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GRADO EN INGENIERÍA EN TECNOLOGÍAS DE TELECOMUNICACIÓN

TRABAJO FIN DE GRADO

Mecánica Pulmonar

Autor: Teresa Conde Germán-Coley

Director: Subramanian Venkat Shastri

Madrid

Declaro, bajo mi responsabilidad, que el Proyecto presentado con el título

Lung Mechanics

en la ETS de Ingeniería - ICAI de la Universidad Pontificia Comillas en el

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Autorizada la entrega del proyecto

EL DIRECTOR DEL PROYECTO

Fdo.: Subramanian Venkat Shastri Fecha: 08/ 06/ 2025

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MECÁNICA PULMONAR

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RESUMEN DEL PROYECTO

El modelado preciso de la dinámica pulmonar es clave para comprender la mecánica respiratoria y ajustar adecuadamente la ventilación mecánica, reduciendo así el riesgo de lesiones inducidas. Este proyecto propone un modelo matemático del sistema respiratorio basado en funciones no lineales, con el fin de representar de forma realista las propiedades dinámicas del parénquima pulmonar durante el ciclo respiratorio.

Palabras clave: Mecánica pulmonar, flujo respiratorio, volumen pulmonar, presión pulmonar, propiedades viscoelásticas, ventilación mecánica.

1. Introducción

En los últimos años, el interés por el estudio del sistema respiratorio y su mecánica ha aumentado notablemente en ámbitos clínicos y de bioingeniería, impulsado por la necesidad de comprender y simular con precisión el comportamiento dinámico de los pulmones. Un ajuste adecuado del flujo de aire en el paciente resulta clave para prevenir lesiones pulmonares asociadas a la ventilación mecánica.

Este proyecto desarrolla un modelo basado en funciones no lineales aplicadas al flujo respiratorio, como la ley de Poiseuille y las formulaciones propuestas por Chatburn (2003) para describir el comportamiento de flujo y volumen durante la espiración. Asimismo, se explora la analogía entre el modelo de Merritt (1978), diseñado para globos de goma, y la mecánica del tejido pulmonar, lo que permite representar de forma más precisa la dinámica respiratoria mediante expresiones matemáticas simplificadas.

El estudio se centra en la simulación de las propiedades viscoelásticas del tejido pulmonar, integrando además fenómenos de fricción e histéresis presentes durante las fases de inspiración y espiración. Una comprensión profunda de la interacción entre flujo, volumen y presión contribuye a mejorar las estrategias de ventilación y a reducir el riesgo de lesiones pulmonares inducidas por el ventilador (VILI), una complicación frecuente en unidades de cuidados intensivos.

2. Definición del proyecto

El comportamiento mecánico de los pulmones, junto con factores como la presión transpulmonar, la elasticidad torácica y las propiedades viscoelásticas del tejido pulmonar, resulta fundamental para entender cómo distintas estrategias de ventilación afectan individualmente a cada paciente. Por ello, comprender los mecanismos mediante los cuales los ventiladores pueden inducir lesiones es clave para una intervención clínica adecuada.

Este proyecto desarrolla un modelo computacional de la mecánica pulmonar basado en ecuaciones no lineales, especialmente diseñadas para representar fenómenos viscoelásticos complejos y efectos de memoria. El modelo se inspira en el comportamiento de globos de goma descrito por Merritt y Weinhaus (1978), adaptado al contexto pulmonar para simular con mayor precisión la dinámica presión-volumen en distintas condiciones respiratorias

Para alcanzar este objetivo se han llevado a cabo las siguientes tareas:

- Estudio y análisis de modelos pulmonares existentes y revisión de nuevos enfoques aplicables a la mecánica respiratoria.
- Diseño de un modelo matemático que integre términos viscoelásticos y relaciones no lineales entre presión y flujo.
- Implementación computacional del modelo mediante scripts en Matlab.
- Simulación de ciclos respiratorios (inspiración y espiración) bajo diferentes configuraciones, y análisis de las respuestas obtenidas.

La simulación se lleva a cabo en Matlab, empleando resolutores numéricos como `ode45` para resolver ecuaciones diferenciales ordinarias. Se realizan diversas simulaciones modificando parámetros del modelo (flujo, rigidez del tejido, resistencia), y se analizan salidas clave como la evolución de la presión y del volumen pulmonar en el tiempo.

Se construye el modelo completo considerando dos escenarios: con flujo constante como entrada, y presión constante como variable de entrada. Para ello, se integran tres aproximaciones distintas:

- La ley de Poiseuille ajustada a flujo turbulento;
- Los modelos de Chatburn (2003) para volumen y flujo;
- El modelo de Merritt (1978) para la relación presión-radio del tejido elástico.

Finalmente, la validación del modelo se realiza mediante una comparación entre los resultados obtenidos de ambos modelos y a su vez con resultados obtenidos en la literatura científica. (Chatburn, 2003; Arnal et al., 2018).

3. Descripción del modelo

El fenómeno fisiológico que se representa es la mecánica pulmonar durante la respiración para ello, se divide principalmente en dos bloques: inspiración y espiración respiratoria. Ambos bloques consisten en modelos matemáticos que recogen las características tanto de la fisiología pulmonar como la del tejido que está en contacto el flujo respiratorio hasta que este llega a los pulmones. Para ello, se han empleado diversas funciones matemáticas para su correcta descripción, algunas propias de la fisiología pulmonar y otra acudiendo a la semejanza entre el comportamiento del tejido de un globo de goma y el tejido pulmonar.

Para ello, se ha hecho uso de parámetros de entrada características de la respiración como son: radio de la tráquea, radio mínimo pulmonar, radio máximo que puede alcanzar, presión mínima (PEEP) y presión máxima (PIP) entre ellos. Las variables de salida del modelo son el tiempo de espiración marcado por el paciente y las funciones que representan la variación de presión, volumen y flujo respiratorio en función del tiempo.

4. Resultados

Los resultados obtenidos tras el desarrollo del modelo son favorables respecto a valores fisiológicos recogidos de pacientes adultos bajo ventilación mecánica, tal como se establece en Arnal et al. (2018). Los valores que se obtienen al finalizar el ciclo respiratorio son equivalentes a los valores de entrada predeterminados, confirmando la validez del modelo.

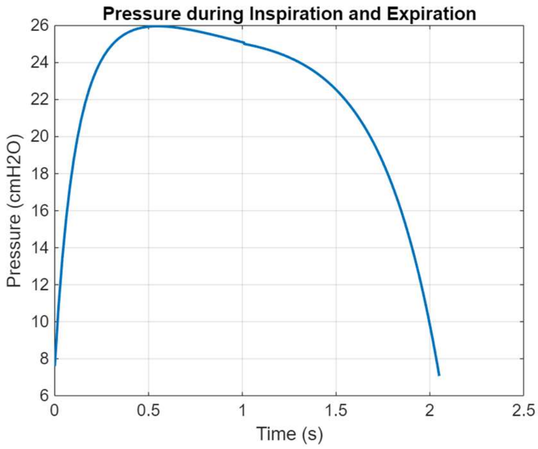

Ilustración 1 Evolución de la Presión durante Ciclo Respiratorio

En el caso de la presión, durante la inspiración, la presión aumenta de forma pronunciada debido a la resistencia al flujo y al retroceso elástico del pulmón, alcanzando un valor máximo de aproximadamente 26 cmH_2O. Durante la espiración, la presión disminuye progresivamente, lo cual refleja la liberación pasiva de la energía elástica acumulada en el sistema. El valor PEEP que se alcanza es 7 cmH_2O.

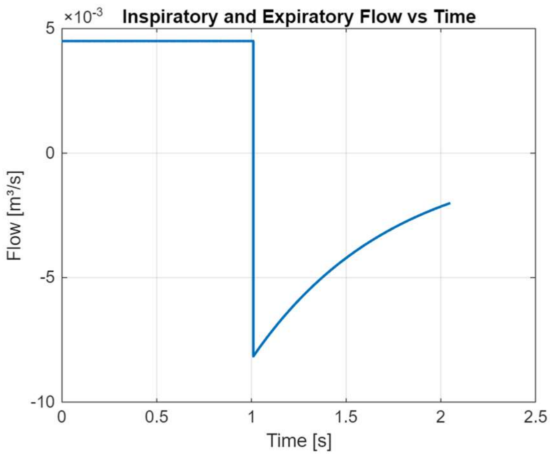

Ilustración 2 Evolución del Flujo durante Ciclo Respiratorio

En la Ilustración 2 Evolución del Flujo durante Ciclo Respiratorio se representa la variación del flujo respiratorio durante las fases de inspiración y espiración. En la

inspiración, se impone un flujo de entrada constante, cuyo valor es de 0.0045 m^3/s, equivalente al volumen tidal dividido entre el tiempo inspiratorio. Esta estimación se realiza aplicando directamente la definición de flujo como el cociente entre volumen y tiempo.
Durante la espiración, al tratarse de una fase pasiva, se utiliza la función definida por Chatburn (2003) para describir la evolución temporal del flujo espiratorio mostrando comportamiento exponencial.

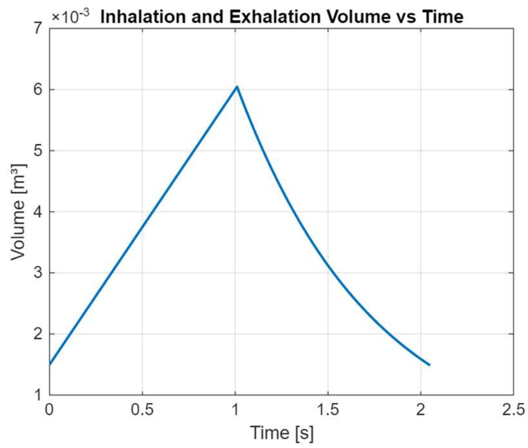

Ilustración 3 Evolución del Volumen durante Ciclo Respiratorio

En la Ilustración 3 Evolución del Volumen durante Ciclo Respiratoriose muestra la variación del volumen pulmonar durante las fases de inspiración y espiración. Durante la inspiración, se observa un incremento lineal del volumen, lo cual es coherente con el hecho de que se ha aplicado un flujo constante de entrada.
En la espiración, se vuelve a emplear el modelo descrito por Chatburn (2003) para representar la evolución del volumen en función del tiempo. Este modelo asume una respuesta exponencial decreciente, que simula el vaciamiento pasivo de los pulmones. En este caso, el volumen oscila entre un máximo de 80 mL/kg y un mínimo de 20 mL/kg, valores fisiológicamente representativos del volumen tidal en pacientes adultos.

5. Conclusiones

A lo largo de este trabajo se ha desarrollado un modelo matemático capaz de simular el comportamiento de la mecánica pulmonar durante un ciclo respiratorio completo, incorporando funciones fisiológicamente realistas tanto para la inspiración como para la espiración. El modelo se ha construido a partir de referencias teóricas consolidadas, como el modelo de Merritt (1978) para el comportamiento viscoelástico del tejido pulmonar, y el modelo de Chatburn (2003) para la dinámica pasiva de la espiración.

Los resultados obtenidos son coherentes con los valores fisiológicos recogidos en la literatura especializada para pacientes adultos en ventilación mecánica, especialmente los descritos por Arnal et al. (2018), lo que refuerza la validez del enfoque seguido.

Este modelo permite representar de forma más realista la evolución de la presión, el volumen y el flujo respiratorio en función del tiempo, lo cual lo convierte en una herramienta útil tanto para la docencia como para la simulación de condiciones clínicas. Como línea futura, se plantea la posibilidad de extender el modelo a contextos patológicos (como EPOC o SDRA), así como la integración de datos reales de paciente para su validación clínica y el desarrollo de un sistema de control.

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LUNG MECHANICS

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ABSTRACT

A precise modeling of pulmonary dynamics is essential to understand respiratory mechanics and properly adjust mechanical ventilation, thereby reducing the risk of ventilation-induced injuries. This project proposes a mathematical model of the respiratory system based on nonlinear functions, aiming to realistically represent the dynamic properties of the lung parenchyma throughout the respiratory cycle.

Keywords: Pulmonary mechanics, respiratory flow, lung volume, lung pressure, viscoelastic properties, mechanical ventilation.

1. Introduction

In recent years, interest in studying the respiratory system and its mechanics has significantly increased in both clinical and bioengineering fields, driven by the need to accurately understand and simulate the dynamic behavior of the lungs. Proper adjustment of airflow in the patient is essential to prevent lung injuries associated with mechanical ventilation.

