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Central Bank Digital Currencies and Financial Stability in a Modern Monetary System*

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Abstract

The aim of this study is to disentangle the effects of introducing an interest-bearing central bank digital currency (CBDC) for financial stability using a Diamond and Dybvig (1983) model in which (i) both CBDC and private bank deposits can be used in exchange and (ii) liquidity is created endogenously. Agents have direct access to a CBDC, which is a claim on the central bank. They use both sight deposits and CBDC to buy goods and commercial banks borrow reserves to cover liquidity needs. The introduction of an interest-bearing CBDC has direct implications for the sight deposit rate and the loan rate of banks. Besides, if the central bank aims to have a positive net worth and the absence of bank runs, a high demand for a CBDC is a necessary condition to achieve both objectives. If this is not the case, financial stability will be endangered.

JEL Classification: E42 E58 G21
Keywords: CBDC, banking sector, financial stability, bank runs

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"*Today, probably more than at any other time in our history, innovation has the potential to profoundly alter banking activities. It is no longer just about transforming our payment systems, it is our very currency that is at stake*" (Villeroy de Galhau, 2019, p.4).

1 Introduction

Traditional means of payment are progressively being replaced by retail payment innovations and electronic payment instruments. This fact combined with the fear of using cash because of the pandemic caused by the Coronavirus disease (COVID-19) (ECB, 2020), suggest an evolution toward a cashless society in payments.[1] As a response, central banks are carefully analysing the possibility of issuing digital forms of money for general use (Boar and Wehrli, 2021; Náñez Alonso et al., 2021), that is, central bank digital currencies (CBDCs). Their ever-mounting attention and the hope they become the means of payment of the future, have led to their interest skyrocketing among not only central bankers but also policymakers, lobbyists and financial services companies.

Although in early stages, many jurisdictions are already focusing on the possible CBDC design (Auer et al., 2020b). The CBDC could be universally accessible - retail CBDC - or be restricted to a particular group or agents - wholesale CBDC.[2] It may bear interest - interest-bearing CBDC - or not, similar to cash or private cryptocurrencies. Besides, the monetary authority could put into practice caps to holdings of the digital currency to prevent undesirable consequences (BIS, 2018). Anonymity vis-à-vis the central bank is another feature. The CBDC may be token-based, in a similar way to private digital tokens, or account-based.[3,4]

The rise of CBDCs – with three countries and one monetary union that have already issued a public money in digital form[5] – demands a thorough investigation of their implications for monetary policy, financial stability and payment systems.[6]

[1]While cash is being substituted as a means of payment, the demand for banknotes has constantly increased. This is known as the *cash paradox* (Jiang and Shao, 2020). Auer et al. (2020a) show evidence of the cash paradox - simultaneous concerns around viral transmission and hoarding of cash.

[2]Auer and Böhme (2020) analyse some of the technical design choices for retail CBDCs and their possible trade-offs. For a detailed discussion regarding the differences between a wholesale CBDC and a retail CBDC, see Pfister (2019).

[3]The main difference between token and account-based CBDCs is that they have a dissimilar form of verification when they are exchanged (Kahn and Roberds, 2009).

[4]For a comparison of a CBDC with cash, reserves and private digital currencies, see Table A1 in Appendix A.1.

[5]In October 2020, the Central Bank of the Bahamas issued the first retail CBDC in the world, the Sand Dollar. That announcement was followed, in March 2021, by the Eastern Caribbean Central Bank (ECCB). The ECCB deployed its DCash, becoming the first monetary union in launching a CBDC project. Nigeria was the first country in Africa issuing a CBDC, the eNaira, in October 2021. Finally, Jamaica issued the JAM-DEX at the end of the second quarter of 2022.

[6]CBDCs may also have an impact in other areas. Lagarde (2018) and Auer et al. (2022) remark that the growth of CBDCs can increase financial inclusion since they will reach people and enterprises in remote zones. In a similar line, Mancini-Griffoli et al. (2018) highlight that a

What impact will the introduction of a retail CBDC have in the financial system? Will an interest-bearing retail CBDC make digital bank runs more likely, destabilising the financial system? Is financial stability compatible with other objectives of the central bank such as aiming at having non-negative net worth? The goal of this chapter is to formally disentangle the effects of introducing an interest-bearing central bank-issued digital currency on financial stability and financial fragility in a modern monetary system.

I develop a tractable model based on the seminal paper of Diamond and Dybvig (1983) with nominal bank contracts (Skeie, 2008; Allen et al., 2014) and the features of a modern monetary system described in Rivero Leiva and Rodríguez Mendizábal (2019), in an environment where both public digital money and private bank deposits are used in exchange and money is created endogenously.[7] Entrepreneurs need to borrow money from commercial banks to pay their workers. Lending to them, commercial banks create inside money (deposits). When they receive their salary, workers save their money in sight deposits that will be remunerated at the end of the period. The following period, they have the chance to transfer money from their commercial bank account to the central bank, at a cost. As Meaning et al. (2018) underscore, commercial banks may react to the competition from central bank deposits by making it more costly to allocate funds out of the bank, that is, establishing or increasing fees. Households – formed by an entrepreneur and a worker – may use both commercial and central bank liabilities (deposits) as means of payment to buy goods and services. At the end of the final period, they must repay the loan to commercial banks.

Unlike the previous literature, one particular feature of the model is that there is no cash.[8] Therefore, a widely accessible CBDC that replaces cash is introduced into this economy.[9] Contrary to the paper by Rivero Leiva and Rodríguez Mendizábal (2019), everyone can open an account at the central bank free of charge.[10] In addition, the central bank has the possibility of remunerating its deposits at a variable rate. Besides being a potential tool for improving the transmission of monetary policy – an aspect beyond the scope of this paper – an interest-bearing CBDC can be used for financial stability reasons and to prevent the monetary authority from becoming

CBDC could encourage financial inclusion and minimise some of the costs and risks associated to the payment system, reduce informality, tax evasion and illegal activities (Rogoff, 2017) and create a more efficient electronic payment system (Marcel, 2019). There are also short-term economic gains derived from the creation of a public digital currency: fees from withdrawing money from the ATM or insurance, storage and transportation costs. Central banks could save printing and coining costs as well. Notwithstanding, apart from benefits, concerns and costs may also arise. Marcel (2019) warns about the need to improve cybersecurity of central banks to prevent cyberattacks and potential fraud.

[7] Deleidi and Fontana (2019) have empirically proved the validity of the endogenous money theory in the Eurozone for the 1999–2016 period.

[8] Engert et al. (2018) find that the disruptions that could be associated with the move to a cashless society are not important and will not cause material and system-wide problems.

[9] We are abstracting from having cash as a third payment option, but one may think in a cash-like CBDC in the case where the interest rate on the CBDC is set to zero.

[10] This may result in social savings if there are gains produced by economies of scale, as Eichengreen (2019) stresses. It may also be the case that central banks will rely on PSPs to perform retail-facing services in CBDC systems.

a significant financial middleman if the CBDC converts into a large-scale store of value. Niepelt (2020) adds that the introduction of a CBDC in combination with the refinancing operations rate may increase transparency, improving public scrutiny of central bank policies and reducing the influence of well-organised lobbies.

My main results are as follows. First, the model allows us to scrutinise how issuing an interest-bearing CBDC affects the interest rates managed by commercial banks. It is shown that the interest rate on deposits offered by commercial banks will react directly depending on the CBDC interest rate. A higher CBDC interest rate set by the central bank will force commercial banks to improve the attractiveness of sight deposits through a higher interest rate.

In addition, the interest rate on the CBDC has a direct impact on the loan rate. Imposing a higher CBDC interest rate will force commercial banks to pay a higher interest rate on sight deposits. As the funding costs are going to increase, they will also have to impose a higher loan rate in order to be solvent in the last period.

Second, the model is also a useful framework to analyse the possibility of digital bank runs. Weidmann (2018, p.4) remarks: "in a digital bank run, all it takes is a few mouse clicks to transfer savings out of the private financial system and into a central bank account. Customers are less likely to think twice about doing that". A digital bank run may befall for two reasons: either because households have a strong preference for retail CBDC deposits or fear bank failure. Nevertheless, it may be the case households think that commercial banks will become insolvent. If that occurs, they will not be able to retrieve their funds. Trying to anticipate this situation, they will be part of a massive withdrawal of deposits from the commercial bank. In this paper, I prove that the central bank can prevent a coordinated digital bank run from occurring by imposing a not relatively high interest rate on the refinancing operations. In other words, the opportunity of a run only becomes visible at moderately high interest rates on the refinancing rate of the central bank. This reinforces the role of the central bank as lender of last resort with the aim of stopping a liquidity crisis from turning into a solvency crisis.

Third, I analyse under which conditions the central bank can guarantee financial stability and aiming at having seigniorage revenues - maintaining non-negative net worth. Note that in reality, central banks have other objectives apart from preserving financial stability such as maintaining price stability and conducting an independent and credible monetary policy. The latter is proxied here by referring to the non-negative net worth of the central bank. If the demand for a CBDC - which does not depend only on the central bank - is high enough and the interest rate on the open market operations is higher than the interest rate on the CBDC, the economy will be in a situation where financial stability and seigniorage revenues can coexist. Nevertheless, I prove that aiming at having seigniorage revenues, the issuance of a CBDC by the central bank imposes a lower bound. The substitution of sight deposits by CBDC forces banks to charge a higher interest rate on loans to avoid being insolvent because of the increase in deposit funding.

