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ASSESING MARKET EFFICIENCY FOR A BIODIVERSITY FOCUSED INDEX USING MULTIFRACTAL DETRENDED FLUCTUATION ANALYSIS (MF-DFA)

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1. INTRODUCTION

1.1 Rationale & Results of the Study

In recent decades, our planet has witnessed an alarming and accelerating decline in biodiversity, casting a shadow over the Earth's ecosystems and the survival of countless species. Statistics reveal a grim reality, with extinction rates growing at an unprecedented rate. The Intergovernmental Science-Policy Platform on Biodiversity and Ecosystem Services (IPBES) reports that approximately one million plant and animal species now face the risk of extinction, many within decades, due to human activities (IPBES, 2019). Habitat loss, driven by deforestation, urbanization, and agricultural expansion, remains a primary driver of biodiversity loss. According to the World Wildlife Fund (WWF), the Earth has lost an estimated 60% of its wildlife population in just over four decades (Worl Wildlife Fund, 2020). These staggering figures underscore the urgency of addressing the global biodiversity crisis. Biodiversity serves as the bedrock of a wide array of ecosystem services, essential for human well-being and the functioning of our societies, like provisioning services, such as food and clean water, to regulating services, including climate regulation and disease control (Daily, et al., 2009). The concept of ecosystem services underscores the intricate relationship between biodiversity and the nourishment of human life. For instance, pollination, predominantly carried out by diverse species of bees, butterflies, and other insects, is a critical ecosystem service supporting agricultural productivity. According to the Food and Agriculture Organization (FAO), 75% of global food crops depend, at least in part, on animal pollinators (Food and Agriculture Organization of the United Nations, 2016). Likewise, wetlands and forests act as natural water purifiers, ensuring the availability of clean water for human consumption. These ecosystem services, contribute not only to human well-being but also to economic activities on a global scale. Recognizing the vital connection between biodiversity and these services is fundamental to understanding the far-reaching implications of biodiversity loss and the urgent need for conservation and sustainable investment.

In response to the critical state of global biodiversity, a multitude of conservation efforts and initiatives have emerged at various levels of governance. At the international level, organizations like the United Nations' Convention on Biological Diversity (CBD) have spearheaded global conservation agreements. The CBD's Aichi Biodiversity Targets and, more recently, the Post-2020 Global Biodiversity Framework, exemplify international commitments to halt biodiversity loss and protect critical ecosystems (CBD, 2021). Non-governmental organizations (NGOs) have been instrumental in conservation efforts, mobilizing resources, advocating for policies, and engaging in fieldwork. The World Wide Fund for Nature (WWF) and The Nature Conservancy are prominent examples of NGOs at the forefront of biodiversity conservation (World Wildlife Fund, 2022) (The Nature Conservancy, 2022). At the corporate level, many large companies have recognized the value of biodiversity and sustainability. Companies are promoting initiatives that preserve nature and its ecosystems; like Unilever who has committed to sourcing all its agricultural raw materials sustainably, promoting biodiversity conservation while meeting consumer demand (Unilever, 2024). These examples showcase how corporate engagement in biodiversity conservation aligns with global goals and underscores the importance of investing in sustainability. National governments have also played pivotal roles, enacting legislation, establishing protected areas, and implementing conservation policies tailored to their unique ecosystems. For instance, Costa Rica's Payment for Ecosystem Services (PES) program has been a pioneering initiative in rewarding landowners for preserving forests and biodiversity (Pagiola, Arcenas, & Platais, 2002).

Conservation Finance refers to the efforts within the financial sector to gather and manage capital specifically for the preservation of biodiversity (Cosma, Rimo, & Cosma, 2023). Over the next decade, the focus of conservation finance should be in bridging the significant funding gap, estimated to be in the hundreds of billions annually. Mobilizing private investment is crucial; however most top-tier finance journals don’t give enough coverage to the financial risks that come with biodiversity loss, or how those risks should be priced, or how the intervention of private financial flows is vital (Karolyi & Tobin de la Puente, 2023). The biggest challenge is measuring the true economic value of biodiversity and accurately assessing the long-term impact of conservation efforts. Unlike traditional financial assets, the value of biodiversity is often intangible and interconnected with multiple ecosystem services; therefore, it is difficult to quantify in monetary terms (Millenium Ecosystem Assessment, 2005). Additionally, establishing universally accepted metrics and methodologies remains a complex task (Balmford, Gaston, Blyth, James, & Kapos, 2003).

Motivated by these pressing issues, the study aims to contribute to the existing and ongoing research surrounding conservation finance, by analyzing the market efficiency of the MSCI World Select Natural Resources Index. The index is used as a proxy for evaluating the behavior and evolution of the biodiversity market; as well as the returns and profitability associated with investing in biodiversity on a broader scale. The results showed that the data presents multifractal characteristics throughout the decade, especially notable persistent behaviors at the begging of the period. However, from 2021 onwards the market starts leaning significantly towards market efficiency, indicating a considerable evolution in market dynamics.

1.2 Data & Methodology

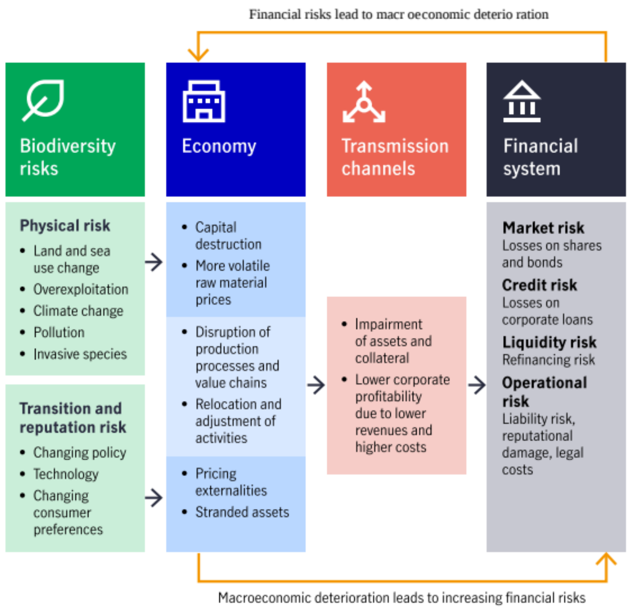
To evaluate the efficiency of the biodiversity market, the study uses the data provided by the MSCI regarding their World Select Natural Resources Index. This includes the list of constituents and the level end of day value of the index for the past decade. Which refers to the cumulative effect of all the day's trading activities on the index constituents. The approach selected to determine the efficiency of the biodiversity market is the Multifractal Detrended Fluctuation Analysis (MF-DFA), the logarithmic returns are calculated and taken as input for the MF-DFA. This technique is quite innovative in the field of finance and econophysics, as it was more commonly used for biomedical or natural phenomena time series. It is a numerical algorithm designed to identify self-similarity within a time series, by analyzing the fluctuations across different time scales. It explores the presence of correlations and the degree of fractality within the series; and it is applicable to both discrete and time-continuous stochastic processes (Gorjão, Hassan, Kurths, & Witthaut, 2021). There is a constantly growing body of literature regarding the application of MF-DFA to financial time series. For example, how Aslam, Ferreira, and Mohti explored the efficiency of frontier stock markets through multifractal detrended fluctuation analysis, as outlined in their study published in the *International Journal of Emerging Markets*; or their study published in the International Journal of Financial Studies (2020) that examined evidence of intraday multifractality in European stock markets during the COVID-19 outbreak. Their research contributes to the understanding of market dynamics during periods of heightened uncertainty, offering insights into the intricacies of market behavior in response to significant global events. However, there aren't that many articles focused on the behavior of green financial time series, particularly in the context of biodiversity investment and neither are there many articles applying MFDFA analysis to green assets' time series. It is precisely to that gap of information where this paper aims to contribute.

2. THE BIODIVERSITY CHALLENGE

Nature plays a crucial role in our global economy; it is vital for human prosperity. However, human activity keeps growing in expense of nature. The UN's Intergovernmental Science-Policy Platform on Biodiversity and Ecosystem Services has warned that if the production and consumption of natural resources keeps increasing at the current rate, nature will not be able to withstand in the future (IPBES, 2019). According to the Dasgupta Review (2021), half of the world's global GDP (US$44 trillion approx..) is moderately to highly dependent on nature. Recent studies show that the value of ecosystem services is estimated to be between USD 125 to 140 trillion per year, which supersedes global GDP (OECD, 2019). These numbers only understate how the risk for an economic disaster keeps rising as nature deteriorates. When talking about the environment, apart from climate change, biodiversity loss is the main concern, and it has gained popularity since the 15 Sustainable Development Goals were set for 2030 (Karolyi & Tobin de la Puente, 2023). Extinction rates are the highest they have averaged over the past 10 million years. Furthermore, species that are not considered endangered are facing a rapid depletion of their genetic diversity. This reduction is primarily due to habitat fragmentation, degradation, and localized extinction events (IPBES, 2019).

The loss of biodiversity threatens not only the health of ecosystems but also consequently the health of the economy (Manulife Investment Managment , 2023). Moreover, it increases credit, market, and operational risk. Up until recently companies had not realized how natural capital could affect financial decision making. However, for the past few years more and more firms are beginning to realize the importance of how biodiversity can affect their portfolios and vice versa. There are two types of factors in biodiversity that can introduce financial risk: physical and transition risks. When a financial entity lends, invests, or ensures a company that depends on ecosystem services then physical risks can arise. On the other hand, transition risks come, according to the NGFS, from being exposed to companies that damage biodiversity. For example: if an area is protected then businesses operating in the area will have to end up moving; or if consumers boycott certain ingredients such as palm oil because rainforests have suffered from its production (Van Toor, Piljic, Schellekens, Van Oorschot, & Kok, 2020).

Figure 1: From biodiversity risks to financial risks



Source: "Indebted to nature. Exploring biodiversity risk for the Dutch financial sector", dnb.nl, June 2020.

The reason why nature and biodiversity loss have captured less attention than climate change is because of how complex it is to measure. It is multidimensional and there are many factors to be considered. For example, the number and distribution of plant and animal species, number of unique species, species at risk of extinction, threats to biodiversity, invasive species etc. All of this across land and aquatic ecosystems. Therefore, a global or even a countrywide measure of biodiversity is hard to achieve. That's why the main challenge nowadays for governments is to design the legal framework, consistent with the Global Biodiversity Framework, so that accurate valuations of nature are carried out (Manulife Investment Managment , 2023). As Brian Kernohan says, "*Quantifying the value in nature is the first step to making it an investable thesis*".

According to the OECD, the value of biodiversity can be decomposed into 5 sections (Biller & Sermann, 2002):

- <u>Direct extractive use</u>: products that are traded or have the potential to be traded like plants, food, and other types of commercial products.

