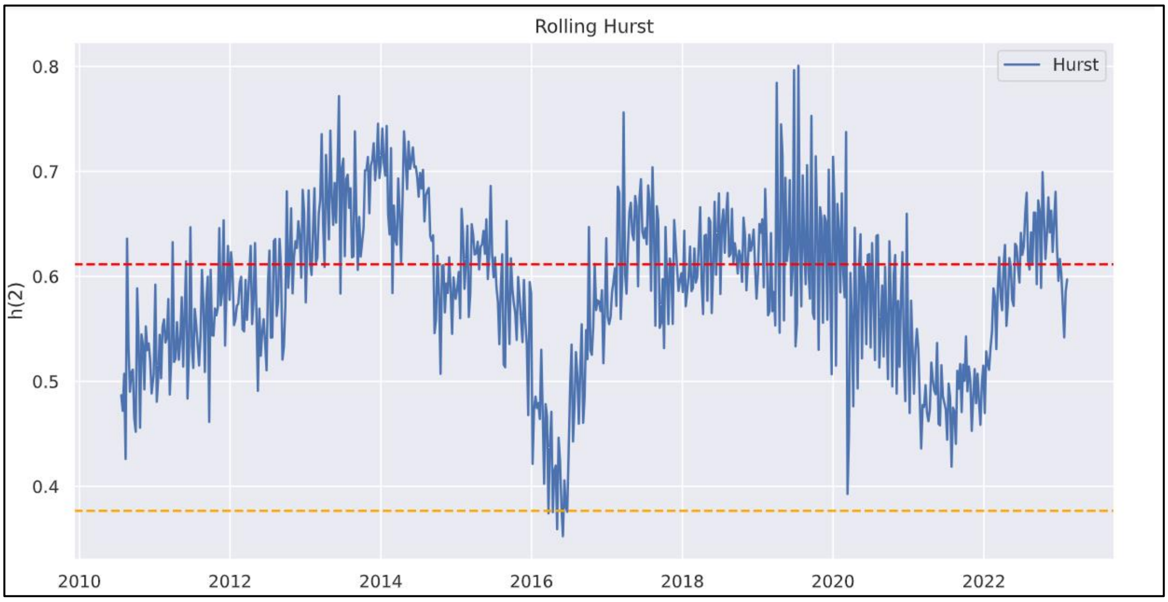
The results obtained indicate that, over its lifespan, the Global X Lithium & Battery Tech ETF has exhibited predominantly random behavior, aligning with the Efficient Market Hypothesis (EMH).

4.2 Rolling window Fractal Analysis

To determine how efficient has the Global X Lithium & Battery Tech ETF price series behaved throughout time, we've estimated the second-order Hurst coefficient on a moving window of 252 days (1 year in trading days). The considered confidence interval has also been recalculated considering only 252 observations as opposed to the total number of data points in the entire series. This has returned a wider confidence interval than the one displayed in Figure 2, as lower number of observations leads to a wider range of possible values for the calculated metrics.

Figure 3 shows the results obtained for the rolling window analysis. The graph shows that the series has remained for most of its lifetime, inside the realms of efficiency. However, in certain periods, the process has moved from efficient behavior to persistency.

Figure 3. Evolution of the second-order Hurst coefficient throughout time, of the log-returns of the Global X Lithium and Battery Tech ETF.



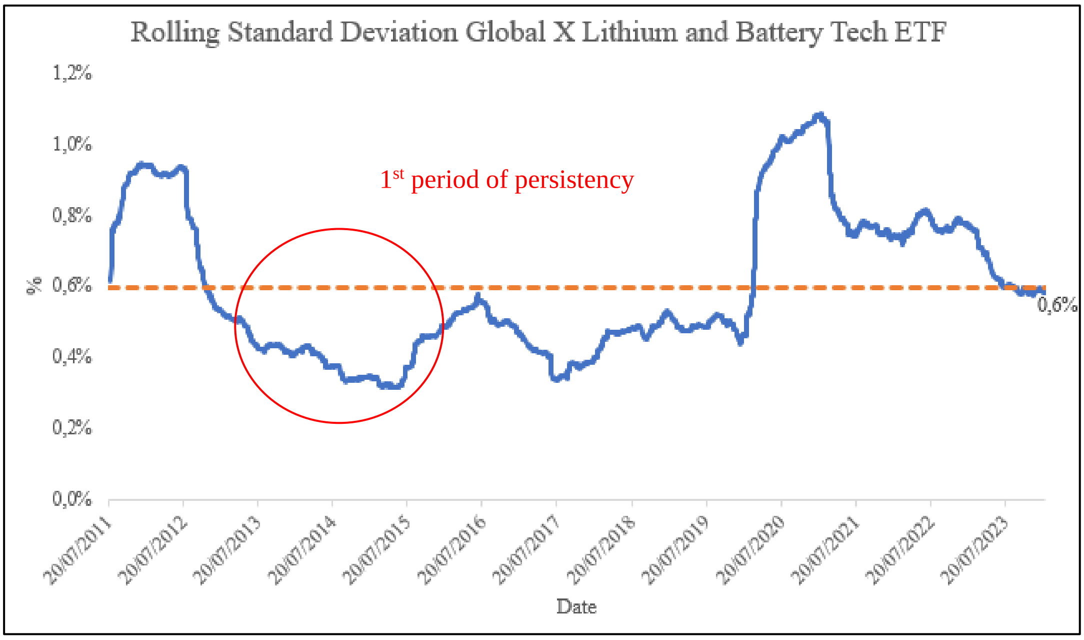
Source: Own elaboration

The first period of persistency occurred from July 2013- July 2015. During this time span, the technology underlying Li-ion batteries underwent a process of continuous improvement (Anthony, 2013) and the viability of electric cars started to take shape as more economies began to adopt this mode of transportation and to finance the construction of related infrastructure (Evarts, 2015; Wald, 2013).

These positive developments signaled to external economic agents the significant role Li-ion batteries were expected to play in the future. However, they also introduced an element of uncertainty into the market regarding the precise consequences these events would have on lithium prices and related players.

