Optimal operation of a hydropower plant in a stochastic environment

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Abstract

Given the currently changing climate conditions it is of primary importance to optimise the management of hydropower resources. This paper proposes a framework in a dynamic setting to determine the water outflow that maximises the value of a water resource for a given reservoir. The model includes two sources of uncertainty, the water inventory determined mainly by the water inflow and the electricity prices. It is implemented under the stochastic optimal control approach and calibrated using monthly data of reservoir characteristics from ResOpsUs. The results indicate that the inventory dynamics are specially important in valuing reservoir resources. The application of optimal management policies guarantees the long term sustainability of the reservoir. The possible effects of climate change are considered in a sensitivity analysis to changes in the price and water inventory dynamics.

Keywords: water management, climate change, hydro power, sustainable reservoir

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1. Introduction

Extreme weather events in the Western US have driven the two largest reservoirs in the U.S. Lake Powell (Utah-Arizona border) and Lake Mead (Nevada-Arizona border), to the lowest points since they were filled in 1963 (Lake Powell) and in 1934 (Lake Mead). Reservoirs that operate in similar conditions may present management difficulties since such scenarios were not expected when operating policies were initially designed.

The term "deadpool" refers to a state in which water in a reservoir drops so low that it cannot flow downstream from the dam. This condition has important implications for water consumers, irrigation services and for hydro power generators[4]. If a reservoir is at "deadpool" state it has previously reached the minimum power pool elevation in which turbines are not longer able to generate power. The word "deadpool" has recently been classified by "The Economist"[5] as one of the keywords that will shape the world in the coming year.

As water draughts and water flood risk increase with climate change, the determination of optimal strategies of reservoir water levels becomes a primary concern among consumers, corporations, intergovernmental institutions and academics. Hydroreservoirs are mainly used for hydropower production and water consumption. They operate under high flexibility and low operating costs. The decision on how to optimally schedule water storage to meet random seasonal demand within minimum cost is of high relevance and has important implications. While the optimal operation of hydro turbines may be challenged by the increasing threat of higher temperatures and droughts, hydro turbines will remain a desirable technology in the generator mix of power system as they have a low carbon impact and they operate under highly flexible and low costs conditions. Hydropower reservoirs can also be used as pumped storage hydropower (PSH) which is a type of hydroelectric storage. According to Uria-Martinez *et al.* (see the 2021 US hydropower market report) hydropower generation represented 6.6% of U.S electricity

[4]See article "What is a deadpool: a water expert explains" available at : The Conversation, 12th of May 2022

[5]See The Economist Podcast "The World Ahead 2023" 19, Dec 2022

generation and 38% of electricity from renewables. The same report shows that PSH capacity grew over the past decade by almost as much as all other U.S storage combined. [6] In this paper we introduce a framework that allows the design of optimal reservoir policies when water resources are exclusively used for hypro-power generation purposes.

Given the currently changing climate conditions, the primary focus for the operation of these reservoirs is to optimise their operation so that there is enough water to keep generating electricity. This will require that the affected areas implement appropriate policies linking the supply and demand side of the water resource so that reservoir dead pool levels can be avoided.

We propose a framework in a dynamic setting to determine the monthly water outflow which maximises the value of the water resource for a given reservoir. We follow the methodology formulated by Pizarro and Schwartz (2021) and implement a stochastic optimal control approach to determine the optimal level of water outflow that maximises the value of the natural resource. This is done by modelling the water inventory or storage process and the electricity price as two independent stochastic processes that affect the value of the reservoir. We assume that operation takes place in a fully deregulated market and therefore work under the assumption that the reservoir is a "price taker." The power generator will set the production level that maximises profit for a given inventory level.

This paper is related to the literature that derives optimal operation strategies of power plants. Thompson *et al.* (2004) apply real option theory to derive nonlinear partial-integro-differential equations (PIDEs) to value and calculate optimal storage levels in hydroelectric and thermal power generators. Their paper discusses the different methods applied in the literature. These include numerical techniques such as trinomial trees and Monte Carlo Methods. Carmona and Ludkovski (2010) analyse the valuation of energy storage facilities using the stochastic control approach applied to natural gas storage and hydroelectric pumped storage. Their model is solved by constructing a robust numerical scheme based on Monte Carlo regressions.

[6] The work of Uria-Martinez *et al.* is published in the 2021 US hydropower market report

Similar approaches are adopted by Latorre *et al.* (2014) who develop a simulation tool to solve the long term hydro-thermal problem to evaluate the outcome from forecasts of either water inflows or future operation situations. The limitations of such approaches from a financial view point is that a) they are calibrated to a single data set b) they fail to efficiently account for the price evolution in the optimisation process. The work of Thompson *et al.* (2004) is an important exception as it applies the PIDES approach with mean reverting prices to address these underlined limitations. Carmona and Ludkovski (2010) consider a Markovian price model and solve a Bellman dynamic programming equation with a for the value function. The problem is translated into a quasi-variational partial differential equation (pde) formulation. However, the empirical application of the proposed solution in these frameworks is based on a numerical analysis with simulated data.

A major contribution of this paper is that we propose a general framework that can be applied to reservoir related variables and electricity price data of any properly defined area. For this purpose we use two particularly rich data sets: i) the GRandD data set which incorporates a cross section of dam related information including reference river name, closest city, maximum capacity, main dam use ii) the ResOpsUs data set which compiles daily time series of reservoir characteristics for 679 major dams in the US. We exploit the existence of rich data availability to study the effect of price and inflow uncertainty on value and optimal storage strategy under realistic settings. Note that our application can easily be extended to the analysis of any reservoir in the database that is solely used for hydropower generation.[7]

Another important contribution is that our framework provides a detailed analysis of electricity price and water inventory dynamics and its effect in the reservoir management. The empirical application is based on monthly data of reservoir characteristics which differs from the empirical exercises based on simulated data. Our results also show that while there are two main sources of uncertainty affecting the optimal solution, inventory uncertainty is more important than price uncertainty. This is the case under the assumption of

[7]The ResOpsus database includes the identifier of the facility. The GRandD data base allows identification of the geographical area that is used to download the corresponding monthly electricity prices. The full code will be provided upon request

stochastic prices with and without mean reversion.

We solve the model using the Value-function iteration approach instead of the more traditional partial differential equation approach. This formulation allows to solve for the optimal dynamic storage policy of a given reservoir and then use this policy to value the natural resource. The solution allows to study how the optimal outflow level relates to the two state variables specified as the inventory and price processes. The model therefore takes explicit account of the managerial control of the outflow rate. It also shows how the outflow process optimally evolves with stochastic changes in the state variables. The framework is used to simulate the price and storage dynamics over specific time horizons based on the optimal policy under different stochastic shocks. Results show that the optimal policy is closely linked to the inventory dynamics and that optimal water storage does never run down to zero.

The framework is additionally extended to analyse the changes in the established state dynamics to perform a sensitivity analysis under various scenarios. The first objective is to address the impact of higher average future electricity prices that could arise due a number of factors leading to long term supply and demand disruptions.

The second objective is to measure the optimal management impact of climate change. We consider for this purpose the change of managerial policies under lower future average water inflow as well as the policy adaptation to extreme weather events.

Our results demonstrate that while shifts in long term average prices do not result in significant policy changes in the long run, the impact of lower average water inflow requires adjustment of optimal long run policies.The possibility of extreme weather events as captured by increased volatility uncertainty suggests that conservative storage adaptation policies should be pursued.

The paper is structured as follows. In section 2 we introduce the theoretical valuation framework. The data description is provided in Section 3. Inventory dynamics and electricity price dynamics are described in sections 4 and 5, respectively. The implementation methodology is provided in Section 6. Results are presented in 7. Section 8 analyses the sensitivity of policy responses with respect to changes in climate and electricity prices dynamics. Finally, we conclude in section 9.

2. Theoretical Framework

In this section we introduce the dynamic stochastic modelling for the reservoir and the mathematical framework used to optimise the management of the facility. The first step in determining the optimal policy is to specify the dynamics of the two state variables: the water inventory and electricity prices.

Figure 1 illustrates the operation of the reservoir. The facility has water inventory which increases with the water inflow (from rains) and decreases with the evaporation and the outflow of water used to generate electricity through one or more turbines. There is a one to one mapping between the elevation from the sea level to the inventory level so that one unit of elevation corresponds to one unit of inventory. The elevation difference between the reservoir level and the downstream river affect the efficiency of the turbine, which is used to generate electricity.