- Direct Non-Extractive Use: refers to services provided by biodiversity, like ecotourism, education, developing of new pharmaceuticals etc.

- Indirect Use: refers to the services provided by ecosystems like water supply, soil conservation, food supply etc.

- Option Values: refers to the possibility that someone may want to use a certain resource in the future.

- Existence Values: refers to the amount of money people are willing to pay to protect and preserve nature.

Products linked with direct extractive use and some others with non-extractive use are usually the ones in which the market might be interested in and therefore might invest in. Products and services that refer to other values are often associated with the public good (Biller & Sermann, 2002).

In late 2022, in Canada, the United Nations Conference on Biological Diversity (COP15) concluded with a historic agreement aimed at guiding global actions in support of nature until 2030. COP15 has paved a clear path for governments worldwide. The commitment from governments is expected to lead to the establishment of explicit regulations and incentive structures that will encourage businesses to act and motivate capital markets to invest. Once the value of nature is measured, it must be factored into accounting. This step is essential to ensure that the environmental impact of economic activities is routinely monitored just like other financial metrics. These initiatives combined with effective government actions, will be instrumental in achieving our conservation goals. (Manulife Investment Managment , 2023).

The financial sector comes in to salvage the shortcomings of governments, whose policies tend to not be effective due to their short-term nature. A solution through conservation finance can be seen as "*an integrated approach to solve a problem or challenge through the specific use of financial and economic instruments*" (Cosma, Rimo, & Cosma, 2023). The main objective of a financial solution is to create a self-sufficient and sustainable economy that can last for years to come and align with different interests (Cosma, Rimo, & Cosma, 2023)

The next challenge for conservation finance is closing the gap between the current amount invested in biodiversity and the amount needed to maintain ecosystems' integrity (Karolyi & Tobin-de la Puente, 2023). According to the United Nations Environment

Programme (2023) "*State of Finance for Nature"*, in 2022 cash inflows for nature-based solutions increased by 11% (an additional US$20 billion compared to the previous year). However, despite this positive trend the funding falls short of what it's needed to achieve the ambitious 30x30 biodiversity target; which aims to conserve 30% of the Earth's land and ocean by 2030. This gap is estimated to be between US$100 billion and US$200 billion (United Nations Enviroment Programme, 2023). Another study carried out by the Paulson Institute, The Nature Conservancy, and the Cornell's Atkinson Center for Sustainability (Deutz, et al., 2020) estimated an even bigger financing gap that falls between $722 billion and $967 billion. The largest part of the investment would be needed for shifting the agricultural sector towards sustainable methods by 2030, with costs ranging from $315 billion to $420 billion annually. Followed by the global rangelands, estimated at $81 billion annually, and then the transition of global fisheries to sustainable practices, which is expected to cost between $23 billion and $47 billion each year (Deutz, et al., 2020).

What everyone can agree on is the idea that return-seeking private capital is a vital component in addressing biodiversity decline as public funding alone has been deemed insufficient. However, significant private capital investment in biodiversity will occur in proportion to the competitiveness of the ROI and adjusted risk. To accomplish this objective, a fundamental shift is necessary in how markets assign value to nature, assets linked to nature and natural resources. It is crucial to discover and implement innovative financial and policy approaches to mobilize the substantial funding. Another big incentive that could motivate the private sector is regulation. Policies such as encouraging companies and financial institutions to reveal risks related to nature, aligning investment portfolios to promote a positive impact on the environment, reinforcing the argument for nature-based solutions through carbon markets, and increasing concessional funding options such as subordinated loans, guarantees, and grants (Karolyi & Tobin de la Puente, 2023)

If the financial sector starts to increase its investments towards companies engaged in active biodiversity conservation measures, it will serve as a catalyst for the rest of the economy; because of the influential role of the financial sector in driving economic activities. When financial institutions prioritize biodiversity-conscious companies through investments and financing, they set a significant precedent for sustainable practices. This shift in priorities within the financial sector has a ripple effect throughout the economy, encouraging other sectors to align with sustainability goals (HM Goverment , 2023). Furthermore, increased investments in biodiversity conservation can spur innovation and the development of green technologies. Companies may explore sustainable practices that benefit the environment and create new markets and job opportunities. This innovation can stimulate economic growth and market expansion in sectors related to clean energy, sustainable agriculture, and conservation technologies (European Comission, 2020) (Jacob & Jänicke, 2006).

There is a wide range of possibilities for investors looking to protect the ecosystem. Biodiversity Bonds have gained traction globally, with countries like Peru issuing "Biodiversity Conservation Bonds" to fund conservation efforts in the Amazon rainforest (Clean Bonds Initiative, 2022). Additionally, Impact Investing has enabled initiatives like the Blue Bonds in Seychelles, which aim to finance the protection and restoration of marine biodiversity (The Nature Conservancy, 2023). There are also several indexes which pool companies that carry out tangible plans for preserving biodiversity, some examples are Wildlife 20 (W20) Index, MSCI World Select Natural Resources Index, Euronext Biodiversity Screened World Index, ISS STOXX Biodiversity indices etc. To conduct our analysis, we have selected the MSCI World Select Natural Resources Index as the optimal proxy for the market of companies investing in biodiversity. This decision is based on the extensive research into the initiatives undertaken by the index's constituents, which has demonstrated their significant involvement in biodiversity investments.

3. THE EFFICIENT MARKET HYPOTHESIS AND FRACTAL THEORY

In 1970, Eugene F. Fama introduced the Efficient Market Hypothesis (EMH) in his acclaimed article "*Efficient Capital Markets: A Review of Theory and Empirical Work*". The EMH posits that a market is efficient when security prices “fully reflect” all available information. Therefore, it defends that price of traded securities such as bonds, equity, derivatives etc. are inherently fair because information is readily available to all investors, who are all assumed to be rational. Consequently, in an efficient market there are no arbitrage opportunities. Neither technical analysis, which involves studying past stock prices to predict future trends; nor fundamental analysis, which means looking through financial data like earnings, debt, or asset valuations to identify stocks that are undervalued, would allow the investor to make abnormal results that would surpass those achievable through a diversified portfolio of stocks with similar risk profiles, selected at random (Malkiel, 2003) (Fama, 1970). The EMH is associated with a random walk model, which in essence suggests that price fluctuations are driven by unpredictable factors making them stochastic in nature. The logic supporting this is the following: if information is available to everyone and reflected immediately on the prices then tomorrow’s price change will only depend on tomorrow's events and will have no relation to today’s prices. This theory democratizes investing, as an expert investor and a beginner have the same opportunities (Malkiel, 2003) (Fama, 1970).

It is important to highlight that Fama carried out a significant amount of empirical research on event studies to see if stock prices did indeed respond effectively to information. His work shows that investors were as likely to underreact to news as they were to overreact; and it was as common for abnormal returns to continue after a certain event as it was for them to reverse. The key is whether the market presents patterns of statistically significant correlation over a long period of time (Malkiel, 2003) (Fama, 1970).

The theory has traditionally assumed that returns follow a normal distribution as the consequence of millions of individual investment decisions. The returns are characterized by its mean and variance; this implies that the errors also follow this distribution. Even though the EMH was widely accepted at the end of the 20th century, in the beginning of the 21st century its universal dominance started to weaken. There have been numerous studies regarding fat-tails, skewness, and kurtosis in the distribution of financial returns (Affleck-Graves & Mcdonald, 1989) (Campbell, Lo, & MacKinlay, 1997) (Jondeau & Rockinger , 2003) (Szego, 2002) (Tockat, Rachev, & Schwartz, 2003) (Dufour, Khalaf, & Beaulieu, 2003) (Costa & Cavaliere, 2005). Other limitations include insider trading and privileged information which undermines the assumption that all investors possess the same amount of information; the power of financial institutions over the market, herding behavior, speculation by financial institutions etc. (Lin, 2023).

Moreover, when testing the efficiency of a market we need an asset pricing model that shows how assets' expected returns are supposed to behave in market equilibrium. We want to see if actual returns in fact perform how they are supposed to according to the market equilibrium model. However, if the test fails, it is not possible to know whether it was due to market inefficiency or a failure of the asset pricing model. This dilemma is widely known as the joint hypothesis problem (Fama, 1970).

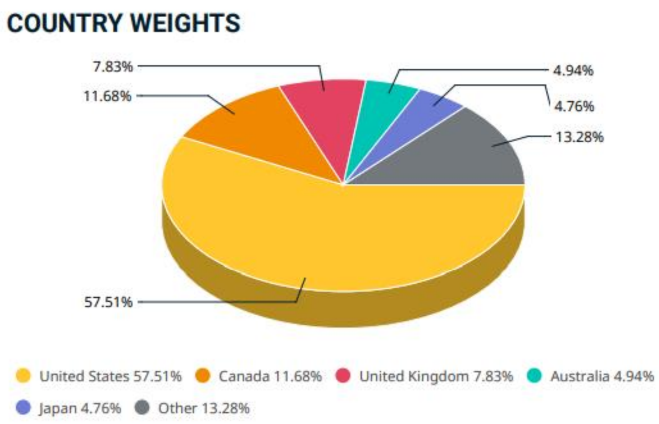
Having in mind the limitations of the Efficient Markets Hypothesis, it's reasonable to look to different approaches to explain the behavior of stock prices and returns. In the 1990s Ed Peters and Benoit Mandelbrot introduced the Fractal Market Theory. Peters published his book called *Fractal Market Theory Analysis: Applying Chaos Theory to Investment and Economics.* The logic behind using chaos theory in finance comes from the non-linear dynamic that characterizes financial markets, therefore solely using statistical models will produce poor results. The capital markets are often referred to as "self-similar", the same as fractals. Fractals refer to geometric objects that can be deconstructed into components mirroring the overall shape. Meaning that stock prices present a similar structure when taking different time samples. This theory shifts the spotlight from market efficiency to market stability, acknowledging both the regular randomness of daily markets and any anomalies present (Oprean, Tanasescu, & Bratian, 2014). This is used as the basis for technical analysis. In accordance with this structure, the FMH examines investor time horizons, the influence of liquidity, and the effects of information across the business cycle. The main challenge is to decide which length of time periods to study, because prices and returns often show that the degree of volatility varies across different time scales (Liberto, n.d.).

Mandelbrot introduced the phenomena of long-term dependence (Mandelbrot, 2005). By applying the foundational concept of Brownian motion to asset prices, one can typically anticipate the extent of their movements. However, if price changes exceed the predictions based on the square root of time law (a cornerstone of the random walk model), questions arise about the underlying mechanisms. Mandelbrot hypothesized that in markets exhibiting long-term dependence, movements in one direction tend to persist over subsequent days and even weeks. This departure from pure randomness suggests a departure from traditional notions of chance. Mandelbrot introduced a quantitative measure, the Hurst exponent H, named after the hydrologist Harold Edwin Hurst, to gauge this tendency. Essentially, the Hurst exponent serves as a tool to assess the long-term memory of a time series, indicating its propensity to either revert strongly to the mean or exhibit directional clustering, thereby revealing underlying trends (Mandelbrot, 2005). In Section 5, we will further explore the theory and interpretation of the Multifractal Detrended Fluctuation Analysis.

