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A Two-Level Fuzzy Multi-Objective Design of ATO Driving Commands for Energy-Efficient Operation of Metropolitan Railway Lines

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Abstract: Policies for reducing CO_2 and other GHG emissions have motivated an increase in electrification in metropolitan areas, mandating reductions in energy consumption. Metro systems are keystone contributors to the sustainability of cities; they can reduce the energy consumption of cities through the use of the economic driving parameters in their onboard automatic train operation systems (ATO) and through the strategic design of efficient timetables. This paper proposes a two-level optimization method to design efficient, comfortable, and robust driving commands to be programmed in all the interstations of a metro line. This method aims to increase the sustainability of metro operations by producing efficient timetables with economic driving for each interstation while considering comfort restrictions and train mass uncertainty. First, in the eco-driving level, an optimal Pareto front between every pair of successive stations is obtained using a multi-objective particle swarm optimization algorithm with fuzzy parameters (F-MOPSO). This front contains optimized speed profiles for different running times considering train mass variations. The global problem is stated as a multi-objective combinatorial problem, and a fuzzy greedy randomized adaptive search procedure (F-GRASP) is used to perform an intelligent search for the optimal timetables. Thus, a global front of interstation driving commands is computed for the whole line, showing the minimum energy consumption for different travel times. This method is analyzed in a case study with real data from a Spanish metro line. The results are compared with the minimum running time timetable and a typical timetable design procedure. The proposed algorithms achieve a 24% reduction in energy consumption in comparison to the fastest driving commands timetable, representing a 4% increase in energy savings over the uniform timetable design.

Keywords: energy saving; sustainability in railway transport; efficient timetable; fuzzy logic; multi-objective combinatorial optimization; train control; train load variation

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Citation: Sánchez-Contreras, G.; Fernández-Rodríguez, A.; Fernández-Cardador, A.; Cucala, A.P. A Two-Level Fuzzy Multi-Objective Design of ATO Driving Commands for Energy-Efficient Operation of Metropolitan Railway Lines. *Sustainability* **2023**, *15*, 9238. https://doi.org/10.3390/su15129238

Academic Editor: Mariano Gallo

Received: 28 April 2023
Revised: 25 May 2023
Accepted: 6 June 2023
Published: 7 June 2023

1. Introduction

The road to decarbonization has been paved by the European Commission, which aims to achieve an 80% reduction in CO_2 and other GHG emissions by 2050. Metropolitan systems are greatly affected by this objective, with their primary sources of carbon emissions coming from industry, transport, and domestic sources. This leads to strategies that focus on replacing fuel-based technologies with clean alternatives, increasing the electrification of cities to progressively reduce these emissions and, therefore, the electricity demand of urban areas.

As an electrical transport mode, metro systems are crucial to achieving sustainable urban mobility. However, metro rail transit contributes significantly to the electricity demand in metropolitan areas, and with growing electricity prices, reducing the energy consumption of metro operations has become crucial for the sustainability of cities. Several strategies have been studied and implemented to improve railway systems' energy efficiency, mainly targeting low-investment measures that do not require changing the train fleet or improving infrastructure in metros.

One of the highest potential energy savings methods comes from eco-driving [1], which involves designing the most efficient driving pattern between two stations to meet the target schedule with minimum energy consumption. The eco-driving implementation is significantly different for metro trains than mainline trains, which employ manual driving.

Many modern metropolitan lines have their trains automatically driven by automatic train operation systems (ATO). Their automatic operation starts with the onboard ATO equipment receiving a set of driving commands at the station from the centralized regulation system. Typically, the driving command sets are pre-programmed in the ATO equipment for each pair of stations and comprise four parameters: holding speed (*vh*), coasting speed (*vc*), re-motoring speed (*vr*), and braking rate (*b*), as shown in Figure 1.

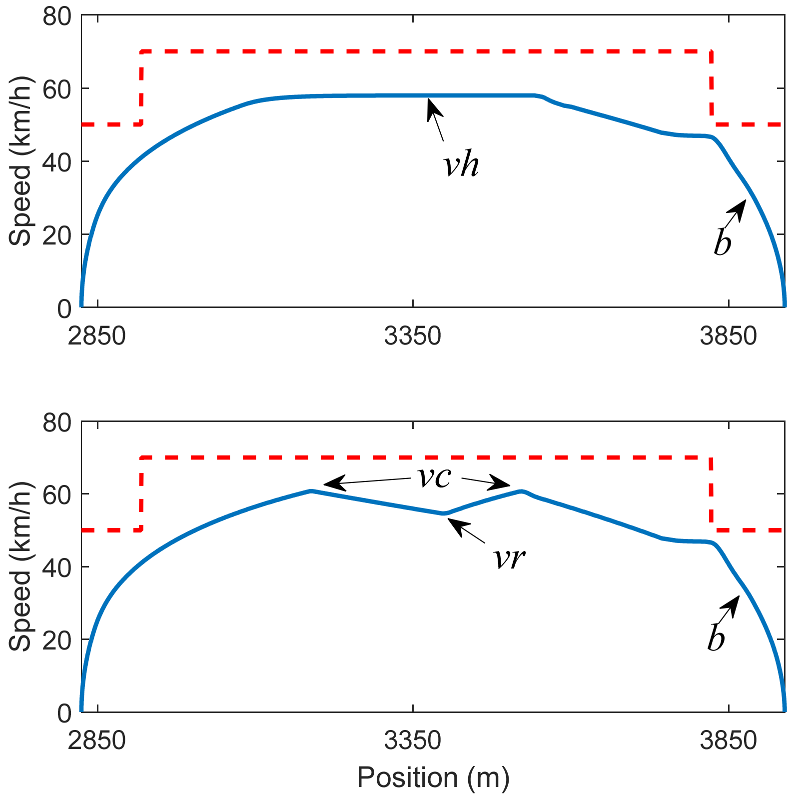

Figure 1. Example of ATO driving commands in a speed profile.

After that, the train implements a specific speed profile to the following station, which results from the ATO driving logic using the driving commands received and the track characteristics as input data. Complex algorithms are part of the ATO driving logic. They are in charge of different functions, including the calculation of braking curves (with variable deceleration to maintain the train's braking level depending on variations in the grade), speed regulation (according to the maximum speed profile, holding speed, and braking curves), the calculation of the start of the braking points (to ensure that the train can follow the braking curve and make a smooth transition from traction to the braking curves), the calculation of the start of the coasting points, and passenger comfort control (limiting the jerk and imposing a hysteresis cycle between traction and braking mode) [2].

Therefore, the eco-driving problem in ATO systems is designing the driving commands to be programmed in the track-side equipment, which defines the speed profile that the onboard equipment will implement during the operation according to the ATO's logic. As said before, the input for the eco-driving problem comes from the timetable where the target times to achieve between stations can be extracted. In metropolitan systems, the operator determines the commercial speed of the line according to passenger demand. The commercial speed determines the journey time between terminal stations and, associated with that, the time margin with respect to the minimum journey time. Then, the running time between stations can be obtained by distributing the complete journey margin time among all the stretches.

The timetable can be designed with energy efficiency objectives by optimizing the distribution of the journey time. With this aim, a greater time margin should be allocated where the benefits in energy consumption associated with increasing the running time are higher. Therefore, energy consumption can be minimized when designing ATO driving commands by means of an integrated approach that takes into account eco-driving and timetable optimization.

This paper presents an optimization method to design CBTC ATO driving commands that minimize the energy consumption of a metro line while simultaneously optimizing the trains driving at each pair of stations and the schedule. Furthermore, this method uses a multi-objective optimization model to obtain the solution for all the possible values of commercial speed (i.e., all the possible journey times between terminal stations).

Multi-objective optimization is carried out with a two-level approach. At the eco-driving level, the multi-objective eco-driving problem is solved at each interstation employing a MOPSO algorithm with fuzzy parameters (F-MOPSO) combined with a high-fidelity ATO simulator. The ATO model presented in this paper has been tested, and its accuracy has been proved in different metro lines [3,4]. A Pareto curve for each interstation is obtained, containing the most efficient driving commands in terms of energy consumption and running time. The passenger load uncertainty is also considered, defining the train mass as a fuzzy number. The comfortability from the passengers' perspective and the robustness of the speed profiles performed by the driving commands against passengers' load variations is ensured by applying comfort and driving pattern robustness restrictions during the F-MOPSO optimization.

At the global level, the timetable design is stated as a multi-objective combinatorial optimization problem (MOCOP). This problem is solved using a greedy randomized adaptive search procedure with fuzzy parameters (F-GRASP). This search algorithm produces a global journey time/energy consumption Pareto front where each solution represents the eco-driving speed profiles (defined by a set of driving commands) for every line interstation. The multi-objective approach has the advantage of providing a front of optimal solutions, where the decision-maker can select the trip time for the driving command design in view of the trade-offs between time and energy. Moreover, the proposed method achieves optimal timetables while accounting for train load uncertainty.