In this paper, I abstract from many important issues. The central bank could implement other strategies to stop the possibility of a bank run. First, it can restrict the decline in bank deposits and lending by setting limits on individual CBDC holdings. Second, it can discourage - through fees or other instruments - convertibility from bank deposits to the retail CBDC. Both may be avenues for future research. In addition, the model is developed to allow for a representation of outside money in the form of a CBDC and inside money in the form of commercial bank deposits and loans. It can be extended by incorporating a private digital currency which competes with both sight and CBDC deposits.

This research is closely related to the very recent literature analysing the impact of issuing a retail CBDC in the banking system. Based on the "new monetarism" approach of Lagos and Wright (2005) and Rocheteau and Wright (2005), Chiu et al. (2019) assess the general equilibrium effects of introducing a CBDC and find that it cannot only improve bank intermediation efficiency but also increase lending – even if the usage of a CBDC is low. In an environment where both CBDC and private bank deposits can be employed in exchange, Keister and Sanches (2019) find that although CBDC promote efficiency in exchange and raises welfare, it may also crowd out bank deposits, lowering investment. Combining the overlapping generations (OLG) framework of Diamond (1965) and the banking models of Klein (1971) and Monti (1972), Andolfatto (2021) examines the consequences of introducing a CBDC on a monopolistic banking sector and interest rates. He underlines that the introduction of an interest-bearing CBDC increases financial inclusion and may affect the equilibrium interest rate on deposits, but not the interest rate on bank lending or the level of investment. Built in the standard neoclassical growth model, Piazzesi and Schneider (2020) show that the competition between private sight deposits and the CBDC will endanger the supply of deposits of commercial banks, which may mean that credit lines become more expensive. Instead, using a new Keynesian dynamic stochastic general equilibrium (NK-DSGE) model, Gross and Schiller (2021) show that although a CDBC may crowd out bank deposits, this effect can be alleviated if the monetary authority tries to disincentive CBDC holdings or provides central bank funds back to banks.

This paper relates also to the strand of the literature that focuses on the effects in financial stability and financial fragility. Kim and Kwon (2019) base their analysis in the OLG framework of Champ et al. (1996) and find that the introduction of a public digital currency decreases private credit supply and increases the nominal interest rate, which may translate to a raise in the likelihood of bank-runs, undermining financial stability. Extending the bank run model of Gertler and Kiyotaki (2015), Bitter (2020) finds that while a CBDC decreases net worth in the banking industry in non-crisis times, it may lessen the risk of a bank run in turmoil periods. In a similar line – but employing a Diamond and Dybvig (1983) model – Fernández-Villaverde et al. (2021) show that although the central bank may dissuade digital bank runs, it can also jeopardise maturity transformation since it will arise as a deposit monopolist. In contrast, Williamson (2022) notes that bank runs could become more frequent but less costly with CBDCs. In a different study, Fernández-Villaverde et al. (2020) present an impossibility result, also known as the CBDC trilemma. The central bank can only achieve at the same time two of the following objectives: efficiency, financial stability (i.e., absence of runs), and price stability. After studying how a

CBDC would facilitate runs out of bank deposits into a CBDC in financial upheaval situations, Bindseil (2020) explores possible solutions to the problem. Böser and Gersbach (2020) add that central banks can use monetary policy with tight collateral requirements to prevent the possibility of bank runs. Nevertheless, after certain periods, such policy will make banking activities unviable. Finally, Berentsen and Schär (2018) argue that a retail CBDC will increase the stability of the financial system because it would have a disciplining effect on commercial banks. Despite the significant contribution of all these papers, the role that outside money plays is not the one it actually does in the current monetary system. Therefore, this paper is the first attempt to combine a realistic view of money creation with the issuance of a retail interest-bearing CBDC.

Policymakers and central bankers are in need of new insights about retail issued CBDCs. A challenge that they have faced over these first years of researching about public digital currencies is finding a consensus on their actual design and their theoretical effects. My findings not only provide new insights into the CBDC and financial stability literature. They also matter for the future design of a CBDC. In a landscape where cash is disappearing, and electronic payments are rising, central banks should be placed at the forefront of the digital transformation.

The paper is structured as follows. Section 2 describes the theoretical model and characterises the equilibrium with both valued sight and CBDC deposits. Section 3 presents under which conditions digital bank runs may happen and when financial stability and seigniorage revenues may coexist. Finally, section 4 concludes.

2 A model of banking with CBDC

The economy represents a geoid whose measure is assumed to be 1. In this geoid, there are three dates: period 0, 1 and 2. Locations are continuously distributed over the geoid and on each location there is a continuum of identical risk averse households and a continuum of banks, both with measure 1. Each household is composed of a worker and an entrepreneur. There is also a central bank, a centralised goods market and a centralised labour market.[11] Both workers and entrepreneurs can access the banking system without incurring in a cost. Doing so, the households can earn an interest rate i^s on sight deposits held in the bank (private bank deposits) and i^d on deposits held in the central bank (digital currency deposits).

2.1 Households

Households are composed of a worker and an entrepreneur. The household's objective is to choose a path for consumption to maximise the sum of utilities where $U(\cdot)$ is bounded, continuously differentiable, strictly increasing, strictly concave, satisfies Inada (1963) conditions, and has a coefficient of relative risk aversion

$$-c \cdot \frac{U''(c)}{U'(c)} > 1. \tag{1}$$

[11] Workers supply labour inelastically.

Households face uncertainty about future liquidity needs in period $t = 0$. With probability λ, household becomes impatient ($h = 1$) and prefers to consume in period $t = 1$, while with probability $(1 - \lambda)$ the household is impatient ($h = 2$) and consumes at $t = 2$. Households observe types at the beginning of period 1 (once the idiosyncratic liquidity shock is realised).

Entrepreneurs

Entrepreneurs hire labour at $t = 0$. This labour is can be used in two risk-free productive technologies: long and short. They employ a fraction α of labour in the short technology and the remaining $1 - \alpha$ in the long technology.

The long technology needs two periods to produce goods. At $t = 0$, the long productive technology starts producing. As a result, it gives $\rho_2 > 1$ units of the good at $t = 2$. If a fraction $y \in [0, 1]$ of the long productive technology is interrupted at $t = 1$, it will produce $\rho_1 \cdot y$ units of the good (with $0 \leq \rho_1 \leq 1$). The remaining fraction left until maturity of the production process will yield $\rho_2(1 - y)$ in period $t = 2$.

The short productive technology produces each period and gives 1 unit of the good as return. The goods produced are sold either at $t = 1$ or $t = 2$. At $t = 1$, if not consumed, the goods can be stored.

When workers are hired, at time $t = 0$, entrepreneurs lack the credibility to convince those workers they will get paid in that period. Besides, assume entrepreneurs cannot employ the worker in their household and need to hire them from other households in a competitive labour market. Then, given that entrepreneurs enter in $t = 0$ with no resources, they need to borrow inside money from a bank located in the same location they live in.

Workers

At $t = 0$, each worker is endowed with a unit of time. They work for an entrepreneur who is not in their same household in exchange for an income W. This income is a wage received as a sight deposit. At $t = 1$, the deposit interest rate (i_1^s) is paid by the financial institution to the deposit account holder. Furthermore, they have the responsibility of buying consumption goods for the household at period 1 or 2, depending on whether the household is impatient or patient.

At $t = 1$, impatient households purchase goods transferring part of their liquid funds (sight deposits) to entrepreneurs of different households in exchange for goods produced. Hence, they are subject to a money in advance (MIA) constraint

$$P_1 \cdot c^1 \leq (1 + i_1^s) \cdot W, \tag{2}$$

with c^1 (c^2) the consumption of impatient (patient) agents, and P_1 (P_2) the nominal price of the consumption good in period 1 (period 2). After goods purchases, the household has to make a portfolio choice allocating the resources that are left plus the revenues from selling goods[12] at $t = 1$ into either public digital currency or sight deposits. Households face two different portfolio constraints depending on whether they are impatient or patient:

$$(1 + \psi) \cdot D_2^1 + S_2^1 \leq (1 + i_1^s) \cdot W - P_1 \cdot c^1 + P_1 \cdot q_1^1 \tag{3}$$

if the household is impatient and

$$(1 + \psi) \cdot D_2^2 + S_2^2 \leq (1 + i_1^s) \cdot W + P_1 \cdot q_1^2 \tag{4}$$

if the household is patient. Variables q_1^1 and q_1^2 are the amount of goods sold by the entrepreneur of a household of type 1 and 2 respectively. D_2^1 (D_2^2) is the public digital currency holdings of impatient (patient) agents[13], and S_2^1 (S_2^2) is the sight deposits of impatient (patient) agents. Meaning et al. (2018) highlight that banks may respond to the competition from a CBDC by making it more costly to transfer funds out of the bank. This feature is captured in my model by the term ψ, which can be interpreted as a commission charged by the bank.

The amount of goods sold by the entrepreneur has to satisfy the following resource constraint:

$$q_1^1 \leq \alpha + \rho_1 \cdot y^1 \tag{5}$$

if the household is impatient, and

$$q_1^2 \leq \alpha + \rho_1 \cdot y^2 \tag{6}$$

if the household is patient. The production of impatient or patient households coming from the short productive technology (α) and the interrupted fraction of the long productive technology ($\rho_1 \cdot y^h$) has to be equal or higher than the total amount of goods sold in the first period (q_1^h).[14]

At $t = 2$, since patient households are the only ones who consume goods, they face a MIA constraint of the form:

$$P_2 \cdot c^2 \leq (1 + i_2^d) \cdot D_2^2 + (1 + i_2^s) \cdot S_2^2 + P_2 \cdot (\alpha + \rho_1 \cdot y^2 - q_1^2). \tag{7}$$

The left hand-side of equation (7) represents the value of the goods consumed. The right hand-side expresses the return on both sight and digital currency deposits and the goods stored in the previous period, that can be consumed today.

