# Single-level Robust Bidding of Renewable-only Virtual Power Plant in Energy and Ancillary Service Markets for Worst-case Profit Optimization

Hadi Nemati, Student, IEEE, Pedro Sánchez-Martín, Ana Baringo, Álvaro Ortega, Member, IEEE

*Abstract*—This paper proposes a novel single-level robust mathematical approach to model the RES-only Virtual Power Plant (RVPP) bidding problem in the simultaneous Day Ahead Market (DAM) and Secondary Reserve Market (SRM). The worst-case profit of RVPP due to uncertainties related to electricity prices, Non-dispatchable Renewable Energy Sources (ND-RES) production, and flexible demand is captured. In order to find the worst-case profit in a single-level model, the relationship between price and energy uncertainties leads to some non-linear constraints, which are appropriately linearized. The simulation results show the superiority of the proposed robust model compared to those in the literature, as well as its computational efficiency.

*Index Terms*—Renewable-only virtual power plant, single-level model, robust optimization, uncertainty, worst-case profit.

#### I. INTRODUCTION

#### A. Motivation

T HE penetration of ND-RESs has experienced a remarkable growth in the last decades. However, the stochastic nature of these sources implies that ND-RESs are less reliable when it comes to predictable and controllable power injection over a given period of time [1]. This makes ND-RESs participation in the energy and Ancillary Service Market (ASM) difficult, as failure to meet with the contracted energy and reserve in the market will lead to penalties if not suspension from future market activities. However, by integrating multiple portfolios of ND-RESs and other flexible assets as an RVPP, the performance and competitiveness of ND-RESs in these markets can be significantly improved [2].

The viability of RVPP depends on its economic performance, related to benefits and costs. Different markets bring different benefits according to the bidding/offering ability of RVPP and its ability to provide what is promised [3]. However, in addition to the internal uncertainties of RVPP units in their production and demand, there are various external uncertainties in the markets, such as the energy and reserve electricity price uncertainties [4]. Therefore, the development of bidding approaches for RVPP participation in different markets taking into account the characteristics of RVPP units, market rules, and internal and external uncertainties has at most important for RVPP operators and researchers [5].

## B. Literature Review

Many papers in the literature use mathematical optimization models to capture different uncertainties associated with

Virtual Power Plant (VPP) due to ease of implementation, convergence to the global optimum, and computational efficiency of these models [6]. In this context, Robust Optimization (RO) programming is an efficient way to deal with different sets of uncertainties that vary in their possible values. The goal of RO is to find the worst case of the optimization problem to minimize the negative impact of uncertainties on the solution [7]. However, the definition of the worst case can vary depending on how the optimization is implemented, whether it is single-level or multi-level, and can lead to different solutions in each approach. The authors in [8]-[15] develop a single-level optimization problem for the VPP market bidding problem to find the worst case of energy of ND-RESs. The literature addresses VPP scheduling and bidding problems by considering different uncertainty characterizations in demand [8]-[10], [13], [15], ND-RES production [9]–[15], and electricity price [11]–[13], [15], focusing on multi-market [8], [9], [11]-[15], multi-objective [10], and multi-energy models [12], [13]. The main advantages of the mentioned single-level RO programming in [8]-[15] are the possibility to consider multiple uncertainties, simplicity of implementation, global optimality, and calculation efficiency. However, a simplified definition of the worst case of energy for the severe scenarios is implemented. In fact, the worst case of energy defined for ND-RESs in the above papers does not lead to the worst condition of profit, considering the possibility of different values of electricity prices. For instance, in a case where the electricity price is low in a certain period, even though the energy of a ND-RES can deviate significantly in this period, the resulting loss for RVPP might not be significant compared to a period with much higher electricity price and average or low energy deviation.

Multi-level RO models provide more flexibility to find the actual worst-case of the VPP bidding problem compared to single-level models. This is due to the definition of a new level for the optimization problem that models the behavior of uncertain parameters (both electrcity price and energy uncertainties). Therefore, the objective function of this level can be defined to find the worst case of energy or profit of VPP. In addition, another level for the problem can be included to define the corrective or remedial actions after the occurrence of uncertainties.

The literature on multi-level models proposes mathematical techniques, including Adaptive [16], [17] and Stochastic [4], [18], [19] RO to account for various uncertainties in ND-RES production [4], [16]–[19], demand [16], [19], electricity prices [4], [18], and reserve deployment or dispatch

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order of Transmission System Operator (TSO) [4], [17], [18]. The proposed models are implemented for the multi-market participation of VPP [4], [16], [18] and for multi-energy VPPs [19]. Different techniques, including Benders and other decomposition techniques [4], [16], the Column Constraint Generation algorithm [18], and improved versions of these algorithms [17], [19], are proposed to accelerate the solution time of the optimization problem. The main limitations of the multi-level approaches in general, and in the above works in particular, are the complexity of programming and the fact that the size of the problem grows with the number of iterations in the solving procedure. In addition, they usually imply long computational times, which can compromise applications such as sensitivity analysis.

## C. Approach and Contributions

To avoid the difficulties of implementing a multi-level model and the computational complexity for practical applications, this paper models the worst-case profit of RVPP against uncertainties by means of a novel single-level Mixed Integer Linear Programming (MILP) problem. The equations related to the uncertain parameters in the objective function of the optimization problem (equations related to the energy and reserve price uncertainties) as well as the equations related to the uncertain parameters in the constraints (equations related to the ND-RES and demand uncertainties) are defined by developing the approach in [11], [15], [20] and by developing on the idea from the big M method [21]. The proposed implementation of robust constraints makes it possible to capture the relationship between different uncertain parameters in the objective and constraints of the optimization problem, and to find the exact worst case profit of RVPP. Finally, defining the relationship between uncertain parameters in order to find the worst case leads to some non-linear constraints, which are linearized by using well-established methods.

The contributions of this paper are threefold:

- Modeling the worst-case profit robustness of an RVPP with a single-level Mixed Integer non-Linear Programming (MINLP) model. As opposed to other single-level models in the literature, the proposed model maximizes the expected profit of RVPP for the simultaneous DAM and SRM participation against the worst-case profit robustness of different uncertainties on prices and energy (ND-RES production and demand).
- Addressing the non-linear couplings between various uncertainties within the optimization problem, and subsequently formulating an equivalent MILP problem for the initial MINLP one.
- The proposed single-level MILP model has high computational efficiency and simpler implementation compared to the multi-level optimization models in the literature.

## D. Paper Organization

The reminder of the paper is organized as follows. A conceptual comparison of energy and profit robustness approaches is presented in Section II. The proposed single-level robust bidding problem of RVPP for DAM and SRM participation is formulated in Section III. An illustrative example is given in Section IV to show the performance of the proposed robust model in finding the worst-case profit. The simulation results are presented in Section V. Finally, the conclusions are drawn in Section VI.

#### **II. COMPARING ENERGY AND PROFIT ROBUSTNESS**

Figure 1 shows the structure of a deterministic RVPP bidding problem and a comparison between the energy and the profit robustness approaches. In the deterministic approach, a single value (usually the median or average) of the forecast data is considered to solve the optimization problem. The constraints are related mainly to the operation of the RVPP units, and supply-demand balance [22]. When considering the uncertainties, depending on whether the uncertainties affect the objective function or the constraints of the optimization problem, different sets of constraints need to be defined in each of the RO approaches. The uncertainties related to the energy/reserve electricity price affect the objective function of the optimization problem, whereas the uncertainties associated with the ND-RES generation and demand consumption affect the constraints.



Fig. 1. A comparison between the energy and profit robustness approaches.