This project develops a model based on nonlinear functions applied to respiratory flow, such as Poiseuille's law and the formulations proposed by Chatburn (2003) to describe flow and volume behavior during expiration. It also explores the analogy between Merritt's model (1978), originally designed for rubber balloons, and the mechanical behavior of lung tissue, allowing for a more accurate representation of respiratory dynamics through simplified mathematical expressions.

The study focuses on simulating the viscoelastic properties of lung tissue, also incorporating friction and hysteresis phenomena that occur during inspiration and expiration. A thorough understanding of the interaction between flow, volume, and pressure contributes to improved ventilation strategies and helps reduce the risk of ventilator-induced lung injury (VILI), a frequent complication in intensive care units.

2. Project Definition

The mechanical behavior of the lungs, along with factors such as transpulmonary pressure, thoracic elasticity, and the viscoelastic properties of lung tissue, is essential to understanding how different ventilation strategies affect each patient individually. Therefore, understanding the mechanisms through which ventilators may induce injury is key to proper clinical intervention.

This project develops a computational model of pulmonary mechanics based on nonlinear equations, specifically designed to represent complex viscoelastic phenomena and memory effects. The model is inspired by the behavior of rubber balloons described by Merritt and Weinhaus (1978), adapted to the pulmonary context to more accurately simulate pressure–volume dynamics under various respiratory conditions.

To achieve this goal, the following tasks have been carried out:

- Study and analysis of existing pulmonary models and review of new approaches applicable to respiratory mechanics.
- Design of a mathematical model that integrates viscoelastic terms and nonlinear pressure–flow relationships.
- Computational implementation of the model using Matlab scripts.
- Simulation of respiratory cycles (inspiration and expiration) under different configurations and analysis of the resulting model behavior.

The simulation is carried out in MATLAB, using numerical solvers such as ode45 to solve ordinary differential equations. Multiple simulations are performed by varying model parameters (flow, tissue stiffness, resistance), and key outputs—such as the evolution of lung pressure and volume over time—are analyzed.

The complete model is built considering two scenarios: one with constant flow as the input and another with constant pressure. To this end, three different approaches are integrated:

- The Poiseuille law, adapted for turbulent flow;
- The Chatburn (2003) models for pressure, volume, and flow;
- The Merritt (1978) model for the pressure–radius relationship of elastic tissue.

Finally, the model is validated through a comparison between the results obtained from both configurations and those reported in the scientific literature (Chatburn, 2003; Arnal et al., 2018).

3. Description of the Model

The physiological phenomenon represented is pulmonary mechanics during respiration, which is divided into two main phases: inspiration and expiration. Both phases consist of mathematical models that incorporate the characteristics of lung physiology as well as those of the tissue in contact with the respiratory flow until it reaches the lungs. Various mathematical functions have been employed for accurate modeling—some specific to pulmonary physiology, and others based on the analogy between the behavior of rubber balloons and lung tissue.

Input parameters characteristic of respiration include: tracheal radius, minimum and maximum lung radius, and minimum (PEEP) and maximum (PIP) pressures. The model's output variables are the patient-specific expiration time and the functions that describe the variation of pressure, volume, and respiratory flow over time.

4. Results

The results obtained from the model are consistent with physiological values observed in adult patients under mechanical ventilation, as established by Arnal et al. (2018). The values achieved at the end of the respiratory cycle match the predefined input values, confirming the validity of the model.

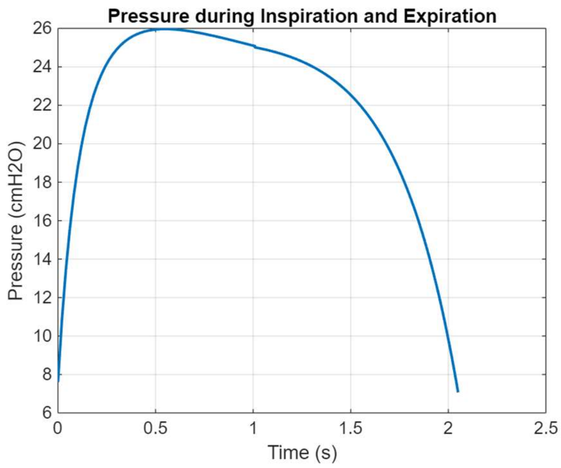

Illustration 1Pressure Function during Respiratory Cycle

In the case of pressure, during inspiration, it increases sharply due to flow resistance and the elastic recoil of the lung, reaching a maximum value of approximately 26 cmH_2O. During expiration, the pressure progressively decreases, reflecting the passive release of the elastic energy stored in the system. The PEEP value reached is 7 cmH_2O.

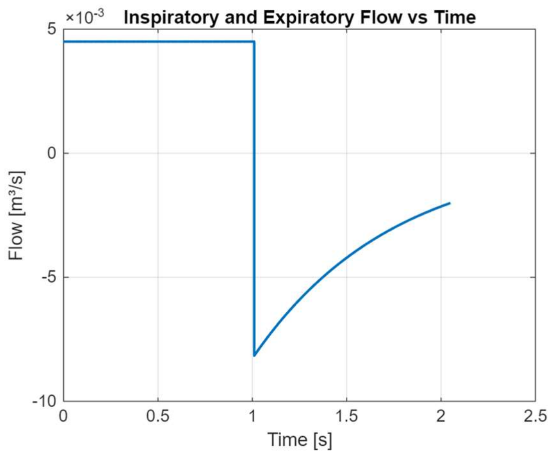

Illustration 2 Flow Function during Respiratory Cycle

Figure 2 shows the variation of respiratory flow during the inspiration and expiration phases. During inspiration, a constant inflow is imposed, with a value of 0.0045 m^3/s, equivalent to the tidal volume divided by the inspiratory time. This estimate is obtained by directly applying the definition of flow as the ratio between volume and time.

During expiration, since it is a passive phase, the function defined by Chatburn (2003) is used to describe the temporal evolution of expiratory flow, exhibiting an exponential behavior.

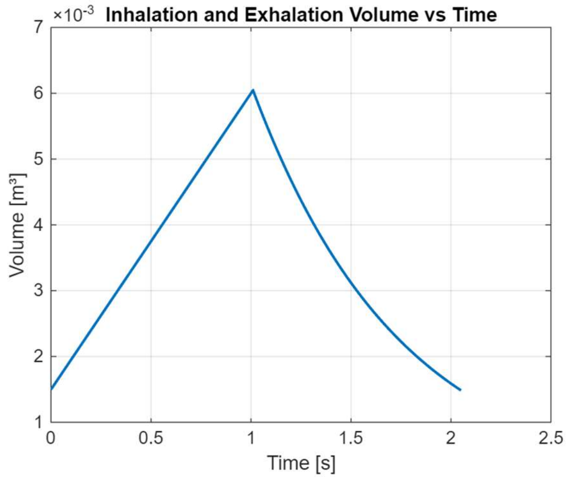

Illustration 3 Volume Function during Respiratory Cycle

Figure 3 shows the variation of lung volume during the inspiration and expiration phases. During inspiration, a linear increase in volume is observed, which is consistent with the application of a constant inflow.

During expiration, the model described by Chatburn (2003) is again used to represent the time-dependent evolution of volume. This model assumes a decaying exponential response, simulating the passive emptying of the lungs. In this case, the volume ranges from a maximum of 80 mL/kg to a minimum of 20 mL/kg, values that are physiologically representative of the tidal volume in adult patients.

5. Conclusions

Throughout this work, a mathematical model has been developed to simulate the behavior of pulmonary mechanics during a complete respiratory cycle, incorporating physiologically realistic functions for both inspiration and expiration. The model has been built based on well-established theoretical references, such as the model by Merritt (1978) for the viscoelastic behavior of lung tissue and the model by Chatburn (2003) for the passive dynamics of expiration.

The results obtained are consistent with physiological values reported in the specialized literature for adult patients under mechanical ventilation, particularly those described by Arnal et al. (2018), which reinforces the validity of the chosen approach.

This model allows for a more realistic representation of the time-dependent evolution of pressure, volume, and respiratory flow, making it a valuable tool for both educational purposes and the simulation of clinical conditions. As a future direction, it is proposed to extend the model to pathological contexts (such as COPD or ARDS), as well as to integrate real patient data for clinical validation and the development of a control system.

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Chapter 1. INTRODUCTION

1.1 MOTIVATION

Nowadays, the use of mechanical ventilation is essential for the treatment and recovery of patients with respiratory diseases, as well as for those with non-respiratory conditions that affect oxygenation and pulmonary function. Its application has proven to be crucial in critical clinical situations, providing not only vital support, but also comfort and stability for patients. Thanks to advances in monitoring and automation, mechanical ventilation has become an indispensable resource in intensive care units (ICUs), complex surgical procedures, and long-term treatment of patients with respiratory failure.

However, if the ventilator settings are not properly adjusted to the patient's physiological characteristics, lung injuries may occur or worsen. The set of damages that a patient may suffer due to improper ventilator settings, mismatched with their actual respiratory needs, is known as Ventilator-Induced Lung Injury (VILI). This condition negatively impacts gas exchange, prolongs hospital stays, and can cause direct damage to lung tissue. These complications are often not caused by device failure, but rather by an oversimplified interpretation of the patient's respiratory dynamics.

Traditionally, linear and time-invariant models have been used to represent the mechanical behavior of the respiratory system, considering only parameters such as airway resistance and lung compliance. While these models provide an initial approximation, they have proven to be inaccurate in complex clinical scenarios, particularly when attempting to simulate phenomena such as the delayed tissue response to constant pressure or the progressive adaptation of lung tissue to repeated efforts.

The introduction of viscoelastic properties of lung tissue offers a robust and realistic alternative. These properties allow for the characterization of dynamic phenomena that more closely resemble real biological behavior, such as the time-dependent relationship between stress and tissue deformation, which is especially relevant in patients with chronic lung diseases, fibrosis, or in natural aging processes. Fractional viscoelastic models provide a powerful framework to describe biological systems that exhibit memory effects and non-instantaneous responses to mechanical stimuli. Furthermore, incorporating these elements enhances the ability of models to adapt to different clinical states and to anticipate potential complications associated with invasive ventilation. (Ganzert et al. (2009))

The key parameters that define human respiration are flow, pressure, and volume. The relationship among these three variables is nonlinear, which means that using overly simplified linear models based on ordinary differential equations may be detrimental for patients in complex clinical conditions.

An inadequate representation of pulmonary behavior can lead to various forms of ventilator-induced lung injury, such as:

- Barotrauma: injury caused by excessive pressure,
- Volutrauma: caused by excessive air volumes,
- Atelectrauma: cyclical collapse and reopening of alveoli,
- Biotrauma: systemic inflammatory response triggered by mechanical stimulation,
- Ergotrauma: a broader term that encompasses the total energy applied to the respiratory system and its cumulative effects.

All of these injury mechanisms are directly related to the mechanical energy delivered during the ventilation process, and their occurrence can be minimized through more accurate and adaptive modeling of pulmonary mechanics. (Venkataraman, s.f.).

Therefore, with the aim of improving the simulation of lung mechanics, this work proposes the incorporation of models that include the viscoelastic properties of lung tissue, allowing a faithful and nonlinear representation of the lung's real behavior in response to stimuli such as pressure, volume, or flow. The modeling approach is inspired by the Fractional Standard Linear Solid (FSLS) model, which has been consolidated as one of the most advanced frameworks for simulating biological systems with memory behavior (Shastri & Narendra, 2020).

However, this project specifically adopts an alternative approach based on the physical model developed for the pressure curve of a rubber balloon, as described by Merritt and Weinhaus (1978). This model relates internal pressure to the balloon radius, accounting for the hyperelastic behavior of the material, and qualitatively reproduces phenomena such as hysteresis and the existence of a maximum pressure point before a rapid collapse, features that are also observed in pulmonary mechanics. The balloon analogy makes it possible to capture the nonlinear response of lung tissue to expansion, as well as effects related to alveolar collapse and overdistension.

Although idealized, this physical approach is based on the similarity between the lungs and a spherical-shaped plastic balloon, and it offers a useful tool for simplifying the study of respiratory system behavior and analyzing the impact of different ventilation conditions.

This represents a significant advantage over classical models, as it allows for the reproduction of effects such as delayed relaxation and pulmonary hysteresis, which are key phenomena for understanding the real interaction between the ventilator and the patient. This approach paves the way for accurate simulations and the potential design of safer and smarter control systems for respiratory support.

Nevertheless, even with the incorporation of advanced viscoelastic models, a certain margin of error must be acknowledged due to the geometric simplifications and the omission of complex anatomical and structural factors, such as the alveolar architecture, which continue to influence the system's final response. The analogy between a rubber balloon and the lung tissue is a valuable qualitative approach to alveolar collapse, overdistension and nonlinear mechanisms, but it remains a simplification.