2.1. Water inventory dynamics

The water dynamics of reservoirs are usually modelled as an Input-Output process. Following Song *et al.* (2022) we define the water inventory dynamics as:

$$dI_t = [\Phi(t) - (E(t) + F(t) + q(t))]dt + \sigma_I \, dW_t \tag{1}$$

Where I_t (m^3) is the inventory of water, t is the calendar (time), $\Phi(t)$ (m^3/s) is the expected rate of water flow into the reservoir, $E(t)$ (m^3/s) is the expected quantity of water that evaporates at time t , $F(t)$ (m^3/s) represents the rate of infiltration of water, and $q(t)$ (m^3/s) is the rate of water flowing

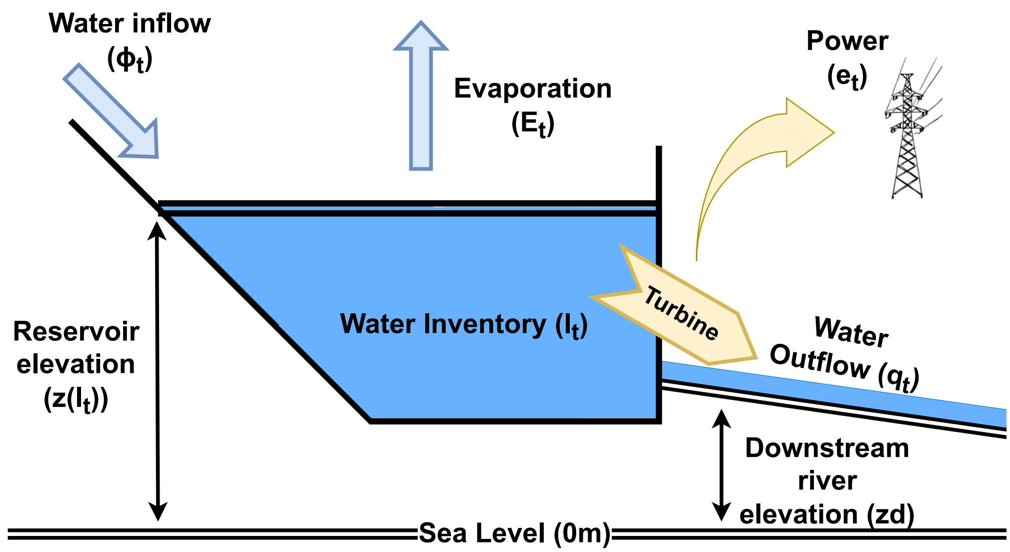


Figure 1: Diagram of a generic hydropower plant.

out of the reservoir into the turbine. The later represents the stochastic control of the problem. W is a Brownian Motion that captures the randomness of the process underlying inventory dynamics.

The reservoir's manager does not have direct influence on the variables $\Phi(t)$, $E(t)$ and $F(t)$. In what follows we treat these variables as stochastic processes but use their expected value for our calculations. This requires that the uncertainty of those processes is captured by $\sigma_I \, dW_t$. Details on the dynamics of each of these variables are provided in section 4, where we estimate a model for every variable separately.

The manager does, nevertheless, have direct control on the water outflow variable $(q(t))$. The objective is to determine the (optimal) value of q(t) (the managerial control) which maximises the value of the resource facility given that he is constrained by the maximum and minimum storage levels defined as (I_{max}) and (I_{min}) respectively

2.2. Electricity price dynamics

The consideration of the stochastic nature of output prices are highly important for determining optimal managerial responses and valuation of natural resources (see Brennan and Schwartz(1985)). As underlined by Thompson *et al.* (2004) the existence of market prices under efficient deregulated markets drives individual generators to choose the output quantity that maximises production. Output prices in this context are specified by the electricity price process. The latter have been extensively modelled in the past decades, especially after the liberalisation of power markets. Lucia and Schwartz (2002) use daily Nord-Pool day ahead electricity prices and find evidence of mean reversion. The also show that the underlying the seasonality is highly relevant in the price process. These trends arise because electricity cannot be stored at sufficiently large scale and therefore must be generated as needed.

Cartea and Figueroa (2005) propose a different approach to modelling daily electricity prices, including the seasonal behaviour while adding jump processes. Their results show that the addition of jump processes produce a well specified model. Further extensions include Borovkova and Schmeck (2017) which extend previous specifications by including a stochastic time change and the temperature as a factor.

The price specification proposed in this paper is highly conditioned by data frequency available for the electricity price series (see details in sections 3 and 6). This restricts us to use average monthly prices of electricity. We shall see in section 3 that the consideration of monthly intervals smooths jumps in the price dynamics. Jumps and seasonal dynamics are also lower in a hydropower application because hydro reservoirs provide means for power storage (see Uria-Martinez *et al.* (2021) for details in the US economy).

Since our secondary objective is to valuate the reservoir facility we require a long time horizons implying that the problem formulation does not require inclusion of short term factors such as temperature. We contend that weather conditions implicitly affect water inflows into the reservoir which are considered under our formulation. Extreme movements in weather conditions will affect the volatility of the inventory process. Changes in weather conditions are also modelled endogenously in section 8.2 and section 8 where we study how optimal management policies are adapted to the different forms of climate change (lower water inflow and higher volatility respectively).

We have followed the literature and considered the specifications introduced by Cartea and Figueroa (2005) as well as the assumptions of Lucia and Schwartz (2002). Our results demonstrate that the best fitting model for the monthly electricity price data is:

$$dP_t = \alpha + \beta\ P_t + \gamma_1\ sin((t+\tau_1)\ \frac{2\ \pi}{6}) + \gamma_2\ sin((t+\tau_2)\ \frac{2\ \pi}{12}) + \sigma_P dZ_t \qquad (2)$$

Where P_t is the electricity price at a given time (t), α is the average growth rate, β is the speed of the mean-reversion, γ_1 and γ_2 measure the amplitude of each seasonal behaviour, τ_1 and τ_2 are shifts in the wave, dZ_t is an increment to a Wiener process and $(dW_t)(dZ_t) = \rho\ dt$, ρ is the correlation between the two Wiener processes corresponding to the price and inventory dynamics respectively.[8] Note that there are two seasonal components, this results in two yearly price peaks and two yearly price lows. The length of a given peak depends on the amplitude and shifts of the waves.

We follow Pizarro and Schwartz (2021) and consider mean reversion as a benchmark case and extend the analysis to the non mean reversion as a robustness test. This alternative specification is given by the following equation (3).

$$dP_t = \alpha + \gamma_1\ sin((t+\tau_1)\ \frac{2\ \pi}{6}) + \gamma_2\ sin((t+\tau_2)\ \frac{2\ \pi}{12}) + \sigma_P dZ_t \qquad (3)$$

When the non mean reversion case is considered we follow Pizarro and Schwartz (2021) and remove the β term which captures the speed of mean reversion in equation 2.

[8]This correlation is used when solving the entire system using the value function iteration approach

2.3. The operation of the Turbine

The next step is to model the electricity generation process. This is captured by the dynamics of the turbine of the reservoir. We follow the formulation introduced by Wu *et al.* (2018) in modelling the turbine process:

$$e_t = \eta \, q(t) \, \left(\frac{z(I_t) + z(I_t + dI_t)}{2} - zd \right)$$

Where e_t (Kwh) is the power generated at a given time (t), η $(Kwh\ s/m^4)$ is the power generating coefficient of the turbine, $z(I_t)(m)$ is the elevation of the reservoir level for a given inventory at a time t. Therefore $\frac{z(I_t)+z(I_t+dI_t)}{2}$ is the average elevation of the reservoir and zd (m) is the elevation of the downstream. In consequence, $\frac{z(I_t)+z(I_t+dI_t)}{2} - zd$ measures the average elevation difference between the reservoir inventory and the downstream level at time t.

The turbine model is therefore specified as the outflow of the water times the pressure (captured by the elevation difference) times the constant η, which converts the underlying factors factors to kwh.

In order to find the correct value of the η parameter we follow the work of Chen *et al.* (2023) which models a series of reservoirs, and assumes η values in the order of 9000 $(Kwh\ s/m^4)$.

Our analysis includes a formulation of the elevation function, z. Its dynamics are specified in section 4.3, where we address the link between the reservoir inventory and the elevation.

We additionally impose the following constraint: the quantity of water in the turbine processes should not be less than zero and is expected to have a maximum capacity. This implies that water outflow levels will lie between q_{min} and q_{max}.

2.4. Risk adjusted Discount rate

We estimate the risk-adjusted discount rate for reservoirs following the Capital Asset Pricing Model (CAPM) approach applied in Pizarro and Schwartz (2021). The purpose is to discount future cash flows using the appropriate rate.

We collect stock price data of a representative company in the hydropower sector. We also construct proxies for the risk free rate variable and the market portfolio for the 2004-2019 period of a company that trades publicly and represents the hydropower production sector. We select Duke Energy as it is the second largest investor in the U.S. hydropower.[9] The company trades publicly and its business is located in the Midwest of the country, which is geographically close to the reservoir analysed in this paper (see section 3).

Our proxy for the market portfolio is obtained from Kenneth R.French's website[10] using the Utilities portfolio of the Industry portfolios, which includes water supply and electricity. We also use the US 13 week Treasury Bill as a proxy for the historical risk free rates. We assume that the future inflation rate is 2%, in line with the current target inflation of the Federal Reserve.[11]

The CAPM β applied for discount rate calculation is constructed by estimating betas for 60-month rolling windows over our sample period. This delivers an estimated median beta of 0.61 for the 2004-2019 period. We plug this value within the CAPM framework and calculate that the real discount rate is 4.8% for the U.S. hydropower plant. Estimation results are reported in table 1.

[9] https://www.duke-energy.com/our-company/environment/renewable-energy/hydroelectric-energy

[10] http://mba.tuck.dartmouth.edu/pages/faculty/ken.french/data_library.html

[11] https://www.federalreserve.gov/faqs/economy_14400.htm

Table 1: Estimation of the discount factor.

β	0.610
$R_m - R_f$	0.092
R_f	0.012
$R^{Nom}_{hydrpower}$	0.069
$R^{Real}_{hydropower} = (1 + R^{Nom}_{hydrpower})/(1 + \pi) - 1 = 0.048$	

2.5. Valuation and optimal operation policy

The starting point for the valuation of any project is the determination of the future cash flows that will be generated . In the current case the cash flow generated by the system for a given outflow ($q(t)$) can be defined as:

$$\pi(I_t, P_t; q(t)) = e_t \cdot P_t - C(q(t)) \tag{4}$$

Where $C(q(t))$ is the cost of operating the plant for a given outflow.

