GRADO EN INGENIERÍA EN TECNOLOGÍAS INDUSTRIALES

TRABAJO FIN DE GRADO

OPTIMAL BLACK-START SEQUENCE OF DISTRIBUTED GENERATION

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Declaro, bajo mi responsabilidad, que el Proyecto presentado con el título

OPTIMAL BLACK-START SEQUENCE OF DISTRIBUTED GENERATION

en la ETS de Ingeniería - ICAI de la Universidad Pontificia Comillas en el

curso académico 2024 es de mi autoría, original e inédito y

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de otros documentos está debidamente referenciada.

Fdo.: Alejandro Lacave Diez Fecha: 4/07/2024

Alejandro Lacave

Autorizada la entrega del proyecto

EL DIRECTOR DEL PROYECTO

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SECUENCIA ÓPTIMA DE ARRANQUE BLACK-START CON GENERACIÓN DISTRIBUIDA

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RESUMEN EJECUTIVO DEL PROYECTO

1) Introducción

Hoy en día las redes eléctricas se operan de formas lo más seguras posibles, aunque pese a tener una gran fiabilidad siempre existen riesgos de apagones o cortes en el suministro eléctrico. Para combatir dichas averías, las redes eléctricas cuentan con métodos de arranque que se conocen como métodos black-start, los cuales consisten en ir conectando generación y demanda de manera secuencial. Dentro de estos métodos existen varios tipos.

El top-down consiste en energizar la red eléctrica afectada mediante la ayuda de una red vecina la cual comienza energizando las líneas de trasmisión hasta alcanzar las zonas de menor voltaje dentro de la red como pueden ser las demandas y partes de la generación. Por otro lado, está el método bottom-up por el cual la propia red consigue energizarse mediante una serie de generadores con capacidad de auto arrancarse o más comúnmente conocida como capacidad black-start. Anteriormente, la secuencia de arranque se realizaba a través del sistema de transmisión, pero avances en la generación distribuida (grupos generadores ubicados en el sistema de distribución de una red eléctrica) han hecho que esta parte de la red eléctrica pueda contribuir en la energización de la propia red.

Para abordar este problema, se plantea un problema de optimización cuyo objetivo principal es buscar la secuencia de arranque más optima posible. Se plantean distintas funciones objetivo dependiendo de los autores, en algunos casos buscando la maximización de la recuperación de cargas (Ding, Wang, Qu, Wang, & Shahidehpour, 2022) o de la recuperación de generación (Cao, Wang, Liu, Azizipanah-Abarghooee, & Terzija, 2017). En otros casos, se busca la minimización de pérdidas durante la secuencia de arranque (Chen, Chen, Wang, & Butler-Purry, 2018). Como todo problema de optimización está sujeto a ciertas restricciones ya se busca emular fielmente la realidad de las redes eléctricas, de esta manera algunas de las restricciones más importante a las que se somete dicho problema de optimización tienen que ver con: límites de voltaje en nudos de la red, flujo de cargas a través de las líneas de corriente, puesta en marcha de los generadores o reconexión de cargas entre otras restricciones.

2) Metodología

Teniendo en cuenta todo lo anterior y centrándonos en un ejemplo de red de distribución, el objeto del proyecto consiste en, partiendo de un programa inicial el cual emula la reconexión de una red de distribución mediante un problema de optimización, ampliar la formulación de dicho problema añadiendo la restricción del "Cold Load Pickup" la cual consiste en un tipo de comportamiento de las cargas durante la reconexión de estas. Dicha restricción se caracteriza por simular como ciertas cargas que en un normal funcionamiento de la red actuarían de manera no simultánea, es decir, trabajarían en momentos distintos dando un sumatorio general de la carga total menor a la suma real de todas ellas. Cuando se produce la avería y hay que energizar la red, dichas cargas necesitan de un nivel mínimo de energía para ganar diversidad y empezar a trabajar por separado por lo que en los primeros instantes de la reconexión se produce un pico de carga el cual se irá mitigando con el tiempo y el que requerirá de más potencia de generación, la siguiente figura de (Gholami & Aminifar, 2017) muestra un gráfico con la secuencia de reconexión anteriormente explicada.

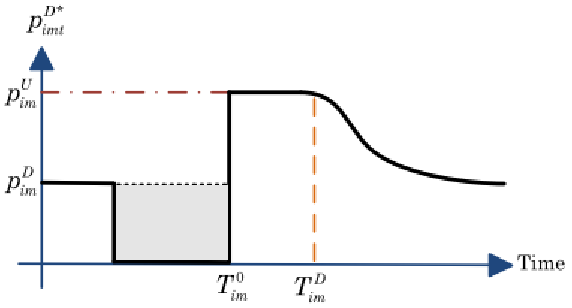

Ilustración (Res. Ejecutivo) 1: Secuencia de arranque CLPU

En la realización del proyecto se procederá de la siguiente manera. Primeramente, se implantará la restricción del Cold Load Pickup mediante la utilización de Matlab, herramienta que se usará para la codificación del programa. Posteriormente se probará el nuevo código con un ejemplo de red de distribución y se compararán los distintos comportamientos de reconexión de la carga, para concluir, se llevará a cabo un análisis de sensibilidad para comprobar como los distintos valores posibles de los parámetros de la restricción del Cold Load Pickup afectan a la secuencia de reconexión de la carga.

3) Formulación Matemática y Resultados

La secuencia de arranque de la red ejemplo se formulará como un problema de optimización como se ha comentado anteriormente y se resolverá mediante la aplicación de un MILP (Mixed Integer Linear Programming). La función objetivo del programa tratará de buscar la secuencia de arranque más optima mediante la maximización de la recuperación de la carga a lo largo del proceso de arranque. En cuanto a las principales restricciones del problema planteado se tendrán en cuenta restricciones acerca de los generadores de la red, relacionadas con las potencias máximas y mínimas de generación de cada uno, así como restricciones lógicas de conexión las cuales no permiten la desconexión de un generador una vez conectado a la red. Entre otras restricciones del problema se encuentran relacionadas con el tipo de comportamiento de las cargas (ya sea con la formulación estándar o con la formulación del Cold Load Pickup), con el flujo de cargas el cual será fijado a un flujo de cargas en corriente continua y con la forma en la que será operada la red (se trata de una red mallada operada de manera radial).

A continuación, se compararán los principales casos de secuencia de reconexión estos son las dos formas de comportamiento de las cargas durante el proceso de restauración. La red en la que se llevará a cabo las pruebas de reconexión, extraída de (Tomás, García, & Sigrist, 2023), se presenta a continuación.

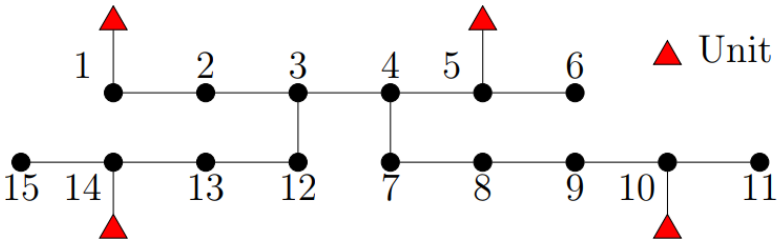

Ilustración (Res. Ejecutivo) 2: Red de Distribución Ejemplo

Como se aprecia, consta de quince nudos de los cuales 4 cuentan con generadores (nudo 1, 5, 10 y 14), los cuales tienen una capacidad máxima de generación de 5MW o 7,5MW dependiendo del generador, catorce de ellos cuentan con bloques de demanda (todos los nudos excepto el nudo 1), los cuales suman una demanda total de 18.903MW y las líneas 3-12, 3-4, 7-8 y 8-9 cuentan con interruptores los cuales pueden estar abiertos o cerrados en función del estado de la reconexión. Cabe resaltar que el único generador con capacidad black-start de la red es el generador del nudo 1, el cual será el que comience la secuencia de arranque en todos los casos.

De entre todos los casos de estudio se presentará el que corresponde con la totalidad de los generadores con una capacidad máxima de 5MW.

En el primer caso, en el que la demanda tiene un comportamiento estándar, esto es, cuando se conecta un bloque de carga se conecta con la totalidad de su demanda en un solo paso o lo que es lo mismo, se conecta con su demanda de régimen permanente. Se tiene una secuencia de arranque como la siguiente.

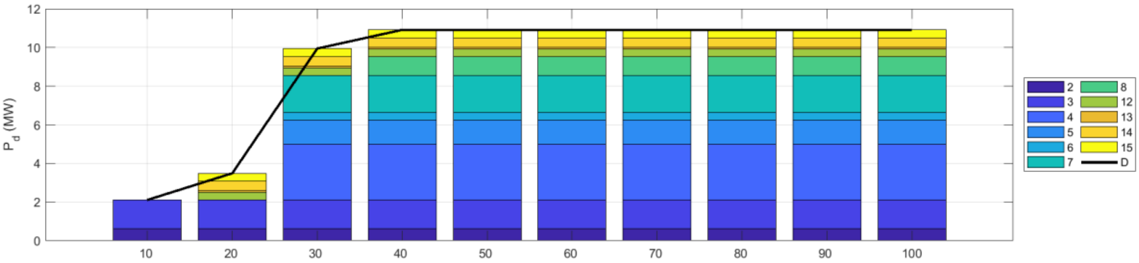

Ilustración (Res. Ejecutivo) 3: Secuencia de reconexión de cargas (Comportamiento estándar)

Cabe resaltar como la función de la demanda total únicamente puede aumentar si se conecta un bloque de cargas nuevo o por el contrario quedarse igual si se ha alcanzado el régimen permanente en la secuencia. Notar también como la demanda total en régimen permanente aumenta hasta los 10.914MW a diferencia de los 18.903MW de carga total. Esto se debe a la naturaleza de los interruptores de las líneas, una vez un interruptor se cierra automáticamente se energizan todos los nuevos nudos que se conectan a la red, así como los bloques de carga que están conectados a los mismos. Sin embargo, los generadores necesitan de un nudo previamente energizado para poder empezar a suministrar potencia. Es por esto por lo que el interruptor de la línea 8-9 produce un bloqueo de demanda de 7.989MW, los generadores 1,5,14 no son capaces de soportar la conexión de los nuevos bloques de carga sin la ayuda del último generador.

En cuanto a la secuencia de arranque de la red con el comportamiento de las cargas en condiciones de Cold Load Pickup caben destacar una serie de cosas. Primeramente, como se explicó anteriormente, con este tipo de comportamiento de los bloques de carga, en los primeros instantes de la reconexión se produce un aumento de la demanda mayor a la demanda real del propio bloque de cargas, esto se produce por un factor de no diversificación inicial que hará aumentar de manera brusca la demanda inicial para posteriormente ir decrementando hasta el valor real de la demanda de dicho bloque. Para el caso estudiado se ha fijado el factor de no diversificación a $S_d^U = 1.2$ y produce una secuencia de reconexión como la siguiente.

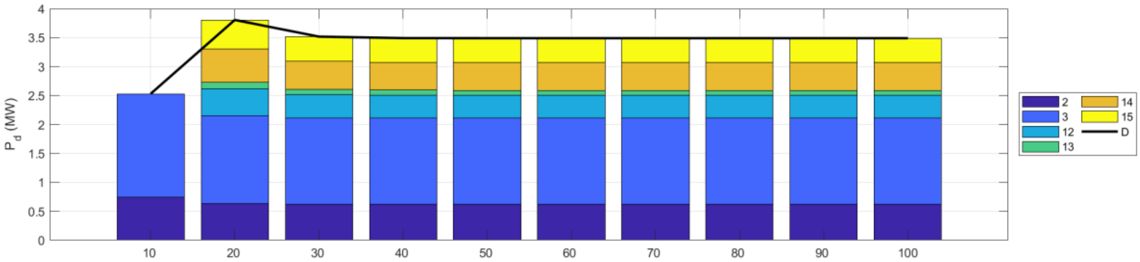

Ilustración (Res. Ejecutivo) 4: Secuencia de reconexión de cargas (Comportamiento CLPU)

Lo primero que se puede apreciar en diferenciación al caso anterior es que el número de bloques de carga que han conseguido reconectarse es menor que en el caso del comportamiento estándar. Esto es por lo descrito anteriormente, al tener picos de demanda mayores, ya que la demanda inicial de cada bloque es 1.2 veces mayor que la normal, los mismos generadores en funcionamiento no son capaces de aguantar el nivel de demanda que supondría la reconexión de las mismas cargas que en caso anterior. En este caso, se produce un pico de demanda de 3.806MW que se va decrementándose en cada instante hasta llegar a un régimen permanente de 3.492MW. Cabe destacar que si se quisiese reconectar el mismo número de bloques de carga que en el caso anterior habría que aumentar la potencia máxima de generación de las unidades de generación distribuida.