As highlighted by Marini and Gattinoni (2018), mechanical ventilation strategies must account for the heterogeneity of lung mechanics and the energy applied to different regions of the lung. Therefore, more sophisticated and individualized models are required to better predict complications associated with invasive ventilation and disease-specific alterations in pulmonary function.

Therefore, a certain degree of uncertainty must be accepted, and the results obtained from the model should always be interpreted and adapted within a clinical framework.

Chapter 2. DESCRIPTION OF THE TECHNOLOGIES

This project has been divided into inspiration and expiration, as these are the two main phases that constitute respiratory mechanics. For the simulation of each phase, and their subsequent adjustment, different approaches were employed, since each required specific conditions and distinct behavior.

Respiratory mechanics can be understood as the result of the interaction between two main forces: on the one hand, the pressure generated by the diaphragm and accessory muscles during contraction, which allows lung expansion; and on the other, the resistance exerted by the thoracic cavity, which acts as a limit to that expansion. This dynamic balance gives rise to pressure differences between the inside of the lungs and the outside environment (mouth and nose), which in turn generates airflow during the inspiration and expiration phases (Chatburn 2003).

During inspiration, the descent of the diaphragm and thoracic expansion create a lower pressure inside the lungs compared to the ambient pressure, facilitating the inflow of air. During expiration, the process reverses: the internal pressure increases, allowing air to be expelled (Chatburn 2003). This behavior can be considered similar to the one of a rubber balloon, where inflation requires overcoming the resistance of the elastic material through input pressure, while deflation occurs due to the recoil pressure exerted by the stretched tissue (Merritt 1978).

Moreover, a mechanical ventilation model should not be limited to representing only the lung tissue but must also consider the behavior of the upper airways, particularly regarding the airflow resistance they introduce. This resistance leads to a pressure drop that significantly affects the development of inspiration, influencing the flow and pressure in the opening of the lungs. (Chatburn 2003)

This is one of the key reasons why it is necessary to distinguish between the inspiration and expiration phases in the simulation. During inspiration, the air must overcome both the resistance of the lung's elastic tissue and the friction within the upper airways, which directly affects the input pressure within the lungs. In contrast, during expiration, the behavior can be simplified by assuming that the internally generated pressure is sufficient to expel the air, without the outflow resistance playing a critical role in adjusting ventilator parameters. Therefore, in this phase, it is sufficient to know the resulting output pressure in order to evaluate the system's behavior. During expiration, the parameters that are involved in this process are: both the pressure of the lung tissue and the pressure of the thoracic cavity and the expansion of the diaphragm. (Chatburn, 2003)

The fundamental physiological variables that determine the behavior of the respiratory system are respiratory flow, lung volume, and pressure. These three quantities are the main variables of the model and are dynamically interrelated throughout the respiratory cycle.

As it is described in Chatburn´s book "Fundamentals of Mechanical Ventilation", different modeling approaches exist depending on which of these variables is used as the control variable. In the present work, flow (or pressure) has been selected as the input variable, which involves defining a function that represents the airflow introduced into the airways during the inspiratory phase.

Based on this flow function, and depending on the lung volume reached, a pressure evolution is generated, allowing the identification of the point at which the peak inspiratory pressure (PIP) is reached. Once this threshold is reached, inspiration is considered complete, and the expiratory phase begins, during which the exhalation of air is simulated until the positive end-expiratory pressure (PEEP) is reached.

Since flow is mathematically the derivative of volume with respect to time, and pressure is modeled as a volume function (through the radius, in the case of the spherical model), it is possible to derive the evolution of respiratory pressure from the resulting volume function. In this way, the model enables the simulation of a complete respiratory cycle controlled by input flow, accurately reproducing the physiological variations of pressure and volume during the inspiration and expiration phases.

Although the complete respiratory cycle can be represented using a unified model, the inspiration and expiration phases show distinct physiological and functional differences that justify a separate treatment in the modeling process. During inspiration, an active force is applied, generated either by the ventilator or by the respiratory muscles in a biological system, whereas expiration can be assumed to be a passive process, determined by the elastic recoil of lung tissue.

For this reason, the model has been divided into two distinct blocks: one for inspiration and one for expiration, with slight variations in the governing equations. The functioning of the inspiration model is described in detail in the following chapters, based on the constant inflow function defined as the variation of volume over the inspiratory time, or constant pressure function, both directly related to the pressure variation generated within the lungs.

Matlab provides a highly accurate environment for function visualization and allows for increased precision when necessary. In addition, the program enabled the development of the model using vectors, allowing for iterative adjustment of parameter values.

The models used for both inspiration and expiration are nonlinear equations. To solve these equations and obtain highly accurate results, Matlab was chosen due to its extensive library of toolboxes and core functions, and because it is an environment specifically designed for solving mathematical problems. The simulations included in this model were carried out using built-in Matlab functions. Ordinary differential equations (ODEs) were simulated, starting with the movement function which is the most simplified functions that defines breathing, and solved using Matlab´s `ode45` function.

The complete mathematical expressions of the model, including the nonlinear pressure-radius relationship based on Merritt's model (1978), are explainned in Chapter 5. Additionally, graphical representation functions such as `plot`, `subplot`, `xlabel`, `ylabel`, `title`, and `sgtitle` were used. Anonymous functions were employed to define the model's mathematical expressions, along with auxiliary functions such as `find`, `diff`, and `max` for general data analysis.

Despite the usefulness of Matlab and the functions employed, the developed model presents inherent limitations due to the simplifications made. The most significant limitation is the geometric idealization of the lung, which has been modeled as a spherical rubber balloon, although real morphology is far more complex and difficult to describe mathematically. Furthermore, lung tissue homogeneity was assumed, and important internal structures such as bronchi, bronchioles, and the actual gas exchange process occurring in the alveoli were omitted.

Nevertheless, these simplifications are appropriate for the goals of this project, which focuses on constructing an idealized physical model that allows for the simulation of breathing and the accurate computation of respiratory pressure, airflow, and lung volume throughout the respiratory cycle.

In summary, this chapter has described the key technologies and concepts that support the development of the model, from the physiological behavior of respiration and the selection of flow as the control variable, to the choice of Matlab as the simulation environment and the balloon-inspired physical model. This technological foundation will enable the implementation and precise analysis of the respiratory system behavior under different ventilation conditions in the following chapters.

Chapter 3. STATE OF THE ART

Before proposing a project, it is necessary to review existing approaches to mechanical ventilation and their effects on patients. It is essential to conduct a study that analyzes the weaknesses of current models and, from there, develop a more accurate and reliable alternative. In this context, the present project was designed based on the second of three Yale Technical Reports focused on fractional-order derivatives. This report describes six models, traditionally represented by linear time-invariant systems, in which the introduction of fractional differential equations enables a more precise representation of the viscoelastic behavior of biological tissues.

The use of such equation's dates to the work of Leibniz and L'Hôpital in 1695. Since then, a theoretical framework has been developed that has proven especially useful for describing systems whose behavior cannot be adequately represented using integer-order derivatives. (Shastri & Narendra, 2020)

Traditionally, the dynamics of physical and biological systems have been described using ordinary differential equations (ODEs), assuming that inertial properties dominate the system's behavior. However, in many cases, it has been shown that using non-integer order derivatives allows for a more accurate representation of complex and nonlinear dynamic behaviors (Shastri & Narendra, 2020). This is particularly evident in viscoelastic tissues, such as lung tissue or the material of a rubber balloon (Merritt & Weinhaus, 1978).

In the report by Shastri and Narendra (2020), six specific cases are presented in which models based on fractional differential equations outperform those built with traditional linear differential equations. Among these applications is the study of pulmonary mechanics, whose accurate modeling is crucial for improving the performance and safety of mechanical ventilators.

In clinical contexts, an accurate description of the viscoelastic behavior of lung function is especially important. Alterations in these properties are present in various respiratory pathologies, such as Acute Respiratory Distress Syndrome (ARDS), Chronic Obstructive Pulmonary Disease (COPD), or pulmonary fibrosis. In all these cases, a misinterpretation of the mechanical properties of the lungs can lead to improper ventilator settings, which in turn may result in complications such as Ventilator-Induced Lung Injury (VILI).

Studies such as those by Dai et al. (2015) have made significant contributions to this line of research. In their work, they compared several models describing pulmonary mechanics: the Standard Linear Solid (SLS) model, the Generalized Model (GM), and the Fractional Standard Linear Solid (FSLS) model. These models are based on simplified mechanical configurations combining elastic elements (springs) and viscous elements (dashpots) and are evaluated based on their ability to predict the actual mechanical response of lung tissue to mechanical stimuli.

Their results demonstrated that the fractional model (FSLS) provided the best fit to experimental data, significantly reducing the mean squared error compared to the other models. This improvement was observed in both the inspiratory and expiratory phases, highlighting the model's ability to capture complex dynamic phenomena such as tissue relaxation and the nonlinear relationship between pressure and volume. These findings reinforce the idea that fractional models are not merely theoretical tools but have practical value in enhancing personalized ventilatory support for patients.

In the present work, those approaches are taken as reference; however, an alternative modeling strategy based on Merritt´s model for rubber balloons is proposed (1978). Unlike the fractional-order approach, this model is based on the nonlinear pressure–volume relationship and does not explicitly use fractional differential equations but considers the viscoelastic properties of rubber balloon material.

This model draws on the experimental and theoretical work of James and Guth on stress in rubber materials, where the material is treated as a network of flexible molecular chains. This formulation allows for the mechanical behavior of deformable elastic structures, such as rubber balloons, to be described accurately, and provides a solid physical foundation for its analogy with the respiratory system.

The Merritt model establishes both a physical and mechanical analogy with lung behavior. Equivalent to the lungs, a rubber balloon represents a closed deformable system with viscoelastic properties. Unlike classical linear differential models, this approach captures phenomena such as elastic behavior in viscoelastic tissues. Moreover, it exhibits a pressure–volume curve with hysteresis, reflecting the nonlinear response of lung tissue (Merritt 1978).

Merritt deduced a nonlinear relationship between the balloon's expansion radius and the applied pressure, using principles of elastic tension and geometry. In the pulmonary context, this formulation allows for the inclusion of parameters such as tissue elasticity and airway resistance, offering a more accurate representation of the real dynamics of the system.

There are several similarities between the mechanics of a rubber balloon and pulmonary mechanics. In both cases, the expansion is nonlinear, which implies that the relationship between input pressure and volume increase is also nonlinear. A minimum pressure is required to initiate expansion: in the lung model, this pressure corresponds to the threshold needed to overcome the initial resistance of the respiratory system, typically referenced to atmospheric pressure or head-level pressure. Once this threshold is surpassed, a rapid expansion phase follows, which eventually stabilizes, requiring an almost linear increase in pressure to continue expansion. This final phase is critical, as it is where the risk of barotrauma due to tissue overdistension must be carefully avoided. (Chatburn, 2013).

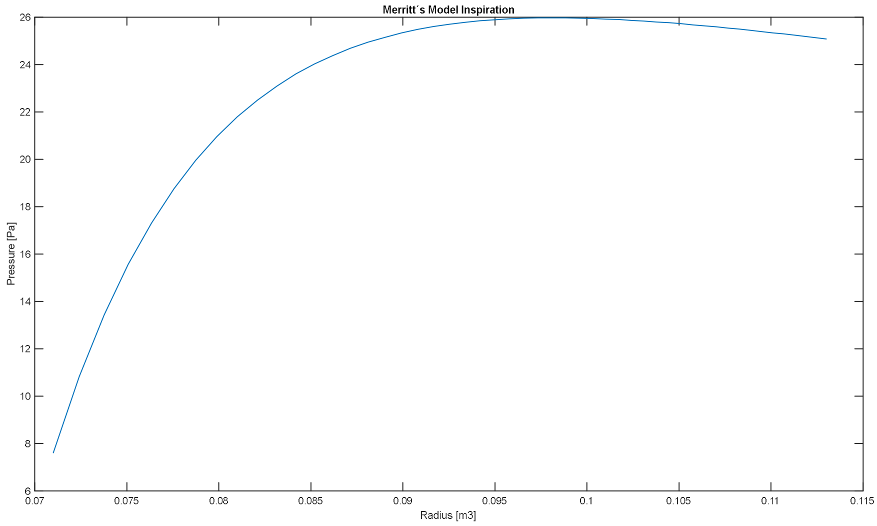

Figure 3. 1. Pressure–radius relationship during exhalation using the function defined by Merritt (Merritt & Weinhaus, 1978)

In Figure 3. 1. Pressure–radius relationship during exhalation using the function defined by Merritt (Merritt & Weinhaus, 1978) during the inspiration and exhalation phase. In this case, the inverse relationship between the internal pressure of the system and the radius is represented, as the volume decreases. These curves reflect the nonlinear relationship between decreasing volume and pulmonary pressure during inspiration and expiration.