After goods are consumed, the household has to pay back the original loan taken by the entrepreneur and the interest rate associated to it (i^l). If the household is

[12] The revenues will come from the short technology and from the liquidation of the long technology.

[13] Assume that $D_2^1 = D_2^{1,raw} \cdot (1 + \Psi)$ and $(D_2^2) = (D_2^{2,raw}) \cdot (1 + \Psi)$, being Ψ a non-pecuniary preference for CBDCs based on functionality or just the fact that it is directly backed by the monetary authority. $D^{h,raw}$ is the pure CBDC holdings without taking into account the non-pecuniary preferences. These non-pecuniary preferences will accrue to the household, and are set in $t = 0$. A higher Ψ - which it is assumed that cannot be negative - means a higher preference for the demand of CBDC. This may have important implications for the preference of CBDC and financial stability. I would like to thank an anonymous referee for this comment.

[14] With $h = 1$ if the household is impatient and $h = 2$ if the household is patient.

impatient, the total amount of available resources has to be equal or higher than the loan and its interest rate:

$$(1+i^l)\cdot W \leq (1+i_2^d)\cdot D_2^1 + (1+i_2^s)\cdot S_2^1 + P_2\cdot\Big[(1-\alpha-y^1)\cdot\rho_2+\alpha+\rho_1\cdot y^1 - q_1^1\Big]. \quad (8)$$

The right hand-side of equation (8) encompasses the income from both the sight and digital currency deposits and the sale of the remaining produced goods. If the household is patient, its loan repayment equation takes the following form:

$$(1+i^l)\cdot W \leq (1+i_2^d)\cdot D_2^2 + (1+i_2^s)\cdot S_2^2 + P_2\cdot\Big[(1-\alpha-y^2)\cdot\rho_2+\alpha+\rho_1\cdot y^2 - q_1^2\Big] - P_2c^2. \quad (9)$$

2.2 Banks

The role of banks is needed to solve the commitment problem between entrepreneurs and workers (Rivero Leiva and Rodríguez Mendizábal, 2019). Banks located in the same location as entrepreneurs lend them inside money to pay their workers. Banks directly deposit the amount of inside money in the worker's account. The account is shared both by the worker and the entrepreneur who belong to the same household. This action produces a double entry in the balance sheet of the bank, allowing us to introduce endogenous liquidity creation as in reality. McLeay et al. (2014, p.14) highlight that "whenever a bank makes a loan, it simultaneously creates a matching deposit in the borrower's bank account, thereby creating new money". Most money in reality is created by commercial banks making loans.

At $t = 1$, the bank has to demand reserves to avoid possible liquidity shortages that it may have. I assume that banks do not need collateral to borrow money from the monetary authority - this assumption can be relaxed in future research. The interest rate that the central bank charges for refinancing commercial banks is i^R. The main purpose of banks in this economy is to maximise their net worth at $t = 2$, being solvent at $t = 1$ (i.e. positive net worth). In the first period, the net worth of the commercial bank (NW_1^B) is equal to the assets it has, i.e. the amount of money lent to the entrepreneurs (W), minus their liabilities, which correspond to the deposits in the commercial bank (S_2^1) and the money that has been transferred to digital currency accounts at the central bank (D_2^1). In addition, the bank also charges a fee (ψ) from transferring the money to a non-commercial bank account. Remember that the amount that the commercial bank receives also depends on the non-pecuniary preference for CBDC that the household sets - which is captured in D_2^1 and D_2^2.[15] The equation of the net worth in the first period takes the following form:

$$NW_1^B = W - [\lambda(D_2^1 + S_2^1 - \psi D_2^1) + (1-\lambda)(D_2^2 + S_2^2 - \psi D_2^2)]. \quad (10)$$

In the second period, $t = 2$, banks receive the interest rate from the loan (asset side) and pay an interest rate on sight deposits to the household and an interest rate on the reserves borrowed to the central bank (liabilities). The rest of the liability part refers to the amount of goods sold by both types of households in the second period ($P_2[\lambda q_2^1 + (1-\lambda)q_2^2]$) minus the use of sight deposits for consumption by patient households ($(1-\lambda)P_2c^2$). Therefore, the net worth will be in the second period will be:

[15] See footnote 13.

$$NW_2^B = (1+i^l)W - (1+i_2^s)[\lambda S_2^1 + (1-\lambda)S_2^2] + (1-\lambda)P_2c^2 \\ -P_2[\lambda q_2^1 + (1-\lambda)q_2^2] - (1+i^R)[\lambda D_2^1 + (1-\lambda)D_2^2]. \quad (11)$$

I assume that there is perfect competition in the banking sector. Commercial banks will make decisions to maximise their net worth in the second period conditioned on the net worth in $t = 1$ being non-negative.

2.3 Central bank

In this economy, the central bank has two policy instruments: the interest rate on the refinancing operations (i^R) and the interest rate on the CBDC (i^d).[16] At the beginning of period $t = 0$, the central bank chooses both interest rates conditioned on the policy objectives it may have, that is, preserving financial stability and having non-negative net worth (non-negative seigniorage revenues).

In period $t = 1$, the net worth of the central bank is composed of the loans it has granted to the banks and the digital currency deposits it holds:

$$NW_1^{CB} = [\lambda D_2^1 + (1-\lambda)D_2^2] - \lambda D_2^1 - (1-\lambda)D_2^2 = 0. \quad (12)$$

In period $t = 2$, the central bank receives its income from the reserves lent to commercial banks and has to pay the CBDC remuneration:

$$NW_2^{CB} = (1+i^R)[\lambda D_2^1 + (1-\lambda)D_2^2] - (1+i_2^d)(1-\lambda)D_2^2 - (1+i_2^d)\lambda D_2^1 \\ = (i^R - i_2^d)[\lambda D_2^1 + (1-\lambda)D_2^2]. \quad (13)$$

It is clear that the level of seigniorage revenues (i.e. $NW_2^{CB} > 0$) that the central bank will have depends on the difference between the refinancing rate (i^R) and the CBDC interest rate (i^d), as long as there is demand of CBDC.

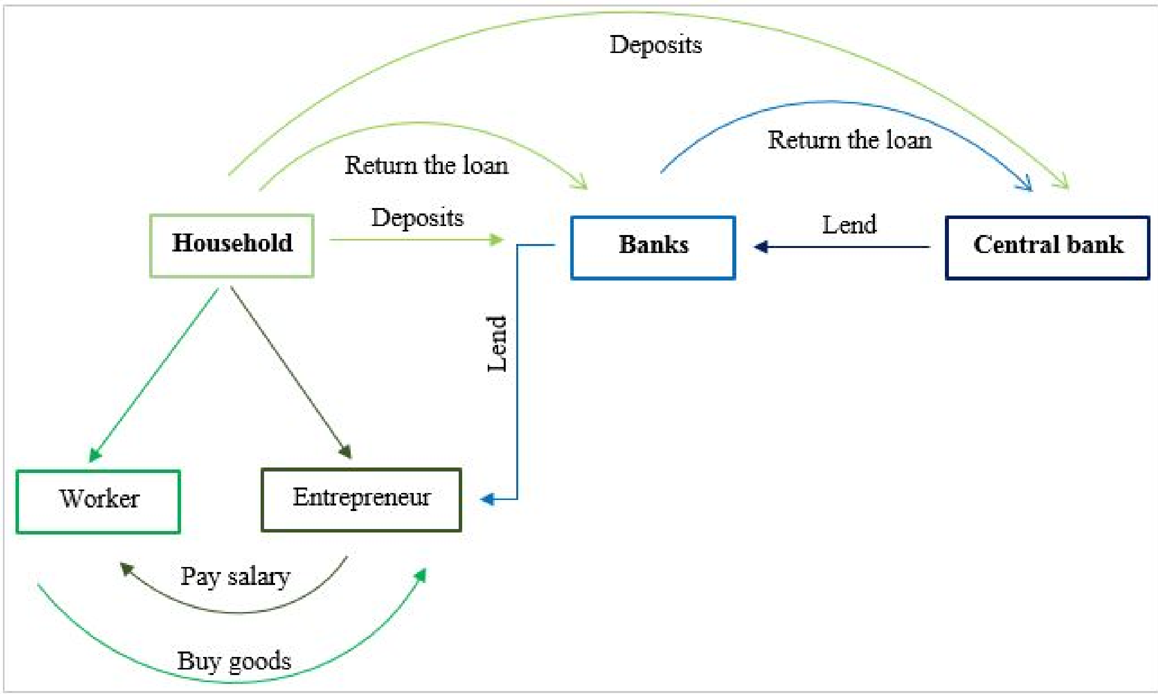
Figure 1 presents a simple illustration of the model and how all the agents interact among them.

2.4 CBDC, sight deposits and fiat reserves

In this economy, cash has been completely replaced by a CBDC issued by the central bank. Thus, the CBDC is a direct claim on the monetary institution. The CBDC is a complete safe and liquid asset, accessible by non-banks (retail CBDC), i.e. it can be held by all types of households without any restriction in an account at the central bank. It implies it can be held by households in unlimited quantities. As in reality, I assume the central bank cannot lend to households. In addition, the central bank will supply as much public digital currency as demanded.

[16] We assume the supply of CBDC is completely elastic to the interest rate on the CBDC, i^d. In addition, in this model it is still possible for the central bank to pay interest on reserves to commercial banks. It is assumed the monetary authority can do that just establishing a negative rate on the refinancing operations (i^R) interest rate.