Por último, se llevó a cabo un análisis de sensibilidad en el que el objetivo principal era comprobar el impacto que tenían los distintos parámetros de la restricción del Cold Load Pickup (los cuales son: el factor de carga no diversificada, el factor de carga diversificada, el ratio de caída de la demanda para cada instante que pasa y el tiempo que tarda cada bloque de cargas en empezar a decrementar su demanda total). Se estudió el impacto de cada uno de estos parámetros por separado y por último se compararon dos casos de CLPU con distintos parámetros cada uno para de esta manera observar el impacto que tienen de manera conjunta.

4) Conclusiones

En el desarrollo de este documento, se expone un modo de conseguir una secuencia óptima de arranque black-start formulada a modo de problema de optimización en la que se pretende maximizar la carga recuperada comparando dos formulaciones de reconexión de los bloques de carga (Comportamiento estándar y comportamiento Cold Load Pickup). Con la implementación de esta restricción del Cold Load Pickup se pretende reflejar más fielmente la realidad de la secuencia de reconexión de las redes eléctricas. A su vez, se trata de hacer una comparación con la formulación estándar a modo de conocer que factores son los que hay que tener en cuenta ya que pueden causar problemas cuando se implementé la formulación del Cold Load Pickup. Finalmente, el análisis de sensibilidad permite conocer cuáles son los parámetros de mayor peso, es decir, los que pueden limitar más la secuencia de reconexión de cargas.

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OPTIMAL BLACK-START SEQUENCE OF DISTRIBUTED GENERATION

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Collaborating Entity: ICAI – Universidad Pontificia Comillas

EXECUTIVE OVERVIEW OF THE PROYECT

1) Introduction

Modern electrical power systems are operated as reliably as possible. Despite the high reliability there are always risks of outages or blackouts. To face those outages, electrical power systems have methods of restoration known as black-start sequence which consist of connecting generation and demand sequentially. Within these restoration methods, there exist various types.

The top-down approach involves energizing the affected network with the help of a neighboring grid, which starts by energizing the transmission lines until reaching the lower voltage areas within the grid, such as demands and some parts of the generation. On the other hand, the bottom-up method is found where the proper grid is the one that becomes energized through a series of generators with self-start capability, more commonly known as black-start capability. Previously, the startup sequence was carried out through the transmission system, but advances in distributed generation (generator sets located in the distribution system of a power grid) have enabled this part of the power grid to contribute to the energization of the grid itself.

To address this issue, an optimization problem is proposed with the primal objective of finding the optimal startup sequence. Various objective functions are proposed depending on the authors, in some cases aiming to maximize load recovery (Ding, Wang, Qu, Wang, & Shahidehpour, 2022) or generation recovery (Cao, Wang, Liu, Azizipanah-Abarghooee, & Terzija, 2017). In other cases, the aim is to minimize losses during the startup sequence (Chen, Chen, Wang, & Butler-Purry, 2018). Like all optimization problems, it is subject to certain constraints to faithfully emulate the reality of power grids. Thus, some of the most important constraints for this optimization problem are related to voltage limits at network nodes, load flow through power lines, generator start-up or load reconnection, among other restrictions.

2) Methodology

Taking all the stated above into account and focusing on an example of a distribution network, the object of this project consists in, starting from an already existing model that emulates with an optimization problem the reconnection of a distribution network, expanding the formulation of the optimization problem by adding the “Cold Load Pickup” constraint which consists of a kind of behavior load blocks have during their reconnection. This constraint is characterized for simulating how certain loads that under normal conditions would act non-simultaneously (i.e, would operate a different time instant resulting in a smaller load sum than the sum that would result if all the loads of the load block would act at the same time instants). When the outage occurs, loads need a certain power to start gaining diversity and start operating non-simultaneously again. This produces a load peak in the initial moments of the reconnection that will be mitigated over time, due to this demand peak a higher generation capacity will be needed to reconnect the same amount of load blocks. The following figure from *(Gholami & Aminifar, 2017)* shows the reconnection sequence explained above.

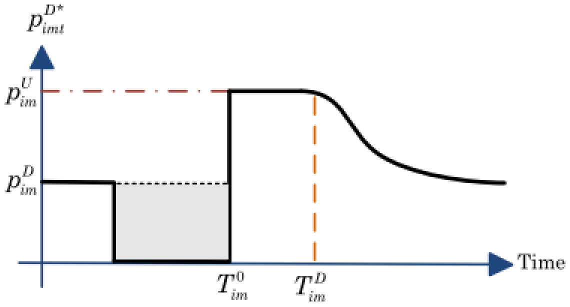

Figure (Exc. Overview) 1: CLPU reconnection sequence

The project will be carried out as follows. Firstly, the Cold Load Pickup constraint will be implemented using MATLAB, a tool that will be used for the programing of the optimization problem. Subsequently, the program with the implementation will be tested with an example of a distribution network where the different load formulations will be compared. Finally, a sensitivity analysis will be conducted to examine how the different values of the CLPU parameters can affect the load-reconnection sequence.

3) Mathematical Formulation and Results

The reconnection sequence of the example network will be formulated as an optimization problem, as previously mentioned. Which will be solved using MILP (Mixed Integer Linear Programming). The objective function of the model will aim to find the optimal restoration sequence by maximizing load-recovery. Regarding the constraints of the proposed problem, constraints related to the distributed generation will be considered, including the maximum and minimum generation capacity of generators, as well as logical connection constraints of generator that imply that once a generator has been connected along the sequence it won’t be able to get disconnected. Among other constraints of the optimization problem restrictions related to the load behavior are taken into account (whether the load respond to the standard formulation or to the Cold Load Pickup formulation), power flow restrictions are also modelled (Direct current power flow is modelled) as well as logical constraints that have to do with the way the network is operated (radially operated meshed distribution network).

Following, the principal cases of restoration sequence will be compared, these are the load-reconnection sequence following both demand formulation. The example network that will be used to test the model will now be presented, note that has been taken from (Tomás, García, & Sigrist, 2023).

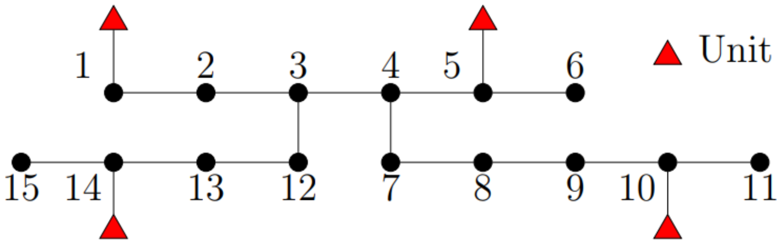

Figure (Exc. Overview) 2: Example of Distribution network

As can be seen, the network consists of fifteen buses, four of which have generators(buses 1,5,10 and 14), each with a maximum generation capacity of 5MW or 7.5MW depending on the generator. Fourteen buses have load blocks (all buses except bus 1), with a total demand of 18.903MW. Additionally, lines 3-12,3-4,7-8 and 8-9 have remote-controlled switches (RCSs) that can be opened or closed depending on the state of the reconnection. It is worth noting that the only generator with black-start capacity is the generator at node 1, which will be responsible for initiating the restoration sequence.

Among all the case studies, the one that corresponds to the maximum capacity of generators of 5MW will be presented.

The first case refers to the load behaving with the standard formulation. This is, when a load block is connected it instantly connects with totality of its demand (with the demand that corresponds to its permanent regime). A restoration sequence like the following can be observed.

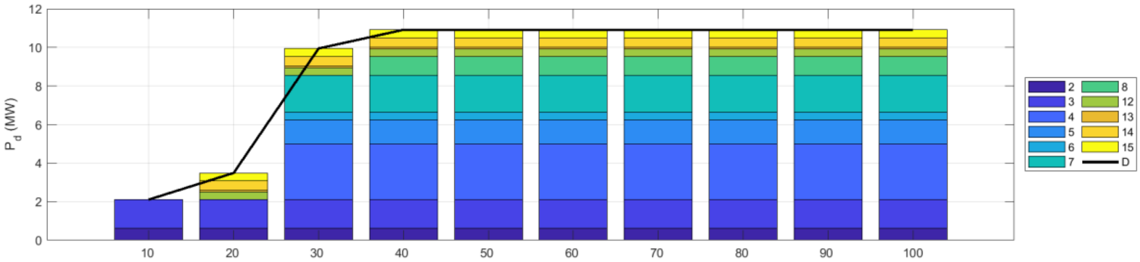

Figure (Exc. Overview) 3: Load-reconnection sequence(standard formulation)

It can be noted how the total demand function can only increase if a new load block is connected or, alternatively, remains the same steady state of the restoration sequence has been reached. Also, can be observed how the total demand in steady state increases to 10.914MW, compared to the total possible demand of 18.903MW. This is due to the nature of the RCSs; once a switch is closed, all new nodes connected to the network are automatically energized along with the load blocks connected to them. However, generators need a previously energized node to start supplying power. This is the reason why the RCS of branch 8-9 causes a demand block of 7.989MW, the generators at nodes 1,5 and 14 are unable to cope with the incoming demand that would result of the connection of the last load blocks without the generation power of generator 10.

Regarding the restoration sequence of the network under Cold Load Pickup conditions a series of things must be considered. Firstly, as it has been previously explained, with this type of demand behavior in the first instants of the reconnection there is an increase in the demand of each block due to a non-diversified factor that causes a sharp increase of the demand which will progressively decrease to the actual demand of the load block. For the case studied, the non-diversified factor has been set to $S_d^U = 1.2$ resulting in the following reconnection sequence.

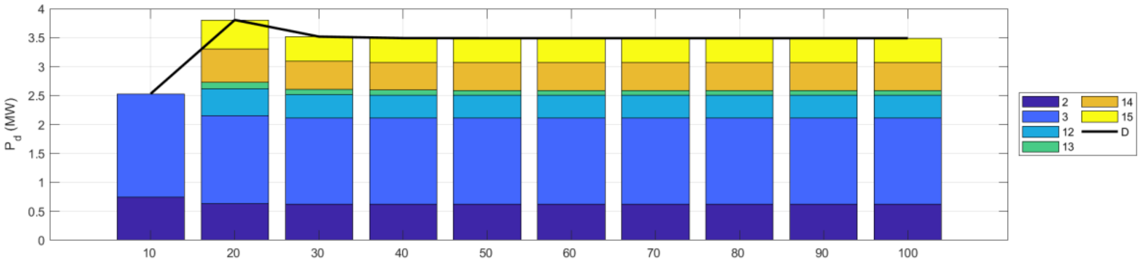

Figure (Exc. Overview) 4: Load-reconnection sequence (CLPU conditions)

The first thing that can be observed, in comparison to the previous case, is that the amount of load blocks that have managed to reconnect is smaller than with a standard load behavior. This is due to the reason previously described, with higher demand peaks (since the initial demand of each block is 1.2 times greater than in normal conditions), the generators that are operative are not capable of handling the demand that would be necessary to reconnect the same amount of load blocks than in standard conditions. In this scenario, there is a demand peak of 3.806MW that decreases progressively until it reaches the steady state at 3.492MW. Note that to achieve the same amount of load reconnection than in standard conditions the maximum generation capacity of the distributed generation units would need to be increased.