Substituting the power generating model specified in section section 2.3 we obtain the following:

$$\pi(I_t, P_t; q_t) = \eta q_t \left(\frac{z(I_t) + z(I_{t+1})}{2} - zd \right) \cdot P_t - C(q(t)) \tag{5}$$

Given that our objective is to optimise with respect to $q(t)$, we derive our solution from (5) where $\pi(I_t, P_t; q_t)$ is the cash flow generated by the resource.

The next step requires that we define the value of the facility for a given time (t), inventory level (I_t), electricity price (P_t) and outflow policy ($q(t)$). This is specified as the actual value of the future cash flows of the resource, using the appropriate risk adjusted discount rate. In consequence, the value of the facility is given by:

$$H(t, I_t, P_t; q(t)) = \mathbb{E}_t \left(\int_{\tau=t}^{\infty} e^{-r(\tau-t)} \pi(P_\tau, I_\tau; q_\tau) \ \ d\tau \right) \tag{6}$$

Where r is the discount rate and $e^{-r(\tau-t)}$ is the discount factor. $H(t, I_t, P_t; q(t))$ is the value of the facility for a given policy $q(t)$. The outflow quantity that maximises the function $H(t, I_t, P_t; q(t))$ is defined as $(q(t))$:

$$V(t, I_t, P_t) = \max_{q(t)} H(t, I_t, P_t; q(t)) \tag{7}$$

Equation 7 can be transformed into the Bellman equation 8 for discrete time specifications:

$$V(t, I_t, P_t) = \max_{q(t)} \left\{ \pi(t, I_t, P_t; q(t)) + e^{-r\Delta t} \mathbb{E}_t \left[V(t + \Delta t, I_{t+\Delta t}, P_{t+\Delta t}) \right] \right\} \tag{8}$$

This optimisation problem can be solved using the value-function iteration approach. This methodology is explained and implemented in section 6.

3. Data

In our empirical application we calibrate our models using data from a specific US reservoir. The key processes to be modelled are the electricity price and the inventory of the reservoir. The latter depends on a number of previously defined variables. The purpose is to address how optimal management responses depend on the variations of the two state variables.[12]

[12]Note that the data used in this manuscript is provided as a supplementary file following the Journal´s data sharing guidance

3.1. Reservoir data

We collect reservoir information from the databases GRanD discussed in Lehner *et al.* (2011) and ResOpsUS which is applied in Steyaert *et al.* (2022). The GRanD database stores dam's general information at the cross sectional level (river name, nearest city, maximum capacity or the main use of the dam). The ResOpsUS database provides information of 679 major reservoirs in the US. Each reservoir has associated identifiers and time series data. The time series data includes daily frequency for the following variables: inventory (I_t), inflow ($\Phi(t)$), evaporation ($E(t)$), elevation of the water level and the outflow ($q(t)$).

In order to select the reservoir for this analysis we have applied a series of filters to the data. It is important to highlight that our objective is to model the operation of the reservoir that generates electricity. For this reason, we have selected dams whose main use is the generation of electricity based on the information available on GRanD database. From the 679 dams considered only 81 fulfilled this condition. Next, we dropped from the sample those dams for which both inflow and outflow variables were not available. Once we apply this second restriction the number of reservoirs with the required data were 26.

The empirical application of the framework introduced in section 2 requires a set of continuous data. The objective is to optimise the operation of the facility in terms of water storage for a given price process. We establish that if the operation managerial decision is not important then there should be a strong correlation between the inflow and outflow variables.[13] We therefore calculated the correlation for each dam and sorted dam sample components from the lowest to the highest correlation values. We excluded those dams for which there were missing data.[14] The applied selection procedure leaves us with a reservoir located in Arkansas as the best candidate for model cali-

[13]We observed that daily correlations where low but they exhibited strong dependence. For example, a dam that exhibits an inflow in time (t) and the same quantity of outflow in time (t+1) indicate high dependence (even if there is low correlation). For this reason the correlation measure was done grouping the data by months.

[14]The first 4 ranked dams had missing the evaporation data while the fifth only presents 0.01% of missing data

bration purposes. The reservoir data is transformed from the daily frequency to monthly in order to match the electricity price data.

3.2. Electricity price data

In order to calibrate the electricity price process we consider the time series of Arkansas monthly electricity prices available from the U.S. Energy Information Administration[15]. Our sample ranges from January 2004 to December 2019. The sample start date is motivated by the liberalization of the US electricity market in the early years of the new century[16] The sample end date is selected on the basis of data availability.

4. Estimation of the water inventory dynamics

We use the ResOpsUS database, to estimate the models for the inventory variables considered in equation 4, excluding the infiltration variable.

4.1. The inventory variables

We first present the model and estimation dynamics for the inventory variables of equation 4. One of the challenges in this formulation is that we need to find an approach to determine the period length to be applied for the seasonal components of the model. We tackled this problem using the discrete Fourier Transform of the data. This transformation decomposes the time-series into waves with different amplitudes taking place in different times within a year. Relevant seasonal effects are expected to exhibit large amplitudes in the Fourier decomposition.

Figure 2 shows the Fourier Transform for multiple variables. The horizontal axis represents different periods of the transformation while the vertical axis

[15] https://www.eia.gov/

[16] https://www.epa.gov/greenpower/understanding-electricity-market-frameworks-policies

Table 2: Estimation results for the Evaporation model.

Parameter	Estimation	Std. Error	t-statistic	P(> $\|t\|$)
α	4.9601	0.0178	278.3951	0.0000
γ	2.7452	0.0252	108.9507	0.0000
τ	-2.5472	0.0175	-145.3087	0.0000
Residual standard error: 0.2469				
Df: 189				

indicates the amplitude of each wave. As stated, we use the peaks observed in this Figure to determine the seasonal periods of the applied formulations. For example, we use a 12-month period for the evaporation specification.

We explain in Appendix A the way in which this transformation is applied to obtain the time-series seasonality frequencies. We also show the equivalence between the Fourier transform and the seasonal functions.

4.1.1. Evaporation ($E(t)$)

We find that the expected evaporation variable exhibits a clear seasonal behaviour with yearly dynamics. The framework used to model this variable is represented in equation 9 below.

$$E(t) = \alpha + \gamma \ \sin\left((t+\tau)\frac{2\,\pi}{12}\right) \tag{9}$$

The model assumes that there is seasonality with a 12-month period based on the Fourier decomposition observed in Figure 2. Parameters are estimated accordingly minimising the mean square error. Results are reported in table 2.[17]

[17] Model results show a high goodness of fit for model estimates. They suggest that the process is highly predictable. This was not expected ex ante as weather related variables are regarded as highly noisy. This raises the possibility that the data may be artificially generated.

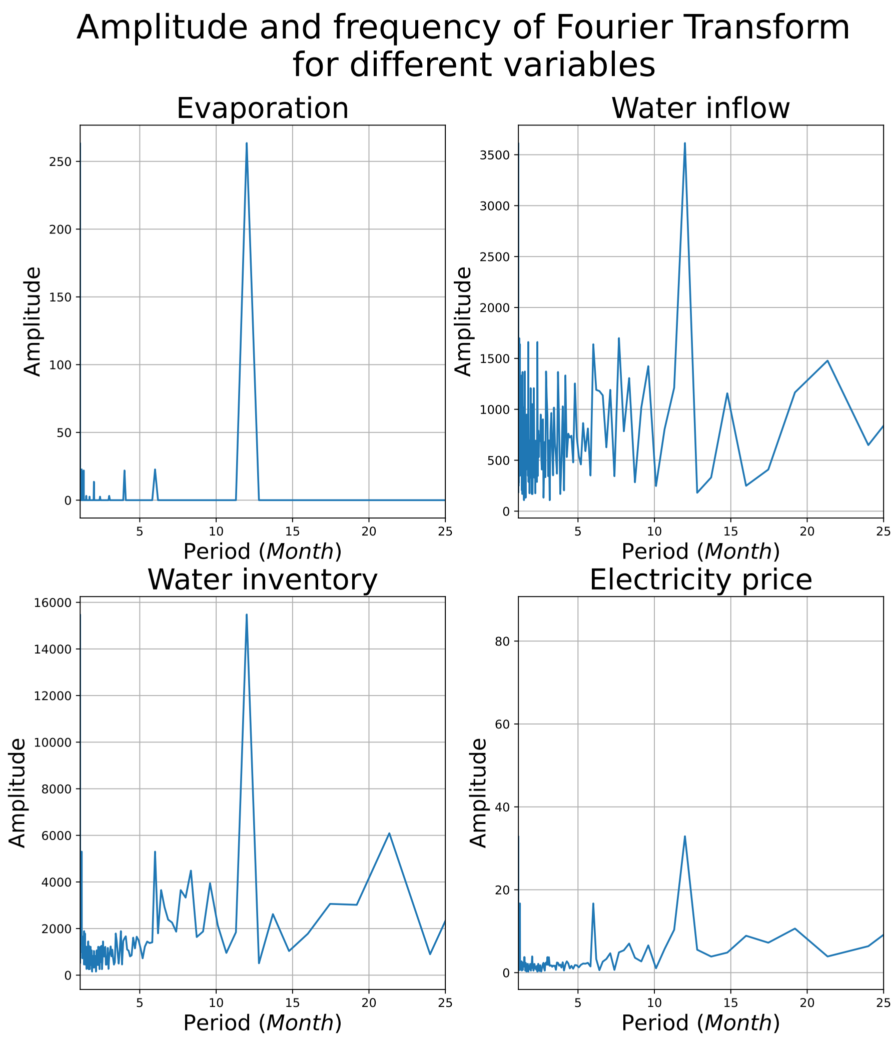


Figure 2: Amplitude of the Fourier Transform for different periods, for different variables: evaporation, water inflow, water inventory and electricity price.