Figure 1: Representation of the model



Source: Author's elaboration

On the other hand, CBDC and sight deposits are used by households to acquire consumption goods. Both pay an interest rate in each period (i_2^d, i_1^s and i_2^s respectively). The demand of CBDC will mainly depend directly on the interest rate paid by the central bank and indirectly on the interest rate on the sight deposits and the fee established by banks (which will determine the demand for commercial sight deposits). In other words, the demand of the public digital currency will depend on how desirable it is in comparison with other means of payment, i.e., private sight deposits.

Although both the CBDC and fiat reserves are liabilities to the central bank, they present some differences. First, the CBDC can be held by households in the form of accounts at the central bank and outside the banking system. On the contrary, reserves can only be held by banks. Second, the CBDC is used to make payments and offset liquidity risk in the household payment system and reserves are used to offset liquidity risk and service payment orders in the banking system. The demand for CBDC has implications on the demand for reserves because it forces banks to demand reserves when sight deposits are converted into a CBDC.

2.5 Household problem

Households choose consumption (c^h) in period $t = h$, with $h = 1$ for impatient households and $h = 2$ for patient households to maximise their utility. They also liquidate the long productive technology (y^h) and choose the portfolio allocation at $t = 1$, between digital currency holdings (D_2^h) and sight deposits holdings (S_2^h). As the problem is neutral with respect to the loan, W, everything is normalised by it. Hence, lower case letters of prices, digital currency holdings and sight deposit

Table 1: Glosary

Variable	Description	Variable	Description
c	consumption	ρ	fraction of the production
y	long productive technology	q	amount of goods sold by the entrepreneur
s	sight deposits	d	CBDC deposits
i^d	CBDC interest rate	i^s	sight deposits interest rate
i^l	loan interest rate	λ	share of impatient agents
ψ	bank commission	Ψ	degree of non-pecuniary preferences for the CBDC
α	fraction of labour used at time $t = 0$ by the household in the short technology		

holdings imply that they have been divided by W.

Let $\nu^1(\alpha)$ be the maximum level of utility that the impatient household is going to obtain as a function of the technology portfolio. α is the fraction of the hired labour in the short technology. The optimisation problem is as follows:

$$\nu^1(\alpha) = \max\left\{U(c^1)\right\} \tag{14}$$

subject to the money in advance constraint:

$$p_1 \cdot c^1 \leq 1 + i_1^s;\ (\eta_1) \tag{15}$$

to the portfolio constraint:

$$d_2^1(1+\psi) + s_2^1 \leq 1 + i_1^s - p_1 \cdot c^1 + p_1 \cdot q_1^1;\ (\kappa_1) \tag{16}$$

to the resource constraints:

$$0 \leq q_1^1;\ (\tau_1) \tag{17}$$

$$q_1^1 \leq \alpha + \rho_1 \cdot y^1;\ (\varphi_1) \tag{18}$$

and to the loan repayment equation:

$$\begin{aligned} 1 + i^l \leq p_2 \cdot [(1-\alpha-y^1)\rho_2 + \alpha + \rho_1 \cdot y^1 - q_1^1] \\ +(1+i_2^d)d_2^1 + (1+i_2^s)\cdot s_2^1;\ (\Xi_1). \end{aligned} \tag{19}$$

In parenthesis at the end of each equation is the corresponding Lagrange multiplier.

For the patient household, let $\nu^2(\alpha)$ be the maximum level of utility that she is going to obtain as a function of the technology portfolio., α is the fraction of the hired labour in the short technology. The optimisation problem is as follows:

$$\nu^2(\alpha) = \max\left\{U(c^2)\right\} \tag{20}$$

subject to the portfolio constraint:

$$(1+\psi)d_2^2 + s_2^2 \leq 1 + i_1^s + p_1 \cdot q_1^2;\ (\kappa_2) \tag{21}$$

to the resource constraints:

$$0 \leq q_1^2;\ (\tau_2) \tag{22}$$

$$q_1^2 \leq \alpha + \rho_1 \cdot y^2;\ (\varphi_2) \tag{23}$$

to the money in advance constraint:

$$p_2 \cdot c^2 \leq (1+i_2^d)d_2^2 + (1+i_2^2)s_2^2 + p_2(\alpha + \rho_1 \cdot y^2 - q_1^2);\ (\eta_2) \tag{24}$$

and to the loan repayment equation:

$$\begin{aligned} 1 + i^l \leq (1+i_2^d)d_2^2 + (1+i_2^2)s_2^2 \\ +p_2(\alpha + \rho_1 \cdot y^2 - q_1^2) - p_2c^2 + p_2(1-\alpha-y^2)\rho_2;\ (\Xi_2). \end{aligned} \tag{25}$$

The first order conditions and the household-labour-technology problem are characterised in Appendix A.2.

2.6 Equilibrium with valued both sight and CBDC deposits

Definition 1. An equilibrium is a collection of allocations $\{\alpha, y^h, c^h, q_1^h, s_2^h$ and $d_2^h\}$ for $h \in \{1,2\}$ and prices $\{p_1, p_2, i_1^s, i_2^s$, and $i^l\}$ such that:

1. given prices, allocations solve individual problems of both households and commercial banks, and,
2. prices are such that goods market clear for $t = 1$ and $t = 2$.

Let us characterise the equilibrium with both CBDC and sight deposits.

Definition 2. *An equilibrium with valued both sight deposits and central bank digital currency deposits is an equilibrium in which* $s_2^1 > 0$, $s_2^2 > 0$, $d_2^1 > 0$, $d_2^2 > 0$.

Proposition 1. *There exists a unique equilibrium with valued both sight deposits and central bank digital currency deposits in which*:

- *the equilibrium value of the consumption basket for impatient households is 1, i.e.,* $c^1 = 1$,
- *the equilibrium value of the consumption basket for patient households is* ρ_2, *i.e.,* $c^2 = \rho_2$,
- *production decisions satisfy:* $y^1 = y^2 = 0$, $\alpha = \lambda$,
- *the amount of goods sold by the entrepreneur satisfies:* $q_1^1 = q_1^2 = \alpha$, $q_2^1 = q_2^2 = (1-\alpha)\rho_2$,
- *the price in period 1 is equal to:* $p_1 = 1 + 2\psi[\lambda d_2^1 + (1-\lambda)d_2^2]$,
- and *interest rates are*:

$$i_1^s = 2\psi[\lambda d_2^1 + (1-\lambda)d_2^2]; \tag{26}$$

$$(1+\psi)(1+i_2^s) = (1+i_2^d) = \rho_2 \cdot \frac{p_2}{p_1}; \tag{27}$$

$$i^l = \left(i^R - \frac{(1-\psi)i_2^d - 2\psi}{1+\psi}\right)[\lambda d_2^1 + (1-\lambda)d_2^2] + \frac{i_2^d - \psi}{1+\psi}. \tag{28}$$

Proof. See Appendix A.3.

Discussion. The nominal interest rate for both CBDCs $(1 + i_2^d)$ and sight deposits $(1 + i_2^s)$, together with the interest rate on sight deposits in period 1 (i_1^s), the price level for the second period (p_2) and the interest rate on the loans (i^l), are not determined individually. The real variables are determined.

From the sight deposits-CBDC interest rate condition (equation (27)), it can be drawn that the relative attractiveness of central bank money relative to bank deposits will rest on the interest rate on both competing means of payment and the fee established by commercial banks. In equilibrium, households will be indifferent between holding commercial bank money or central bank money. Besides, if banks make free to transfer funds out of their institution $(\psi = 0)$, in equilibrium, the interest rate on sight deposits and the CBDC in the second period will be the same, i.e., neither sight nor CBDC deposits will dominate in rate of return to each other.

The interest rate on sight deposits in the first period will depend on the commission and the amount of central bank digital currency demanded by households. This has two main implications. First, as there is no aggregate uncertainty, i.e, banks behave under perfect foresight in aggregate, the bank will anticipate the revenues it will have in the first period as a consequence of the conversion of sight-deposits to a CBDC. The competition in the banking sector forces it to remunerate deposits in period $t = 1$. Again, if banks make free to transfer funds out of their institution $(\psi =0)$, in equilibrium, the interest rate on sight deposits in period $t = 1$ will be zero. These levels ensure that the net worth of banks is zero in the first period. Second, the central bank may affect - through changes in the interest rate on the CBDC - sight deposits interest rates and prices.

These interest rate levels ensure that the net worth of banks is zero in both the first and the second period. Thus, banks are solvent in the second period and no runs will happen. This means that maintaining sight deposits in commercial banks as well as CBDC deposits at the central bank is an equilibrium outcome.

Implications on the loan rate. Until now, I have characterised the equilibrium conditions with valued both sight and CBDC deposits. Let us determine how the interest rate on loans varies depending on the different monetary instruments (equation (28)):

- The higher the refinancing rate of the central bank is, the higher the lending rate in equilibrium will be. In particular:

$$\frac{\partial i^l}{\partial i^R} = (\lambda d_2^1 + (1-\lambda)d_2^2) \geq 0 \tag{29}$$

Higher refinancing rates imply an increase in the costs of commercial banks. To compensate this, they need to increase their revenues through a higher loan interest rate.

In addition, the impact of the interest rate on the refinancing operations will be even higher the higher the demand of the CBDC is.[17] The mechanism is straightforward. A higher amount of CBDC deposits will imply that the commercial bank should refinance that amount, borrowing more reserves from the central bank. However, the interest rate i^R would have no effect if the household chooses to not demand CBDC.