Finally, a sensitivity analysis has been conducted to examine the impact of the different parameters of the Cold Load Pickup constraint (these are: the non-diversified load factor, the diversified load factor, the decay rate of load at each instant of time and the time to gain diversity of each load block). The impact of these parameters was studied separately, and lastly, two CLPU cases with different parameters were compared to see the impact they have combined.

4) Conclusions

Along this document, a method to achieve an optimal black-start sequence is presented, formulated as an optimization problem its main aim is to maximize the load recovery comparing two formulations of load-reconnection (standard behavior and Cold Load Pickup behavior). The implementation of this CLPU constraint aims to reflect in a more accurate way the reality of reconnection sequences of power grids. Additionally, a comparison with the standard formulation is made to achieve a better understanding of the factors that need to be considered when implementing the CLPU constraint. Finally, the sensitivity analysis helps identifying the most significant parameters, this is, the ones that can limit most the load-reconnection sequence.

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Nomenclature of optimization problem

Sets & Indexes

$\mathcal{B}$	Set of buses
$\mathcal{D}$	Set of demands
$\mathcal{G}, \mathcal{G}_{BS}, \mathcal{G}_{NBS}$	Set of all generators, BS generators and non-BS generators
$\mathcal{L}, \mathcal{L}_{SW}, \mathcal{L}_{NSW}$	Set of all branches, switchable branches and non-switchable branches
$\mathcal{T}$	Set of time intervals
b	Index of buses
d	Index of demands
g	Index of generators
l	Index of branches
t	Index of time intervals

Parameters

$\beta_{\boldsymbol{d}}$	Priority of re-connection for load *d*
$\mathbf{A}$	Incidence matrix
$\overline{\mathcal{P}_g}$	Maximum generation capacity of generator *g* [MW]
$\overline{\mathcal{P}_l}$	Maximum active power capacity of line *l* [MW]
$\overline{\mathcal{R}_g}$	Maximum ramp-up of generator *g* [MW]
$\overline{S_l}$	Maximum power capacity of line *l* [MVA]
$\underline{\mathcal{P}_g}$	Minimum generation capacity of generator *g* [MW]
$\underline{\mathcal{R}_g}$	Minimum ramp-down of generator *g* [MW]
D_d	Demand of load *d* [MW]
H_g	Inertia of generator *g* [s]
S_{base}	Power base of the system [MVA]
K_g	Governor gain of generator *g* [pu]
T_g	Turbine-governor time constant of generator *g* [s]
Div_d	Time for load *d* to start gaining diversity
S_d^U	Undiversified load factor of load *d*

S_d^D	Diversified load factor of load *d*
α_d	Decay rate of Cold Load Pickup curve for load *d*

Variables

P_b	Active power injection to bus *b* [MW]
P_g	Active power generation of generator *g* [MW]
P_l	Active power flow through branch *l* [MW
$P_{d,t}$	Active power demand of load *d* at time instant *t* [MW]
x_b	Connection status of bus *b* [$\in \{0,1\}$]
x_g	Connection status of generator *g* [$\in \{0,1\}$]
x_l	Connection status of branch *l* [$\in \{0,1\}$]
x_d	Connection status of load *d* [$\in \{0,1\}$]

1 Introduction

1.1 Problem statement

Modern electrical power systems are operated as reliably as possible. Despite the high reliability there are always risks of blackouts or outages occurrences (Feltes & Grande-Moran, 2008). Once fully or partially blacked-out, the power system needs to be restored. The mechanism of response to these blackouts is known as black-start (BS) and consists of connecting generation and load sequentially. The kind of process of black-start sequence used mainly depends on the nature of the blackout.

When facing a partial outage, one way of acting is by looking for assistance in nearby power grids where the tie lines from neighbor's power grids are used to energize the high voltage transmission lines and once a certain amount of transmission subsystems are energized bringing generation back is relatively direct (Feltes & Grande-Moran, 2008). This way of restoring the power of a network is known as top-down restoration as it starts by the initialization of the high voltage lines without the previous energization of the low voltage grids.

On the other hand, another method of restoration of a blacked-out grid consists of a more internal approach where the proper grid throughout BS generating units (hydro or combustion turbine units) restores the service. BS units are generators that can restart themselves with no external help. Among this approach of black-start sequence, the grid is separated in several islanded grids, that are initially restored in parallel by starting-up their BS generating units and feeding loads, and that are subsequently synchronized by re-connecting transmission tie lines. This approach is known as the bottom-up approach as it starts with individual generating units and subsequently, they deliver cranking power to nonblack-start (NBS) generating units so that the overall generating power is maximized (Sun, Liu, & Liu, 2011) and the full capacity of the grid is achieved in the less time possible.

Distribution systems can support the bottom-up approach to restore the power after blackout events. Distribution systems have evolved and are still evolving in the recent years from a passive system, where they performed as intermediates between the transmission lines and the low and medium voltage feeders, to a more active form of networks by the implementation of what it is known as the distributed generation (DG), also known as decentralized generation. This new way of generation benefits from small renewable energy generation whose resources are usually wind, water or sun. The implementation of the DG is gaining importance as it results in advantages such as environmental benefits, improvements in efficiency or transmission and distribution (T&D) line losses (Khamis, Shareef, Bizkevelci, & Khatib, 2013). However, the implementation of the DG must consider the appearance of some problems that must be solved such as the frequency synchronization and stabilization among others.

Moving on, DG can be incorporated in the BS sequence as an improvement in the method of restoration of the power after a blackout. Previously to DG, the optimization of the BS mainly focused on the transmission network as the distribution network is commonly simplified as load blocks, not considering the possible impact of the distribution network (DN) in the restoration of the bulk power system. However, with this new approach of the DG it leaves the possibility of a complementary power restoration system. Starting with the bottom-up approach, thanks to the DG, the DN can also be separated into islands to cooperate with the restoration of the bulk power system, to be able to do that, the DG must have BS capabilities to make the procedure in a stable manner among all islands of the system.

1.2 State of art

Regarding the problem cited before, there have been various ways in which scholars have tried to tackle the issue of finding an optimal BS sequence for auxiliary power grids, in which different methods have been applied as discussed before (Bottom-up & Top-down). The power system restoration process through the BS sequence can be formulated and analyzed as an optimization problem where the main goal is to find the most efficient path to restore the power of a grid when an outage occurs.

Focusing on our issue, the main ways in which it has been modelled is by the application of a mixed integer linear programming (MILP) (Gholami & Aminifar, 2017) or on fewer occasions a mixed integer second-order cone programming (MISOCP) (Ding, Wang, Qu, Wang, & Shahidehpour, 2022). The objective function is to restore the power system as completely and as fast as possible. A first objective function that would focus on maximizing the value of the loads recovered in the restoration sequence among the grid has been proposed by (Ding, Wang, Qu, Wang, & Shahidehpour, 2022) & (Bassey & Butler-Purry, 2020). Whereas other sources as (Cao, Wang, Liu, Azizipanah-Abarghooee, & Terzija, 2017) & (Gu, Zhou, Li, & Liu, 2019) opt for maximizing the generation capacity under the proper grid this is trying to make the NBS units to work in the less time possible, although all the approaches we have seen are with maximization OF there are some references as (Chen, Chen, Wang, & Butler-Purry, 2018) & (Gholami & Aminifar, 2017) suggesting a minimization OF where the main goal is to minimize the load outage costs or in the contrary as expressed in (Patsakis, Rajan, Aravena, Rios, & Oren, 2018) where they model a OF trying to minimize the load shedding of the proper system to incentivize the energization of the grid. The decision of using one type of OF or another depends merely on how the problem and the restoration sequence have been modeled or planned.

All these models try to emulate the real functioning of the actual sequence and due to that they must respond to a series of constraints, some of them come from the proper nature of the grid analyzed, and others to respect the laws of physics involved in the process. Regarding constraints that involve the well-functioning of the grid some of those that must be considered are the following:

- Voltage limits of the grid: The buses cannot withstand any level of voltage and during the restoration sequence it must be limited to ensure a safe margin of voltage.
- Power flow: Another limiting factor regarding the grid is the level of power that can flow through the lines, depending on the line of the grid the limit of each line must be considered and modelled as a constraint. This is important in the sequence as it marks the level of energization of the system.
- Frequency: To ensure the stability of the grid during the restoration it is of vital importance to state a constraint that would set frequency limits.

Constraints are not only applied to the functioning of the whole network, but they also can affect specific components, for example the generators. Generators are classified into BS or NBS units (includes the ones of the generation system as well as the DG) , this distinction needs to be expressed as a constraint as not all the generators have BS capacity. For the ones that count with the BS capacity the proper functioning of the generators must be considered that is the active and reactive power of each BS unit has to be under the operation margins of the proper generator. The starting of system also includes a series of constraints such as the generator ramp, as at the beginning of the sequence, in the interval of time Δt, the generation capacity of the BS unit won't be at full capacity. This can be observed in the graphic of the generator startup curve shown in (Sun, Liu, & Zang, 2011).

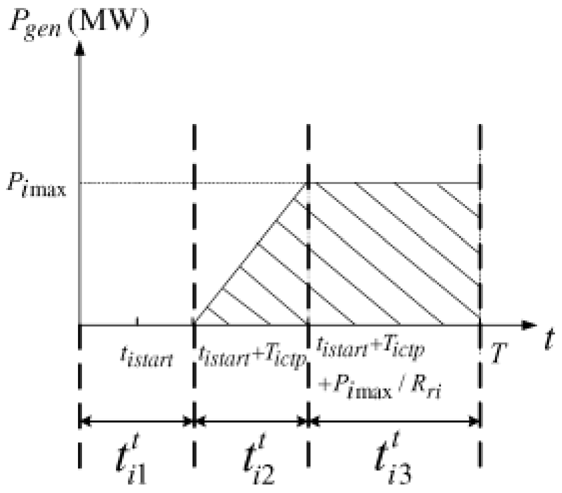


Figure 1: Generator startup ramp and generation capability.

Another constraint that must be taken into account when modeling the problem is the Cold Load Pickup (CLPU). This issue refers to a temporary overload of the system as load groups that tend to consume power non-simultaneously start consuming power simultaneously. When the outage occurs and the restoration of the load block starts, more loads of the load block would need of the power supply so the aggregate demand would be greater than in normal working conditions.

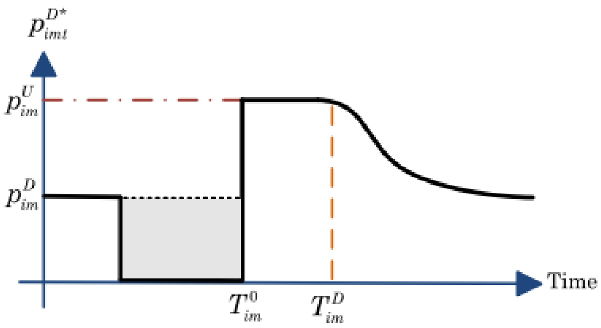


Figure 2: Cold Load Pickup(CLPU) restoration sequence.

As the graph shows (Gholami & Aminifar, 2017), the sequence of the CLPU restoration can be represented as a first order dynamic model prior to which there is a step that represents the outage and the power recovery. This issue can cause major problems and that is why it must be modeled, due to the increase in load and hence the active power it can provoke the frequency to go under the minimums permitted.

According to (Liu, Lin, Wen, & Ledwich, 2013), an acceptable range of load pickup must be found to maintain the frequency constraint in a safe interval of ±0,5 Hz that is what a normal operating bulk system permits although some control devices adjust the limits to even safer margins.