In the energy robustness approach, those periods that result in more deviation of the energy/reserve electricity price variance multiplied by the total traded energy/reserve of RVPP are selected as the worst-case scenarios of the electricity price [15]. In the energy robustness constraints, the periods that have higher deviation of energy are selected as the worst case of ND-RESs production or demand regardless of the electricity price.

In the profit robustness approach proposed in this paper and for the uncertain parameters in the objective function of the optimization problem (energy/reserve electricity price), the worst case is defined according to the final value of the energy/reserve electricity price by means of binary variables. The final value of the energy electricity price is also used to calculate the worst case of profit/cost of each unit (uncertainty of ND-RES and demand in the constraints of the optimization problem). For this purpose, the final energy electricity price is multiplied by the energy variable of ND-RES/demand and is limited by the profit reduction effect due to ND-RES/demand uncertainty.

In the following section, the proposed profit robustness approach is formulated as a single-level optimization problem. In Section IV, these two approaches are compared using an illustrative example.

# **III. PROFIT ROBUSTNESS FORMULATION**

# A. Nomenclature

This subsection presents the notation and nomenclature used in the remainder of the paper.

# **General Notation Concepts**

- An uncertain parameter with a tilde symbol denotes the median value in the forecast distribution, representing a point where half of the observations are lower ( $\tilde{A}$ );
- the hat/inverse hat symbol on uncertain parameters signifies the greatest positive/negative permitted deviation from the forecast's median ( $\hat{A}$ ,  $\check{A}$ );
- parameters with an upper/lower bar represent their upper/lower bounds of parameter  $A(\bar{A}, \underline{A})$ ;
- upward/downward arrows indicate up/down direction of regulation in variables and parameters  $(a^{\uparrow}, A^{\uparrow}/a^{\downarrow}, A^{\downarrow})$ .

#### **Indexes and Sets**

$d \in \mathcal{D}$	Set of demands
$p \in \mathcal{P}$	Set of daily load profiles

 $r \in \mathcal{R}$  Set of ND-RESs

 $t \in \mathcal{T}$  Set of time periods

 $\Xi^{DA+SR}$  Set of decision variables of DAM and SRM

#### **Parameters**

$C_r^R$	Operation and maintenance costs of N	D-RES $r$
	[	€/MWh]
$C_{d,p}$	Cost of load profile $p$ of demand $d$	[€]
$E_d$	Energy consumption of demand $d$ through	ghout the
	planning horizon	[MWh]
M	Very big positive value	[€]
$P_d$	Power consumption of demand $d$	[MW]
$P_r$	Power production of ND-RES r	[MW]
$P_{d,p,t}$	Profile $p$ of demand $d$ prediction during	g period $t$
		[MW]
$P_{r,t}$	ND-RES r production prediction during	; period $t$
,		[MW]
$R_d$	Ramp rate of demand d [N	/W/hour]
$R_{r(d)}^{\mathrm{SR}}$	Secondary Reserve (SR) ramp rate of N	D-RES r
<i>r</i> ( <i>a</i> )	(demand d) []	MW/min]
$T^{\rm SR}$	Required time for SR action	[min]
$\Delta t$	Duration of periods	[hour]
$\Gamma^{\rm DA/SR}$	DAM/SRM price uncertainty budget	[-]
$\Gamma_{r(d)}$	ND-RES $r$ production (demand $d$ ) un	ncertainty
	budget	[-]
$\kappa$	User-defined parameter to set the lim	it of up
	reserve traded in the SRM as a percentage	ge of total
	power capacity of RVPP	[%]
ε	Very small positive value	[€]
$\varrho_t$	Coefficient to calculate the ratio of do	wn-to-up
	reserve requested by the TSO during per	iod t [%]
$\beta_{d,t}$	Percentage of flexibility of demand $d$ duri	ng period
	t	[%]
$\lambda_t^{\mathrm{DA/SR}}$	DAM/SRM price prediction during	period t
	[€/MWh]	/[€/MW]

#### **Continuous Variables**

$p_t^{\rm DA}$	Total traded power by RVPP in the DAM of	luring		
D.	period t	[MW]		
$p_{r(d),t}^{\mathrm{DA}}$	Production of ND-RES $r$ (consumption of de	emand		
	d) in the DAM during period $t$	[MW]		
$r_t^{ m SR}$	Total SR traded by RVPP for different TSC	calls		
	on conditions during period $t$	[MW]		
$r_{r(d) t}^{\mathrm{SR}}$	SR provided by ND-RES $r$ (demand $d$ ) for	differ-		
7 (u),0	ent TSO calls on conditions during period $t$	[MW]		
$y_t^{(\prime)\mathrm{DA}}$	RVPP profit affected by DAM negative (po	sitive)		
- 0	price uncertainty during period t	[€]		
$y_t^{\mathrm{SR}}$	RVPP profit affected by SRM price uncer	tainty		
	during period t	[€]		
$y_{r(d),t}$	RVPP profit (cost) affected by ND-RES $r$ pr	oduc-		
0. (1),-	tion (demand $d$ ) uncertainty during period $t$	[€]		
$\eta_t^{(\prime)\mathrm{DA}}$	Dual variable to model the negative (positive)	) price		
10	uncertainty of DAM during period $t$	[€]		
$\eta_t^{\mathrm{SR}}$	Dual variable to model the price uncertain	nty of		
	SRM during period $t$	[€]		
$\eta_{r(d),t}$	Dual variable to model the ND-RES $r$ produced	uction		
. (.))	(demand $d$ ) uncertainty during period $t$	[€]		
$\nu^{\mathrm{DA/SR}}$	Dual variable to model the price uncertain	nty of		
	DAM/SRM	[€]		
$\nu_{r(d)}$	Dual variable to model the ND-RES $r$ produced	uction		
. (=)	(demand $d$ ) uncertainty during period $t$	[€]		
Binary Variables				
$u_{d,p}$	Indicator of selection of profile $p$ of demand	1 d [-]		
$\gamma_{i}^{(\prime)DA}$	Binary variable that is 1 if DAM negative	(posi-		
$\Lambda t$	tive) price robustness constraints are active of	luring		
	period t, and 0 otherwise	[-]		
$\gamma_{i}^{SR}$	Binary variable that is 1 if SRM price robu	stness		
$\Lambda t$	constraints are active during period $t$ and $t$	0 oth-		
	constraints are active during period 0, and	0.001		

erwise[-] $\chi_{r(d),t}$ Binary variable that is 1 if ND-RES r (demand d)<br/>robust constraints are active during period t, and<br/>0 otherwise

#### B. Price Robustness (Objective Function)

The objective function of simultaneous RVPP participation in the DAM and SRM, as well as the associated robust constraints, are presented in this section.