It can be observed that as the lung radius decreases, the internal pressure of the organ also decreases, meaning that the effort required to continue exhalation is reduced, which is consistent with the final stages of expiration in pulmonary models.

To reinforce this analogy, Figure 3. 2. Representation of pulmonary volume and pressure behavior during the processes of inspiration and expiration (Chatburn 2003) includes a classical representation of the temporal behavior of pressure and volume during the processes of inspiration and expiration, expressed as a function of the number of time constants of the respiratory system. This image, extracted from Chatburn's book (2003), shows how volume and pressure decrease exponentially during passive exhalation, reaching

36.8% of the equilibrium value after one time constant. Although it is a different representation, focused on the time axis, the shape of the descending curve bears a remarkable qualitative similarity to the one obtained in the Merritt model, reinforcing the validity of using this model as a physical analogy of pulmonary behavior.

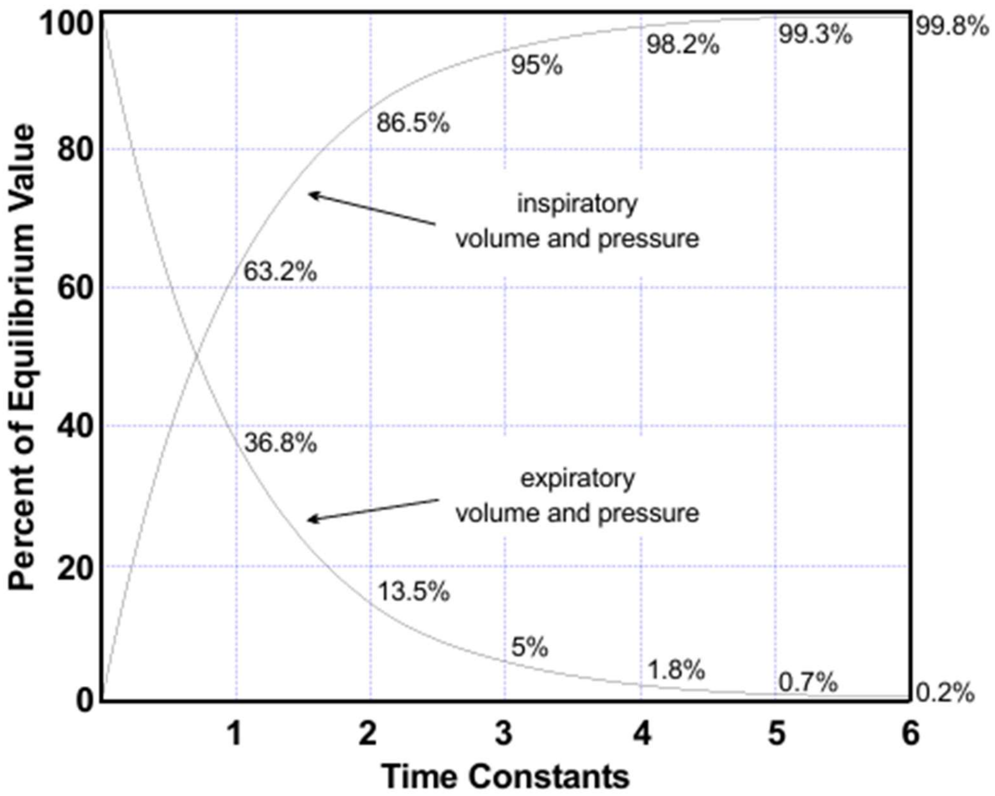

Figure 3. 2. Representation of pulmonary volume and pressure behavior during the processes of inspiration and expiration (Chatburn 2003)

Although this model does not include fractional-order differential equations, and therefore does not explicitly represent tissue memory effects, the results obtained through its application show a notable similarity to clinical data recorded during pressure–volume curve monitoring in patients undergoing mechanical ventilation.

In this work, the model is used as a base due to its simplicity, interpretative value, and its ability to qualitatively reproduce typical pressure–volume curves observed in clinical settings. Its implementation enables the development of a simulation tool that, while not directly addressing the temporal dynamics of the system, contributes to a better understanding of the mechanical behavior of the lungs under varying pressure levels, and may serve as a starting point for more complex future extensions.

Chapter 4. PROJECT DEFINITION

4.1 *JUSTIFICATION*

The analysis carried out in the previous section highlights the limitations of simplified models used to describe respiratory mechanics. The model selected and developed in this project is based in the model defined by Merritt (1978) for rubber balloons and the model for expiration defined by Chatburn (2003). The combination of both, provides an accurate approximation of experimental physiological values recorded in real patients, particularly during the expiration phase. Its formulation, based on physical principles applied to a closed deformable system, offers a precise representation of the viscoelastic phenomena present in lung tissue.

While the model relies on simplifications, it aims to deliver precise and reproducible results for normal respiratory mechanics. It is not intended for simulating complex pathological conditions, but rather as a qualitative tool to understand the key dynamics of pulmonary ventilation with more precision in comparison to other models.

4.1.1 POINT 1

The first technical issue identified in simplified models is their inability to accurately describe the viscoelastic properties that characterize lung tissue and are present during both inspiration and expiration. These properties, which define the lung's response to mechanical stimuli, prevail over the inertial properties typical of other physical systems and represent a significant challenge when attempting to correctly model the dynamic behavior of the respiratory system.

Viscoelasticity implies that the lung's response to an applied pressure is neither immediate nor linear but rather depends on the previous deformation history. This time-dependent behavior gives rise to phenomena such as hysteresis, progressive tissue relaxation, and dynamic flow resistance (Birzle & Wall, 2019). None of these effects are adequately captured by classical models based on linear, integer-order systems. For this reason, it becomes necessary to turn to alternative models with different formulations, capable of more accurately approximating the real behavior of the system.

Achieving high precision between model results and physiological behavior is particularly important in the context of the respiratory system, as inaccurate ventilator settings can lead to clinical complications, worsen the patient's condition, or even induce new pathologies such as Ventilator-Induced Lung Injury (VILI).

4.1.2 POINT 2

Another important feature developed in this model is the inclusion of airflow friction in the upper airways. It has been shown by Dai et al. (2015), two flow regimes exist: laminar flow and turbulent flow. Before introducing exact friction values into the model, hypothetical values were used to analyze the effect of different pressure profiles on the system.

In many traditional models, the resistance of the airways to airflow is either omitted or represented in an idealized manner, which limits their validity, especially in dynamic conditions such as forced inspiration or in patients with obstructive diseases.

However, it is important to incorporate air-tissue friction, as it directly affects the inlet pressure of respiratory flow into the lungs. This, in turn, impacts the effort required by the patient during each inspiration or the parameter adjustment when using a mechanical ventilator.

4.1.2.1 First Approach

In this step, the incoming flow into the respiratory system is described, considering not only the resulting pressure but also the forces acting in favor of (F_{vent}) and against (F_{musc}) respiratory flow. The function that describes this behavior is:

$$F_{vent} + F_{musc} = b \cdot \frac{dr}{dt} + k \cdot r - k \cdot r_0$$

Equation 4. 1 Friction Model (Chatburn, 2003)

The sum of both forces must overcome the resistance to volume change in the system. The friction within the system is represented by the term $b \cdot \frac{dr}{dt}$, while the term $k \cdot r - k \cdot r_0$ corresponds to the elastic component of the model.

Although this is a simplified function of the flow, it serves to qualitatively analyze the effects of friction within the system.

4.1.2.2 Laminar Flow

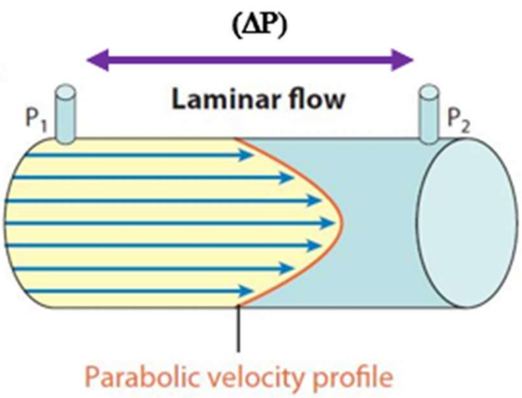

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Figure 4. 1. Representation of Laminar Flow (Bossé et al. 2010).

Laminar flow is characterized by being ordered and stratified, with parallel streamlines. In the respiratory system, this type of flow occurs predominantly in the smaller bronchi and presents low resistance to the movement of air. Its behavior is modeled using a pressure that varies linearly, as represented in the following equation:

$$\frac{dr}{dt} + k_1 \cdot r = P_i - k_2 \cdot r^2 \cdot \frac{dr}{dt}$$

Equation 4. 2 Laminar Flow Model

This model allows for the representation of parabolic velocity profiles and is associated with regions where the flow is more stable. Despite its simplicity, laminar flow is not the dominant regime in the airway entry zones and therefore has been discarded as the primary reference flow in this study.

4.1.2.3 Turbulent Flow

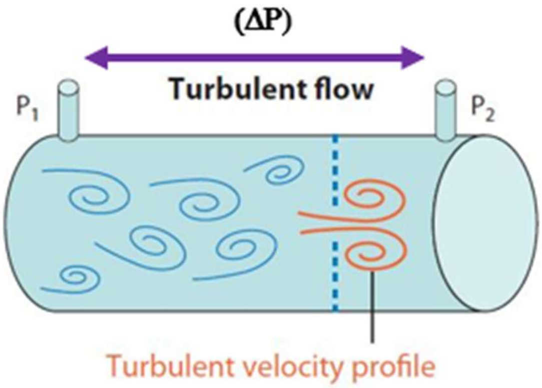

Figure 4. 2. Turbulent Flow Representation (Bosse et al., 2010)

Turbulent flow is a chaotic flow regime that presents greater resistance compared to laminar flow and is typically found in the trachea and during forced breathing. It is defined as follows:

$$\frac{dr}{dt} + k_1 \cdot r = P_i - k_2 \cdot r^4 \cdot \frac{d^2r}{d^2t}$$

Equation 4. 3 Turbulent Flow Model

As can be observed from the function, turbulent flow exhibits a nonlinear pressure variation and greater losses compared to laminar flow. The incorporation of this behavior allows the model to more realistically represent the effects of airflow under demanding clinical conditions, such as in patients with airway obstruction or during controlled mechanical ventilation.

These functions will be presented and analyzed in Chapter 5.

4.1.3 POINT 3

The most accurate approach for capturing the relationship between inspiration and expiration involves the use of fractional-order differential functions. This type of function offers approximations that are very close to experimental data, as they can capture hysteresis effects and memory-dependent behavior. These types of tissues do not depend solely on the current stimulus but also on the deformation history.

However, fractional-order models have not been used in this project, as their proper implementation would require a rigorous and complex study to precisely determine the parameters that define them.

Instead, this work adopts the model developed by Merritt (1978), which describes the nonlinear relationship between volume variation inside a rubber balloon and the pressure applied at its inlet. This analogy is valid, as the behavior of balloon material exhibits viscoelastic properties similar to those in the lung tissue, and its response to airflow closely resembles the respiratory dynamics. This choice enables a simpler, more interpretable, and practical formulation without compromising the ability to represent the fundamental aspects of the system.

Additionally, functions described in Chatburn's model (2003) have been used to simulate the variation curves of volume, pressure, and respiratory flow during expiration as a function of time. These models were selected due to their nonlinear nature and their explicit incorporation of resistivity and compliance, which are intrinsic to lung tissue. This allows the analysis to be enhanced and complemented by integrating a physiological perspective into the proposed modeling approach.

4.1.4 POINT 4

Finally, the last aspect to be analyzed is the separation of the model into inspiration and expiration phases. This division was deemed necessary, as each respiratory phase involves different mechanical behavior and therefore requires a specific analysis.

In the inspiratory phase, two models have been developed. The first assumes a constant flow function as the inlet, while the second assumes a constant pressure function. The aim is to simulate the operation of ventilators controlled either by pressure or by volume. Once the inlet function was selected, it was necessary to evaluate the type of flow occurring in the airways, specifically in the trachea. For this reason, a simulation was conducted using constant turbulent flow values and laminar flow values applied to Merritt´s model function. After applying both configurations, it is observed that the turbulent flow model yields a response that closely matches the expected theoretical, nonlinear pattern.