1.3 Objectives and methodology of the project

With all the stated in previous points and starting from an already existing model this project aims to determine the optimal black-start sequence of distributed generation within an islanded part of a distribution system. Regarding this, the main objectives to achieve are the following:

- Firstly, the main objective of the project will be to implement in the current optimization problem the sequence of the CLPU to extend the current model.
- Once implemented, we will continue by verifying the well-functioning of the new model and its impact on the BS sequence. The model to determine the BS sequence will be applied to different practical cases. This is, it will be checked with different grid configurations.
- Finally, to conclude the analysis we will conduct a sensitivity analysis on how the different parameters of the CLPU model affect the BS sequence.

Along the realization of the project these objectives will be fulfilled progressively until the completion of the entire project. To do so, following a schedule diligently is crucial to achieve the objectives in their proper deadlines. The following monthly methodology chart shows the main deadlines of the project objectives.

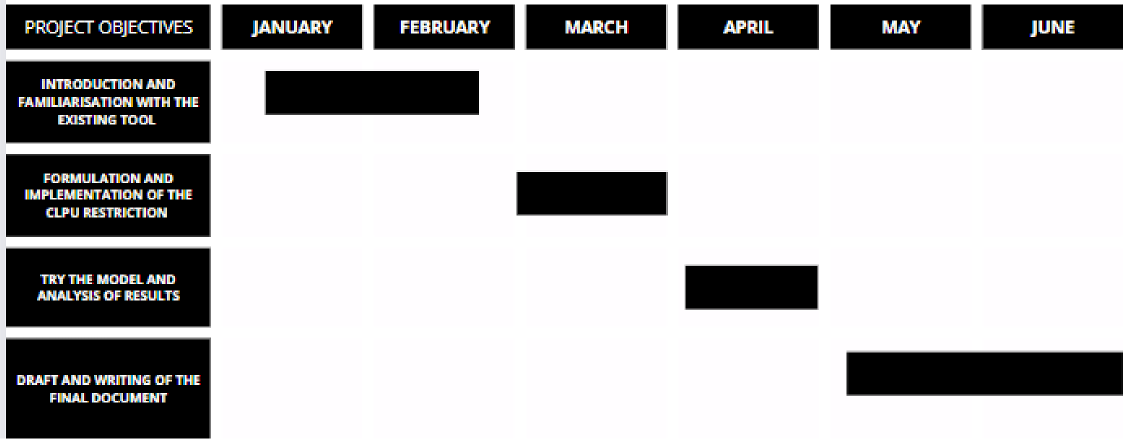


Figure 3: Timetable and methodology of the project.

The initial familiarization with the current model will be crucial to formulate and implement the following constraints of the CLPU to the existing tool. To do so, the programming tool it will be used is Matlab, which consists of a user-friendly program that will enable the formulation of the optimization program and to reach a set of results that will be later analyzed. To code this optimization problem a specific toolbox of the program will be needed, this is, matlab optimization toolbox.

1.4 Sustainable developmental goals

As a part of the development of the project, and to contribute in some way to the sustainability of the planet, the project will need to be aligned with some of the goals presented by the UN for sustainable development. These goals are the ones shown below.

Figure 4: Sustainable developmental goals.

Analyzing the different goals presented and seeing the nature of this project, the main objectives to which this project could help would be the goals more connected to an affordable and sustainable energy (these are goals number 7 & 11). This project could contribute to the cheapening of energy in cases of outages in cities or communities making energy more accessible.

In another way, the optimization of this BS sequence model could also contribute to the achievement of having clean energy along our power system

2 Optimal Black-Start Sequence Formulation

In the following sections the mathematical formulation related with the well-functioning of the program will be stated.

2.1 Formulation of the problem Overview

As it has been stated previously, the black-start sequence has been usually formulated as an optimization problem where the optimal start-up sequence of the DGs is ought to be found. Figure 5 illustrates the load re-connection at different instants of time depending on the priority of each demand block.

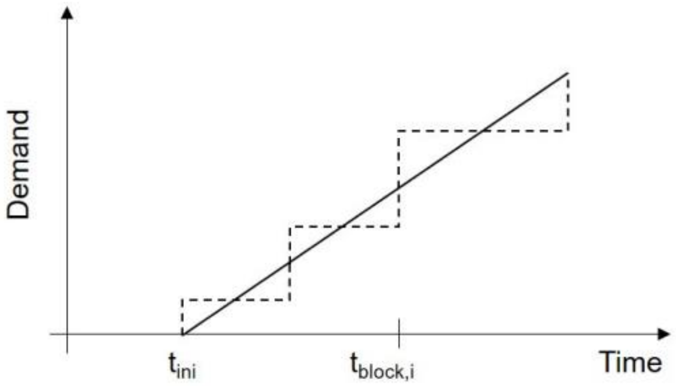


Figure 5: Load re-connection sequence explanation.

Note that there is only one generation unit that has BS capacities meanwhile the rest of the generators of the network require of an already energized grid to synchronize with.

2.2 Objective Function

In the field of power system restoration, the primary objective of our black-start sequence tool is to re-energize the grid efficiently, to do so, an objective function to maximize the load recovery at any instant of the restoration process is formulated:

$$max \sum_{d \in D, t \in T} \beta_d \cdot P_{d,t} \qquad (1)$$

2.3 DG-related constraints

Regarding the distributed generation of the grid, the constraints that regulate the functioning of generators are the following. Firstly, (2) imposes that during the restoration period a generation unit can be started up but not shut down. (3) reflects the ramping up and down of the generation units. (4) shows the minimum generation constraint meanwhile (5) states the maximum generation capacity of each generator.

$$x_{g,t} - x_{g,t-1} \geq 0 \quad g \in \mathcal{G} \qquad (2)$$

$$-\underline{R_g} \leq P_{g,t} - P_{g,t-1} \leq \overline{R_g} \quad g \in \mathcal{G} \tag{3}$$

$$P_{g,t} \geq \underline{\mathcal{P}_g} \cdot x_{g,t} \quad g \in \mathcal{G} \tag{4}$$

$$P_{g,t} \leq \overline{\mathcal{P}_g} \cdot x_{g,t} \quad g \in \mathcal{G} \tag{5}$$

2.4 Load-Related Constraints

2.4.1 Standard Demand Formulation

Regarding the standard demand formulation of the loads, (6) shows that the actual load re-connection order is imposed by the distribution system operator (DSO), $X_{d,t}$, that is determined by the priority of each load. Independent of the re-connection order of the loads, (7) states that loads can only be connected to the grid and not disconnected. The mathematical formulation is as follows:

$$x_{d,t} \geq X_{d,t} \; d \in \mathcal{D} \tag{6}$$

$$x_{d,t} - x_{d,t-1} \geq 0 \tag{7}$$

2.4.2 Cold Load Pickup Formulation

Regarding the formulation of the CLPU constraint, a linear CLPU model will be implemented to model the behavior of load at different time steps, following a given CLPU curve. The curve it will be used to implement the proper CLPU constraint is represented below in Figure 6 and has been taken from (Chen, Chen, Wang, & Butler-Purry, 2018).

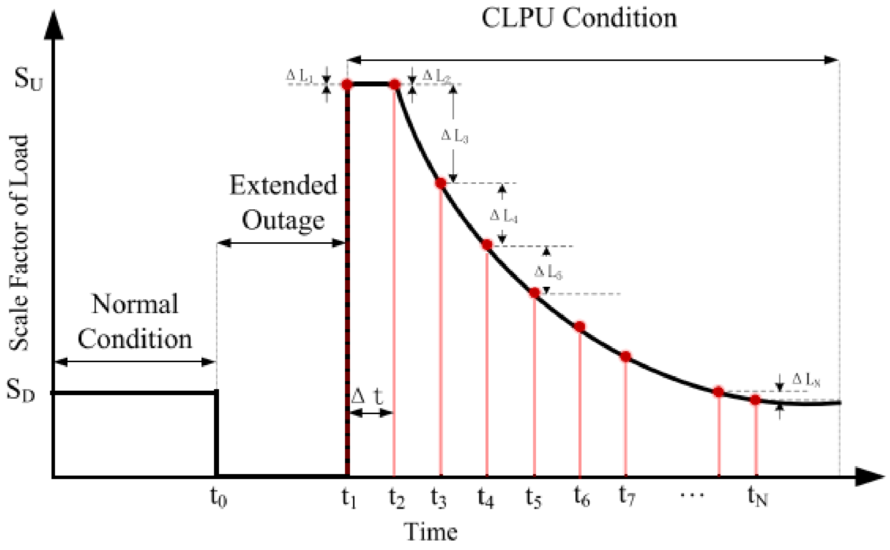


Figure 6: CLPU curve to be implemented in the model.

The figure shows a delayed exponential CLPU curve in which t_0 represents the instant of the outage and t_1 shows the moment of time in which the power is restored. Due to the outage, a loss of diversity is produced and therefore the undiversified loading factor at the moment of time t_1 is S^U. After a period, the load starts gaining diversity and decreases as an exponential function until the post-outage diversified load factor S^D, which normally stands for the pre-outage level of the load. Given the CLPU curve we can calculate the demand at each sampling time, this is, at each instant t_k where there is a unique sampling interval of length Δt. Assuming a total of *N* intervals in the CLPU curve. It can be noted $L_d(k)$ as the scale factor of the demand at an instant kth and Δ $L_d(k)$ as the difference between two consecutive scale factor instants of the curve, kth and (k-1)th. In this case study, the function $L_d(k)$ will be replaced by P_d as it will be the active power what will be modelled with the CLPU conditions. The expressions of the stated functions are represented in the following equations (8) and (9):

$$\Delta L_d(k) = \begin{cases} 0 & k = 1 \\ L_d(k) - L_d(k-1) & 1 < k \leq N \end{cases} \qquad (8)$$

Note that $\Delta L_d(k)$ should be calculated prior to the distribution system restoration algorithm.

For the CLPU delayed exponential curve, $L_d(k)$ can be calculated as follows:

$$L_d(k) = (S^D + (S_d^U - S_d^D)e^{-\alpha_d \cdot c_{d,k}})u(c_{d,k}) + S^U(1 - u(c_{d,k}), 1 < k \leq N \qquad (9)$$

$$c_{d,k} = (k-1)\Delta t - Div_d \qquad (10)$$

$$u(i) = \begin{cases} 1, & i > 0 \\ 0, & i \leq 0 \end{cases} \qquad (11)$$

The $c_{d,k}$ function represents the duration between the instant where the demand starts gaining diversity and the kth time step. Having introduced all this, the final formulation of the CLPU constraint considering all the above is the following:

$$P_{d,t} = P_d \cdot \left(S^U x_{d,t} - \sum_{k=1}^{t} \Delta P_d(k) x_{d,t-k+1} \right), \qquad d \in \mathcal{D}, \qquad t \in \mathcal{T} \qquad (12)$$

Note that P_d stands for the pre-outage active power and that $\Delta P_d(k)$ is calculated with the equations above (Equations from (9) to (11)).