The objective function (1) maximizes the benefits of RVPP in the DAM and SRM. The first and second lines of (1) calculate the expected RVPP incomes from bidding in the DAM and from up and down SR provision, respectively, considering the corresponding robustness cost of asymmetric electricity price uncertainties. The third line in (1) defines the operation costs of ND-RESs, and the costs of selecting a particular load profile. Note that the implementation of variables  $y_t^{DA}$ ,  $y_t'^{DA}$ ,  $y_t^{SR,\uparrow}$ , and  $y_t^{SR,\downarrow}$  in the objective function is one of the main differences between the proposed model and the common approach to model the price robustness in the literature [11], [15]. By means of these variables, the final value of the DAM/SRM electricity price and the traded energy/reserve of RVPP are used to calculate the worst-case scenarios.

$$\max_{\Xi^{\mathrm{DA}+\mathrm{SR}}} \sum_{t\in\mathfrak{T}} \left[ \tilde{\lambda}_t^{\mathrm{DA}} p_t^{\mathrm{DA}} \Delta t - y_t^{\mathrm{DA}} - y_t'^{\mathrm{DA}} \right] \\ + \sum_{t\in\mathfrak{T}} \left[ \tilde{\lambda}_t^{\mathrm{SR},\uparrow} r_t^{\mathrm{SR},\uparrow} + \tilde{\lambda}_t^{\mathrm{SR},\downarrow} r_t^{\mathrm{SR},\downarrow} - y_t^{\mathrm{SR},\uparrow} - y_t^{\mathrm{SR},\downarrow} \right] \\ - \sum_{t\in\mathfrak{T}} \sum_{r\in\mathfrak{R}} C_r^R p_{r,t}^{\mathrm{DA}} \Delta t - \sum_{d\in\mathfrak{D}} \sum_{p\in\mathfrak{P}} C_{d,p} u_{d,p}$$
(1)

The set of constraints (2) is related to the uncertainties of the DAM electricity price and are written by developing the approach in [11], [15], [20] and elaborating on the big M method [21].

$$\lambda_t^{DA} = \tilde{\lambda}_t^{DA} - \check{\lambda}_t^{DA} \chi_t^{DA} + \hat{\lambda}_t^{DA} \chi_t^{\prime DA} , \qquad \forall t \ (2a)$$

$$\nu^{DA} + \eta_t^{DA} \ge \check{\lambda}_t^{DA} p_t^{DA} \Delta t \quad , \qquad \qquad \forall t \ \text{(2b)}$$

$$\nu^{DA} + \eta_t^{\prime DA} \ge -\hat{\lambda}_t^{DA} p_t^{DA} \Delta t \quad , \qquad \qquad \forall t \quad (2c)$$

$$y_t^{DA} \ge \nu^{DA} + \eta_t^{DA} - M(1 - \chi_t^{DA}) \quad \forall t \ (2d)$$

$$y_t'^{DA} \ge \nu^{DA} + \eta_t'^{DA} - M(1 - \chi_t'^{DA})$$
,  $\forall t \ (2e)$ 

$$\varepsilon(\chi_t^{DA}) \le \eta_t^{DA} \le M(\chi_t^{DA}) \quad \forall t \quad (2f)$$

$$\mathcal{L}(\chi_t) \ge M_t \ge M(\chi_t)$$
,  $\forall t (2g)$   
 $M(1 \to DA) < \tilde{\lambda} DA_m DA \wedge t \to DA < M(\Delta DA) \quad \forall t (2h)$ 

$$-M(1-\gamma_{t}^{DA}) \leq \lambda_{t}^{DA} p_{t}^{DA} \Delta t - \nu^{DA} \leq M(\chi_{t})^{DA} \forall t \quad (2i)$$

$$\sum \left(\chi_t^{DA} + \chi_t^{\prime DA}\right) = \Gamma^{DA} , \qquad (2j)$$

$$\chi_t^{t\in\mathcal{I}} + \chi_t'^{DA} < 1 \quad , \qquad \qquad \forall t \ (2\mathbf{k})$$

$$\nu^{DA}, n_{\ell}^{DA}, n_{\ell}^{DA}, u_{\ell}^{DA}, u_{\ell}^{\prime DA} > 0$$
.  $\forall t$  (21)

$$\chi_t^{DA}, \chi_t^{\prime DA} \in \{0, 1\}$$
 ,  $\forall t \,(2\mathbf{m})$ 

Constraint (2a) determines the DAM electricity price in each time period according to the condition of binary variables  $\chi_t^{\rm DA}$ and  $\chi_t^{\prime \text{DA}}$ , which are related to the negative and positive price volatility, respectively. Constraints (2b) and (2c) model the impact of the absolute value of negative and positive price volatility on profit reduction when the electricity price takes its worst condition. Constraints (2d) and (2e) set a lower bound for the profit reduction variables  $y_t^{\text{DA}}$  and  $y_t'^{\text{DA}}$  due to the negative and positive price uncertainty, respectively. When the binary variable  $\chi_t^{\text{DA}}$  ( $\chi_t^{\prime\text{DA}}$ ) is 1, constraint (2d) (constraint (2e)) is active. Depending on whether RVPP sells or buys electricity on the market, the worst DAM price conditions occur at the price values  $\tilde{\lambda}_t^{\text{DA}} - \tilde{\lambda}_t^{\text{DA}}$  and  $\tilde{\lambda}_t^{DA} + \hat{\lambda}_t^{\text{DA}}$ , respectively. The dual variables  $\eta_t^{DA}$  and  $\eta_t^{\prime\text{DA}}$ , related to the negative and positive deviations of the electricity price, are logically constrained by (2f) and (2g), respectively, based on the active or non-active status of the periods to comply with the robustness budget defined in (2i). Constraints (2h) and (2i) define the lower and upper bounds for the differences between the possible profit reductions (due to the negative price deviation  $\check{\lambda}_t^{DA} p_t^{DA} \Delta t$  and the positive price deviation  $-\hat{\lambda}_t^{DA} p_t^{DA} \Delta t$ ) and the dual variable  $\nu^{DA}$ . According to these constraints, the possible profit reductions must be greater than or equal to the dual variable  $\nu^{DA}$  for those periods that the electricity price fluctuates to its worst case. Constraints (2h) and (2i) are thus essential to avoid selecting incorrect periods for the worst case of profit deviations, especially when other uncertain parameters such as ND-RESs production and demand (see Sections III-D and III-E) affect the total power traded by RVPP  $(p_t^{DA})$ . The robustness budget  $\Gamma^{DA}$  in (2j) is a user-defined parameter that determines the number of periods in which the electricity price can deviate to its worst condition.  $\Gamma^{DA}$  is thus a particularly relevant parameter in this model. Constraint (2k) prevents positive and negative electricity price deviations in the same period. Constraints (21) and (2m) define the nature of positive dual variables and binary variables, respectively. The use of two binary variables to define the negative and positive price deviations and the implementation of constraints (2h) and (2i) to avoid illogical condition for the price deviations is another significant improvement compared to previous robust formulations in the literature [11], [15].

The set of constraints (3) related to the uncertainty in the up and down SRM price is defined similarly to (2). The only difference is that for both up and down SRM price, only the negative SRM price deviations due to uncertainty are considered in (3a) and (3b), respectively. This is due to the fact that the positive SRM price deviations always result in more benefit for RVPP. Therefore, the maximum possible profit deviations  $\check{\lambda}_t^{SR,\uparrow}r_t^{SR,\uparrow}$  and  $\check{\lambda}_t^{SR,\downarrow}r_t^{SR,\downarrow}$  are calculated based on the negative upward and downward SRM price deviations  $\check{\lambda}_t^{SR,\uparrow}$  and  $\check{\lambda}_t^{SR,\downarrow}$  in constraints (3c) and (3d), respectively.