4.1.2. The Inflow process $(\Phi(t))$

The calibration of the expected inflow variable is complex, as it exhibits seasonal behaviour (see Figure 2) and erratic variations across months. This behaviour reflects multiple sources of noise, that emerge due to random fac-

Table 3: Estimation results for the Inflow model.

Parameter	Estimation	Std. Error	t-statistic	P(> $\|t\|$)
α	51.7650	6.4057	8.0810	0.0000
β	0.2048	0.0716	2.8610	0.0047
γ	31.3252	6.5748	4.7644	0.0000
τ	1.3272	0.3918	3.3878	0.0009
Residual standard error: 60.7318				
Df: 187				

tors such as the early melting of snow or the variance in rain falls. We used an autoregressive framework (AR) with a seasonal component similar to that specified for the evaporation variable. The model is thus defined as:

$$\Phi(t+1) = \alpha + \beta\ \Phi(t) + \gamma\ \sin\left((t+\tau)\frac{2\ \pi}{12}\right) \tag{10}$$

The parameters of this equation were estimated following the same approach as under the evaporation model. The results of the optimisation process are reported in table 3.

4.1.3. The Infiltration Process ($F(t)$)

We do not have access to infiltration data. We also failed in identifying a clear pattern when calculating values as implied by equation 4. We therefore followed Dessie *et al.* (2015), Song *et al.* (2016) and Dang *et al.* (2020), and dropped this variable from the analysis by assuming that its value in negligible.

4.2. The Inventory model (I_t)

Once we have completed calibration of model parameters we proceed to estimate the proposed inventory specification (11) using the data for the selected reservoir.

$$dI_t = [\Phi(t) - (E(t) + F(t) + q(t))]dt + \sigma_I \, dW_t \tag{11}$$

We take expected values for the inflow and evaporation variables estimated in the previous subsections. Note that the outflow variable ($q(t)$) is not estimated as it is defined as the control variable. Figure 3 illustrates the time series evolution of the actual and estimated inventory values both measured in million cubic meters (MCM). Thus, studying the differences between the estimated and the measured we obtain residuals standard deviation (σ_I).

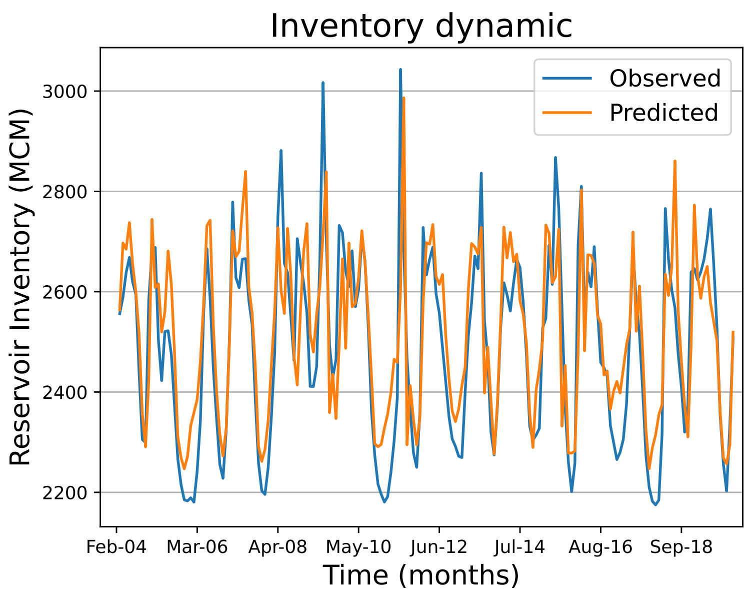


Figure 3: Inventory model and observed data.

Reported values show that both series move very closely. This suggests that the proposed model specified under (11) does deliver inventory dynamics that are highly related to the observed process. Inventory dynamics are stable during the sample analysed, suggesting that there were not important shifts in the management process during the last two decades

4.3. Elevation function ($z(I_t)$)

The ResOpsUS database also provides time series data on elevation. The relationship between elevation and inventory as underlined in the turbine model (12). The elevation has a direct effect on mean values of the inventory equation. The elevation does therefore affect the managerial problem specified in equation 5 through the inventory state variable. The power generating equation is:

$$e_t = \eta \; q(t) \; \left(\frac{z(I_t) + z(I_t + dI_t)}{2} - zd \right) \tag{12}$$

In our optimisation process elevation is calculated for every inventory value. Because the elevation variable is required to evaluate the profit function, we estimate the relationship between elevation and inventory using historical data. We use this data to define $z(I_t)$ as a real-valued function given that the model inputs I_t, $z(I_t)$ are defined as the interpolation between the elevation of the two closest registered inventories This implies that if $I_t = 2500$ (MCM), the two closest inventories recorded are 2498 (MCM) and 2505 (MCM) with 175.19(m) and 175.23(m) as elevations registered for those inventories, respectively. This implies that $z(2500) = 175.20$ (m)).

5. Estimation of the electricity price dynamics

We have established in Section 2 that the the benchmark literature on electricity price modelling uses daily prices and includes jumps in the electricity process reflecting the absence of storability of this commodity. Here we contend that the same specification may not be optimal for monthly data. Indeed, we have stated that the evolution of the monthly price series analysed in this paper is smoother implying that there are no jumps in our data. In consequence, the electricity price model proposed does not include jump dynamics.

In what follows we fit two different models to our data. The first model includes a seasonal component. The second model incorporates a seasonal

Table 4: Estimation results for the non-mean-reverting model.

Parameter	Estimation	Std. Error	t-statistic	P(> $\|t\|$)
α	0.0073	0.0148	0.4926	0.6229
γ_1	0.1871	0.0210	8.9277	0.0000
τ_1	0.9250	0.1069	8.6494	0.0000
γ_2	0.1695	0.0210	8.0879	0.0000
τ_2	-2.0302	0.2362	-8.5962	0.0000
σ_P: 0.2053				
Df: 187				

component as well as mean-reversion. These features are represented in equation 13 and 14, respectively. The non mean reverting model will be estimated as a robustness test of the results obtained in the canonical specification model (14).

$$dP_t = \alpha + \gamma_1 \ \sin((t+\tau_1)\ \frac{2\ \pi}{6}) + \gamma_2 \ \sin((t+\tau_2)\ \frac{2\ \pi}{12}) + \sigma_P dZ_t \tag{13}$$

$$dP_t = \alpha + \beta \ P_t + \gamma_1 \ \sin((t+\tau_1)\ \frac{2\ \pi}{6}) + \gamma_2 \ \sin((t+\tau_2)\ \frac{2\ \pi}{12}) + \sigma_P dZ_t \tag{14}$$

Note that in these two models the seasonal components are defined under a 6-month and a 12-month period. Estimated parameters of equations 13 and 14 are reported in Table 4 and Table 5, respectively.

Reported results demonstrate that the seasonal and mean reverting components are statistically significant. The estimated seasonal parameters (γ_1, τ_1, γ_2 and τ_2) show that there are two price peaks, dated in July and in January. As we shall see under the simulations performed in Section 7, the price peak registered in July is higher than the one reported in January. The process corresponding to the price dynamics exhibits two minimum values in April and October, respectively. Reported results therefore reveal that periods with extreme temperatures (winter and summer) give rise to the the highest electricity prices, but not necessarily the same peak price level. As

Table 5: Estimation results for the mean-reverting model.

Parameter	Estimation	Std. Error	t-statistic	P(> $\|t\|$)
α	0.3634	0.1535	2.3671	0.0190
β	-0.0394	0.0169	-2.3323	0.0208
γ_1	0.1828	0.0208	8.8352	0.0000
τ_1	0.8948	0.1086	8.2393	0.0000
γ_2	0.1659	0.0208	7.9934	0.0000
τ_2	-2.1829	0.2477	8.8138	0.0000
σ_P: 0.2030				
Df: 186				

expected, periods with comfortable temperatures (April and October) exhibit the lowest prices.

6. Model Implementation

We solve the Bellman equation (8) introduced in section 2. We solve the recursive problem using the value-function iteration approach. Model parameters are summarised in tables 2, 3 and 5.[18].

6.1. The value-function iteration approach

We implement the value-function iteration algorithm in order to solve for the optimal policy under different states. The problem solution involves defining a discrete state space for the inventory, price and time variables. The manager's optimal policy is the determination of the quantity of water used to generate electricity that maximises the value of the facility. This decision depends on the time period (or season) within a given year, the electricity price and the water inventory.