4. MSCI WORLD SELECT NATUAL RESOURCES INDEX

Following the objective of studying the efficiency of the biodiversity market, the first step lies in selecting the appropriate index to serve as a reliable proxy for this complex and constantly evolving market landscape. After thorough research of existing literature, green indexes, financial markets, and environmental conservation efforts the index used as proxy is the MSCI World Select Natural Resources Index. This index is based on its parent index MSCI World IMI Index, that consists of a wide range of large, mid, and small cap companies across 23 developed countries. This index is specifically structured to mirror the performance of publicly traded companies in said developed markets that are involved in the ownership, processing, or development of natural resources (MSCI, 2024).

Figure 2: Country Weights



Source: MSCI World Natural Resources Index Factsheet, https://www.msci.com/documents/10199/5836f428-12d1-43a9-a857-5ed2b8f62585 , March 2024.

As seen in figure 2, most of the investments are located in the United Stated, followed by Canada and the UK. Regarding companies in the European Union, after looking through the data, it showed that approximately 2% of companies were based in European countries. The index is developed by choosing companies included in the following sub-industries:

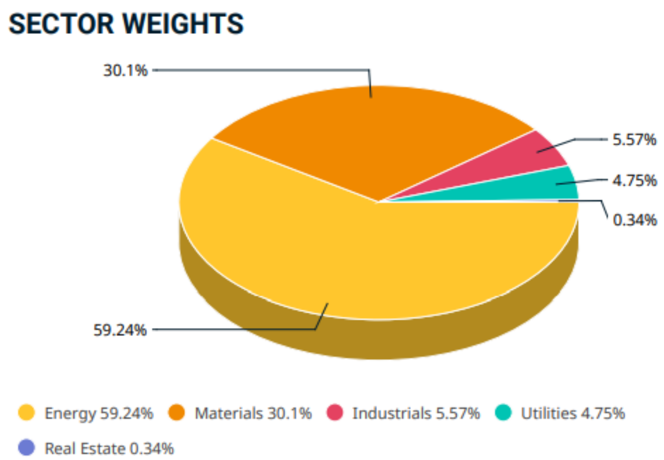
- Energy: Coal & Consumable Fuels, Integrated Oil Gas, Oil Gas Drilling, Oil Gas Equipment Services, Oil Gas Exploration Production, Oil Gas Refining Marketing, Oil Gas Storage Transportation
- Materials: Aluminum, Commodity Chemicals, Construction Materials, Copper, Diversified Chemicals, Diversified Metals & Mining, Fertilizers & Agricultural

Chemicals, Forest Products, Gold, Industrial Gases, Metal & Glass Containers, Paper Packaging, Paper Products, Precious Metals & Minerals, Specialty Chemicals, Steel, Specialized REITs

- Utilities: Electric Utilities, Gas Utilities, Multi-Utilities, Renewable Electricity, Water Utilities.
- Industrials: Agricultural Farm Machinery, Construction Engineering, Construction Machinery, Industrial Machinery (MSCI, 2024).

There are 1,170 companies in the index. In figure 3 we can see the distribution between sectors: energy, materials, industrials, utilities, and real state. Additionally, to mitigate concentration risk, the weight of each sub-industry is limited to 20%, and individual security weights are capped at 5%. The rebalancing of the Index occurs semi-annually in May and November, aligning with the Semi-Annual Index Reviews of the parent index (MSCI, 2024).

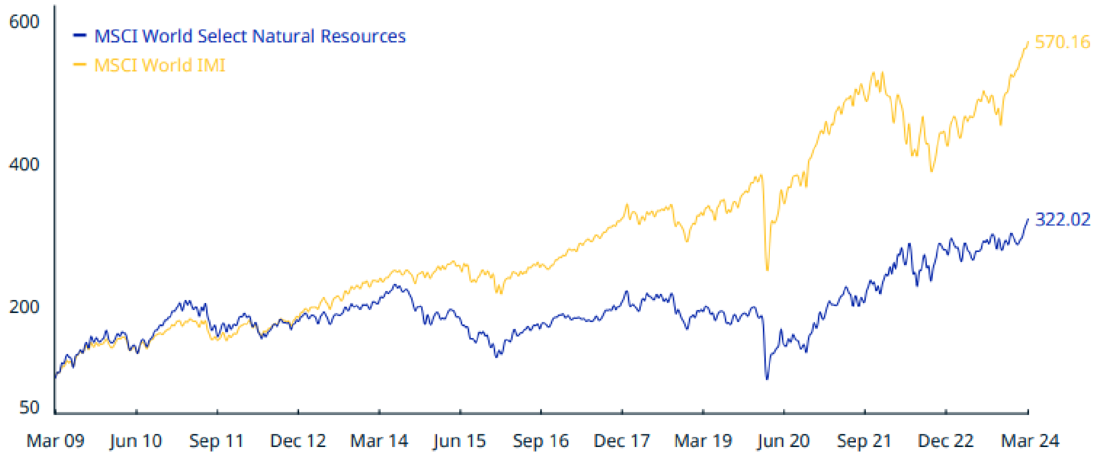
Figure 3: Sector Weights



Source: MSCI World Natural Resources Index Factsheet, https://www.msci.com/documents/10199/5836f428-12d1-43a9-a857-5ed2b8f62585 , March 2024.

In the graph below we can see the evolution of the net returns from 2008 to March 2024 in comparison to its father index MSCI World IMI. We can see that it tends to move along the parent index, only below.

Figure 5: Cumulative Index Performance – Net Returns (USD) (Sep 2008 – March 2024)



Source: MSCI World Natural Resources Index Factsheet, https://www.msci.com/documents/10199/5836f428-12d1-43a9-a857-5ed2b8f62585 ,March 2024

In figure 6, we see the top 10 constituents of the index, all of them carry out tangible and measured plans to help preserve and promote biodiversity. The data used in this paper consists of the level EOD daily net returns of The MSCI World Select Natural Resources Index spanning the last decade, from February 6th, 2014, to February 6th, 2024.The data was provided by the MSCI. For more information, the reader is referred to the annex where there is detailed information about the top 10 constituents and examples on their biodiversity preserving initiatives.

Figure 6: Top 10 Index Constituents

	Country	Index Wt. (%)	Parent Index Wt. (%)	Sector
EXXON MOBIL CORP	US	5.08	0.64	Energy
CHEVRON CORP	US	4.02	0.39	Energy
CONOCOPHILLIPS	US	3.30	0.21	Energy
SHELL	GB	3.07	0.30	Energy
LINDE (NEW)	US	2.19	0.31	Materials
TOTALENERGIES	FR	2.12	0.21	Energy
CANADIAN NAT RESOURCES	CA	1.78	0.11	Energy
SCHLUMBERGER	US	1.71	0.11	Energy
ENBRIDGE	CA	1.68	0.11	Energy
MARATHON PETROLEUM	US	1.67	0.11	Energy
Total		26.63	2.48	

Source: MSCI World Natural Resources Index Factsheet, https://www.msci.com/documents/10199/5836f428-12d1-43a9-a857-5ed2b8f62585 ,March 2024

5. MF-DFA THEORETICAL BACKGROUND

This study uses the multifractal detrended fluctuation analysis (MFDFA) to explore the delicate intricacies of financial markets. Fractal methods encompass both single fractal and multifractal methods, with the latter being considered superior due to its ability to describe the multi-scale and subtle sub-structures of fractals in complex systems (Mandelbrot, 2005); (Arneodo, Bacry, & Muzy, 2008). The Multifractal Detrended Fluctuation Analysis (MF-DFA) was created with the purpose of exploring multifractality's dynamic characteristics, specifically focusing on examining the interplay between non-linear temporal correlation and the presence of fat-tailed distribution (Kantelhardt, et al., 2002). The application of MF-DFA spans across various markets characterized by the random walk hypothesis (RWH); which posits that asset prices are stochastic and unpredictable, making future prices completely random. The random-walk hypothesis, popularized by Bachelier's work in the early 20th century, is a base concept in finance (Bachelier, 1900).

On one side of the spectrum, multifractality has been employed in agricultural and commodities markets, such as energy and metals, revealing departures from the RWH (Memon, Yao, & Naveed, 2022) (STOSIC, NEJAD, & STOSIC, 2020). At the same time, in capital markets MF-DFA has highlighted the existence of fractal properties (Kantelhardt, et al., 2002) (Di Matteo, 2007) (Aslam, Ferreira, & Mohti, 2023) (Zunino, y otros, 2008)As a result of the identification of multifractal characteristics within financial markets over the past decade, there has been a growing interest in employing MF-DFA for analyzing stock markets. Nowadays, multifractality stands as one of the most popular and actively explored subjects within the field of econophysics (Milos, Hatiegan, Milos, Barna, & Botoc, 2020). In particular, MF-DFA has gained popularity as it measures the efficiency of market price returns for a long period of time. This technique involves computing the average volatility within various intervals of the time series, using these statistical points to derive volatility functions, and subsequently determining generalized Hurst exponents based on the power law of these functions (Wang, Liu, & Qin, 2014). One significant benefit of MF-DFA in comparison to alternative methods lies in its capacity to identify long-term correlations within time series that are not stationary. We will now provide an overview of the fundamental procedures and mathematical expressions that form the basis of this analysis (Aslam, Mohti, & Ferreira, 2020)

(Kantelhardt, et al., 2002) provided the structure of MFDFA which consists in the following steps:

Step 1: Take a possibly non-stationary time series $X(t)$ (in time or space t) and find the "detrended" profile of the process by doing:

$$Y_i = \sum_{k=1}^{i}(X_k - \mu_X), \text{ for } i = 1, 2, \dots, N, \qquad [1]$$

i.e. the cumulative sum of its observations subtracting the mean of the data.

Step 2: Afterwards we section the data into non-overlapping segments of length s, which results in N_s = int (N/s) segments. Because the length of the data is not always a multiple of the segment's size, discard the last points of the data. So, this part of the data is not disregarded, the same process is repeated but starting from the opposite side of the series. Therefore, there will be $2N_s$ segments of the time series (see Kantelhardt et al., 2002 for a deeper description).

Step 3: For each of these segments calculate the local trend by a least-squares fit of the series. This means fitting a polynomial Y_v of order m and calculate the variance of the difference between the data and the polynomial fit. For this paper, m=1 which is equivalent to linear detrending.

$$F(v,s) = \frac{1}{s}\sum_{i=1}^{s}\left[Y_{(v-1)s+i} - y_{(v-1)s+1}\right]^2 \qquad [2]$$

for $v = 1, 2, \dots, N_s$, where $y_{(v-1)s+i}$ is the polynomial fitting for the segment $Y_{(v-1)s+i}$ of length s, fitted via least squares.