$$\lambda_t^{SR,\uparrow} = \tilde{\lambda}_t^{SR,\uparrow} - \check{\lambda}_t^{SR,\uparrow} \chi_t^{SR,\uparrow} , \qquad \forall t \ \text{(3a)}$$

$$\lambda_t^{\star} = \lambda_t^{\star} = \lambda_t^{\star} + \lambda_t^{\star} - \lambda_t^{\star} + \chi_t^{\star} + \chi_t^$$

$$\nu + \eta_t \ge \lambda_t - \eta_t , \qquad \forall t \ (3c)$$
$$\nu^{SR,\downarrow} + \eta_t^{SR,\downarrow} > \check{\lambda}_t^{SR,\downarrow} r_t^{SR,\downarrow} . \qquad \forall t \ (3c)$$

$$y_t^{SR,\uparrow} \ge \nu^{SR,\uparrow} + \eta_t^{SR,\uparrow} - M(1 - \chi_t^{SR,\uparrow}) , \qquad \forall t \text{ (3e)}$$

$$y_t^{SR,\downarrow} \ge \nu^{SR,\downarrow} + \eta_t^{SR,\downarrow} - M(1 - \chi_t^{SR,\downarrow}) , \qquad \forall t \ (3f)$$

$$\varepsilon(\chi_t^{-n}) \leq \eta_t^{-n} \leq M(\chi_t^{-n}), \qquad \forall t \ (3g)$$

$$\varepsilon(\chi^{SR,\downarrow}) \leq \gamma^{SR,\downarrow} \leq M(\chi^{SR,\downarrow}) \qquad \forall t \ (3g)$$

$$-M(1 - \chi_t^{SR,\uparrow}) \leq \tilde{\lambda}_t^{SR,\uparrow} r_t^{SR,\uparrow} - \nu^{SR,\uparrow} \leq M(\chi_t^{SR,\uparrow}) \forall t \quad (3i)$$

$$M(1 - \chi_t^{SR,\downarrow}) \leq \tilde{\lambda}_t^{SR,\downarrow} r_t^{SR,\downarrow} - \kappa^{SR,\downarrow} \leq M(\chi_t^{SR,\downarrow}) \forall t \quad (3i)$$

$$\sum \chi_t^{SR,\uparrow} = \Gamma^{SR,\uparrow} , \qquad (3k)$$

$$\sum_{t\in\mathcal{T}}\chi_t^{SR,\downarrow} = \Gamma^{SR,\downarrow} , \qquad (31)$$

$$t \in \mathcal{T}$$
  
$$SR_{\uparrow} = SR_{\downarrow} = SR_{\uparrow} = SR_{\downarrow} = SR_{\uparrow} = SR_{\downarrow} =$$

$$\begin{array}{ll} \nu & \gamma, \nu & \gamma, \eta_t & \eta_t & g_t & g_t & \geq 0 \\ \chi_t^{SR,\uparrow}, \chi_t^{SR,\downarrow} \in \{0,1\} & , & \forall t \text{ (3n)} \end{array}$$

$$\chi_t \quad \gamma, \chi_t \quad \gamma \in \{0, 1\} \quad , \qquad \forall t \quad (3n)$$

# C. Supply-demand and Traded Constraints

The supply-demand balancing constraint of RVPP units is defined in (4a). All RVPP units are assumed to be connected to a single node. The variable  $r_t^{SR}$  related to the total traded reserve of RVPP and the variables  $r_{r,t}^{SR}$  and  $r_{d,t}^{SR}$  related to the reserve of RVPP units are defined according to different reserve activation scenarios similar to [15]. Constraints (4b) and (4c) assign the upper and lower bounds of traded energy and reserve by RVPP, respectively. Constraint (4d) defines the proportion of down and up reserve requested by TSO. The up reserve provided is limited by (4e) to a fraction of the total capacity of the generating units of RVPP.

$$\sum_{r \in \mathcal{R}} (p_{r,t}^{DA} + r_{r,t}^{SR}) - \sum_{d \in \mathcal{D}} (p_{d,t}^{DA} - r_{d,t}^{SR}) = p_t^{DA} + r_t^{SR} , \forall t$$
(4a)

$$p_t^{DA} + r_t^{SR,\uparrow} \le \sum_{r \in \mathcal{R}} \bar{P}_r , \qquad \forall t \ (4b)$$

$$-\sum_{d\in\mathcal{D}}\bar{P}_d \le p_t^{DA} - r_t^{SR,\downarrow} , \qquad \forall t \ (4c)$$

$$r_t^{SR,\uparrow} = \varrho_t r_t^{SR,\downarrow} , \qquad \forall t \ (4d)$$
$$r_t^{SR,\uparrow} < \kappa \sum \bar{P} \qquad \forall t \ (4e)$$

$$T_t \leq \kappa \sum_{r \in \mathcal{R}} T_r, \qquad \forall t \in \mathcal{C}$$

## D. ND-RES Profit Robustness

The profit robustness formulation of ND-RESs is given in (5). Constraint (5a) is the lower bound on the ND-RESs output power. Constraint (5b) sets the ND-RESs output through the median forecast generation of ND-RESs  $P_{r,t}$  and the possible negative power deviation  $\chi_{r,t}\check{P}_{r,t}$  (active when  $\chi_{r,t} = 1$ ). The binary variable  $\chi_{r,t}$  is determined according to the profit robustness constraints (5c)-(5j) proposed in this work. Constraint (5c) limits the profit of ND-RESs for each time period by considering the robustness of the problem against uncertain parameters of ND-RESs production. The upper bound of this constraint is computed as the median profit minus the profit reduction due to the negative deviation of power forecast of ND-RESs,  $y_{r,t}$ . The median profit is calculated by multiplying the electricity price  $\lambda_t^{DA}$  and the median production of ND-RESs minus the provided up reserve, both multiplied by the time period duration  $(\tilde{P}_{r,t} - \tilde{r}_{r,t}^{SR,\uparrow})\Delta t$ . Constraint (5c) is a non-linear expression that is linearized in Section III-F. Constraint (5d) assigns the upper bound of the dual variable  $y_{r,t}$  to the negative profit deviation  $\lambda_t^{DA} \dot{P}_{r,t} \Delta t$ of each ND-RESs due to uncertainty. To model the worst-case scenarios of profit reduction for each ND-RES, only negative power deviations are considered in this constraint, since positive deviations will usually benefit the RVPP. Constraint (5e) determines the lower bound of the dual variable  $y_{r,t}$  according to the dual variables  $\nu_r$  and  $\eta_{r,t}$ , and the condition of the binary variable  $\chi_{r,t}^{DA}$ . Constraint (5f) assigns the lower bound of the sum of the dual variables  $\nu_r$  and  $\eta_{r,t}$  to the maximum profit reduction for each ND-RES in each time period. According to constraint (5g), the dual variable  $\eta_{r,t}$  is defined based on the active or non-active status of the profit reduction due to the robustness of the production of ND-RESs. Constraint (5h) defines the profit robustness budget for each ND-RES.

$$\underline{P}_{r} \leq p_{r,t}^{DA} - r_{r,t}^{SR,\downarrow} , \qquad \forall r,t \quad (5a)$$

$$p_{r,t}^{DA} + r_{r,t}^{SR\uparrow} = \dot{P}_{r,t} - \chi_{r,t}\dot{P}_{r,t} , \qquad \forall r,t \quad (5b)$$

$$\lambda_t^{DA} p_{r,t}^{DA} \Delta t \le \lambda_t^{DA} (P_{r,t} - r_{r,t}^{DA,\dagger}) \Delta t - y_{r,t} , \quad \forall r, t \quad (5c)$$

$$y_{r,t} \le \lambda_t^{--} P_{r,t} \Delta t , \qquad \forall r,t \quad (5d)$$
  
$$y_{r,t} \ge \nu_r + \eta_{r,t} - M(1 - \chi_{r,t}) , \qquad \forall r,t \quad (5e)$$

$$y_{r,t} \ge \nu_r + \eta_{r,t} - M(1 - \chi_{r,t}), \qquad \forall r, t \quad (50)$$

$$\nu_r + \eta_{r,t} \ge \lambda^{DA} \check{P}_{r,t} \Lambda t \qquad \forall r \ t \quad (51)$$

$$\nabla r + \eta_{r,t} \ge \Lambda_t - r_{r,t} \bigtriangleup t , \qquad \forall r, t \quad (51)$$

$$\sum_{k=1}^{n} \sum_{i=1}^{n} \sum_{k=1}^{n} \sum_{i=1}^{n} \sum_{k=1}^{n} \sum_{i=1}^{n} \sum_{i$$

$$\sum_{t\in\mathcal{T}}\chi_{r,t}-1_r,\qquad\qquad\forall 7$$

$$\begin{aligned} \nu_r, \eta_{r,t}, y_{r,t} &\geq 0 , & \forall r, t \quad \text{(51)} \\ \chi_{r,t} &\in \{0, 1\} , & \forall r, t \quad \text{(55)} \end{aligned}$$

