

## DMA-SAP-230 INTRODUCTION TO DIFFERENTIAL EQUATIONS

**SEMESTER:** Fall / Spring  
**CREDITS:** 6 ECTS (4 hrs. per week)  
**LANGUAGE:** English  
**DEGREES:** SAPIENS program

### Course overview

Techniques and applications of ordinary differential equations, including Fourier series and boundary value problems, linear systems of differential equations and an introduction to partial differential equations. Intended for engineering majors and other who require a working knowledge of differential equations.

The aim of this course is to provide you with:

- techniques, both explicit and numerical, to solve important classes of ordinary differential equations;
- practice in understanding how differential equations model physical phenomena;
- the ability to interpret the solutions that are found;
- numerical methods implemented in Matlab.

### Prerequisites

Basic knowledge of calculus and linear algebra.

### Course contents and methodology

#### Methodology

Lecture, solving calculation problems during exercises. There will be lab sessions with Matlab implementing Euler's and Runge-Kutta's methods.

## Contents

### 1. INTRODUCTION TO DIFFERENTIAL EQUATIONS (Chapter 1 of [1])

Basic definitions and terminology. Initial value problems. Differential equations as mathematical models: population dynamics, radioactive decay, Newton's law of cooling, spread of a disease, chemical reactions, falling bodies and air resistance, etc.

### 2. FIRST-ORDER DIFFERENTIAL EQUATIONS (Sections 2.1-2.4, 3.1-3.2, 9.1-9.2 of [1])

Direction fields and autonomous first order differential equations. Separable equations. Differences between linear and nonlinear equations. Linear first-order differential equations. Exact equations and integrating factors. Modeling with first-order differential equations: population dynamics, mixtures, chemical reactions, series circuit, etc. Numerical solutions of differential equations: Euler and Runge-Kutta methods.

### 3. SYSTEMS OF LINEAR FIRST-ORDER DIFFERENTIAL EQUATIONS (Chapter 8 and Section 3.3 of [1])

Definitions and preliminary theory. Homogeneous linear systems and fundamental matrices. Nonhomogeneous linear systems. Variation of parameters. Superposition Principle. Homogeneous linear systems with constant coefficients. Matrix exponential. Nonhomogeneous linear systems with constant coefficients. Undetermined coefficients. Modeling with systems of first-order linear differential equations: mixtures, competition models, electrical networks, etc.

### 4. HIGH-ORDER LINEAR DIFFERENTIAL EQUATIONS (Sections 4.1-4.7 and 5.1 of [1])

Definitions and examples. Homogeneous and nonhomogeneous linear differential equations. General solution of homogeneous and nonhomogeneous equations. Homogeneous and nonhomogeneous linear equations with constant coefficients. Undetermined coefficients and superposition principle. Variation of parameters. The Cauchy-Euler equation. Modeling with higher-order linear differential equations: mechanical and electrical vibrations, forced vibrations, frequency response and resonance.

### 5. THE LAPLACE TRANSFORM (Sections 7.1-7.6 of [1])

Definition of the Laplace Transform. Inverse transforms and transforms of derivatives. Operational properties. Systems of linear differential equations.

## 6. SYSTEMS OF NONLINEAR FIRST-ORDER DIFFERENTIAL EQUATIONS (Sections 10.1-10.4 of [1])

Autonomous systems. Stability of linear systems. Linearization and local stability. Autonomous systems as mathematical models.

## 7. PARTIAL DIFFERENTIAL EQUATIONS (Sections 11.1-11.4, 12.1-12.7 of [1])

Orthogonal functions and Fourier series. Fourier Cosine and Sine series. Eigenvalue problems. Separation of variables. The Heat equation. Vibrating strings and the Wave equation. Steady-state temperature and the Laplace's equation.

### Textbooks

- [1] *Differential Equations with Boundary-Value Problems*, 8th edition, Dennis G. Zill, Warren S. Wright. (basic bibliography)
- [2] *Differential Equations with Boundary Value Problems*, 2<sup>nd</sup> edition, James R. Brannan, William E. Boyce. (additional bibliography)
- [3] *Differential Equations with MATLAB*, Brian R. Hunt, Ronald L. Lipsman, John E. Osborn, 3<sup>rd</sup> edition, Wiley, 2019.

### Grading

The overall grade will be obtained as follows:

- Final examination (40%)
- Two midterms (25% each)
- Participation and homework (10%)

Moreover, at the end of each unit the students will be allowed to submit some optional problems from a selected list. This optional work can increase the final grade up to 10%.

The exams are all closed notebook, closed textbook and no calculator. The course will not be graded on a curve, i.e., there is no bound on the numbers A's, B's, C's, etc.

Students will have the chance to retake the exam. The resulting grade will be computed as follows: 30% of midterms and 70% of the final exam.