[18]We assume that the cost function takes a value of 0 due to lack of data availability. Reservoir operation costs are typically very low see Latorre *et al.* (2014)

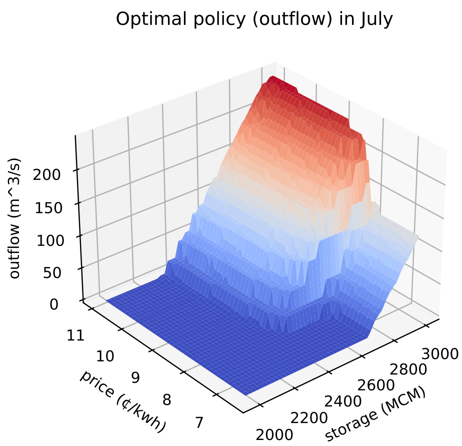
The algorithm defined under 1 is implemented for this purpose. The estimation delivers two outputs in the discrete state space i) the tensor V, which calculates the optimal value of the facility given the state ii) the policy, which returns the optimal strategy with managerial responses to variations in inventory levels, prices and time (season). The cost of operation in the profit function is assumed to be negligible.

Figure 4 reports estimation results for every given value of inventory and price. [4a] illustrates the optimal policy for the facility on the first of July while the value of the resource is presented in figure [4b]. Figure 4a, the optimal policy implies that low electricity price value delivers lower outflow of water to produce electricity. In a similar way, the water outflow is positively related to the quantity of water stored.[19] A close look at this figure shows that the optimal outflow ranges from zero in states of low to medium storage to 200 $[MCM/month]$ in state of high storage and high prices. The optimal policy is a non linear function of the state variables. For example, the water outflow policy in a high price state ranges from zero to 200 $[MCM/month]$ in high price states and from zero to 100 $[MCM/month]$ in low price states.

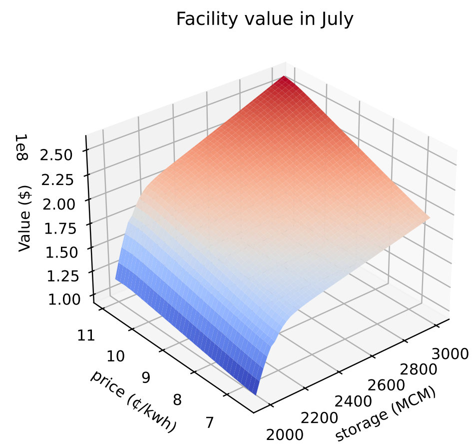
Figure 4b shows the value of the facility at a given time illustrating the economic trade off of the model as the marginal increase of the value of the resource per unit increase in price and storage. It is clear that the value of the facility is positively related to the electricity price. However the relationship is non linear over the state space. An extra unit of storage becomes more valuable in high price states when compared to low price states, as it improves current and future profitability. In agreement with results reported by Pizarro and Schwartz (2021) and Carmona and Ludkovski (2010) when the resource price increases the current value and the incentives to produce more output also increase. The resource value is also positively related to the level of water inventory. However, an increase in the water outflow diminishes the storage level reducing the future value of the facility.

Reported results suggest that for the current state of the reservoir (Decem-

[19]Note that electricity production depends on the the elevation of the reservoir since this variable determines the pressure of the water flowing into the turbine, as modelled in section 2.3



(a) Optimal outflow policy in July



(b) Facility value for different states at a fixed time (July).

Figure 4: Optimal policy and value of the facility when the time is fixed to July. Electricity price and inventory, the other state variables, are not fixed.

ber 2019) with a storage of 2,624 (MCM) and a price of 8.45 (¢/kwh) the value of the resource is 224 million US dollars. It is important to underline that the calculated resource value only represents the value of the water used to generate electricity. The reservoir could potentially be used for different purposes such as irrigation or man´s use that may provide additional value. For instance in an analysis applied to the Aswan High Dam Strzepek *et al.* (2008) show that there are large premiums derived from the terrain downstream available for growing crops and the benefits of flood control . This analysis is exclusively focused on hydro production.

Our findings also suggest that the reservoir should optimally reduce water outflow to zero if the water storage ranges between 2000 and 2400.

In what follows we provide a series of simulation exercises that illustrate how the reservoir is optimally managed so that the deadpool related states can be avoided under future management.

Algorithm 1 Value-function iteration, assuming the non-mean-reverting electricity price model.

$\lambda^I \leftarrow$ probability of state for the Inventory
$Z^I \leftarrow$ States for the Inventory (normal shocks)
$\lambda^P \leftarrow$ probability of state for the electricity Price
$Z^P \leftarrow$ States for the electricity price (normal shocks to log-price)
$\rho \leftarrow$ The correlation imposed between λ^I and λ^P
$r \leftarrow$ discount factor
$N \leftarrow$ length of Storage values grid
$M \leftarrow$ length of Prices values grid
$Q \leftarrow$ Outflow grid (policies)
$T \leftarrow$ Latest time step (in months)
$V \leftarrow$ Value of the states (it depends on time, price and inflow)
while **not** stop condition **do**
  for t **in** [latest_date + 1, ... , T] **do**
    for (n,m) **in** [(1,1),..., (N,M)] **do**

$$V(t,n,m) \leftarrow \max_{q \in Q} \pi(I_n, P_m, q) + \frac{1}{1+r} \sum_{k,j} \lambda_k^I \lambda_j^P V\left(t + \Delta t, I_n + \Delta I_n, P_m + \Delta P_m\right)$$

Where:

$$t + \Delta t = t + 1$$
$$I_n + \Delta I_n = I_n + \Phi(t, \cdot) - E(t) - q + \sigma_I Z_k^I$$
$$P_m + \Delta P_m = P_m + \alpha + \beta P_m + \gamma_1 \ \sin\left((t+\tau_1)\frac{2\ \pi}{6}\right) + \gamma_2 \ \sin\left((t+\tau_2)\frac{2\ \pi}{12}\right)$$

$$\text{Policy}(t,n,m) \leftarrow q$$

    end for
  end for
end while

7. Results

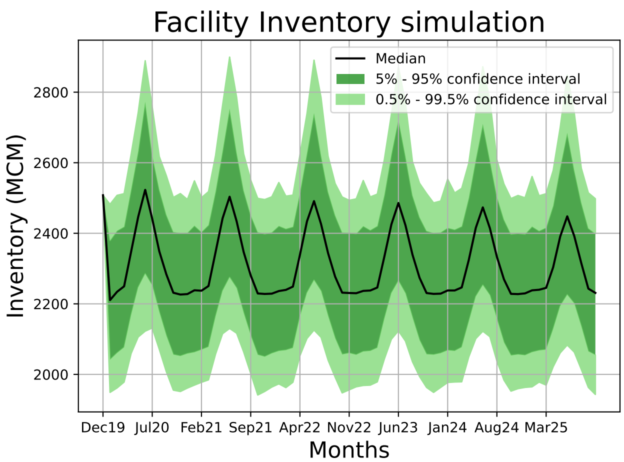
This section illustrates how the water inventory, the electricity price and the optimal outflow evolve if the optimal reservoir policy is implemented. We simulate 10,000 paths over a period of 6 years of monthly observations for the price and inventory processes combining their respective distributions with the optimal policy. This requires that the evaporation and the water inflow processes are also simulated. The simulations are performed assuming normality of the residuals in every equation specification.

We use the outflow variable of the inventory model (1) in every step as the optimal policy derived in section 6. Expected values are used for the evaporation and inflow variables therefore assuming that their variability will be captured by the random process of the inventory. All simulations start from the current state (defined under the last observation, December 2019). The following initial values values are assumed for the and inventory variables; price = 8.45 and inventory = 2624.

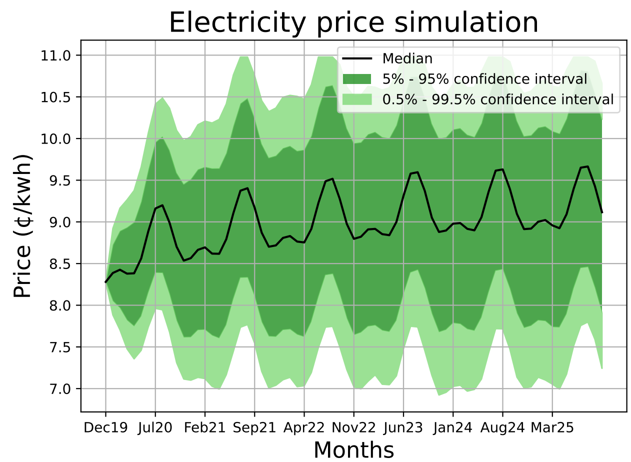
Figure 5 illustrates simulation results for the underlying stochastic variables. Simulations of median paths and corresponding 99% and 90% confidence interval levels for inventory, electricity price and optimal outflow and inflow variables are presented in Figs [5a], [5b], [5d] and [5c], respectively. Reported simulation results suggest that the manager should store the water and use it in periods in which the electricity price is highest. This is consistent with the natural resource valuation literature (see Brennan and Schwartz(1985)) which uses stochastic control theory to explicitly account for output rates which are explicitly linked to the output price. The outflow rate is the analogue to the output variable in the valuation literature. The inventory level (which can be regarded as the input variable) does always affect the policy. The effect of storage on water outflow is highly conditioned by the defined storage boundaries which are I_{max} and I_{min}. These boundaries induce limitations in the stored water levels of the resource. Hence, the dispatch of water has to be carefully managed depending on the current level of water stored in order to guarantee the optimal amount of water in future states.

