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Assessing the Neutral Interest Rate for the U.S. Economy: Extension of the Holston Laubach Williams Model, combined with the Kalman Filter and Machine Learning

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Abstract:

This study aims to estimate the neutral interest rate for the U.S. economy by examining its correlation with crucial macroeconomic variables and assessing its implications for economic policy. This research employs an extension of the Holston, Laubach, and Williams (HLW) model and the Kalman filter to provide a robust framework for estimating the neutral rate. Applying machine learning techniques, specifically K-fold cross-validation, enhances the model's predictive accuracy. Several key variables, including inflation, unemployment, GDP growth, yield spreads, and non-farm payroll data, are integrated to understand the factors influencing the neutral rate comprehensively.

A notable contribution of this study is incorporating variables validated through heuristic rules, such as the Sahm Rule, employing non-farm private payrolls to provide timely recession signals as a proxy of the unemployment rate. The incorporation of yield spreads further enhances the model's predictive capability. The findings indicate that the natural interest rate frequently declines before economic downturns, including the 2008 financial crisis and the 2020 COVID-19 pandemic. This phenomenon serves as an early warning indicator of impending economic challenges. Moreover, in anticipation of this year's Fed rate cut, the estimated neutral interest rate has already been revised downwards, acting as a leading indicator.

The analysis indicates that as the natural interest rate declines, it signals potential economic weaknesses, which precede increases in the unemployment rate. This predictive capability, supported by the Sahm Rule, emphasises the significance of the neutral interest rate in guiding monetary policy decisions. The study emphasises the necessity for policymakers to monitor the natural rate meticulously to provide timely indications of economic stress, thereby enabling the implementation of preemptive measures to mitigate the risk of recession.

The findings indicate the necessity for policy recommendations to maintain economic stability and growth. In particular, monitoring the natural interest rate and labour market metrics to anticipate and mitigate future economic downturns is emphasised. Furthermore, the study examines the historical context of interest rates and the theoretical foundations of the neutral rate, providing a comprehensive analysis of the interrelationship between macroeconomic variables and monetary policy.

Key words: Interest Rate, Natural Interest Rate, HLW, KF, GDP, Unemployment, Spread

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1. Introduction

i. Overview and scope of the study

This study aims to analyse the interest rate variables and implications with the final objective of establishing a method to estimate a possible neutral interest rate for the United States economy. However, it could also be adapted to be applied to other economies. The study employs the Holston, Laubach, and Williams (2017) model combined with the Kalman filter, a statistical tool that helps estimate unobservable variables in a dynamic setting (Holston et al., 2017; Kalman, 1960).

Additionally, the study utilises K-fold cross-validation to ensure the robustness and reliability of the model, providing a comprehensive performance assessment across multiple subsets of the data. A significant incremental value of this study is the integration of additional variables into the existing models, specifically the yield spreads and labour market metrics such as non-farm payroll data, which enhance the predictive accuracy and comprehensiveness of the analysis based on both the results obtained and the previous literature like the Sahm Rule (2019).

The monetary authorities are endowed with a clear mandate that they are obliged to achieve and maintain. In the case of the United States, the Federal Reserve is the responsible authority, established in 1913 and tasked with regulating the economy with the dual mandate following the Employment Act of 1946 and, subsequently, the Federal Reserve Reform Act of 1977 (Broz, 1997). As Congress prescribes, the Federal Reserve's dual mandate is to foster the conditions that achieve both stable prices and maximum sustainable employment within the United States economy (Thorbecke & Levy, 2016). In contrast, in Europe, the European Central Bank (ECB) is responsible for monetary policy to maintain price stability by aiming for a symmetric inflation target of 2% over the medium term (restated in 2021).

According to Gardner (1983), the interest rate is an instrument that monetary authorities have to regulate some aspects of the economy. Using interest rates to address inflation is a crucial aspect of monetary policy. Interest rates significantly impact various macroeconomic components, including supply, demand, and investment (Taylor, 1993;

Clarida et al., 2000), as they are the primary and most efficient instruments that monetary authorities use.

The setting of interest rates and the interpretation of the implications require a clear understanding of the economic situation. This enables the achievement of the economic targets set by policymakers. Monitoring a range of macroeconomic variables is essential to assess the business cycle. This helps identify potential economic trends and the appropriate policy responses (Bernanke & Blinder, 1990).

Historically, adjustments to interest rates have not been executed promptly, resulting in the economy entering a recessionary phase and failing to transition into an expansionary phase when feasible (Mishkin, 2007). This indicates that economies are not operating at their full potential, and society has had several adverse consequences.

Over the past two decades, monetary policy has been employed significantly to navigate financial crises and maintain economic stability (Federal Reserve, n.d.). In the early 2000s, central banks, particularly the Federal Reserve, leveraged interest rate adjustments to counteract the economic downturn triggered by the bursting of the dot-com bubble. The onset of the 2008 financial crisis further expanded the scope of monetary interventions, with the Federal Reserve and the ECB implementing quantitative easing alongside traditional rate cuts (even reaching negative nominal interest rates). The measures mentioned earlier were designed to stimulate economic activity, prevent a protracted recession, and stabilise financial markets. This followed the National Bureau of Economic Research (NBER) for the United States and the European Central Bank (ECB) for Europe, identifying the period as recessionary.

In early 2020, in response to the economic disruptions caused by the COVID-19 pandemic, the Federal Reserve implemented a series of unprecedented monetary policies, including reducing interest rates to near-zero levels and reintroducing extensive quantitative easing programs as Auerbach et al. (2020) and Beckworth (2020) expose. Aguilar et al. (2020) describe how similar strategies were adopted globally as central banks, for example, in Europe by the ECB, endeavoured to mitigate the economic impact of lockdowns and a downturn in consumer activity. Direct fiscal supports, such as consumer checks and business subsidies to firms, further reinforced these strategies. These actions demonstrate the malleability of monetary policy, which can be employed to manage traditional objectives such as inflation and employment and address broader systemic risks that threaten economic stability.

The hypothesis of the neutral interest rate, which forms the central tenet of this study, suggests that there is a point where the economy remains stable and neutral. This means that monetary policy is neither expansionary nor contractionary when the economy achieves full employment and maintains stable inflation. This hypothesis demonstrates the necessity of precisely estimating the neutral interest rate to maintain economic equilibrium (Archibald & Hunter, 2001).

This study aims to develop a robust methodological framework to estimate the neutral interest rate for the U.S. economy. The analysis involves examining historical data and previous research to establish a reliable estimate of the neutral interest rate. The study will focus on how changes in key macroeconomic variables such as inflation, unemployment, and GDP growth impact the neutral interest rate (Holston et al., 2017; Beyer & Wieland, 2019) combined with new variables proposed based on related studies as the 10-2 and 10-3 spread and the Non-Farm Payrolls.

ii. Significance of the Neutral Interest Rate

The neutral interest rate, also known as the natural rate of interest or the equilibrium interest rate, is a theoretical concept used in monetary policy (Archibald & Hunter, 2001). Numerous papers and studies have been on this topic, as the neutral interest rate represents an ideal equilibrium point for economies. The main reason for all the research is that reaching this neutral interest rate could have prevented severe social consequences (during recession periods) and facilitated economic growth more stably. Recent research underscores its relevance in guiding central banks' policy decisions, especially during periods of economic uncertainty (Holston et al., 2017; Beyer & Wieland, 2019).

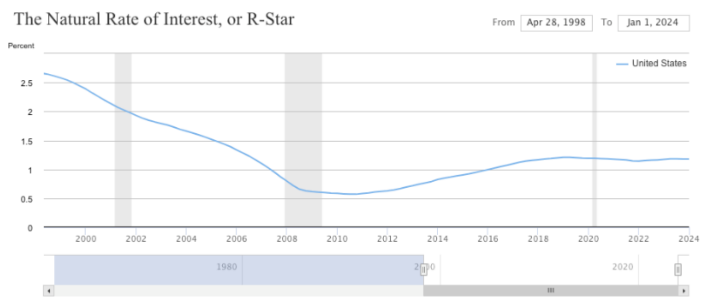
The term "neutral interest rate" was first used by Wicksell (1936) in 1898 in his book "Interest and Prices." In "Interest and Prices," Wicksell introduces the "natural interest rate." He defines this as the equilibrium interest rate at which the level of savings exactly equals the investment demand, thereby ensuring price stability within an economy.

Wicksell also postulates that deviations from this natural rate can have significant macroeconomic consequences. The author postulates that keeping interest rates below the equilibrium point (currently referred to as an expansionary cycle) could lead to increased levels of investment and consumption, ultimately leading to inflationary pressures. Conversely, interest rates above the natural rate could reduce economic activity and deflationary tendencies (currently defined as a recession or contractionary cycle).

