A Dynamic Approach to Control for Cohort Differences in Maturation Speed Using Accelerated Longitudinal Designs

Pablo F. Cáncer[1], Eduardo Estrada[1*], and Emilio Ferrer[2]

1. Department of Social Psychology and Methodology. Universidad Autónoma de Madrid (Spain)

2. Department of Psychology. University of California, Davis

*Correspondence should be sent to: eduardo.estrada.rs@gmail.com

Abstract

Accelerated longitudinal designs (ALD) allow studying developmental processes usually spanning multiple years in a much shorter time framework by including participants from different age cohorts, which are assumed to share the same population parameters. However, different cohorts may have been exposed to dissimilar contextual factors, resulting in different developmental trajectories. If such differences are not accounted for, the generating process will not be adequately characterized. In this paper, we propose a continuous-time latent change score model as an approach to capture cohort differences affecting the speed of maturation of psychological processes in ALDs. This approach fills an important gap in the literature because, until now, no method existed for this goal. Using a Monte-Carlo simulation study, we show that the proposed model detects cohort differences adequately, regardless of their size in the population. Our proposed model can help developmental researchers control for cohort effects in the context of ALDs.

Keywords: Accelerated longitudinal designs; cohort differences; speed of maturation; latent change score models; continuous time models; state space models

A Dynamic Approach to Control for Cohort Differences in Maturation Speed Using

Accelerated Longitudinal Designs

The study of developmental phenomena typically involves taking multiple repeated measures across many years. Accelerated longitudinal designs (ALDs; Bell, 1953, 1954; Duncan et al., 1996), also known as cohort-sequential (Nesselroade & Baltes, 1979) or cross-sequential designs (Schaie, 1965), allow studying processes that unfold over long periods of time, but in a much shorter time frame. The key aspect of ALDs is that participants enter the study at different ages—that is, they come from different *age* or *birth cohorts*. Consider, for example, a study examining the development of cognitive abilities from ages 5 to 25. In a conventional longitudinal design, such a study would require following each participant for 20 years, which is unfeasible in most cases. In contrast, in an ALD the researcher can select a sample of participants ranging in age from 5 to 22, and measure them once a year during four consecutive years. This combination of cross-sectional and longitudinal information from different age cohorts results in a complete coverage of the target age range, even though each participant provides information for only a fraction of the total study period. Figure 1 illustrates a hypothetical example of individual longitudinal data of participants measured every year for 20 years (left panel) and data from an ALD in which participants are measured every year for four years (right panel).

[FIGURE 1]

ALDs have been most extensively used in the field of cognitive development, especially during childhood, adolescence, and early adulthood (e.g., Estrada et al., 2019; Fandakova et al., 2017; Ferrer et al., 2009; Ferrer & McArdle, 2004; Green et al., 2017; Wendelken et al., 2017), but also covering the entire life-span (McArdle et al., 2002; McArdle & Woodcock, 1997). Particularly during childhood and adolescence, one of the most relevant aspects of developmental

change concerns the speed of maturation. Certain individuals mature faster than others—that is, they reach the peak of their development at earlier ages. For example, previous studies on cognitive abilities have identified groups of children that reach similar levels of reading and verbal performance, but at different speeds (Ferrer et al., 2010; Holahan et al., 2018).

The main assumption of ALDs is that all the cohorts share a common developmental trajectory, and this trajectory can be studied by linking together all the segments of data provided by each cohort. This assumption, originally termed *convergence* (Bell, 1953, 1954), is referred to as *cohort equivalence* in this manuscript. We define two or more cohorts as being *equivalent* when their trajectories can be described with the same set of parameter values—that is, when they come from the same population. In populations with little or no changes over the years, this assumption is reasonable. Nevertheless, human societies are dynamic and constantly exposed to potentially transformative events such as technological and medical advances, financial or health crisis, migrations, or changes in the educational system, among many others. In such scenarios, differences across cohorts may emerge due to differences in some of these social and economic factors. If such cohort differences are not accounted for, ALDs are not capable of disentangling age-related changes from cohort differences, leading to substantially biased estimates (Estrada & Ferrer, 2019).

In the present study, we propose an approach to identify and control for cohort differences in the speed of maturation of a developmental process in the context of ALDs. For this, we use a latent change score approach using state-space equations in continuous time. Latent change score models, also called latent difference models (LCS; Ferrer & McArdle, 2003; McArdle, 2001, 2009; McArdle & Hamagami, 2001) are a dynamic approach to the study of

developmental data in which the changes, instead of the levels, are the focus and are modeled as latent variables. This approach offers several advantages over other methods in the literature. In the remainder of the article, we review previous approximations to the modeling of cohort differences and the speed of maturation. Next, we introduce latent change score models, continuous-time models, and their joint implementation within a state-space modeling framework. We describe the proposed model and evaluate its performance under different sampling conditions in a simulation study. Finally, we discuss the results and elaborate on the strengths and limitations of the model.

Previous approaches to examine cohort differences in the speed of maturation

One of the first successful attempts to account for cohort differences in developmental change came from Miyazaki & Raudenbush (2000), who proposed a hierarchical model that captured cohort differences in an intercept, a linear slope, and a quadratic slope. This approach was later adopted by empirical researchers and extended to latent growth curves (LGC) and other multilevel models (e.g., Finkel et al., 2007; Gerstorf et al., 2011; Hoffman et al., 2011; Orth et al., 2015). Research based on these approaches has considered cohort differences in two aspects of the model: a) the intercept, which usually corresponds to the level in the first measurement occasion, and b) the rate of change, which includes all the linear and higher order components that account for change in the variable of interest[1]. LGC and multilevel approaches, however, have at least two important limitations. First, linear and higher order components are not capable of separating the speed of the changes from their size. For example, larger scores in a linear component involve steeper slopes (i.e., faster changes), but also higher scores in the long-term

---

[1] Quadratic and higher order components are also sometimes interpreted a rate of acceleration (e.g., Finkel et al., 2007; Gerstorf et al., 2011).

(i.e., larger changes). Adding polynomial terms to these models can help to better capture the shape of the trajectories, but they are not interpretable in terms of development. The second limitation is that LGC and multilevel models are not dynamic models. These models are useful to describe change as a function of time, but they cannot explain the mechanisms that bring about change (see, for example, McArdle, 2009; Voelkle et al., 2018). In psychological science, most developmental theories describe individuals and environmental influences on them as a dynamic system, inasmuch it unfolds continuously over time and its changes are (at least partially) determined by the past history of the system. In order to disentangle the components of change, models are needed that can capture the dynamic nature of developmental processes.

In a recent study, Estrada et al. (2021) proposed using LCS models as a dynamic approach to the study of cohort differences. In fact, they proposed an extension of these models that successfully captured cohort differences in the initial and maximum level of the trajectories. However, they did not consider potential differences in the speed of maturation.

LCS models have been frequently used for the study of development from childhood to early adulthood. In this age range, most traits (e.g., cognitive abilities) show a rapid growth during the first years followed by a progressive deceleration, until they reach a maximum level between 20 and 30 years of age (e.g., Figure 1)—the exact age depends on the specific ability and the individual (McArdle et al., 2002). LCS models are particularly useful for the study of this type of trajectories because they capture three key aspects of exponential functions: 1) the initial state, representing the level at the onset of the process (typically, the first measurement occasion), 2) the asymptote, representing the maximum level to which the trajectories tend, and 3) the speed of maturation (or decay), which determines how fast the process is reaching the

asymptote. Thus, unlike LGCs and multilevel models, LCS models can separate the speed of maturation from the overall growth, the latter being indicated by the position of the asymptote. Figure 2 depicts various examples of trajectories in which all cohorts reach the same maximum level in the long-term (captured by the asymptote in the model), but older cohorts (i.e., individuals born earlier) show slower maturation.

[FIGURE 2]

Importantly, the speed of maturation in univariate LCS models is explicitly captured by a self-feedback or auto-proportion parameter. In the empirical and methodological literature, this parameter is described as the extent to which the state of a process at any given time t is determined by its previous state at $t-1$. However, to the best of our knowledge, no previous empirical studies have used it as an indicator of the speed of maturation, and very few methodological papers have acknowledged it as such (see Cáncer et al., 2021; Estrada et al., 2021). Furthermore, and to our best knowledge, the model proposed in this manuscript is the first approach to controlling for cohort differences in the speed of maturation. Therefore, although cohort differences in the so-called *rate of change* (i.e., overall change, typically captured with linear and higher order components in multilevel and growth models) have been widely reported in the literature (e.g., Cole, 2000, 2003; Drewelies et al., 2018; Estrada et al., 2021; Finkel et al., 2007; Gerstorf et al., 2011; Vainikainen & Hautamäki, 2022; Zhang et al., 2020), it is hardly possible to find empirical examples of cohort differences in the speed of maturation specifically. This does not mean, however, that such differences do not exist at all (see, for example, Eckert-Lind et al., 2020). Note that, when researchers report cohort differences in certain features of the trajectories, they are actually reporting differences in certain parameters of their model. Some

specific aspects of development, such as the speed of maturation, are not captured by the parameters of static models typically used in developmental studies. Thus, cohort differences in maturation speed may go unnoticed in such studies, or confounded with other aspects of development.

A mathematical description of latent change score models

In a typical specification of the LCS model, the so-called *dual* LCS model, latent changes in a process *y* are a function of: (a) the latent state of the process at the previous occasion *t*−1, through a self-feedback parameter β, and (b) an additive component (sometimes termed slope), representing a linear effect on the system. Therefore, the changes for each individual *i*, at any time *t*, are expressed as:

$$\Delta y_{i[t]} = \beta \cdot y_{i[t-1]} + y_{a,i} \tag{1}$$

The left panel of Figure 3 represents the path diagram of a univariate LCS model. This model combines information from the initial level, the additive component, and the self-feedback to generate specific trajectories for each individual. Equation 1 specifies two sources of between-individual variability: a) the initial level, which captures the mean μ_0 and variance σ^2_0 in the latent level at the first measurement occasion, and b) the additive component, which captures the mean μ_a and variance σ^2_a of the latent linear component added at each repeated occasion. The initial level and additive component are usually allowed to be correlated, with covariance $\sigma_{0,a}$ (or correlation $\rho_{0,a}$). The mean and variance in the maximum level of the trajectories (or the asymptotes, parameters μ_{As} and σ^2_{As}, respectively) are not directly estimated in LCS models. Instead, they are obtained through the following equations:

$$\mu_{As} = \mu_a / (-\beta) \tag{2}$$

$$\sigma^2_{As} = \sigma^2_a / (-\beta)^2 \tag{3}$$

where it is shown that the variance of the additive component σ^2_a captures individual differences in the maximum level of the trajectories. Once the initial state and additive component are specified, all within-individual variability is determined by the change equation (Equation 1), and any observed deviations from the implied latent trajectories are considered measurement error (with variance $\sigma^2_e)^2$.

[FIGURE 3]

In LCS models, the self-feedback parameter has a dual interpretation: in the *short-term*, it represents the extent to which changes from $t{-}1$ to t are determined by the state of the process at $t{-}1$, whereas in the *long-term*, it represents the rate (or speed) at which the process moves with respect to the asymptote. As described previously, most developmental processes from childhood to early adulthood can be assumed to follow exponential trajectories of decelerated growth. In such trajectories, the self-feedback is always negative, representing a deceleration effect—that is, the process tends to an equilibrium or asymptote[3]. Therefore, larger (i.e., more negative) self-feedbacks will result in trajectories that approach the asymptote more quickly, implying a higher maturation speed (for a detailed interpretation of the LCS model parameters, see Cáncer et al., 2021).