The main contribution of the proposed work Is a multi-objective integrated methodology to solve the ATO driving command design problem of a complete metro line by the optimization of train driving and timetables. Furthermore, the method proposed covers the main uncertainty of operation (train mass variation) using fuzzy modelling and applying driving pattern robustness restrictions in the optimization process. Finally, the quality of the service is guaranteed in the results by the comfort restrictions in place and by including detailed ATO modelling to improve the accuracy of the results and the applicability to real systems.

The paper is organized as follows. In Section 2, the relevant literature is reviewed. Section 3 describes the procedure to obtain robust and efficient driving commands. The F-MOPSO algorithm for obtaining efficient speed profiles is presented in Section 4. Section 5 describes the F-GRASP algorithm to design and optimize the timetable for a metro line. A case study using data from a real metro line is analysed in Section 6. Finally, conclusions are drawn in Section 7.

2. Literature Review

The optimal regimes of train driving were obtained in the first eco-driving studies applying Pontryagin's maximum principle, including traction, holding speed, coasting, and braking [5,6]. Thus, an optimal speed profile can be obtained by calculating the switching points between the optimal regimes [7]. These regimes can also be applied in constrained eco-driving models with speed limits, gradients, variable traction forces, and trip time restrictions [8,9]. Furthermore, they can include the negotiation of steep uphills and downhills [10].

The methods proposed in the literature to calculate efficient driving patterns can be classified into mathematical optimization and simulation-based models.

Different approaches have been proposed to obtain eco-driving solutions. Among them, we can find constructive algorithms [7–10], dynamic programming [11,12], mixed-integer programming models with approximations [13,14] or sequential quadratic programming [11,15], and the discretization of the problem with the application of Lagrange multipliers [16].

The eco-driving problem in ATO systems involves the design of the driving commands to be programmed. Thus, it is crucial to include in the model the ATO logic to obtain an acceptable degree of accuracy because trains' energy consumption and running times critically depend on the automatic driving rules. However, these rules are frequently nonlinear, hindering the analytical resolution of the models. Mathematical models usually apply simplifications to obtain solutions. In some cases, these simplifications could affect the accuracy of the solutions obtained. This effect could be significant in metro lines where the running time differences between the nominal and fastest driving could be 2–3 s.

Simulation-based methods are fundamental in this regard because of their flexibility in being combined with accurate models of train operations, including train motion, line infrastructure, and ATO logic.

Compared with the traditional fixed block signalling systems, the newest signalling systems are based on moving block communication-based train control (CBTC) [17], which uses radio communications with higher bandwidth. Thanks to these communication improvements, the driving parameters can take practically continuous values, increasing the possible speed profile space and making impractical the use of exhaustive searches. Therefore, the application of eco-driving in CBTC systems demands more sophisticated optimization models. These types of eco-driving methods include artificial intelligent techniques such as artificial neural networks (ANN) [18], nature-inspired algorithms such as genetic algorithms (GA) [19,20], ant colony optimization (ACO) [21], simulated annealing (SA) [22], differential evolution (DE) [23], multi-objective particle swarm optimization (MOPSO) [24], and non-dominated sorting genetic algorithm II (NSGA-II) [25].

Previous studies mainly focused on eco-driving designs between two stations, where the timetable was predefined and running times were fixed. However, energy savings can also be obtained by optimizing the timetable in the complete line.

With timetable optimization, the metro operator determines the running time between terminal stations in a line regarding passenger demand. Many approaches to this problem have been made without considering eco-driving optimization through dynamic programming coupled with a simulation model to compute the utilized energy [26]. Other methods involve using the ε-constraint method and applying multi-objective optimization with distance-based methods [27]. Several mixed-integer programming models have been proposed to solve the scheduling problem for single-track railways [28], to integrate train-stop planning and timetabling in high-speed railways [29], to solve the timetable and train-set circulation plans in transition times for high-speed lines [30], to solve the problem of multi-periodic train timetabling and routing high-speed trains at stations [31], to integrate the line planning, timetable and rolling stock allocation problem [32], to integrate the timetable problem with passengers' train booking decisions [33], and to solve the schedules of trains on a railway network to determine the best stop locations for both technical and religious services [34]. Passenger waiting time has also been included in optimization [35] by treating passenger behaviour as deterministic. In [36], a method for determining the optimal train running interval and routing scheme is proposed based on analyzing the spatiotemporal distribution of passenger flow to control the train load factor. Contrary to the common fixed timetables, an operational concept to produce schedules with ad hoc individual passenger demands was presented in [37].

Other approaches aim to produce timetables that increase regenerative braking recovery. In [38], a mixed integer programming model was proposed to optimize the timetable to maximize the exchange of regenerative braking between powering and braking trains. The authors of [39] used a GA to maximize the coincidences of departures and arrivals to stations in a line. An artificial bee colony algorithm was applied in [40] to modify train intervals and dwell times to maximize the overlap in the timetable of tractioning and braking processes. In [41], a mixed-integer programming model was applied to increase the synchronization of power demands from tractioning trains and the power supply of braking trains. In [42], a mixed-integer linear programming model to synchronize traction and braking events combined with a local search algorithm to shave power peaks was proposed.

Maximum energy savings can be accomplished using integrated optimization methods. These methods combine efficient speed profiles with optimal timetables while minimizing traction energy consumption. Approaches to this problem include dynamic programming with energy consumption and service quality as objectives [43], multi-train simulation with a single train speed profile optimization model [44], two-level optimization models coupled with a genetic algorithm to search for the optimal solution [45], an integer programming model to minimize total net energy consumption [46], and dynamic integrated optimization with adaptive cycle time based on passenger demand [47]. In [48], an iterative algorithm was applied to integrate trip time distribution and improve the driving strategy. In [49,50], pseudospectral and indirect methods were applied to allocate time supplements within interstations to achieve energy-efficient train operations within a railway timetable. These methods used the acceleration–holding speed–coasting–braking sequence and were applied for non-ATO regional trains. An integrated optimization approach was proposed by [51] using a three-dimensional grid network to reduce traction energy consumption. In [52], available time supplements were redistributed over the trips and were further optimized in [53] by including uncertain delays using fuzzy numbers and constraints applied to punctuality. In the approach presented in [54], the timetable design problem was solved using a model that considers headway between successive trains, trip time distribution, and passenger demand. In this work, a two-level algorithm was presented by applying dynamic programming for the train control level to obtain energy-efficient driving with a given trip time. At the timetable level, the results for the dynamic programming step are fed into a SA algorithm to optimize the headway of trains and trip times.

The integrated methods previously stated are single-objective optimization methods. Thus, a single solution is obtained for a trip time or a time–energy balance. Other papers have formulated multi-objective travel time and energy consumption problems. In [55], the proposed model included metaheuristics to improve the optimal consumption curves and use historical data. This study proposed a long-distance train model with speed profiles that do not correspond to ATO systems. Gao et al. [56] formulated a bi-objective linear program, which considers energy consumption and passenger travel time for metro lines, but an ATO model is not included. Yang et al. (2019) [57] proposed a multi-objective problem solved by NSGA-II to optimise timetables in a metro line considering cost, passenger waiting time, and robustness. The driving model consisted of an acceleration phase, a coasting point, and a braking phase.

These previous studies state a deterministic problem and do not include uncertainty in their models, which has a relevant influence in real-world applications. The main sources of uncertainty in ATO traffic operation are train delays and train load [3]. Other studies have modelled uncertainty in different parameters, such as train traction [58].

Given the high precision of ATO equipment in train driving, mass variations caused by passenger load variations are the only relevant factor affecting the execution of pre-programmed driving parameters [59]. The climate conditions considered in [60] are irrelevant in metro operations where the train is protected by the tunnel in most track sections.

Passenger mass fluctuates with the operation period, primarily between peak and off-peak hours. Unsteady passenger flow and train headway cause variations in the mean value of passenger load for each period. Several authors have tackled the eco-driving problem considering train mass uncertainty. In [24], the proposed model optimized ATO speed profiles using a mean value of train mass for a specific period. By controlling the driving pattern against train mass variations, a Pareto set of robust and efficient speed profiles were obtained with accurate ATO models [3]. Moreover, the set of driving commands to be programmed into the ATO equipment was obtained through an optimization model that considered statistical information about delays to minimize the expected energy consumption. Another approach proposed in [25] generates a Pareto optimal curve by applying an NSGA-II algorithm, which was modified to consider fuzzy values of train mass. Energy consumption was minimized with the design of optimal ATO speed profiles along a pair of stations. Both of these studies were limited to computing speed profiles for the route between every two stations for given running times, assuming that the timetable is fixed. However, the interstation running time, which is determined by the train timetable, also influences energy consumption. Normally, a longer interstation running time implies less energy consumption.