(5j)

E. Demand Cost Robustness

the demand for each period to predefined demand profiles, taking into account the median and positive demand forecasts. Only positive demand deviations are considered for the worst-case cost robustness scenarios, since the negative demand deviations (i.e., lower consumption) usually result in lower costs for RVPP. Constraint (6b) ensures that the algorithm selects only one demand profile among several profiles. When the binary variable  $\chi_{d,t}$  in (6a) for a certain period is 1, the possible positive deviation of the demand becomes active. The binary variable  $\chi_{d,t}$  is determined according to the cost robustness constraints (6c)-(6h) proposed in this work. Constraint (6c) sets the lower bound on the cost of buying electricity from DAM to supply demand, which equals the cost of buying electricity for the median demand forecast  $\lambda_t^{DA} \sum_{p \in \mathcal{P}} (\tilde{P}_{d,p,t} u_{d,p}) \Delta t$  plus the additional cost of positive demand fluctuation due to uncertainty represented by the dual variable  $y_{d,t}$ . The additional cost of buying electricity for positive demand fluctuation due to uncertainty  $\lambda_t^{DA} \sum_{d} (\hat{P}_{d,p,t} u_{d,p}) \Delta t$  is assigned as the upper bound of the dual variable  $y_{d,t}$  by constraint (6d). On the other hand, the lower bound of the dual variable  $y_{d,t}$  is given by constraint (6e) to find the worst cases of demand cost robustness. The dual variables  $\nu_d$  and  $\eta_{d,t}$  are logically constrained in (6f) and (6g) to determine those periods that positive demand deviations lead to the worst cost robustness scenarios. Constraint (6h) assigns the user-defined parameter of the robustness budget  $\Gamma_d$ to set the number of periods allowed for positive deviations in demand due to cost robustness. Constraints (6i) and (6j) confine the demand up reserve according to the percentage of downward demand flexibility and the minimum possible demand, respectively. Constraints (6k) and (6l) are similarly defined to limit the down reserve considering the opposite direction of demand flexibility and the maximum possible demand. The worst conditions of ramp-up and ramp-down in two consecutive periods considering the reserve activation are defined in constraints (6m) and (6n), respectively. The capability of demand to provide up and down reserve is defined by constraints (60) and (6p), respectively. Constraint (6q) limits the minimum energy that each demand should use for the entire period. Constraints (6r) and (6s) describe the nature of positive dual variables and binary variables, respectively.

The demand cost robust formulation is illustrated in (6), which is based on the deterministic model presented in [23].

$$p_{d,t}^{DA} = \sum_{p \in \mathcal{P}} \left( \tilde{P}_{d,p,t} u_{d,p} + \chi_{d,t} \hat{P}_{d,p,t} u_{d,p} \right) , \qquad \forall d, t$$
 (6a)

$$\sum_{P \in \mathcal{P}} u_{d,p} = 1 , \qquad \qquad \forall d \quad (6b)$$

$$\lambda_t^{DA} p_{d,t}^{DA} \Delta t \ge \lambda_t^{DA} \sum_{p \in \mathcal{P}} (\tilde{P}_{d,p,t} u_{d,p}) \Delta t + y_{d,t} , \quad \forall d, t$$
 (6c)

$$y_{d,t} \le \lambda_t^{DA} \sum_{p \in \mathcal{P}} (\hat{P}_{d,p,t} u_{d,p}) \Delta t , \qquad \forall d, t$$
 (6d)

$$y_{d,t} \ge \nu_d + \eta_{d,t} - M(1 - \chi_{d,t}) , \qquad \forall d, t$$
 (6e)

$$\nu_d + \eta_{d,t} \ge \lambda_t^{DA} \sum_{p \in P} (\hat{P}_{d,p,t} u_{d,p}) \Delta t , \qquad \forall d, t \quad (6f)$$

$$\varepsilon \chi_{d,t} \le \eta_{d,t} \le M \chi_{d,t} , \qquad \forall d,t$$
 (6g)

$$\sum_{t\in\mathfrak{T}}\chi_{d,t}=\Gamma_d , \qquad \qquad \forall d \quad (6h)$$

$$r_{d,t}^{SR,\uparrow} \leq \underline{\beta}_{d,t} \sum_{p \in \mathcal{P}} \left( \tilde{P}_{d,p,t} u_{d,p} \right) , \qquad \forall d,t \quad (6i)$$

$$r_{d,t}^{SR,\uparrow} \le p_{d,t}^{DA} - \underline{P}_d , \qquad \qquad \forall d, t \quad (6j)$$

$$r_{d,t}^{SR,\downarrow} \leq \beta_{d,t} \sum_{p \in \mathcal{P}} \left( P_{d,p,t} u_{d,p} \right) , \qquad \forall d,t \ (\mathbf{6k})$$

$$r_{d,t}^{SR,\downarrow} \le \bar{P}_d - p_{d,t}^{DA} , \qquad \qquad \forall d, t \quad (61)$$

$$(p_{d,t}^{DA} + r_{d,t}^{SR,\downarrow}) - (p_{d,(t-1)}^{DA} - r_{d,(t-1)}^{SR,\uparrow}) \le R_d \Delta t , \quad \forall d, t \text{ (6m)}$$

$$(p_{d,(t-1)}^{2n} + r_{d,(t-1)}^{2n}) - (p_{d,t}^{2n} - r_{d,t}^{2n}) \le \underline{R}_d \Delta t , \quad \forall d, t \text{ (6n)}$$

$$r_{d,t}^{SR,+} \leq T^{SR} \underline{R}_{d}^{SR}, \qquad \forall d, t \ (60)$$

$$\forall d, t$$
 (6p)

$$\underline{E}_d \le \sum_{t \in \mathcal{T}} (p_{d,t}^{DA} \Delta t - r_{d,t}^{O(t)}) , \qquad \qquad \forall d \quad (6q)$$

$$\nu_d, \eta_d, \eta_d, \eta_d \neq 0$$
,  $\forall d, t$  (6r)

$$\chi_{d,t} \in \{0,1\}$$
 ,  $\forall d,t$  (68)

## F. Coping with Non-linear Constraints

This section discusses the derivation from the non-linear terms in sets of equations (5) and (6) to obtain a single-level MILP problem with an exact solution. The set of equations (5) contains two non-linear terms on the left and right hand sides of (5c) due to the multiplication of the electricity price variable  $\lambda_t^{DA}$  and the continuous variables  $p_{r,t}^{DA}$  and  $r_{r,t}^{SR,\uparrow}$ . By substituting the electricity price from constraint (2a) into the non-linear terms, the profit robustness constraint (5c) can be rewritten as (7a). In equation (7a), the non-linear terms  $\check{\lambda}_t^{DA} \chi_t^{DA} (p_{r,t}^{DA} + r_{r,t}^{S\hat{R},\uparrow}) \Delta t \text{ and } \hat{\lambda}_t^{DA} \chi_t'^{DA} (p_{r,t}^{DA} + r_{r,t}^{SR,\uparrow}) \Delta t \text{ are } \lambda_t^{SR,\uparrow} \Delta t \text{ are } \lambda_t^{SR,\downarrow} \Delta t \text{ are } \lambda_t^{SR,\uparrow} \Delta t \text{ are } \lambda_t^{SR,\downarrow} \Delta$ the multiplications of binary and continuous variables. Note that the consideration of discrete rather than continuous values for the electricity price in (2a) is relevant to the robustness concept, since the worst-case scenarios occur in the boundary values of electricity price. Finally, the non-linear equation (7a) can be replaced by the set of linear constraints (7b)-(7h) using the method in [21].