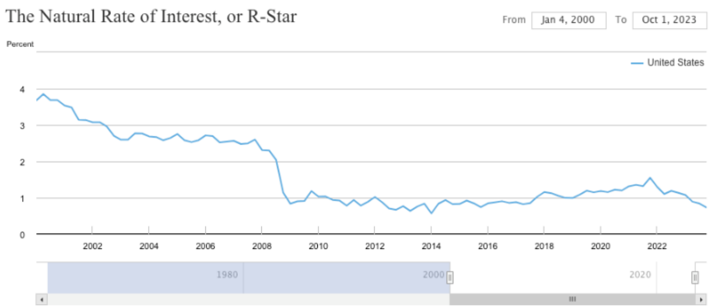
In contrast, John Maynard Keynes (2018) argued in "The General Theory of Employment, Interest and Money" that the neutral interest rate should be considered in the context of full employment. According to Keynes, it is the rate at which investment and saving are in equilibrium at full employment level without causing inflation. This redefinition shifts the focus from merely achieving price stability to maintaining employment, thereby broadening the range of economic objectives that the neutral interest rate seeks to achieve.

It is essential to recognise that the neutral interest rate has not been fully identified within a given economy as a theoretical construct that has evolved in response to implementing different economic policies (Archibald & Hunter, 2001). Consequently, all published estimates and research findings are inherently provisional, particularly given the unique characteristics of each economy (and country) that will influence the specific variables and the resulting interest rate as, for example, the case described by Kang and Do (2024). This variability is further complicated by global economic integration, which impacts domestic interest rates through international capital flows and monetary policy spillovers (Ferreira & Shousha, 2023).

Conversely, the New York FED has calculated the natural interest rate (R-Star) using both the base Lauran Williams model and the improved Holston Laubach Williams model publicly available.



Source: Measuring the natural rate of interest. (2024). FEDERAL RESERVE BANK of NEW YORK. https://www.newyorkfed.org/research/policy/rstar



Source: Measuring the natural rate of interest. (2024). FEDERAL RESERVE BANK of NEW YORK. https://www.newyorkfed.org/research/policy/rstar

Nevertheless, direct comparison is not possible since this study places particular emphasis on the incorporation of new variables, including NFP and 10-3 and 10-2 spreads, in addition to the fact that the source of the data is not replicable, and the assumptions used in the model are not public. Consequently, the two results are not homogeneous in methodology and, therefore, not comparable.

In any case, the difficulty of determining the neutral interest rate has recently increased, as the complexity of economic systems has increased exponentially since Wicksell first proposed his theory (Wicksell, 1936). Nevertheless, numerous studies have been carried out and are still being carried out to assess the neutral interest rate for different economies. For instance, the use of advanced econometric models and real-time data analysis has improved the precision of these estimates, although challenges remain (Holston et al., 2017; Beyer & Wieland, 2019).

iii. Methodology Overview

This study aims to estimate the interest rate for the U.S. economy. In order to accurately estimate the potential interest rate for the United States economy, our study will utilise a combination of quantitative metrics based on previous studies and existing models, in addition to other newly derived variables derived from qualitative case studies and other variables derived from the literature review. In particular, the Sahm Rule will be employed as a heuristic indicator to identify the onset of recessions, informing the analysis of business cycles and interest rate adjustments. The objective is to activate automatic stabilisers (Sahm, 2019).

This study will utilise historical data from the Federal Reserve of St. Louis and additional reputable sources for the investigation (for example, Bloomberg, FactSet, and Refinitiv Eikon) to track the progression of interest rates and analyse their effects throughout history. On the other hand, the data used for the empirical analysis with Python will be extracted using the FRED API (Federal Reserve of St. Louis) through an API key requested explicitly for this purpose. Using this extraction method, we will ensure that all the data is homogeneous and will reduce the data source disparity risk.

Furthermore, the study will employ the Holston, Laubach, and Williams (2017) model, which is specifically designed to estimate the natural rate of interest and utilise the Kalman filter to extract unobservable components from the data (Holston et al., 2017; Kalman, 1960). The Kalman filter is a powerful statistical tool that enables the estimation of the underlying state of a variable over time, even in the presence of noise and other inaccuracies in the data.

Moreover, the study will integrate machine learning techniques to enhance the accuracy of predictions by identifying patterns and trends within the data (Bajari et al., 2015). In particular, the K-fold cross-validation technique will be employed to guarantee the model's robustness and reliability, thereby providing a comprehensive performance assessment across multiple subsets of the data. This method involves dividing the dataset into k equal-sized subsets, training the model on k-1 subsets, and testing it on the remaining subset. The results are then averaged to estimate the model's accuracy (Refaeilzadeh et al., 2009; Vu et al., 2022).

Additionally, the study will extend the traditional Taylor rule model by incorporating additional explanatory variables, such as yield spreads and labour market metrics. This will provide a more comprehensive understanding of the factors influencing interest rates. The extended Taylor rule model enhances predictive accuracy by considering the inertia of interest rate adjustments and broader market conditions. In particular, the incorporation of the Non-Farm Payrolls following the analysis conducted by Claudia Sahm, in conjunction with 10-2 and 10-3 yield spreads, enables a more comprehensive approach to setting interest rates. These variables reflect broader economic conditions and labour market health, thereby providing crucial insights into future economic activity and inflationary pressures. These variables enable a more comprehensive approach to setting interest rates, reflecting historical trends and current economic conditions.

The study employs advanced econometric models and real-time data analysis to provide more precise and reliable estimates of the neutral interest rate (Holston et al., 2017). This is achieved through econometric tools and FRED's flexibility and real-time data. Furthermore, using Python as the programming language ensures the potential for future enhancement and utilisation. This is due to the increasing popularity of Python as a programming language (Rashed & Ahsan, 2018).

iv. Structure Overview

This study offers a comprehensive analysis of the neutral interest rate, demonstrating its theoretical and practical significance. The introduction provides an overview of the study's scope, the neutral rate's importance, and the methodology. The introduction defines the primary and secondary research objectives.

The literature review examines the conceptualisation of the neutral interest rate, its historical evolution, and its significance in economic theory. The review also considers previous models and empirical findings, focusing on the relationship between interest rates and business cycles. The key frameworks, such as the Phillips curve and the Sahm Rule, are discussed to underscore recent advances in estimating the neutral interest rate.

The research methodology section provides a detailed account of the analytical framework, including the Extended Taylor Rule (1993) Model and the Holston-Laubach-Williams (HLW) model (2017). The text explains using the Kalman filter and machine learning techniques for real-time data analysis. It justifies the selection of key macroeconomic variables, including inflation, unemployment, GDP growth, yield spreads, and non-farm payroll data. The methodology incorporates proxy variables validated thrugh Sahm Rule to provide timely recession signals and integrates yield spreads to capture market expectations. This section also presents the rationale for variable selection and their expected correlation and impact on the neutral interest rate.

In the empirical analysis, the study employs correlation and impact analysis in conjunction with econometric modelling in order to estimate the neutral interest rate. The models are populated with historical data from the Federal Reserve and other sources, incorporating advanced statistical tools and machine learning techniques to enhance predictive accuracy. Integrating the Sahm Rule and yield spreads provides a comprehensive framework for analysing business cycles and interest rate fluctuations.

The discussion of results interprets the model's outcomes, elucidating the economic and policy implications. The analysis compares the estimated neutral interest rate to actual economic conditions, emphasising the importance of monitoring the natural rate for early signals of economic stress.

The conclusions summarise the main insights and policy recommendations, discuss the study's limitations, and suggest future research directions. The appendices include the Python code used for data analysis and model implementation, ensuring transparency and reproducibility. Finally, the references section lists all sources consulted during the study.

v. Previous investigations and possible investigations

The concept of the "neutral interest rate" was first articulated by Knut Wicksell (1936) in 1898 and represented a seminal development in monetary policy. Wicksell's introduction of this concept laid the theoretical foundations for identifying the balance between saving and investment required to achieve economic price stability. This fundamental contribution has underpinned subsequent academic research and informed policy formulation.

Expanding on Wicksell's initial framework, John Maynard Keynes (2018) introduced an additional dimension—employment—into the analysis. In his seminal text, Keynes argued that the neutral interest rate should facilitate equilibrium at full employment without triggering inflation. This concept expansion broadened the scope of the neutral interest rate to include a broader range of economic objectives, thereby increasing its relevance and utility in economic planning and policymaking.

Contemporary academic work has continued to explore this complex concept, mainly using econometric models to estimate the neutral interest rate for specific economies. For instance, Gómez-Valle and Martínez-Rodríguez (2016) and Teodoru and Toktonalieva (2020) have employed sophisticated models to better understand and estimate the neutral interest rate. Such empirical investigations are essential because they confirm historical and theoretical constructs with modern data and adapt these theories to the complex dynamics of contemporary economies.

Recent studies have further refined these models, incorporating advanced econometric techniques and real-time data analysis to improve the accuracy of estimates. For example, Beyer and Wieland (2019) examined the instability and imprecision of equilibrium real interest rate estimates, highlighting the need for more robust methodologies. Lunsford and West (2018) investigated secular drivers of U.S. safe real rates, providing valuable insights into long-term trends and their implications for monetary policy.