- The impact of choosing a high CBDC interest rate on the loan rate will depend on the amount of the CBDC demanded[18] - weighted by their respective types of agents:

$$\frac{\partial i^l}{\partial i^d} = \frac{1}{1+\psi} - \frac{1-\psi}{1+\psi}(\lambda d_2^1 + (1-\lambda)d_2^2). \tag{30}$$

  The only case where the derivative is zero happens when:

$$\lambda d_2^1 + (1-\lambda)d_2^2 = \frac{1}{1-\psi}.$$

  In this economy, the amount of CBDC will always be lower than the threshold, $\frac{1}{1-\psi}$. Hence, an increase of the CBDC interest rate would make the commercial bank to increase the loan rate:

$$\frac{\partial i^l}{\partial i_2^d} > 0. \tag{31}$$

  The intuition is as follows. Higher CBDC interest rates will increase the cost of deposit funding for commercial banks, which will directly lead to higher lending rates.

The central bank's policy through its digital currency has implications not only for prices in the economy but also for how expensive borrowing is.

3 Digital bank runs and financial stability with a CBDC

[17]By extension, the impact of the interest rate on the refinancing operations will also depend on the degree of non-pecuniary preferences for CBDC (Ψ) that the household sets at the beginning of the period.

[18]As in the previous case, the impact of choosing a high CBDC interest rate on the loan rate will also depend on the non-pecuniary preferences for CBDC (Ψ) that the household sets at $t = 0$

3.1 Digital bank run

It may be the case that households have concerns about the solvency of their bank. Both impatient and patient households need their funds at the financial institution to repay the loan in $t = 2$. If some customers of the banking system withdraw their funds of one bank or of a set of banks with mass zero, the probability of default by banks increases. I assume that in $t = 1$, some households decide to withdraw their sight deposits[19], that is, agents cause a coordinated digital bank run. This would occur immediately.

It will take households only a few seconds - through an electronic device - to transfer savings out of some commercial banks and into their central bank account. Because of online banking, households have also the potential to withdraw their deposits within a few seconds from a device to an account at another bank or a non-bank e-money provider. Since there is no cash in this economy, households can put their funds into a CBDC deposits at the central bank, goods stored or deposits in other commercial banks.

To make our analysis comparable to the one in Diamond and Dybvig (1983) and Rivero Leiva and Rodríguez Mendizábal (2019), I assume that the coordinated digital bank run happens when the liquidity preference shock has already been realised, at the beginning of period 1. This also means that the loan rate and the interest rate on the sight deposits in period $t = 1$ have already been established. However, households have not been able to purchase goods yet.

Proposition 2. *The possibility of a digital run will become reality if the following condition is fulfilled*:

$$i^R > i^l. \tag{32}$$

Proof. See the first part of Appendix A.4.

Discussion. A self-fulfilling digital bank run will occur in equilibrium if the refinancing rate established by the central bank is high enough. Households will evaluate this condition ex-ante and will run on their bank because that banks will not be solvent in the second period.

In times of economic turmoil, issuance of a retail CBDC could endanger financial stability by transferring funds from bank deposits to the CBDC at the central bank. Conversely to condition (32), as long as the refinancing rate of the central bank is not really high (equation (32) is not satisfied), there will not be a digital bank run. That implies the following:

$$i^R \leq i^l.$$

[19] A bank run on a group of banks with positive mass or a systemic digital bank run are not evaluated in this paper.

As in equilibrium, $i^l = \left(i^R - \frac{(1-\psi)i_2^d - 2\psi}{1+\psi}\right)[\lambda d_2^1 + (1-\lambda)d_2^2] + \frac{i_2^d - \psi}{1+\psi}$, I use expression (32) to determine when a self-fulling digital bank will not be triggered.

In particular, a self-fulling digital bank run will not be triggered in equilibrium as long as[20]

$$i^R \leq \frac{i_2^d - \psi - ((1-\psi)i_2^d - 2\psi)[\lambda d_2^1 + (1-\lambda)d_2^2]}{(1+\psi)(1 - \lambda d_2^1 - (1-\lambda)d_2^2)}. \tag{33}$$

3.2 Central bank policy objectives

Central banks have different policy objectives such as preserving financial stability and maintaining price stability through conducting an independent and credible monetary policy. The latter aspect is considered in this paper by referring to the non-negative net worth of the central bank. The credibility and independence of the monetary policy of the central bank is significantly influenced by the financial strength of the institution (Lonnberg and Stella, 2008). Hence, the issuance of a retail CBDC may have important financial stability and central bank balance sheet management considerations. In particular, an interest-bearing CBDC would give the monetary authority an additional instrument: the interest rate on the CBDC (i^d). This instrument and the interest rate on the refinancing operations (i^R), are the central bank's tools for achieving its policy objectives: in this case, financial stability and aiming non-negative net worth.[21]

However, for each i^R, i^d, and the demand for a CBDC - which does not depend only on the central bank -, there will be an equilibrium where the net worth of both commercial banks and the central bank and the likelihood of a digital bank run will be different.

If the objective of the monetary policy of the central bank is to ensure that:

- the net worth of commercial banks is positive, i.e., $nw_2^B > 0$,
- the net worth of the central bank is positive, i.e., $nw_2^{CB} > 0$,
- and there is an absence of digital bank run, then,

it must be the case that the refinancing rate of the central bank is bounded. To avoid a digital bank run, that interest rate has an upper-bound (equation (33)). To avoid central bank losses, the refinancing rate has a lower-bound, that is:

$$i^R > i_2^d.$$

[20]See the second part of Appendix A.4.

[21]Note that under this framework, the central bank cannot perform its monetary policy aiming at improving the welfare of households. Monetary policy does not affect consumption levels, i.e., it is not possible to characterise the welfare-maximising monetary policy with respect to consumption. Other papers have shown that a CBDC can impact consumption. For instance, Davoodalhosseini (2021) computes the benefits of a public digital currency to be around 0.16 percent of total consumption.

Thus, the refinancing rate of the central bank has both a lower and a upper bound:

$$i^R \in \left[i_2^d, \frac{i_2^d - \psi - ((1-\psi)i_2^d - 2\psi)[\lambda d_2^1 + (1-\lambda)d_2^2]}{(1+\psi)(1-\lambda d_2^1 - (1-\lambda)d_2^2)} \right]. \tag{34}$$

I have to determine under which conditions there is a range of values of the interest rate on a CBDC deposits and the CBDC demand that allow the net worth of the central bank and commercial banks to be positive and avoid a digital bank run. Thus, it must be the case that:

$$i_2^d < \frac{i_2^d - \psi - ((1-\psi)i_2^d - 2\psi)[\lambda d_2^1 + (1-\lambda)d_2^2]}{(1+\psi)(1-\lambda d_2^1 - (1-\lambda)d_2^2)}.$$

Proposition 3. *The central bank will achieve financial stability and will be solvent - i.e, will have seigniorage revenues, at the same time, for any reasonable value of i_2^d, where $i_2^d < i^R$, as long as*:

$$\lambda d_2^1 + (1-\lambda)d_2^2 > 0.5. \tag{35}$$

Proof. See Appendix A.5.

Notice this result is independent on the value of the fee established by commercial banks.

Discussion. Introducing an interest-bearing retail CBDC is often seen as an additional instrument of central bank policy. Nevertheless, issuing a CBDC imposes an additional constraint. This happens because the interest rate on the CBDC deposits cannot be independent from the refinancing rate. The connection between the two arises from the competition between means of payment, private deposits and CBDC by agents, together with the solvency bounds of commercial banks and the required seigniorage revenues of the central bank.

If the demand for a CBDC is high enough, the economy will be in a situation where financial stability and seigniorage revenues can coexist. Aiming at having a positive net worth, i.e., positive seigniorage revenues, the issuance of a CBDC by the central bank imposes an additional constraint. The lower bound of the refinancing interest rate has relevant implications for financial stability since the remuneration of central bank digital currency deposits has direct impact on the lending rate. Establishing a relatively high CBDC interest rate will force the central bank to set also a high refinancing rate if it aims at positive seigniorage revenues. As a result, there will be an upward pressure on the loan rate of commercial banks. At the same time, the opportunity of a coordinated digital bank run only becomes visible at moderately high interest rates on the CBDC deposits.

Financial stability and positive seigniorage revenues can coexist as long as the demand of the CBDC is large enough. In this economy, the higher the demand of the CBDC is, the higher the loan rate will be. Such demand affects the loan rate through two different channels. First, directly, depending on the difference between the refinancing rate of the central bank and the CBDC interest rate:

$$\frac{\partial i^l}{\partial(\lambda d_2^1 + (1-\lambda)d_2^2)} = i^R - \frac{(1-\psi)i_2^d - 2\psi}{1+\psi}. \tag{36}$$

Second, indirectly, through the refinancing rate (equation 29). When the demand of the CBDC is higher than the demand of sight deposits in the economy, that is, when $d_2^1 + (1-\lambda)d_2^2 > 0.5$, the loan interest rate will be higher than the refinancing rate of the central bank. Hence, a high demand of the CBDC does not endanger financial stability as long as the refinancing rate is not high enough. The substitution of sight deposits by CBDC deposits, however, will force banks to charge a higher interest rate on loans to avoid being insolvent.

However, if the demand of CBDC is:

$$\lambda d_2^1 + (1-\lambda)d_2^2 < 0.5,$$

and the open market operations interest rate is higher than the CBDC interest rate (positive seigniorage revenues), in equilibrium, the lending rate established by commercial bank would be lower than the refinancing rate of the central bank. Therefore, a self-fulfilling digital bank run would be triggered.