The models used for this evaluation are presented in Equation 4. 2 Laminar Flow Model and Equation 4. 3 Turbulent Flow Model both defined by Bosse et al. (2010). The results of these simulations indicate that the flow at the entrance of the airways is turbulent.

Subsequently, the function developed by Merritt (1978) was used to determine the variation of pressure and volume over time. In the case of inspiration, both the inlet function and the flow application time were predefined.The results obtained will be presented and analyzed in Chapter 5.

On the other hand, expiration is addressed from a different perspective, modeled as a passive process. The aim is to validate the values previously defined during the inspiratory phase. The simulation combines the pressure model proposed by Merritt (1978) and the flow and volume models described by Chatburn (2003).

Based on these models and the predefined maximum and minimum pressure values, the variation in respiratory pressure as a function of lung volume is recalculated by simulating a reduction in lung radius during the air-outflow phase. Once the pressure–volume relationship for expiration is established, the results are used as inputs for Chatburn's flow and volume equations.

The objective of this analysis is to obtain the corresponding volume, pressure, and flow profiles during expiration and to verify that the model's behavior remains consistent with the initial physiological conditions.

4.2 OBJECTIVES

This project has focused on the development of a mathematical model capable of accurately representing pulmonary mechanics throughout the respiratory cycle. To this end, several processes have been implemented and various methodological approaches applied, with the objective of achieving a representation that closely reflects real physiological behavior.

The primary issue addressed in this work is the need to overcome the limitations of traditional methods, which often lack the capacity to accurately represent pulmonary dynamics, particularly in complex clinical scenarios. This lack of precision can result in incorrect ventilator settings, potentially worsening the patient's condition, especially in critical contexts such as patients with low pulmonary compliance or high airway resistance.

The main improvement introduced by the model developed in this project is the explicit incorporation of key physiological properties, such as elasticity, hysteresis, and tissue resistivity. Particular attention has been given to the viscoelastic characteristics of the lung, which are reflected in phenomena such as deformation memory and the nonlinear response of lung tissue to pressure and volume changes.

The proposed system has been designed with adaptive control logic, so that, based on a constant flow input function, it can adjust the pressure or flow signal generated by a mechanical ventilator. Thanks to this logic, the model enables the simulation of both normal and pathological conditions, allowing its application across various patient profiles.

Initially, the project considered the use of fractional-order differential equations, given their ability to represent systems with memory and complex dynamics. However, due to the computational complexity involved in solving these equations, a hybrid approach was adopted, combining two complementary strategies:

On the one hand, the model described by Merritt (1978) was used, which establishes an analogy between the behavior of the lungs and that of an elastic balloon, leveraging the parallels between the viscoelastic properties of lung tissue and those of rubber balloon material. This model has proven useful for representing both the inspiratory phase, by analyzing the relationship between pressure and volume within the lungs, and passive expiration, in which the ventilator is not actively involved.

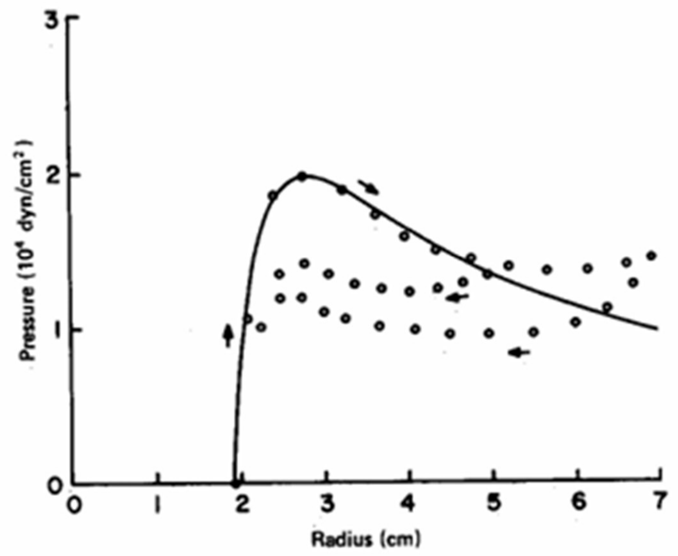

Figure 4. 3 Pressure-Radius Curves for Rubber Balloons defined by Merrit

Figure 4. 3 Pressure-Radius Curves for Rubber Balloons defined by Merrit shows the pressure–radius curve of a rubber balloon obtained from the equation proposed by Merritt (1978). Although no quantitative analysis was performed against experimental data, a qualitative similarity can be observed with Figure 3. 1. Pressure–radius relationship during exhalation using the function defined by Merritt (Merritt & Weinhaus, 1978).

In both graphs, a characteristic nonlinear behavior is evident, in which pressure reaches a maximum value at a certain radius (inspiration) before decreasing (expiration), reflecting the presence of pulmonary hysteresis and the viscoelastic response that depends on the tissue's deformation state. This similarity in the general shape of the curves reinforces the analogy between the behavior of an elastic balloon and lung tissue, as proposed by Merritt, and suggests that the model used can capture the overall behavior of the respiratory system during passive expiration with reasonable accuracy.

In addition, to complement Merritt's model during the expiration phase, a system based on the approach proposed by Chatburn has been implemented. This system allows for management of the output pressure or flow signal, yielding accurate values comparable to experimental data, and enabling the adjustment of a mechanical ventilator to the model.

Finally, the developed model has been designed to be scalable and adaptable to different physiological conditions, making it suitable for integration into clinical simulators or educational and research support tools.

Moreover, several specific objectives have been defined for the model, including the selection of the development environment, which in this case is Matlab. Matlab was chosen due to its powerful mathematical capabilities, as well as its availability of specialized libraries oriented towards solving dynamic systems and differential equations, making it an ideal platform for the mathematical modeling of complex physiological systems.

Another core objective was the analysis and detailed representation of the two fundamental phases of the respiratory cycle: inspiration and expiration. Each phase exhibits distinct mechanical characteristics and therefore requires different modeling approaches. Inspiration, being actively controlled by a mechanical ventilator in assisted patients, was modeled as an active dynamic system in which either pressure or flow is regulated. Expiration, on the other hand, was treated as a passive phenomenon, governed by the viscoelastic properties of lung tissue.

Furthermore, it was necessary to adapt existing models to the context of respiratory mechanics, such as Merritt's function for rubber balloons, whose formulation was adjusted to represent the behavior of the pulmonary system during the expiratory phase.

4.3 METHODOLOGY

This project has progressed iteratively, adapting to the results and conclusions obtained at each stage. The methodology followed a structure based on four main phases: analysis, design, implementation, and validation, with the aim of verifying that the methods and functions used were appropriate to achieve the expected outcomes. This process included a necessary adjustment of the project's approach upon recognizing the complexity involved in solving fractional-order differential equations.

As mentioned before, the project is divided into two parts: the inspiration phase and the expiration phase. The work began with the study of the inspiratory phase, aiming to examine the response of more simplified models used to describe pulmonary mechanics. In this analysis phase, the goal was to explore the outcome of different arbitrarily selected functions and evaluate the response of Matlab´s built-in `ode45` solver to these inputs.

As a starting point, a simplified equation of pulmonary mechanics was used, in which airway pressure (P(t)) is modeled as the sum of two components: a resistive term, proportional to airflow Q(t), and an elastic term, proportional to lung volume V(t). This formulation is known as the equation of motion of the respiratory system(Chatburn, 2003). Originally, this equation arises from a force balance principle, representing a direct application of Newton's Third Law: "For every action, there is an equal and opposite reaction".

This equation is associated with a simplified physical model of the respiratory system. In its most common form, the model represents the system as a rigid tube conducting airflow, connected to an elastic compartment (Papamoschou, D. (1995)). The rigid tube symbolizes the airways and is related to resistance (R) to gas flow. The elastic compartment represents the combined behavior of the lungs and chest wall, whose capacity to expand is characterized by elastance (E) or, equivalently, by its inverse, compliance (C) (Chatburn, 2003).

This simplified approach typically neglects the inertance of both the gas and the tissues, as under normal physiological conditions and typical breathing frequencies, this component is negligible. Therefore, the model relies solely on the relationship between pressure, flow, and volume, enabling a practical and sufficiently accurate description of respiratory dynamics for simulating pulmonary behavior under standard clinical conditions.

$$P(t) = R \cdot Q(t) + E \cdot V(t)$$

Equation 4. 4 Equation of Motion for the Respiratory System

Subsequently, friction components were introduced into the model to observe how the system would respond under increased complexity. In this case, the equation of motion for the respiratory system is expressed in terms of pressure, rather than force.

This adjustment was made by dividing the input pressure into two forces: the force generated by the ventilator (positive inlet pressure) and the pressure generated by the respiratory muscles. The resulting model is equivalent to the one expressed in Equation 4. 1 Friction Model (Chatburn, 2003), but adapted to pressure units.

The remaining components of the equation correspond to respiratory flow ($\frac{dr}{dt}$), volume $(r - r_0)$, where r represents the instantaneous radius of the airways and r_0 their resting radius; resistivity (R) and elasticity (E).

This initial model does not yet include lung tissue, but rather focuses on the upper airways.

$$P_{vent} + P_{musc} = R \cdot \frac{dr}{dt} + E \cdot (r - r_0)$$

Equation 4. 5 Equation for Respiratory System under Friction Conditions.

This expression represents the mathematical model commonly used to describe the interaction between the patient and the ventilator (Chatburn, 2003). Its formulation helps to understand how ventilators regulate the variables of pressure, flow, and volume, and how these variables relate to the mechanical properties of the respiratory system, particularly resistance and compliance. In this way, it establishes the foundation for the development of pressure-controlled, volume-controlled, or flow-controlled mechanical ventilators, depending on the dominant variable in each clinical case (Chatburn, 2013).

Once it was verified that the system's theoretical response was consistent under input functions such as pulse or exponential, the model was refined by distinguishing between laminar and turbulent flow, in order to adjust the system's behavior to more closely match experimentally observed data.

$$P_2 - P_1 = \frac{8 \cdot l \cdot \mu \cdot V'}{\pi \cdot r^4}$$

Equation 4. 6 Equation for Respiratory System with Laminar Flow (Bosse et al., 2010)

$$P_2 - P_1 = \frac{8 \cdot l \cdot \rho \cdot V^{2'}}{\pi \cdot r^4}$$

Equation 4. 7 Equation for Respiratory System with Turbulent Flow (Bosse et al., 2010)

The flow expression under laminar conditions is based on Poiseuille's Law, a fundamental principle in fluid dynamics. This law forms the basis of the physical model underlying the definition of resistance (R) in the equation of motion of the respiratory system. Through this formulation, it becomes evident how the geometry of the airways and the viscosity of the gas directly influence the pressure required to generate flow.

On the other hand, when turbulent flow is considered, the expression used is also derived from Poiseuille's Law but adapted to represent the nonlinearity of the system under high-velocity or high-resistance conditions, typical of phases such as forced expiration.

These expressions allow us to observe how pressure losses and resistance vary depending on the type of flow and the degree of airway expansion. In both cases, k_1 and k_2 are adjustable constants associated with the mechanical properties of the system, and P_i represents the inlet pressure applied by the ventilator.

During this initial analysis phase, arbitrary functions were used to represent the input pressure (step, pulse, exponential…), with the goal of observing the model's response and qualitatively adjusting the shape of the pressure curve. As the shape of the input signal was refined, the system was shown to produce results consistent with expected behavior. It is important to note that, at this early stage, physiological variables such as the time vector, elasticity and resistivity constants, and pressure amplitude were selected arbitrarily, with no intention of reproducing specific clinical cases.

To complete the inspiration phase, real physiological input values were introduced into the model, such as tidal volume, peak inspiratory pressure, and characteristic resistances of the respiratory system. This allowed the model's behavior under these conditions to be assessed for consistency with expected physiological responses. The results obtained from this simulation are presented and analyzed in Chapter 5.

Subsequently, the expiration phase was addressed. The main objective of this section is to calculate the time vector corresponding to the duration of expiration based on the initial physiological parameters introduced into the model. In both inspiration and expiration, the function proposed by Merritt (1978) was used, which describes the pressure–volume relationship in a rubber balloon, establishing an analogy with the viscoelastic behavior of lung tissue.