In the previous years to the 1st persistency period, the volatility of returns for the Global X Lithium and Battery Tech ETF surpassed its historical average of 0.6%, by approximately 0.4% portraying the aforementioned increased uncertainty. However, once these growth expectations started to materialize, the standard deviation of the returns rapidly decreased, as the series underwent successive trading days with positive constant returns which translated into trend-reinforcing behavior. Figure 4 illustrates the fluctuations in the volatility of the series, depicted by the standard deviation of returns for the Global X Lithium and Battery Tech ETF, tracked under a 252-day moving average.

Figure 4. Evolution of the standard deviation throughout time, of the log-returns of the Global X Lithium and Battery Tech ETF.



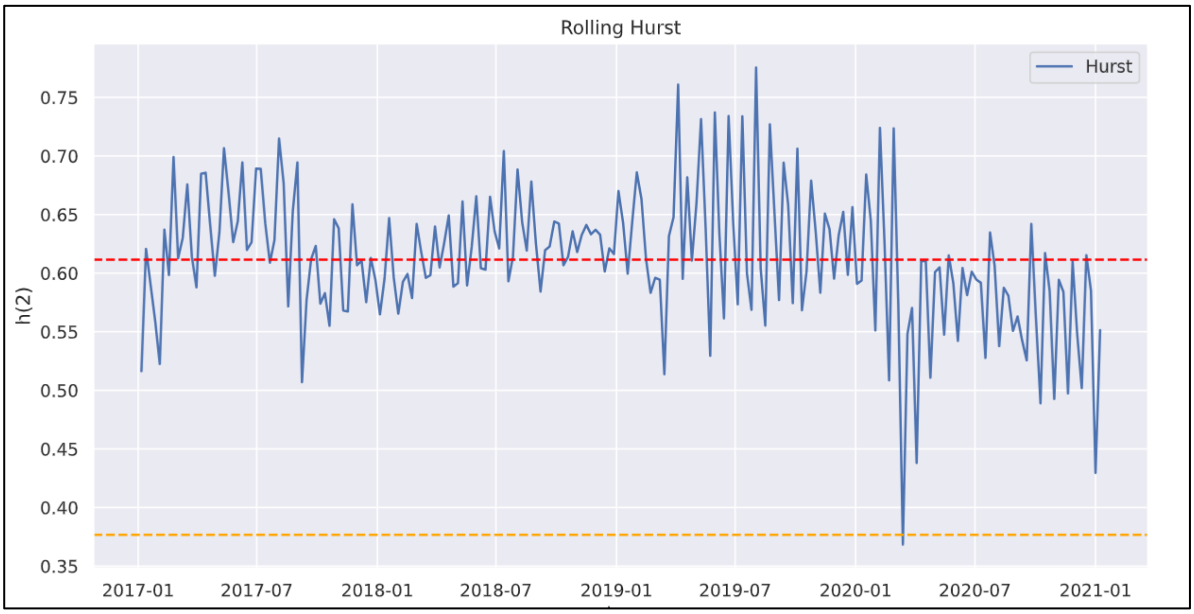
Source: Own elaboration

Overall, these developments led the Global X Lithium and Battery Tech ETF to exhibit long-term memory and thus persistent behavior. However, efficiency was soon restored by August 2016, a situation that continued until July 2017, when the ETF began to display long-term memory again.

From July 2017 - July 2020, the ETF entered its second period of persistency with the series' Hurst coefficient approaching and surpassing the upper limit of the confidence interval. To better understand the fractal dynamics of the series during this period, we've conducted a complementary rolling window fractal analysis, for this specific period. The motivation behind the latter was that, compared to the other persistency periods, from

2017-2020 the series' Hurst coefficient behaved in a more random manner, consistently surpassing the upper limit of the confidence interval and then rapidly reverting within efficiency realms. As such, a closer look was warranted to better understand the underlying behavior of the series The results obtained for this specific time period are shown in Figure 5.

Figure 5. Evolution of the second-order Hurst coefficient between 2017-2020, of the log-returns of the Global X Lithium and Battery Tech ETF.