3. Two-Level Fuzzy Optimization Model for the Design of Robust and Efficient Driving Commands and Timetables

Designing and optimizing an efficient metro line timetable is a complex problem where opposing objectives, running time and energy consumption, must be improved to achieve an optimal solution. The approach presented in this paper to solve this problem consists of a two-level optimization model, as shown in Figure 2. In the lower level (eco-driving model), the F-MOPSO algorithm is applied to search for a set of optimal speed profiles for each interstation. Once a Pareto front is computed for train operations at every interstation in the line, the timetable problem becomes a combinatorial problem. The F-GRASP algorithm is used for the higher level (global level) to obtain a global Pareto front, where each solution represents the total running time of a train from the first to the last station along the line and the corresponding total energy consumption of an optimal timetable. This total running time (and its associated timetable) and the total energy consumption are obtained by selecting one solution (a speed profile) from the Pareto curve of each individual interstation. Thus, a member of the global Pareto front is an optimal set of driving commands to be programmed from the first to the last station that provides a timetable that aims to minimize total running time and energy consumption.

3.1. Global Optimization Level

The global fitness evaluation function for the Pareto front computed in the F-GRASP algorithm is a fuzzy extension of the crisp problem shown in the following equations:

$$\text{Minimize } \hat{f}_g(\hat{x}_g) = \left(f_{g1}(\hat{x}_g),\ f_{g2}(\hat{x}_g)\right) \tag{1}$$

$$f_{g1}(\hat{x}_g) = \sum\nolimits_1^S f_1(\hat{x}_s) \tag{2}$$

$$f_{g2}(\hat{x}_g) = \sum\nolimits_1^S f_2(\hat{x}_s) \tag{3}$$

where $f_{g1}(\hat{x}_g)$ and $f_{g2}(\hat{x}_g)$ are the total running time and energy consumption of the entire metro line, obtained with a sequence of optimal speed profiles $\hat{x}_g = (\hat{x}_1,\ \hat{x}_2, \ldots,\ \hat{x}_S)$ from interstation 1 to S computed in the F-GRASP algorithm. Each element s in $\hat{x}_g$ is a solution $(\hat{x}_s)$ of driving commands for every interstation s. The possible solutions for $\hat{x}_s$ are the Pareto fronts computed at the eco-driving level. $f_1(\hat{x}_s)$ and $f_2(\hat{x}_s)$ are the running time and the energy consumption of the interstation s, respectively.

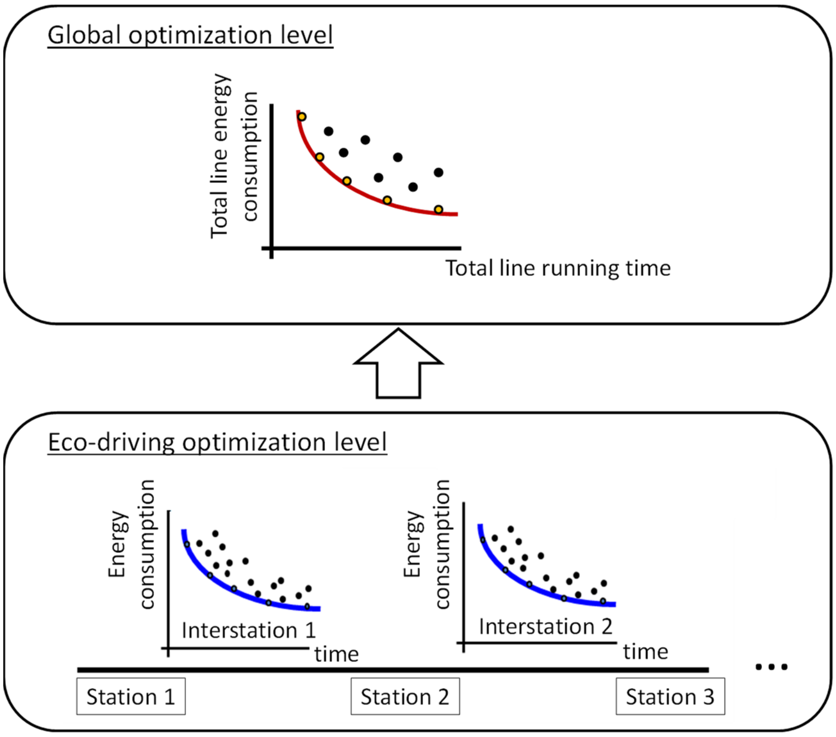

Figure 2. Two-level optimization level.

3.2. Eco-Driving Optimization Level

At the eco-driving level, speed profile optimization at each interstation s is a multi-objective problem, where the objectives are minimizing running time and energy consumption in a route, with its objective function shown in

$$\text{Minimize } \hat{f}(\hat{x}_s) = (f_1(\hat{x}_s),\ f_2(\hat{x}_s)) \tag{4}$$

A detailed simulator of train motion and ATO equipment is used to compute $f_1(\hat{x}_s)$ and $f_2(\hat{x}_s)$ [2]. The design of these speed profiles dictates the driving modes in which a train can be operated between two stations. Four decision variables, the driving parameters of an ATO configuration, determine the speed profiles: coasting speed (vc), re-motoring speed (vr), holding speed (vh), and braking rate (b). Solutions obtained in the F-MOPSO algorithm are defined as a configuration vector of driving parameters $\hat{x}_s = (vc, vr, vh, b)$. The computed Pareto curve contains a set of non-dominated solution vectors where at least one objective has a lower-performing value than other solutions from the solution space. The other objective is likewise of lower or equal value. Every speed profile obtained in the Pareto front is a feasible solution that complies with the comfort criteria defined by the metro operator. These criteria dismiss solutions that include uncomfortable rides with traction cut-offs in steep slopes, several re-motoring periods, short traction periods, and low-speed rides. Restrictions related to passenger comfort are relevant for implementations in a real-world operation and complement the jerk control implemented in the ATO driving logic.

One comfort criterion is the maximum slope value for traction cut-off restrictions, which limits the application of coasting during steep uphill segments,

$$p < p_{\max} \text{ if } F_m = 0 \text{ and } F_m^{\text{prev}} > 0 \tag{5}$$

where p is the slope, $p_{\max}$ is the maximum slope permitted at the start of a coast, F_m is the current traction effort, and F_m^{prev} is the traction effort in the previous instant. Cutting off traction during a steep slope results in an uncomfortable sensation for passengers because of the sudden change in acceleration rates [61].

Another restriction is the maximum number of re-motoring occurrences in transit,

$$n_{\text{remotoring}} < n_{\max} \tag{6}$$

where $n_{\text{remotoring}}$ is the number of re-motoring instances executed along an interstation and $n_{\max}$ is the maximum number of re-motoring instances permitted for passenger comfort. A limitation regarding the minimum traction time of a train during traction drive mode is also in place. This limitation avoids rapid successive changes in traction modes during the train's journey, which leads to an uncomfortable perception of constant variation in motion due to jounces.

Train speed is also limited with a coasting speed restriction that does not allow the coasting speed to fall below a certain threshold, a minimum speed constraint throughout the journey (7), and a minimum speed limit along the curves to avoid the excessive wear of the wheels and track.

$$v > v_{\min} \tag{7}$$

All feasible solutions must ensure the driving pattern in the interstation remains stable, that is, not affected by variations in the transported mass or passenger load, to minimize variability in running times and energy consumption. This additional robustness restriction is imposed using a filter to exclude those speed profiles, which modifies its driving pattern when the train mass varies [3].

3.3. Fuzzy Mass Model

The model proposed in this paper accounts for uncertainty regarding passenger load in train transits between stations [25]. Given the vagueness of knowledge regarding mass variations of a train during different times of the day (peak hours, off-peak hours), the additional passenger mass is modelled using fuzzy numbers [62]. Fuzzy mass values will also make running time and energy consumption fuzzy.

The use of fuzzy values for the passenger load allows for modelling the train mass uncertainty during a certain period in order to devise the planning of the train timetable.

Let M be the train mass corresponding to the passengers' load that is represented by a fuzzy number, $\widetilde{M}$. This fuzzy number is modelled as a pseudo-triangular fuzzy number. The triangular fuzzy number is truncated (being, thus, a pseudo-triangular fuzzy number) because of technical reasons, as the shaved-off side indicates that it is not possible to exceed the maximum passengers' mass value expressed by the train manufacturer (maximum load). The proposed model could be applied as well for other possibility membership functions (such as exponential, hyperbolic, and piece-wise linear functions [63] and S-shaped membership functions [64,65].