---

[2] Further within-individual variability could be incorporated into the model in the form of prediction errors (i.e., innovation or dynamic error) at the latent level (Oravecz et al., 2011; Voelkle et al., 2012; Voelkle & Oud, 2015). This specification, however, is very uncommon in the LCS framework.

[3] LCS models are also capable of capturing exponential trajectories of accelerated (instead of decelerated) change, which are defined by a positive self-feedback. In such scenarios, scores further from the initial level lead to larger subsequent changes, resulting in a pattern of "explosive" change, where even small increases in time lead to dramatic changes in the variable of interest.

Importantly, the standard LCS model (Equation 1) depicts a system in which changes occur in discrete time steps, which has problematic implications in the context of ALDs. LCS models in discrete time (LCS-DT) assume that all participants are measured at the exact same time points—that is, measurement intervals are equal across participants and occasions. This assumption is rather difficult to hold in empirical applications, and even more so in ALDs, where participants are rarely measured at the same age. A typical scenario is, for example, one in which a participant is measured at ages 5.32, 6.41, and 7.23, whereas another is measured at ages 5.84, 6.36, and 7.58. Although both belong to the same cohort, the exact time interval is not constant across participants, nor across occasions for each participant. Furthermore, previous research has shown that LCS-DT models do not provide accurate estimates of the generating parameters when time intervals are unevenly spaced (see Estrada & Ferrer, 2019). In the next section, we introduce continuous-time modeling as a solution to this problem.

Continuous time models

Continuous-time (CT) models have been proposed as a powerful approach for the study of psychological processes in longitudinal research (de Haan-Rietdijk et al., 2017; Deboeck & Preacher, 2016; Oud & Delsing, 2010; Oud & Jansen, 2000; Oud & Singer, 2008; Ryan et al., 2018; van Montfort et al., 2018; Voelkle et al., 2012; Voelkle & Oud, 2013, 2015). These models use differential equations to describe change in the process of interest, which is assumed to unfold in continuous time. Equation 1, where the change in y occurs over a time lag of Δt=1 (the left-hand side of the equation could be specified as $\Delta y/1$), could be considered a crude approximation of the underlying continuous process. In CT, this equation can be re-expressed as a first order ordinary differential equation that provides the change in y (dy) as a function of the level $y(t)$, for an infinitesimally brief time lag dt (Brown, 2007):

$$\frac{dy_i(t)}{dt} = \beta \cdot y_i(t) + y_{a,i} \tag{4}$$

Continuous-time models account for the exact occasions of measurement to estimate parameters that are independent of the time lag and can be transformed to any specific time interval. Therefore, they can naturally account for time intervals of any length, be they equal or unequal across occasions or participants. From a practical point of view, this allows comparing parameters that are estimated using different time intervals (see Voelkle et al., 2012). Also, one could argue that most, if not all, psychological processes are assumed to unfold continuously over time (i.e., they do not stop existing between observations). In this regard, CT models provide a more theoretically accurate representation of the seamless nature of developmental processes.

In the context of ALDs, where each individual provides only few observations and time intervals can differ widely across individuals, CT models are a more adequate approach. In fact, previous studies comparing LCS models in discrete and continuous time with data from ALDs found that CT models provide a much better recovery of the generating parameters when the observations are unevenly spaced (see Estrada et al., 2021; Estrada & Ferrer, 2019)).

Modeling change in continuous time: state-space models

In this study, we use a state-space approach in continuous time (SSM-CT) to detect and control for cohort differences in the speed of maturation. SSMs were originally developed in the field of mechanical and electrical engineering with the purpose of detecting, predicting, and separating random signals from noise (Kalman, 1960; Kalman & Bucy, 1961). In recent years, they have been introduced to the study of dynamics in psychological research (Chow et al., 2010; Gu et al., 2014; Hunter, 2018; Ji & Chow, 2019; Oud & Jansen, 2000). In the context of

psychological processes, this approach allows detecting temporal dynamics, predicting future states of the system, and separating the latent relevant process from measurement error.

SSMs have two main components: the state equation and the output equation. In continuous-time, the *state (or transition) equation* is a first-order ordinary differential equation that describes change in a vector of latent variables for an infinitesimally brief time interval (dt):

$$dy_i(t) / dt = Ay_i(t) + Bu_i(t) + q_i(t) \tag{5}$$

Where $y_i(t)$ is a $l \times 1$ vector of latent states for each individual i, u_i is a $m \times 1$ vector of observed covariates, q_i is a $l \times 1$ vector of dynamic noise (i.e., prediction error) with mean zero and covariance Q, A is a $l \times l$ matrix of auto-regressive dynamics, and B is a $l \times m$ matrix of covariate effects on the latent state y_i. In this framework, the latent states at $t=0$ (namely, initial conditions; see Ji & Chow, 2019) are defined by a latent initial mean vector, noted as x_0, and a latent initial covariance matrix, noted as P_0 (Hunter, 2018).

The second component of the SSM model is the *output equation*, which is equivalent to the measurement structure in structural equation models (SEM). It links the latent level to the time-specific observations and separates latent scores from measurement error:

$$Y_i(t) = Cy_i(t) + Du_i(t) + r_i(t) \tag{6}$$

where Y_i is a $n \times 1$ vector of observed (or manifest) variables, r_i is a $n \times 1$ vector of observation noise (i.e., measurement errors) with mean zero and covariance R, C is a $n \times l$ matrix of factor loadings, and D is a $n \times m$ matrix of covariate effects on the observed state Y_i.

The parameters in SSM-CT models are estimated using a recursive algorithm called Kalman Filter, consisting of a series of alternating prediction and correction steps (Chow et al.,

2010; Hunter, 2018). In the prediction step, the filter uses information from the state vector and its covariance matrix at time $t-1$ to create a forecast for the state vector and covariance at time t. In the correction step, it uses the observed data and measurement model to update the forecast from the previous step. As the Kalman Filter iterates across individuals and time, it produces Kalman scores (similar to the latent scores in SEM) that can be substituted into a log-likelihood function. This function, called the *prediction error decomposition function*, iteratively reduces prediction errors to obtain maximum likelihood estimates of the model parameters (for further details, see Boker et al., 2018; Chow et al., 2010; Hunter, 2018; Kalman, 1960; Kalman & Bucy, 1961; Oud & Jansen, 2000).

The right panel of Figure 3 illustrates a univariate LCS model in continuous-time specified as a SSM. In a SSM-CT framework, the auto-regressive dynamics of the continuous-time LCS model (LCS-CT) described in Equation 4 can be respecified into a state equation as:

$$\frac{d}{dt}\begin{bmatrix} y_{l,i} \\ y_{a,i} \end{bmatrix} = \begin{bmatrix} \beta & 1 \\ 0 & 0 \end{bmatrix}\begin{bmatrix} y_{l,i} \\ y_{a,i} \end{bmatrix} \tag{7}$$

where y_l represents the time-varying latent level for each individual, and y_a represents the time-invariant latent linear component —note that its mean and variance do not change over time. This latter component influences the latent level y_l but it is not influenced by it. Both latent variables are defined at time zero as a multivariate normal distribution with mean vector and covariance matrix:

$$\begin{bmatrix} y_{l0} \\ y_a \end{bmatrix} \sim N\left(\boldsymbol{x}_0 = \begin{bmatrix} \mu_0 \\ \mu_a \end{bmatrix}, \boldsymbol{P}_0 = \begin{bmatrix} \sigma_0^2 & \sigma_{0,a} \\ \sigma_{0,a} & \sigma_a^2 \end{bmatrix} \right) \tag{8}$$

Similarly, the output equation of a LCS model in which the process of interest is measured with a single indicator[4] can be expressed as:

$$[Y_i] = \begin{bmatrix} 1 & 0 \end{bmatrix} \begin{bmatrix} y_{l,i} \\ y_{a,i} \end{bmatrix} + [e_i] \tag{9}$$

where the additive component $y_{a,i}$ is not linked to any observation and e_i represents the measurement error, with mean 0 and time-invariant variance σ^2_e.

Note that the SSM-CT in Equation 7 is a continuous time specification of the LCS-DT described in Equation 1 and has the same number of parameters. However, the interpretation of the parameters differs in two key aspects: a) the time metric and b) the definition of the initial conditions. In a LCS-DT model, the parameters β, μ_a, σ^2_a, and $\sigma_{0,a}$ are scaled for a time lag $\Delta t=1$, whereas in a SSM-CT model they are scaled for an infinitesimally brief time lag (dt). Similarly, in a LCS-DT model, μ_0 and σ^2_0 are the mean and variance at the first measurement occasion ($t=1$), whereas in SSM-CT they represent the state of the system at time zero, which can refer to any arbitrary point in time (not necessarily the first occasion). For further details on their mathematical relation, see Chow et al. (2010), Estrada & Ferrer (2019), Hunter (2018), Oud & Jansen (2000), or Voelkle & Oud (2015). In the next section, we present an extension of the SSM-CT model that allows cohort differences in the self-feedback parameter (i.e., the speed of maturation).

Modeling cohort differences in the speed of maturation: A SSM-CT model with moderators

---

[4] When the latent level of *y* is measured by multiple indicators, Equation 9 can be extended to include them. In that case, the dimensions of the vector of observed variables, the matrix of factor loadings, and the vector of measurement errors are rescaled according to the number of indicators.

Consider a study examining the development of fluid reasoning from childhood to early adulthood in which participants were born between 2000 and 2009. As researchers, we may want to test whether younger generations (i.e., individuals born later) may be experiencing a faster maturation in fluid reasoning abilities, perhaps due to recent technological advances or changes in the educational system. To represent such an effect, we can use the year of birth to create an observed variable *coh* with values from 0 to 9 (from the youngest to the oldest cohort), representing the cohort to which each participant belongs. In the context of LCS models, differences in the speed of maturation of fluid reasoning across cohorts will involve cohort differences in the self-feedback parameter. In order to capture such differences, we propose an extension of the LCS model that uses the cohort as a moderator of the auto-regressive dynamics across latent states. In our model, between-cohort variability in the self-feedback parameter can be modeled by specifying the following state equation:

$$\frac{d}{dt}\begin{bmatrix} y_{l,i} \\ y_{a,i} \end{bmatrix} = \begin{bmatrix} \beta + \lambda_\beta \cdot coh_k & 1 \\ 0 & 0 \end{bmatrix}\begin{bmatrix} y_{l,i} \\ y_{a,i} \end{bmatrix} \tag{10}$$

where coh_k is the observed value in the cohort variable for each cohort k, β determines the value of the self-feedback when $coh_k=0$ (in our example, the youngest cohort), and λ_β represents the change in the self-feedback for each unit of change in the cohort. This formulation is an extension of Equation 7 that allows capturing variability in the speed of maturation (i.e., the self-feedback β) due to differences in the cohort variable.

In Equation 10, β and λ_β are freely estimated, whereas coh_k is considered a fixed parameter. In some statistical software, such as OpenMx (Boker et al., 2018), observed variables can be inserted into the state-space matrices to specify statistical models at the individual level. These variables (usually termed *definition variables*) are specified as fixed parameters that take,

for each individual, their corresponding observed value in that variable (e.g., coh_k in Equation 10). In the context of ALDs, with individuals grouped into cohorts, we use this functionality to specify models at the cohort level, with all individuals within a specific cohort sharing the same model (i.e., the same set of parameter values).