Furthermore, Holston, Laubach, and Williams (2017) extended the Laubach and Williams (2003) model to an international context, analysing trends and determinants of the natural interest rate across multiple advanced economies. This 2017 model was chosen for its comprehensive approach and improved accuracy in estimating the natural interest rate in a globalised economy. Incorporating international data provides a more robust framework for understanding the factors influencing the natural rate, which is critical for developing effective monetary policy.

Additionally, Carvalho, Ferrero, and Nechio (2016) explored demographic transitions and their impact on the natural interest rate, proving that ageing populations can significantly influence neutral rate estimates. Del Negro et al. (2017) investigated the influence of global factors, such as international capital flows and foreign economic conditions, on the U.S. natural rate of interest, emphasising the interconnected nature of modern economies.

These developments reinforce the continuing relevance of the foundational work of Wicksell and Keynes. They demonstrate that modern economic research is still developing and improving our understanding of the neutral interest rate in an increasingly complex world economy, building on the foundations they laid more than a century ago.

2. Research Objectives

i. Primary Aim: Determination of the Neutral Interest Rate

The primary objective of this study is to ascertain the neutral interest rate for the United States economy. The study will draw upon previous research on interest rates in various economies to achieve this objective. The neutral interest rate represents an equilibrium where monetary policy neither stimulates nor contracts the economy, thereby ensuring no expansionary or contractionary biases (Atesoglu, 2007). This rate is critically important for achieving full employment and stable inflation, thereby preventing inflation and deflation and supporting economic stability (Chetwin & Wood, 2013).

Maintaining a neutral interest rate is essential to prevent recessions and the severe societal impacts they can cause. Consequently, the primary objective is to develop a robust methodological framework for estimating the neutral interest rate. This framework will contribute to establishing a balanced and sustainable economic environment, reducing the likelihood of recessionary or inflationary cycles (Woodford, 2011).

ii. Secondary Aims: Variable Selection and Rationale

This study's secondary aim is to comprehensively examine the pivotal macroeconomic variables that shape the neutral interest rate. The selected variables include inflation, unemployment, and GDP growth, which directly influence the business cycle and interact with monetary policy. The variables will be selected carefully, following the guidelines set forth by the National Bureau of Economic Research (NBER). These indicators provide crucial insights into the efficacy of monetary policies in achieving price stability and maximum employment.

Furthermore, additional variables and concepts, such as consumer confidence and investment trends, will be considered. Consumer confidence is a key indicator of public sentiment towards the economy and is a significant factor influencing spending patterns and demand (Curtin, 2007). Investment trends indicate the levels of capital allocation and their impact on economic productivity (Baker et al., 2016).

The rationale behind selecting these variables is based on their proven correlation with interest rates and broader economic implications. Integrating traditional and novel variables is intended to enhance understanding of the neutral interest rate and its relationship with overall economic conditions.

In particular, the study incorporates two variables: NFP based on the Heuristic rule Sahm Rule (2019) and yield spreads (10-2 and 10-3 spreads) to enhance the model's predictive accuracy. The Sahm Rule (2019) offers timely indications of impending recessions based on fluctuations in the unemployment rate. Conversely, yield spreads are indicative of market expectations and economic conditions.

3. Literature Review and Theoretical Context

i. Conceptualising the Neutral Interest Rate

The neutral interest rate, or the natural or equilibrium interest rate, is a pivotal concept in monetary policy. The neutral interest rate represents an ideal equilibrium point where the economy operates at its full potential without expansionary or contractionary pressures. This rate is crucial as it aligns savings with investment, maintaining price stability and fostering sustainable economic growth (Vargas, 2016).

Knut Wicksell (1936) first introduced the concept of the neutral interest rate in his 1898 book Interest and Prices. He defined it as the rate at which the supply of savings is equal to the demand for investment, ensuring that inflation remains stable. According to Wicksell, deviations from this neutral rate could give rise to significant macroeconomic imbalances. If the interest rate is set below the neutral level, it may result in excessive investment and consumption, which could give rise to inflationary pressures. Conversely, setting the rate above the neutral level could result in a suppression of economic activity, which in turn could lead to deflation and recessionary conditions.

Subsequently, John Maynard Keynes (2018) advanced the concept by establishing a correlation between the neutral interest rate and full employment. In his seminal work, The General Theory of Employment, Interest and Money, Keynes proposed that the neutral rate is not merely a price stability indicator but also a means of achieving equilibrium wherein employment is maximised without inflationary pressures. This broader interpretation emphasises that the neutral interest rate helps to balance various economic objectives, including full employment and stable prices.

The neutral interest rate is a theoretical construct that challenges precise estimation. Many factors influence the neutral interest rate, including productivity growth, demographic trends, and global economic conditions. Consequently, the neutral interest rate fluctuates over time and across different economies. The intricacies of contemporary economic systems further compound the challenge of accurately determining this rate.

Economists employ a variety of models in order to estimate the neutral interest rate. One of the most prominent models is the Holston, Laubach, and Williams (2017) (HLW) model, which decomposes the observed real interest rate into a trend component (the neutral rate) and a cyclical component. This model is predicated on the assumption that observable macroeconomic variables, such as GDP growth and inflation, can be used to infer the unobservable neutral rate. The Kalman filter (1960) is frequently employed within these models to provide real-time estimates by continuously updating as new data becomes available.

The theoretical foundation of the neutral interest rate is also tied to the Phillips curve, which depicts an inverse relationship between unemployment and inflation. According to this framework, deviations from the neutral interest rate affect inflation and employment levels. For instance, a reduction in interest rates may result in a decline in unemployment, but this could lead to an increase in inflation. Conversely, an increase in interest rates may decrease inflation but also increase unemployment.

Despite the development of econometric modelling, the estimation of the neutral interest rate remains challenging due to the dynamic nature of economic conditions and the influence of global factors. Nevertheless, ongoing research in this area remains of critical importance. As evidenced by studies such as those of Vargas (2016), achieving the neutral interest rate can prevent severe societal consequences and promote more stable economic growth. The neutral interest rate is a reference point for policymakers when formulating monetary policies. These policies aim to stabilise the economy, ensure sustainable growth, and prevent severe economic fluctuations.

ii. History of interest rates and business cycle**s**

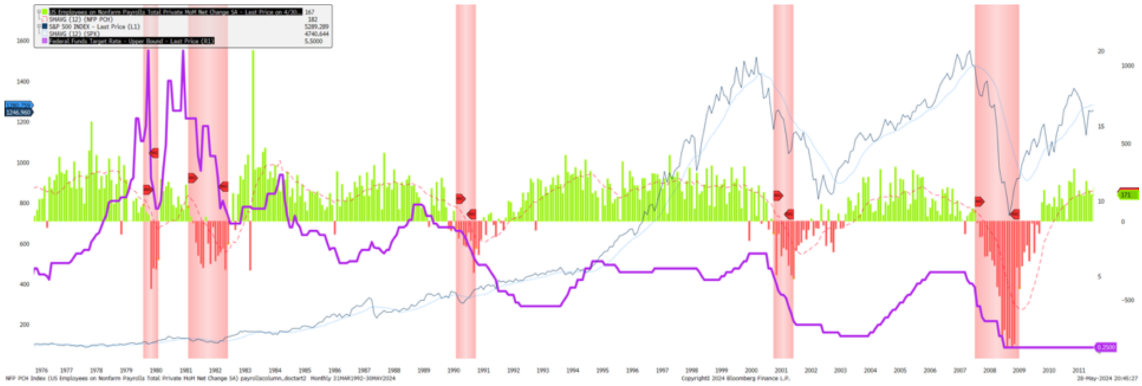
Throughout the 20th century, interest rates in the United States exhibited a direct responsiveness to prevailing economic conditions and the implementation of economic policies. During the First World War, the Federal Reserve implemented an increase in interest rates in order to control wartime inflation, which reached its peak after the cessation of hostilities. In the aftermath of the Great Depression, the Federal Reserve maintained low interest rates to stimulate borrowing and investment (Stucki & Homer, 1964). Nevertheless, it was demonstrated that low interest rates alone were insufficient to offset the severity of the Great Depression. To counteract the deflationary spiral, interest rates were temporarily increased in 1933 (Stucki & Homer, 1964).

According to Rosenblum and Strongin (1983), in the post-World War II era, interest rates were lowered in order to facilitate economic recovery and growth. The 1950s saw the establishment of stable interest rates, averaging approximately 3%, which facilitated a period of economic prosperity. However, the subsequent decades were characterised by mounting inflationary pressures, partly due to increased government spending on social programmes and the Vietnam War. By the 1970s, the period of stagflation had begun, with interest rates rising to combat inflation, which occasionally reached double-digit levels. The period ended in the early 1980s when Federal Reserve Chairman Paul Volcker's aggressive rate hikes were employed to finally tame inflation. This resulted in a severe recession, but the economy was stabilised.