Results also suggest that median values of inventory and policy are stable over time implying that storage is above 2000 with 95% probability level. Storage therefore never runs down to zero. In the same line, water inventory is always under 3000 (its upper boundary), hence, the risk of flood emergence is also controlled. The simulated median price path increases smoothly over time crossing the \$10 threshold with 0.5% probability under normal conditions and with 5% probability only in the summer peaks. The degree of uncertainty as signalled by the 99.5% and 95% confidence intervals increases after the first six months for the three simulated series but remains stable thereafter. As a result there is a 0.5% probability that prices are above \$10.5 after 1.5 years. Water outflow is zero just before the summer inventory peaks. Outflow levels reach the 150 ($MCM/month$) threshold with 5% probability and the 200 ($MCM/month$) level only with 0.5% probability.

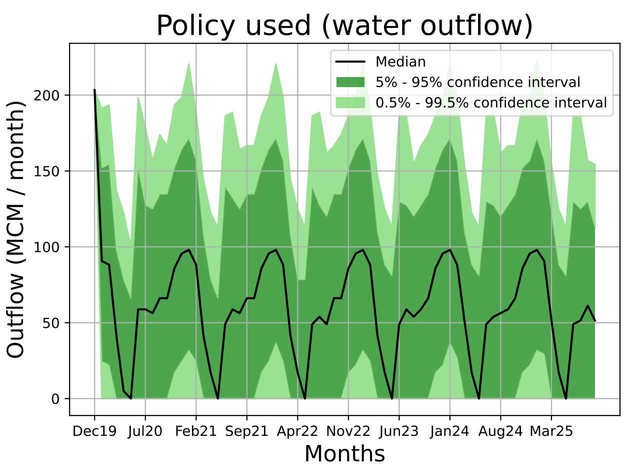
Appendix B reports robustness test results when the electricity price process does not exhibit mean-reversion. Results are highly consistent with those delivered under the benchmark canonical model specified in equation (14).



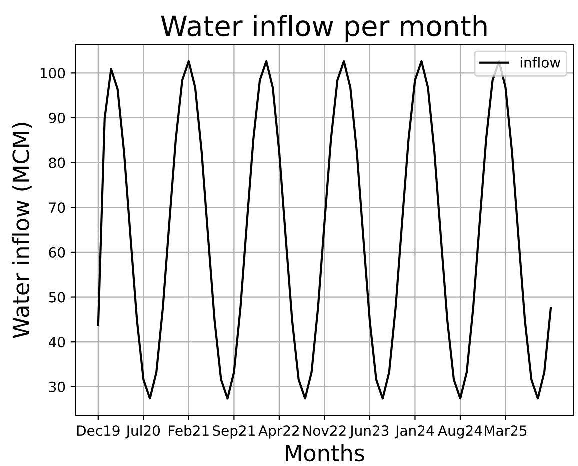
(a) Inventory level simulated



(b) Electricity price simulated



(c) Optimal policy used in the simulations



(d) Water inflow used, state variable.

Figure 5: Simulating the inventory and electricity price dynamic applying the optimal policy obtained in section 6. The price dynamic follows the non-mean-reverting model proposed in (13).

8. Sensitivity analysis

The results reported in the previous section were obtained assuming that the parameters of the stochastic processes for the inventory and price remain the same in the future as they were under the estimation period. However, the quantity and volatility of rain falls in the future may be different than in the past due to climate change. Average future prices may also be higher in the future due to a number of reasons such as the increase of geopolitical disruptions or the emergence of regulatory or technical changes related to the energy transition.

To evaluate the impact of future changes in the price and in the inflow processes on our results we solve the valuation problem using different assumptions about model parameters. Our focus will be on addressing the sensitivity with respect to the parameters in the electricity price model and climate change (as captured by changes in water inflow and inventory uncertainty) on the optimal policy.

8.1. Sensitivity analysis to changing electricity market price process

The analysis in the previous section has shown that prices are an important driver of optimal policies. The proposed model illustrates the managerial control over the outflow rate which responds to the electricity price. It is therefore of great relevance to analyse the effects of future price scenarios on the optimal policy. We therefore analyse model solutions for parameter changes in the electricity price dynamics. The process of the electricity price is defined under (15). In what follows, we illustrate model results under the assumption that prices in the future behave differently than they have done in the past. This involves analysing the model under two different α parameter values. Note that α is the average growth of the process (which also influences the mean reversion level). As previously underlined, prices may follow a different growth rate in the future in response to a number of factors. Supply side disruptions arising from geopolitical tensions, natural resource scarcity or the need for market re-design to support the ongoing energy transition are potential drivers of future average price regime shifts.

We analyze the different scenarios by considering two α parameter values representing benchmark and high price conditions.

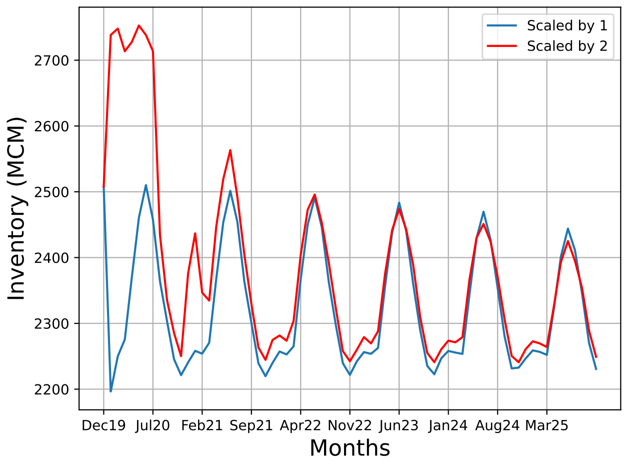
$$dP_t = \alpha + \beta\ P_t + \gamma_1\ \sin((t+\tau_1)\ \frac{2\ \pi}{6}) + \gamma_2\ \sin((t+\tau_2)\ \frac{2\ \pi}{12}) + \sigma_P\ dZ_t \quad (15)$$

Specifically, we follow the same method as in Section 6 and provide optimal solutions for the benchmark case, using the estimated α, and for the high price scenario, multiplying α by 2.

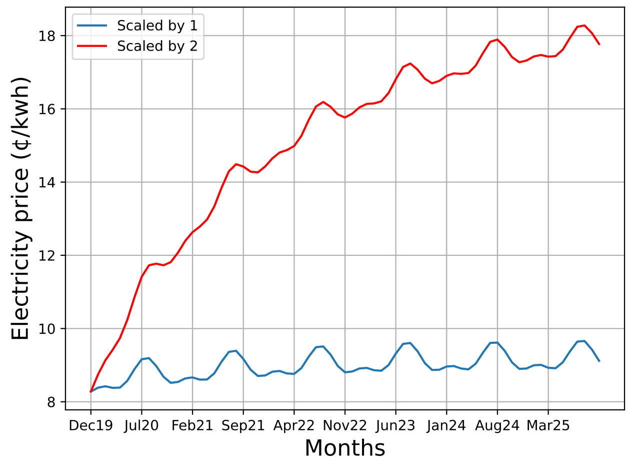
Figure 6 reports median of the simulated paths for inventory, prices and water outflow under the two different scenarios considered.

Figure 6a, illustrates the projected inventory for the benchmark and high price scenarios. It shows that the electricity price is a key decision making variable that will affect the optimal inventory level. Depicted results show that during the first 12 months the storage is much higher than in the long term. This is specially the case for the scenario in which the expected price is high (see the red line) in which case it is most profitable to store water to sell electricity at the highest price and thus obtain higher profitability. There are two peaks in the simulated inventory process during the first year, mirroring the seasonal component of prices. Projected inventory decreases monotonically for lower price scenarios. This effect slowly disappears in the long term (after month 12) where we see similar levels of inventory for the two projected price scenarios (with more scenarios simulated the pattern of converging in the long run stands). This implies that the policy applied responds to a stabilised price process under both scenarios. More importantly, Figure 6c shows that the optimal outflow policies are similar in the long run. Thus, the long term optimal operation of the reservoir is not too sensitive to the changes in the estimated α of the price model.

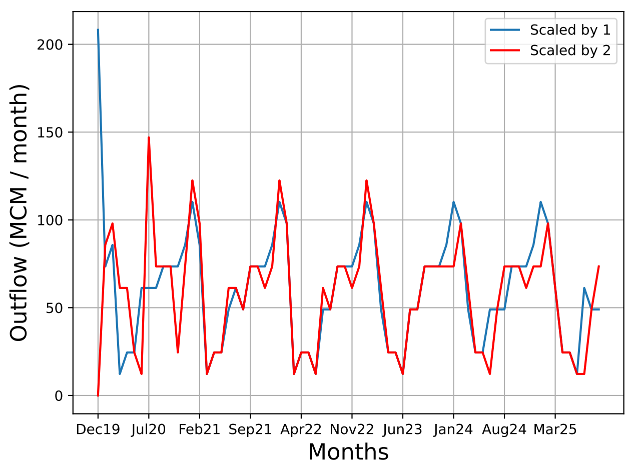
Figure 6c shows the outflow optimal policy under the benchmark and high price scenarios. Under the highest price scenario (red line) inventory is maximised during the first 12 months under the expectation of higher prices. This delivers lowest outflow levels during the first six months when compared to the benchmark case. The outflow rate is however highly related for



(a) Median water inventory of the simulations for different parameters of the electricity price model.



(b) Median electricity price of the simulations under different values of average growth for the electricity price model.



(c) Median water outflow of the simulations for different parameters of the electricity price model.

Figure 6: Simulations for the water inventory, electricity price and water outflow under different average growth specifications of the electricity price model.

all price scenarios from month 12 onward suggesting that in the long run the optimal policy is independent from the emergence of different trends in the price process.