As described previously, the self-feedback is related to the additive component and the asymptotes through Equations 2 and 3. Such relations imply that, for a fixed asymptote, differences in the self-feedback will lead to differences in the mean (μ_a) and variance (σ^2_a) of the additive component. Also, because covariances are sensitive to the metric of the variables, changes in the additive component variance will lead to differences in its covariance with the initial state ($\sigma_{0,a}$), but not necessarily in their correlation ($\rho_{0,a}$), as this is independent of the metric. Therefore, capturing cohort differences in the self-feedback parameter will involve modeling such differences also in: a) the vector of initial latent means (x_0) and b) the initial latent covariance matrix (P_0). Figure 3 shows, marked with an asterisk, the parameters that may vary due to cohort differences in the speed of maturation. In our model, cohort differences in the vector of initial means are specified as:

$$x_0 = \begin{bmatrix} \mu_0 \\ \mu_a + \lambda_{\mu a} \cdot coh_k \end{bmatrix} \tag{11}$$

where all parameters (except coh_k) are freely estimated, μ_0 captures the mean state of the process at $t=0$, μ_a is the additive component mean when $coh_k=0$, and $\lambda_{\mu a}$ captures the change in the additive component mean for each unit of change in the cohort. This specification of the latent states allows capturing variability in the mean of the additive component due to differences in the cohort variable.

Cohort differences in the initial latent covariance matrix are modeled by first decomposing this matrix as $\boldsymbol{P}_0 = \boldsymbol{DRD}$, where $\boldsymbol{D}$ is the diagonal matrix of standard deviations and $\boldsymbol{R}$ is the correlation matrix, and then specifying cohort effects in the standard deviation of the additive component:

$$\boldsymbol{P}_0 = \begin{bmatrix} \sigma_0 & 0 \\ 0 & \sigma_a + \lambda_{\sigma a} \cdot coh_k \end{bmatrix} \begin{bmatrix} 1 & \rho_{0,a} \\ \rho_{0,a} & 1 \end{bmatrix} \begin{bmatrix} \sigma_0 & 0 \\ 0 & \sigma_a + \lambda_{\sigma a} \cdot coh_k \end{bmatrix} \tag{12}$$

where all parameters (except coh_k) are freely estimated, σ_0 is the standard deviation of the latent initial state, σ_a is the standard deviation of the additive component when $coh_k=0$, $\rho_{0,a}$ is the cohort-invariant correlation between the initial state and the additive component, and $\lambda_{\sigma a}$ captures the change in the standard deviation of the additive component for each unit of change in the cohort variable[5]. This specification of the latent covariance matrix captures cohort differences in the additive component variance and, at the same time, accounts for the resulting differences in the metric of the covariance with no need for additional parameters.

Importantly, in the previous example on fluid reasoning, the values of the cohort variable from the youngest to the oldest cohort were {0, 1, 2, 3, 4, 5, 6, 7, 8, 9}, representing a linear change. However, these values are arbitrarily chosen by the researcher, and other sequences are possible depending on particular hypotheses. If the change in β is expected to be non-linear across cohorts, the sequence {0, 1, 4, 9, 16…} can be used to represent a quadratic cohort effect. Other transformations of the cohort variable can be applied to account for relations of various forms (i.e., cubic, square root, logarithmic, or exponential, among others). In the supplementary

---

[5] Note that the dual LCS model described in previous sections and the LCS model with moderators from Equations 10-12 differ in the interpretation of some parameters. In the former, the parameters β, μ_a, and σ_a describe the trajectories of the whole sample of participants, whereas in our model they refer to participants with a value of zero in the cohort variable (i.e., when $coh_k=0$).

materials, we include annotated R code for estimating the SSM-CT model with moderators, as well as a SEM version of the model in discrete-time.

Method

The goal of the present study is to examine the ability of a SSM-CT model with moderators to identify and control for cohort differences in the speed of maturation in the context of ALDs. For that purpose, we generated repeated measures of a latent process y that unfolds over 15 years. The process was generated according to the LCS model described in Equation 1. In empirical longitudinal studies, time intervals between assessments typically vary across both time points and participants (e.g., see Voelkle et al., 2012). We reproduced these unequal intervals by dividing each year into 52 weeks. That is, we used a semi-continuous LCS model to generate 52 waves of data per year, for a total of 780 waves. Then, for each year, we chose one of the waves at random with equal probability for each wave (i.e., a probability of 1/52). This resulted in a 15-wave database with unevenly spaced time intervals of random lengths between and within participants. Because the key feature of ALDs is that participants are only measured during a fraction of the target age range under study, the number of waves per cohort was trimmed according to different patterns of planned missing data (see the section *Sampling Schedule*, below).

The parameters of the model were chosen to represent trajectories that are typical of the development of cognitive abilities from childhood to early adulthood (e.g., Kail & Ferrer, 2007; Schmitt et al., 2017; Van Der Maas et al., 2006). These parameters are reported in Table 1, and were based on previous empirical studies (Ferrer et al., 2007, 2010; Shaywitz et al., 1990). One of the strengths of continuous-time models is that the resulting CT parameters can be

transformed into DT parameters for any specific time interval. For a detailed description of such transformations, see Voelkle & Oud (2015) and Estrada & Ferrer (2019).

[TABLE 1]

The parameters in Table 1 were used as a baseline to generate 100 replications for each of the 48 simulation conditions, which were created by fully crossing the following three factors, explained in the next sections:

1. Size of cohort effect: four conditions ($d = \{0, .1, .2, .4\}$)

2. Sampling schedule: four conditions (D0, D1, D2, and D3)

3. Sample size: three conditions ($N = \{125, 200, 500\}$)

Size of cohort effect

In this manuscript we present the first approach to modeling cohort differences in the speed of maturation captured by a self-feedback parameter. To the best of our knowledge, there are no previous references of how large such differences may be in empirical data, although their size will probably vary widely depending on the construct, the developmental period under study, or the distance in year of birth across cohorts. As guidelines for our simulation, we used a study by Ferrer et al. (2010) that examined the development of reading abilities and IQ in a representative sample of schoolchildren from ages 6 to 18. In this study, the authors used LCS models to evaluate differences in reading abilities between groups. They found differences in the self-feedback parameters of around .2 in reading abilities and .1 in IQ across groups. Based on this study, we included four different effect sizes in our simulation: $d = \{0, 0.1, 0.2, 0.4\}$.

In the baseline condition ($d = 0$), all cohorts were defined by the same set of parameters reported in Table 1 (i.e., no cohort effects). The remaining conditions implied small, medium,

and large cohort effects. The size of the effect was defined as the absolute difference in the self-feedback between the youngest and the oldest cohort. Cohort differences were meant to reproduce a pattern of differences where the youngest cohort had the largest (or most negative) self-feedback (i.e., the fastest maturation speed), and this value decreased linearly down to the oldest cohort. For example, with an effect of $d=.4$ and 11 cohorts, the values of the self-feedback parameter from the youngest to the oldest cohort were $\{-.45, -.41, -.37, -.33, -.29, -.25, -.21, -.17, -.13, -.09, -.05\}$. The range of self-feedback parameter values across all conditions of cohort effect size is reported in Table 2. Note that, across all conditions, the value of the self-feedback in the middle cohort (or the average of the two middle cohorts) was $-.25$.

Because of the dependence of the asymptote on both the self-feedback and the additive component (see Equations 2 and 3), differences in the self-feedback also involved differences in the mean and variance of the additive component to ensure that the mean and variance of the asymptote were cohort-invariant. Thus, in conditions with an effect of $d=.4$ and 11 cohorts, the parameter values were $\{13.5, 12.3, 11.1, 9.9, 8.7, 7.5, 6.3, 5.1, 3.9, 2.7, 1.5\}$ for the additive component mean, and $\{5.06, 4.20, 3.42, 2.72, 2.10, 1.56, 1.10, .72, .42, .20, .06\}$ for the additive component variance. The ranges of these parameters across all conditions are also reported in Table 2.

[TABLE 2]

Sampling schedule

There are many possible sampling schedules for an accelerated longitudinal design, depending on the desired length of the study, the average time interval between assessments, or the available budget. For this study, we chose sampling designs that could be carried out in a

maximum of five years. We based our choice on the findings of Estrada & Ferrer (2019), who evaluated the performance of the LCS-CT model under seven different sampling designs. Among them, we selected the three designs (D1, D2, and D3) that showed the best performance and cost-efficiency. These designs are summarized in Figure 4.

[FIGURE 4]

As a reference, we added a benchmark design (D0) consisting of full trajectories for each participant in each cohort. It involved 13 cohorts in which every participant was measured yearly from ages 5 to 19. Although this design is not very feasible in empirical studies, it allows examining the ability of the model to recover cohort effects when full trajectories are available, and serves as a baseline against which to compare the performance of the other designs.

Sample size

In a previous study, Estrada et al. (2021) found that only 125 participants were required to recover cohort effects in the mean latent initial level and asymptote of the trajectories using LCS models. Here we included the following three sample sizes: 125, 200, and 500 individuals per sample.

Estimation and analysis

For each sample in each condition, we fitted the SSM-CT model with cohort-related parameters described in Equations 10-12 and freely estimated the following ten parameters: initial state mean and standard deviation (μ_0 and σ_0), additive component mean and standard deviation (μ_a and σ_a), correlation between initial state and additive component ($\rho_{0,a}$), self-feedback parameter (β), measurement error variance (σ^2_e), and three cohort-related parameters (λ_β, $\lambda_{\mu a}$, and $\lambda_{\sigma a}$). The model was estimated with the functions

mxExpectationStateSpaceContinuousTime and *mxFitFunctionML* from the R package OpenMx

(Boker et al., 2018; Hunter, 2018; Neale et al., 2016). The R code for generating the data sets and

estimating the model is available at: https://github.com/PFernandez-Cancer/ALD_cohort_effects.

Results

The model converged for all samples and conditions. We evaluated the relative bias,

variability, and rates of 95% confidence interval (CI) coverage of the parameter estimates across

all simulation conditions. These results are reported in Figures 5-7. All numerical results can be

found in the Supplemental Materials.

Relative bias

The relative bias of the estimates in each condition was computed as $RB = (\overline{\theta}_{est} - \theta) / \theta$,

where $\overline{\theta}_{est}$ is the average estimate across all replications in a given condition, and θ is the true

parameter value[6]. Values of RB closer to 0 imply unbiased estimates, positive values imply

overestimation, and negative values imply underestimation. Consistent with previous literature,

we considered estimates to be non-trivially biased if |RB|>.10 (Flora & Curran, 2004). For a

general overview of the model performance, we also report the Root Mean Square of the

Relative Bias, computed as $RMS(RB) = \sqrt{\sum_{k=1}^{K} RB_k^2 / K}$, where *K*=10 is the number of

parameters in the model.

The top panel of Figure 5 depicts the RB for all parameters across all simulation

conditions. Overall, the recovery of the generating parameters was excellent. As expected,

---

[6] When cohort effects are null (*d*=0), the parameters λ_β, $\lambda_{\mu a}$, and $\lambda_{\sigma a}$ have populational values of zero
(θ=0), and thus computing the *RB* would imply dividing by zero. In such conditions, *RB* was computed by
dividing the absolute bias by the minimum value for θ in our study, that is, the population value of λ_β, $\lambda_{\mu a}$,
and $\lambda_{\sigma a}$ in conditions with small cohort differences (*d*=.1).