Step 4: Seeing how the $F(v,s)$ is a function of each variance for each v-segment of the data and of the different s-length segments chosen. Define the $q - th$ order fluctuation function by doing the average of the $2N_s$ variances of the segments of size s.

$$F_q(s) = \left\{\frac{1}{2N_s}\sum_{v=1}^{2N_s}[F(v,s)]^{\frac{q}{2}}\right\}^{1/q} \qquad [3]$$

The fluctuation function $F_q(s)$ depends on two parameters: the segment size s and $q - th$ power. Being $F_q(s)$ the increasing function of s. The index variable q can take any real value except zero. For q=2, the analysis is the same as a standard DFA.

The parameter q is useful to differentiate between segments with small and large fluctuations. If q has a negative value it enhances small fluctuations, whereas a positive value enhances large fluctuations.

Step 5: after consecutively doing the procedure for different values of s, we can see that there is a relationship between equation 3 and s. To determine the scaling behavior of the fluctuations function we use the log-log plots of $F_q(s)$ vs s for every value of q. Where $h(q)$ is the generalized Hurst exponent or the self-similarity exponent.

$$F_q(s) \sim s^{h(q)} \qquad [4]$$

When the scale parameter s increases it indicates how rapidly the value of $F_q(s)$ grows, meaning how fast the local fluctuations grow. A consistent $h(q)$ across different q values suggest a monofractal time series, whereas variability in $h(q)$ indicates multifractality. In multifractal time series the relationship between

Step 6: from equation 5 we can estimate the Hurst exponent. After taking logarithms in both sides, we get:

$$log\, F_q(s) \;=\; h(q) \;*\; log\, s \qquad [5]$$

When the scale parameter s increases it indicates how rapidly the value of $F_q(s)$ grows, meaning how fast the local fluctuations grow.

The Hurst exponent can be used to measure market efficiency (Feder, 1988) (Kroha & Skoula, 2024). If it falls within the range of 0.5 to 1, it suggests positive autocorrelation, indicating a persistent behavior with long memory effects across various time scales. Values closer to 1 signify the presence of substantial and sudden changes. Conversely, a Hurst exponent value within the range of 0 to 0.5 signifies negative autocorrelation, representing a shift in the trend, or anti-persistent behavior (Kroha & Skoula, 2024). Anti-persistence implies more frequent reversals, covering shorter distances compared to a random process. As noted by Edgar Peters in his book "*Fractal Market Analysis, applying chaos theory to investments and economics*", a Hurst exponent of 0.5 (q = 0) corresponds to a Brownian time series, commonly known as a classical random walk, which suggests efficiency. In a multifractal context, a series is considered multifractal if the Hurst exponent $h(q)$ varies with q and steadily decreases as q increases. On the other hand, the series is monofractal when $h(q)$ remains independent of q (Ihlen, 2012).

Another alternative to determine whether a time series is characterized by multifractality is through multifractal spectrum analysis, which is based on the following relationship of the Renyi exponent:

$$\tau(q) \;=\; qh(q) \;-1 \qquad [6]$$

After, through a Legendre transform we get:

$$\alpha = \tau(q)' \;=\; h(q) \;+\; qh'(q) \qquad [7]$$

and the multifractal spectrum given by

$$f(\alpha) = q[\alpha - h(q)] + 1 \quad [8]$$

Where α is the Hölder exponent:

$$\alpha = h(q) + q\frac{dh(q)}{dq} - \tau(q) \quad [9]$$

Having in mind that the higher the variability of $h(q)$, the richer the multifractality, from the equation in step 6 we can define that the multifractality degree is the following

$$\Delta hq = max[h(q)] - min[h(q)] \quad [10]$$

A higher Δhq indicates a more multifractal series, with $h(q)$ decreasing as q increases (Zunino et al., 2008). And the intermittency degree as:

$$\Delta\alpha = max[\alpha] - min[\alpha] \quad [11]$$

Since multifractality and market efficiency are negatively correlated, the lower the value of these two parameters, the more efficient the time series (Domino, 2011); (Caraiani, 2012).

A python script has been used for the MFDFA analysis. Following the preparatory setup, the notebook dives into the core analytical procedures. Initially, graphical representations are generated, clearly visualizing the financial metrics over time using seaborn-enhanced plots, set against an SVG format for high-quality output. Data cleansing ensures the integrity and applicability of the dataset for multifractal analysis. Next the focus shifts to a quantitative analysis, where logarithmic returns on closing prices are computed to capture the relative changes in financial metrics. The latter part of the notebook focuses on the multifractal analysis, first taking all the data and then each year individually, and applying the defined function get_mfdfa, which orchestrates the multifractal detrended analysis. By varying parameters such as lag and the q-order, the function meticulously computes multifractal metrics, including Hurst exponents and multifractal spectra. In its concluding sections, the notebook employs advanced visual and statistical tools to present the results of the analysis. Functions are defined to generate plots that highlight the complex dynamics captured by the analysis. Additionally, the application of a rolling Hurst calculation provides a dynamic view of the data's long-term dependencies, offering a window into evolving market conditions over time. To determine the threshold upon which we can reject the hypotheses that a market is efficient, we performed a bootstrapping of the returns of the index, calculating the Hurst exponent of classified as efficient returns. Afterwards with a significance level of 5% two thresholds were defined to set limits on the efficient hypothesis. Hurst exponents lower than 0.392 are not efficient, and values higher than 0.603 either. These values will help us analyze the results and determine the efficiency of our data.

6. RESULTS

In the results section of the study, we present the outcomes of the multifractal detrended fluctuation analysis (MF-DFA) of the index data, focusing on the variability of the Hurst exponents and multifractal spectrum across different market conditions, observed in the past decade and throughout each year individually. By coupling detailed visualizations with quantitative metrics, we effectively demonstrate the multifaceted nature of financial time series, shedding light on their complexity and the underlying dynamics that guide their movements. These insights not only enhance our understanding of financial markets' fractal properties but also underscore the robustness of multifractal methods in uncovering subtle characteristics of economic data.

Figure 7: Evolution of the level end of day value of the MSCI World Select Natural Resources Index for the last ten years.

Source: author's elaboration

Figure 7 represents the evolution of the daily net returns of the MSCI World Select Natural Resources Index over the last decade, from the beginning of 2014 until February 2024. From the graph we can interpret that the overall trend of the index is upward, indicating a general increase in the index returns over the last 10 years. There is a significant fall around 2020, where the index presents its maximum drawdown, this is consistent with the Covid-19 crisis and the economic downturn that followed. However, after this period the index shows a strong recovery, reaching values higher than pre-

2020, which indicates a period of growth. From the average line we can see that the index is quite volatile with many peaks and downs throughout the last 10 years. The volatility is particularly high during the years leading up to 2020 and less so in the following years.

Figure 8 displays the evolution of the logarithmic returns of the index throughout the last ten years. The returns fluctuate around the zero line which indicates that on any day the returns can be positive or negative, which is typical for a stock index. There are periods of increased volatility, consistent with the previous graph; particularly around 2020. The impact of the COVID-19 on the global economy and the natural resources sector specifically can be perceived by these fluctuations. As the time series advanced towards 2024 the seems to stabilize.

Figure 8: Evolution of the daily logarithmic returns

Source: author's elaboration

In Table 1, we can see the main financial descriptives for each year. It is noticeable how the Sharpe ratio, which measures the risk-adjusted return, has been improving since 2020. The Jarque Bera tests suggests that the logarithmic returns deviate from the normal distribution most of the time.

Table 1: Main financial metrics of the time series

Year	Annual Return (%)	Annualized Volatility (%)	Sharpe Ratio	Skewness	Kurtosis	Jarque-Bera Test Statistic	Jarque-Bera p-value
2014	-5.273%	13.518%	-0.470	-0.683	3.037	107.616	0.000
2015	-20.531%	18.989%	-1.199	-0.045	1.378	20.658	0.000
2016	25.628%	20.007%	1.080	-0.214	0.885	10.468	0.005
2017	11.986%	9.703%	1.079	-0.145	0.126	1.084	0.581
2018	-18.076%	16.074%	-1.229	-0.419	0.455	9.889	0.007
2019	16.750%	13.464%	1.073	-0.255	0.913	11.906	0.003
2020	-12.262%	40.530%	-0.323	-1.440	10.807	1365.506	0.000
2021	31.441%	19.309%	1.341	-0.248	0.100	2.790	0.248
2022	20.973%	25.482%	0.705	-0.521	0.941	21.344	0.000
2023	8.408%	16.002%	0.458	-0.253	1.419	24.597	0.00

Source: author's elaboration

After carrying out this preliminary descriptive analysis, we applied the MF-DFA to the logarithmic returns time series of the index. This analysis was carried out in Python using the mfdfa library. The results shown in Table 2 illustrate significant variations in the fractal properties of the time series, captured by the different orders of q. As q shifts from negative to positive there is a notable decrease in the slope and the generalized Hurst exponent $h(q)$, which highlights the transition from a more persistent to a more antipersistent behavior in the time series data. The scaling exponent $\tau(q)$ serves to understand the prevalence and impact of fluctuations within different scales. It transitions from negative to positive as q increases. This change signifies a switch in the dominant fluctuations, from smaller to bigger ones, as q grows. Negative $\tau(q)$ values indicate that smaller fluctuations are more influential in the dataset's fractal structure, when lower moments of q are studied. On the other hand, as q moves into the positive range $\tau(q)$ becomes positive suggesting that larger fluctuations are becoming more significant. Moreover, the singularity strength α decreases, indicating that the data becomes less complex and more homogeneous as q increases. The width of the singularity spectrum Dα, which measures the richness of the multifractal spectrum, narrows as q increases, suggesting a reduction in the multifractal complexity (Kantelhardt, et al., 2002); (Ihlen, 2012).