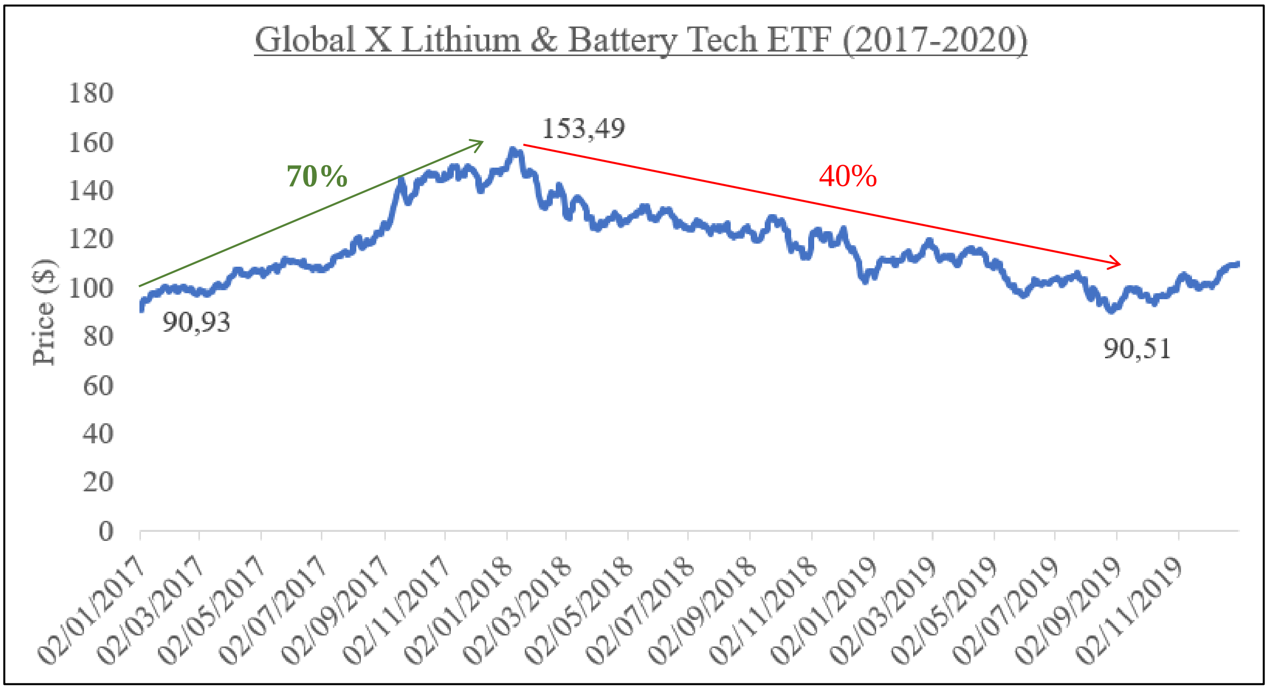


Source: Own elaboration

Figure 5 illustrates that between 2017 and 2020, the Global X Lithium and Battery Tech ETF exhibited persistent behavior on several occasions.

During this timeframe, the prices of ETFs followed a constant slow-varying upward trend, beginning in 2017 and continuing into early 2018, ultimately surging by 70% in value. The subsequent 2 years however, ETF prices reversed, causing the series to relinquish most of its gains and experience a cumulative -40% return by the time it reached its trough in October 2019. Figure 6 visually portrays this price evolution of the series, over the mentioned period.

Figure 6. Graphical representation of the price series of the Global X Lithium & Battery Tech ETF from 2017-2020



Source: Own elaboration

These price developments, occurred as a direct consequence of a speculative boom fueled by optimistic projections on the various applications of lithium, and the emergence of oversupply concerns as the announcement of forthcoming investments in the extraction process of the commodity increased.

To begin with, in 2017, the long-term prospects of lithium as a commodity, and lithium related players was becoming increasingly optimistic. This was due to double digit growth in electric vehicle sales (Global X Research Team, 2018), supported by government subsidies, and a deflationary trend in renewable energy costs (Saefong, 2018), which seemed to mark the start of a growth cycle for lithium. This favorable demand backdrop was accentuated by some of the most prominent institutional investors, such as Blackrock, who began to invest in companies involved in the extraction and processing processes of the commodity (Sanderson, 2017). Altogether, this contributed to a herding behavior (Banerjee, 1992) amongst investors and the consequent price hike and speculative boom experienced by lithium prices and lithium related players within that period.

However, in January 2018, Australia, seeking to capitalize on the positive outlook of the commodity, announced an upcoming expansion of great scale, of mineral mines across the country, aided by Chinese investments (Jamasmie, 2018). At the time, this forecasted growth in capacity would effectively make Australia the largest global lithium supplier, closely followed by Chile. Therefore, the market anticipated the outbreak of a potential oversupply of lithium, (Jamasmie, 2018) which caused the commodity price to decline sharply by 16% (Smith, 2019) and eventually reversed the totality of the gains achieved by the ETF throughout 2017.

Overall, these circumstances made the Global X Lithium & Battery Tech ETF follow pronounced positive and negative trends, behaving in a trend-reinforcing manner and thus departing from the EMH, as is shown in Figure 5.

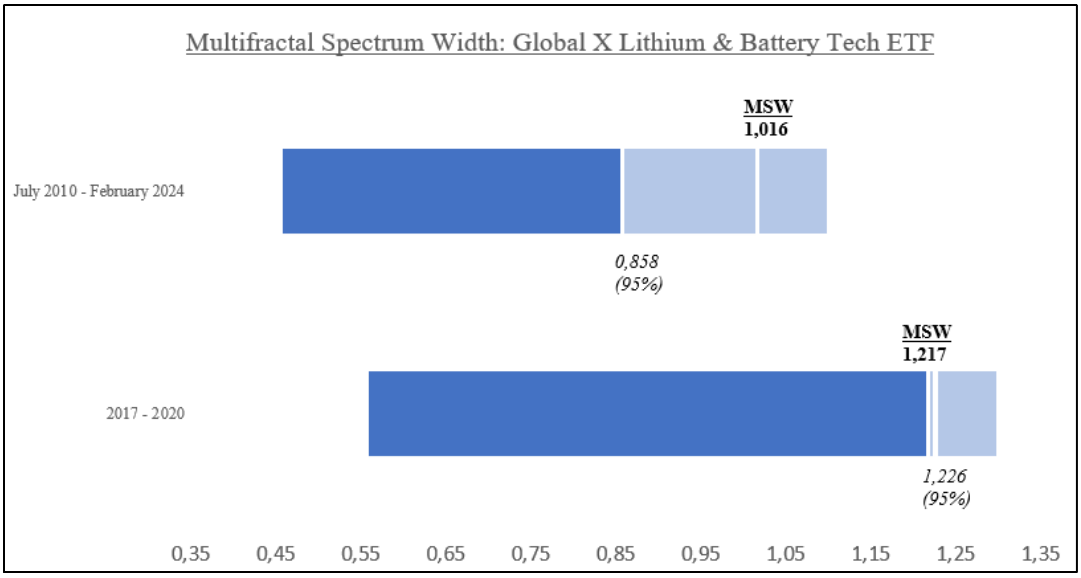
In July 2020, efficiency within the series was restored, and remained that way until July 2023, when the ETF briefly displayed a persistent conduct. The latter can likely be attributed to the financial turmoil experienced by global markets due to the rise of inflation levels, interest rates, the outbreak of the Russo-Ukraine war, and the energy crisis which followed.

One of the implications drawn from this analysis is that during the years that followed the outbreak of the SARS-CoV-19 and the ensuing economic and financial difficulties, the ETF continued to behave efficiently. This finding contradicts (Aslam, Mohti, & Ferreira, 2020), who by studying the multifractal properties of the European stock market during the COVID-19 pandemic, found that the outbreak of this crisis caused stock prices to exhibit multifractality and thus weak-form inefficiency.