Design 0 yielded the least biased estimates, with RB ranging from −.05 to .04. This design has the highest data density (i.e., 15 repeated measures per participant), and thus it is used as a proxy for optimal model performance. In Design 1, the parameters capturing cohort effects (i.e., λ_β, $\lambda_{\mu a}$, and $\lambda_{\sigma a}$) were slightly underestimated with samples of 125 participants and small cohort differences (d=.1), with RB ranging from −.19 to .004. Also, the parameter capturing cohort effects in the standard deviation of the additive component ($\lambda_{\sigma a}$) was slightly underestimated in conditions with 200 participants and equivalent cohorts (d=0), with RB between −.15 and .01. In Designs 2 and 3, nearly all the estimates were unbiased, regardless of cohort effect size and sample size. Importantly, cohort differences in the self-feedback parameter were accurately captured by λ_β across all conditions in Designs 2 and 3 (RB range from −.08 to .07), whereas in Design 1 a minimum of 200 participants was required (RB range from −.07 to .02).

[FIGURE 5]

The *RMSRB* for each design and condition is reported in the bottom panel of Figure 5. In general, the total bias was very low for all designs. Model performance was mostly affected by sample size (i.e., larger samples led to better performance), and it was generally better in conditions with larger cohort effects. In conditions with 125 and 200 participants, Design 1 showed the largest total bias, whereas Designs 2 and 3 performed very similarly to the benchmark design (D0). With samples of 500 participants, the differences between designs were negligible.

Variability of the estimates

We evaluated the variability of the estimates by computing the standard deviation of the relative bias in each condition: $SDRB = SD[(\overline{\theta} - \theta) / \theta]$. This index captures the variability of the

parameter estimates on the same scale for all parameters. Unlike the relative bias, there are no standard criteria for determining an excessive degree of variability. This index is always positive and values closer to zero imply less variability of the estimates. For a general overview of the model's precision, we provide the total standard deviation of the relative bias, computed as:

$$\text{mean}(SDRB) = \sum_{k=1}^{K} SDRB_k / K .$$

The top panel of Figure 6 depicts the *SDRB* across all simulation conditions. As expected, larger samples led to less variability in the parameter estimates. However, the *SDRB* was most affected by the size of the cohort differences, with larger differences leading to smaller *SDRB*. In the benchmark design (D0), the variability of the estimates was slightly higher for $\lambda_{\sigma a}$, with *SDRB* ranging between .05 and .50, and very low for the remaining parameters (range .01-.19). The performance of Designs 1, 2, and 3 reproduced a similar pattern: the parameters capturing cohort effects (i.e., λ_β, $\lambda_{\mu a}$, and $\lambda_{\sigma a}$) had larger *SDRB* compared to the remaining parameters of the model. In particular, the variability was always higher for $\lambda_{\sigma a}$ (range .08-1.26), followed by λ_β (range .05-1.13), and $\lambda_{\mu a}$ (range .04-.89).

[FIGURE 6]

The total variability of the SSM-CT model with moderators is reported in the bottom panel of Figure 6. Design 0 showed the best performance, with mean *SDRB* values between .03 and .13. In contrast, the largest variability in the estimates was found for Design 1, with mean *SDRB* values between .08 and .46. The amount of total variability in Designs 2 and 3 was almost identical across all conditions (ranges 05-.27 and .05-.31, respectively), with a marginally better performance of Design 3.

Coverage

We computed the rates of coverage as the proportion of 95% confidence intervals around the point estimate that included the true parameter value. As such, 95% is the optimal value of coverage, and coverage below 90% is considered inadequate (Collins et al., 2001; Enders & Peugh, 2004). As a measure of the global coverage of the model for each design, we computed the mean coverage as: $\text{mean}(coverage) = \sum_{k=1}^{K} coverage_k / K$.

The coverage results were excellent, with rates of 95%CI coverage above 86% (average 94.6%) across all parameters and conditions. Importantly, the coverage for the parameters capturing cohort differences was always above 89% (average 94.3%). There were no meaningful differences in mean coverage across designs: all designs performed very similarly to the benchmark design (D0), with mean coverage rates around 94%. Figure 7 shows the 95%CI coverage rates for all parameters (top panel) and the mean coverage across designs (bottom panel). Complete numerical results are included in the supplemental materials.

[FIGURE 7]

Discussion

In the present work, we introduced a latent change score approach to detect and control for cohort differences in accelerated longitudinal designs. In particular, we focused on differences in the speed of maturation of one developing process, which is captured by a self-feedback parameter. This is an important goal as, to the best of our knowledge, no methods are available in the literature for detecting cohort differences in speed of maturation independently of the maximum level of the trajectories. For this, we used a latent change score model in continuous-time that includes the cohort as a moderator of the auto-regressive dynamics across

latent states. In this section, we provide an overview of the model's performance and elaborate on the limitations and methodological considerations derived from the present study.

Summary of findings

The performance of the LCS model with cohort-related moderators was evaluated under various conditions of sampling design, sample size, and cohort effect size. As expected, the best model performance was achieved for the benchmark design (D0), representing participants who were followed yearly during the complete time range of the study. Nevertheless, differences in performance between the benchmark design and the other designs were very small. That is, the performance of any of the ALDs evaluated in this paper is not much worse than an optimal design with complete trajectories for each individual. In Design 1, where participants were measured only two times in alternative years, the total length of the study was three years. This design led to slightly larger bias and variability in the estimations, probably because the overlap between cohorts was minimal (see Figure 4). In fact, it required at least 200 participants to accurately capture cohort effects in the speed of maturation. Despite these marginal amounts of bias, the rates of coverage were excellent for all generating parameters, even in conditions with 125 participants.

The model performance in Designs 2 and 3 was almost identical. In Design 2 participants were measured three times in alternative years (i.e., the length of the study was five years), whereas in Design 3 they were measured four times in consecutive years (i.e., the length of the study was four years). In both designs, the recovery of the generating parameters was excellent in terms of bias and coverage, regardless of sample size and cohort effect size. In terms of variability, Design 2 showed a marginally better performance. In sum, both designs were equally adequate for recovering features of the generating process and cohort effects in maturation speed.

Given these similar results, the choice of one over the other should be decided by the researcher based on the desired number of assessments (three or four measurements) and the desired length of the study (five or four years).

Overall, the accuracy and coverage of the parameters capturing cohort differences in the speed of maturation was excellent. However, these parameters showed more variability in the estimations compared to the remaining parameters of the LCS model. An interesting finding is that sample size had little impact on the variability of the estimations. Of course, larger samples led to less variability, but this was mostly affected by the size of cohort differences and the type of sampling design. Across all cohorts, larger cohort effect sizes consistently led to lower variability in the parameter estimates. This implies that larger cohort effects are somewhat easier to estimate in any given sample. Similarly, the ALD with the least estimation variability was Design 2, followed by Design 3 and Design 1, in that order (we are not considering D0 in this comparison because it is not an ALD). Based on these findings, we would recommend using our model with Design 1 and 200 participants or more, or with Designs 2 or 3 and 125 participants or more.

Theoretical and methodological considerations

The procedure presented in this paper extends previous work on the study of cohort differences in accelerated longitudinal designs (cf. Estrada et al., 2021; Miyazaki & Raudenbush, 2000). Our main contribution is the introduction of one of the first approaches to the study of cohort differences using dynamic models (along with Estrada et al., 2021), and the first approach to the study of differences in the speed of maturation of developmental processes. For this, we used an extension of LCS models because: a) they are suited to model exponential trajectories, common in many developmental processes and b) they incorporate a self-feedback parameter

that explicitly captures the speed of maturation of the process of interest. Importantly, despite the great popularity of LCS models, their ability to capture the speed of maturation has apparently gone unnoticed in the literature until now. In this manuscript, we introduced and emphasized this feature of LCS models, and proposed using the cohort variable as a moderator to account for differences in maturation speed.

The present approach will provide further understanding on how the evolution of the social, cultural, or economic conditions affects the speed of development of particular populations, and how this development looks after controlling for cohort differences. In fact, although we focused on growth trajectories from childhood to early adulthood, our approach could potentially be applied to trajectories of decelerated or accelerated decline, typically found in the study of physical and cognitive development in late life (e.g., Dodge et al., 2014; Finkel et al., 2007; Gerstorf et al., 2011; Hoffman et al., 2011; Zhang et al., 2020). In this context, the self-feedback can be used as an indicator of the speed of decay, where larger self-feedbacks (i.e., further from zero) imply faster declines.

A key aspect of the proposed model is that the set of values defined in the cohort variable reflects the expectations of the researcher about the direction and type of cohort effect. In this manuscript, we used a linear effect in favor of the younger cohorts as a proof of principle, but other effects are possible. The values of the cohort variable work in a way similar to the slope loadings in a LGC model. In a LGC model, the slope loadings can be modified to represent different functional forms, and the researcher can use nested model specifications to select the functional form that better captures the shape of the trajectories. Similarly, the values of the cohort variable can be changed to represent different types of cohort effects, and the researcher

can use nested model comparisons to select the type of effect that better captures the differences across cohorts.

In developmental studies, differences across groups are often modeled via a multiple-group specification, where different sets of parameters are estimated for each group. This is a reasonable approach when the number of groups is small, and the sample size within each group is sufficiently large. However, in ALDs each cohort may include only a few participants (e.g., 125 participants divided into 12 cohorts leads to 10-11 participants per cohort). In these situations, a multiple-group approach may not have enough power to detect cohort differences across groups. Furthermore, the number of parameters in a multiple-group model increases linearly with the number of groups. For example, an ALD with 12 cohorts would require estimating 40 parameters (12 self-feedbacks, 12 additive component means, 12 additive component variances, 1 initial mean, 1 initial variance, 1 correlation, and 1 error variance). In contrast, our model required only 10 parameters, regardless of the number of cohorts, thus providing a more parsimonious approach.

Although we implemented the LCS model with moderators using state-space equations in continuous-time, other approaches are also possible. For example, despite the increasing popularity of SSMs, many empirical researchers are more familiar with SEM. The state and output equations in SSMs are mathematically equivalent to the structural and measurement equations in SEM (for details, see Chow et al., 2010 and Hunter, 2018). Equations 10, 11, and 12 can be implemented into a SEM framework to account for cohort differences in the time-lagged dynamics, means, and variances of the model. In the supplemental materials, we provide R code in OpenMx for the formulation and estimation of the model both in continuous-time SSM and

discrete-time SEM. Also, in Appendix A we provide a brief description of a discrete-time SEM version of the LCS model with cohort-related moderators.

In this manuscript, we proposed an approach based on parameter moderation using latent change score model in continuous-time to address a specific problem, namely the detection of, and control for, cohort effects in the context of ALDs. Importantly, this approach can be applied to any type of (non-accelerated) longitudinal designs and can be used to test for moderating effects besides those related to cohorts. For example, Hu et al. (2014) used a latent differential equation model in SEM to examine the extent to which the coupling relations between emotional eating and estradiol were moderated by a self-reported measure of negative affect. In the context of psychometrics, parameter moderation has also been proposed as a valuable tool for exploring measurement invariance and item differential functioning (see Bauer, 2017).