If we are in a situation where $d_2^1 + (1-\lambda)d_2^2 > 0.5$ but the interest rate on CBDC deposits is higher than the interest rate on the open market operations, the central bank will have losses, i,e., a negative net worth. But, the reader may be wondering whether there is a need of having seigniorage revenues. The objective of the central bank usually is far beyond being profitable. It is not a profit-maximising enterprise. In fact, having negative net wealth is not uncommon, specially in developing countries. For instance, Stella and Lonnberg (2008) show that at least 15 Latin American central banks had losses for five or more years between 1987 and 2005. As long as an automated and fully credible rule of re-capitalisation by the government of the monetary authority in case of negative worth is implemented (Bindseil et al., 2004), losses do not necessarily jeopardise the central banks' monetary policy targets. In this paper, both having monetary losses and preserving financial stability may coexist. However, in practice, central banks are more likely to report slightly positive profits than negative ones (Goncharov et al., 2020). Goncharov et al. (2020) highlight that the political environment - fear of losing operational independence - and behavioural and agency frictions are related to loss avoidance. Lonnberg and Stella (2008) and Reis (2015) also remark that the credibility and independence of the monetary policy of a monetary authority is significantly influenced by the financial strength and solvency of the institution.

4 Conclusion

Innovation in the payments arena is rapidly evolving and modifying the current monetary landscape. This digital shift has reached central banks and monetary institutions resulting in a race to develop and issue a new form of digital money: a central bank digital currency. In this paper, I have offered an examination of the effects of introducing an interest-bearing central bank-issued digital currency on financial stability in a modern monetary system where both public digital money and private bank deposits can be used interchangeably. To do so, I employ a Diamond and Dybvig (1983) model with nominal bank contracts (Skeie, 2008; Allen et al., 2014) and the features of a modern monetary system (McLeay et al., 2014; Rivero Leiva and Rodríguez Mendizábal, 2019).

In equilibrium with both valued sight and CBDC deposits, agents are indifferent to hold commercial bank money or central bank money. The relative attractiveness of central bank money relative to bank deposits will mainly rest on the interest rate on both competing means of payment and the fee established by commercial banks. Examining the impact of introducing an interest-bearing CBDC is of interest from a variety of policy perspectives. I show that the central bank's policy through its digital currency has implications not only for prices in the economy but also for how expensive borrowing is. A higher CBDC interest rate will force commercial banks to impose a higher lending rate in order to compensate the increase of deposit funding. I also find that, conditioning on having a high demand of the CBDC, the central bank can guarantee both financial stability and seigniorage revenues. The second target, however, imposes a lower bound on the refinancing rate of the central bank that may endanger financial stability and may have spillover consequences for monetary policy.

In this paper, for the sake of simplicity, I abstract from many relevant issues. Suppose that the banking system is not permitted to borrow all the reserves they want. This imposes an additional constraint on the bank's constrained maximisation problem. How is this new equilibrium different from the equilibrium in which there are no reserve requirements? Moreover, I assume that the central bank just sets the price of the CBDC. It may be also the case that it would like to set the quantity of the CBDC instead of the price as an strategy to stop the possibility of a bank run because the monetary authority is restricting the decline in bank deposits allowing limits on CBDC holdings. In addition, the model is developed to allow for a representation of outside money in the form of a CBDC and inside money in the form of commercial bank deposits and loans. In the reality, in economies prone to currency crises, private digital currencies are also surging as an alternative store of value. The model could be extended by incorporating a private digital currency which competes with both sight and CBDC deposits. Finally, another limitation of my model is that it has been set in the context of a closed economy. Bank runs of the previous decades and centuries have befallen in an open environment. Households may have the chance to migrate their deposits from their local bank to a foreign bank or to an external central bank – if that is permitted. In fact, the current financial and payments systems share widespread cross-border linkages. It may be the case that a poorly designed retail CBDC issued in one country endangers the stability of the financial system of other countries. This leaves another avenue to further investigation.

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A Appendix

A.1 Design alternatives for a CBDC

Table A1 provides a classification of the different types of money and the features they share.

Table A1: Classification of money*

	Universally accessible	Electronic	Central bank issued	Interest bearing	Caps
Cash	✓	✗	✓	✗	✗
Reserves	✗	✓	✓	✓	✗
Private digital currencies	✓	✓	✗	✗	✗
Wholesale CBDC	✗	✓	✓	✓	✓
Retail CBDC	✓	✓	✓	✓	✓

Source: Author's elaboration based on the classification of Bech and Garratt (2017) and BIS (2018). *A check mark means that it is possible to implement the feature.

Bordo and Levin (2018) highlight that a CBDC would fulfill all the fundamental tenets of currencies established by Jevons (1875) a century and half ago. It can perfectly be a unit of account, a medium of exchange with no cost, a secure store of value (Bordo and Levin, 2018) and a standard of deferred payment (Shirai, 2019).

A.2 First order conditions

Solving the maximisation problem, the first order conditions for consumption, central bank digital currencies, sight deposits, the amount of goods sold by the entrepreneur of a household of type 1 and 2 and the long technology are:

Consumption:

$$[c^1] : U'(c^1) = p_1 \cdot (\eta_1 + \kappa_1).$$
$$[c^2] : U'(c^2) = p_2 \cdot (\eta_2 + \Xi_2).$$

CBDC deposits:

$$[d_2^1] : d_2^1 \cdot [\Xi_1(1 + i_2^d) - \kappa_1(1 + \psi)] = 0.$$
$$[d_2^2] : d_2^2 \cdot [\Xi_2(1 + i_2^d) + \eta_2(1 + i_2^d) - \kappa_2(1 + \psi)] = 0.$$

Sight deposits:

$$[s_2^1] : s_2^1 \cdot [\Xi_1(1 + i_2^s) - \kappa_1] = 0.$$
$$[s_2^2] : s_2^2 \cdot [(\Xi_2 + \eta_2)(1 + i_2^s) - \kappa_2] = 0.$$

Amount of goods sold by the entrepreneur:

$$[q_1^1] : q_1^1 \cdot [\tau_1 - \varphi_1 - p_2\Xi_1 + p_1\kappa_1] = 0.$$
$$[q_1^2] : q_1^2 \cdot [\tau_2 - \varphi_2 - p_2(\eta_2 + \Xi_2) + p_1\kappa_2] = 0.$$

together with the slackness conditions:

$\tau_1 \cdot q_1^1 = 0.$
$\tau_2 \cdot q_1^2 = 0.$
$\varphi_1(\alpha + \rho_1 \cdot y^1 - q_1^1) = 0.$
$\varphi_2(\alpha + \rho_2 \cdot y^2 - q_1^2) = 0.$

Long productive technology:

$[y^1] : [\varphi_1\rho_1 + \Xi_1 p_2(\rho_2 - \rho_1)]y^1 = 0.$
$[y^2] : [\varphi_2\rho_1 + \Xi_2 p_2(\rho_2 - \rho_1) + p_2\rho_1\eta_2]y^2 = 0.$

There are also the envelope conditions with respect to α:

$\frac{d\nu^1(\alpha)}{d\alpha} = \varphi_1 + \Xi_1 p_2(1 - \rho_2).$

$\frac{d\nu^2(\alpha)}{d\alpha} = \varphi_2 + p_2\eta_2 + p_2\Xi_2(1 - \rho_2).$

Besides, on period $t = 0$, the household chooses the split of hired labour between both the short and long technology to solve the problem:

$$\nu(\alpha) = \max\left\{\lambda\nu^1(\alpha) + (1 - \alpha)\nu^2(\alpha)\right\} \tag{37}$$

subject to

$$0 \le \alpha \le 1. \tag{38}$$

Thus, the first order condition is:

$$\lambda\frac{d\nu^1(\alpha)}{d\alpha} + (1 - \lambda)\frac{d\nu^2(\alpha)}{d\alpha} + \beta_0 - \beta_1 = 0, \tag{39}$$

with β_0 and β_1 the Lagrange multipliers associated to left-hand side and right-side of the constraint (equation (38)) respectively.

A.3 Equilibrium with valued both sight and CBDC deposits. Proof of proposition 1

I prove that there exists a unique equilibrium with valued both sight deposits and central bank digital currency deposits. To do that, I obtain the interest rate conditions, the consumption levels and the production decisions, given the size of the short term technology, α, chosen by the entrepreneurs.

Let us obtain the equilibrium values of the consumption baskets and the production decisions:

Assuming that the amount of goods sold by the entrepreneur is positive in the first ($q_1^1 > 0$) and the second period ($q_1^2 > 0$)[22]:

[22] $q_1^1 = 0$ and $q_1^2 = 0$ cannot be a equilibrium because impatient households consume in period 1.

$$\tau_1 - \varphi_1 = p_2\Xi_1 - p_1\kappa_1. \tag{40}$$

$$\tau_2 - \varphi_2 = p_2(\eta_2 + \Xi_2) - p_1\kappa_2. \tag{41}$$

Using equations (40) and (50):

$$\frac{p_1}{p_2}\frac{(1+i_2^d)}{(1+\psi)} = 1 + \frac{\varphi_1 - \tau_1}{p_2\Xi_1}. \tag{42}$$

Employing equations (41) and (51):

$$\frac{p_1}{p_2}\frac{(1+i_2^d)}{(1+\psi)} = 1 + \frac{\varphi_2 - \tau_2}{p_2(\Xi_2 + \eta_2)}. \tag{43}$$

From equations (42) and (43), two possibilities arise:

i) If $\varphi_1 = \varphi_2 = 0$ and $\tau_1 = \tau_2 = 0$, then:

$$p_1\frac{(1+i_2^d)}{(1+\psi)} = p_2. \tag{44}$$

This would imply that $q_1^1 > \alpha$, $y^1 = 0$, $q_1^2 > \alpha$, $y^2 = 0$.

ii) If $\varphi_1 > 0, \varphi_2 > 0$ and $\tau_1 = \tau_2 = 0$, then, the following two options may happen:

$$p_1\frac{(1+i_2^d)}{(1+\psi)} = p_2 + \frac{\varphi_1}{\Xi_1}.$$

$$p_1\frac{(1+i_2^d)}{(1+\psi)} = p_2 + \frac{\varphi_2}{\Xi_2 + \eta_2}.$$

Therefore, they both may be summarised in:

$$p_1\frac{(1+i_2^d)}{(1+\psi)} > p_2 \tag{45}$$

This would imply that $q_1^1 = \alpha + \rho_1 y^1$, and $q_1^2 = \alpha + \rho_1 y^2$.