4.3 Multifractal Spectrum Analysis

Figure 7 presents the multifractal spectrum width of the Global X Lithium and Battery Tech ETF, for the entire sample period and the specific period of 2017-2020. Given the varying number of observations in both series, distinct confidence intervals have been constructed using white noise simulations with sample sizes equivalent to those of the respective data. The outcomes of these simulations are also detailed in Figure 7.

Figure 7. Multifractal spectrum width and Confidence Interval (95%) for the Global X Lithium and Battery Tech ETF for the complete time period under study (2010-2024) and the specific period ranging from 2017-2020.



Source: Own elaboration

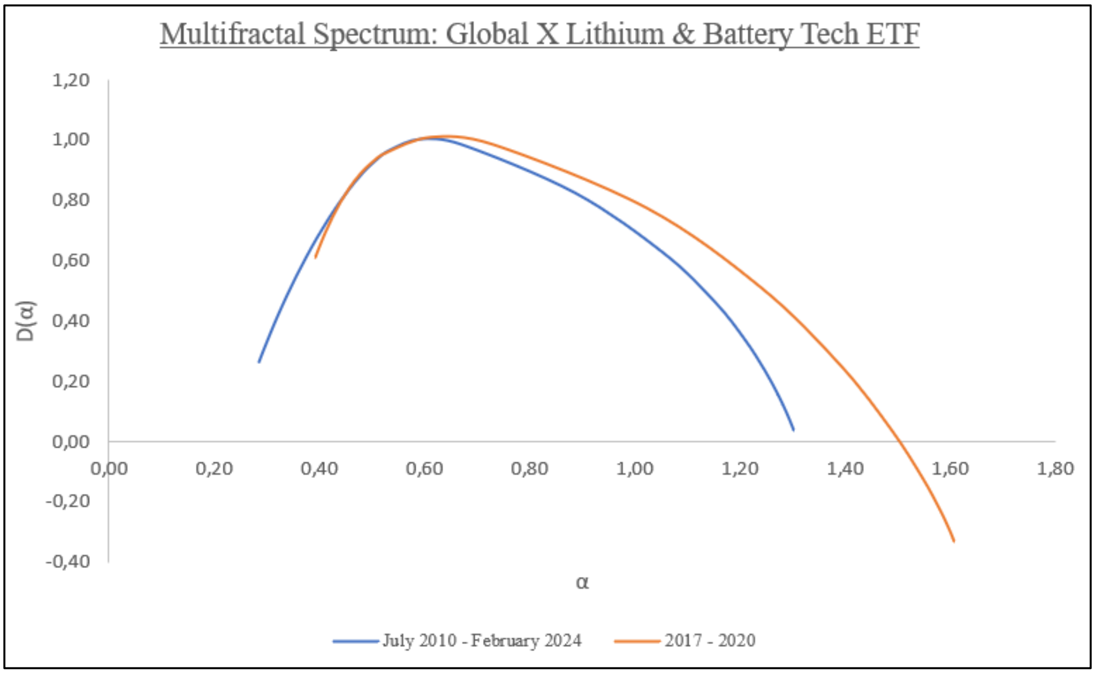
These results appear to challenge those derived from the Hurst coefficient analysis. Specifically, the multifractal spectrum width of the Global X Lithium and Battery Tech ETF lies beyond its corresponding confidence interval, suggesting inefficiency throughout, whilst during the period 2017-2020, the multifractal spectrum width falls within the confidence interval, implying adherence to the Efficient Market Hypothesis.

However, upon contrasting these findings with those of the rolling window analysis conducted later, the conclusions drawn from both the Hurst coefficient and the multifractal spectrum width align. Furthermore, the rolling window analysis reveals significant volatility in the Multifractal Spectrum Width results, thereby explaining the initial large level obtained for the aggregate metric.

To supplement the multifractal spectrum width analysis, we've plotted the multifractal spectra of the two series, as depicted in Figure 8. These spectra provide additional insights, particularly highlighting the larger spectrum width of the Global X Lithium and Battery Tech ETF during the 2017-2020 period, consistent with the results obtained from the Hurst coefficient analysis.

Furthermore, both ETF series exhibit right truncations in their singularity spectra, indicating consistency with respect to small fluctuations within the series. This suggests that periods of persistence have primarily occurred during significant movements in the series' returns rather than through minor variations.

Figure 8. Multifractal singularity spectrum of the Global X Lithium and Battery Tech ETF for the entirety of the series and the period encompassing the years 2017-2020.