Limitations and future directions

In this study, we focused on the speed of maturation of a single construct that unfolds over time. In principle, our model could be extended to multivariate systems including dynamic interrelations between several latent processes. In univariate systems, the specific shape of the exponential trajectories is defined by the interaction between the self-feedback and the additive component over time. However, in bivariate systems the nonlinear behavior of the system is defined by the interaction between all parameters representing self-feedbacks, the couplings, and the additive components, and the resulting trajectories may not have an exponential form. Consequently, the self-feedback parameter cannot be directly interpreted as the rate (or speed) of reduction between the initial and peak levels (i.e., speed of maturation or decay) in such bivariate systems (for further details, see Cáncer et al., 2021). Future research should investigate how to

extend the current proposed approach to multivariate systems and examine ways to capture cohort differences in the cross-lagged dynamics between latent processes.

Consistent with the patterns of change typically found in cognitive development from childhood to early adulthood, we analyzed exponential trajectories of decelerated change. However, in the context of aging, declines in well-being or cognitive abilities may exhibit patterns of accelerated decay, which are defined by positive self-feedbacks. In principle, accelerated longitudinal designs and bivariate latent change score models could be applied to examine such trajectories, but it must be noted that their performance in that setting has not been investigated. Future research should examine whether cohort differences can be detected and controlled for with the methods proposed in this paper.

The proposed model uses a discrete variable as a moderator of the dynamic relation between latent states, but using continuous moderators is also possible in the SSM and SEM frameworks. One key difference between discrete and continuous moderators lies in the number of cases per level of the moderator. In the ALDs used in this study, cohorts were defined by the year of birth, and there were no fewer than seven participants per cohort. If cohorts were defined by the day (instead of year) of birth, many levels of the cohort variable would contain only one participant. However, using such continuous moderators may not be possible in LCS models. This is due to the dependency among the parameters representing self-feedback, additive component mean, and additive component variance. In the sampling schedules used in the present study, each cohort (i.e., each level of the moderator variable) had enough individuals to estimate a mean and variance for the latent additive component. With a continuous moderator, one would impose individual differences in the mean and variance of the additive component, but the estimation of between-individual variance parameters for single individuals would not be

possible. This has relevant implications in the context of ALDs: a) a sampling schedule must be chosen to ensure that no cohorts have one case only, as this would lead to estimation problems, and b) if cohorts are defined by the date of birth, it is necessary to create wide enough bins before using it as a moderator. In the present paper, we used bins of one year, but it may be reasonable to use bins of two, three, or more years so as to ensure that the number of participants in each group is large enough.

In our modeling approach, we account for differences in the self-feedback parameter due to differences in the cohort variable. However, individual variability in the speed of maturation may not be fully explained by the cohorts. Driver & Voelkle (2018) developed a Bayesian framework for estimating hierarchical continuous-time models that allows the specification of random effects in any of the model parameters. This is a promising approach for detecting cohort differences such as those studied in this paper, and it could potentially capture the variance in a given parameter that is not explained by the cohort variable. However, future research should examine the performance of this approach under conditions commonly found in ALDs, such as high percentage of data incompleteness, few participants per cohort, few repeated measures per participant, and random time intervals between observations.

As a final note, the sampling schedules evaluated in this manuscript are planned missing data designs where the source of missingness is controlled by the researcher. However, non-planned missing data (e.g., participant attrition) is common in longitudinal studies, and can be particularly harmful when missingness is related to the variables being measured—that is, when missingness is "not at random" (MNAR). The impact of MNAR in longitudinal analyses has been documented, and various methods have been proposed for dealing with this problem (e.g., Enders, 2011; Gottfredson et al., 2014; Jeličić et al., 2009, 2010; Laird, 1988; Wang et al., 2008).

It must be noted that, in the presence of unplanned missing data, especially MNAR, the sampling requirements will likely be more demanding than those proposed in this study. Future research should investigate for the effect of unplanned data missingness on the performance of continuous-time dynamic models, particularly in the context of ALDs.

Conclusion

In the present study, we proposed a LCS model with cohort-related moderators that allows identifying cohort differences in the speed of maturation of developmental processes. This is an important contribution because it allows other researchers to address the question of whether differences in the speed of maturation exist across different cohorts, independently of whether such cohorts reach the same maximum level on average. We focused on data gathered through accelerated longitudinal designs because a) ALDs are an efficient solution for studying process that unfold over long periods of time, and b) they heavily rely on the assumption that cohorts are equivalent. However, the approach proposed here can be applied to any study including different cohorts, regardless of whether the design is accelerated.

Our results suggest that the proposed model can adequately capture such cohort differences in the context of accelerated longitudinal designs. Based on our findings, researchers should note that:

a) In empirical research, the assumption of cohort equivalence is typically unknown. If researchers suspect that there might be cohort effects affecting the maturational speed of the process under study, they should consider including cohort-related moderators in their models. Our findings indicate that the proposed model is able to: a) capture cohort differences ranging from very small to very large, and b) estimate cohort-related

parameters as null when cohort differences are zero in the population (i.e., when the assumption of cohort equivalence is met).

b) All the sampling designs evaluated showed excellent performance in terms of bias, variability, and coverage. The choice of one over the rest will depend on the desired number of evaluations, length of the study, and sample size available. Design 1 was the shortest, with two evaluations per participant and a total length of three years, but it required at least 200 participants. The performance of Design 2 (three evaluations during five years) and Design 3 (four evaluations during four years) was virtually identical, and both required only 125 participants.

c) The performance of the benchmark design (D0) was only slightly better than that of the three ALDs studied in this paper (D1, D2, and D3). This suggest that, despite the large amount of (planned) missing data, ALDs are not only very similar in performance to conventional longitudinal designs (with complete trajectories for each individual), but also represent a much more cost-efficient alternative.

We have shown that the continuous-time LCS model with cohort-related moderators is a reliable tool to account for cohort differences in the speed of maturation of a developmental process. We encourage researchers to use this model for developmental research, and particularly in the context of ALDs. We hope these findings will provide useful information for the design of longitudinal studies and the analysis of potential cohort differences in such studies.

References

Bauer, D. J. (2017). A more general model for testing measurement invariance and differential item functioning. Psychological Methods, 22, 507–526. https://doi.org/10.1037/met0000077

Bell, R. Q. (1953). Convergence: An Accelerated Longitudinal Approach. *Child Development*, *24*(2), 145–152. https://doi.org/10.2307/1126345

Bell, R. Q. (1954). An Experimental Test of the Accelerated Longitudinal Approach. *Child Development*, *25*(4), 281–286. https://doi.org/10.2307/1126058

Boker, S. M., Neale, M. C., Maes, H. H., Wilde, M. J., Spiegel, M., Brick, T. R., Estabrook, R., Bates, T. C., & Mehta, P. (2018). *OpenMx User Guide*. 195.

Brown, C. (2007). *Differential Equations: A Modeling Approach*. SAGE.

Cáncer, P. F., Estrada, E., Ollero, M. J. F., & Ferrer, E. (2021). Dynamical Properties and Conceptual Interpretation of Latent Change Score Models. *Frontiers in Psychology*, *12*, 2801. https://doi.org/10.3389/fpsyg.2021.696419

Chow, S.-M., Ho, M. R., Hamaker, E. L., & Dolan, C. V. (2010). Equivalence and Differences Between Structural Equation Modeling and State-Space Modeling Techniques. *Structural Equation Modeling: A Multidisciplinary Journal*, *17*(2), 303–332. https://doi.org/10.1080/10705511003661553

Cole, T. J. (2000). Secular trends in growth. *Proceedings of the Nutrition Society*, *59*(2), 317–324. https://doi.org/10.1017/S0029665100000355

Cole, T. J. (2003). The secular trend in human physical growth: A biological view. *Economics & Human Biology*, *1*(2), 161–168. https://doi.org/10.1016/S1570-677X(02)00033-3

Collins, L. M., Schafer, J. L., & Kam, C.-M. (2001). A comparison of inclusive and restrictive strategies in modern missing data procedures. *Psychological Methods*, *6*(4), 330–351. https://doi.org/10.1037/1082-989X.6.4.330

de Haan-Rietdijk, S., Voelkle, M. C., Keijsers, L., & Hamaker, E. L. (2017). Discrete- vs. Continuous-Time Modeling of Unequally Spaced Experience Sampling Method Data. *Frontiers in Psychology*, *8*. https://doi.org/10.3389/fpsyg.2017.01849

Deboeck, P. R., & Preacher, K. J. (2016). No Need to be Discrete: A Method for Continuous Time Mediation Analysis. *Structural Equation Modeling: A Multidisciplinary Journal*, *23*(1), 61–75. https://doi.org/10.1080/10705511.2014.973960

Dodge, H. H., Zhu, J., Lee, C.-W., Chang, C.-C. H., & Ganguli, M. (2014). Cohort Effects in Age-Associated Cognitive Trajectories. *The Journals of Gerontology: Series A*, *69*(6), 687–694. https://doi.org/10.1093/gerona/glt181

Drewelies, J., Deeg, D. J. H., Huisman, M., & Gerstorf, D. (2018). Perceived constraints in late midlife: Cohort differences in the Longitudinal Aging Study Amsterdam (LASA). *Psychology and Aging*, *33*(5), 754–768. https://doi.org/10.1037/pag0000276

Driver, C. C., & Voelkle, M. C. (2018). Hierarchical Bayesian continuous time dynamic modeling. *Psychological Methods*, *23*(4), 774–799. https://doi.org/10.1037/met0000168

Duncan, S. C., Duncan, T. E., & Hops, H. (1996). Analysis of longitudinal data within accelerated longitudinal designs. *Psychological Methods*, *1*(3), 236–248. https://doi.org/10.1037/1082-989X.1.3.236

Eckert-Lind, C., Busch, A. S., Petersen, J. H., Biro, F. M., Butler, G., Bräuner, E. V., & Juul, A. (2020). Worldwide Secular Trends in Age at Pubertal Onset Assessed by Breast

Development Among Girls: A Systematic Review and Meta-analysis. *JAMA Pediatrics*, *174*(4), e195881. https://doi.org/10.1001/jamapediatrics.2019.5881

Enders, C. K. (2011). Missing not at random models for latent growth curve analyses. *Psychological Methods*, *16*, 1–16. https://doi.org/10.1037/a0022640

Enders, C. K., & Peugh, J. L. (2004). Using an EM Covariance Matrix to Estimate Structural Equation Models With Missing Data: Choosing an Adjusted Sample Size to Improve the Accuracy of Inferences. *Structural Equation Modeling: A Multidisciplinary Journal*, *11*(1), 1–19. https://doi.org/10.1207/S15328007SEM1101_1

Estrada, E., Bunge, S. A., & Ferrer, E. (2021). Controlling for cohort effects in accelerated longitudinal designs using continuous- and discrete-time dynamic models. Psychological Methods. Advance online publication. https://doi.org/10.1037/met0000427

Estrada, E., & Ferrer, E. (2019). Studying developmental processes in accelerated cohort-sequential designs with discrete- and continuous-time latent change score models. *Psychological Methods*, *24*(6), 708–734. https://doi.org/10.1037/met0000215

Estrada, E., Ferrer, E., Karama, S., Román, F. J., & Colom, R. (2019). *Time-Lagged Associations Between Cognitive and Cortical Development From Childhood to Early Adulthood*. *55*(6), 1338–1352. http://dx.doi.org/10.1037/dev0000716

Fandakova, Y., Selmeczy, D., Leckey, S., Grimm, K. J., Wendelken, C., Bunge, S. A., & Ghetti, S. (2017). Changes in ventromedial prefrontal and insular cortex support the development of metamemory from childhood into adolescence. *Proceedings of the National Academy of Sciences*, *114*(29), 7582–7587. https://doi.org/10.1073/pnas.1703079114