2.5 Network-related Constraints

2.5.1 DC Power Flow (DC-PF)

In the model proposed, the power flow constraints are based on a so-called direct current power flow (DC-PF) approximation of the power flow for being a linear formulation. DC-PF assumes that the network is inductive, and voltages are close to their nominal values. Other linear formulations exist, more suitable for distribution networks, but this is not the main focus of the project. To implement this in the model, the following formulation will take place. (13) will control the power injections in all buses as well as the active power flow through branches of the grid. (14) states the active power injection balance at each bus, where $\mathcal{D}_b$ and $\mathcal{G}_b$ stand for the subsets of distributed generation and demands at bus b. Finally, the active power flow through the branches of the grid is limited to the connection of the proper branches, this is, it is only possible to have a power flow through a branch that has been connected prior to the power flow, as it is noted in (15).

$$\mathbf{P}_{l,t} = \mathbf{A} \cdot \mathbf{P}_{b,t} \qquad (13)$$

$$P_{b,t} = \sum_{g \in \mathcal{G}_b} P_{g,t} - \sum_{d \in \mathcal{D}_b} P_{d,t} \qquad b \in \mathcal{B} \qquad (14)$$

$$-x_{l,t} \cdot \overline{\mathcal{P}_l} \leq P_{l,t} \leq x_{l,t} \cdot \overline{\mathcal{P}_l} \qquad (15)$$

2.5.2 Radially Operated Meshed DSs

Although the grid topology may be meshed, the grid is operated radially mainly due to protection and economic reasons. The formulation must thus consider the radial operation by imposing a series of logical constraints that will be announced next. (16) states that the generator with BS capabilities will determine the connection status of the bus where is connected, meanwhile NBS generators to start functioning the bus where they are connected must be energized prior to the functioning of these, this is reflected in (17). Where $\mathcal{B}_g$ stands for the subset of buses that have generation capabilities. (18) represents that the buses at the end of switchable lines will only be energized if the line is switched on before, no way will an end-line bus of a switchable line be energized without a prior connection of the line to the grid, in the same way buses that are connected with non-switchable lines will automatically be energized when it exists an active power flow along the branch, this is shown in (19). Where $\mathcal{B}_l$ notes the subset of buses that are end-lines of branch l. To continue, (20) notes the existing relation between the status of a bus and its corresponding demand connected to it. This is, if a bus is energized the demand will be connected immediately to the grid, where $\mathcal{B}_d$ is the subset of buses that contain the demands. Finally, (21) states that there is an existing boundary in the radially operated DS problem, considering there is only one generator with BS capacities.

$$x_{b,t} = x_{g,t} \quad g \in \mathcal{G}_{BS}, b \in \mathcal{B}_g \tag{16}$$

$$x_{b,t} \geq x_{g,t} \quad g \in \mathcal{G}_{NBS}, b \in \mathcal{B}_g \tag{17}$$

$$x_{l,t} \leq x_{b,t} \; b \in \mathcal{B}_l, l \in \mathcal{L}_{SW} \tag{18}$$

$$x_{l,t} = x_{b,t} \; b \in \mathcal{B}_l, l \in \mathcal{L}_{NSW} \tag{19}$$

$$x_{b,t} = x_{d,t} \; d \in \mathcal{D}, b \in \mathcal{B}_d \tag{20}$$

$$\sum_{b \in \mathcal{B}} x_{b,t} - \sum_{l \in \mathcal{L}} x_{l,t} = 1 \tag{21}$$

3 Analysis of Results

3.1 Description of the test system

All the mathematical formulation of the problem presented above will be tested in an example grid that is shown in Figure 7, (Tomás, García, & Sigrist, 2023). This network counts with the following characteristics: The distribution system counts with four generation units, where only one of them has BS capacities, these are located on nodes one, five, ten and fourteen, being the one located in bus 1 the DG unit with BS capacities.

Regarding the network reduction and the connectivity of the branches, branches 3-4, 3-12, 7-8 and 8-9 are equipped with RCS. The total generation capacity of the network is 26MW, that is distributed with a maximum generation capacity of 5MW in DG units 5 and 10 meanwhile generators 1 and 4 have a maximum generation capacity of 8MW, this generation capacity is likely to be modified along the different case studies. The load and generation parameters will be specified further ahead.

Different case studies will be carried out with this example grid, where different aspects of the formulation will be changed. By doing this it will be possible to compare the different results obtained. These case studies will analyze the impact of the selection of the load behavior, this is, the load can behave as standard demand or as the CLPU load behavior. To analyze more deeply the CLPU behavior different cases of the CLPU formulation will be run by conducting a sensitivity analysis with the different parameters.

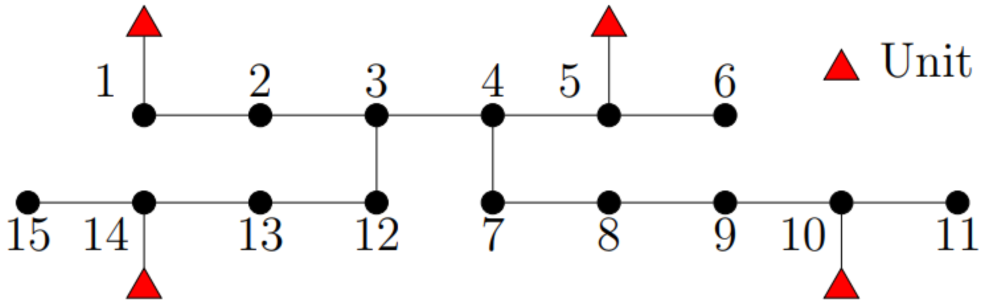


Figure 7: Case study grid.

As has been previously said, the grid parameters that correspond to the load demand, the DG system or the general technical parameters of the network will now be presented. Note that the pre-outage demand amounts to 18.903MW and that the power rating for the branches parameters stands for $S_{BASE} = 30MW$. Regarding the time span of the simulation, it will consist of ten instants with a step time between them of 10 minutes. The parameters of each branch of the network and the load demands are represented in Table 1 below (note that each demand is connected to the bus "To" of each branch of the grid):

Line	From	To	**R (p.u)**	**X (p.u)**	**P_d (MW)**	**Load Priority**
L1	1	2	0.00315	0.007507	0.624	100
L2	2	3	0.00033	0.001849	1.485	100

L3	3	4	0.00667	0.030808	2.874	200
L4	4	5	0.00579	0.014949	1.266	300
L5	5	6	0.01414	0.36549	0.399	300
L6	4	7	0.008	0.036961	1.914	200
L7	7	8	0.009	0.041575	0.969	500
L8	8	9	0.007	0.032346	0.639	400
L9	9	10	0.00367	0.01694	0.84	400
L10	10	11	0.009	0.415754	6.51	400
L11	3	12	0.0275	0.127043	0.396	500
L12	12	13	0.315	0.081405	0.087	500
L13	13	14	0.03965	0.102984	0.483	500
L14	14	15	0.01061	0.004153	0.417	500

Table 1: Grid and Demand parameters

To continue, the DG units also have a series of parameters that must be considered, these are listed in the following table:

Bus	$\overline{\mathcal{P}_g}$**(MW)**	$\underline{\mathcal{P}_g}$**(MW)**	$\overline{\mathcal{R}_g}$**(MWh/h)**	$\underline{\mathcal{R}_g}$**(MWh/h)**	Type
1	7.5	0.1	45	-45	Bs generator
5	5	0.1	30	-30	Common generator
10	5	0.1	30	-30	Common generator
14	7.5	0.1	45	-45	Common generator

Table 2: DG unit's parameters

Note that the $\overline{\mathcal{P}_g}$ will always remain the same except when it is indicated otherwise.

3.2 Analysis of different case studies

3.2.1 Standard Demand Load-restoration

As to start, in this subsection it will be analyzed the impact that has in the network restoration the standard demand behavior. This will be considered the base case from which further changes will be added to the network.

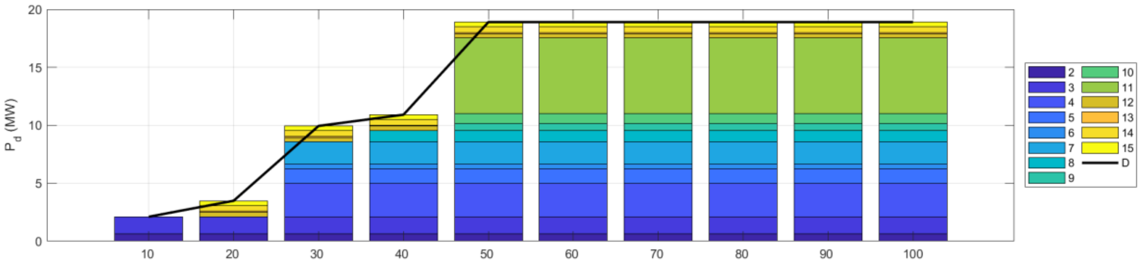


Figure 8: Load-Reconnection sequence (standard load formulation).

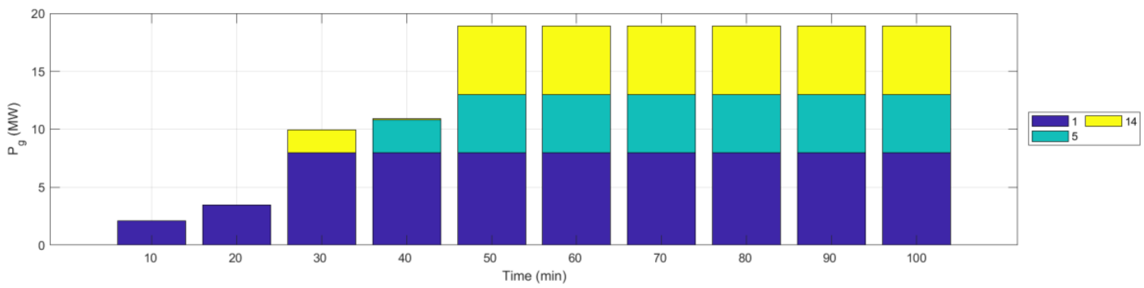


Figure 9: DG units start-up sequence (standard load formulation).

In Figure 8 it can be seen how the load-reconnection evolves along the timelapse, it shows both the individual load-reconnection, this is, which demand is connected in each instant of time as well as the total demand recovered. Note that the total demand recovered amounts to 18.903MW, which indicates that all loads have been able to be recovered. Due to the excess in generation capacity of the DG units all RCSs have been able to get turned on and consequently the entire network has been able to get energized. Note how the total demand function can only increase if new load blocks get connected or, alternatively, remain the same if the steady state of the reconnection sequence has been reached.

Figure 9 shows the start-up sequence of the generators, where generator 1 is the one that has the BS capacity and as it is shown is the one that starts the reenergization. Only three generators from the four available are used for the post-outage energization of the grid.

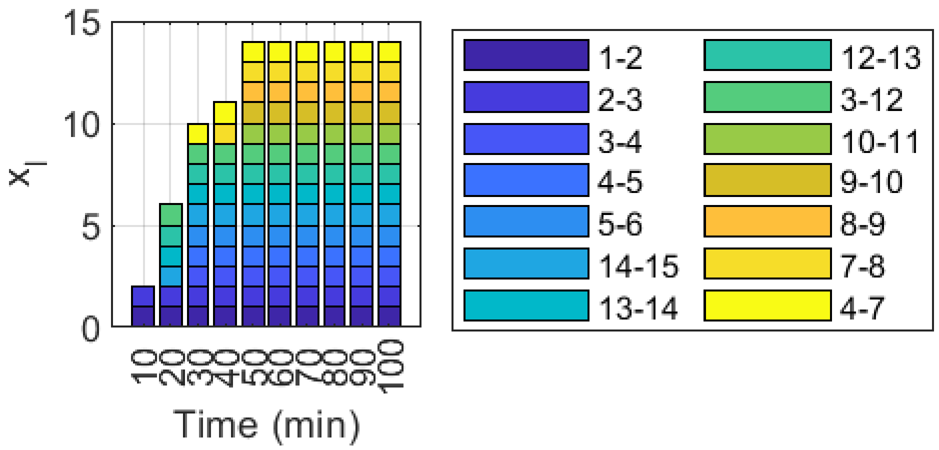


Figure 10: Binary Variable of branch connections.

Figure 10 illustrates the reconnection of the branches through the recovery sequence, from the hole grid all branches are reconnected within a time span of 50 minutes as it can be observed.

3.2.2 CLPU model Load-restoration

In this section, the behavior of the load is changed as stated in previous point 2.4.2. To do so, we must start by adding a series of parameters that will define the behavior of the load-reconnection sequence. The values of this parameters will now be presented in Table 3 down below:

Load	S_d^U	S_d^D	α_d	$Div_d[min]$

D1	1.2	1	0.3	2
D2	1.2	1	0.3	2
D3	1.2	1	0.3	2
D4	1.2	1	0.3	2
D5	1.2	1	0.3	2
D6	1.2	1	0.3	2
D7	1.2	1	0.3	2
D8	1.2	1	0.3	2
D9	1.2	1	0.3	2
D10	1.2	1	0.3	2
D11	1.2	1	0.3	2
D12	1.2	1	0.3	2
D13	1.2	1	0.3	2
D14	1.2	1	0.3	2

Table 3: Parameters CLPU

Note that the values of the parameters have been randomly selected between a range of possible values. And that the value of the parameters common to both cases will remain equal to the ones presented above for now.