The possible values of the mass are shown in Figure 3. Negative values of passengers' load are not allowed, and m_l is the least passenger load possible. The greatest possible value is m_a, and the maximum value $\widetilde{M}$ is m_b, which has a possibility of λ. The maximum load is indicated by the train manufacturer and corresponds to the maximum passenger load permitted in the train. The value of λ is the possibility associated with this maximum load for the considered scenario. Therefore, the function that indicates membership to the fuzzy mass is indicated in (8).

$$\mu_{\widetilde{M}}(m) = \begin{cases} \left(\dfrac{1}{m_a - m_l}\right)m - \dfrac{m_l}{m_a - m_l}, & m_l \le m \le m_a \\ \left(\dfrac{\lambda - 1}{m_b - m_a}\right)m - \dfrac{\lambda m_a + m_b}{m_b - m_a}, & m_a \le m \le m_b \\ 0, & m < m_l \\ 0, & m > m_b \end{cases} \tag{8}$$

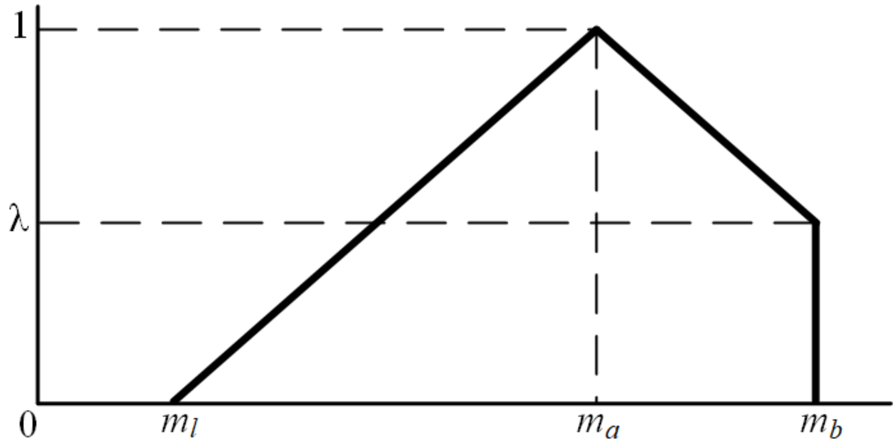

Figure 3. Fuzzy passenger load mass.

Total train mass $\widetilde{M_t}$ is the empty train mass M_0 plus the additional mass from the passenger load $\widetilde{M}$. The fuzzy total train mass necessarily produces fuzzy values for the resulting running time $\widetilde{T}$ and energy consumption $\widetilde{E}$ of a global train journey. Equation (4) is expressed in fuzzy terms as

$$\text{Minimize } \widetilde{F}(\hat{x}) = \left(\widetilde{T}(\hat{x}_s),\ \widetilde{E}(\hat{x}_s)\right) \tag{9}$$

where $\widetilde{T}$ and $\widetilde{E}$ are the fuzzy running time and fuzzy energy consumption, respectively, for station s.

Similarly, Equation (1) can be written in fuzzy terms as:

$$\text{Minimize } \widetilde{F_g}(\hat{x}) = \left(\widetilde{T_g}(\hat{x}_g),\ \widetilde{E_g}(\hat{x}_g)\right) \tag{10}$$

where $\widetilde{T_g}$ and $\widetilde{E_g}$ are the fuzzy total running time and fuzzy total energy consumption, respectively, for a complete metro line.

3.4. Fuzzy Dominance

Applying multi-objective optimization algorithms to compute a Pareto curve of optimal solutions requires the determination of dominance rules to identify dominating solutions among the population with better values for each objective. With fuzzy objectives, fuzzy dominance must be established to rule out dominated solutions in the solution space [66]. The fuzzy Pareto dominance concept is used to compare two solutions, where an individual solution $\widetilde{A}$ fuzzy-dominates another solution $\widetilde{B}$ if each objective k is fuzzy-dominated [67]. A t-norm function is created in the fuzzy intersection of each objective, determining the corresponding level of dominance, as shown in Equation (11).

$$\mu^{\text{dom}} = \left(\widetilde{A} <_{\text{k}}^{\text{F}} \widetilde{B}\right) = \cap\ \mu_{\text{k}}^{\text{dom}}\left(\widetilde{A} <_{\text{k}}^{\text{F}} \widetilde{B}\right) \tag{11}$$

where the objective k is transformed into a measure in [0, 1] with the fuzzy-k dominance function.

Given the two objectives of the problem, the fuzzy numbers $\widetilde{T}$ for running time and $\widetilde{E}$ for energy consumption are compared using a *min* t-norm operation. Therefore, dominance is computed using strong fuzzy dominance, defined in [25], by means of possibility and necessity measures [68].

The strong fuzzy dominance used in this paper is calculated by applying the necessity measure, as shown in (12). n^{SD} is the required level of necessity for the fuzzy comparison.

$$\min\left(N\left(\widetilde{T}_B > \widetilde{T}_A\right),\ N\left(\widetilde{E}_B > \widetilde{E}_A\right)\right) \geq\ n^{SD} \tag{12}$$

The necessity measure for the running time and energy consumption can be calculated with regard to the possibility measure as follows:

$$N\left(\widetilde{T}_B > \widetilde{T}_A\right) = 1 - \Pi\left(\widetilde{T}_B < \widetilde{T}_A\right) \tag{13}$$

$$N\left(\widetilde{E}_B > \widetilde{E}_A\right) = 1 - \Pi\left(\widetilde{E}_B < \widetilde{E}_A\right) \tag{14}$$

Therefore, applying alpha-cuts arithmetic, A dominates B if $t_{\overline{\alpha}}(A) < t_{\underline{\alpha}}(B)$ and $e_{\overline{\alpha}}(A) < e_{\underline{\alpha}}(B)$, comparing lower and upper limits with $\alpha = 1 - n^{SD}$.

This type of fuzzy dominance can be applied because of the relationship between time, energy consumption, and train mass. This dependency follows an increasing monotone pattern, where an increase in train mass necessarily increases the running time and energy consumption of the journey.

4. Eco-Driving Level: F-MOPSO Algorithm

The first level of the procedure detailed in this paper involves using a nature-inspired algorithm such as MOPSO to obtain efficient driving commands for every interstation. The MOPSO algorithm has been selected because it has been proven that it outperforms other commonly applied algorithms in the speed profile optimization problem [24]. These driving commands are obtained using a fuzzy mass model to include mass variations related to passenger load. The result is an optimal Pareto curve for every interstation that contains speed profiles that are comfortable and robust and serve as an input for the next level of optimization.

F-MOPSO Algorithm

The design of ATO speed profiles for the initial step of the efficient timetable design algorithm involves the application of an F-MOPSO algorithm to find the optimal solutions for economical driving at each interstation.

The search for solutions in particle swarm optimization algorithms is carried out using a population of individuals [69]. Each individual $\hat{x}$ is called a particle, and the population of particles is referred to as a swarm. Particles fly to regions inside the search space where the best results are obtained; in this case, the goal of the particles is to find solutions that minimize each objective with lower functional values. A particle's flight is modified iteratively through its individual experience and is also influenced by the results the other particles in the swarm have obtained. Iteratively, each particle communicates its findings to the swarm and adjusts its trajectory based on its personal best result (*pbest*) and the swarm's global best result (*gbest*), converging on an optimum or near-optimal point of the solution space.

Every partIcle i of the swarm represents a vector of decision variables containing the driving parameters for the ATO commands $\hat{x}_s = (vc, vr, vh, b)$. The position of the particles of the swarm is updated according to the following equations:

$$v_i(n) = wv_i(n-1) + c_1 r_1 (p_i - x_i(n-1)) + c_2 r_2 (p_g - x_i(n-1)) \tag{15}$$

$$x_i(n) = x_i(n-1) + v_i(n) \tag{16}$$

The motion of each particle is driven by its velocity $\hat{v} = (v_{vc}, v_{vr}, v_{vh}, v_b)$. This velocity is computed every n iterations using the distance between its current position x and the previously obtained *pbest* and *gbest* position values, stored in a vector $\hat{p} = (p_{vc}, p_{vr}, p_{vh}, p_b)$. The influence of the personal and the swarm experience is modulated through the social factors c_1 and c_2, respectively, and r_1 and r_2 are random numbers between 0 and 1. The coefficient w represents the inertia weight of the particle and decreases from an initial w_1 to

w_2, helping the swarm search based on global experiences on the first iterations to search in new areas and progressively orient themselves towards their local experience to discover optimal points in their vicinity.

The need to guide the particles through the search space with the local and global best findings requires selecting these drivers correctly. Once the particle swarm has converged on its solutions, fuzzy dominance is applied to dismiss solutions that are dominated. Mutually non-dominated solutions are saved to an external archive A, becoming candidates for the global guide. The best speed profile guide is selected using a crowding distance mechanism (CD) [70], and a specific probability is denominated as Top select probability. Crowding distance ensures variability in a Pareto curve by calculating the average distance of a speed profile to its neighboring solutions. Solutions with the highest crowding distance values are selected with the Top select probability from a sorted list by descending crowding distance to encourage the exploration of the least crowded areas in the solution space.