Table 2: MF – DFA Metrics

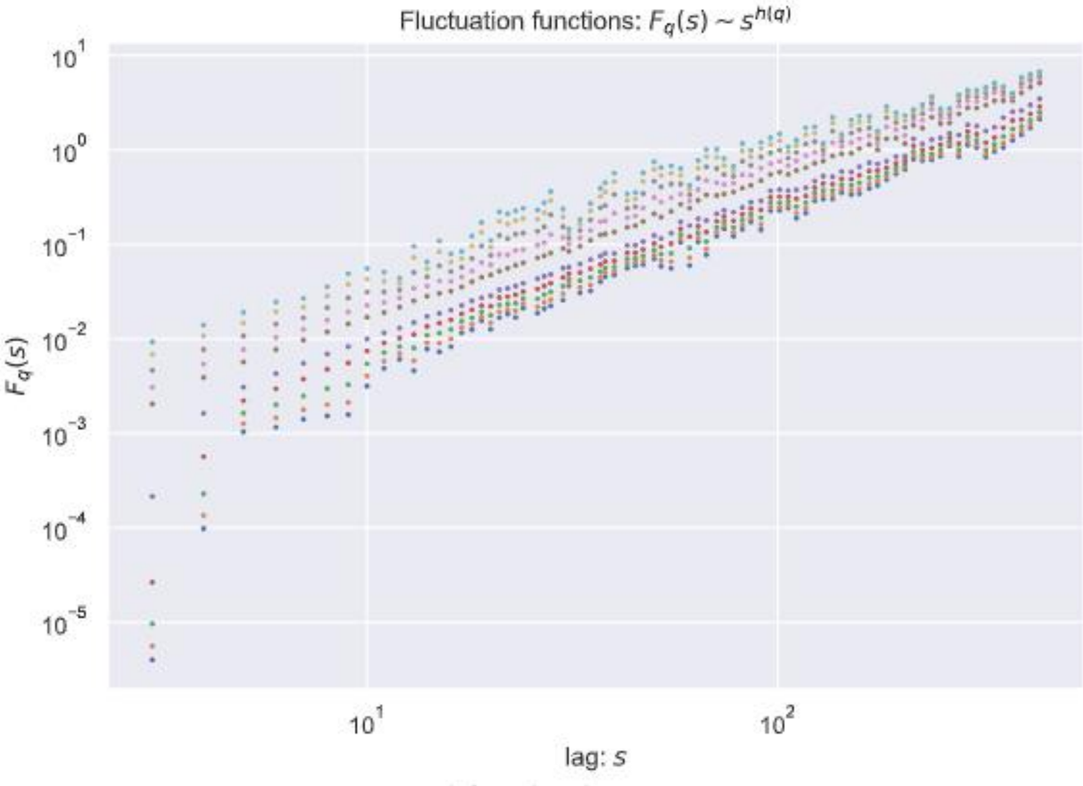
q	Slope	hq	$\tau(q)$	α	Dα[1]
-5.0	2.216	1.216	-7.083	1.469	-0.264
-4.0	2.153	1.153	-5.613	1.459	-0.224
-3.0	2.054	1.054	-4.164	1.403	-0.046
-2.0	1.903	0.903	-2.806	1.233	0.339
-1.0	1.697	0.697	-1.697	0.782	0.915
1.0	1.540	0.540	-0.459	0.588	1.047
2.0	1.533	0.533	0.066	0.500	0.934
3.0	1.514	0.514	0.542	0.423	0.728
4.0	1.478	0.478	0.913	0.339	0.444
5.0	1.444	0.444	1.221	0.307	0.318

Source: author's elaboration

Figure 9 shows the fluctuation functions on a log-log scale which is an output from applying the MF-DFA analysis to a time series. The fluctuation function is expected to scale with the lag s according to a power law which is shown by the dot lines in the plot. Here q represents different orders of the fluctuation function. The dot lines in the high part of the graph represent the positive q's and the low ones the negative. From the graph we interpret that the time series has multifractal properties as there are multiple lines with different slopes, which indicates that there are several scaling behaviors captured by different moments of q. However, the lines are not very far away from each other which can mean that it's not a high degree of multifractality.

Figure 9: MF-DFA Results: Fluctuation functions for q= -5 to q=5

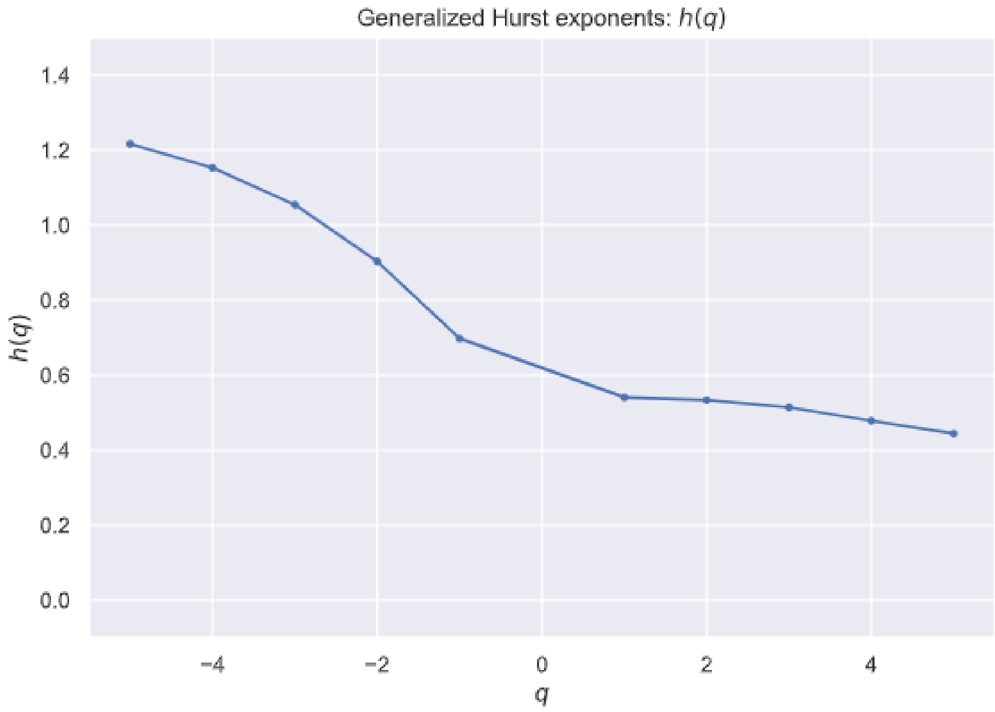
[1] In Kanderhalt, et al., 2002 it's the same as $f(\alpha)$: multifractal spectrum.



Source: author's elaboration

Figure 10 displays the Generalized Hurst Exponent as a function of q. The Hurst exponent measures the long-term memory of a time series. From the graph we see that the Hurst exponent, $h(q)$, varies for different values of q, particularly it decreases as q increases, which indicates patterns of multifractality. For positive q's the values of the Hurst are lower, which indicates that segments with large fluctuations have a noise like structure. Moreover, their Hurst's range between 0.6 and 0.4 which means that larger fluctuations lean towards efficiency. For negative q's the Hurst values are higher suggesting that smaller fluctuations have a relatively persistent behavior (Ihlen, 2012).

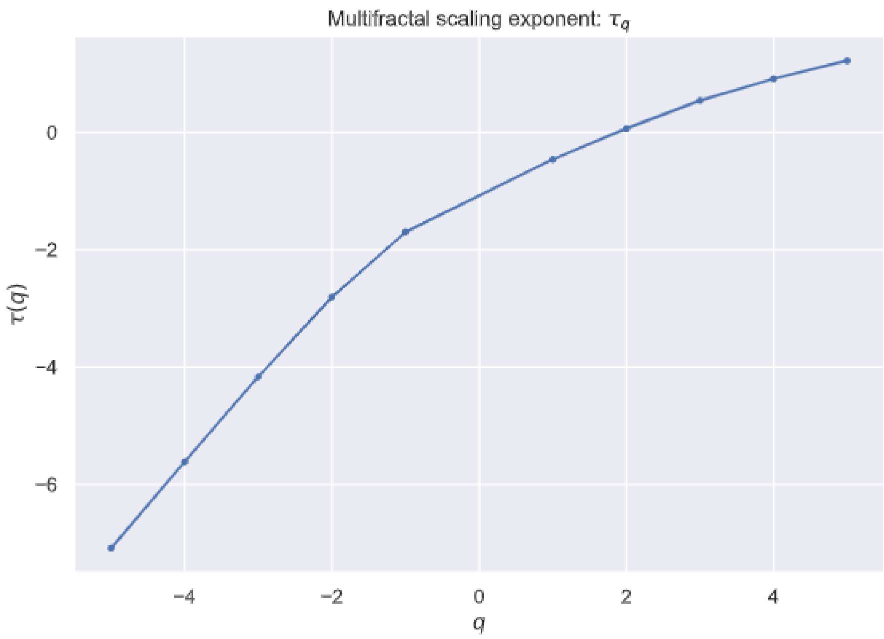
Figure 10: Function of Hurst exponent in terms of q



Source: author's elaboration

Figure 11 illustrates the Renyi exponent, which is linear for monofractal time series and non-linear for multifractal. We can see that it has more of an exponential shape which indicates multifractality and therefore a complex structure of the time series that can be due to several factors (Ihlen, 2012).

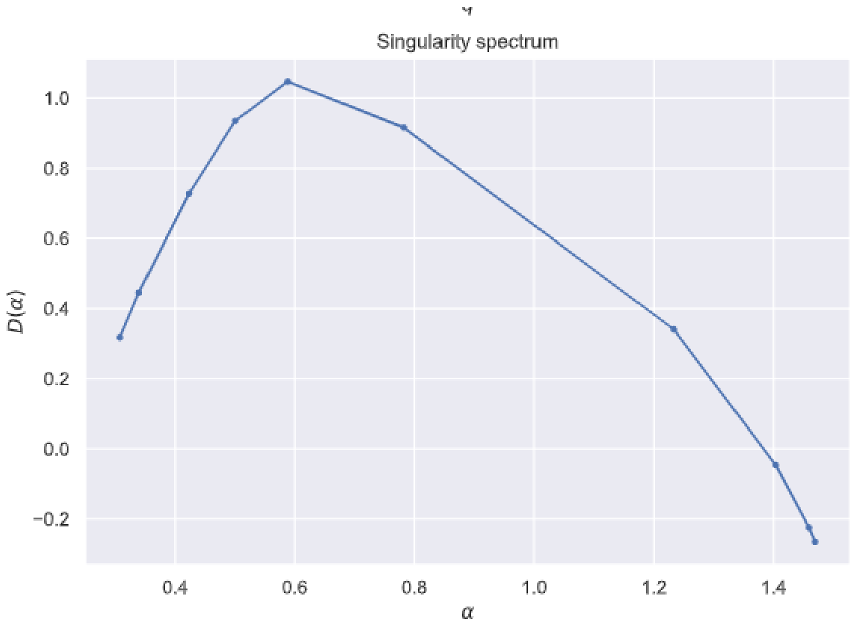
Figure 11: Renyi Exponent



Source: author's elaboration

Figure 12 displays the singularity spectrum obtained from the Holder Exponent and the Lagendre transformation that was covered above. Consistent with the previous figures, the width of the bell-shaped spectrum indicates multifractality. A wider spectrum suggests a greater diversity in the local scaling behaviors (Ihlen, 2012) .

Figure 12: Singularity spectrum



Source: author's elaboration

However, the time series is long, and the behavior of the net returns can have different patterns throughout the decade. In order to study the efficiency more specifically we calculated a classical q = 2 rolling Hurst Exponent and a metrics table for each year.