The first step in the load-reconnection sequence with the CLPU approach is to build up the CLPU curve applying the parameters presented above. To do so, a time interval between consecutive points of $\Delta t = 10\ min$ will be used as well as a number of intervals of $N = 5$. The curve will now be presented in Figure 11.

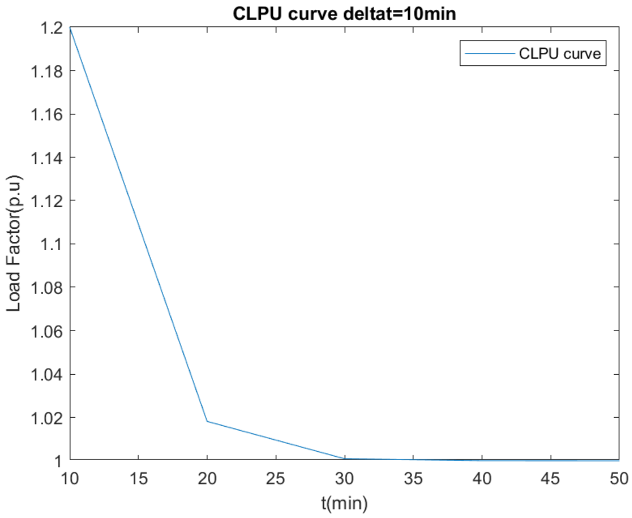


Figure 11: CLPU curve for Δt=10min

State how the curve is a lineal approximation between consecutive instants of time to the one presented above. Once the curve has been built up, the load sequence can be performed, and the result of the reconnection is as follows:

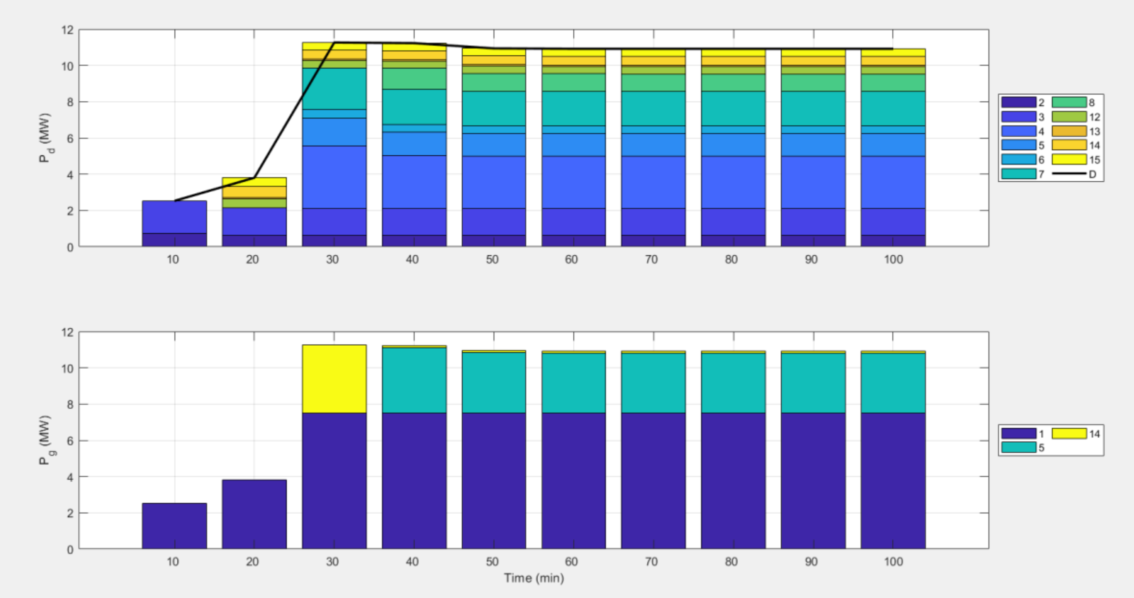


Figure 12: Load-reconnection in CLPU conditions.

As it is shown in Figure 12, when applying the CLPU conditions to the restoration sequence the number of demands that have been able to reconnect in comparison to the initial case is much lower. This is because the amount of power needed to satisfy the demands in the first instants of the reconnection due to the non-diversified load factor is much higher than in the base case, and with a pgmax of 7.5MW is not sufficient to cover the demand of the different loads that would connect if the RCS of branch 8-9 was closed. Due to this, the connected lines remain as shows Figure 13.

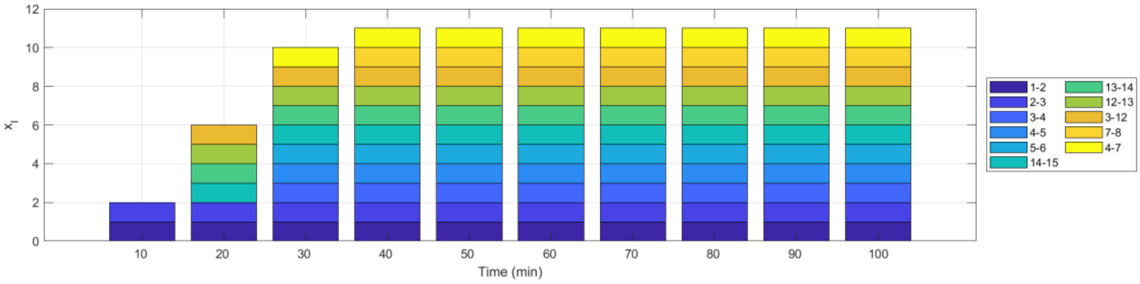


Figure 13: Branch connections in CLPU conditions.

Regarding the reconnection sequence of Figure 12, and in comparison with the initial case where the loads got connected with a fixed demand level, here it can be observed how the different loads start with a demand level much higher than their real demand in permanent regime. This reduction is produced following the curve presented in Figure 11. In this case, there is a final demand in the steady state of 10.914MW while there is a peak of demand in the transitory regime of 11.262MW.

3.3 Load formulation comparison (Pgmax=5MW in all generators)

To broaden the case study the pgmax of the generators 1 and 14 will be decreased to 5MW to see the impact it has on the load restoration with both formulations presented above.

3.3.1 Standard demand Load-restoration sequence

It is interesting to see what happens if the generation capacity decreases substantially as it can affect directly to the load-reconnection. The restoration sequence with these conditions is presented below.

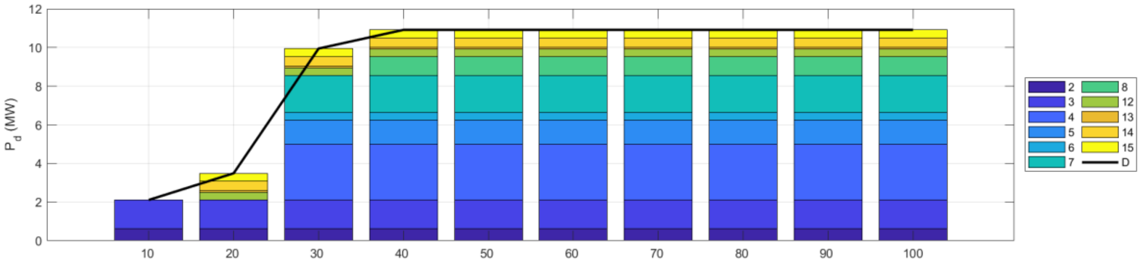


Figure 14: Load-reconnection sequence in standard conditions (pgmax=5MW)

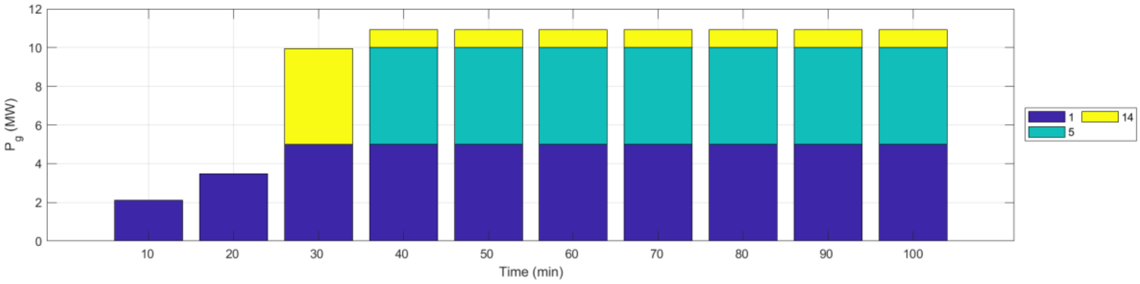


Figure 15: Distribution of generation among generators in standard conditions (pgmax=5MW)

Note that the total demand recovered amounts to 10.914MW, that is 7.989MW less than the pre-outage demand, which indicates that not all loads have been able to be recovered even though the maximum generation capacity is higher than the pre-outage demand. This happens due to the nature of the RCSs, once a switch is turned on all the buses that get connected to the network are immediately energized as well as the loads connected to them. And as the generator cannot start to function until the bus it is connected to is energized, it is impossible to supply enough power with the already functioning generators. This is the main reason why the RCS of branch 8-9 cannot be turned on creating a load block of the remaining 7.989MW.

3.3.2 CLPU formulation load-reconnection sequence

Maintaining the same values of the parameters presented in section 3.2.2 it will be analyzed how the load-reconnection sequence changes with the decrease in generation capacity. The restoration sequence is presented in Figure 16.

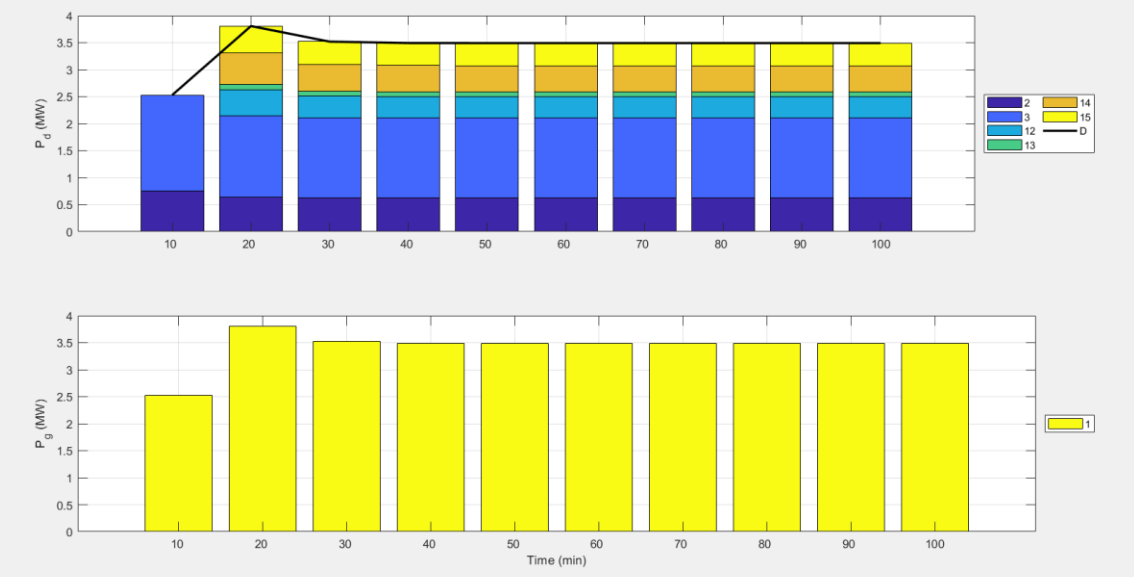


Figure 16: Load-reconnection & DG sequence in CLPU conditions (pgmax=5MW)

It can be observed how the number of load blocks has decreased considerably from the case of standard demand formulation with pgmax=5MW. In this case, not only RCS of branch 8-9 cannot get turned on but the ones in braches 7-8 and 3-4 neither having a total demand in permanent regime of 3.492MW. As it is following the CLPU formulation, the reconnection has a transitory regime where the total demand reaches 3.8068MW and starts decreasing until reaching the steady state.