As Hillenbrand (2020) exposes, the turn of the century presented new challenges, with the dot-com bubble bursting in 2000, causing significant economic downturns. In response, the Federal Reserve, under the guidance of Chairman Alan Greenspan, implemented a series of rate cuts, reducing the federal funds rate from 6.5% in 2000 to 1% in 2003 (Federal Reserve, n.d.). The reduction was intended to forestall a protracted economic downturn by making borrowing more affordable and encouraging investment and consumption. Nevertheless, the historically low interest rates inadvertently contributed to the housing market bubble, as cheap credit-fuelled excessive speculative investments in real estate. The subsequent collapse of the housing bubble precipitated the 2008 financial crisis. In response to the housing market bubble, the Federal Reserve, under the leadership of Chairman Ben Bernanke, implemented a series of measures to stimulate the economy (Federal Reserve, n.d.). These included a reduction in the federal funds rate to near-zero levels (0-0.25%) by December 2008 and the implementation of quantitative easing programmes involving purchasing government securities to increase the money supply and liquidity in the financial system (Federal Reserve, n.d.).

According to the FED announcements and summaries, the Federal Reserve began normalising interest rates in December 2015 as the economy began to recover. This marked the commencement of a gradual and meticulous upward adjustment. This normalisation continued until 2018, when the interest rates approached 2.5%. However, the outbreak of the COVID-19 pandemic in 2020 prompted a swift reversal of this policy, with rates again being reduced to near zero. Furthermore, a new round of quantitative easing was implemented to mitigate the economic fallout of the pandemic, as Auerbach et al. (2020) and Beckworth (2020) expose.

By 2022, with the economy showing signs of recovery and inflation beginning to rise sharply, the Federal Reserve initiated a new cycle of rate increases. By mid-2023, the federal funds rate had been increased to a range of 5.25% to 5.5%. The rapid increase was designed to curb inflation by cooling the overheated economy without precipitating a recession. This is a delicate balancing act known as a soft landing. (All as described in (Federal Reserve, n.d.)).



Source: Bloomberg, Ramón Bermejo and own elaboration.

The Federal Reserve's recent statements indicate a commitment to further rate increases if inflation remains above its long-term target of 2%, reflecting a proactive stance in managing economic overheating while monitoring employment levels (Monetary Policy: What Are Its Goals? How Does It Work?, n.d.). This approach illustrates the Federal Reserve's dual mandate to foster maximum employment and price stability, with interest rate adjustments representing a primary tool for navigating complex and rapidly changing economic landscapes.

iii. Review of Previous Models and Empirical Findings

Estimating the neutral interest rate has been a central focus of economic research, with various models developed to capture its dynamics. Initially, models were based on fundamental macroeconomic variables, yet frequently proved inadequate for accurately forecasting long-term economic trends. This section reviews several key models employed over the years, including simple linear regressions and more complex dynamic stochastic general equilibrium (DSGE) models. It highlights the methodological frameworks of these models and their contributions to understanding interest rates, for example, Duan and Jacobs (1996) and Gómez-Valle and Martínez-Rodríguez (2016).

Recent developments in econometric modelling have led to the introduction of more sophisticated tools that incorporate financial market indicators, expectations data, and global economic factors. For example, Beyer and Wieland (2019) addressed the instability and imprecision in equilibrium real interest rate estimates, advocating for more robust methodologies that account for global economic linkages and financial market variables. These models have enhanced the precision of neutral rate estimates, highlighting the inherent complexity and uncertainty in predicting a rate that balances economic conditions without explicit data. Holston, Laubach, and Williams (2017) extended the Laubach and Williams (2003) model by incorporating international data, thus providing a more robust framework for understanding the factors influencing the natural rate. Additionally, Carvalho, Ferrero, and Nechio (2016) examined demographic shifts and their implications for the natural rate, emphasising the role of ageing populations in lowering the neutral interest rate. Del Negro et al. (2017) explored global factors such as international capital flows and foreign economic conditions, illustrating the interconnected nature of modern economies.

Furthermore, Lunsford and West (2018) provided insights into the secular drivers of U.S. safe real rates, focusing on long-term trends and their implications for monetary policy. Including financial market indicators and global economic variables in recent models has underscored the necessity of a comprehensive approach to estimating the neutral rate. The critique of these models will focus on their assumptions, their responsiveness to economic shocks, and their utility in real-time policymaking.

As previously stated, the New York FED (Measuring the Natural Rate of Interest, 2024) has employed the LW and HLB techniques to calculate the natural interest rate (R-Star). However, direct comparisons with other studies are challenging due to the differing methodologies employed. This study, for instance, emphasises incorporating new variables such as non-farm payrolls (NFP) and the 10-3 and 10-2 yield spreads. Incorporating these variables provides additional insights into labour market conditions and financial market expectations, which are not fully captured by the models used by the New York Fed. Additionally, the data sources and assumptions used in the New York Fed's models are not entirely replicable, making methodological comparisons difficult. Consequently, the results from different models are not homogeneous and, therefore, not directly comparable.

iv. The Role of Unemployment and Business cycles in Interest Rate Theory and the Relevance of Interest Rates Spreads

The Unemployment Rate and the Business cycle

The unemployment rate is a crucial indicator of economic activity, closely linked to business cycles, and profoundly impacts interest rate determination. This section examines the theoretical and empirical relationships between unemployment rates, business cycles, and monetary policy. Furthermore, it examines how central banks utilise interest rate adjustments to regulate unemployment, particularly during economic downturns.

The Sahm Rule, developed by Claudia Sahm, is a crucial heuristic for detecting recessions. According to this rule, a recession is signalled when the three-month moving average of the national unemployment rate rises by 0.5 percentage points or more relative to its low in the previous 12 months (Sahm, 2019). Empirical observation has demonstrated that the Sahm Rule is a timely and reliable recession indicator, allowing policymakers to take pre-emptive actions, such as activating automatic stabilisers, to mitigate economic downturns.

The Phillips curve, which postulates an inverse relationship between unemployment and inflation, represents a pivotal theoretical framework that informs the formulation of monetary policy (Schorfheide, 2008). The theory posits that a reduction in unemployment levels leads to an increase in inflation due to increased demand for goods and services. This implies that workers' bargaining power increases as unemployment rates decline, resulting in higher wages and prices.

Nevertheless, the applicability of the Phillips curve in the post-global financial crisis era has been the subject of considerable debate. Recent empirical studies indicate a weakening of the traditional Phillips curve relationship, suggesting that other factors, such as global supply chains, technological advancements, and labour market dynamics, may also significantly impact inflation and unemployment (Holston et al., 2017; Beyer & Wieland, 2019). For example, globalisation has introduced novel dimensions to labour markets, where outsourcing and international competition can influence wage-setting behaviour and inflationary pressures.

Central banks like the Federal Reserve employ interest rate adjustments to influence economic activity and manage unemployment. Central banks typically reduce interest rates in economic downturns to stimulate borrowing and investment. This boosts economic activity and reduces unemployment (Mishkin, 2007). Lower interest rates reduce the cost of borrowing for businesses and consumers, thereby encouraging investment in capital goods and consumer spending. This increase in demand mitigates the negative impacts of economic recessions, promoting job creation and reducing unemployment levels.

Conversely, in periods of economic expansion and low unemployment, central banks may raise interest rates to prevent the economy from overheating and control inflation. Higher interest rates make borrowing more expensive, dampening investment and spending and slowing economic activity. This preemptive measure is designed to maintain inflation within target ranges and ensure sustainable economic growth.

The incorporation of the Sahm Rule into models that estimate the neutral interest rate is of paramount importance. Empirical analysis has demonstrated the effectiveness of the Sahm Rule in identifying the onset of recessions at an earlier stage than other indicators, thereby allowing for more proactive policy responses (Sahm, 2019). When the Sahm Rule threshold is met, it often correlates with significant economic slowdowns, justifying the need for immediate monetary policy interventions, such as interest rate cuts, to support economic stability.

The incorporation of unemployment data into models estimating the neutral interest rate is of paramount importance. For instance, the Holston, Laubach, and Williams (2017) model incorporates labour market conditions to dynamically adjust the natural interest rate. This incorporation allows for more precise real-time policy adjustments that reflect the prevailing economic circumstances. The model employs a state-space approach, whereby the natural interest rate is estimated by filtering out the cyclical components from observed data on real interest rates, GDP growth, and inflation. Incorporating variables related to the labour market, such as unemployment rates and labour force participation, enables the model to more accurately capture the underlying economic trends that influence the neutral rate.

In addition to the Phillips curve (Schorfheide, 2008), other theoretical models, such as Okun's law (Porras-Arena & Martín-Román, 2023), describe the relationship between unemployment and economic output. Okun's law postulates an inverse relationship between changes in the unemployment rate and an economy's GDP growth rate. In particular, for each percentage point increase in the unemployment rate, GDP is estimated to be an additional two percentage points below its potential (Porras-Arena & Martín-

Román, 2023) (Porras-Arena & Martín-Román, 2023). This relationship highlights the impact of labour market conditions on overall economic performance and, by extension, on setting interest rates.