For stationary series, the benchmark to study efficiency is at q=2 and values above 0.6 indicate a persistent behavior and values below 0.392 indicate anti-persistent behavior. The rolling starts in 2015 because it is calculated with the last 252 days. We can see that between 2015 to 2017 the behavior of the Hurst indicated a period of market efficiency. From 2017 to 2018 the market displayed anti-persistent patterns. Meaning that if the market fell the index tended to go up. From 2018 until 2021, with some oscillations due to covid, the index showed a persistent behavior. However, since 2021 the time series behaves quite efficiently.

Figure 13: Rolling Hurst

Source: author's elaboration

To enhance our analysis, we initially examined the behavior of the data over the entirety of the decade. After, we conducted a year-by-year investigation to meticulously dissect the complex dynamics within the dataset. For a detailed inspection of the specific values, readers are referred to the tables provided in the annex. These tables contain the year-by-year metrics that are key for examining the data's multifractal properties. The key aspects of our individual analysis reveal a consistent pattern in slope values, albeit with some notable yearly variations.

The results from 2014, show higher values of $h(q)$, particularly for negative q's, which suggests persistence in the market. Meaning, that past price movements had a more significant influence on future prices during this time, which deviates from the Efficient Market Hypothesis. As we progress into the decade, there is a gradual shift observed in the slope, Hurst Exponents. The values start slightly decreasing, which can be indicative of a decrease in long-term correlations and a subtle move towards market efficiency, although the market stills shows significant multifractal characteristics. In 2021, the world was still adjusting to the turmoil of the Covid-19 pandemic, and its during this year that we see significant deviations in some of the metrics. The results showed notably higher slope values alongside with reduced Dα values, indicating an improvement in market efficiency. The higher slope values and lower Dalpha (multifractal spectrum) suggest a more uniform scaling behavior and greater homogeneity within the market. Together these metrics point to a market where pricing mechanisms are becoming more efficient. During this time, financial markets were being influenced by unprecedented levels of government intervention in the economy, changes in monetary policy, countries going into state of alarm, shifts in investor's behavior, multiple casualties etc. All these factors could have contributed into pushing the market towards being more efficient. From 2021 onwards the behavior presented continued much on the line of market efficiency.

7. CONCLUSION

If the finance sector shifted investment towards more environmentally conscious companies, it would play a pivotal role for the 2030, Environmental, Social and Governance objectives. To achieve that, research into sustainable finance and the behavior of green assets such as the one studied in this paper plays a critical role in making investing in nature more attractive for potential investors; as well as helping to clarify its value, which is one of the main barriers in these types of investments. The establishment of more investment vehicles that help people to invest in companies that actively protect and help restore the environment, biodiversity, water etc. also assumes a pivotal role into transforming the finance sector.

In this study we have focused specifically on biodiversity, its importance, and the performance of the companies that allocate resources to preserving it. The study takes as proxy for the biodiversity market the MSCI World Natural Resources Index and uses its returns time series to understand the behavior and profitability of investing in biodiversity. The study is centered on determining whether this index meets the criteria of an efficient investment, thus making it appealing to potential investors. In essence, if investors perceive a market as efficient, attempting to achieve abnormal results becomes absurd since the market price already reflects the potential of the companies. Consequently, for investors to create investment vehicles, such as an exchange traded fund (ETF) tracking this index, it needs to be backed by quantitative data that proves that active management and independent research doesn't yield abnormal results long term.

To determine whether this index is efficient we have taken the data and applied a Multifractal Detrended Analysis which has showed us deep insights into the price movements dynamics and the efficiency in question. The analysis allows us to understand how smaller and larger fluctuations impact the overall market behavior. The results show that the time series of the MSCI index exhibits significant multifractal properties, a reflection of the complex and evolving market conditions over the last decade. We reach these conclusions by looking at the Hurst exponents and the multifractal spectrum. The observed shift towards market efficiency post-2021, as reflected by the more stabilized nature of the index, matches with the recovery phases of the global economy and the increased confidence of investors in sustainable assets. This trend towards market efficiency is crucial as it grows the potential for green assets to attract mainstream investors and to be more considered for investment vehicles like for example an ETF that tracks the MSCI World Natural Resources Index, which is the end goal.

Moreover, the consistent upwards trend in the index, despite a major drawdown during Covid-19, underscores the resilience and long-term profitability of investing in biodiversity. This resilience coupled with the improvement of the Sharpe Ratios post 2020, further highlights the growth of the market and its potential to offer competitive, risk-adjusted returns to investors. Such quantitative backing is essential for a broader participation in green investments.

In this context, applying an MF-DFA analysis is especially innovative as academics hadn't started applying it to finance until quite recently. However green investments haven't quite been the focus yet, so this study is contributing to expanding new techniques into sustainable finance.

Declaración de Uso de Herramientas de Inteligencia Artificial Generativa en Trabajos Fin de Grado

ADVERTENCIA: Desde la Universidad consideramos que ChatGPT u otras herramientas similares son herramientas muy útiles en la vida académica, aunque su uso queda siempre bajo la responsabilidad del alumno, puesto que las respuestas que proporciona pueden no ser veraces. En este sentido, NO está permitido su uso en la elaboración del Trabajo fin de Grado para generar código porque estas herramientas no son fiables en esa tarea. Aunque el código funcione, no hay garantías de que metodológicamente sea correcto, y es altamente probable que no lo sea.

Por la presente, yo, Lucía Grau Torres , estudiante de E2 Analytics de la Universidad Pontificia Comillas al presentar mi Trabajo Fin de Grado titulado " ASSEING MARKET EFFICIENCY FOR A BIODIVERSITY FOCUSED INDEX USING MULTIFRACTAL DETRENDED FLUCTUATION ANALYSIS (MF-DFA)", declaro que he utilizado la herramienta de Inteligencia Artificial Generativa ChatGPT u otras similares de IAG de código sólo en el contexto de las actividades descritas a continuación [el alumno debe mantener solo aquellas en las que se ha usado ChatGPT o similares y borrar el resto. Si no se ha usado ninguna, borrar todas y escribir “no he usado ninguna”]:

1. **Crítico**: Para encontrar contra-argumentos a una tesis específica que pretendo defender.

2. **Referencias**: Usado conjuntamente con otras herramientas, para identificar referencias preliminares que luego he contrastado y validado.

3. **Interpretador de código**: Para realizar análisis de datos preliminares.

4. **Estudios multidisciplinares**: Para comprender perspectivas de otras comunidades sobre temas de naturaleza multidisciplinar.

5. **Corrector de estilo literario y de lenguaje**: Para mejorar la calidad lingüística y estilística del texto.

6. **Sintetizador y divulgador de libros complicados**: Para resumir y comprender literatura compleja.

Afirmo que toda la información y contenido presentados en este trabajo son producto de mi investigación y esfuerzo individual, excepto donde se ha indicado lo contrario y se han dado los créditos correspondientes (he incluido las referencias adecuadas en el TFG y he explicitado para que se ha usado ChatGPT u otras herramientas similares). Soy consciente de las implicaciones académicas y éticas de presentar un trabajo no original y acepto las consecuencias de cualquier violación a esta declaración.

Fecha: 23/04/2024

Firma: ____Lucia Grau Torres______________________

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ANEX I: INFORMATION ABOUT THE TOP TEN CONSTITUENTS OF THE MSCI WORLD SELECT NATURAL RESOURCES INDEX

In order to demonstrate the index's unequivocal focus on companies undertaking concrete and precise measures to uphold the conservation of nature and biodiversity, we will conduct an examination of the top ten constituents and the specific initiatives they undertake.

Exon Mobil Corp, founded in 1870, the American oil company operates across 60 countries and maintains a strong commitment to the preservation of nature and biodiversity. Their foundational principle, "Protect Tomorrow. Today," serves as the driving force behind their environmental conservation initiatives and their unwavering adherence to stringent standards. They create customized land and habitat management plans to suit each operational location and scale. In 2021, ExxonMobil introduced an updated and enhanced set of Project Environmental Standards for Land Utilization and Marine Sound, with the primary goal of averting or mitigating adverse environmental consequences associated with land usage, including those occurring in critical habitats, and efficiently managing activities producing marine sound. Regularly, they evaluate the proximity of their major facilities to ecologically protected zones, utilizing this information to enhance safety measures and emergency response strategies. Notably, almost 30% of their primary operational sites are situated within a five-kilometer radius of designated protected areas. Furthermore, ExxonMobil employs diverse assessment procedures for evaluating environmental, social, and health-related risks and challenges, applying best practices such as the Cross Sector Biodiversity Initiative's mitigation hierarchy. They actively incorporate nature-based solutions into project design considerations, with a specific focus on land and habitat management. This commitment extends to providing comprehensive training for their employees in this field. (Exxon Mobil, 2024)

Chevron Corp, founded in 1911, the American oil company is deeply committed to the preservation of biodiversity and has a long history of collaborating with various stakeholders, including communities, industry bodies, regulators, and conservation groups, to protect biodiversity in regions where the company conducts its operations. The company prioritizes safety, efficient operations, and environmental responsibility, especially in areas with sensitive biological characteristics. Chevron actively works to avoid and mitigate significant impacts on species, habitats, and ecosystems, integrating biodiversity into their decision-making process. Their communication and engagement efforts involve sharing information about their biodiversity-related activities with both internal and external stakeholders, including government, local communities, and industry associations. Chevron also seeks to participate in external policy-making activities that affect their operations. Some examples of innovative solutions to safeguard biodiversity are the following: transforming platform jackets into reefs in Thailand, restoring wetlands and switching to solar-powered mobile lighting in Australia, using comprehensive drilling plans in Colorado etc. In their website they have a space meant for success stories where they explain with great detail some of their initiatives and their results, as well as their joint ventures. Overall, a company that cares about the environment and takes pride in effectively communicating their efforts. (Chevron, 2024)

ConocoPhillips, one of the world’s largest energy and oil company that operates over 13 countries. They have a commitment to factor biodiversity into their global operations. Consequently, they don’t explore, develop, drill, or produce in habitats of endangered species unless there are measures to be taken that can ensure the safety of the ecosystem. They use a management system based on their Sustainable Development Risk Management Standard to assess for biodiversity risks onshore and offshore. They focus on developing best practices while considering the cumulative effects of their operations. To do this, they conduct site assessments and baseline studies to gather data on local biological diversity. They also create indicators and metrics to monitor the impacts on biodiversity and their risk management performance. In addition, Chevron places a strong emphasis on implementing innovative and sustainable solutions for biodiversity conservation. They actively engage in stewardship and habitat conservation practices on lands they own. They leverage programs like the SPIRIT of Conservation Program and migratory bird joint ventures to support habitat conservation and restoration. (ConocoPhillips, 2024)

Shell is a global group that provides energy and other petrochemicals. Shell has a Respecting Nature goal with 4 priorities, one of them being biodiversity. They have new projects in areas rich in biodiversity (critical habitats) that started implementation in 2021 and their objective was to have a net positive impact in said areas. In 2021 they also launched nature-based solutions to protect and restore land. Along with activities to combat deforestation starting 2022. Moreover, they have stated that they won’t explore nor develop activities in natural and mixed World Heritage Sites. Shell's approach to biodiversity emphasizes avoiding negative impacts, minimizing disturbances to sensitive species, restoring habitats, and making positive contributions to conservation. They are dedicated to safeguarding protected areas, working with the International Union for

Conservation of Nature (IUCN) and other partners, and collaborating on guidelines for mitigating the impact of renewable energy projects on biodiversity (Shell, 2024).