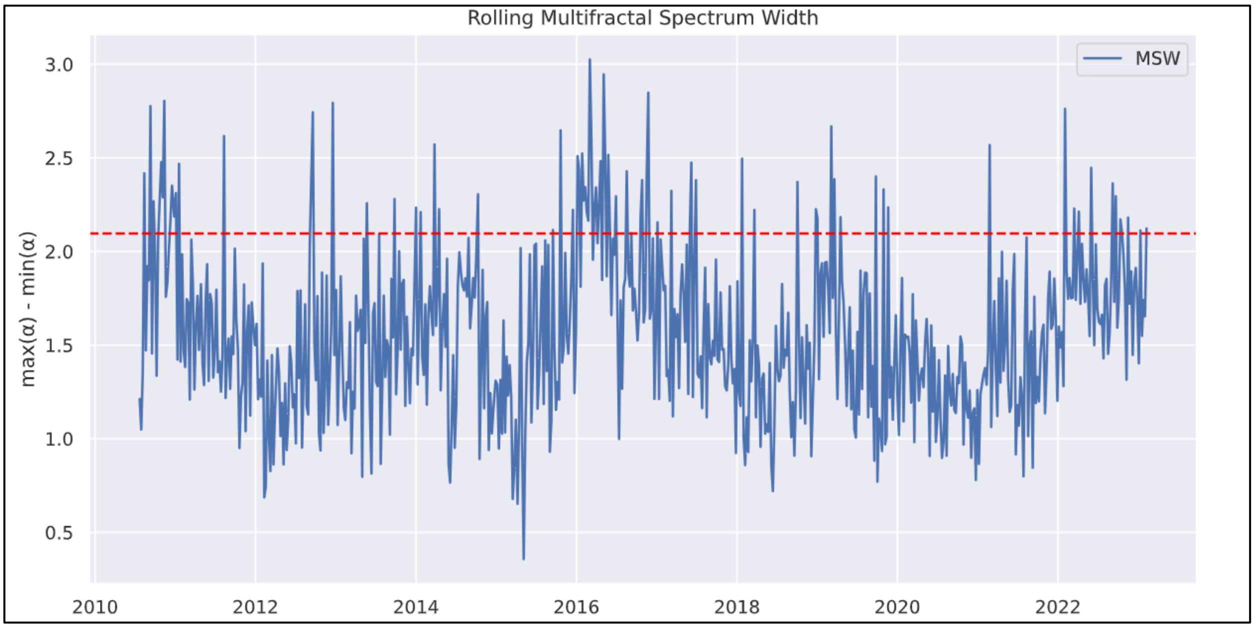


Source: Own elaboration

4.4 Rolling window multifractal spectrum width

The rolling window multifractal spectrum width analysis exhibits a close resemblance to the results and conclusions obtained using the Hurst coefficient. Figure 9 displays the results of the rolling window analysis for the Global X Lithium and Battery Tech ETF.

Figure 9. Evolution of the Multifractal Spectrum Width throughout time, of the log-returns of the Global X Lithium and Battery Tech ETF.



Source: Own elaboration

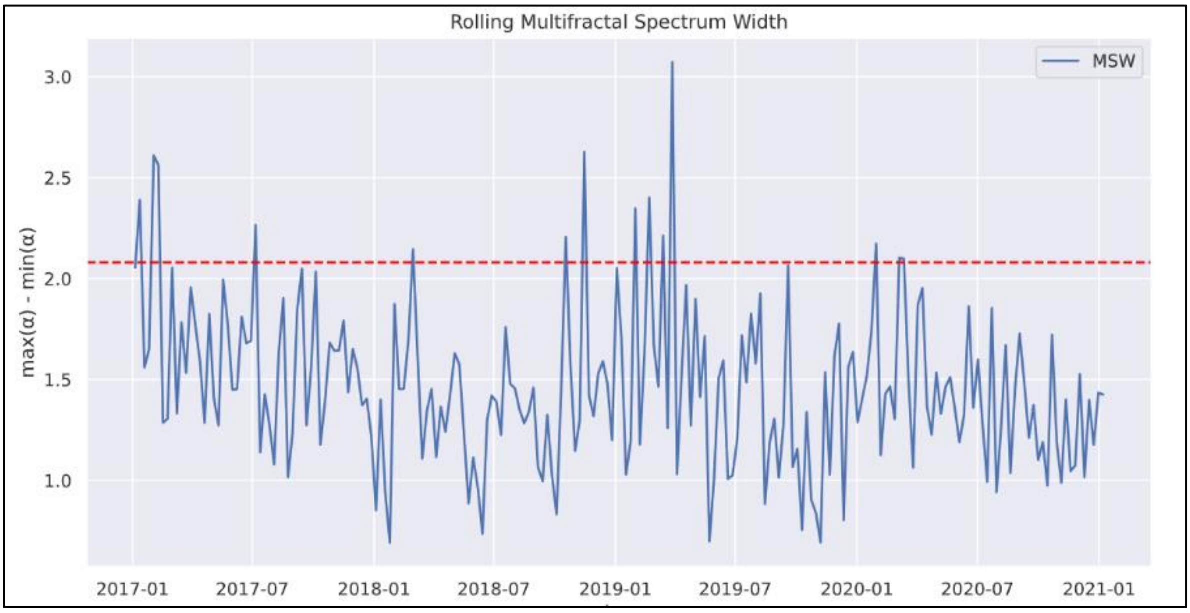
Comparing the rolling window distribution of the Hurst coefficient with that of the multifractal spectrum width, both appear similar in shape. As such, the latter graph highlights how the series has predominantly stayed within efficiency levels, with the exception of two periods of inefficiency in July 2011 and January 2017, which align considerably with those identified using the Hurst coefficient. In addition, it seems that through the use of the multifractal spectrum width, we're able to identify periods of inefficient behavior within the series, months before the rolling Hurst signals their existence. Consequently, this suggests the multifractal spectrum width might be a leading indicator of the Hurst coefficient.

The rolling window analysis further illustrates the multifractal spectrum width's high volatility when applied to the ETF, thus providing additional insights into the initial findings.

The multifractal spectrum width rolling analysis for the 2017-2020 period also bears resemblance to the Hurst coefficient analysis.

Figure 10 displays the results of the rolling window analysis for the Global X Lithium and Battery Tech ETF for the specific period ranging from 2017-2020. The figure depicts an initial period of inefficiency followed by a consistent approximation to the upper boundary of the confidence interval.

Figure 10. Evolution of the multifractal spectrum width between 2017-2020, of the log-returns of the Global X Lithium and Battery Tech ETF.



Source: Own elaboration

Overall, both conducted analyses indicate that the Global X Lithium & Battery Tech ETF has predominantly demonstrated throughout its lifetime weak-form informational efficiency. As such, it demonstrates that the lithium market is efficient and presents suitable investment opportunities for public and private investors looking to gain exposure to lithium and Li-ion batteries, and thus support the efforts of the upcoming global energy transition.

Furthermore, the use of the multifractal spectrum width, as a measure of efficiency throughout the series, yields promising results yet warrants further research. The rolling window analysis returns similar insights to those obtained by the Hurst coefficient and suggests the multifractal spectrum width might serve as a leading indicator to the latter, thus encouraging its use as a complementary tool, as it provides more information on the scale-invariant structure of the series.

However, when considering the entire series , the findings of both metrics contradict each other, likely due to the significant volatility exhibited by the multifractal spectrum width when applied to the Global X Lithium & Battery Tech ETF series. Therefore, before deeming it a reliable measure of inefficiency, it's imperative to thoroughly examine the intricacies of this metric's construction and validate its results by applying it to other series.