The F-MOPSO methodology to compute the optimal speed profiles between stations can be summarized in the following steps:

- Step 1: Initialize the swarm particles' positions and velocities and evaluate the objective functions to obtain the first *pbest* value.
- Step 2: Start the search loop with the initialized particles for N iterations.
- Step 3: Update the velocity and position of every particle using Equations (15) and (16).
- Step 4: Evaluate the particles in their new positions using the fuzzy fitness function, resulting in fuzzy running time and energy consumption values.
- Step 5: Apply comfort criteria and driving pattern robustness to filter speed profiles that do not comply with the comfort requirements. Pattern robustness checks if the driving pattern is maintained with the upper and lower alpha-cut values of the fuzzy train mass.
- Step 6: Apply fuzzy dominance to dismiss dominated and unfeasible solutions and add non-dominated speed profiles to the external archive A.
- Step 7: Calculate the crowding distance for each objective and sort the solutions.
- Step 8: Update *gbest* and *pbest*.
- Step 9: Repeat steps 3–8 until the last iteration.

5. Global Level: F-GRASP Algorithm

The speed profiles computed with the F-MOPSO algorithm provide the driving commands of the most efficient speed profiles that the train can perform in every transit between stations to allow for the flexible selection of a specific running time and energy consumption. Thus, a combinatorial approach is needed to achieve an optimal Pareto curve of solutions for the entire line. Every solution in this global Pareto front contains a vector of driving commands for each interstation, defining all the speed profiles a train will follow in a complete cycle of operation. The associated timetable is obtained from each speed profile's trip time, including their respective dwell times.

Considering the bi-objective nature of the problem, efficient timetable design is a multi-objective combinatorial optimization problem (MOCOP). Using a finite set of objects represented by the speed profile solutions from the F-MOPSO algorithm, an optimal running time and energy consumption value is obtained. MOCOPs can be approximated by classic literature problems, which share a similar mathematical formulation. Proposed problems include the assignment problem, cutting stock problem, job shop problem, travelling salesman problem, vehicle routing problem, and knapsack problem. The efficient timetable design combinatorial problem can be considered a variation of the 0/1 multi-objective knapsack problem [71,72] by studying the similarities between both problems. Given a set of items (speed profiles) with a specific weight and value (running time and energy consumption), the objective is to determine which items must be included in the collection (timetable) to maximize total value while keeping the total weight below or equal to the maximum weight limit (minimizing energy consumption while keeping total running time below or equal to a certain threshold).

The literature has numerous propositions to solve multi-objective combinatorial problems. Greedy algorithms are widely used because of their low computational time, making them suitable for real-time implementation in train operations. These algorithms obtain near-optimal solutions with a memory-less iterative procedure. GRASP [73] is a multi-start metaheuristic two-phase procedure involving construction and a local search. A greedy randomized criterion creates a feasible solution, which is then explored iteratively in the local search phase to improve the solution until the best overall solution is found in the vicinity of the initially proposed solution. The constructive heuristic creates a restrictive candidate list (RCL) where each member is added with a greedy function, selecting the best items from the item pool. A partial solution is estimated by selecting random candidates from the RCL to increase the diversity of solutions.

For our problem, a similar formulation considered by Soares Vianna and Claudio Arroyo [74] is used, with a series of modifications to adapt it to the timetable design problem. The main formulation is

$$\begin{array}{l} \textit{Minimize} \\ \lambda_1^n\left(\frac{f_{g1}(\hat{x}_g)-T_{min}^g}{T_{max}^g-T_{min}^g}\right)+\lambda_2^n\left(\frac{f_{g2}(\hat{x}_g)-E_{min}^g}{E_{max}^g-E_{min}^g}\right) \end{array} \tag{17}$$

$$\begin{array}{l} \textit{subject to} \\ \hat{x}_g=(\hat{x}_1,\ \hat{x}_2,\ldots,\ \hat{x}_S) \\ \hat{x}_s\in\left\{\hat{x}_1^s,\hat{x}_2^s,\ldots,\ \hat{x}_j^s\right\} \\ s\ \in\{1,2,\ldots,\ S\} \\ j\in\{1,2,\ldots,\ P^s\} \\ \sum\limits_{w=1}^{2}\lambda_w=1,\ \ 0\le\lambda_w\le 1 \\ w\ \in\{1,2\} \end{array} \tag{18}$$

where λ_1^n and λ_2^n are the weights of the running time and energy consumption objectives for each iteration n, $f_{g1}(\hat{x}_g)$, and $f_{g2}(\hat{x}_g)$ are the evaluated running time and energy consumption, T_{max}^g, T_{min}^g, E_{max}^g, and E_{min}^g are the boundaries of the problem, $\hat{x}_g$ is the solution vector of the driving commands, $\hat{x}_s$ is the set of driving commands for the Pareto front of interstation s, S is the total number of interstations in the line, j represents a solution from an interstation's Pareto front, and P is the total number of solutions in the Pareto front of an interstation.

The first set of solutions is created using randomized solutions selected from the RCL. Each fuzzy solution obtained from the previous step for each interstation is ranked using a similar formulation to (17). The RCL is populated with the computed rankings and sorted in ascending order. The ranking formula is

$$r_j^s=\lambda_1^n\left(\frac{f_1\left(\hat{x}_j^s\right)-T_{min}^s}{T_{max}^s-T_{min}^s}\right)+\lambda_2^n\left(\frac{f_2\left(\hat{x}_j^s\right)-E_{min}^s}{E_{max}^s-E_{min}^s}\right) \tag{19}$$

where $f_1\left(\hat{x}_j^s\right)$ and $f_2\left(\hat{x}_j^s\right)$ are the evaluated running time and energy consumption for the solutions of driving commands $\hat{x}_j^s$; T_{max}^s, T_{min}^s, E_{max}^s, and E_{min}^s are the boundaries of the running time and energy consumption for an interstation s.

The weights λ_1^n and λ_2^n are modified in every iteration of the F-GRASP algorithm to create a diversity of solutions.

Both (17) and (19) are normalized with the boundaries of the problem and the interstation, respectively. These are the limits of the objectives and represent the global maximum and minimum running time and energy consumption of the timetable design problem and the maximum and minimum running time and energy consumption a train can achieve with the computed speed profiles from the F-MOPSO algorithm for a single interstation. The global boundaries are obtained by evaluating the solution composed of the maximum and minimum running time speed profiles and the maximum and minimum energy consumption speed profiles and obtaining the total running time and energy consumption of these profiles. The time and energy boundaries for a single interstation are obtained by comparing the lowest and highest values for each interstation and saving the minimum and maximum values for all interstations. Normalization is carried out by subtracting the minimum running time and energy consumption from the evaluated values and dividing by the range between the maximum and minimum bounds of these values.

The parameter δ is used to prune the CL and obtain the RCL, limiting the number of candidates that are available to be selected for the creation of a solution. Candidates at the top of the ranking have the lowest evaluated values and are more likely to be selected in the construction phase. In contrast, the worst-ranked values can be excluded from the construction phase with low enough values of δ.

A random solution from the initial constructed phase is explored in the local search phase to improve the evaluation function value. The worst-ranked $\hat{x}_s$ component of the solution $\hat{x}_g$ is selected for improvement using a loop where a neighbouring solution from the interstation Pareto front with better ranking is explored. A new global solution $\hat{x}_g^*$ is evaluated, and the process is repeated until the exploration for a solution $\hat{x}_s$ is exhausted and $\hat{f}_g\left(\hat{x}_g^*\right)$ is no longer improved. A new component $\hat{x}_s$ is selected, and the previous one is marked as *EXPLORED* to avoid future explorations. This step is completely greedy. The quality of the final solution $\hat{x}_g$ determines the number of iterations of the local search step.

Once the best solution is obtained, $\hat{x}_g$ is stored as a result of the F-GRASP iteration, and the algorithm computes the next iteration. When the maximum number of iterations N has been reached, fuzzy dominance is applied to the saved F-GRASP solutions to dismiss dominated solutions, as explained in Section 3.2. The output of the optimization algorithm is the global Pareto curve for the line.

The pseudocode of the fuzzy GRASP algorithm is presented in Algorithm 1.

Algorithm 1 F-GRASP

```
Input: N, δ, Speed Profiles
Output: PF (Pareto Front)
  T^g_min, T^g_max, E^g_min, E^g_max ← Evaluate max and min time and energy speed profiles and
  compute total values
  T^s_min, T^s_max, E^s_min, E^s_max ← Sort time and energy values for each interstation and retrieve max
  and min value
  Initialize λ_w with random value between 0 and 1
  F-GRASP archive ← Ø
  while iteration <= N
    CONSTRUCTION PHASE
      Rank all speed profiles
      Create candidate list with ranked speed profiles
      Sort candidate list in ascending order
      Let RCL be a list of δ % of candidate list
      x̂_g ← Random solution from RCL
      Compute initial f̂_g(x̂_g)
    LOCAL SEARCH PHASE
      Mark all components of x̂_g as NON-EXPLORED
      while Exists any x̂_s marked as NON-EXPLORED
        x̂_s ← Worst − ranked component in solution x̂_g
        if Rank (x̂_s) is worse than best rank in Pareto front s
          x̂_s*= Better − ranked neighbour solution to x̂_s
          Compute f̂_g(x̂_g*)
          if f̂_g(x̂_g*)<f̂_g(x̂_g)
            x̂_g ← x̂_g*
          else
            Mark x̂_s as EXPLORED
```

Algorithm 1 *Cont.*

Save $\hat{x}_g$ in the F-GRASP archive
Update λ_w with random value between 0 and 1
iteration ← iteration + 1
PF ← Compute fuzzy dominance on FGRASP archive
return PF

6. Case Study

The proposed procedure described in Sections 2–4 was applied to a case study of urban rail transit of Metro de Madrid. The metro line consists of 16 stations and 12 km that were modelled with speed limits, curves, tunnels, and gradients for each interstation. This line is representative of a typical metro line because the interstations present different cases of uphill and downhill gradients and different speed limitations due to track curvature. It is operated as a loop starting and ending at the same station. The train studied is a class-3000 train of Metro de Madrid, with 160 tons of empty train mass, a maximum passenger load of 78 tons, and a total length of the train of 90 m. The maximum power of the train is 1500 kW, and the traction network voltage is 1.5 kV DC.