Ferrer, E., & McArdle, J. J. (2003). Alternative Structural Models for Multivariate Longitudinal Data Analysis. *Structural Equation Modeling: A Multidisciplinary Journal*, *10*(4), 493–524. https://doi.org/10.1207/S15328007SEM1004_1

Ferrer, E., & McArdle, J. J. (2004). An Experimental Analysis of Dynamic Hypotheses About Cognitive Abilities and Achievement From Childhood to Early Adulthood. *Developmental Psychology*, *40*(6), 935–952. https://doi.org/10.1037/0012-1649.40.6.935

Ferrer, E., McArdle, J. J., Shaywitz, B. A., Holahan, J. M., Marchione, K., & Shaywitz, S. E. (2007). Longitudinal models of developmental dynamics between reading and cognition from childhood to adolescence. *Developmental Psychology*, *43*(6), 1460–1473. https://doi.org/10.1037/0012-1649.43.6.1460

Ferrer, E., O'Hare, E., & Bunge, S. (2009). Fluid reasoning and the developing brain. *Frontiers in Neuroscience*, *3*, 3. https://doi.org/10.3389/neuro.01.003.2009

Ferrer, E., Shaywitz, B. A., Holahan, J. M., Marchione, K., & Shaywitz, S. E. (2010). Uncoupling of Reading and IQ Over Time: Empirical Evidence for a Definition of Dyslexia. *Psychological Science*, *21*(1), 93–101. https://doi.org/10.1177/0956797609354084

Finkel, D., Reynolds, C. A., McArdle, J. J., & Pedersen, N. L. (2007). Cohort Differences in Trajectories of Cognitive Aging. *The Journals of Gerontology: Series B*, *62*(5), P286–P294. https://doi.org/10.1093/geronb/62.5.P286

Flora, D. B., & Curran, P. J. (2004). An Empirical Evaluation of Alternative Methods of Estimation for Confirmatory Factor Analysis With Ordinal Data. *Psychological Methods*, *9*(4), 466–491. https://doi.org/10.1037/1082-989X.9.4.466

Gerstorf, D., Ram, N., Hoppmann, C., Willis, S. L., & Schaie, K. W. (2011). Cohort differences in cognitive aging and terminal decline in the Seattle Longitudinal Study. *Developmental Psychology*, *47*(4), 1026–1041. https://doi.org/10.1037/a0023426

Green, C. T., Bunge, S. A., Briones Chiongbian, V., Barrow, M., & Ferrer, E. (2017). Fluid reasoning predicts future mathematical performance among children and adolescents. *Journal of Experimental Child Psychology*, *157*, 125–143. https://doi.org/10.1016/j.jecp.2016.12.005

Gottfredson, N. C., Bauer, D. J., & Baldwin, S. A. (2014). Modeling Change in the Presence of Nonrandomly Missing Data: Evaluating a Shared Parameter Mixture Model. *Structural Equation Modeling: A Multidisciplinary Journal*, *21*(2), 196–209. https://doi.org/10.1080/10705511.2014.882666

Gu, F., Preacher, K. J., Wu, W., & Yung, Y.-F. (2014). A Computationally Efficient State Space Approach to Estimating Multilevel Regression Models and Multilevel Confirmatory Factor Models. *Multivariate Behavioral Research*, *49*(2), 119–129. https://doi.org/10.1080/00273171.2013.866537

Hoffman, L., Hofer, S. M., & Sliwinski, M. J. (2011). On the confounds among retest gains and age-cohort differences in the estimation of within-person change in longitudinal studies: A simulation study. *Psychology and Aging*, *26*(4), 778–791. https://doi.org/10.1037/a0023910

Holahan, J. M., Ferrer, E., Shaywitz, B. A., Rock, D. A., Kirsch, I. S., Yamamoto, K., Michaels, R., Marchione, K. E., & Shaywitz, S. E. (2018). Growth in Reading Comprehension and Verbal Ability From Grades 1 Through 9. *Journal of Psychoeducational Assessment*, *36*(4), 307–321. https://doi.org/10.1177/0734282916680984

Hu, Y., Boker, S., Neale, M., & Klump, K. L. (2014). Coupled latent differential equation with moderators: Simulation and application. *Psychological Methods*, *19*(1), 56–71. https://doi.org/10.1037/a0032476

Hunter, M. D. (2018). State Space Modeling in an Open Source, Modular, Structural Equation Modeling Environment. *Structural Equation Modeling: A Multidisciplinary Journal*, *25*(2), 307–324. https://doi.org/10.1080/10705511.2017.1369354

Jeličić, H., Phelps, E., & Lerner, R. M. (2009). Use of missing data methods in longitudinal studies: The persistence of bad practices in developmental psychology. *Developmental Psychology*, *45*, 1195–1199. https://doi.org/10.1037/a0015665

Jeličić, H., Phelps, E., & Lerner, R. M. (2010). Why Missing Data Matter in the Longitudinal Study of Adolescent Development: Using the 4-H Study to Understand the Uses of Different Missing Data Methods. *Journal of Youth and Adolescence*, *39*(7), 816–835. https://doi.org/10.1007/s10964-010-9542-5

Ji, L., & Chow, S.-M. (2019). Methodological Issues and Extensions to the Latent Difference Score Framework 1. In E. Ferrer, S. M. Boker, & K. J. Grimm (Eds.), *Longitudinal Multivariate Psychology* (pp. 9–37). Routledge. https://doi.org/10.4324/9781315160542-2

Kail, R. V., & Ferrer, E. (2007). Processing Speed in Childhood and Adolescence: Longitudinal Models for Examining Developmental Change. *Child Development*, *78*(6), 1760–1770. https://doi.org/10.1111/j.1467-8624.2007.01088.x

Kalman, R. E. (1960). A New Approach to Linear Filtering and Prediction Problems. *Journal of Basic Engineering*, *82*(1), 35–45. https://doi.org/10.1115/1.3662552

Kalman, R. E., & Bucy, R. S. (1961). New Results in Linear Filtering and Prediction Theory. *Journal of Basic Engineering*, *83*(1), 95–108. https://doi.org/10.1115/1.3658902

Laird, N. M. (1988). Missing data in longitudinal studies. *Statistics in Medicine*, *7*(1–2), 305–315. https://doi.org/10.1002/sim.4780070131

McArdle, J. J. (2001). A latent difference score approach to longitudinal dynamic structural analysis. In R. Cudeck, S. du Toit, & D. Sörbom, *Structural equation modeling, present and future: A Festschrift in honor of Karl Jöreskog* (pp. 7–46). Scientific Software International.

McArdle, J. J. (2009). Latent Variable Modeling of Differences and Changes with Longitudinal Data. *Annual Review of Psychology*, *60*(1), 577–605. https://doi.org/10.1146/annurev.psych.60.110707.163612

McArdle, J. J., Ferrer-Caja, E., Hamagami, F., & Woodcock, R. W. (2002). Comparative longitudinal structural analyses of the growth and decline of multiple intellectual abilities over the life span. *Developmental Psychology*, *38*(1), 115–142. https://doi.org/10.1037/0012-1649.38.1.115

McArdle, J. J., & Hamagami, F. (2001). Latent difference score structural models for linear dynamic analyses with incomplete longitudinal data. In L. M. Collins & A. G. Sayer (Eds.), *New methods for the analysis of change.* (pp. 139–175). American Psychological Association. https://doi.org/10.1037/10409-005

McArdle, J. J., & Woodcock, R. W. (1997). Expanding test–retest designs to include developmental time-lag components. *Psychological Methods*, *2*(4), 403–435. https://doi.org/10.1037/1082-989X.2.4.403

Miyazaki, Y., & Raudenbush, S. W. (2000). Tests for linkage of multiple cohorts in an accelerated longitudinal design. *Psychological Methods*, *5*(1), 44–63. https://doi.org/10.1037/1082-989x.5.1.44

Neale, M. C., Hunter, M. D., Pritikin, J. N., Zahery, M., Brick, T. R., Kirkpatrick, R. M., Estabrook, R., Bates, T. C., Maes, H. H., & Boker, S. M. (2016). OpenMx 2.0: Extended Structural Equation and Statistical Modeling. *Psychometrika*, *81*(2), 535–549. https://doi.org/10.1007/s11336-014-9435-8

Nesselroade, J. R., & Baltes, P. B. (1979). *Longitudinal research in the study of behavior and development*. Academic Press. https://pure.mpg.de/pubman/faces/ViewItemOverviewPage.jsp?itemId=item_3005291

Oravecz, Z., Tuerlinckx, F., & Vandekerckhove, J. (2011). A hierarchical latent stochastic differential equation model for affective dynamics. Psychological methods, 16(4), 468.

Orth, U., Maes, J., & Schmitt, M. (2015). Self-esteem development across the life span: A longitudinal study with a large sample from Germany. *Developmental Psychology*, *51*(2), 248–259. https://doi.org/10.1037/a0038481

Oud, J. H. L., & Delsing, M. J. M. H. (2010). Continuous Time Modeling of Panel Data by means of SEM. In K. van Montfort, J. H. L. Oud, & A. Satorra (Eds.), *Longitudinal Research with Latent Variables* (pp. 201–244). Springer Berlin Heidelberg. https://doi.org/10.1007/978-3-642-11760-2_7

Oud, J. H. L., & Jansen, R. A. R. G. (2000). Continuous time state space modeling of panel data by means of sem. *Psychometrika*, *65*(2), 199–215. https://doi.org/10.1007/BF02294374

Oud, J. H. L., & Singer, H. (2008). Continuous time modeling of panel data: SEM versus filter techniques. *Statistica Neerlandica*, *62*(1), 4–28. https://doi.org/10.1111/j.1467-9574.2007.00376.x

Ryan, O., Kuiper, R. M., & Hamaker, E. L. (2018). A Continuous-Time Approach to Intensive Longitudinal Data: What, Why, and How? In K. van Montfort, J. H. L. Oud, & M. C. Voelkle (Eds.), *Continuous Time Modeling in the Behavioral and Related Sciences* (pp. 27–54). Springer International Publishing. https://doi.org/10.1007/978-3-319-77219-6_2

Schaie, K. W. (1965). A general model for the study of developmental problems. *Psychological Bulletin*, *64*(2), 92–107. https://doi.org/10.1037/h0022371

Schmitt, S. A., Geldhof, G. J., Purpura, D. J., Duncan, R., & McClelland, M. M. (2017). Examining the relations between executive function, math, and literacy during the transition to kindergarten: A multi-analytic approach. *Journal of Educational Psychology*, *109*(8), 1120–1140. https://doi.org/10.1037/edu0000193

Shaywitz, S. E., Shaywitz, B. A., Fletcher, J. M., & Escobar, M. D. (1990). Prevalence of Reading Disability in Boys and Girls: Results of the Connecticut Longitudinal Study. *JAMA*, *264*(8), 998–1002. https://doi.org/10.1001/jama.1990.03450080084036

Vainikainen, M.-P., & Hautamäki, J. (2022). Three Studies on Learning to Learn in Finland: Anti-Flynn Effects 2001–2017. *Scandinavian Journal of Educational Research*, *66*(1), 43–58. https://doi.org/10.1080/00313831.2020.1833240

Van Der Maas, H. L. J., Dolan, C. V., Grasman, R. P. P. P., Wicherts, J. M., Huizenga, H. M., & Raijmakers, M. E. J. (2006). A dynamical model of general intelligence: The positive manifold of intelligence by mutualism. *Psychological Review*, *113*(4), 842–861. https://doi.org/10.1037/0033-295X.113.4.842

van Montfort, K., Oud, J. H. L., & Voelkle, M. C. (2018). *Continuous Time Modeling in the Behavioral and Related Sciences*. Springer.