In this case the number of lines connected it is maintained during practically the entire reconnection as only one RCS is switched on. The lines reconnection is presented below.

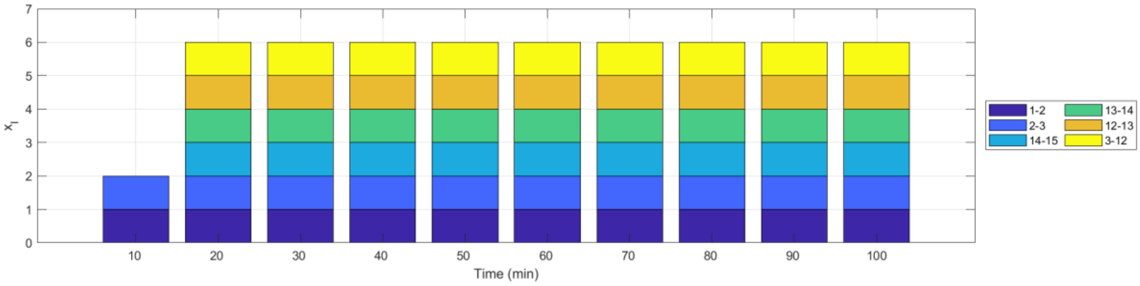


Figure 17:Brach connections in CLPU conditions (pgmax=5MW)

3.4 Sensitivity Analysis

In this section of the document, once the CLPU constraints have been implemented and tested a sensitivity analysis is going to be conducted where it will be studied how changes in the different parameters of the CLPU affect to the load-reconnection sequence as well as to the proper CLPU curve.

Note that in each subsection, the parameters that are not object of study will remain invariant in relation to the main case study.

3.4.1 Impact of load factors

As a first object of study, it will be tested the impact of varying both the non-diversified load factor and the diversified load factor. The new values of both parameters will now be shown Table 4.

	S_d^D	S_d^U
Base case study	1	1.2
Sensitivity analysis	1	1.7

Table 4: Load factors values

Regarding the impact of both factors on the CLPU curve, the curve decreases at the same speed as the presented in the previous points. As scalars, the only contribution they have to the curve is of movement in the vertical axis, comparison of curves is presented in Figure 18.

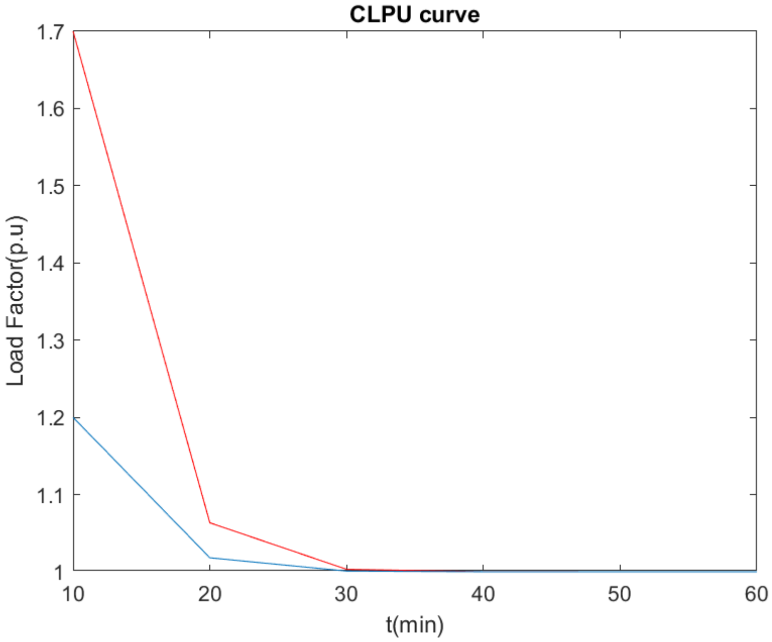
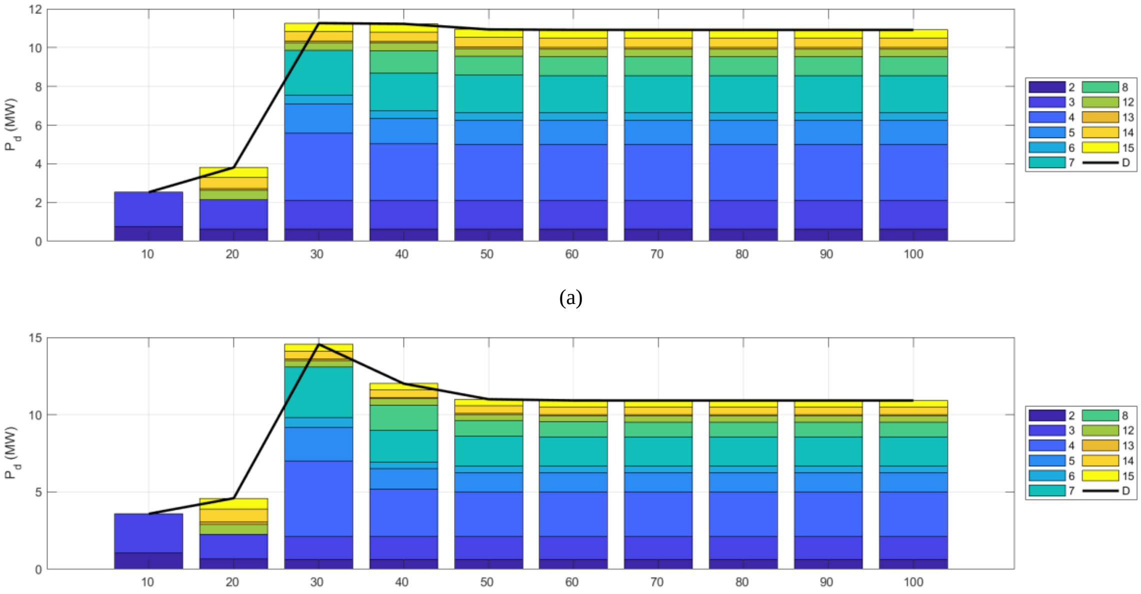


Figure 18: Comparison of CLPU curves(load factor parameters)

Now, it will be compared the load reconnection sequence with both load factors. It can be noted that with the new values of the non-diversified load factor the sequence in each case will differ mainly in the transitory regime as in the case of $S_d^U = 1.7$ it will reach higher peaks of demand as it will be shown in Figure 19.



(a)

(b)

Figure 19: Load-reconnection comparison: (a) $S_d^U = 1.2$ *(b)* $S_d^U = 1.7$

As can be seen in both figures, the restoration sequence changes significantly from one case to another. In case (a), which corresponds to the base case, it can be observed how the peak of demand is not much bigger than the steady state due to the non-diversified load factor that is close to one. While in case (b), by increasing substantially the non-diversified load factor the peak of demand increases a lot in comparison to case (a). Being the peak of demand for cases (a) & (b) is of 11.26MW and 14.55MW respectively. It is remarkable that an increase of 0.5 in the non-diversified load factor produces an increase of 3.29MW.

3.4.2 Impact of duration to gain diversity parameter

To continue with the sensitivity analysis, it will now be tested the impact it has in the CLPU restoration sequence the parameter of “time to gain diversity” (Div_d). Set to 2min in the base case the new value is presented below.

	$Div_d[min]$
Base case value	2
New value	10

Table 5: Time to gain diversity parameter values

This parameter represents the time that the demand is fixed in the non-diversified load factor, once this timelapse has passed the demand starts decreasing following the CLPU curve. The comparison of both curves is presented in Figure 20.

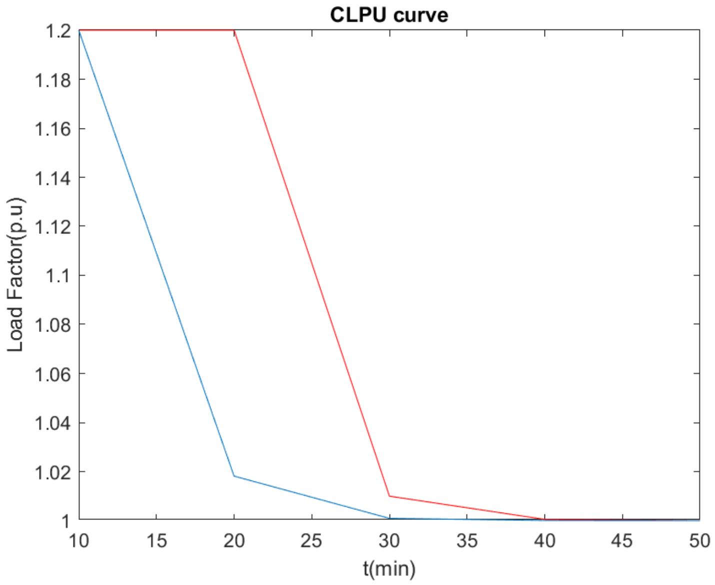
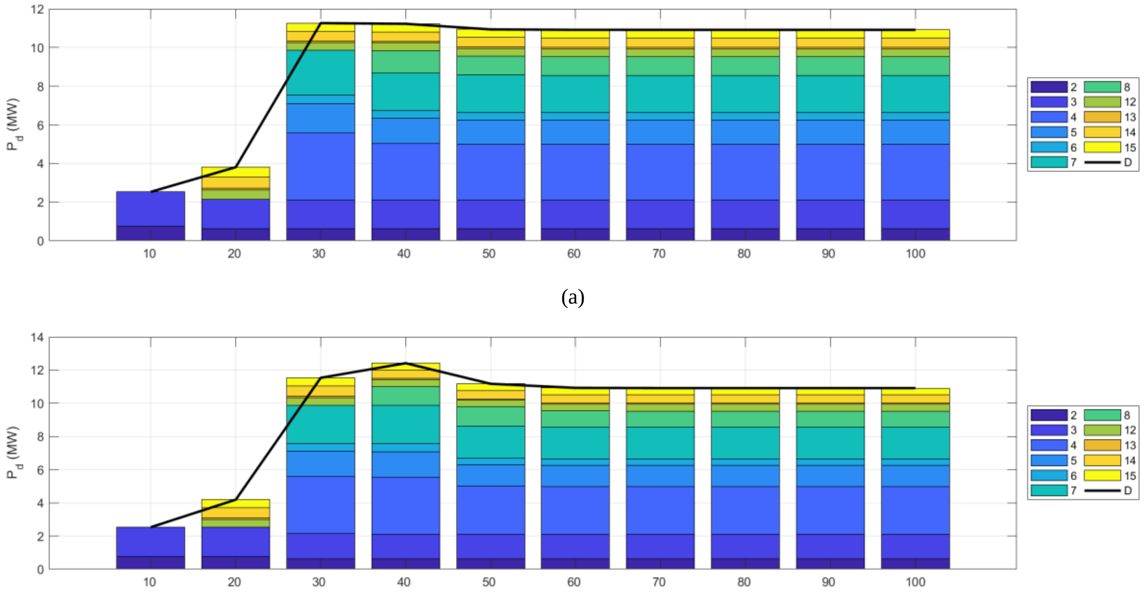


Figure 20: CLPU curve comparison (Time to gain diversity parameter)

The red curve is displaced to left by 10 minutes which is the parameter demand, the load-reconnection will be postponed by one time instant in comparison to the restoration sequence with $Div_d = 2min$. Both sequences will now be presented.



(a)

(b)

Figure 21: Load-reconnection sequence: (a)Div=2min (b)Div=10min

As represented in Figure 21 the peak of demand with a $Div_d = 10min$ is displaced an instant of time, where in the base case it occurs at time instant 30 and in the case of $Div_d = 10min$ the peak occurs at time instant 40. Note that due to the increase in time to gain diversity of the load blocks the demand peak not only shifts in time but it also increases to 12.41MW as a bigger accumulation of demand is produced.