8.2. Sensitivity analysis: shift in water inflow due to climate change

The second objective is to assess the effect of different water inflow supply scenarios on the optimal policy. This is an important challenge for the optimal management of the reservoir. For instance, the Financial Times[20] has recently underlined that nearly one-fifth of humanity lives in stressed river basins. The decrease in water supplies is largely determined by climate change. In what follows we analyse the future consequences of change weather conditions by considering different scenarios of water inflows in optimal policy solutions.

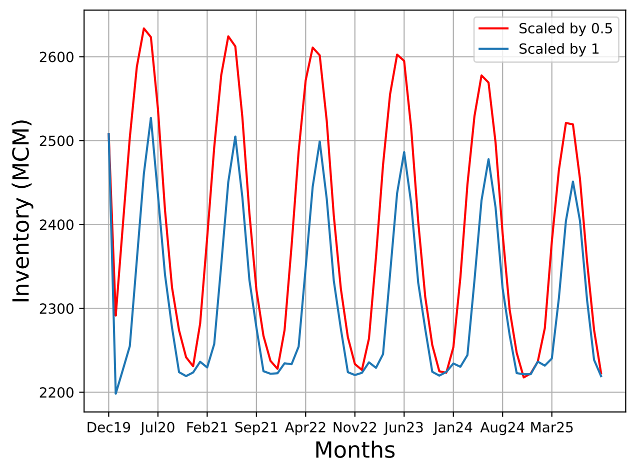
The quantity of water flowing into the reservoir is modelled by (16) and it is defined as a mean reverting process. We follow the same approach as in the previous section and address the sensibility of the optimal policy to different scenarios of water inflows by considering a scenario with lower value in the parameter α, α is scaled by 0.5, is compared to the benchmark case, α scaled by 1.

$$\Phi(t+1) = \alpha + \beta\ \Phi(t) + \gamma\ \sin\left((t+\tau)\frac{2\,\pi}{12}\right) \tag{16}$$

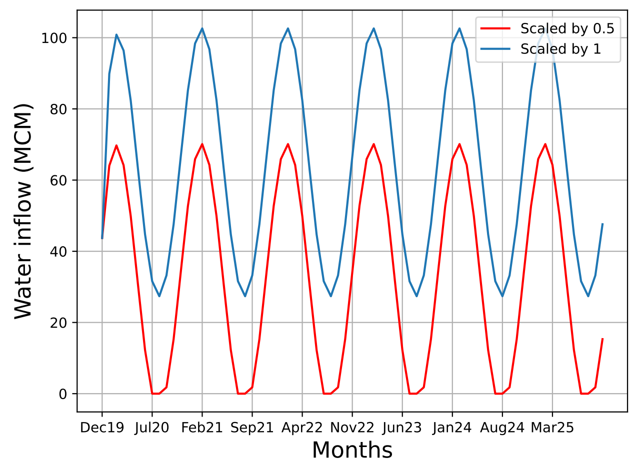
We solve the optimisation problem deriving the optimal policy for each future state and use this policy to simulate future scenarios. Figure 7 shows resulting median inventory, water inflow and optimal policy for the simulated paths for two different α values, the benchmark case and the the lower water inflow scenario.

Figure 7a shows that there is a clear difference of resulting simulated inventory paths that respond to the two water inflow scenarios illustrated in

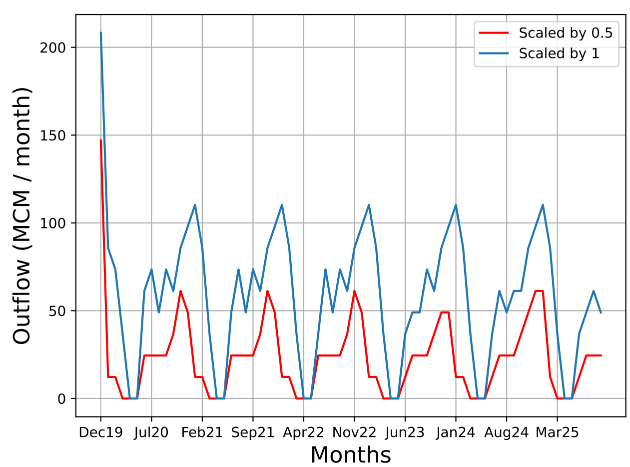
[20]Water pricing is the key to equitable access, 12th of March 2023



(a) Median water inventory of the simulations for different parameters of the water inflow model.



(b) Median water inflow of the simulations under different values of average growth for the water inflow model.



(c) Median water outflow of the simulations for different parameters of the water inflow model.

Figure 7: Simulations for the water inventory, water inflow and water outflow under different average growth specifications of the water inflow model.

Figure 7b. Optimal policies store water for the high price season (summer) in order to have water during the peak times in electricity prices. The scenario that represents the possibility of lower water inflows due to less frequent rains (see blue line) indicate that the optimal policy is to start storing water earlier than under the benchmark case (see the orange line). This implies that the policy is more conservative in the use of the resource.

Figure 7c illustrates the outflow policy that is applied under the two scenarios. A close look at this picture shows that under the the scenario with water scarcity (represented by the blue line) a more conservative policy is applied which concentrates the highest production during the period with the highest electricity peak, identified during the summer season.

In the previous analysis we showed that the electricity price does not affect the inventory in the long run. Note that this is not the case for the water inflow analysis under climate change as illustrated in 7a. This give rise to different long term patterns reflecting the impact of extreme weather events.

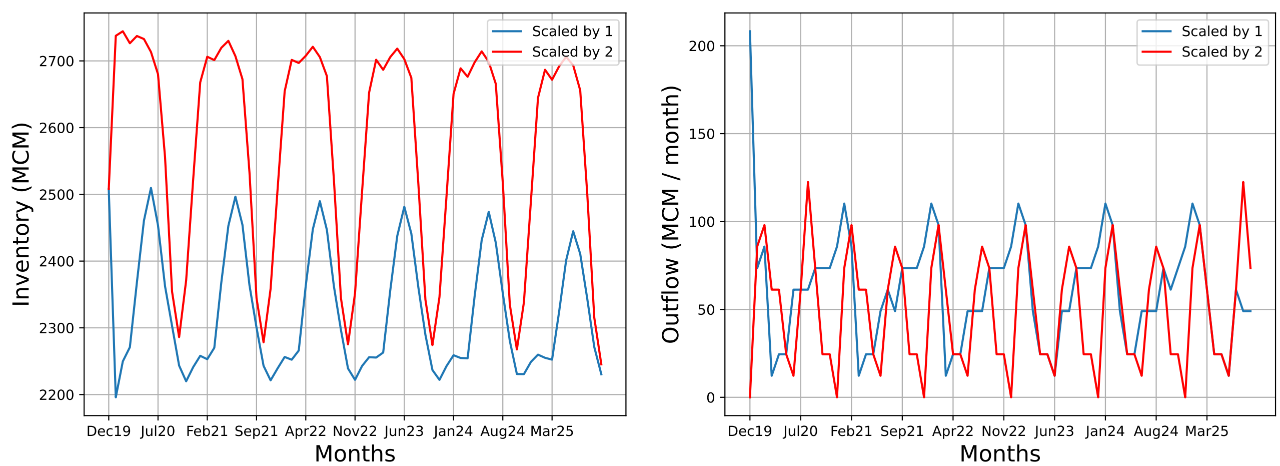
8.3. Increased inventory uncertainty due to climate change

This section assesses the impact of uncertainty on estimated results by solving the model under different assumptions related to the volatility of the inventory process. We are particularly interested in addressing the effect of extreme weather events in the form of storms or floods, exceptionally high heat and draughts. This is important at the time of writing when drought conditions in western states of America are replaced by abnormally wet weather as stated by government scientists.[21]

The adaptation of management policies to extreme weather events is therefore considered under shifts in the inventory uncertainty dynamics. The purpose of this analysis to provide management guidance to avoid situations in which emergency measures are adopted to deal with abnormal heat and drought. Such scenarios can lead to a supply chain problems that have to

[21]see article "Climate graphic of the week: flood risk replaces drought across western US states" 1st of April 2023

be deeply considered. For instance, emptying rivers and reservoirs have recently hit hydropower plants supplying electricity in China forcing companies such as Toyota and Foxconn to suspend operations.[22] In order to design the optimal adaptation policy we have incorporated the possibility of abnormal weather characteristics by analysing the effects of scaling the volatility value by a factor 2. We follow the same procedure as in section 8.1 and 8.2 for this purpose. Figure 8 shows the estimated median paths for the water inventory and the water policy outflow.



(a) Median water inventory of the simulations for different parameters of the inventory volatility.

(b) Median water inflow of the simulations under different values of inventory volatility.

Figure 8: Simulations for water inventory and water outflow under different volatility scenarios

A close look at Fig. 8a demonstrates that under high volatility conditions it is optimal to increase storage and keep its level high during longer periods than under benchmark case. The optimal management adapts to extreme weather events by following a conservative inventory policy. Fig. 8b shows that the derived optimal median water outflow policy is more variable under the high volatility scenario than under the benchmark case. Specifically the median water outflow policy reaches a very low level twice a year just before the winter and summer electricity price peaks. Illustrated results also show that

[22]See the FT article "The trickle-down effect of empty rivers and reservoirs", 22th August 2022)

there are two annual outflow peaks. These may be reflecting the possibility of facing floods in the future.[23]

The overall results within this section do, therefore, demonstrate that when considering the possibility of adapting to extreme abnormal weather events it is optimal to store more water during longer periods so that to future production can be guaranteed. Because the effect of draughts cannot be mitigated via immediate management, optimality requires adaptation of the storage policies so that they become more conservative or risk averse.