Total Energies is a multi-energy company that produces and markets energies globally: oil, biofuels, natural gas, green gases… The company has pledge to support and work towards the SDGs, including protecting biodiversity. They follow in all their projects and operations the Mitigation Hierarchy: avoidance, minimization, restoration, and offsetting. They propose 4 commitments: respecting their voluntary exclusion zones, like Artic Sea ice areas; managing biodiversity in their new projects with the aim of producing a net positive impact. They also commit to manage biodiversity at their existing sites and promoting biodiversity regarding publicity, awareness programs, sharing data etc. They provide detailed plans on how to deliver on these 4 commitments (Total Energies, 2021).

Schlumberger (SLB), founded in 1926, is a global technology company in the oil and gas industry. They prioritize the safety of ecosystems and biodiversity in their operations. Thay have and Environmental Management Process, which includes a section on biodiversity. This process involves risk assessments before starting any new endeavor and a guide on mitigation efforts. They focus on preserving wildlife, preventing land contamination, land rehabilitation and restoration and decommissioning and abandonment. Some examples are detailed on the website along with last's year sustainability report. In 2020, employee volunteers helped plant 2000 fruit trees near Prune, India. In Ankara, Turkey SLB worked with the General Directorate of Forestry to plant 10,000 trees to restore a local ecosystem (Schlumberger Limited, 2023).

Linde is a global industrial gases and engineering company, whose operations don't have a big impact on biodiversity. In its planning for new sites, Linde ensures minimal harm to biodiversity, following international guidelines like the UN's Voluntary Guidelines on Biodiversity-Inclusive Impact Assessment. Their approach is to avoid operations near protected areas, and periodic assessments confirm this. The latest evaluation covering 600+ sites found no proximity to protected areas. Linde will continue periodic reevaluations and biodiversity impact analysis in 2023 (Linde, 2022).

Canadian Nat Resources is a prominent player in the Canadian crude oil and natural gas industry. Over the past two decades, investments in technology and research within the sector have grown by over tenfold, resulting in decreased environmental impact and reduced expenses. Canadian innovation and collaborative efforts within the industry are poised to maintain its sustainability, competitiveness, and productivity well into the future. They focus more on preserving the environment as a whole rather than having specific initiatives for biodiversity. However, they do have plans to counteract deforestation (as of 2022 they have planted 8.6 million trees) and reduce the impact of oil and gas operations on the land so they don't harm healthy ecosystems (Canadian Natural, 2024).

Enbridge Gas is North America's largest natural gas utility. In their 2022 Sustainability report we can see that they have a section to address biodiversity and land. Their technical services teams integrate biodiversity when carrying out an operation. Via Enbridge Fueling Futures, they allocate resources to support biodiversity through initiatives such as the RGV Reef study, initiated in 2022, as well as by offering financial assistance and enabling their staff to engage in endeavors like tree planting and the restoration of habitats. They work with landowners and local communities to hear their concerns regarding the effect of their operations on their environment. Enbridge is actively observing the progress of emerging guidelines and suggested reporting requirements, which encompass the Global Biodiversity Framework (GBF) and the Taskforce on Nature-related Financial Disclosures (TNFD). They are presently collaborating with the Wildlife Habitat Council to evaluate their performance related to biodiversity in line with these evolving standards, and they intend to disclose the results in our forthcoming Sustainability Report (Enbridge, 2022).

Marathon Petroleum (MPC) is an American downstream energy company, it refines, markets and transports petrol. MPC shows its dedication to protecting biodiversity through several proactive strategies to conserve natural habitats and species. Its Operational Excellence Management System ensures the integration of ecological considerations into the operations. They collaborate with several regulatory agencies like U.S. EPA and U.S. Fish and Wildlife Service to design necessary mitigation measures (Marathon Petroleum, 2024).

ANEX II: MF-DFA METRICS YEAR BY YEAR

Metrics Year 2014

	slope	**hq**	**tau**	**alpha**	**Dalpha**
-5.0	2.779323	1.779323	-9.896614	2.100897	-0.607869
-4.0	2.698929	1.698929	-7.795717	2.090955	-0.568101
-3.0	2.571568	1.571568	-5.714705	2.032988	-0.384258
-2.0	2.364871	1.364871	-3.729742	1.835248	0.059246
-1.0	2.044208	1.044208	-2.044208	1.147892	0.896316
1.0	1.713935	0.713935	-0.286065	0.793380	1.079446
2.0	1.667966	0.667966	0.335932	0.574238	0.812543
3.0	1.620803	0.620803	0.862410	0.478256	0.572358
4.0	1.573111	0.573111	1.292443	0.395053	0.287769
5.0	1.530503	0.530503	1.652516	0.360072	0.147846

Metrics Year 2015

	slope	hq	tau	alpha	Dalpha
-5.0	2.095046	1.095046	-6.475230	1.258731	0.181573
-4.0	2.054125	1.054125	-5.216499	1.249360	0.219059
-3.0	1.992170	0.992170	-3.976510	1.213270	0.336701
-2.0	1.894979	0.894979	-2.789959	1.118401	0.553156
-1.0	1.739707	0.739707	-1.739707	0.768422	0.971285
1.0	1.515308	0.515308	-0.484692	0.568116	1.052808
2.0	1.482321	0.482321	-0.035358	0.434269	0.903896
3.0	1.461282	0.461282	0.383846	0.408416	0.841404
4.0	1.445369	0.445369	0.781475	0.390779	0.781642
5.0	1.433081	0.433081	1.165404	0.383929	0.754242

Metrics Year 2016

	slope	hq	tau	alpha	Dalpha
-5.0	3.026352	2.026352	-11.131760	2.347459	-0.605537
-4.0	2.946075	1.946075	-8.784301	2.356152	-0.640308
-3.0	2.806485	1.806485	-6.419456	2.343066	-0.609742
-2.0	2.549084	1.549084	-4.098169	2.166995	-0.235822
-1.0	2.085465	1.085465	-2.085465	1.190514	0.894951
1.0	1.473373	0.473373	-0.526627	0.642940	1.169567
2.0	1.421677	0.421677	-0.156645	0.370112	0.896869
3.0	1.404532	0.404532	0.213596	0.367602	0.889211
4.0	1.394640	0.394640	0.578559	0.351078	0.825751
5.0	1.383150	0.383150	0.915751	0.337192	0.770209

Metrics Year 2017

	slope	hq	tau	alpha	Dalpha
-5.0	2.703690	1.703690	-9.518452	1.888312	0.076892
-4.0	2.657535	1.657535	-7.630140	1.892040	0.061980
-3.0	2.578124	1.578124	-5.734372	1.906748	0.014129
-2.0	2.408322	1.408322	-3.816645	1.892545	0.031555
-1.0	1.949282	0.949282	-1.949282	1.102381	0.846901
1.0	1.490498	0.490498	-0.509502	0.620936	1.130438
2.0	1.456763	0.456763	-0.086474	0.408884	0.904243

3.0	1.436089	0.436089	0.308267	0.382839	0.840249
4.0	1.419801	0.419801	0.679204	0.359399	0.758392
5.0	1.405413	0.405413	1.027065	0.347861	0.712242

Metrics Year 2018

	slope	hq	tau	alpha	Dalpha
-5.0	2.469351	1.469351	-8.346753	1.646005	0.116725
-4.0	2.425187	1.425187	-6.700747	1.657597	0.070361
-3.0	2.343853	1.343853	-5.031560	1.671742	0.016333
-2.0	2.178632	1.178632	-3.357263	1.569426	0.218411
-1.0	1.892708	0.892708	-1.892708	0.999725	0.892983
1.0	1.641910	0.641910	-0.358090	0.709656	1.067746
2.0	1.618131	0.618131	0.236261	0.578127	0.919993
3.0	1.599388	0.599388	0.798164	0.543532	0.832433
4.0	1.580832	0.580832	1.323326	0.508242	0.709641
5.0	1.562930	0.562930	1.814648	0.491322	0.641962

Metrics Year 2019

slope	hq	tau	alpha	Dalpha	
-5.0	2.604854	1.604854	-9.024270	1.843831	-0.194884
-4.0	2.545110	1.545110	-7.180439	1.851492	-0.225531
-3.0	2.440428	1.440428	-5.321285	1.830584	-0.170467
-2.0	2.259635	1.259635	-3.519271	1.656321	0.206629
-1.0	2.008643	1.008643	-2.008643	1.058201	0.950442
1.0	1.655331	0.655331	-0.344669	0.724767	1.069436
2.0	1.582830	0.582830	0.165659	0.473012	0.780365
3.0	1.533785	0.533785	0.601355	0.411375	0.632770
4.0	1.497102	0.497102	0.988409	0.370718	0.494462
5.0	1.468558	0.468558	1.342791	0.354382	0.429119

Metrics Year 2020

slope	hq	tau	alpha	Dalpha	
-5.0	2.542981	1.542981	-8.714904	1.814883	-0.359514
-4.0	2.475005	1.475005	-6.900020	1.811969	-0.347857
-3.0	2.363655	1.363655	-5.090965	1.776007	-0.237055
-2.0	2.174003	1.174003	-3.348007	1.593456	0.161095
-1.0	1.904053	0.904053	-1.904053	0.993977	0.910077
1.0	1.633924	0.633924	-0.366076	0.729608	1.095684
2.0	1.642385	0.642385	0.284770	0.632830	0.980890
3.0	1.633195	0.633195	0.899584	0.582002	0.846422
4.0	1.612193	0.612193	1.448774	0.529776	0.670330
5.0	1.591827	0.591827	1.959136	0.510362	0.592674

Metrics Year 2021

	slope	hq	tau	alpha	Dalpha
-5.0	3.397778	2.397778	-12.988889	2.690913	-0.465678
-4.0	3.324494	2.324494	-10.297976	2.698548	-0.496215
-3.0	3.197265	2.197265	-7.591794	2.693663	-0.489194
-2.0	2.955325	1.955325	-4.910650	2.607956	-0.305262
-1.0	2.375881	1.375881	-2.375881	1.463999	0.911882
1.0	1.481347	0.481347	-0.518653	0.736258	1.254911