From this relationship, it is possible to determine the pressure and volume vectors during expiration, which subsequently allows the calculation of compliance and resistance values for lung tissue, necessary to complete the model. The equation formulated by Merritt (1978) applied in this context is as follows. In this model, it is assumed that the lung has perfect spherical morphology, to simplify the volume and pressure calculations based on the radius:

$$P = P_0 + \frac{C}{{r_0}^2 \cdot r}[1 - (\frac{r_0}{r})^6]$$

Equation 4. 8 Pressure-Volume Model defined by Merritt (1978)

This expression describes the evolution of pressure (P) as a function of lung radius (r), using a resting radius (r_0) and an elastic constant (C) as reference parameters. The formula is inspired by the analogy between the lung and a rubber balloon, a classical model in respiratory biomechanics, where the pressure required to maintain a certain degree of expansion does not follow a linear relationship, but rather a highly nonlinear one with respect to the radius.

In this model, the first term P_0 represents an initial or baseline pressure, while the second term accounts for the elastic component of the system. The presence of the factor $(\frac{r_0}{r})^6$ implies that small changes in radius can lead to significant variations in resulting pressure, particularly when the radius approaches values near collapse. This behavior qualitatively reflects the effect of alveolar recruitment during the early stages of inspiration and overdistension during more advanced phases.

Its structure is consistent with the principles described in the works of Chatburn (2003) and Ionescu et al. (2009), where the nonlinear dependency between pressure, volume, and radius is emphasized, along with the critical influence of airway geometry on resistance and the ventilatory effort required. This formulation can be interpreted as an approximation of the nonlinear elastic component of the respiratory system, associated with the viscoelastic behavior of lung tissue.

Once the pressure and volume vectors are determined, the values of compliance (C) and resistance (R) are calculated, which allow for the application of the set of equations proposed by Chatburn (2003) to model the respiratory system behavior during expiration. These expressions enable the calculation of pressure, volume, and flow over time during this phase:

For pressure:

$$P = \frac{VT}{C} \cdot e^{\frac{-t}{RC}}$$

Equation 4. 9 Pressure Model defined by Chatburn (2003)

This formula represents the pulmonary pressure due to the elastic load of the system at the beginning of expiration. The initial value, $\frac{VT}{C}$, corresponds to the accumulated pressure after a full inspiration, and decreases exponentially over time as lung volume diminishes. This behavior reflects the gradual release of elastic energy stored in the lungs.

This expression is supported by Chatburn (2003), who describes passive expiration as an exponential discharge governed by the time constant of the respiratory system.

For volume:

$$V = C \cdot (PIP - PEEP) \cdot e^{\frac{-t}{RC}}$$

Equation 4. 10 Volume Model defined by Chatburn (2003)

This second formulation is based on the difference between inspiratory pressures $(PIP - PEEP)$ to estimate lung volume during expiration. It describes an exponentially decreasing evolution of lung volume over time.

This formula is interpreted as an approximation of volume behavior during passive expiration, under the assumption that the initial volume at the start of expiration is $V = C \cdot (PIP - PEEP)$ that is, the theoretical maximum volume reached at the end of inspiration, assuming it had been sufficiently prolonged.

For flow:

$$F = \frac{-(PIP - PEEP)}{R} \cdot e^{\frac{-t}{RC}}$$

Equation 4. 11 Flow Model Defined by Chatburn (2003)

This expression describes airflow during passive expiration. The initial term $\frac{-(PIP-PEEP)}{R}$ represents the maximum flow at the beginning of expiration, generated solely by the residual elastic pressure. As the lungs empty, the flow decreases exponentially, also governed by the time constant. The negative sign indicates that it is an expiratory flow, i.e., directed outward. This formulation is consistent with that described by Chatburn (2003) and other authors in characterizing passive flow behavior in mechanical ventilation.

These equations allow for the simulation of passive expiration behavior, under the assumption that lung tissue empties following an exponentially decreasing dynamic, governed by the elastic and resistive properties of the system. The time vector is determined by the moment the expiration model reaches the minimum lung volume. The maximum (PIP) and minimum (PEEP) pressure values are also considered.

Finally, it is important to highlight the inherent limitations of the model developed using this methodology. First, a passive expiration is assumed, meaning the possible active involvement of expiratory muscles or alveolar collapse under specific clinical conditions is not considered. Second, a geometric simplification is applied, modeling the lung as a perfect sphere, which facilitates the mathematical treatment of lung volume but does not accurately reflect the real morphology of the respiratory system.

Lastly, although the viscoelastic properties and memory effect of lung tissue are acknowledged, they have not been fully implemented using fractional-order differential equations due to the complexity involved. This aspect is left as a potential line for future development of the model.

4.4 ECONOMICAL ESTIMATION AND PLANIFICATION

Every project requires temporal and financial planning to ensure its feasibility and to enable efficient management of time and available resources.

The following section presents the time and cost planning for this project. First, the different phases carried out throughout the project are broken down, and then an estimate of the costs associated with its execution is provided.

4.4.1 TEMPORAL SCHEDULING

Phase	*Description*	*Duration*	*Period*
Investigation	Literature review on pulmonary mechanics and mechanical ventilator design; review of the state of the art.	1 month	December 2024
Study of Mathematical Models	Study of various existing mathematical models and selection of the reference model to be used; analysis of its applicability to pulmonary mechanics.	1 month	January 2025
Development and Implementation (Inspiration)	Implementation of the previously studied models in Matlab; creation of a simulation environment and data validation.	2 months	February y March 2025

Development and Implementation (Expiration)	Selection of the model for the expiration phase and creation of a simulation environment with experimental data.	1 month	April 2025
Unificación. Redacción y entrega final	Integration of results, writing of the final report, and preparation for submission.	1 month	May 2025

Tabla 4. 1 Temporal Scheduling of the Project

4.4.2 ECONOMICAL SCHEDULING

Concept	*Description*	*Estimated Cost*
Matlab´s License	Use of Matlab environment and toolboxes for mathematical simulations	0€ (university licence)
Computer Equipment	Personal computer used for programming, simulations, and report writing	600€ (partial amortization during the project)
Electricity and Internet	Estimated cost of energy consumption and internet connection during project development	50€
Dedication Time	300 hours	3000€
Bibliography and Documentation	Access to scientific articiles and books	0€ (university licence)
Total Cost		3650€

Tabla 4. 2 Economical Scheduling of the Project

The time planning has made it possible to structure the different stages of the project in an orderly manner, ensuring progressive and coherent development. On the other hand, the cost estimate, although approximate, provides a realistic view of the resources involved, both material and human. Both analyses reflect the commitment to the objectives established from the beginning of the project.

Chapter 5. MODEL DEVELOPMENT

The purpose of this project is to develop a mathematical model capable of autonomously simulating a complete pulmonary breathing cycle, based on three fundamental variables: respiratory flow, pressure, and lung volume. The system has been designed following the physiological operation of mechanical ventilators, incorporating an input function, either pressure or flow, during the inspiratory phase, and producing as output the characteristic variables of expiration: flow, volume, pressure, and expiration duration.

The operating principle is based on evaluating whether the results produced by the model match the expected theoretical values under normal physiological conditions. For this purpose, the model uses the reference ranges described in the article Parameters for Simulation of Adult Subjects During Mechanical Ventilation (Arnal et al., 2018). If consistency with these values is observed, the model is considered to accurately represent the behavior of a healthy respiratory system. In future developments, this approach could serve as the basis for indirect assessment of pulmonary health in real patients.

The model has been constructed with the goal of faithfully reproducing the physiology of the respiratory process. To achieve this, development was divided into two distinct phases: inspiration and expiration. In the inspiratory phase, a constant flow function is introduced to represent the patient's breathing pattern, and pressure and volume values are calculated over time. To determine the pressure–volume relationship during this phase, the model defined by Merritt is used, which draws an analogy between the lung's viscoelastic behavior and that of a rubber balloon.

During the expiratory phase, the system autonomously determines the duration of lung emptying. This is achieved by combining Merritt's balloon equation with the expressions proposed by Chatburn, which describe the exponential decay of pressure, volume, and flow as functions of time during passive expiration.

At the end of expiration, the output parameters, such as final lung volume, expiratory flow rate, and residual pressure, can be analyzed to determine whether the simulated respiratory behavior aligns with expected physiological patterns. This comparison enables not only the validation of the model under normal conditions but also provides a basis for the preliminary estimation of the patient's functional respiratory state, particularly in terms of lung compliance, airway resistance, and ventilatory efficiency. Such an assessment is critical for identifying potential deviations from healthy respiratory mechanics and could inform early detection of pathophysiological conditions in future applications.

It is important to highlight the methodological separation between the two phases: while inspiration is modeled as a controlled input that generates a mechanical response in the lungs, expiration is driven by the passive release of elastic energy stored in lung tissue. This distinction allows for the application of specific mathematical models to each phase, adapted to both the type of flow (laminar or turbulent) and the active or passive nature of the process.

The project integrates multiple levels of modeling, combining physical approaches, such as Merritt’s pressure–deformation model, with mathematical formulations derived from fluid mechanics, including Poiseuille’s law for laminar flow, turbulent flow equations, and Chatburn’s expiration models.

Overall, the model developed in this work represents a valuable tool for understanding pulmonary dynamics under normal physiological conditions, allowing for detailed simulation of a complete respiratory cycle using a modular and parameterized approach. Its flexible structure makes it a solid foundation for future extensions, such as the simulation of respiratory pathologies, personalized ventilator adjustment, or the development of intelligent control algorithms for mechanical ventilation assistance.

5.1 *ANALYSIS OF SIMPLIFIED MODELS*

5.1.1 EQUATION OF MOTION OF THE RESPIRATORY SYSTEM

Before implementing the complete model of pulmonary inspiration and expiration, it is essential to analyze the behavior of simplified models traditionally used to describe respiratory mechanics under standard conditions. The study of these models has enabled a better tuning of both phases, and a more accurate understanding of the results obtained during system development.

In this initial stage, the capability of Matlab to solve ordinary differential equations (ODEs) is verified, and the output resulting from the introduction of arbitrary input functions into classical models is analyzed. This verification step is critical to ensure that the mathematical foundation of the system is solid before proceeding with the full implementation of the respiratory model.

As shown in Figure 4. 3 Pressure-Radius Curves for Rubber Balloons defined by Merrit, the first model analyzed is based on a classical formulation of pulmonary mechanics, in which the airway pressure is defined as the sum of two components: one associated with airflow (related to resistance), and another associated with lung volume (related to system elasticity).

In order to analyze this equation in Matlab, a formal reformulation of the function was required. The differential expression considered in this first part is as follows:

$$\mathrm{dr} = \left(\frac{1}{b}\right) \cdot (\mathrm{f} + \mathrm{k} \cdot (r_0 - \mathrm{r}))$$

Where r is the pulmonary radius at each time instant, b represents the resistivity, equivalent to the parameter R, k is the elastic constant of the tissue, equivalent to E, f is the input force, interpreted in this case as constant flow, and r_0 is the resting radius of the pulmonary system.

This formulation allows for the simulation of the dynamics of the pulmonary radius during the inspiratory phase, integrating the combined effect of an external force (flow) and the viscoelastic behavior of lung tissue.