$$\begin{split} \tilde{\lambda}_{t}^{DA}(p_{r,t}^{DA} + r_{r,t}^{SR,\uparrow})\Delta t \\ &- \tilde{\lambda}_{t}^{DA}\chi_{t}^{DA}(p_{r,t}^{DA} + r_{r,t}^{SR,\uparrow})\Delta t \\ &+ \hat{\lambda}_{t}^{DA}\chi_{t}^{\prime DA}(p_{r,t}^{DA} + r_{r,t}^{SR,\uparrow})\Delta t \\ &\leq \lambda_{t}^{DA}\tilde{P}_{r,t}\Delta t - y_{r,t} , \quad \forall r,t \quad (7a) \end{split}$$

$$\begin{split} \tilde{\lambda}_{t}^{DA}(p_{r,t}^{DA} + r_{r,t}^{SR,\uparrow})\Delta t - \check{\lambda}_{t}^{DA}p_{r,t}^{DA,Q}\Delta t \\ + \hat{\lambda}_{t}^{DA}p_{r,t}^{\prime DA,Q}\Delta t \leq \lambda_{t}^{DA}\tilde{P}_{r,t}\Delta t - y_{r,t} , \quad \forall r,t \quad (7b) \end{split}$$

$$p_{r,t}^{DA,Q} = p_{r,t}^{DA} + r_{r,t}^{SR,\uparrow} - p_{r,t}^{DA,A} , \qquad \forall r,t \quad (7c)$$

$$\underline{P}_{r}\chi_{t}^{\mathcal{I}II} \leq \underline{P}_{r,t}^{\mathcal{I}III} \leq P_{r,t}\chi_{t}^{\mathcal{I}III}, \qquad \forall r, t \quad (\mathsf{/d})$$

$$\underline{P}_r(1-\chi_t^{DA}) \le p_{r,t}^{DA,A} \le P_{r,t}(1-\chi_t^{DA}) , \quad \forall r,t \quad (7e)$$

$$p_{r,t}^{DA} = p_{r,t}^{DA} + r_{r,t}^{TA} - p_{r,t}^{DA}, \qquad \forall r, t \quad (/f)$$

$$\underline{P}_{r,\chi_{t}^{\prime}} \leq p_{r,t}^{\prime D \prime \prime} \leq P_{r,t} \chi_{t}^{\prime D \prime \prime}, \qquad \forall r,t \quad (\mathbf{/g})$$

$$\underline{P}_{r}(1 - \chi_{t}^{\prime DA}) \le p_{r,t}^{\prime DA,A} \le \tilde{P}_{r,t}(1 - \chi_{t}^{\prime DA}) , \quad \forall r, t \quad (7h)$$

The auxiliary variables  $p_{r,t}^{DA,Q}$  and  $p_{r,t}^{DA,A}$  with the same possible lower and upper bounds as the term  $p_{r,t}^{DA} + r_{r,t}^{SR,\uparrow}$  are defined to determine the final result of the non-linear term  $\tilde{\lambda}_t^{DA}\chi_t^{DA}(p_{r,t}^{DA} + r_{r,t}^{SR,\uparrow})\Delta t$ . When the binary variable  $\chi_t^{DA}$  related to the negative electricity price deviation is 1, equations (7c)-(7e) set  $p_{r,t}^{DA,Q} = p_{r,t}^{DA} + r_{r,t}^{SR,\uparrow}$  and  $p_{r,t}^{DA,A} = 0$ . On the other hand, for  $\chi_t^{DA} = 0$ , equations (7c)-(7e) lead to  $p_{r,t}^{DA,Q} = 0$  and  $p_{r,t}^{DA,A} = p_{r,t}^{DA} + r_{r,t}^{SR,\uparrow}$ . Similarly, the auxiliary variables  $p_{r,t}^{\prime DA,Q}$  and  $p_{r,t}^{\prime DA,A}$  in equations (7f)-(7h) can define the final result of the non-linear term  $\hat{\lambda}_t^{DA}\chi_t^{\prime DA}(p_{r,t}^{DA} + r_{r,t}^{SR,\uparrow})\Delta t$  in (7a). Therefore, the linear equations (7b)-(7h) can replace the non-linear constraint (7a).

The demand robust cost formulation proposed in (6) includes non-linear terms in (6a), (6c), (6d), and (6f). The non-linear term  $\lambda_t^{DA} p_{d,t}^{DA} \Delta t$  in (6c) can be linearized in the same way as in (7) by introducing new auxiliary variables. In addition, each of the non-linear terms  $\sum_{p \in \mathcal{P}} (\chi_{d,t} \hat{P}_{d,p,t} u_{d,p})$ in (6a), and, by including the expanded term of the electricity price  $\lambda_t^{DA}$  from constraint (2a), the non-linear terms  $\lambda_t^{DA} \sum_{p \in \mathcal{P}} (\tilde{P}_{d,p,t} u_{d,p}) \Delta t$  in (6c), and  $\lambda_t^{DA} \sum_{p \in \mathcal{P}} (\hat{P}_{d,p,t} u_{d,p}) \Delta t$ in (6d) and (6f) includes only the multiplication of two binary variables. To linearize these binary multiplication terms, three new binary variables  $z_{d,p,t}, w_{d,p,t}, w'_{d,p,t}$  are introduced as the final result of binary multiplications of  $\chi_{d,t} u_{d,p}, \chi_t^{DA} u_{d,p}$ , and  $\chi_t^{DA} u_{d,p}$ , respectively. Furthermore, the set of linear constraints (8) is added to (6), which simulate the possible results of multiplying two binary variables by the newly defined binary variables  $z_{d,p,t}, w_{d,p,t}, w'_{d,p,t}$ .

$$z_{d,p,t} \le \chi_{d,t}$$
,  $\forall d, p, t$  (8a)

$$\begin{split} z_{d,p,t} &\leq u_{d,p} , & \forall d, p, t \quad (8b) \\ z_{d,p,t} &+ 1 \geq \chi_{d,t} + u_{d,p} , & \forall d, p, t \quad (8c) \\ w_{d,p,t} &\leq \chi_t^{DA} , & \forall d, p, t \quad (8d) \\ w_{d,p,t} &\leq u_{d,p} , & \forall d, p, t \quad (8e) \\ w_{d,p,t} &+ 1 \geq \chi_t^{DA} + u_{d,p} , & \forall d, p, t \quad (8f) \\ w'_{d,p,t} &\leq \chi'^{DA} , & \forall d, p, t \quad (8g) \\ w'_{d,p,t} &\leq u_{d,p} , & \forall d, p, t \quad (8g) \\ w'_{d,p,t} &\leq u_{d,p} , & \forall d, p, t \quad (8h) \\ \end{split}$$

$$w'_{d,p,t} + 1 \ge \chi'^{DA}_t + u_{d,p} , \qquad \qquad \forall d, p, t \qquad (8i)$$

Finally, by substituting the linear equivalent of constraints (5) and (6) with (7) and (8), problem (1)-(6) can be written as an MILP problem solvable with available MILP solvers such as CPLEX.