3.4.3 Impact of the decay rate of CLPU curve

Following with the analysis, the next parameter to study refers to the decay rate of the CLPU curve (α_d) which will mainly be reflected in changes in the CLPU curve as changes in the slope of the curve will be observed. Below the new value of the parameter α_d.

	α_d
Base case value	0.3
New value	0.1

Table 6: Decay rate parameter values

As the parameter name suggests, it is the responsible of fixing the CLPU curve slope between two instants of time where a higher value of α_d is reflected as a higher slope of the curve as it can be stated in Figure 22.

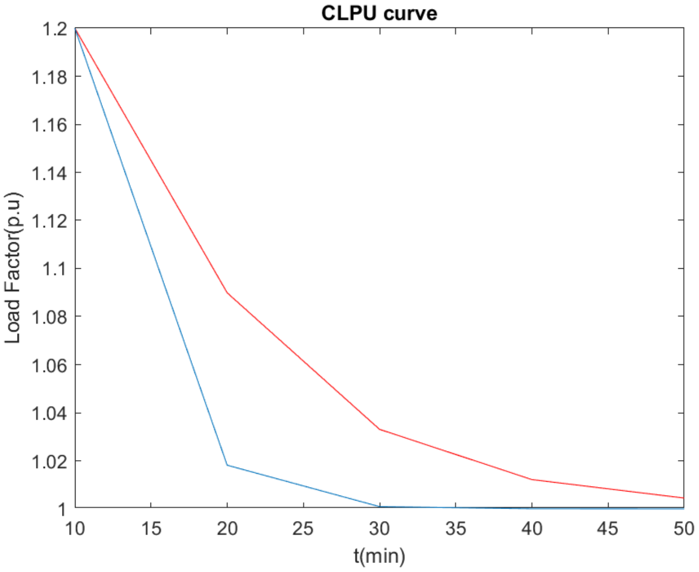
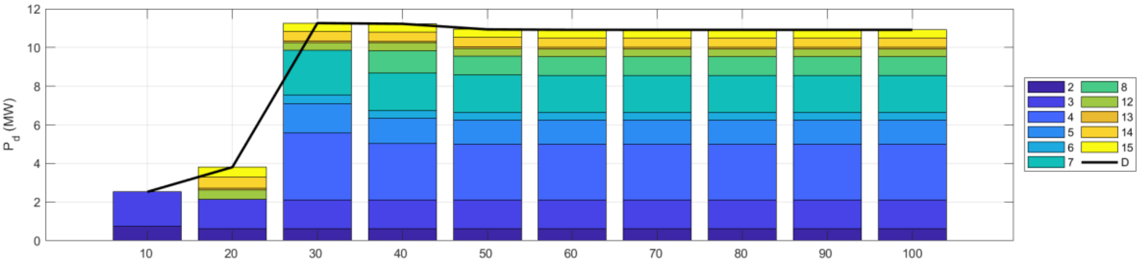
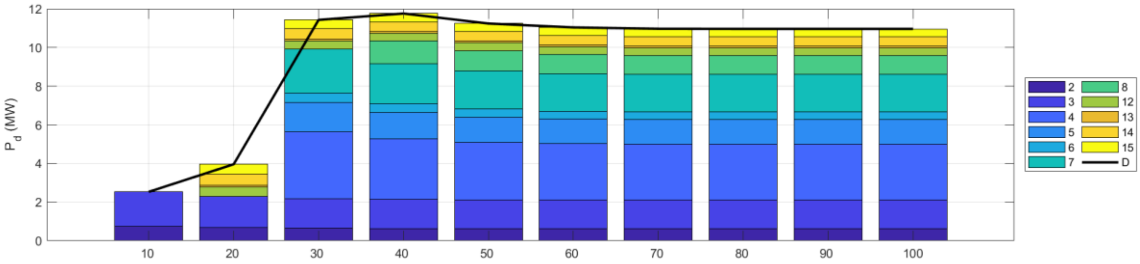


Figure 22: CLPU curve comparison (Decay rate parameter)

As it can be seen in Figure 22 the curve of $\alpha_d = 0.1$ has less slope and consequently the curve decreases less between two consecutive instants. It can be observed how at the last time instant the curve of $\alpha_d = 0.1$ has not been able to reach the diversified load factor meanwhile the curve of $\alpha_d = 0.3$ reaches the diversified load factor in the second time step. This will be reflected in the permanent regime of the load-reconnection sequence which will be shown next in Figure 23.



(a)



(b)

Figure 23: Load-reconnection sequence: (a) α_d=0.3 (b) α_d=0.1

Comparing both cases, it is seen how in case (b) due to a less steep curve the peak of demand is reached later in the reconnection sequence. Although the main thing to highlight is how the permanent regime of case (b) differs from the one of case (a), this is mainly because the CLPU curve of case (b) shown in Figure 22 does not reach the $S_d^D = 1$ consequently the total demand in permanent regime for case (b) increases to 10.962MW instead of the 10.914MW of case (a).

3.4.4 Comparison of CLPU cases

Once every parameter of the CLPU constraints have been analyzed separately, a comparison between to different cases of CLPU will be proposed and analyzed to study how do the parameters of the CLPU act. To do so, two cases with different parameters will be presented.

	S_d^U	S_d^D	Div_d	α_d
Case 1	1.8	1	7	0.2
Case 2	1.6	1	10	0.15

Table 7: Parameters of CLPU condition for both cases

With the parameters presented above a simulation for each case will be runed and results will be presented. The CLPU curves that each case will follow during the restoration sequence are now shown.

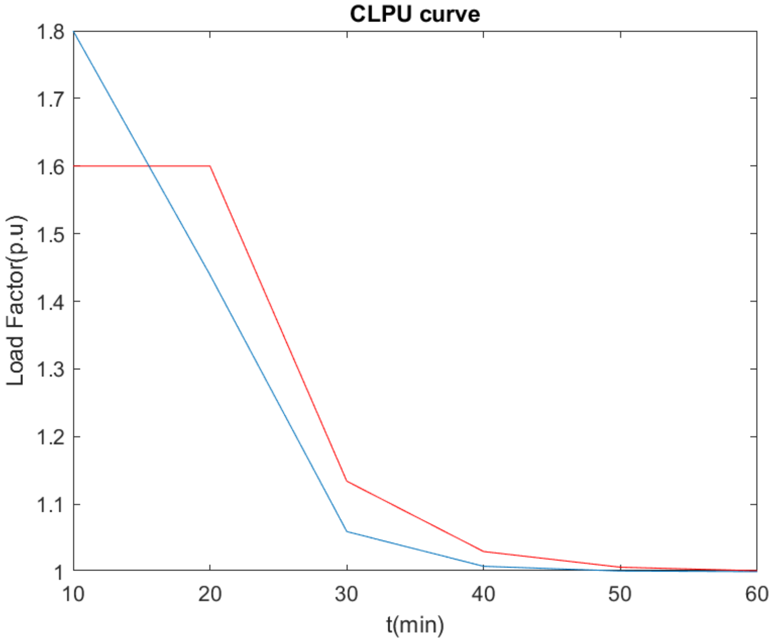
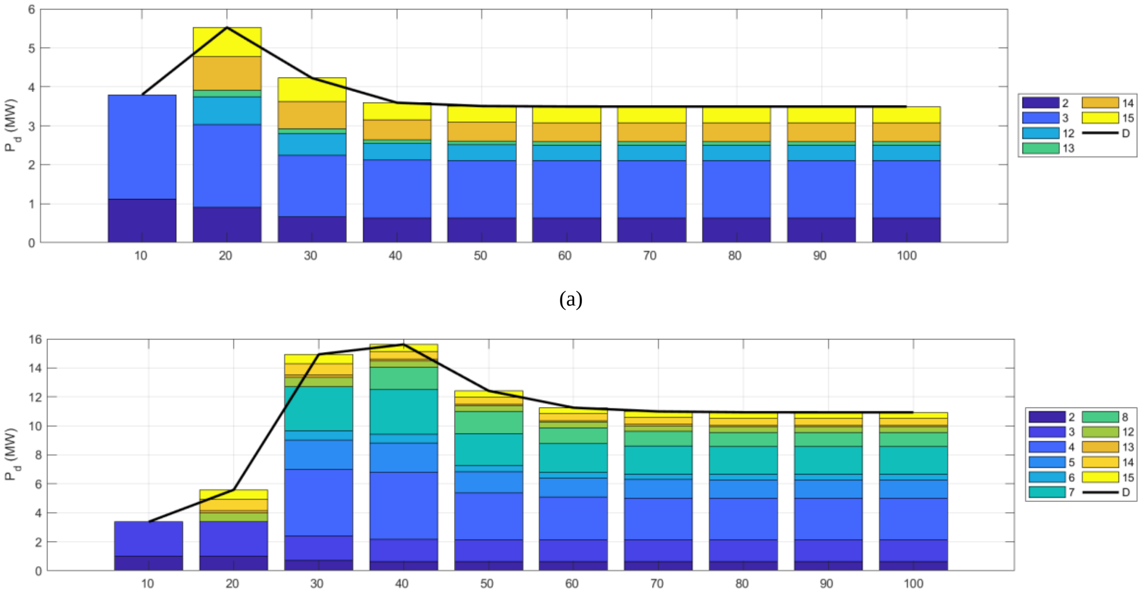


Figure 24: CLPU curve comparison: Blue(case 1) Red(case 2)

Following both curves the restoration sequence will take place in both cases. Next, the main differences between both cases will be announced and represented in Figure 25.



(a)

(b)

Figure 25: Comparison of CLPU cases: (a) Case 1 (b) Case 2

The first thing that can be noted by comparing both sequences has to do with the reconnection of the load blocks, it can be observed how in case 1 due to an excessively high load factor the RCS of branch 3-4 is not able to switch on. This difference of $\Delta S^U = 0.2$ provokes a difference in the peak of demand of $\Delta P_d = 10.09MW$ being the peak of case 1 of 5.52 and the one of case 2 of 15.61.

In relation to all the stated above, the parameter Div_d is responsible of moving the peak of demand to a subsequent instant in comparison to case 1. This is, in case 1 as there is only one set of load blocks that reconnect the peak is achieved in time instant $t = 20mins$ while in case 2 as there are two set of load blocks that reconnect to the grid and it has a higher time to gain diversity is achieves its peak of demand in time instant $t = 40mins$.

Regarding the parameter of the decay rate of the CLPU curve, even though the decay rate in both cases is similar there are differences that can be spotted in the reconnection sequence in case 2 the decrease in demand is done in one more time step, this is, the steady state is reached later than in case 1 as it can be seen in Figure 24.

4 Conclusions

This document has presented a method to achieve an optimal black-start sequence of an islanded distribution system. The distribution system restoration methodology proposed can generate a reconnection sequence which is able to coordinate the actions of the start-up sequences of DGs, the loads under CLPU conditions and the switching of the RCSs within the different branches of the grid.

The optimal BS sequence is formulated as an optimization problem whose objective function maximizes the load recovery along the restoration sequence. The restoration sequence is subject to a series of constraints that include the generation capacities of the DGs, the reconnection of loads among the restoration sequences, the power flow through branches and the dependency and connectivity of the network. The optimization problem formulated as a MILP has been tested with an example grid where the impact of the different constraints has been studied.

Along the document, a comparison between the reconnection sequence with a standard load behavior and a CLPU load behavior is studied. By implementing the CLPU condition a more accurate reflection of reality is achieved as it models more faithfully the load restoration sequence. This CLPU constraint imposes a higher generation capacity for the same demand restoration in permanent regime because of the non-diversified load factor that is active in the first instants of the load restoration. Finally, a sensitivity analysis has been conducted to analyze the impact of the different parameters of the CLPU constraint in the load-reconnection sequence

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