The simulation of the train's motion along the metro lines to compute running times and energy consumption using different speed profiles was carried out using an accurate model of train motion and a real ATO driving control, described exhaustively in [2].

6.1. GRASP Algorithm Validation

The proposed F-GRASP algorithm was used to obtain a Pareto front of efficient driving commands for timetable design. This Pareto curve contains a solution for every iteration of the F-GRASP algorithm and represents the driving commands of the ATO for every interstation in the metro line.

The optimization algorithm receives the Pareto front obtained using the F-MOPSO algorithm for each interstation, as detailed in Section 3. All of its solutions become the input for the algorithm at the global level, thus applying its methodology to obtain a global Pareto front with all the driving commands for the complete line journey. A Pareto front of driving commands is obtained for each of the 32 interstations in the line. The first 16 curves correspond to the interstations in track 1, while the successive 16 curves belong to track 2. The solutions obtained vary according to interstation track features, such as speed limits, gradient, and track length. These driving commands take discrete values and are limited with an upper and lower value and a fixed increment. These values are the same as used in [24] and are shown in Table 1. Comfort restrictions are applied to dismiss unfeasible solutions. These comfort constraints include a restriction to a minimum coasting speed of 20 km/h during the trip, a maximum of 3 re-motoring phases, and a limitation of traction cut-off for uphill segments with a slope greater than 25 mm.

Table 1. Possible ATO driving commands.

	Deceleration Rate (m/s^2)	Speed Holding (km/h)	Coasting Speed (km/h)	Re-Motoring Speed (km/h)
Minimum	0.6	30	30	5
Maximum	0.8	80	80	50
Increase	0.05	0.25	0.5	1

The GRASP algorithm was selected among other optimization methods because of its low computational time and near-optimal solutions. To validate it, the proposed algorithm was compared with an optimization algorithm similar to the one used to obtain the efficient speed profiles, multi-objective particle swarm optimization [75] (MOPSO), and was also compared with the most popular algorithm for MOCOPs [76], the nondominated sorting genetic algorithm II (NSGA-II) [77], based on elitism ranks and nondominated selection. In addition, GRASP was compared with four innovative optimization algorithms based on different techniques: weighted optimization framework [78] (WOF), which applies problem transformation through the use of weighted variables; sparse evolutionary algorithm 2 [79] (SparseEA2), a similar algorithm to NSGA-II but with a sparse genetic operator; large-scale evolutionary multi-objective optimization assisted by direct sampling [80] (LMOEA-DS), which reformulates the problem to identify search directions and guiding solutions; and a cooperative coevolution framework applied to generalized differential evolution [81] (CCGDE3), which uses multiple populations with random grouping of decision variables. All of these algorithms were executed on a computer equipped with an Intel Core i7-8700 CPU at 3.20 GHz and 32 GB of RAM.

Once the optimal driving commands per interstation $\hat{x}_s$ were obtained, the previously mentioned algorithms were applied to obtain the Pareto front of efficient driving commands for the entire line. A comparison between the GRASP and the rest of the algorithms is presented in Figure 4. Each algorithm was applied using crisp datasets for ease of comparison, with a precise mass value corresponding to a passenger load of 75%.

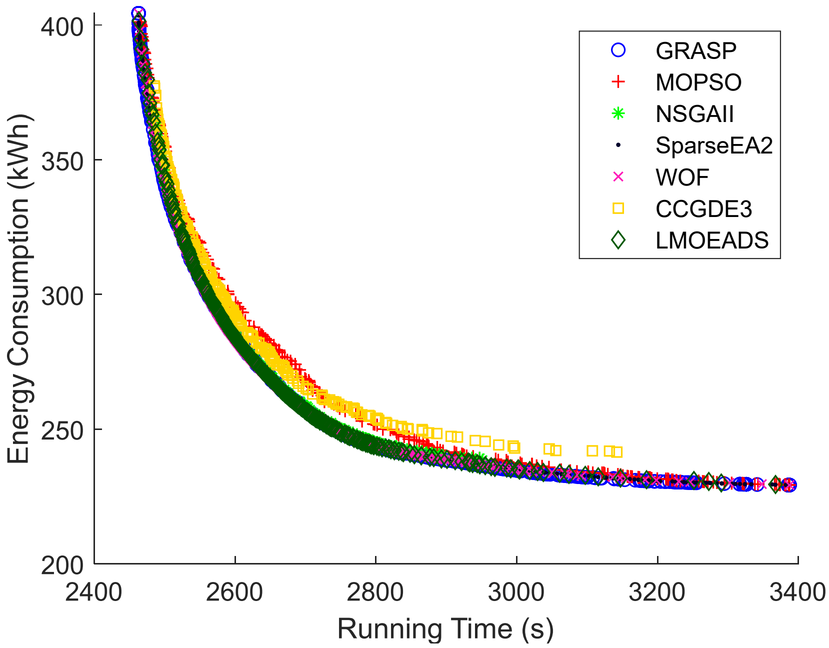

Figure 4. Comparison of the Pareto fronts obtained by the algorithms.

The adjustment parameters of all of the algorithms can be found in Table 2. These parameters were obtained through trial and error, per usual in literature, and showed the best results for optimality in our case study.

The GRASP algorithm is better suited for the case study presented in this paper due to two factors: diversity and optimality. The Pareto fronts obtained by all the tested algorithms can be observed in Figure 4. GRASP reaches the boundary values of the metro line both in running time and energy consumption, whereas algorithms such as NSGA-II and CCGDE3 tend to concentrate their solutions on the central part of the curve. The reason behind this behaviour lies in the weight allocation of the objectives in the GRASP algorithm. These weights change in every iteration randomly, forcing the solutions to become diverse as the algorithm tries to improve them by focusing on one objective over the other.

The evolutionary mechanisms of NSGA-II and CCGDE3 have difficulties providing solutions in the extremes of the Pareto front and generate less diversity of solutions in our case study. Algorithms such as WOF, SparseEA2, and LMOEA-DS present more diverse solutions along the curve but lack in the density of solutions, generating more dispersed points toward the extremes of the curve.

Table 2. Tuned parameters of the algorithms.

MOPSO	**Population**	**Number of generations**	**Cognitive coefficient**	**Social coefficient**	**Mutated genes**	**Mutation rate**
	400	1000	2	2	20	30%
NSGA-II	**Population**	**Number of generations**	**Mutation probability**	**Crossover probability**	**Mutated genes**	
	200	1000	40%	60%	7	
SparseEA2	**Population**	**Number of generations**				
	400	1000				
WOF	**Population**	**Number of generations (original/transformed)**	**Grouping method**	**Transf. function**	**Number of groups**	
	400	1000/500	Ordered	Interval	4	
CCGDE3	**Population**	**Number of generations**	**Number of subpopulations**			
	400	1000	2			
LMOEA-DS	**Population**	**Number of generations**	**Cluster number**	**Number of random sampling along each guiding direction**		
	400	1000	10	30		

The difference in optimality can be explained by resorting to hypervolume computation. Hypervolume indicates the volume of the space that a Pareto front dominates in the objective space [82]. This indicator is used to measure the diversity and convergence of a multi-objective metaheuristic algorithm. Low values of this indicator reveal a higher percentage of the solution space covered by a Pareto front.

The hypervolume values associated with the final result for the different algorithms are displayed in Table 3. The proposed GRASP algorithm has the lowest value of hypervolume, followed by WOF, SparseEA2, and LMOEA-DS. The WOF algorithm shows the second-lowest value because of its optimality in the central part of the Pareto front, but the solutions obtained in the extremes of the curve are less optimal than the ones obtained by the GRASP, thus resulting in a slightly worse hypervolume value. The hypervolume values of NSGA-II and MOPSO are close. However, MOPSO solutions do not reach the optimal front, penalizing its optimality, and NSGA-II does not provide solutions on the extremes, penalizing its diversity. CCGDE3 obtained worse results in both optimality and diversity, which reveals that this algorithm is not suitable for the proposed problem.