#### IV. PROFIT ROBUSTNESS EXAMPLE

This section presents a simple illustrative example to show the performance of the proposed robust formulation in finding the worst-case profit robustness scenarios by considering the asymmetry of the DAM electricity price. The example provides a detailed description of how the worst cases of the electricity price deviations affect the worst cases of energy deviations. In this context, an RVPP with two ND-RESs and one demand in a sample period of 5 hours is considered. The forecast bounds of production and demand of the RVPP units and the DAM electricity price are shown by the dashed/solid lines in Figure 2. Five cases are defined below to compare different conditions for the values of energy and price uncertainty budgets and to compare the results of the proposed model by the model in [15]:

- Case 1: Deterministic case (i.e.,  $\Gamma^{DA} = \Gamma_r = \Gamma_d = 0$ );
- Case 2: Only the DAM electricity price uncertainty is considered. It is assumed that the values of the DAM electricity price can deviate from the median to the worst case values in three periods (i.e.,  $\Gamma^{DA} = 3$  and  $\Gamma_r = \Gamma_d = 0$ );
- Case 3: Only the uncertainty of ND-RES units energy and demand is considered. It is assumed that the production values of ND-RES 1 and ND-RES 2 and the demand can deviate from the median to the worst case values in three, one, and two periods, respectively (i.e.,  $\Gamma^{DA} = 0$  and  $\Gamma_{r1} = 3$ ,  $\Gamma_{r2} = 1$ , and  $\Gamma_d = 2$ );
- Case 4: Both price and energy uncertainties are considered. The electricity price, the production of ND-RES 1 and ND-RES 2, and the demand values can deviate from the median to the worst case values ( $\Gamma^{DA} = 3$  and  $\Gamma_{r1} = 3$ ,  $\Gamma_{r2} = 1$ , and  $\Gamma_d = 2$ ).
- Case 5: The energy robustness problem presented in [15] is solved for the same uncertainty budgets as in Case 4.

Figure 2 shows the final results of DAM electricity price, ND-RESs energy, and demand for different cases proposed in this example. The final values of the above variables, corresponding to the whole period in each hour, are shown by different bars in this figure. If the value of a variable is equal to the median of the forecast (solid black line in the figure), it means that the corresponding period is not selected as the worst case. Figure 3 shows the values of the dual variables  $y_t^{(\prime)DA}$  and  $y_{r(d),t}$  related to the profit/cost affected by different uncertainties for all defined cases except for Case 5. Note that, in the model in [15], these variables are either not defined or defined for energy robustness; therefore, the comparison is only provided for the first four cases.

A. Case 1: The RVPP obtains a profit of  $\bigcirc$ 56 by bidding its median values of ND-RES production according to Figure 2. The final values for the DAM electricity price are also obtained as the median values as the length of all bars is equal to the median. As shown in Figure 3, due to not considering the robustness, all dual variables  $y_t^{(\prime)DA}$  and  $y_{r(d),t}$  are equal to zero, since the problem is a deterministic optimization one.

B. Case 2: In Case 2, the RVPP profit in the DAM is  $- \in 12$ , where the negative value means that the cost of buying electricity to supply demand is higher than the profit obtained by selling electricity on the market. The algorithm chooses periods 3 and 4 for the negative price fluctuation and period 2 for the positive price fluctuation. Therefore, the final electricity prices in periods 3 and 4 (2) are decreased (increased) to their minimum (maximum) values compared to Case 1. Note that the maximum possible profit reduction in each period can be calculated by finding the maximum value of  $\lambda_t^{DA} p_t^{DA} \Delta t$  for the negative price deviation and  $-\lambda_t^{DA} p_t^{DA} \Delta t$  for the positive price deviation. Therefore, the algorithm correctly identifies the periods that lead to the worst cases of profit reduction due to price uncertainty.

C. Case 3: In Case 3, the RVPP profit in the DAM is -  $\in 166$ . The maximum possible profit reduction for each period can be calculated by  $\lambda_t^{DA} \check{P}_{r,t} \Delta t$  for ND-RES and the maximum possible cost increase for demand can be calculated by  $\lambda_t^{DA} \hat{P}_{d,t} \Delta t$ . The worst cases of profit reductions for ND-RES 1 occur in periods 3, 4, and 5, whereas for ND-RES 2 this occurs in period 4. The worst cases of demand cost occur in periods 2 and 5, resulting in maximum demand in these periods. It can be easily verified that the algorithm correctly selects the worst periods in terms of profit reduction for ND-RES or cost increase for demand.

D. Case 4: The RVPP profit in the DAM is -€279. This case shows one of the significant differences between the proposed model and the models in the literature [15] (by comparing the black (Case 4) and white (Case 5) bars), where instead of selecting the periods with higher energy reductions, the proposed algorithm selects the periods that result in higher profit reductions. For instance, the worst case of ND-RES 2 production occurs in period 4 with profit reduction of €60 and energy reduction of 4 MW. However, the period 3 with the highest amount of energy deviation (5 MW) in Case 5 is selected as the worst case.

In Case 5, those periods that result in more deviations of ND-RES production and demand are selected as the worst cases. Moreover, the worst cases of electricity price deviations are determined according to the final values of ND-RES production and demand. Considering the different selection of worst-case periods for Cases 4 and 5, the RVPP obtains a profit of -€279 in the former, which is lower than the profit of -€223 obtained in Case 5. Note that the profit obtained is for bidding in the market and is different from the profit from clearing the market. Suppose the RVPP uses the bidding strategy proposed in this paper, even though its profit is lower. In this condition, it reduces the risk of significant losses and penalties (e.g., due to buying energy in real time or penalties for the energy it promised to provide but cannot) for not considering the actual worst cases. Moreover, the results indicate that the energy robustness approach cannot fully cope with the actual worst cases for both energy (ND-RESs production and demand) and price uncertainty. On the contrary, the profit robustness approach proposed in this paper considers the worst cases of profit/cost deviations for ND-RESs/demand instead of the maximum energy deviation. As a final remark, illustrative results indicate that the proposed algorithm accurately selects the worst-case profit for different uncertainty budgets, and shows better performance in finding the worst-case scenarios compared to the model in [15]. These finding will be thoroughly analyzed in successive sections.

# V. SIMULATION RESULTS

This section presents the simulation results of the proposed single-level robust bidding model for different case studies. The RVPP is located in southern Spain and includes a wind farm, two solar PV plants, and a flexible demand. The production forecast data of the wind farm and the solar PV plants, representing a sample day of the spring season in Spain, are taken from [24], [25]. The solar PV plants and the wind farm each have a rated capacity of 50 MW and operating costs of 5



Fig. 2. Final values of DAM electricity price and RVPP units output energy in different case studies.



Fig. 3. The profit/cost dual variables affected by different uncertainties in different case studies.

€/MWh and 10 €/MWh, respectively. A residential aggregator profile for the flexible demand is considered according to [23]. The demand owner allows a 10% tolerance for additional demand flexibility, which is allocated for the possible SR provision. All energy forecast data related to RVPP units is shown in Figure 4. The price forecast data for DAM and SRM are taken from the Red Eléctrica de España (REE) website, and are shown in Figure 5 for illustration purposes [26].

Two case studies are performed to analyze the performance of the proposed model. In the first case study, different values for all uncertain parameters related to the DAM and SRM electricity prices, ND-RESs energy, and demand are considered to show the behaviour of the proposed model in different uncertain environments. In the second case, by means of an out-of-sample assessment, the bidding approach of this paper is compared with two models in the literature. The detailed description of the input parameters for the above cases is highlighted below:

• Case 1.1: Deterministic case  $(\Gamma^{DA/SR} = \Gamma_r = \Gamma_d = 0);$ 



Fig. 4. The energy forecast data.