Empirical findings have demonstrated that the responsiveness of unemployment to interest rate changes can vary depending on the economy's structural characteristics. For instance, in economies with inflexible labour markets, where wages and employment conditions are less malleable, the impact of interest rate changes on unemployment may be less pronounced. Conversely, in economies with more flexible labour markets, interest rate adjustments can more immediately affect employment levels.

Furthermore, the influence of unemployment on interest rate decisions is compounded by the presence of other macroeconomic variables, including inflation expectations, productivity growth, and external economic shocks. Central banks must consider the interplay of these factors when designing and implementing monetary policy. For example, during periods of stagflation, where high unemployment and high inflation coexist, traditional monetary policy tools may be less effective, necessitating a more nuanced approach to interest rate adjustments.

Interest Rate Spread, Interest Rates, and the Business cycle

Interest rate spreads, such as the 10-2 and 10-3 spreads, are valuable indicators of business cycles and future economic conditions. The 10-2 spread is defined as the difference between the yields on 10-year Treasury notes and 2-year Treasury notes. The 10-3 spread is similarly defined as the difference between the yields on 10-year Treasury notes and 3-month Treasury bills. Historically, an inversion of these spreads (where short-term interest rates exceed long-term rates) has been a reliable predictor of economic recessions (Estrella & Mishkin, 1996). This inversion indicates that investors anticipate a deterioration in future economic conditions, prompting central banks to reduce interest rates to stimulate the economy. Incorporating these spreads into the model facilitates a more comprehensive comprehension of market expectations and their consequences for future economic activity.

The significance of interest rate spreads extends to their influence on monetary policy and business cycles. A yield curve that is steepening (where long-term rates are rising relative to short-term rates) is typically indicative of expectations of more robust economic growth and rising inflation. This may prompt central banks to raise interest rates to prevent the economy from overheating. Conversely, a flattening or inverted yield curve can signal an economic slowdown or recession, prompting central banks to lower interest rates to support economic activity (Rudebusch & Williams, 2009).

4. Research Methodology

i. Analytical Framework and Conceptual Framework

This research builds on established economic theories of interest rates, focusing on the neutral or natural interest rate initially introduced by Knut Wicksell. The neutral interest rate is a theoretical equilibrium where monetary policy neither stimulates nor restricts economic activity, thus maintaining economic stability. The main objective of this research is to adapt this theoretical framework to the U.S. economy to estimate its neutral interest rate, which is crucial for assessing the effectiveness and direction of monetary policy (Archibald & Hunter, 2001).

Analytical Techniques

To estimate the neutral interest rate for the U.S. economy, this study employs several sophisticated analytical techniques:

a. Extended Taylor Rule Model

The Taylor rule, proposed by economist John B. Taylor in 1993, provides a systematic way to determine the nominal interest rate set by the central bank based on economic conditions. The basic Taylor rule suggests that the nominal interest rate should be adjusted in response to deviations of actual inflation from the target inflation rate and actual GDP from potential GDP. The basic formula is:

$$\boldsymbol{i_t = r^* + 0.5(\pi_t + \pi^*) + 0.5(y_t - y^*)}$$

Here, $\boldsymbol{i_t}$ is the nominal interest rate, $\boldsymbol{r^*}$ is the real equilibrium interest rate, $\boldsymbol{\pi_t}$ is the current inflation rate, $\boldsymbol{\pi^*}$ is the target inflation rate, $\boldsymbol{y_t}$ is the logarithm of real GDP, and $\boldsymbol{y^*}$ is the logarithm of potential GDP. The coefficients (0.5) suggest equal weighting to inflation and output gaps, implying that the central bank should raise rates when inflation is above target or the economy is above potential and lower rates in the opposite situations.

The extended Taylor rule model incorporates additional explanatory variables to enhance its predictive accuracy. This extended version considers the inertia of interest rate adjustments and other relevant financial indicators, such as yield spreads and labour market metrics. The extended formula is:

$$\boldsymbol{i_t = r_t + \pi_t + \rho(i_{t-1} - \pi_{t-1}) + (1-\rho)(r_t^* + \pi_t^*) + \beta_1 Spread_{10-2,t} + \beta_2 Spread_{10-3,t} + \beta_3 NFP_t}$$

This formula includes the smoothing parameter $\boldsymbol{\rho}$ to reflect the inertia in interest rate adjustments and additional terms for the spread between long-term and short-term bond yields $\boldsymbol{Spread_{10-2,t}}$ and $\boldsymbol{Spread_{10-3,t}}$ and non-farm payroll data $\boldsymbol{NFP_t}$. These additions help capture broader market conditions and labour market health, providing a more comprehensive approach to setting interest rates. The β is calculated using the Ordinary Least Square Method (Liu, 2009) to get a more accurate result than the fixed parameters.

As previously stated, the Sahm Rule, developed by economist Claudia Sahm, is a heuristic tool to identify the onset of economic recessions. This is why non-farm private payrolls (unemployment rate measurement) are aggregated. The rule is based on the observation that significant increases in the unemployment rate are reliable indicators of a recession. In particular, the Sahm Rule signals a recession when the three-month moving average of the national unemployment rate rises by 0.5 percentage points or more relative to its low during the previous 12 months. This rule has gained attention for its accuracy and timeliness in signalling economic downturns, making it a valuable addition to the analytical toolkit for estimating the neutral interest rate. Incorporating the NFP variable as a proxy of Sahm Rule into the analysis facilitates the provision of early warnings of recessions, thereby enabling more proactive monetary policy adjustments (Sahm, 2019). The added value of these new variables will be measured in the β coefficients that will vary the Natural Rate output.

b. Holston-Laubach-Williams (HLW) model (2017)

The Laubach and Williams model is designed to estimate the natural rate of interest by decomposing the observed real interest rate into a trend component (the natural rate) and a cyclical component. This model uses observable macroeconomic variables, such as GDP growth and inflation, to infer the unobservable components. The core equations are:

$$r_t = r_t^* + z_t$$

$$z_t = \emptyset z_{t-1} + \epsilon_{zt}$$

$$r_{t-1}^* = r_{t-1}^* + g_t + \epsilon_{rt}$$

$$g_t = g_{t-1} + \epsilon_{gt}$$

Here, r_t is the real interest rate, r_t^* is the natural rate of interest, $+z_t$ is the cyclical component, and $g_t$is the trend growth rate. $\emptyset$ Is the persistence parameter of the cyclical component, ϵ_{zt} and ϵ_{rt} are error terms for the cyclical component and natural rate, respectively, and ϵ_{gt} is the error term for the trend growth rate.

The model's strength lies in its real-time adaptability, continuously updating the neutral interest rate estimates as new data becomes available. This allows for a dynamic understanding of the economy's natural rate, which is essential for setting appropriate monetary policy.

Justification and advantages of the Houston, Laubach and Williams 2017 model.

The 2017 Holston, Laubach, and Williams (HLW) model enhances the 2003 Laubach and Williams model. It incorporates a trend growth rate component and considers global economic influences, making it more suited for capturing the complexities of modern economic conditions. The inclusion of international data and additional refinements provide a more accurate and dynamic estimation of the natural rate of interest, which is essential for setting appropriate monetary policy in an increasingly interconnected global economy.

c. Kalman Filter

The Kalman filter, developed by Rudolf E. Kalman in 1960, is a mathematical algorithm for estimating a dynamic system's state from noisy measurements. It is advantageous in fields like control systems, navigation, and econometrics, where real-time updating of estimates is crucial.

Critical Components of the Kalman Filter:

- State Variables: These represent the quantities we are interested in estimating (e.g., the natural interest rate).
- Measurement Variables: These are the observed data points that contain noise and inaccuracies (e.g., observed interest rates, inflation rates).
- Prediction and Update Steps: The Kalman filter alternates between predicting the future state based on a model and updating this prediction based on new measurements.

The stages are:

- **Prediction Step:**
  - Predicted state estimate: $\hat{x}_{t/t-1} = A\hat{x}_{t-1/t-1}$
  - Predicted covariance estimate: $P_{t/t-1} = AP_{t-1/t-1}A^T + Q$
- **Update Step:**
  - Innovation or measurement residual: $y_t - H\hat{x}_{t/t-1}$
  - Innovation covariance: $S_t = HP_{\frac{t}{t}-1}H^T + R$
  - Optimal Kalman gain: $K_t = P_{t/t-1}H^T S_t^{-1}$
  - Updated state estimate: $\hat{x}_{t/t} = \hat{x}_{t/t-1}K_t(y_t - H\hat{x}_{t/t-1})$
  - Updated covariance estimate: $P_{t/t} = (I - K_t H)P_{t/t-1})$

The Kalman filter effectively filters out noise from the observed data, providing more accurate estimates of the underlying economic variables (Refaeilzadeh et al., 2009). It is beneficial for real-time estimation as it updates estimates continuously with the latest available data.

d. Incorporating K-Fold Cross-Validation

K-fold cross-validation enhances the robustness of our analytical framework by systematically validating the model's performance across multiple subsets of the data . This method divides the dataset into k equal-sized folds, trains the model on k-1 folds, and tests it on the remaining fold. The results are averaged to provide a comprehensive performance assessment.