One of the two possibilities should be rule out. Using the envelope conditions, if $\alpha \in (0,1)$[23] and $\beta_0 = \beta_1 = 0$, then:

$$\varphi_1 - p_2\Xi_1(\rho_2 - 1) = 0.$$

$$\varphi_2 - p_2\Xi_2(\rho_2 - 1) + p_2\eta_2 = 0.$$

Solving them for the expression in the right side of equations (42) and (43) respectively (knowing that $\tau_1 = \tau_2 = 0$):

[23]First, it is assumed that α is an interior solution. But α can also be a corner solution. This will be ruled out later.

$$1 + \frac{\varphi_1}{p_2 \Xi_1} = \rho_2.$$

$$1 + \frac{\varphi_2}{p_2(\Xi_2 + \eta_2)} = \rho_2 \cdot \frac{\Xi_2}{(\Xi_2 + \eta_2)}.$$

Plugging them in equations (42) and (43) respectively:

$$\frac{p_1}{p_2} \frac{(1 + i_2^d)}{(1 + \psi)} = \rho_2.$$

$$\frac{p_1}{p_2} \frac{(1 + i_2^d)}{(1 + \psi)} = \rho_2 \cdot \frac{\Xi_2}{(\Xi_2 + \eta_2)}$$

To hold, it must be the case that **the money in advance constraint of patient households at $t = 2$ is slack**. In other words, $\eta_2 = 0$. Hence, the **interest rate condition** in period 2 is:

$$(1 + i_2^d) = \rho_2 \cdot \frac{p_2}{p_1}(1 + \psi). \tag{46}$$

Assume now that $y^1 = y^2 = 0$. This would imply the following conditions:

$$\varphi_1 \rho_1 > \Xi_1 p_2 (\rho_1 - \rho_2).$$

$$\Xi_2 p_2 (\rho_2 - \rho_1) + \rho_1 p_2 \eta_2 > \varphi_2 \rho_1.$$

Using the loan repayment equation (budget constraint) for both agents at $t = 2$ (equations (19) and (25)) and knowing that $\Xi_1 > 0$, $\Xi_2 > 0$, $y^1 = y^2 = 0$, and $q_1^1 = q_1^2 = \alpha$, I get:

$$1 + i^l = p_2 \cdot (1 - \alpha)\rho_2 + (1 + i_2^d) d_2^1 + (1 + i_2^s) \cdot s_2^1.$$

$$1 + i^l = (1 + i_2^d) d_2^2 + (1 + i_2^2) s_2^2 - p_2 c^2 + p_2 (1 - \alpha) \rho_2.$$

Equating both expressions:

$$(1 + i_2^d) d_2^1 + (1 + i_2^s) \cdot s_2^1 = (1 + i_2^d) d_2^2 + (1 + i_2^2) s_2^2 - p_2 c^2.$$

Making use of equations (16) and (21):

$$(1 + i_2^d)(d_2^1 - d_2^2) - (1 + i_2^s) p_1 c^1 = (1 + i_2^s)(1 + \psi)(d_2^1 - d_2^2) - p_2 c^2.$$

As $(1 + i_2^s) = \rho_2 \cdot \frac{p_2}{p_1}$, then:

$$[(1 + i_2^d) - (1 + i_2^s)(1 + \psi)](d_2^1 - d_2^2) + p_2 c^2 = \rho_2 p_2 c^1. \tag{47}$$

From the market clearing, it is known that:

$$\lambda c^1 = \lambda q_1^1 + (1-\lambda) q_1^2.$$

$$\lambda c^1 = \lambda \alpha + (1-\lambda)\alpha; c^1 = 1.$$

The **equilibrium value of the consumption basket for impatient households** is 1.

Now, I obtain the sight deposits-CBDC interest rate equivalence. From the first order conditions of sight deposits and CBDC deposits:

$$\Xi_1(1+i_2^s) - \kappa_1 = 0. \tag{48}$$

$$(\Xi_2+\eta_2)(1+i_2^s) - \kappa_2 = 0. \tag{49}$$

$$\Xi_1(1+i_2^d) - \kappa_1(1+\psi) = 0. \tag{50}$$

$$\Xi_2(1+i_2^d) + \eta_2(1+i_2^d) - \kappa_2(1+\psi) = 0. \tag{51}$$

Combining both equations (48) and (50), I obtain the following condition:

$$(1+\psi)(1+i_2^s) = (1+i_2^d).$$

The previous expression is the **sight deposits-CBDC interest rate condition** of the impatient agents.

Using equations (49) and (51), let us now prove that the same expression is achieved for the patient agents.

Remember the first order conditions of CBDC and sight deposits of the patient agent:

$$(\Xi_2+\eta_2)(1+i_2^s) - \kappa_2 = 0.$$

$$\Xi_2(1+i_2^d) + \eta_2(1+i_2^d) - \kappa_2(1+\psi) = 0.$$

Combining both, I get:

$$(\Xi_2+\eta_2)(1+i_2^s)(1+\psi) = (\Xi_2+\eta_2)(1+i_2^d).$$

Simplifying:

$$(1+i_2^s)(1+\psi) = (1+i_2^d),$$

which is exactly the same as the sight deposits-CBDC interest rate equivalence of the impatient agents.

If I plug the sight deposits-CBDC interest rate condition (equation (27)) in equation (47), it is obtained the **equilibrium value of the consumption basket for patient households**:

$$c^2 = \rho_2 \cdot c^1 = \rho_2.$$

The interior solution implies $y^1 = y^2 = 0$, $\alpha = \lambda$, $c^1 = 1$, $c^2 = \rho_2$ together with the interest rate conditions:

$$(1 + i_2^s) = \frac{(1 + i_2^d)}{(1 + \psi)} = \rho_2 \cdot \frac{p_2}{p_1}. \tag{52}$$

However, apart from the previous interior solution, equation (39) could also hold if:

$$\varphi_1 - \Xi_1 p_2(\rho_2 - 1) > 0 > \varphi_2 - \Xi_2 p_2(\rho_2 - 1) - p_2\eta_2$$

In this case, if

$$\varphi_1 > \Xi_1 p_2(\rho_2 - 1) > 0,$$

by possibility (ii), it should be the case that $q_1^h = \alpha + \rho_1 y^h$ for both types of households.

Solving equation (42) for φ_1 and plugging it in the previous expression:

$$\frac{p_1}{p_2}(1 + i_2^d) > \rho_2.$$

The other case is:

$$\varphi_2 < \Xi_2 p_2(\rho_2 - 1) + p_2\eta_2.$$

Solving equation (43) for φ_2 and plugging it in the previous expression:

$$\frac{p_1}{p_2}\frac{\Xi_2 + \eta_2}{\Xi_2}(1 + i_2^d) < \rho_2.$$

If the MIA constraint of patient households is slack, I am under a contradiction. Consequently, this interior solution is ruled out.

Moreover, equation (39) could also hold if:

$$\varphi_1 \Xi_1 p_2(\rho_2 - 1) < 0 < \varphi_2 - \Xi_2 p_2(\rho_2 - 1) - p_2\eta_2.$$

Following the same procedure as in the previous case, I obtain that:

$$\frac{p_1}{p_2}(1 + i_2^d) < \rho_2.$$

$$\frac{p_1}{p_2}\frac{\Xi_2+\eta_2}{\Xi_2}(1+i_2^d) > \rho_2.$$

If the MIA constraint of patient households is slack, I am under a contradiction again.

Notwithstanding, if $\eta_2 \neq 0$, the previous interior solutions may hold. Following Rivero Leiva and Rodríguez Mendizábal (2019, p.28), it can be easily shown that this is not a possible solution.

In addition, α could also be a corner solution either at $\alpha = 0$ or $\alpha = 1$. Both are ruled out and cannot be an equilibrium solution.[24]

All in all, the only solution with both valued sigh deposits and CBDC holdings involves $\alpha = \lambda, c^1 = 1, c^2 = \rho_2, y^1 = y^2$ and the interest rates obey equation (52).

Let us compute now the equilibrium interest rates:

Equating to zero the net worth of a bank at period 1:

$$nw_1^B = 1 - \lambda(d_2^1 + s_2^1 - \psi d_2^1) - (1-\lambda)(d_2^2 + s_2^2 - \psi d_2^2) = 0. \tag{53}$$

Substituting equation (16) and equation (21):

$$\begin{aligned} nw_1^B = 1 &- \lambda(d_2^1 - \psi d_2^1 + i_1^s - p_1 \cdot c^1 + p_1 \cdot q_1^1 - (1+\psi)d_2^1) \\ &-(1-\lambda)(d_2^2 - \psi d_2^2 + 1 + i_1^s + p_1 \cdot q_1^2 - (1+\psi)d_2^2) = 0; \end{aligned}$$

$$nw_1^B = -i_1^s + 2\psi\lambda d_2^1 + (1-\lambda)2\psi d_2^2 = 0;$$

$$i_1^s = 2\psi[\lambda d_2^1 + (1-\lambda)d_2^2]. \tag{54}$$

The interest rate on sight deposits in the first period will depend on the commission and the amount of central bank digital currency deposits demanded.