9. Summary and conclusion

This paper develops and implements a stochastic optimal control approach to value a hydropower plant. The model includes two stochastic variables, water inventory and electricity price and one deterministic factor calendar date or time. This specification gives rise to a Bellman equation that is solved using the Value-Function Iteration approach. The solution is implemented using estimated parameter values for a US reservoir located in Arkansas. We compute the optimal policy for the problem and analyse the future development of the facility under 10,000 simulated paths. Finally, we analyse the possibility that the parameters in the stochastic inventory and price processes change in the future.

The Bellman equation is solved for seasonal mean reverting price dynamics. However, as a robustness test solutions are also provided under the non mean reversion assumption. Estimation results are robust to the specification of non mean reverting prices.

Reported results highlight the (non linear) strong link between the value of the reservoir, the optimal policy and the inventory and price state variables. Specifically they show that with a 90% probability, inventory levels of the facility evolve within a narrow region. Moreover, With more than 99%

[23] The analysis of confidence intervals (which can be provided upon request) confirms that, as expected ex ante the 5% and 95% bands are significantly wider for the high volatility case than under the benchmark case.

probability the reservoir will not reach the "deadpool" state. The reservoir maximum threshold level is never reached implying that the flooding risk is controlled. Nevertheless, the optimal storage varies significantly between scenarios suggesting that a change in the optimal solution has a large impact on the management of the facility.

This paper presents two major contributions. First, it provides a detailed analysis of electricity price and water inventory uncertainty and its effect in the reservoir management. The empirical application is based on monthly data of reservoir characteristics which differs from the empirical exercises based on simulated "toy" data applied in the literature (see Thompson *et al.*(2004)). Second, the proposed framework allow analysis of long term management policy. The sensitivity analysis shows that that changes in average prices will not have a significant effect in the long run policy. More interestingly, climate change does affect the optimal policy that has to be implemented. Higher inventory will be required under lower water inflow levels. Increased uncertainty in inventory dynamics reflecting the possibility of extreme weather events lleads to higher storage during longer periods.

The results in this paper have important policy implications for regulators and managers of reservoirs. First, optimal policies will need to be adjusted under the presence of climate change. The optimal policy adapts to lower future water inflow levels by providing higher and earlier water storage. Higher inventory uncertainty also leads to more conservative and long lasting storing policy. Changes in average prices will not have a long term policy effect.

The methodology presented in this paper can be extended other reservoirs acting as price takers with main focus in hydro production. However, given that a significant number of dams are used for irrigation as well as for human consumption purposes a natural extension of this analysis will involve the multi use case. This should allow the consideration of future energy transition related applications such as the consumption of water for green hydrogen production purposes or the use of water as means of electricity storage under PSH technologies. The practical implementation of future extensions will require the use of improved cost estimates of the facility including the value of water before and after its use for electricity generation.

Appendix A. The Fourier approach for frequency computations

Given a function $f(x)$ with reasonably good properties it is possible to decompose it as:

$$f(x) = \sum_{k=-\infty}^{\infty} c_k \, e^{ikx}$$

Note that e^{ikx} is a periodic function (spins around the unit circle in the complex plane with a frequency of k) and c_k is the amplitude of the wave. Using the Euler formula[24] it is intuitive that the trigonometric functions form a base of the functional space, if we rotate this base by a factor ρ_k it is possible to make the decomposition in terms of just one trigonometric function. It is done as follows:

$$\begin{aligned} f(x) = \sum_{k=-\infty}^{\infty} c_k \, e^{ikx} &= c_0 + \sum_{k=1}^{\infty} a_k \, \cos(kx) + b_k \, \sin(kx) \\ &= c_0 + \sum_{k=1}^{\infty} \sqrt{a_k^2 + b_k^2} \, (\sin(kx - \rho_k + \frac{\pi}{4})) \end{aligned}$$

Furthermore, it is known that $\rho_k = \arccos \frac{a_k^2}{\sqrt{a_k^2 + b_k^2}}$ and $\sqrt{a_k^2 + b_k^2}$ is the value represented in the vertical axis of Figure 2. So it is clear the equivalency between the seasonal models we propose (see, for example, equations 13 or 14) and the Fourier approach and how appropriate it is to obtain the **period** of the seasonal components.

Although values such as ρ_k or the amplitude are known, we estimate those values via statistical methods in sections 4 and 5 in order to obtain other information such as the statistical significance.

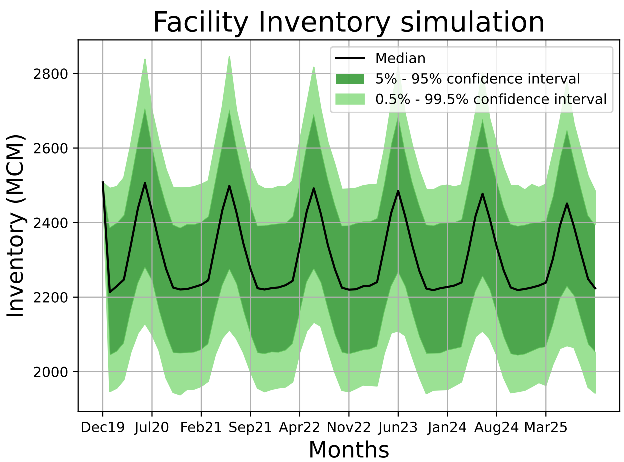
[24] $e^{ix} = \cos(x) + i \, \sin(x)$

Appendix B. Results assuming electricity price is non-mean-reverting

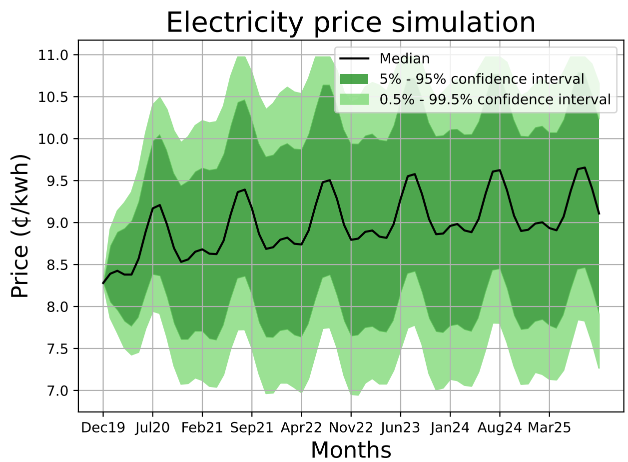
Section 7 shows the results using the optimal policy and assuming a mean-reverting electricity price model. For robustness we have obtained the same results assuming that the electricity price is non-mean-reverting, as introduced in equation (B.1).

$$dP_t = \alpha + \gamma_1 \; sin((t + \tau_1)\,\frac{2\,\pi}{6}) + \gamma_2 \; sin((t + \tau_2)\,\frac{2\,\pi}{12}) + \sigma_P dZ_t \tag{B.1}$$

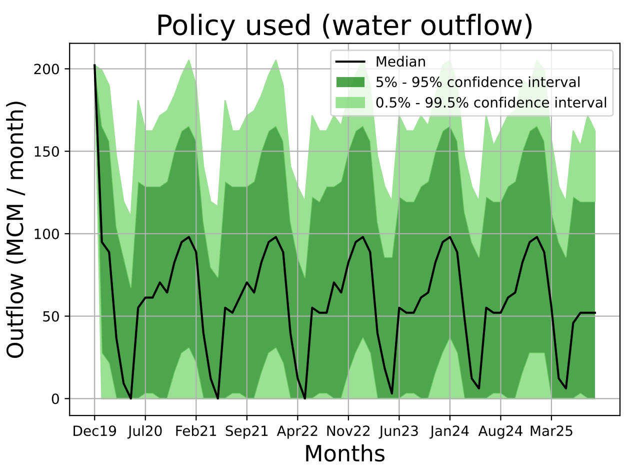
Figure B.9 shows the results under these assumptions. The simulations are similar to those observed for the canonical model.



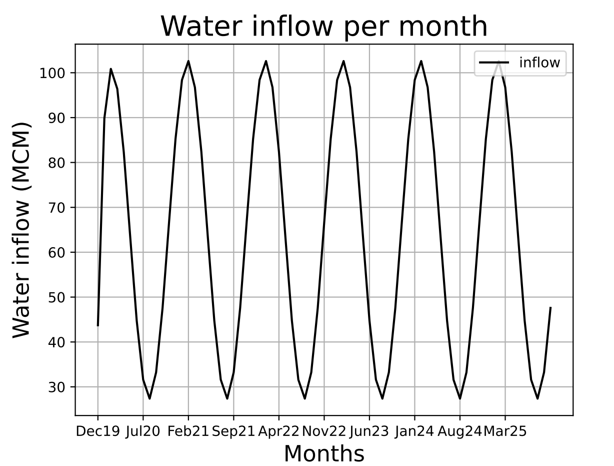
(a) Inventory level simulated.



(b) Electricity price simulated.



(c) Optimal policy used in the simulations.



(d) Water inflow used, state variable.

Figure B.9: Simulating the inventory and electricity price dynamic applying the optimal policy obtained in section 6. The price dynamics follows the non-mean-reverting model proposed in (B.1).

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