Steps for K-Fold Cross-Validation:

1. Data Partitioning: Divide the dataset into k equal-sized folds.
2. Model Training and Testing: For each fold:
   a. Train the model on k-1 folds.
   b. Test the model on the remaining fold.
3. Performance Averaging: Calculate the average performance across all k folds to robustly estimate the model's accuracy.

Data Interpretation and Validation

The interpretation of data focuses on estimating the neutral interest rate for the U.S. economy, assessing whether the economy is operating at its potential equilibrium state, and evaluating the implications of monetary policy based on these findings. To ensure the validity and reliability of the results, the study includes consistency checks with historical data and comparative analyses with previous models and empirical studies. This multifaceted validation approach confirms the study's findings and places them in the broader context of economic research, thereby enhancing their scientific value.

ii. Data Collection, Selection of Variables, and Justification

The variables selected for this study are based on an extensive literature review and empirical analysis, identifying those that have been shown to have a significant impact on the dynamics of interest rates and business cycles:

- Inflation: Indicators such as the Consumer Price Index (CPI) and the Producer Price Index (PPI) measure inflation and provide insight into price stability and the economy's purchasing power. Understanding inflation is critical because higher inflation typically leads to higher interest rates as central banks attempt to curb price increases (Holston et al., 2017). In this analysis, the variable used to measure inflation will be CPI. These metrics are reported monthly by the Bureau of Labor Statistics.

  We used a 2% following the Federal Reserve disclosures for the inflation target assumption.
- Unemployment: Unemployment rates, including metrics such as nonfarm payrolls and job creation statistics, are crucial for assessing labour market health and economic productivity. These rates offer insight into how employment levels interact with and are influenced by monetary policy (Curtin, 2007). In this study, the non-farm Payroll is one of the essential variables as we try to improve the Natural interest rate estimation by incorporating that variable with the Interest rate spread. These metrics are reported monthly by the Bureau of Labor Statistics.
- GDP Growth: This indicator reflects the overall health and vitality of the economy, which is essential for understanding business cycles and their implications for monetary policy. GDP growth rates help determine whether the economy is expanding or contracting, directly influencing interest rate decisions (Ferreira & Shousha, 2023). This metric is reported quarterly by the Bureau of Economic Analysis.

  For the GDP growth target assumption, we have used 3% as a proxy for the historical 1947-2024 average US GDP growth rate.

- Interest Rate Spread: The difference between short-term and long-term interest rates provides insight into market sentiment and expectations. It indicates potential mismatches between market reactions and policy actions. Examples include the 10-2 and 10-3 spreads, often used to predict economic downturns or expansions (Gómez-Valle & Martínez-Rodríguez, 2016). The 10-2 spread, which measures the difference between the yield on 10-year Treasury notes and 2-year Treasury notes, is particularly significant because an inversion of this spread has historically been a reliable predictor of recessions. The 10-3 spread, which measures the difference between the yield on 10-year Treasury notes and 3-month Treasury bills, serves a similar purpose and provides additional insight into short-term economic expectations and monetary policy impact (Estrella & Mishkin, 1996; Rudebusch & Williams, 2009). These variables are available daily from the US Department of the Treasury.

Data Source

Several sources have been used to conduct this study to ensure robustness and reliability. The main ones are:

- Federal Reserve: The primary data source on interest rates, inflation, and other macroeconomic indicators, providing a solid foundation for empirical analysis.
- Bloomberg, Factset, and Refinitiv Eikon: These platforms offer extensive and detailed data on various economic indicators, such as unemployment rates and GDP growth, enriching the dataset and increasing the comprehensiveness of the analysis.
- National Bureau of Economic Research (NBER): The NBER's methodology for identifying economic recessions and expansions is critical for framing the selected economic indicators within the cyclical nature of the U.S. economy.
- Literature Review Sources: This source extracts valuable insights on both variables and the results of previous studies, helping assess the correct selection of variables, analysis, and methodology implementation and interpretation. Some include specific sources such as Elsevier or other reputable sources such as Google Scholar.

- The Python code has been created using PyPI · El Índice de paquetes de Python. (n.d.). PyPI. https://pypi.org/ . All the libraries have been extracted from that source.

In the Python code analysis, the source employed is the Federal Reserve Data through the FRED API, which offers a reliable source of both historical and actual data suitable for this study. This FRED API has been used to request a specific API key, indicating the purpose of the study, and will solely be used for this study. All the data can be extracted similarly, allowing the analysis to be replicated (by simply requesting an API key and adding it in the Python code section).

iii. Advantages and Disadvantages of the proposed methodology

Advantages:

- <u>Comprehensive Framework:</u> By combining the extended Taylor rule model with the Laubach and Williams model, the Kalman filter, and machine learning techniques, the study benefits from a robust and adaptable framework that integrates both traditional and modern analytical approaches (Laubach & Williams, 2003; Kalman, 1960).
- <u>Real-Time Adaptability:</u> The Kalman filter allows for continuously updating estimates, ensuring that the neutral interest rate reflects the most current economic conditions (Kalman, 1960).
- <u>Enhanced Predictive Accuracy:</u> Machine learning techniques improve the accuracy of predictions by identifying complex patterns and relationships in the data that might be missed by traditional econometric models (Athey & Imbens, 2019).

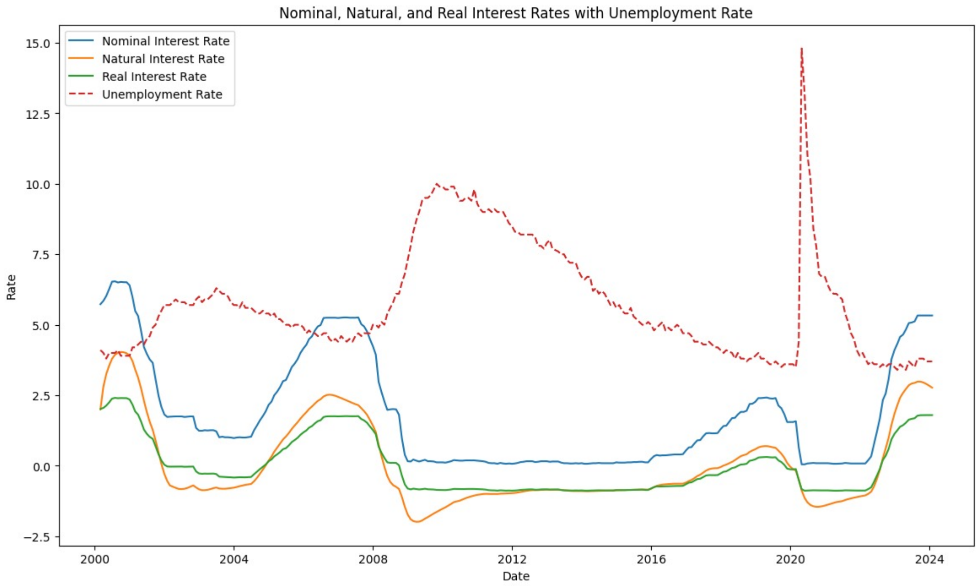
Disadvantages:

- <u>Complexity:</u> The methodological framework requires sophisticated statistical and computational tools, which may limit its accessibility to researchers without advanced technical skills. We have used Google Collab to execute the code to solve this, as it is a powerful virtual machine on the cloud.
- <u>Data Sensitivity:</u> The accuracy of the estimates heavily depends on the quality and timeliness of the input data. Inconsistent or outdated data can lead to erroneous

conclusions. In order to solve this potential problem, we have used a unique, reputable source (FRED API), making all the data homogeneous and reliable.

- Model Assumptions: The reliability of the results is contingent on the validity of the model assumptions. If the underlying assumptions do not hold, the estimates of the neutral interest rate may be biased or inaccurate (Laubach & Williams, 2003). As exposed previously, the assumptions have been extracted from historical data or official estimates such as the Federal Reserve Estimates.

5. Results, Conclusions and Future Research



Results: Curves Trends

This study aimed to estimate and analyse the neutral interest rate for the United States economy, focusing on its behaviour concerning historical economic crises and the Sahm Rule as an early indicator of recessions. The analysis reveals that the natural interest rate has exhibited a downward trend over the past two decades, reflecting structural changes in the economy. This decline was particularly notable before the 2008 financial crisis and the 2020 COVID-19 pandemic, suggesting that the natural interest rate began setting lower levels prior to the onset of these downturns. This pattern indicates that the natural interest rate can be an early warning signal of economic vulnerabilities.

As the natural interest rate declines, it signals potential economic weaknesses, which precede increases in the unemployment rate, while the nominal interest rate is not reacting yet. This correlation aligns with the predictive aspect of the Sahm Rule, which triggers a recession signal when the three-month moving average of the national unemployment rate rises by 0.5 percentage points or more relative to its low during the previous 12 months. The Sahm Rule's correlation with periods of declining natural interest rates underscores its utility as an early warning indicator for policymakers.