Voelkle, M. C., Gische, C., Driver, C. C., & Lindenberger, U. (2018). The Role of Time in the Quest for Understanding Psychological Mechanisms. *Multivariate Behavioral Research*, *53*(6), 782–805. https://doi.org/10.1080/00273171.2018.1496813

Voelkle, M. C., & Oud, J. H. L. (2013). Continuous time modelling with individually varying time intervals for oscillating and non-oscillating processes. *British Journal of Mathematical and Statistical Psychology*, *66*(1), 103–126. https://doi.org/10.1111/j.2044-8317.2012.02043.x

Voelkle, M. C., & Oud, J. H. L. (2015). Relating Latent Change Score and Continuous Time Models. *Structural Equation Modeling: A Multidisciplinary Journal*, *22*(3), 366–381. https://doi.org/10.1080/10705511.2014.935918

Voelkle, M. C., Oud, J. H. L., Davidov, E., & Schmidt, P. (2012). An SEM approach to continuous time modeling of panel data: Relating authoritarianism and anomia. *Psychological Methods*, *17*(2), 176–192. https://doi.org/10.1037/a0027543

Wang, C. Y., Huang, Y., Chao, E. C., & Jeffcoat, M. K. (2008). Expected Estimating Equations for Missing Data, Measurement Error, and Misclassification, with Application to Longitudinal Nonignorable Missing Data. *Biometrics*, *64*(1), 85–95. https://doi.org/10.1111/j.1541-0420.2007.00839.x

Wendelken, C., Ferrer, E., Ghetti, S., Bailey, S. K., Cutting, L., & Bunge, S. A. (2017). Frontoparietal Structural Connectivity in Childhood Predicts Development of Functional Connectivity and Reasoning Ability: A Large-Scale Longitudinal Investigation. *Journal of Neuroscience*, *37*(35), 8549–8558. https://doi.org/10.1523/JNEUROSCI.3726-16.2017

Zhang, P.-D., Lv, Y.-B., Li, Z.-H., Yin, Z.-X., Li, F.-R., Wang, J.-N., Zhang, X.-R., Zhou, J.-H.,

   Wu, X.-B., Duan, J., Mao, C., & Shi, X.-M. (2020). Age, Period, and Cohort Effects on

   Activities of Daily Living, Physical Performance, and Cognitive Functioning Impairment

   Among the Oldest-Old in China. *The Journals of Gerontology: Series A*, *75*(6), 1214–

   1221. https://doi.org/10.1093/gerona/glz196

Appendix A

In this section, we describe how to specify a LCS model with cohort-related moderators in a discrete-time SEM framework. The path diagram of such model is depicted in Figure A1. In this figure, the change scores have been removed. In consequence, the self-feedback has become an auto-regression coefficient with a value of $\beta^* = 1+\beta$. Removing the change scores is not necessary, but it simplifies the path diagram.

[FIGURE A1]

In Figure A1 the moderation structure connecting latent states is composed by a direct and an indirect pathway. The direct pathway from y_{t-1} to y_t takes a value of β^* that is freely estimated. The indirect pathway is formed by three components: a) a coefficient *coh* that is fixed to the observed value in the moderator, b) a coefficient λ_β that is freely estimated, and c) a phantom variable D with mean and variance zero. The indirect pathway is used to capture the moderating effect $coh \times \lambda_\beta$. This time-invariant structure accounts for differences in the self-feedback parameter due to differences in the moderator variable.

As described previously, if the speed of maturation is affected by cohort differences, it is necessary to account for such differences not only in the self-feedback parameter, but also in the mean and variance of the additive component and its covariance with the initial state. In the present paper (both the CT model presented in the main text and the DT model presented in this Appendix), we accounted for cohort effects in the covariance matrix by decomposing it into a correlation matrix and a diagonal matrix of standard deviations (see Equation 12). Because such effects cannot be represented correctly in a path diagram, they are not included in Figure A1. However, some SEM programs such as OpenMx (Boker et al., 2018) provide functions that

allow manipulating the equations of the model to account for moderator effects in any parameter.

In the supplemental materials, we provide annotated R code to specify and estimate the LCS

model with cohort-related moderators within a discrete-time SEM framework.

Table 1. Baseline generating parameters

| Parameter	Value in CT
Self-feedback (β)	$-.25$
Initial mean (μ_0)	10
Additive component mean (μ_a)	7.5
Initial variance (σ^2_0)	25
Additive component variance (σ^2_a)	1.5625
Initial-Additive component covariance ($\sigma_{0,a}$)	4.375
Measurement error variance (σ^2_e)	2
Implied values	
Asymptotic level mean (μ_{As})	30
Asymptotic level variance (σ^2_{As})	25
Initial-Additive component correlation ($\rho_{0,a}$)	.7

Table 2. Parameter values across cohort effects (in continuous time)

Parameter	Effect size of the cohort	Value in the oldest cohort	Value in the youngest cohort
Self-feedback β	$d = 0$	$-.25$	$-.25$
	$d = .1$	$-.20$	$-.30$
	$d = .2$	$-.15$	$-.35$
	$d = .4$	$-.05$	$-.45$
Additive component mean μ_a	$d = 0$	7.5	7.5
	$d = .1$	6	9
	$d = .2$	4.5	10.5
	$d = .4$	1.5	13.5
Additive component variance σ^2_a	$d = 0$	1.5625	1.5625
	$d = .1$	1	2.25
	$d = .2$	.5625	3.0625
	$d = .4$	.0625	5.0625

Figure 1. Examples of complete and partial trajectories in an accelerated longitudinal design
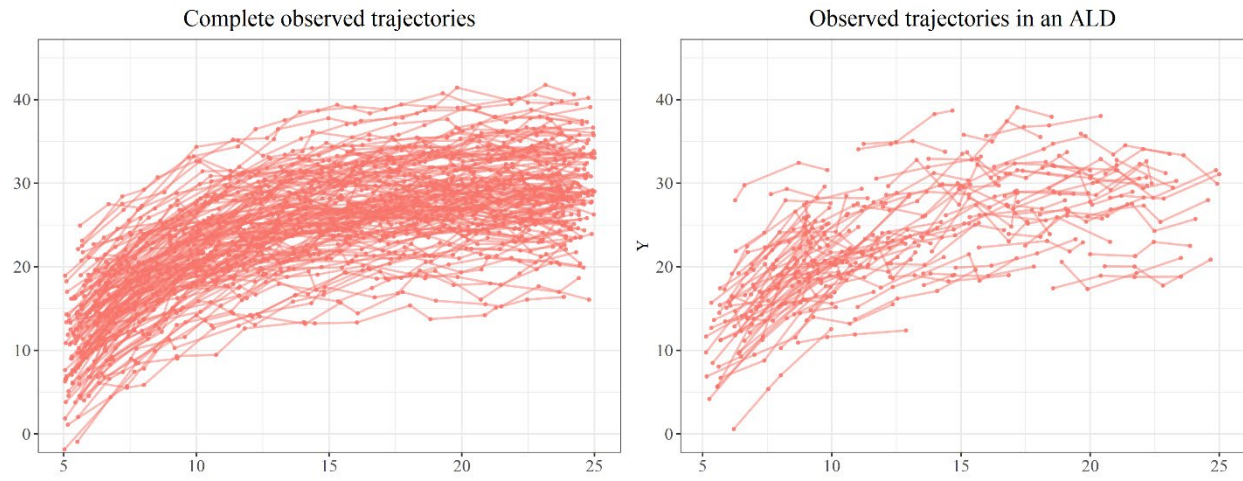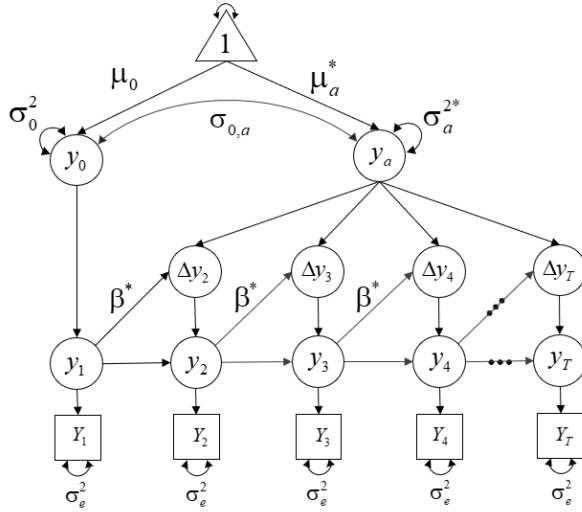
Figure 2. Example of cohort effects in the speed of maturation in a 20-year follow-up (left panel) and a hypothetical 5-year ALD with five cohorts.
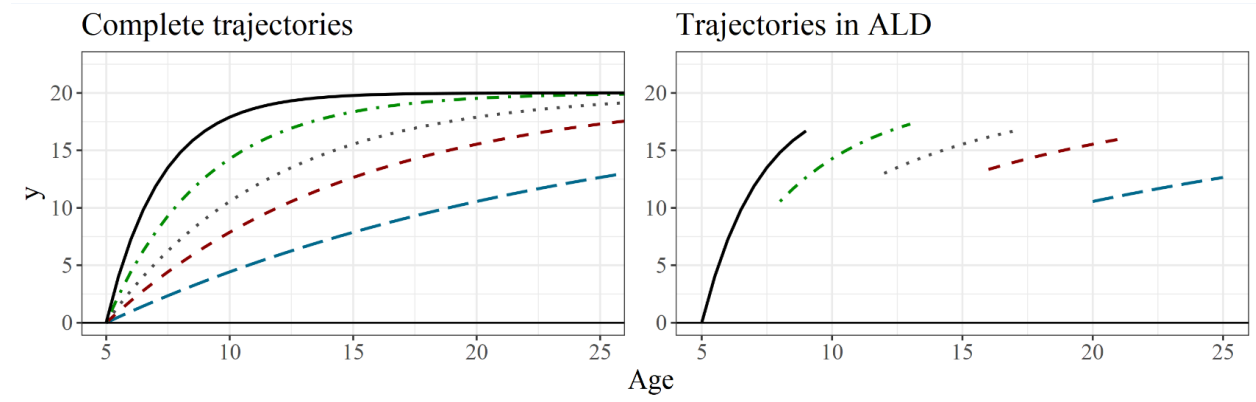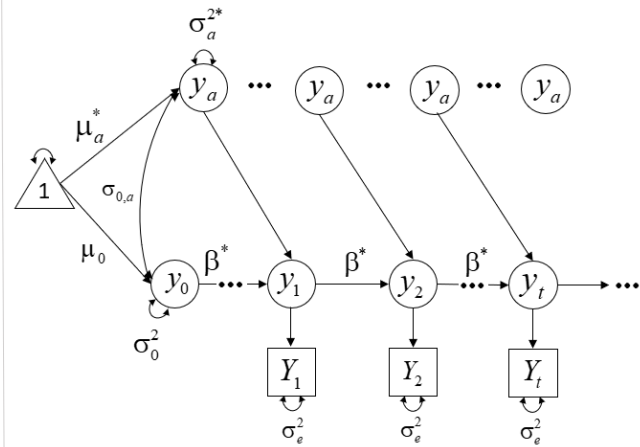
Figure 3. Path diagram of a univariate latent change score model specified as a SEM in discrete-time (left panel) and as a state-space model (SSM) in continuous-time (right panel). All unnamed paths have values fixed at 1.