To evaluate the system's behavior and the stability of its numerical solution, arbitrary input functions with random amplitude and duration were introduced. The equation was solved using Matlab´s `ode45` solver for the following input functions: step, pulse, trapezoidal, and sinusoidal. The results corresponding to each of these inputs are shown below:

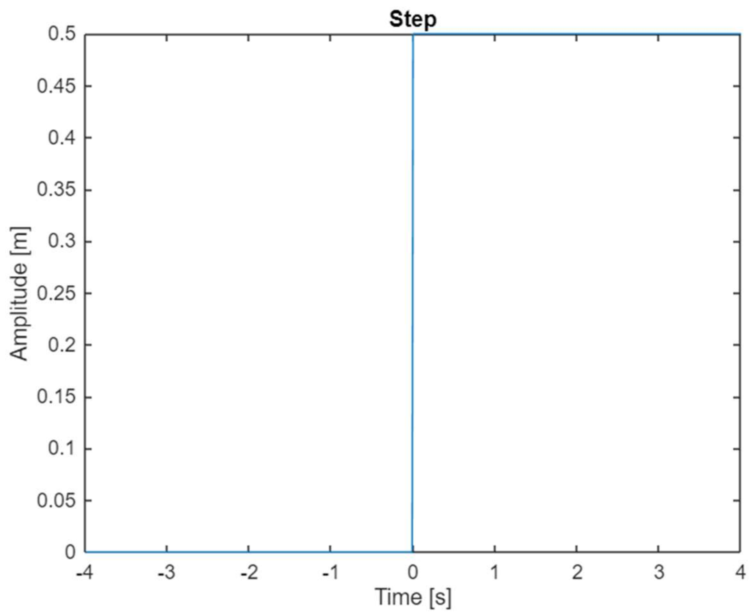

Figure 5. 1 Step Function

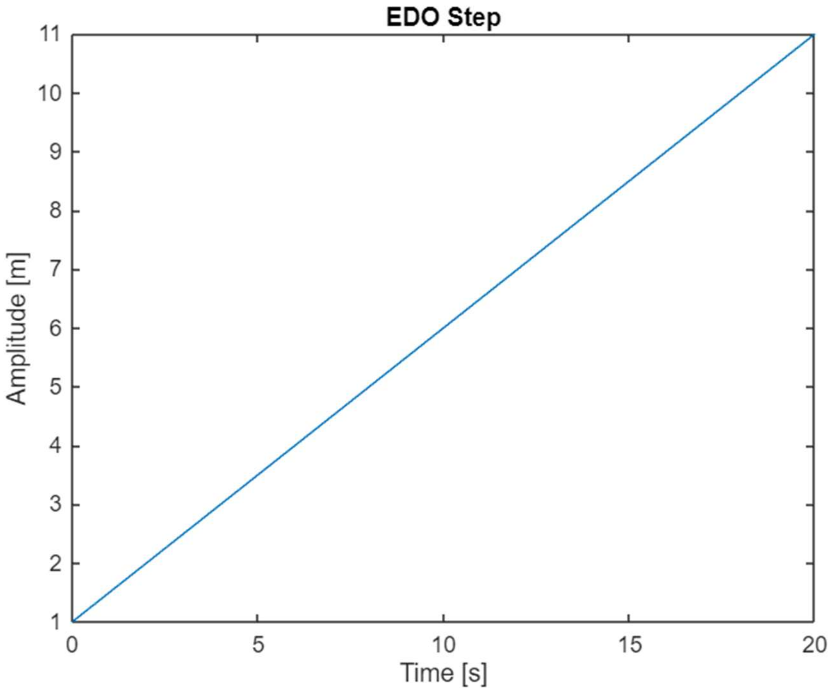

Figure 5. 2 EDO of Step Function

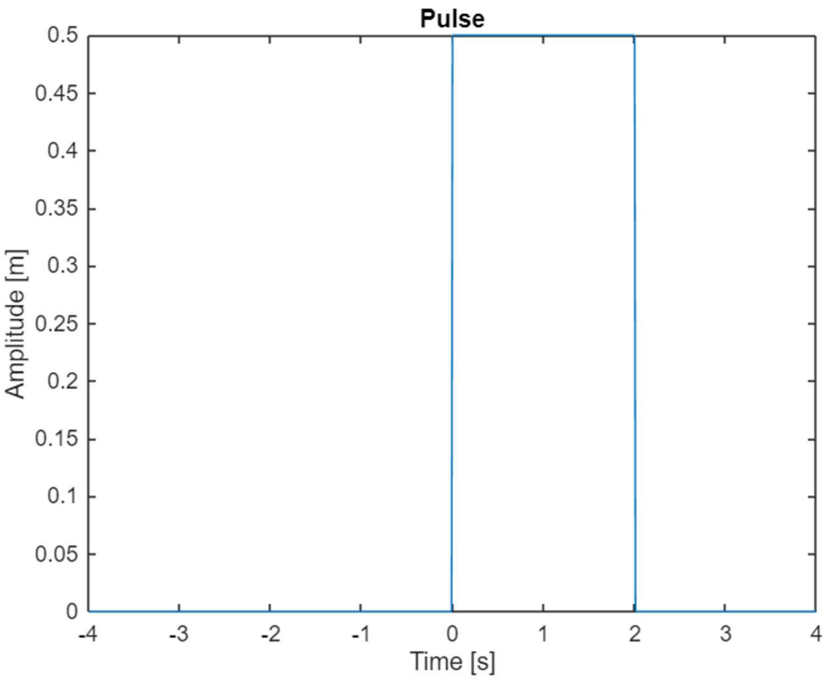

Figure 5. 3 Pulse Function

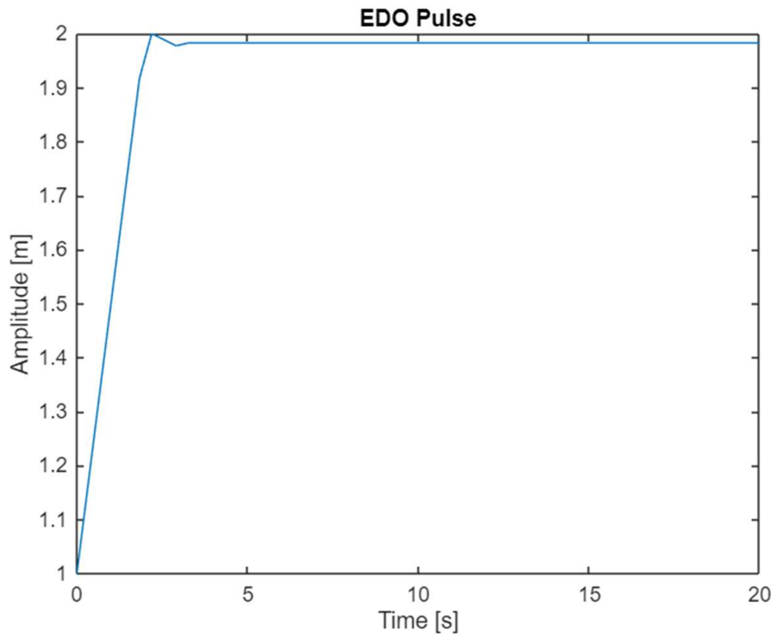

Figure 5. 4 EDO of Pulse Function

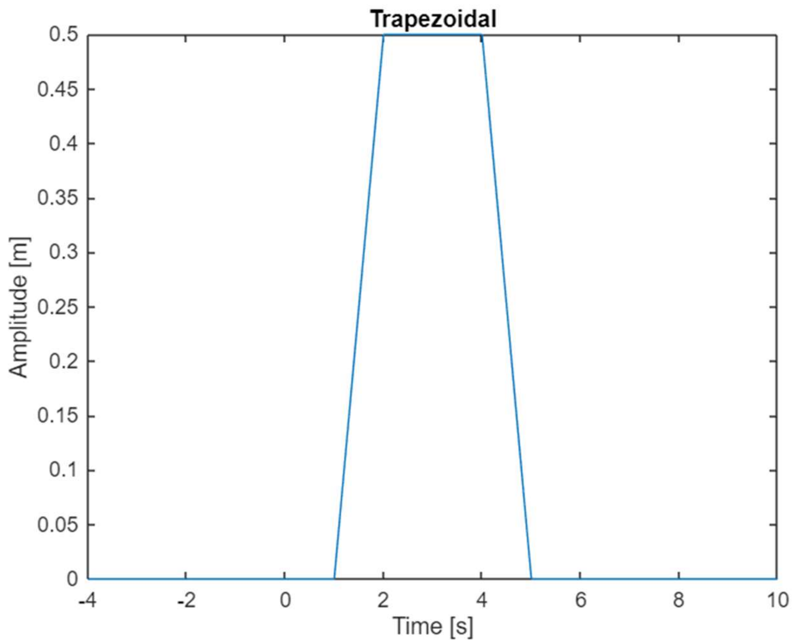

Figure 5. 5 Trapezoidal Function

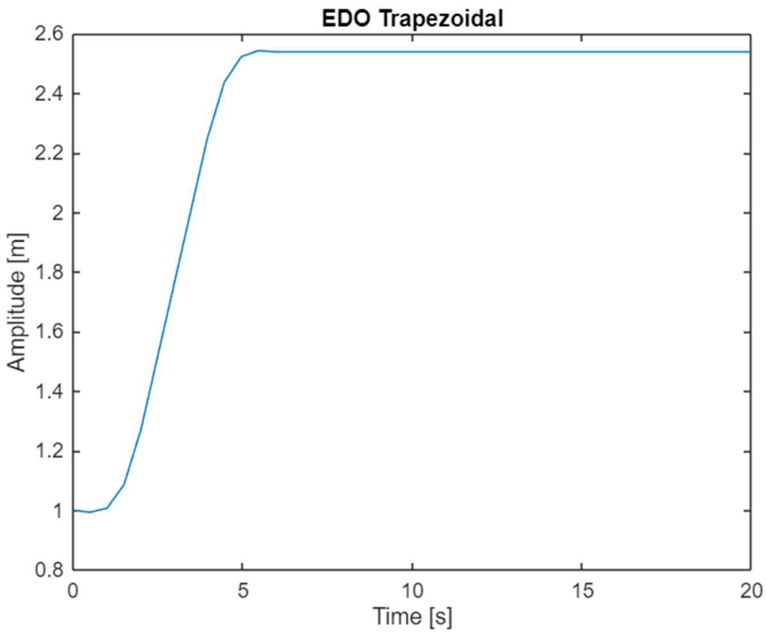

Figure 5. 6 EDO of Trapezoidal Function

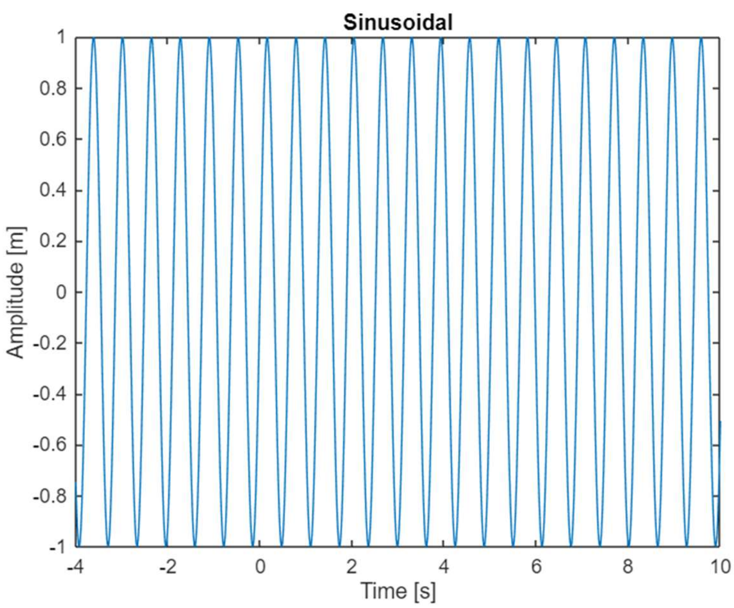

Figure 5. 7 Sinusoidal Function

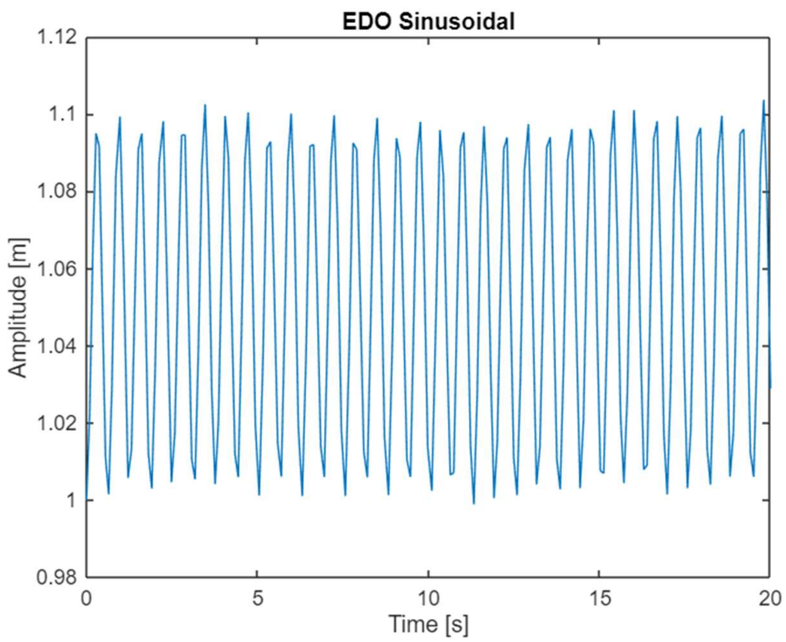

Figure 5. 8 EDO of Sinusoidal Function

In the case of the step function (Figure 5. 1 Step Function), the input consists of a constant signal from the initial time. The output response is a linear function in which the pulmonary radius increases indefinitely, without any limitation, due to the absence of saturation mechanisms in the simplified model. This behavior reflects an idealized response, lacking physiological constraints such as hysteresis.

The next input function used for the analysis is the pulse function (Figure 5. 3 Pulse Function). When applied to the simplified model, it produces a step-like output. The pulmonary radius increases rapidly until it reaches a stable value, representing a rapid expansion followed by a steady-state phase. This result exhibits a more physiological behavior compared to the response obtained for the step function.