Table 3. Execution time for 1000 iterations and hypervolume values.

	GRASP	**NSGA-II**	**MOPSO**	**SparseEA2**	**WOF**	**CCGDE3**	**LMOEA-DS**
Time (s)	8.58	18.39	26.01	68.81	52.15	39.05	52.69
HV	0.1343	0.168	0.1653	0.136	0.135	0.1992	0.1415

To analyse the computational burden of the algorithms, Table 3 shows the execution time for 1000 iterations, and Figure 5 depicts the hypervolume evolution along the computational time. Table 3 shows that, for the execution of 1000 iterations, the GRASP algorithm is the fastest, running in 8.58 s, while the slowest is the SparseEA2 algorithm, taking 68.81 s to execute. In addition, Figure 5 shows that the GRASP algorithm is the fastest in reaching hypervolume convergence, while the SparseEA2 algorithm requires more execution time to obtain similar hypervolume values.

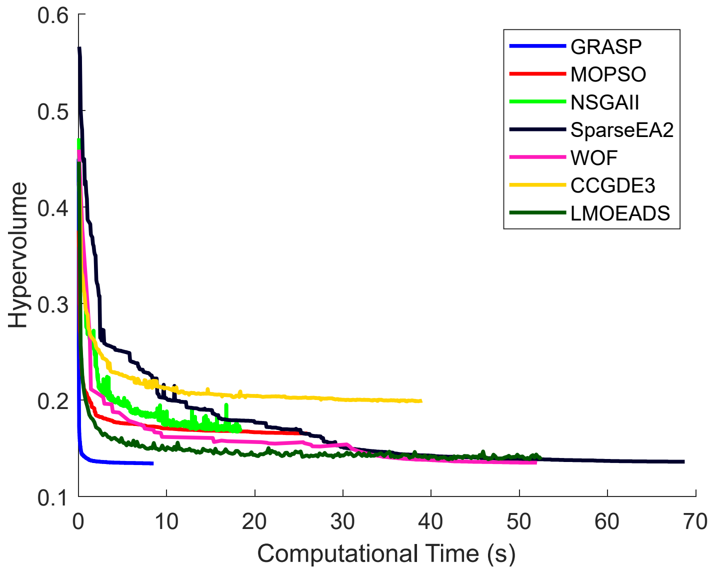

Figure 5. Comparison of the execution time and the hypervolume obtained by the algorithms.

Thus, for all the reasons detailed above, the proposed GRASP algorithm presents the best performance for this combinatorial problem.

6.2. GRASP Application with Fuzzy Mass Model

The design of timetables for the metro line case study was carried out by including uncertainty for the passengers' load in its computation. Following the membership function described in Section 3.3 and Equation (8), a fuzzy mass model was applied (see Figure 3).

The core value (m_a) was 120 tons, representing 75% of the total train mass, and it was the value with the highest possibility. A passenger load equal to the maximum train mass (m_b), 160 tons (100%), was modelled with a possibility (λ) of 0.5. The considered operational scenario has high passenger demand, and the selected λ value affects the shape of the fuzzy number but does not affect the application of the proposed optimization algorithm. The lower train mass value corresponding to 0 possibility (m_l) was equal to 40 tons (25%). An α-cut of 0.7 was applied in this example where lower and upper limits of 60% and 90% passenger load are obtained.

As the train mass was modelled a fuzzy value with associated uncertainty, the resulting speed profiles, running times, and energy consumptions must necessarily be fuzzy values. Therefore, fuzzy dominance must be applied to obtain a fuzzy Pareto front. Strong dominance theory, detailed in Section 3, was used to filter and dismiss dominated solutions in the solution space. This dominance filtering was applied to the sets of speed profiles obtained in the F-MOPSO algorithm for each interstation, resulting in an optimal fuzzy Pareto front for every interstation. Our proposed fuzzy GRASP algorithm uses these fronts to compute a global Pareto front that contains the running times and energy consumption of the different optimal timetables.

The global Pareto front is presented in Figure 6. This figure displays the lower, upper, and core values of the Pareto curve solutions obtained for each F-GRASP iteration. Applying strong fuzzy dominance reveals that many solutions, which would be dominated in the crisp case, cannot be filtered because of the train mass uncertainty modelled by fuzzy numbers.

6.3. Driving Commands Selection

To obtain the timetable associated with a solution from the global Pareto front, a goal running time for the line must be established. Running times obtained in the Pareto curve are fuzzy and require a punctuality restriction over the target running time for the timetable. The running time fuzzy constraints for the timetable design are illustrated in Figure 7. This figure shows the fuzzy running time $\widetilde{T}$. The upper running time is restricted by the goal running time established by the metro operator, T_{obj}. In our case, T_{obj} = 2495 s , which accounts for 5% of the time margin from the fastest journey time.

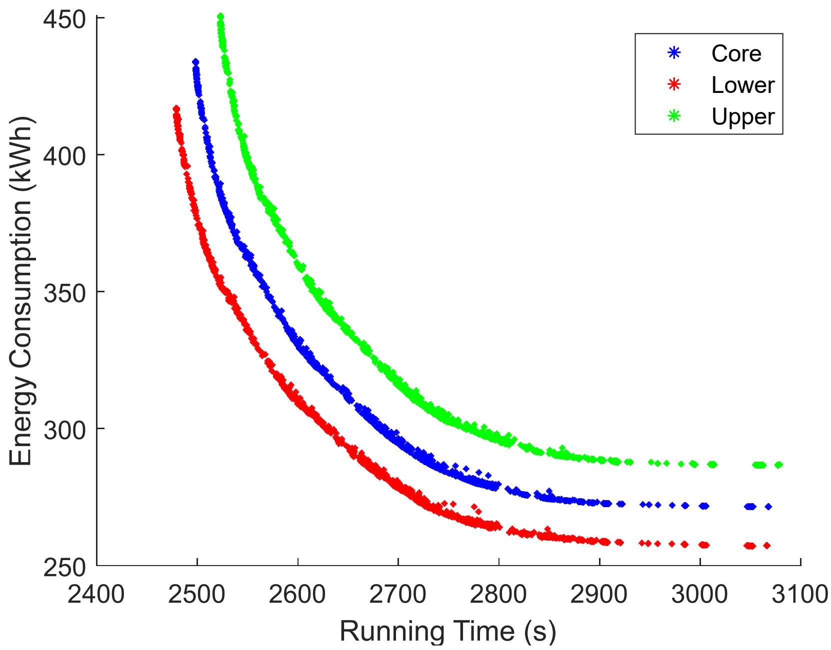

Figure 6. Pareto Front with Fuzzy Mass Model.

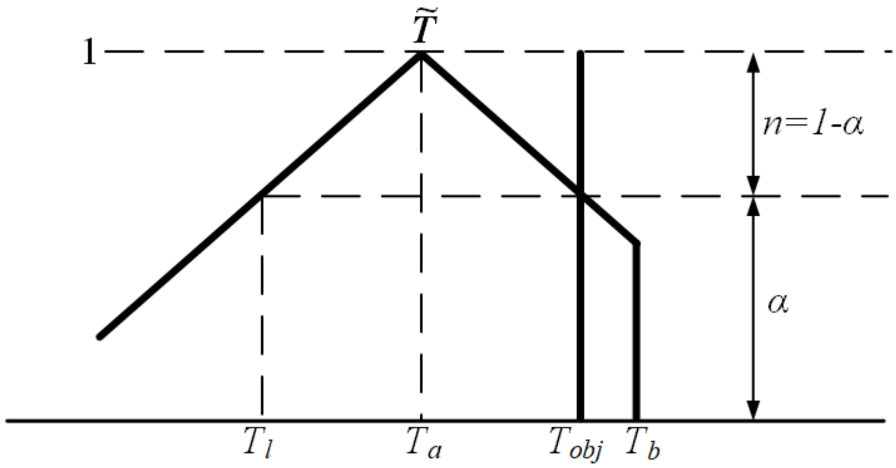

Figure 7. Running time fuzzy constraints for timetable design.

The punctuality restriction was applied by employing a necessity measure $N\left(\widetilde{T} < T_{obj}\right) \geq 0.3$, which is more restrictive than the possibility measure. This type of constraint provides lower running time values for the line to fulfil the constraint. The algorithm could be applied similarly by imposing a possibility measure if more flexibility was given to the punctuality constraint, resulting in a higher running time for the line. Nevertheless, the algorithm is equally efficient, and the quality of solutions is similar in both cases. The selected value of the necessity measure in this case study was $n = 0.3$. With $n = 1 - \Pi\left(\widetilde{T} > T_{obj}\right) = 1 - \alpha$, the value of α in Figure 7 was obtained as $\alpha = 1 - 0.3 = 0.7$.

The chosen values for n and, consequently, α, allow the railway administrator to tune the degree of punctuality that must be accomplished. If the selected necessity measure were equal to 1, there would be complete certainty that the imposed constraint is fulfilled. With a value of $n = 0.3$, an intermediate level of certainty is permitted. This parameter can be freely selected by the railway administrator and does not affect the efficient execution of the algorithm.