- Case 1.2: Only the uncertainties of the energy of the ND-RES units and the demand are considered  $(\Gamma^{DA/SR} = 0 \text{ and } \Gamma_r = \Gamma_d = 5);$
- Case 1.3: Only the DAM and SRM electricity price uncertainties are considered (Γ<sup>DA/SR</sup> = 5 and Γ<sub>r</sub> = Γ<sub>d</sub> = 0);
- Case 1.4: Both DAM and SRM electricity price and energy uncertainties are considered ( $\Gamma^{DA/SR} = \Gamma_r = \Gamma_d = 5$ );
- Case 2: The results of the proposed model for  $\Gamma^{DA/SR} = \Gamma_r = 0, 1, 2, ..., 9$  are compared with models in [15] and [18] using an out-of-sample assessment.

Simulations are performed on a Dell XPS with an i7-1165G7 2.8 GHz processor and 16 GB of RAM using the CPLEX solver in GAMS 39.1.1.

#### A. Case 1

Figure 5 shows the RVPP traded energy and reserve versus the electricity price for Cases 1.1 through 1.4. The general results for all cases show that between hours 8-11, when the demand is high and the production of ND-RES units is not enough to supply all demand, the RVPP is an energy buyer in the electricity market. Between hours 12-15, although the demand is high, the production and demand of RVPP are approximately equal, and RVPP does not trade too much energy in most cases. However, in these hours the consideration of different uncertain parameters in Cases 1.1 through 1.4 has a significant effect on the trading direction of RVPP (whether RVPP is a seller or a buyer of energy). Between hours 16-19, as the demand decreases, the RVPP becomes a seller of energy in most of the cases. The results for traded SR shows that between hours 9-20, that RVPP has high production, it provides more up and down SR to the market.

The total sold energy of RVPP in Cases 1.2 through 1.4 is decreased by 52.0%, 0%, and 51.7%, respectively, compared to Case 1.1, whereas the total bought energy of RVPP is increased by 74.2%, 0%, and 66.3%, respectively. The total up (down) SR provided by RVPP in Cases 1.2 through 1.4 is decreased by 0 (0)%, 2.2 (1.5)%, and 7.5 (7.1)%, respectively, compared to Case 1.1.

The results for each case study show that in the deterministic case (Case 1.1) and in the hours when RVPP is an energy seller in the market, RVPP usually sells more energy and reserve than in Cases 1.2 and 1.4. However, if RVPP is an energy buyer, the energy bought in Case 1.1 is usually less than in Cases 1.2 and 1.4. The reason is that in the deterministic case, the RVPP always takes an optimistic approach because it does not consider any uncertain parameter. In Case 1.2, considering the energy deviation of ND-RESs and demand results in a lower amount of energy sold and a higher amount



Fig. 5. RVPP traded energy and reserve versus electricity price in different case studies.

of electricity purchased from the market compared to Case 1.3, where only DAM and SRM electricity price uncertainties are considered. According to the comparison of Cases 1.1 and 1.3, considering only the electricity price uncertainty results in a lower amount of purchased energy only in some hours, e.g. hour 10, compared to Case 1.1. The reason is that this hour is one of the hours in which the electricity price goes to its worst case. Therefore, the RVPP prefers to supply its demand with its production and also to provide less SR. In other hours (except hours 1-6 and 12 with small differences) there are not too many differences between RVPP traded energy in Cases 1.3 and 1.1. The reason is that although the electricity price goes to the worst cases in some hours in Case 1.3, the RVPP must supply its demand or it can sell energy to the market with lower benefit. Considering all energy and price uncertainties in Case 1.4 results in a different bidding approach in some hours (e.g., hours 9, 10, 12, 14, and 15) compared to Case 1.2, which considers only energy uncertainty. Hours 9, 10, and 14 are exactly the hours where the worst cases of electricity price occur, forcing the RVPP to increase or decrease its bid amount. Note that the worst cases of DAM electricity price occur in hours 8-11 and 14 (which are different from the worst cases of electricity price in Case 1.3).

#### B. Case 2

Figure 6 compares the results of an out-of-sample assessment for the proposed model and models in [15] and [18] for different values of uncertainty budgets between 0 and 9. In the figure,  $\Pi^{av}$  represents the operating profit (no penalization applied),  $K^{av}$  is the penalization cost for not complying with the energy bid, and the net profit of RVPP is represented by  $\Pi^{av} - K^{av}$ . In [15], the energy robustness approach is adopted. The authors in [18] use a multi-level optimization problem which implements an RO approach to model the ND-RES units uncertainties, and a Stochastic Optimization (SO) to capture the electricity price uncertainties. Therefore, the uncertainty budget in Figure 6 for model [18] refers only to the production of ND-RES units. To model the price uncertainty in their SO model, 200 scenarios are considered according to the REE website [26]. For the out-of-sample assessment, 1000 scenarios are generated based on the hourly

distributions of uncertain parameters related to the DAM and SRM electricity prices and ND-RESs production. The Weibull distribution, with its ability to model different degrees of skewness and tails, is used to generate scenarios to better capture the asymmetric behavior of uncertain parameters. Note that an equal value for all time periods, such as in [15] and [18], can be considered for the penalty cost. However, the penalty cost related to the energy that is not provided is set to three times the DAM median price forecast in this paper. In this way, the deviation in the hours when the electricity price is higher leads to more penalty for RVPP.

The net profit of RVPP for uncertainty budget 0 in the proposed model and model [15] is the same as the deterministic components in both models are the same. However, the model in [18] results in a lower value of net profit for uncertainty budget 0 compared to the proposed model and [15] due to the use of a different reserve provision strategy. In this paper and in [15], the production plus the reserve provided by each RVPP unit is limited by the maximum production of each unit, while in [18] this constraint is not defined and only the reserve provision limit by the entire RVPP is considered. Therefore, for an uncertainty budget of 0, using the proposed model or the model in [15] results in a lower energy bid in the DAM in several hours compared to [18]. By increasing the uncertainty budget, the proposed model leads to a higher net profit obtained compared to model [15].

The better results in terms of net profit by using the model [18] compared to the proposed model is, to some extent, expected. This is due to the use of a more sophisticated approach to find the worst case of uncertainties of ND-RESs production, and the consideration of the possibility of rescheduling the RVPP units in the third level of the model [18]. However, the proposed model shows a closely aligned results compared to [18] even in some cases the obtained results in the proposed model are better than model in [18] (see e.g. results for uncertainty budgets 0, 3, 4, and 5).

From the computational standpoint, the simulation time of different cases of the model [15] is less than 2 s due to the simplified approach to identify the worst case of the optimization problem. The simulation time of the model proposed in this paper is less than 90 s in all cases, which meets the



Fig. 6. The out-of-sample assessment for proposed model and the models in [15] and [18] ( $\Gamma^{DA/SR} = \Gamma_r = 0, 1, 2, ..., 9$ ).

acceptable criteria for the RVPP bidding problem when, e.g., different strategies need to be analyzed and compared before submitting the bid to the market operator. The simulation time of the model [18] reaches 90 min in some cases. In summary, the proposed approach demonstrates outstanding computational performance against more intricate approaches such as [18], while providing results that compete in terms of profits with the model in [18] and reducing the risk of penalization compared with [15].

## VI. CONCLUSION

In this paper, a novel, computationally efficient, singlelevel robust bidding method is proposed to capture multiple uncertainties in the DAM and SRM electricity prices as well as ND-RES production and demand of an RVPP. The non-linear couplings between different uncertainties in the objective function and constraints of the optimization problem are addressed by developing an accurate linear model based on the big M method. The obtained results show that the uncertainty of ND-RES and demand has the highest impact on the bidding approach of RVPP compared to the electricity price uncertainty. Furthermore, the sensitivity analysis shows that the RVPP operator can significantly increase its net profit by considering even a low or median value for the risk measure parameter (uncertainty budget). In addition, the simulation results show the computational efficiency of the proposed model as well as high consistency results with more complicated multi-level models. In the future works, the authors aim to optimize the risk measure parameter so that the RVPP operator obtains a certain desired profit.

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