Discrete-time latent change score model as SEM   Continuous-time latent change score model as SSM

Figure 4. Sampling designs

|		Age in years														% of cases entering in each cohort
	DESIGN	5 6 7 8 9 10 11 12 13 14 15 16 17 18 19														
All cases measured at all occasions	D0	13 cohorts														100/13
Two occasions per case	D1	13 cohorts														13.3 13.3 6.7 6.7 6.7 6.7 6.7 6.7 6.7 6.7 6.7 6.7 6.7
Three occasions per case	D2	11 cohorts														18.8 18.8 9.4 9.4 6.3 6.3 6.3 6.3 6.3 6.3 6.3
Four occasions per case	D3	12 cohorts														24.5 12.2 8.2 6.1 6.1 6.1 6.1 6.1 6.1 6.1 6.1

Figure 5. Relative bias of the parameter estimates across all conditions (top panel) and Total Root Mean Square of the relative bias (bottom panel). The cut-offs of −.1 and .1 for the RB are represented by solid black horizontal lines.
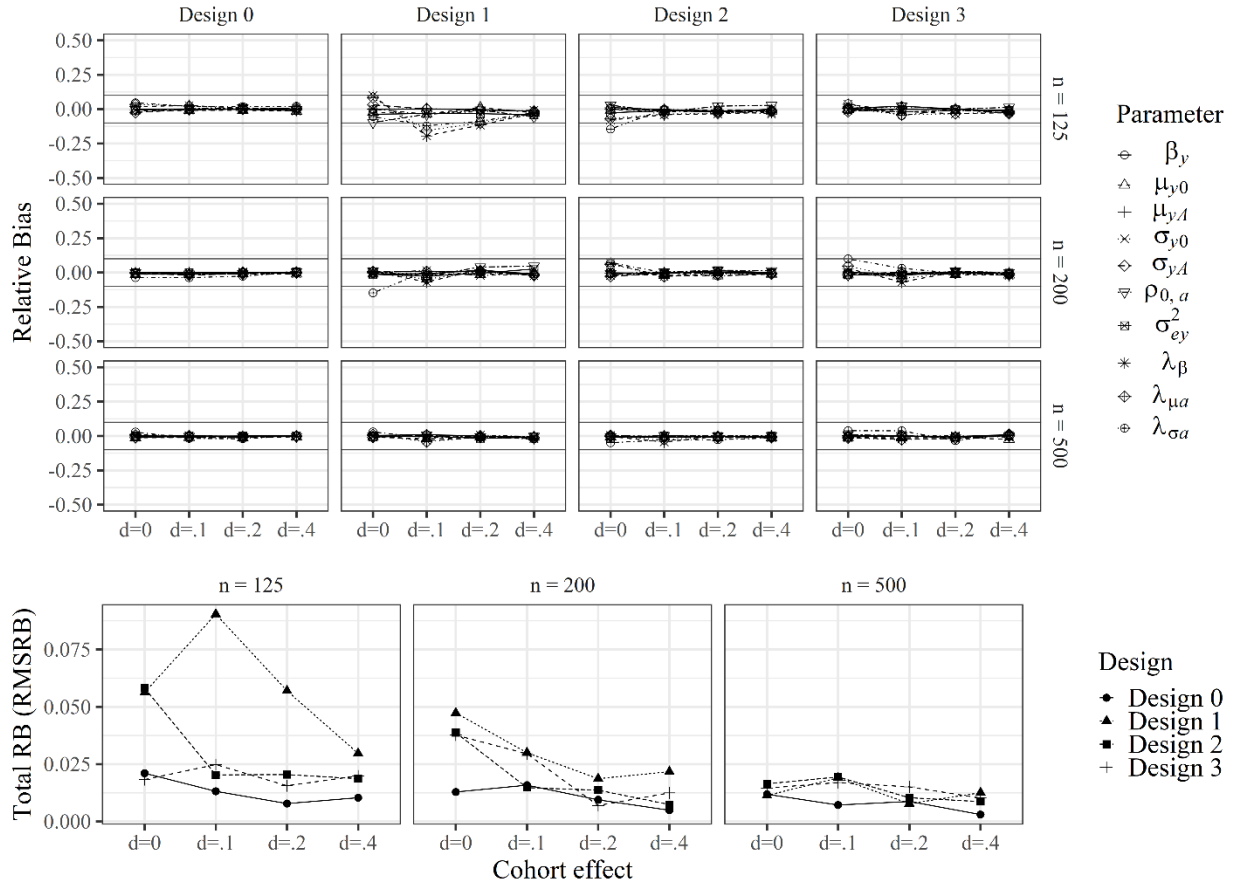
Figure 6. Standard Deviation of the Relative Bias (SDRB) of the parameter estimates across all conditions (top panel) and Total SDRB (bottom panel).
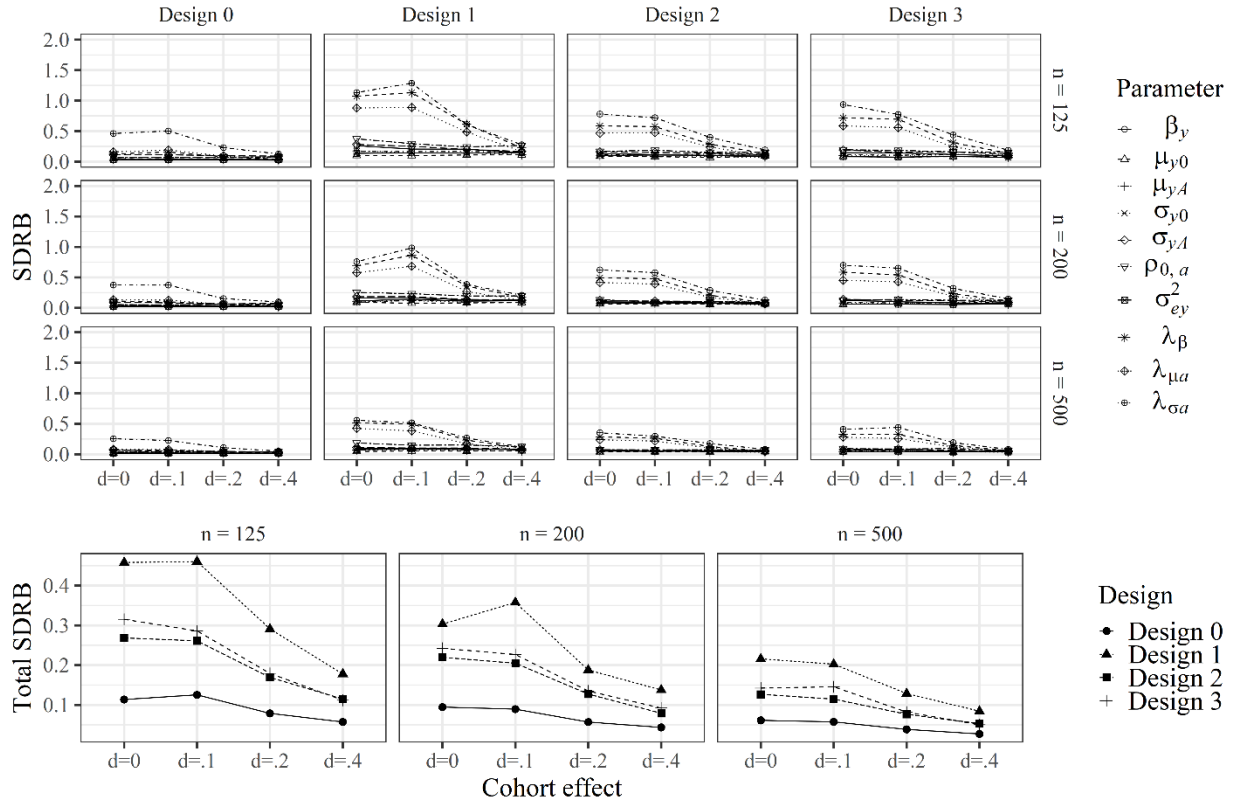
Figure 7. 95%CI coverage of the parameter estimates across all conditions (top panel) and mean coverage (bottom panel).
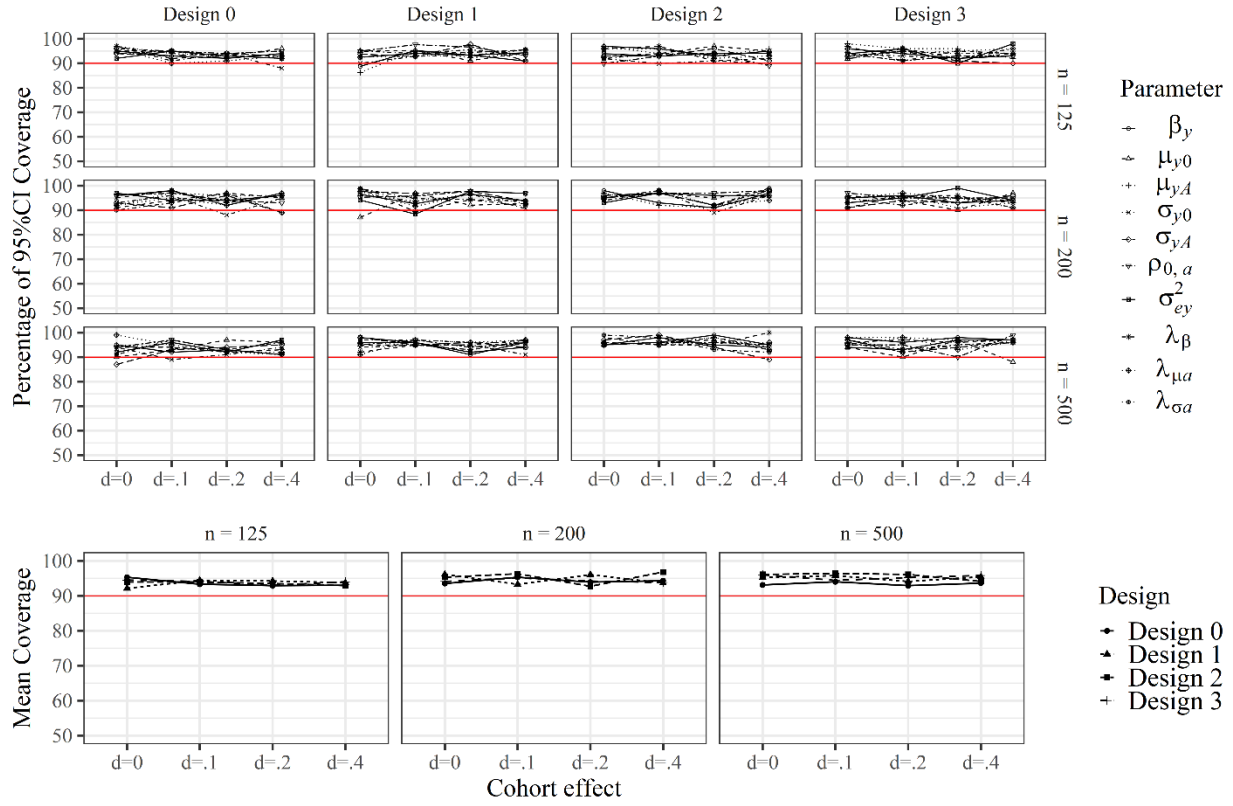
Figure A1. Latent change score model in discrete-time with cohort-related moderators across latent states