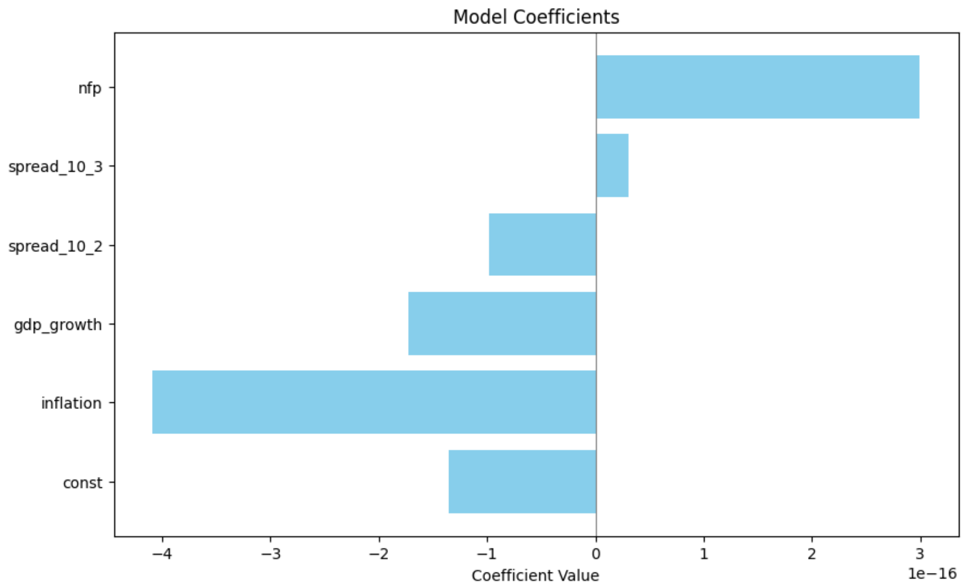
Impact of Economic Crises

The nominal interest rate experienced significant drops during the 2008 financial crisis and the 2020 pandemic, reaching near-zero levels as the Federal Reserve implemented aggressive monetary easing to counteract the economic downturns. These periods also coincided with spikes in the unemployment rate, highlighting the central bank’s response to severe recessions. As the natural interest rate decreases in the face of rising unemployment, it prompts a reduction in the nominal interest rate to stimulate economic activity.

The real interest rate, adjusted for inflation, also followed a downward trend during these crises, reflecting the reduced cost of borrowing and the central bank's efforts to stimulate economic activity. This study introduces new variables into the existing models to enhance the predictive accuracy and comprehensiveness of the analysis. Notably, the inclusion of the unemployment rate, measured through non-farm private payrolls, is linked to the Sahm Rule. This heuristic rule signals a timely recession when the unemployment rate significantly rises. By integrating non-farm private payroll data, the model gains a more granular understanding of labour market conditions, which are crucial for setting interest rates and predicting business cycles. The enhanced model, therefore, offers a more robust framework for estimating the neutral interest rate and anticipating economic downturns.

The natural interest rate's behaviour, as illustrated in the provided graph, often declines before recessions become evident. This decline in pre-recession can be observed before the 2008 financial crisis and the 2020 pandemic when the natural rate set lower levels before these downturns. Advances in the natural interest rate are crucial for policymakers as they provide early warnings of upcoming economic stress. This predictive capability is further supported by the Sahm Rule, which uses changes in the unemployment rate to signal recessions. The model coefficients underscore the significant impact of non-farm private payrolls on predicting business cycles, further validating the integration of labour market metrics into the analysis.

Explanation of Model Coefficients



The model's beta coefficients represent each variable's impact on the interest rate. Each plays a significant role in determining the natural interest rate. The real federal funds rate (RFP) variable exhibits the highest positive coefficient, underscoring its critical importance in the model's calculations and predictions. This solid positive coefficient indicates that the federal funds rate is crucial in determining the natural interest rate.

The spread between 10-year and 3-month Treasury bills (spread_10_3) also exhibits a positive coefficient, indicating that an increase in this spread is associated with a higher natural interest rate. This positive relationship implies that a wider spread between long-term and short-term interest rates indicates market optimism about future economic conditions and growth prospects.

Conversely, the spread between 10-year and 2-year Treasury notes (spread_10_2) exhibits a negative coefficient, indicating that an inversion of this spread predicts a lower natural interest rate. This negative relationship often indicates potential economic downturns or recessions, reflecting market expectations of weaker economic performance.

The GDP growth variable exhibits a positive coefficient, indicating that higher GDP growth is typically associated with higher natural interest rates. This positive coefficient reflects that an expanding economy, characterised by robust GDP growth, tends to push the natural interest rate as demand for capital increases.

The inflation variable exhibits a negative coefficient, indicating that higher inflation rates tend to reduce the real interest rate. This is consistent with the central bank's objective of controlling inflation by adjusting nominal interest rates. Higher inflation typically leads to lower accurate interest rates, intended to stabilise prices in such a scenario.

Finally, the constant term in the model is positive, representing the baseline level of the natural interest rate when all other variables are zero. This positive value indicates a base adjustment in the model's calculations, providing a reference point for the natural interest rate independent of the other variables considered.

This comprehensive analysis of the beta coefficients elucidates the multifaceted influences on the natural interest rate, elucidating the intricate interplay between monetary policy, market expectations, economic growth, and inflation.

Policy Implications and Future Research Directions

The decline in the natural interest rate before major economic crises suggests that policymakers must closely monitor this indicator as part of their economic surveillance. The natural rate's trend can provide early signals of economic stress, allowing for preemptive measures to mitigate recession risks. The effectiveness of the Sahm Rule in signalling recessions, combined with the observed behaviour of the natural interest rate, highlights the importance of integrating these tools into the policy framework. This approach can enhance the responsiveness and precision of monetary policy interventions.

Future research should refine the models to estimate the natural interest rate, particularly by incorporating additional variables that capture the evolving economic landscape. One key area for future exploration is the further integration and analysis of heuristic variables, such as the Sahm Rule. This rule has demonstrated significant predictive power in signalling economic downturns and warrants deeper investigation. Researchers could study the Sahm Rule's implications in greater detail to understand how its early warning signals can be used to predict business cycles more accurately and inform monetary policy adjustments.

Investigating the structural factors contributing to the declining natural interest rate, such as demographic changes, technological advancements, and global economic integration, will also be crucial. These factors have profound implications for long-term economic stability and monetary policy, and understanding their interplay with the natural interest rate could help develop more sustainable economic policies.

Another promising avenue for future research is the application of cross-country comparative studies to evaluate the effectiveness of the Sahm Rule and other heuristic indicators in different economic contexts. Such studies could provide insights into the universal applicability of these tools and help identify country-specific modifications that enhance their predictive accuracy.

In summary, the study underscores the importance of the natural interest rate as a vital monetary policy benchmark. Its declining trend before recent economic crises, coupled with the predictive power of the Sahm Rule, provides valuable insights for anticipating and mitigating future economic downturns. Policymakers are encouraged to closely monitor these indicators to enhance economic stability and ensure timely interventions. Integrating new variables, particularly labour market metrics like non-farm private payrolls, further strengthens the model's predictive capability and policy relevance. By refining models and incorporating additional heuristic variables, future research can contribute to more accurate predictions of business cycles and more effective monetary policy interventions, ultimately enhancing economic stability and growth in the face of evolving global economic challenges.

6. Annex – Python Code

Step 1: Install and Import Required Libraries: This section ensures that all necessary libraries are installed and imported, facilitating the data analysis.

```
pip install pandas numpy pykalman pandas_datareader statsmodels fredapi
import numpy as np
import pandas as pd
import matplotlib.pyplot as plt
from sklearn.model_selection import KFold
import statsmodels.api as sm
from fredapi import Fred
from pykalman import KalmanFilter
```

Step 2: Initialise the FRED API Client: The Federal Reserve Economic Data (FRED) API client is initialised using a specific API key, enabling data retrieval from the FRED database.

```
# Initialise the FRED API client
FRED_API_KEY = 'API_KEY' #Requested for this study
fred = Fred(api_key=FRED_API_KEY)
```

Step 3: Define Date Range Variables: This section defines the period for data analysis, spanning from January 1, 2000, to January 1, 2024.

```
# Date range variables
START_DATE = '2000-01-01'
END_DATE = '2024-01-01'
```