CHAPTER 5. CONCLUDING REMARKS

Lithium is considered one of the critical minerals of the energy transition due to its widespread role in the production process of electric vehicle batteries. However, according to current studies, the commodity is poised to encounter supply shortages due to the existing supply being insufficient to meet the anticipated demand. Therefore, public and private investments in this commodity are considered necessary to ensure the adequate evolution of the energy transition.

This investigation uses the MFDFA methodology alongside the multifractal spectrum width to study the efficiency of a Lithium Future, quoted on the Guangzhou Future Exchange (GFE), and the Global X Lithium and Battery Tech ETF, both considered proxies for the behavior of the Lithium commodity.

According to the results obtained from the analysis of the Global X Lithium and Battery Tech ETF, the lithium market is efficient, having exhibited only some departures from efficiency in certain situations. The latter, correspond to moments where market reactions to news regarding the potential applications and possible oversupply of lithium translated into ETF returns in the form of trend-reinforcing behavior. However, all these departures were short-lived, as market efficiency was soon restored once the effect of the news had dissipated. Therefore, there is no reason to believe that financial barriers in the form of market inefficiencies pose an obstacle for public and private investments in companies involved in the lithium market and consequently, to the gain in exposure to the commodity, and the subsequent necessary development of the latter.

Overall, these results are reassuring for private and public investors in the lithium commodity, as well as for regulators and society. This is mainly because market efficiency is a prerequisite for the efficient allocation of resources, and as such, by demonstrating that the lithium market is an efficient one, this study acknowledges the viability of potential investments in this critical mineral, which are currently paramount for the development of electric vehicles and consequently, for the global energy transition.

Nevertheless, as the role of lithium in the modern economy solidifies, we expect more countries to develop and issue an independent tradable asset for this commodity, enabling the realization of more concrete studies of the efficiency of the market prices of the asset.

The investigation’s findings also suggest further research is needed to validate the multifractal spectrum width as a suitable efficiency metric. The latter, when applied using a rolling window analysis, presents similar evidence regarding market efficiency to the results obtained by the MFDFA methodology and may serve as a leading indicator to the Hurst Coefficient. Furthermore, its use is promising for it provides more information regarding the scale-invariant structure of the series.

However, the volatility of the metric makes it yield different conclusions to the ones obtained using the Hurst coefficient when applied to the series in its entirety. As such, further study regarding its construction and, its validation through application to other processes is required before it can be considered a reliable efficiency measure.

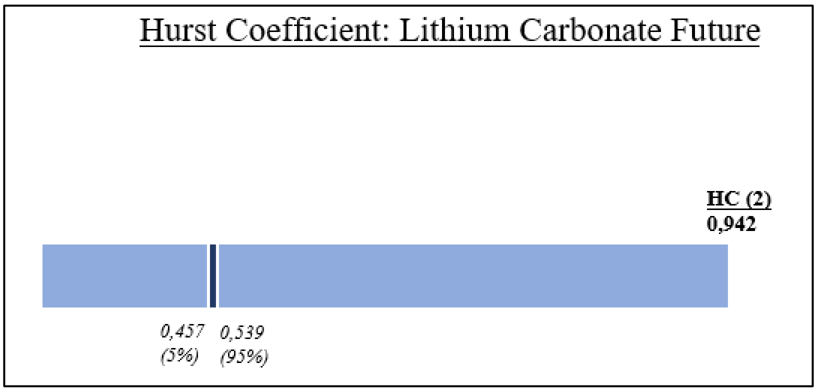
Regarding the lithium carbonate future, it proved inadequate for MFDFA analysis purposes due to illiquidity issues and measurement errors incurred when retrieving data from said series. The leading cause behind these issues are long-lasting financial barriers for foreign agents trading assets quoted in Chinese Exchanges. Hence, to attract the necessary investment for investors seeking exposure to the lithium commodity through this instrument, and thus make the series suitable for analysis, international cooperation and deregulation between China and foreign nations should be promoted.

To the best of our knowledge, this study is the first to analyze the long-term memory of lithium using the MFDFA methodology and to expand on the latter by considering the multifractal spectrum width as a potential measure of efficiency.

APPENDIX. LITHIUM FUTURE ANALYSIS

The second-order Hurst coefficient calculated using the MFDFA methodology, for the Lithium carbonate future is displayed in Figure 11.

Figure 11. Second-order Hurst coefficient and Confidence Interval (5%,95%) for the Lithium Carbonate Future for the period spanning January 2019 - February 2024.



Source: Own elaboration

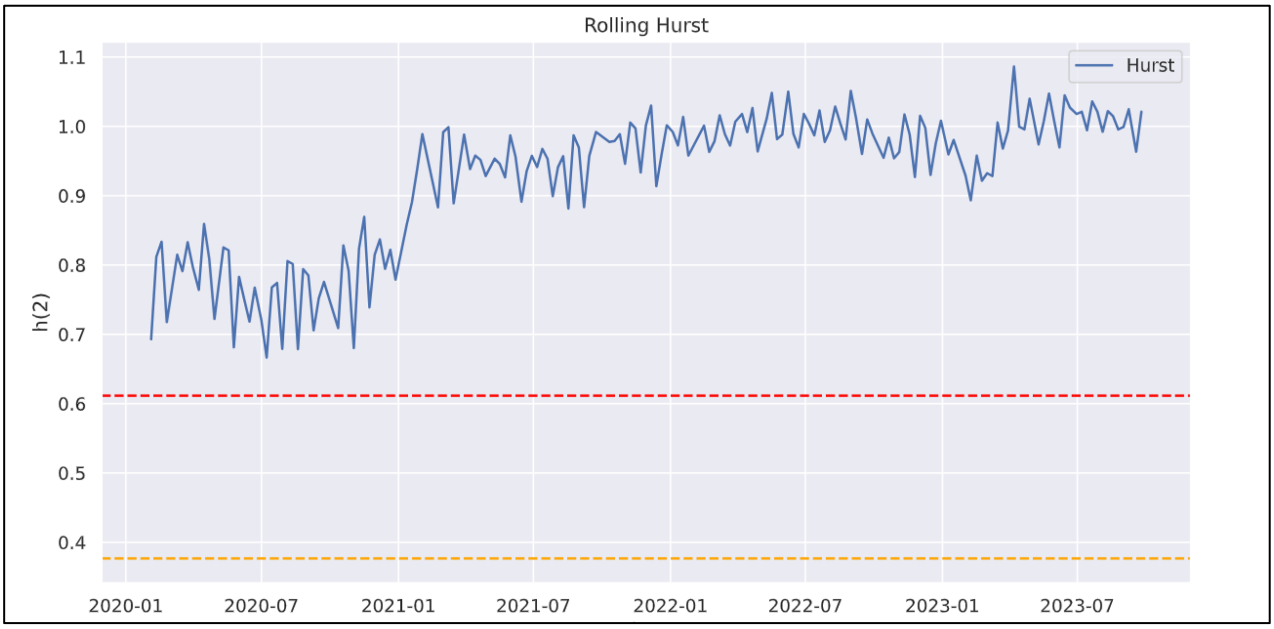
The latter shows the Hurst exponent of the lithium carbonate future is approximately 0.9, a value significantly outside the confidence interval of efficiency, which suggests a pronounced persistent behavior within it. The leading cause of these findings is the discussed horizontal trend within the series, which in turn is most likely a consequence of measurement errors and trading illiquidity in the asset, due to financial barriers.