Finally, I can get the price in period 1. The budget constraint of impatient agents was binding. Hence, since consumption of the impatient agents was 1, price in the first period will be higher than 1 - as long as the fee and the demand of the CBDC are both positive - and equal to:

$$p_1 = 1 + i_1^s = 1 + 2\psi[\lambda d_2^1 + (1-\lambda)d_2^2]$$

Equating to zero the net worth of a bank at period 2.

[24]See Rivero Leiva and Rodríguez Mendizábal (2019, p.29).

$$nw_2^B = (1+i^l) - (1+i_2^s)[\lambda s_2^1 + (1-\lambda)s_2^2] + (1-\lambda)p_2c^2 \\ -p_2[\lambda q_2^1 + (1-\lambda)q_2^2] - (1+i^R)[\lambda d_2^1 + (1-\lambda)d_2^2]. \tag{55}$$

Substituting both MIA constraints:

$$nw_2^B = (1+i^l) - (1+i_2^s)[\lambda(1+i_1^s - p_1c^1 + p_1q_1^1 - (1+\psi)d_2^1) + \\ (1-\lambda)(1+i_1^s + p_1q_1^2 - (1+\psi)d_2^2)] + (1-\lambda)p_2c^2 - p_2[\lambda q_2^1 + \\ (1-\lambda)q_2^2] - (1+i^R)[\lambda d_2^1 + (1-\lambda)d_2^2].$$

After some simplifications:

$$nw_2^B = 1 + i^l + (1+i_2^s)(1+\psi)[\lambda d_2^1 + (1-\lambda)d_2^2] - (1+i_2^s)(1+i_1^s) \\ -(1+i^R)[\lambda d_2^1 + (1-\lambda)d_2^2].$$

Since:

$$(1+i_2^s) = \frac{1+i_2^d}{1+\psi}.$$

I obtain the following:

$$nw_2^B = 1 + i^l + (1+i_2^d)[\lambda d_2^1 + (1-\lambda)d_2^2] - 1 - i_1^s - i_2^s - i_1^s \cdot i_2^s - (1+i^R)[\lambda d_2^1 + (1-\lambda)d_2^2].$$

Rearranging some terms

$$nw_2^B = i^l + (i_2^d - i^R)[\lambda d_2^1 + (1-\lambda)d_2^2] - i_1^s - i_2^s - i_1^s \cdot i_2^s,$$

and plugging Equation (54), I obtain the following expression:

$$nw_2^B = i^l + (i_2^d - i^R)[\lambda d_2^1 + (1-\lambda)d_2^2] - 2\psi[\lambda d_2^1 + (1-\lambda)d_2^2] - i_2^s - i_2^s \cdot 2\psi[\lambda d_2^1 + (1-\lambda)d_2^2];$$

$$nw_2^B = i^l + (i_2^d - i^R)[\lambda d_2^1 + (1-\lambda)d_2^2] - \frac{i_2^d - \psi}{1+\psi} - \frac{2\psi}{1+\psi} \cdot (1+i_2^d) \cdot [\lambda d_2^1 + (1-\lambda)d_2^2];$$

$$nw_2^B = i^l + (i_2^d - i^R - \frac{2\psi \cdot (1+i_2^d)}{1+\psi})[\lambda d_2^1 + (1-\lambda)d_2^2] - \frac{i_2^d - \psi}{1+\psi}.$$

Simplifying:

$$nw_2^B = i^l + \left(\frac{(1-\psi)i_2^d - 2\psi}{1+\psi} - i^R\right)[\lambda d_2^1 + (1-\lambda)d_2^2] - \frac{i_2^d - \psi}{1+\psi}.$$

In equilibrium, the loan rate of commercial banks would be adjusted to fulfill the following condition:

$$i^l = \left(i^R - \frac{(1-\psi)i_2^d - 2\psi}{1+\psi}\right)[\lambda d_2^1 + (1-\lambda)d_2^2] + \frac{i_2^d - \psi}{1+\psi}. \tag{56}$$

A.4 Digital bank run. Proof of proposition 2

If there is a digital bank run, commercial banks will not have to pay interest rate on sight deposits. Household will value whether the net worth of the commercial bank in the second period is positive or negative, i.e., whether the bank is solvent. The net worth of a commercial bank in the second period is:

$$\begin{aligned} nw_2^B = (1+i^l) + (1-\lambda)p_2c^2 - p_2[\lambda q_2^1 + (1-\lambda)q_2^2] \\ -(1+i^R)[\lambda d_2^1 + (1-\lambda)d_2^2]. \end{aligned}$$

Rearranging some terms:

$$nw_2^B = (1+i^l) - [(1+i^R)][\lambda d_2^1 + (1-\lambda)d_2^2] + (1-\lambda)p_2c^2 - p_2[\lambda q_2^1 + (1-\lambda)q_2^2];$$

$$nw_2^B = (1+i^l) - [(1+i^R)][\lambda d_2^1 + (1-\lambda)d_2^2].$$

As I am evaluating a situation in which all funds are withdrawn, it means that:

$$\lambda d_2^1 + (1-\lambda)d_2^2 = 1.$$

Therefore:

$$nw_2^B = (1+i^l) - (1+i^R).$$

A commercial bank will remain solvent as long as:

$$nw_2^B = (1+i^l) - (1+i^R) \geq 0.$$

Analogously:

$$i^l \geq i^R. \tag{57}$$

As long as the refinancing rate of the central bank is lower than the loan rate established by commercial banks, there will not be a digital bank run.

Households will not coordinate in a run in their bank, given that the rest of the banks are solvent, triggering a digital bank run, as long as the refinancing rate of the central bank is not particularly high. In particular, as long as

$$i^R \leq \left(i^R - \frac{(1-\psi)i_2^d - 2\psi}{1+\psi}\right)[\lambda d_2^1 + (1-\lambda)d_2^2] + \frac{i_2^d - \psi}{1+\psi};$$

$$i^R - i^R[\lambda d_2^1 + (1-\lambda)d_2^2] \leq \left(-\frac{(1-\psi)i_2^d - 2\psi}{1+\psi}\right)[\lambda d_2^1 + (1-\lambda)d_2^2] + \frac{i_2^d - \psi}{1+\psi};$$

$$(1 - \lambda d_2^1 - (1-\lambda)d_2^2) \cdot i^R \leq \frac{i_2^d - \psi}{1+\psi} - \frac{(1-\psi)i_2^d - 2\psi}{1+\psi}[\lambda d_2^1 + (1-\lambda)d_2^2];$$

$$i^R \leq \frac{\frac{i_2^d-\psi}{1+\psi} - \frac{(1-\psi)i_2^d-2\psi}{1+\psi}[\lambda d_2^1 + (1-\lambda)d_2^2]}{(1-\lambda d_2^1 - (1-\lambda)d_2^2)};$$

$$i^R \leq \frac{i_2^d - \psi - ((1-\psi)i_2^d - 2\psi)[\lambda d_2^1 + (1-\lambda)d_2^2]}{(1+\psi)(1-\lambda d_2^1 - (1-\lambda)d_2^2)}.$$

This condition must be satisfied. Otherwise, a self-fulling digital bank run will occur in equilibrium.

A.5 Central bank policy objectives. Proof of proposition 3

I should determine the conditions under which, i_2^d will allow that the net worth of the central bank and commercial banks is positive and there is an absence of a digital bank run. To show this, I know that:

$$i_2^d < \frac{i_2^d - \psi - ((1-\psi)i_2^d - 2\psi)[\lambda d_2^1 + (1-\lambda)d_2^2]}{(1+\psi)(1-\lambda d_2^1 - (1-\lambda)d_2^2)};$$

$$i_2^d(1+\psi)(1-\lambda d_2^1 - (1-\lambda)d_2^2) < i_2^d - \psi - ((1-\psi)i_2^d - 2\psi)[\lambda d_2^1 + (1-\lambda)d_2^2];$$

$$i_2^d(1+\psi)(1-\lambda d_2^1-(1-\lambda)d_2^2) < i_2^d-\psi-(1-\psi)i_2^d[\lambda d_2^1+(1-\lambda)d_2^2]+2\psi[\lambda d_2^1+(1-\lambda)d_2^2];$$

$$i_2^d(1+\psi)(1-\lambda d_2^1-(1-\lambda)d_2^2)-i_2^d+(1-\psi)i_2^d[\lambda d_2^1+(1-\lambda)d_2^2] < -\psi+2\psi[\lambda d_2^1+(1-\lambda)d_2^2];$$

$$[(1+\psi)(1-\lambda d_2^1-(1-\lambda)d_2^2)+(1-\psi)[\lambda d_2^1+(1-\lambda)d_2^2]-1]\cdot i_2^d < 2\psi[\lambda d_2^1+(1-\lambda)d_2^2]-\psi;$$

$$(\psi - 2\psi[\lambda d_2^1 + (1-\lambda)d_2^2) \cdot i_2^d < 2\psi[\lambda d_2^1 + (1-\lambda)d_2^2] - \psi;$$

$$(1 - 2[\lambda d_2^1 + (1-\lambda)d_2^2) \cdot i_2^d < 2[\lambda d_2^1 + (1-\lambda)d_2^2] - 1.$$

The previous inequality will be fulfilled in the following situations:

- $\lambda d_2^1 + (1-\lambda)d_2^2 > 0.5$, for all values of i_2^d, as long as, $i_2^d > -1$.
- $i_2^d < -1$, for all values of the demand of the CBDC, i.e., $\lambda d_2^1 + (1-\lambda)d_2^2$. This is not realistic situation. The central bank will not establish a CBDC deposits interest rate of -100%.