Given the resulting global Pareto front of the optimization algorithm, a solution can be drawn fulfilling previously explained punctuality criteria and with the minimum energy consumption, as shown in Figure 6.

A solution set of line-driving commands were selected from the F-GRASP outcome and compared with two alternative timetable designs to evaluate the benefits of the F-GRASP solution. These timetables are a flat-out timetable (or minimum running time timetable) and a timetable computed according to the typical timetable design applied by metro operators. This procedure consists of deciding on a total margin time to be added to the fastest journey time and distributing this margin equally across all interstations whenever possible. The selected speed profile for each interstation uses a running time that is lower or equal to the fastest time plus the margin for that interstation.

Table 4 presents the total savings and margins of the previously described timetables. This table displays the lower and upper values of the cycle's running time and energy consumption for the α-cut considered and the core savings and margins of the considered timetables in relation to the minimum running time timetable. The fuzzy GRASP algorithm achieves similar lower and upper running times compared to the typical timetable for metro operators. However, the intelligent search of the F-GRASP algorithm produces more significant reductions in energy consumption due to its exhaustive search in interstations with more significant potential savings. The typical timetable design reduces the energy consumption with respect to the flat-out design by 20.48%, while the proposed algorithm obtains an additional 4% reduction for up to 24.17% savings. Margins for both strategies are near 4.2%, indicating that the optimized approach to select slower speed profiles and distributing non-uniform margins yields improved energy efficiency and maintains delay absorption capabilities. The savings and margins obtained for each interstation are presented in Figure 8. This figure displays the values obtained using the train mass core value, representing the most possible scenario. F-GRASP generally obtains savings equal to or higher than those gained by designing a timetable with typical metro operator criteria. With the intelligent search performed by the F-GRASP algorithm, savings of up to 70% in energy consumption can be achieved for an interstation. These savings are distributed unequally, selecting slower speed profiles in the interstations where greater global energy savings can be achieved, such as interstations 2, 3, 5, 24, and 31. Other interstations have lower potential savings, such as interstations 7, 8, 11, 15, 28, and 32, and faster speed profiles are selected to reduce the total running time of the timetable. The typical timetable design obtained maximum savings of 62% in an interstation but aimed to achieve savings in every interstation, often compromising the total running time to reduce total energy consumption.

Table 4. Comparison of total savings and margin with F-GRASP.

	$\underline{t}_{\alpha=0.7}$ (s)	$\overline{t}_{\alpha=0.7}$ (s)	$\underline{e}_{\alpha=0.7}$ (kWh)	$\overline{e}_{\alpha=0.7}$ (kWh)	Core Savings (%)	Core Margin (%)
Min. Run. Time	2357.6	2399.0	397.1	429.6	-	-
Typical	2459.3	2495.3	315.8	344.6	20.48	4.18
F-GRASP	2465.6	2494.1	298.1	330.7	24.17	4.21

The margin figure is connected to the savings achieved in every interstation. Margins represent the spare running time compared to the minimum running time. As displayed in Figure 8, F-GRASP margins are notably higher in interstations 2, 3, 5, 9, 24, 29, and 31. These margins are related to the energy savings in those interstations where higher potential savings can be obtained. Therefore, the running times associated with the trip increase proportionally to the energy savings due to the selection of slower speed profiles that involve less traction effort, revealing the trade-off between both energy consumption and running time objectives. The typical timetable design obtains more equally distributed margins across the interstations because of its non-intelligent selection of speed profiles. The design strategy of this timetable selects speed profiles locally and does not account for the global effect on the timetable.

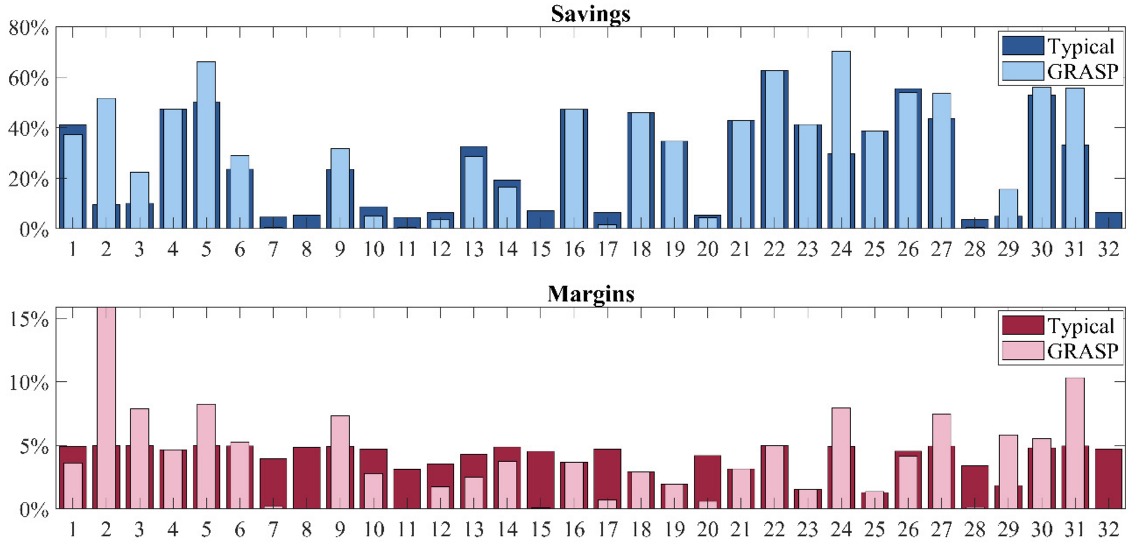

Figure 8. Savings and margins for each interstation.

7. Conclusions

This paper presents an integrated optimization method to design efficient driving commands for all the interstations of a metro railway line and an associated efficient timetable with high-fidelity ATO modelling, comfort restrictions, driving pattern robustness, and train mass uncertainty. This method aims to contribute to the sustainable operations of metropolitan railways by enabling the use of more efficient driving commands.

The proposed method is composed of two-level optimization. At the eco-driving level, a nature-inspired algorithm is used to perform a multi-objective optimization of the speed profiles for each interstation trip. F-MOPSO obtains a Pareto front of solutions, where each one represents a speed profile defined by a set of driving commands with an associated running time and energy consumption. These solutions are obtained considering uncertainty in the train operation by modelling the passengers' load as fuzzy numbers. Moreover, the computed driving commands ensure comfortable and robust speed profiles thanks to the comfort restrictions applied in the optimization.

The speed profiles obtained for each interstation in the first step are fed into a GRASP algorithm with fuzzy parameters (F-GRASP), which performs an intelligent search to optimize the line's total journey time and energy consumption. A global Pareto front is computed, allowing the traffic operator to select a suitable set of speed profiles and the associated timetable in view of the trade-offs between energy consumption and travel time.

Our optimization method has been applied to a case study that uses real data from a Spanish metro line. A real train and 32 interstations have been accurately modelled to test the algorithms. To validate the optimization algorithm, GRASP has been compared with other popular optimization methods for multi-objective problems. The performance of these algorithms has been evaluated, revealing that our F-GRASP algorithm is the most suitable candidate given the quality of the solutions regarding optimality and diversity metrics, as well as computational complexity.

The results from our case study reveal that the proposed method allows the design of driving commands that optimize the global energy consumption of the line journey. Consequently, the driving commands obtained produce 24% energy savings with a 5% increase in travel time compared to the fastest trip time. Furthermore, compared with the typical timetable design, which equally shares the time margin in all the interstations, the proposed design procedure obtained 4% more energy savings. These extra savings are the result of the intelligent distribution of the time margin achieved by the F-GRASP algorithm by using more time margin in the interstations where more global energy savings can be obtained.

Regarding future works, the following suggestions can be highlighted: the application of the driving commands in actual operations, extending the presented concept to design not only the driving commands for nominal operations but also design alternative driving commands at each interstation to recover delays or to reduce the speed to maintain regular train intervals, and the development of traffic regulation algorithms that control trains to meet timetables associated with driving commands.

Author Contributions: Conceptualization, A.F.-R., A.F.-C. and A.P.C.; formal analysis, G.S.-C.; investigation, G.S.-C., A.F.-R., A.F.-C. and A.P.C.; methodology, A.F.-R., A.F.-C. and A.P.C.; project administration, A.P.C.; software, G.S.-C.; validation, G.S.-C., A.F.-R., A.F.-C. and A.P.C.; writing—original draft, G.S.-C. and A.F.-R.; writing—review and editing, A.F.-R., A.F.-C. and A.P.C. All authors have read and agreed to the published version of the manuscript.

Funding: This research received no external funding.

Data Availability Statement: The data presented in this study are available on request from the corresponding author. The data are not publicly available due to privacy restrictions.

Conflicts of Interest: The authors declare no conflict of interest.

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