Trading in Chinese exchanges is restricted in several ways to foreign investors. For example, some securities traded in these exchanges are only made available through regulation, to mainland Chinese investors (Shuye Wang & Jiang, 2004). This decreases the liquidity of the securities traded in Chinese exchanges and poses transaction barriers for foreign investors who must decide whether to face these additional risk considerations.

As mentioned earlier, the studied Lithium Carbonate Future is traded on the Guangzhou Futures Exchange (GFE) in China, and access to data from this exchange is somewhat restricted for foreign investors. This limitation opens the possibility of measurement errors in the available figures. Additionally, the aforementioned financial barriers for foreign agents trading on Chinese exchanges lead to market illiquidity, restricting the daily number of operations and consequently limiting changes in price quotes, independent on the availability of new information. Such behavior causes the series to exhibit trend-like patterns with no return, contributing to the persistence highlighted by the Hurst coefficient. Notably, ample liquidity is a relevant characteristic of efficient markets (Chung & Hrazdil, 2010), aligning with the results obtained and the explanations provided.

Figure 12 shows the results obtained for the Lithium Carbonate Future of the rolling window fractal analysis. As expected, the graph shows that the series has consistently behaved in a persistent manner returning a Hurst coefficient greater than the upper limit of the confidence interval, for the entirety of the period under study.

Figure 12. Evolution of the second-order Hurst coefficient throughout time, of the log-returns of the Lithium Carbonate Future.



Source: Own elaboration

CHAPTER 6. REFERENCES

Annis, A. A., & Lloyd, E. H. (1976). The expected value of the adjusted rescaled Hurst range of independent normal summands. *Biometrika, 63*, 111-116.

Anthony, S. (2013, April 17). New lithium-ion battery design that's 2,000 times more powerful, recharges 1,000 times faster. *ExtremeTech*.

Aslam, F., Mohti, W., & Ferreira, P. (2020). Evidence of Intraday Multifractality in European Stock Markets during the Recent Coronavirus (COVID-19) Outbreak. *International Journal of Financial Studies*.

AuAg. (2024, April 11). *AuAg Funds Research Centre*. Retrieved from Investing in Lithium: https://www.auagfunds.com/research-centre/resources/investing-in-lithium#benefits

Banerjee, A. V. (1992). A Simple Model of Herd Behavior. *The Quarterly Journal of Economics*, 797-817.

Cannon, M. J., Percival, D. B., Caccia, D. C., Raymond, G. M., & Bassingthwaighte, J. B. (1997). Evaluating scaled windowed variance methods for estimating the Hurst coefficient of time series. *Physics A, 241*, 606-626.

Ceballos, R. F., & Largo, F. F. (2017). On The Estimation of the Hurst Exponent Using Adjusted Rescaled Range Analysis, Detrended Fluctuation Analysis and Variance Time Plot: A Case of Exponential Distribution. *Imperial Journal of Interdisciplinary Research, 3*(8), 424-434.

Chung, D., & Hrazdil, K. (2010). Liquidity and market efficiency: A large sample study. *Journal of Banking & Finance*(34), 2346-2357.

Corzo Santamaría, T., Martin-Bujack, K., Portela, J., & Sáenz-Diez, R. (2022). Early market efficiency testing among hydrogen players. *International Review of Economics and Finance, 82*, 723-742.

Degiro. (2024, April 11). *Degiro Investing Commodities*. Retrieved from Lithium: https://www.degiro.com/uk/investing/commodities/lithium

Díaz M, H. A., & Córdova, F. (2022). On the meaning of Hurst entropy applied to EEG data series. *Procedia Computer Science, 199*, 1385-1392.

Egbue, O., & Long, S. (2012). Critical Issues in the Supply Chain of Lithium for Electric Vehicle Batteries. *Engineering Management Journal, 24*(3), 52-62.

Energy Information Administration. (2021). *EIA projects global conventional vehicle fleet will peak in 2038.*

Energy Information Administration. (2023). *EIA projections indicate global energy consumption increases through 2050, outpacing efficiency gains and driving continued emissions growth.* Washington DC.

Energy Tranistions Commission. (2023). *Materials and Resource Requirements for the Energy Transition.* London.

Evarts, E. C. (2015, October 28). Lithium batteries: To the limits of lithium. *nature, 526*, 93-95.

Fama, E. F. (1965, September). Random Walks in Stock Market Prices. *Financial Analysts Journal*, 55-59.

Galluccio, S., Caldarelli, G., Marsili, M., & Zhang, Y.-C. (1997). Scaling in currency exchange. *Physics A, 245*, 423-436.

Global X Research Team. (2018, February 1). Benchmark: What Drove Lithium Prices in December? *Global X*.