2.0	1.416447	0.416447	-0.167106	0.320711	0.808527
3.0	1.374256	0.374256	0.122769	0.269458	0.685606
4.0	1.342953	0.342953	0.371811	0.234408	0.565822
5.0	1.318317	0.318317	0.591585	0.219774	0.507286

Metrics Year 2022

	slope	hq	tau	alpha	Dalpha
-5.0	2.505728	1.505728	-8.528640	1.717501	-0.058867
-4.0	2.452785	1.452785	-6.811138	1.724930	-0.088582
-3.0	2.359593	1.359593	-5.078780	1.729588	-0.109985
-2.0	2.175981	1.175981	-3.351962	1.613269	0.125424
-1.0	1.852241	0.852241	-1.852241	0.961528	0.890714
1.0	1.532621	0.532621	-0.467379	0.626533	1.093912
2.0	1.513678	0.513678	0.027357	0.507940	0.988524
3.0	1.516167	0.516167	0.548502	0.530167	1.041999
4.0	1.521923	0.521923	1.087690	0.536572	1.058597
5.0	1.524329	0.524329	1.621645	0.533955	1.048130

Metrics Year 2023

slope	hq	tau	alpha	Dalpha	
-5.0	2.397150	1.397150	-7.985752	1.626704	-0.147767
-4.0	2.339762	1.339762	-6.359048	1.625142	-0.141522
-3.0	2.245156	1.245156	-4.735467	1.590221	-0.035196
-2.0	2.089303	1.089303	-3.178606	1.430467	0.317672
-1.0	1.874533	0.874533	-1.874533	0.922043	0.952490
1.0	1.587523	0.587523	-0.412477	0.642963	1.055441
2.0	1.527179	0.527179	0.054358	0.441101	0.827844
3.0	1.489908	0.489908	0.469724	0.402694	0.738357
4.0	1.464936	0.464936	0.859745	0.380559	0.662492
5.0	1.446168	0.446168	1.230842	0.371097	0.624645

ANEX III: PYTHON CODE

```
# %%

%config InlineBackend.figure_format ='svg', #‘retina’, ‘jpeg’, ‘svg’, ‘pdf’, 'svg'

import pandas as pd

from pathlib import Path

import numpy as np

import matplotlib.pyplot as plt

from MFDFA import MFDFA

from numpy.polynomial.polynomial import polyfit

import seaborn as sns

sns.set()

from pathlib import Path

from scipy.stats import skew, kurtosis, jarque_bera

# %%

prices = pd.read_excel("Index Levels.xlsx", sheet_name = 'Sheet1')

prices = prices.drop(columns=['MSCI INDEX CODE','ISO CURRENCY SYMBOL','INDEX VARIANT TYPE'])

prices=prices.set_index('CALC DATE')

prices['log_rets']=0

prices['log_rets'] = np.log(prices['LEVEL EOD'] / prices['LEVEL EOD'].shift(1)) #precios / precios desplazados uno hacia abajo para hacer la variacion. Al hacer esto metes un valor perdido

prices.head()

all_data = prices.dropna()

all_data.head()

data = all_data #nos centramos en un periodo anual para acotar.

risk_free_rate = 0.005

results = []

for i in range(2014, 2024):

    data = all_data.loc[all_data.index.year == i]

    if not data.empty:

        # Calculate annual return

        ann_return = (np.exp(data['log_rets'].sum()) - 1) * 100

        # Calculate annualized volatility

        volatility = data['log_rets'].std() * np.sqrt(252) * 100  # Scale to percentage

        # Calculate Sharpe Ratio (adjusting for risk-free rate)

        sharpe_ratio = (mean_log_return_annualized - risk_free_rate) / (data['log_rets'].std() * np.sqrt(252))

        #Calculate skewness

        skewness = skew(data['log_rets'])

        # Calculate kurtosis

        kurt = kurtosis(data['log_rets'])

        # Calculate Jarque-Bera test statistic

        jb_test_statistic, jb_p_value = jarque_bera(data['log_rets'])

        # Append results

        results.append({

            'Year': i,

            'Annualized Return (%)': f"{ann_return:.3f}%",

            'Annualized Volatility (%)': f"{volatility:.3f}%",

            'Sharpe Ratio': f"{sharpe_ratio:.3f}",

            'Skewness': f"{skewness:.3f}",

            'Kurtosis': f"{kurt:.3f}",

            'Jarque-Bera Test Statistic': f"{jb_test_statistic:.3f}",

            'Jarque-Bera p-value': f"{jb_p_value:.3f}"

        })

results_df = pd.DataFrame(results)

print(results_df)

## MF-DFA

def get_mfdfa(data,

              lag_start=None,
```

```
        lag_end=None,
        lag_steps=None,
        q=None,
        order=1,  #m segmetos en linea recta
        integrate=True,
        window=None):
    """
    Wrapper around the MFDFA package
    :param data: time series
    :param lag_start: minimum lag
    :param lag_end: maximum lag (if not provided, len(data)//6)
    :param lag_steps: how many lags to generate
    :param q: power q
    :param order: order of polynomial fitting
    :return:
    """
    def _integrate(data):
        mean = data.mean()
        return (data - mean).cumsum()
    def _power_variations(q_min, q_max, q_steps):
        q = np.linspace(q_min, q_max, q_steps)
        q = q[q != .0]
        return q
    # Let's start
    data_length = len(data)
    if lag_start is None:
        # if lag_start is not provided explicitly,
        lag_start = 3
    if lag_end is None:
        # if lag_end is not provided explicitly,
        lag_end = data_length // 6
    if lag_steps is None:
        # if lag_steps is not provided explicitly,
        lag_steps = min(100, data_length // 6)

    lags = np.unique(np.logspace(np.log10(lag_start), np.log10(lag_end), lag_steps).astype(int))
    # q power variations, removing the 0 power
    if q is None:
        q = _power_variations(-5, +5, 10+1)
    # integrate series
    if integrate:
        integr_data = _integrate(data)
    else:
        # passthrough
        integr_data = data
    # Obtain the (MF)DFA as
    extensions = {}
    if window:
        extensions['window'] = window
    # using MFDFA package to calculate the multifractal DFA
    lag, dfa = MFDFA(integr_data, lag=lags, q=q, order=order, extensions=extensions)
    # This "fine-tuning" should be done in a more robust way,
    # by truncating the lag_start and lag_end values before and not here.
    start = int(0.00 * lag.size)
    end   = int(0.60 * lag.size)
    # And now we need to fit the line to find the slope. Don't
    # forget that since you are plotting in a double logarithmic
    # scales, you need to fit the logs of the results
    slope = polyfit(np.log10(lag[start:end]), np.log10(dfa[start:end]), 1)[1]
    # Hurst exponent
    hq = slope - 1
```

```
    # q-order mass exponent
    tau = q * hq - 1  # slopes or hq??
    # q-order singularity exponent (aka alpha)
    alpha = np.gradient(tau) / np.gradient(q)
    # Dimension (aka D(alpha))
    Dalpha = q * alpha - tau
    metrics = pd.DataFrame(index=q,
                        data={'slope': slope,
                              'hq': hq,
                              'tau': tau,
                              'alpha': alpha,
                              'Dalpha': Dalpha})
    spectral_width = metrics['Dalpha'].max() - metrics['Dalpha'].min()
    return q, lag, dfa, slope, hq, tau, alpha, Dalpha, metrics, spectral_width
# %%
for i in range(2014,2025):
    data = all_data.loc[all_data.index.year == i]
    q, lag, dfa, slope, hq, tau, alpha, Dalpha, metrics, spectral_width = get_mfdfa(
    data['log_rets'].values,
    lag_start=None,
    lag_end=None,
    lag_steps=None,
    q= None,
    order=1,
    integrate=True,
    window=None)
def plot_multifractal(q, lag, dfa, q_results, filename=None):
    figure, axes = plt.subplots(2, 2, figsize=(18, 12))

    # Fluctuation function: log(dfa) vs. log(lag) per q
    for i, qn in enumerate(q):
        axes[0][0].loglog(lag[:], dfa[:, i], '.', markersize=3, label=f'$q={qn}$')
        axes[0][0].set(xscale='log', yscale='log', title=r'Fluctuation functions: $F_q(s) \sim s^{h(q)}$',
                    xlabel=r'lag: $s$', ylabel=r'$F_q(s)$')


    # Hurst exponent per q
    axes[0][1].plot(q_results.index, q_results.hq, '.-')
    axes[0][1].set(title='Generalized Hurst exponents: $h(q)$', xlabel=r'$q$', ylabel=r'$h(q)$', ylim=[-0.1, 1.5])

    # Mass exponent
    # axes[1][0].plot(q_results.index, q_results.hq)
    axes[1][0].plot(q_results.index, q_results.tau, '.-')
    axes[1][0].set(title=r'Multifractal scaling exponent: $\tau_q$', xlabel=r'$q$', ylabel=r'$\tau(q)$')

    # spectral
    axes[1][1].plot(q_results.alpha, q_results.Dalpha, '.-')
    axes[1][1].set(title='Singularity spectrum', xlabel=r'$\alpha$', ylabel=r'$D(\alpha)$')

    if filename:
        #print(f'saving multifractal plot to: {filename}')
        plt.savefig(filename)
        plt.close(figure)

    return figure

# %%
# multifractal plot
filename = Path('demo_multifractal.svg')
```

```
plot_multifractal(q, lag, dfa, metrics, filename=filename)


# ## Rolling Hurst


# %%

def hurst_rolling(log_rets, window, step):

    def _calculate_hq(x):

        # print(x.index[0])

        q, lag, dfa, slope, hq, tau, alpha, Dalpha, metrics, spectral_width = get_mfdfa(

            x.values,

            q=[-2, 2],

            window=False)

        return metrics.loc[2, 'hq']


    result = log_rets.rolling(window=window, min_periods=window).apply(_calculate_hq) #hemos quitado step

    return result


# %%

WINDOW = 252

S = 10

roll_hurst = hurst_rolling(data, WINDOW, S) ##tengo que hacer drop del close

roll_hurst = roll_hurst.clip(lower=0.0, upper=1.0)


# %% [markdown]

# ## Plot rolling hurst


# %%

def plot_hurst_timeseries(rolling_hurst, filename=None, figsize=(12, 6)):


    figure, ax = plt.subplots(1, 1, figsize=figsize)

    # rolling hurst

    sns.lineplot(rolling_hurst)

    ax.set(title='Rolling Hurst', xlabel=r'date', ylabel=r'h(2)')


    if filename:

        #print(f'saving plot to: {filename}')

        plt.savefig(filename)

        plt.close(figure)


    return figure


# %%

# currency plot

filename = Path('demo_rolling.svg')

plot_hurst_timeseries(roll_hurst, filename=filename)
```