Step 4: Data Collection and Preprocessing: This function retrieves economic data from FRED, resamples it to a monthly frequency, fills missing values, and calculates key metrics such as real interest rate. The data is compiled into a data frame for further analysis.

```
# Data Collection and Preprocessing
```

```python
def collect_and_preprocess_data(start_date, end_date):
    # Fetching data from FRED
    inflation = fred.get_series('CPIAUCSL', observation_start=start_date, 
observation_end=end_date)
    unemployment = fred.get_series('UNRATE', observation_start=start_date, 
observation_end=end_date)
    gdp_growth = fred.get_series('A191RL1Q225SBEA', observation_start=start_date, 
observation_end=end_date)
    spread_10_2 = fred.get_series('T10Y2Y', observation_start=start_date, 
observation_end=end_date)
    spread_10_3 = fred.get_series('T10Y3M', observation_start=start_date, 
observation_end=end_date)
    nfp = fred.get_series('PAYEMS', observation_start=start_date, 
observation_end=end_date)
    interest_rate = fred.get_series('FEDFUNDS', observation_start=start_date, 
observation_end=end_date)

    # Resample data to monthly frequency if necessary and fill forward missing values
    inflation = inflation.resample('M').ffill()
    unemployment = unemployment.resample('M').ffill()
    gdp_growth = gdp_growth.resample('M').ffill()
    spread_10_2 = spread_10_2.resample('M').ffill()
    spread_10_3 = spread_10_3.resample('M').ffill()
    nfp = nfp.resample('M').ffill()
    interest_rate = interest_rate.resample('M').ffill()

    # Creating a DataFrame
    data = pd.DataFrame({
        'inflation': inflation.pct_change().dropna() * 100,
        'unemployment': unemployment,
        'gdp_growth': gdp_growth.pct_change().dropna() * 100,
        'spread_10_2': spread_10_2,
        'spread_10_3': spread_10_3,
        'nfp': nfp.pct_change().dropna() * 100,
```

```
        'interest_rate': interest_rate
    }).dropna()

    # Calculate the real interest rate
    data['real_interest_rate'] = data['interest_rate'] - data['inflation']

    return data
```

Step 5: Extended Taylor Rule Model with Coefficient Calculation: This function implements an extended Taylor Rule model, which calculates the target interest rate based on inflation, GDP growth, and other economic indicators. The coefficients are estimated using Ordinary Least Squares (OLS) regression.

```
# Extended Taylor Rule Model with Coefficient Calculation
def extended_taylor_rule(data, target_inflation=2, target_gdp=3):
    # Calculate coefficients using OLS regression
    X = data[['inflation', 'gdp_growth', 'interest_rate', 'spread_10_2', 'spread_10_3', 'nfp']]
    y = data['interest_rate']
    X = sm.add_constant(X)
    model = sm.OLS(y, X).fit()
    betas = model.params

    # Basic Taylor Rule
    data['taylor_rule_rate'] = betas['const'] + betas['inflation'] * (data['inflation'] -
target_inflation) + betas['gdp_growth'] * (data['gdp_growth'] - target_gdp)

    # Extended Taylor Rule
    data['extended_taylor_rule_rate'] = (
        betas['const'] + data['inflation'] + betas['interest_rate'] *
(data['interest_rate'].shift(1) - data['inflation'].shift(1)) +
        (1 - betas['interest_rate']) * (betas['const'] + data['inflation']) + betas['spread_10_2']
* data['spread_10_2'] +
        betas['spread_10_3'] * data['spread_10_3'] + betas['nfp'] * data['nfp']
    )
```

```
    return data
```

Step 6: Improved Holston-Laubach-Williams Model (Natural Interest Rate): This function uses a Kalman Filter to implement the HLW model, which estimates the natural interest rate. The state-space model helps capture the dynamics of interest rate changes over time.

```
# Improved Holston-Laubach-Williams Model (Natural Interest Rate)
def hlw_model(data):
    # Implement a state-space model for HLW
    kf = KalmanFilter(
        transition_matrices=[[1, 0.1], [0, 1]],
        observation_matrices=[[1, 0]],
        initial_state_mean=[0, 0],
        initial_state_covariance=np.ones((2, 2)),
        observation_covariance=1,
        transition_covariance=np.eye(2) * 0.1
    )

    state_means, state_covariances = kf.filter(data['interest_rate'].values)
    data['hlw_rate'] = state_means[:, 0] + state_means[:, 1]
    return data
```

Step 7: Apply Kalman Filter to Smooth the Interest Rate Series: This function applies a Kalman Filter to smooth the interest rate series, providing a clearer view of the underlying trends.

```
# Apply Kalman Filter to smooth the interest rate series
def apply_kalman_filter(data):
    kf = KalmanFilter(initial_state_mean=0, n_dim_obs=1, n_dim_state=1)
    kf = kf.em(data['interest_rate'], n_iter=10)
    state_means, state_covariances = kf.filter(data['interest_rate'].values)
    data['kalman_ir'] = state_means.flatten()
```

```
    return data
```

Step 8: K-Fold Cross-Validation: K-Fold Cross-Validation is applied to evaluate the performance of the OLS regression model. This technique divides the data into k folds, training the model on k-1 folds and testing on the remaining fold iteratively to ensure robustness.

```
# K-Fold Cross-Validation
def k_fold_cross_validation(data, k=5):
    kf = KFold(n_splits=k)
    results = []

    for train_index, test_index in kf.split(data):
        train, test = data.iloc[train_index], data.iloc[test_index]
        model = sm.OLS(train['interest_rate'], sm.add_constant(train[['inflation',
'unemployment', 'gdp_growth', 'spread_10_2', 'spread_10_3', 'nfp']])).fit()
        predictions = model.predict(sm.add_constant(test[['inflation', 'unemployment',
'gdp_growth', 'spread_10_2', 'spread_10_3', 'nfp']]))
        results.append(predictions)

    return results
```

Step 9: Plotting Function: This function visualises the results by plotting the nominal interest rate, natural interest rate, smoothed interest rate, real interest rate, and unemployment rate over time.

```
# Plotting function
def plot_interest_rates(original_data, data):
    plt.figure(figsize=(14, 8))
    plt.plot(original_data.index, original_data['interest_rate'], label='Nominal Interest
Rate')
    plt.plot(original_data.index, data['hlw_rate'], label='Natural Interest Rate')
    plt.plot(original_data.index, data['kalman_ir'], label='Smoothed Interest Rate')
```

```
    plt.plot(original_data.index, data['real_interest_rate'], label='Real Interest Rate',
linestyle='--')
    plt.plot(original_data.index, original_data['unemployment'], label='Unemployment
Rate', linestyle='--')
    plt.xlabel('Date')
    plt.ylabel('Rate')
    plt.title('Nominal, Natural, and Real Interest Rates with Unemployment Rate')
    plt.legend()
    plt.show()
```

Step "Menu": Creates the menu to execute the main formula. Also, it adds a step where all the data is extracted, saved and exported as an Excel Document

```
# Main function
def main():
    # Step 1: Collect and preprocess data
    original_data = collect_and_preprocess_data(START_DATE, END_DATE)

    # Keep a copy of the original data for plotting
    data = original_data.copy()

    # Normalize the data for modeling
    data = (data - data.mean()) / data.std()

    # Step 2: Apply the Extended Taylor Rule Model
    data = extended_taylor_rule(data)

    # Step 3: Apply the Holston-Laubach-Williams Model (Natural Interest Rate)
    data = hlw_model(data)

    # Step 4: Apply the Kalman Filter to smooth the interest rate series
    data = apply_kalman_filter(data)
```

```
    # Step 5: Apply K-Fold Cross-Validation
    results = k_fold_cross_validation(data)

    # Step 6: Print the first few rows of the processed data
    print(data.head())

    # Step 7: Save the results to an Excel file
    with pd.ExcelWriter('interest_rate_analysis.xlsx') as writer:
        data.to_excel(writer, sheet_name='Interest Rates')
        original_data.to_excel(writer, sheet_name='Original Data')

    # Step 8: Plot the interest rates
    plot_interest_rates(original_data, data)

if __name__ == "__main__":
    main()
```

The plot of the model's coefficients (except for the interest rate, which is excluded for visual reasons as it represents the base of the result and, therefore, weights 1.00). The coefficients have been extracted from the previous analysis results.

```
# Coefficients from the model
coefficients = {
    'const': -1.357354e-16,
    'inflation': -4.091081e-16,
    'gdp_growth': -1.729867e-16,
    'spread_10_2': -9.832889e-17,
    'spread_10_3': 3.036230e-17,
    'nfp': 2.996163e-16
}

# Extracting keys and values
labels = list(coefficients.keys())
```

```
values = list(coefficients.values())

# Creating the bar plot
fig, ax = plt.subplots(figsize=(10, 6))
ax.barh(labels, values, color='skyblue')
ax.set_xlabel('Coefficient Value')
ax.set_title('Model Coefficients')
plt.axvline(x=0, color='grey', linewidth=0.8)
plt.show()
```

7. References

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Por la presente, yo, Luis Mansilla Roca de Togores, estudiante de E2+Business Analytics de la Universidad Pontificia Comillas al presentar mi Trabajo Fin de Grado titulado " Assessing the Neutral Interest Rate for the U.S. Economy: Correlation with Macroeconomic Variables and Implications for Economic Policy", declaro que he utilizado la herramienta de Inteligencia Artificial Generativa ChatGPT u otras similares de IAG de código sólo en el contexto de las actividades descritas a continuación [el alumno debe mantener solo aquellas en las que se ha usado ChatGPT o similares y borrar el resto. Si no se ha usado ninguna, borrar todas y escribir "no he usado ninguna"]:

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