Gomes, J. F. (2022). *Clean energy ETFs - A new sustainable trend.* Universidad Europeia.

Graham, J. D., Rupp, J. A., & Brungard, E. (2021, October 13). Lithium in the Green Energy Transition: The Quest for Both Sustainability and Security. *Sustainability, 13*(20).

Huang, E. (2019, August 7). Beijing's subsidy cut is starting to hurt China's largest electric car company. *Quartz*.

Hurst, H. E. (1951, January). Long-Term Storage Capacity of Reservoirs. *Transactions of the American Society of Civil Engineers, 116*(1), 770-808.

Ihlen, E. A. (2012). Introduction to multifractal detrended fluctuation analysis in Matlab. *Frontiers in Physiology, 3*(141).

International Energy Agency. (2023). *Global EV Outlook 2023.*

Jamasmie, C. (2018, June 12). Australia takes over Chile as world's No.1 lithium producer. *Mining.com*.

Jamasmie, C. (2018, May 17). China's Tianqi Lithium buys minority stake in SQM for $4.1bn. *Mining.com*.

Jamasmie, C. (2019, October 23). Most Chilean mines disrupted as Codelco workers join in country-wide strike. *Mining.com*.

Kristoufek, L., & Vosvrda, M. (2014, March). Commodity futures and market efficiency. *Energy Economics, 42*, 50-57.

Mandelbrot, B. B. (1975). Limit Theorems on the Self-Normalized Range for Weakly and Strongly Dependent Processes. *Z. Wahrscheinlichkeitstheorie verw*, 271-285.

Mandelbrot, B., & Wallis, J. (1969). Some long-run properties of geophysical records. *Water Resources Research, 5*(2), 321-340.

Mielniczuk, J., & Wojdyllo, P. (2007, May). Estimation of Hurst exponent revisited. *Computational Statistics & Data Analysis, 51*(9), 4510-4525.

Moreno, M. M., & Gil-Alana, L. A. (2018). LITHIUM INDUSTRY IN THE BEHAVIOUR OF THE MERGERS AND ACQUISITIONS IN THE U.S. OIL AND GAS INDUSTRY. *Energy Sources*.

Pagan, A. (1996). The rise and fall and rise... of the business cycle. *CEPR Discussion Papers* (349), 36.

Palágyi, Z., & Mantegna, R. N. (1999). Empirical investigation of stock price dynamics in. *Physica A, 269*, 132-139.

Qian, B., & Rasheed, K. (2005). *HURST EXPONENT AND FINANCIAL MARKET PREDICTABILITY.*

Restrepo, N., Uribe, J. M., & Guillen, M. (2023). Price bubbles in lithium markets around the world. *frontiers*.

Sadique, S., & Silvapulle, P. (2001). Long-term memory in stock market returns: international evidence. *International Journal of Finance & Economics*.

Saefong, M. P. (2018, February 2). How lithium and cobalt are getting a boost from Tesla, Apple batteries. *Market Watch*.

Sanderson, H. (2017, September 15). BlackRock fund bets big on electric future for lithium. *Financial Times*.

Sanin, M. E. (2022, June 16). Why are lithium prices skyrocketing? *Polytechnique insights*.

Scalas, E. (1998, May). Scaling in the market of futures. *Physica A: Statistical Mechanics and its Applications, 253*(1-4), 394-402.

Shuye Wang, S., & Jiang, L. (2004). Location of trade, ownership restrictions, and market illiquidity: Examining Chinese A- and H-shares. *Journal of Banking & Finance, 28*(6), 1273-1297.

Siokis, F. M. (2014, February 1). European economies in crisis: A multifractal analysis of disruptive economic events and the effects of financial assistance. *Physic A: Statistical Mechanics and its Applications, 395*, 283-292.

Smith, T. (2019, November 4). Lithium Stocks Look Poised to Charge Higher . *Investopedia*.

Solactive AG. (2023). *Index Guideline; Solactive Global Lithium Index.* Frankfurt.

Solactive German Index Engineering. (2024). *FACTSHEET Solactive Global Lithium Index.* Frankfurt.

Sprott Asset Management LP. (2023). *A New Era: How Critical Minerals are Driving the Global Energy Transition.* White Paper. Retrieved from https://sprottetfs.com/media/6491/critical-minerals-etf-whitepaper.pdf

Stokes, L. C., & Breetz, H. L. (2018, October 26). Politics in the U.S. energy transition: Case studies of solar, wind, biofuels and electric vehicles policy. *Energy Policy, 113*, 76-86.

Syed Aun R. Rizvi, G. D. (2014). An analysis of stock market efficiency: Developed vs Islamic stock markets using MF-DFA. *Physic A, 407*, 86-99.

The Economist. (2023, September). Keep digging. How to avoid a green-metals crunch. *The Economist*.

Turvey, C. G. (2007). A note on scaled variance ratio estimation of the Hurst exponent with application to agricultural commodity prices. *Physic A: Statistical Mechanics and its Applications, 377*(1), 155-165.

U.S. Department of Transportation. (2022). *Value of Electric Vehicles for Rural Communities.*

United Nations Framework Convention on Climate Change. (2023). *Yearbook of Global Climate Action 2023.*

Wald, M. L. (2013, October 24). 8 States Teaming Up to Support Electric Cars. *The New York Times*.

Wang, Yu-Zhi; Li, Bo; Wang, Ren-Qing; Su, Jing; Rong, Xiao-Xia. (2011, April). *Computers & Mathematics with Applications, 61*(8), 2129-2131.

Weron, R. (2002, September). Estimating long-range dependence: finite sample properties and confidence intervals. *Physic A: Statistical Mechanics and its Applications, 312*(1-2), 285-299.

Woodyard, C. (2014, September 4). Nevada, Tesla announc huge 'Gigafactory' deal. *USA Today*.

World Data Lab. (2023). *World Emissions Clock*. Retrieved from https://worldemissions.io/

Wulandari, F. (2022, July 9). Lithium price forecast: Will the price keeps its bull run? *Capital.com*.