When a trapezoidal function is used as input (Figure 5. 5 Trapezoidal Function), the result is similar to that of the pulse, but with a more gradual and realistic transition during the expansion phase. This input shape resembles more closely the real operational pattern of a mechanical ventilator, making it a good approximation for simulating more clinically realistic breathing cycles.

Finally, when applying a sinusoidal function (Figure 5. 7 Sinusoidal Function), the output response, although slightly distorted compared to the original signal, preserves the periodicity of the input. This behavior demonstrates the model's ability to track periodic inputs, although with certain limitations inherent to its structural simplicity.

5.1.2 EQUATION OF MOTION OF THE RESPIRATORY SYSTEM WITH FRICTION

After conducting the initial simulations and observing the behavior of the most simplified model, friction is introduced in the form of driving and opposing forces, as explained in Chapter 4.3.

In this phase, pulse and exponential input functions are applied to the updated model to qualitatively compare their responses with theoretical patterns, using as reference the work of Chatburn (2013), which presents idealized curves of volume, pressure, and respiratory flow during the inspiration and expiration phases generated by mechanical ventilators.

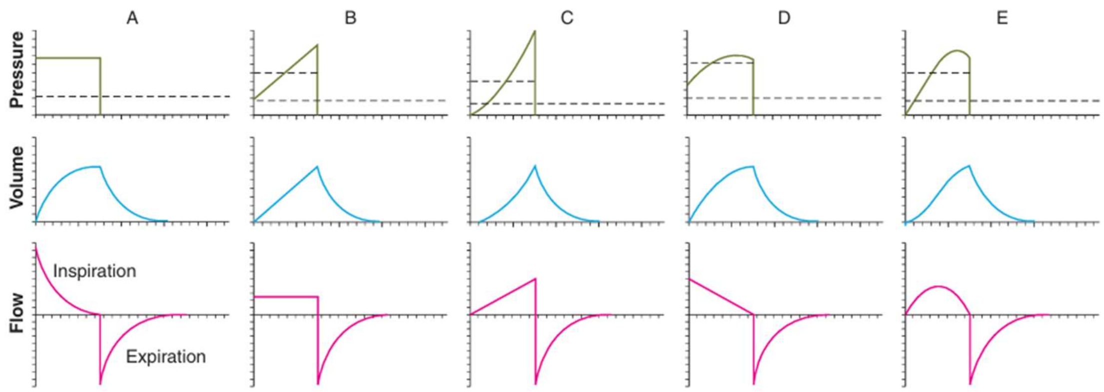

Figure 5. 9 Idealized representation of pressure, volume, and flow curves generated by a mechanical ventilator under different control conditions (pressure-controlled or volume-controlled). Adapted from Chatburn (2013)

The respiratory function introduced here is Equation 4. 1 Friction Model, which represents respiratory behavior by incorporating frictional forces. This equation has been reformulated for proper implementation in Matlab using the following function:

$$\mathrm{dr} = \left(\frac{1}{b}\right) \cdot (F_{vent} - F_{musc} + \mathrm{k} \cdot (r_0 - \mathrm{r})$$

Where F_{vent} represents the force applied by the ventilator, F_{musc} corresponds to the muscular force exerted by the patient (in the opposite direction), b is the resistance of the respiratory system, corresponding to R in the equation, and dr is the derivative of the pulmonary radius with respect to time ($\frac{dr}{dt}$), which represents how lung volume varies as a result of the interaction between external forces and tissue properties.

k is the elasticity constant, equivalent to E, r is the instantaneous pulmonary radius, and r_0 is the resting pulmonary radius.

The functions introduced into the model and their corresponding responses are as follows:

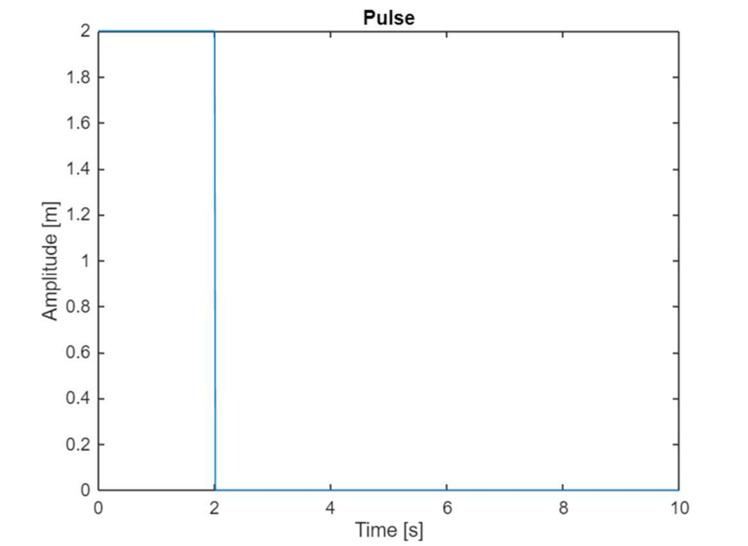

Figure 5. 10 Pulse Function

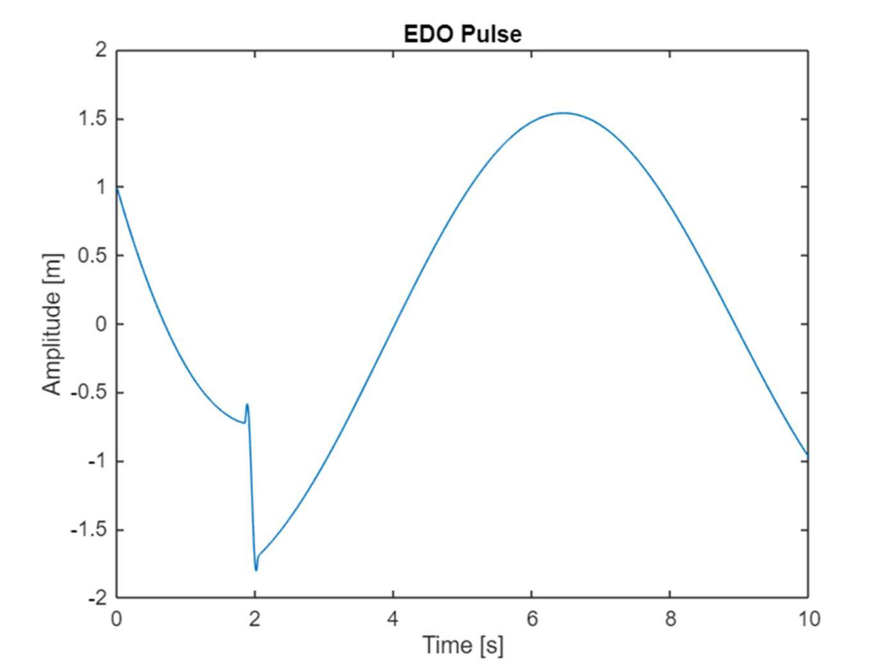

Figure 5. 11 Evolution Pulse Function

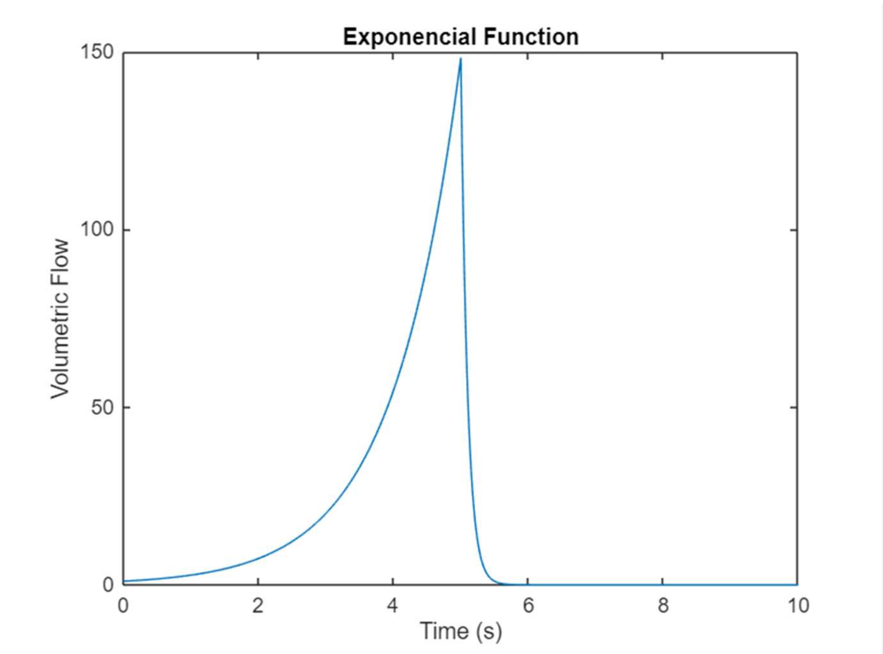

Figure 5. 12 Exponencial Function

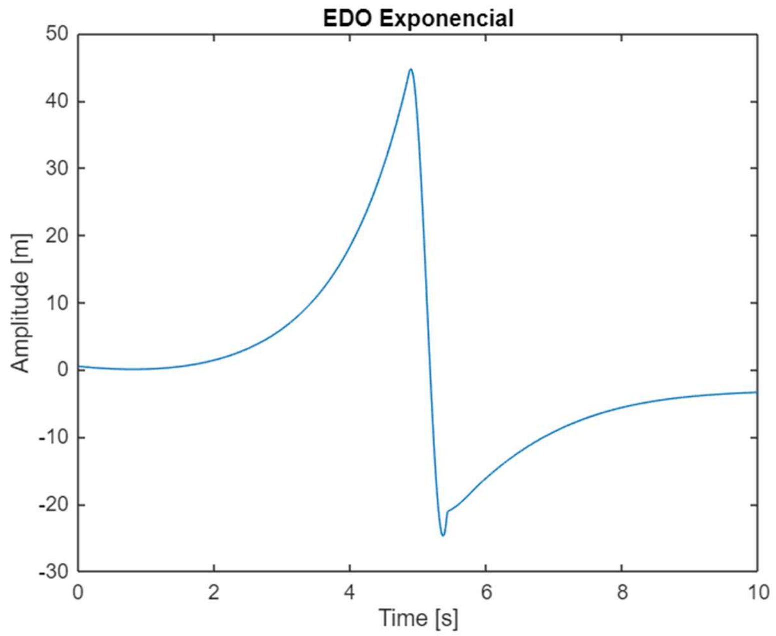

Figure 5. 13 Evolution Exponencial Function

As shown in Figure 5. 9 Idealized representation of pressure, volume, and flow curves generated by a mechanical ventilator under different control conditions (pressure-controlled or volume-controlled) the responses obtained from the input functions used exhibit a similar shape to the theoretical responses generated by mechanical ventilators, as described in the literature.

In the case of the pulse function, unlike the previous analysis with the simplified model, the output of the differential equation displays a characteristic oscillatory behavior, associated with the abrupt termination of the stimulus.

Regarding the exponential function, the system response again exhibits a less pronounced oscillation. The behavior of the output function is dynamic and nonlinear, which brings the result closer to a realistic experimental scenario. This highlights the model's sensitivity to variations in the signal input.

5.1.3 LAMINAR FLOW MODEL AND TURBULENT FLOW MODEL

The model is now analyzed by introducing laminar and turbulent flow functions to determine the airflow entering the lungs. This airflow may exhibit characteristics of either laminar or turbulent flow.

From this point on, the inspiratory model has been divided between the behavior of flow, pressure, and volume in the upper airways, and the mechanical response of lung tissue when subjected to pressure or flow.

The functions have been slightly adapted to ensure proper resolution within the Matlab environment. The resulting formulation is derived from the relationship between the pressure difference across the airways, from the external environment (e.g., mouth and nose) to the end of the trachea, and the corresponding airflow. This leads to the expression presented in Equation 4. 2 Laminar Flow Model, which represents the Laminar Flow Model.

This function computes the output pressure (y) in the airways under laminar flow conditions. The equivalent formulation for turbulent flow is given in Equation 4. 3 Turbulent Flow Modelwhich represents Turbulent Flow Model, with the following parameter correspondences:

- P_1 is the pressure at the end of the trachea,
- P_0 is the pressure at the mouth and nose, which is assumed to be zero,
- x represents the length of the trachea,
- φ is the flow friction constant, and
- r is the radius of the trachea.

At this stage of the project, values consistent with experimental data from real patients begin to be introduced. For the simulations carried out in Matlab, the following physiological and geometric parameters were used, based on standard clinical references and literature for adult patients:

- Tracheal length (x or l): 12 cm = 0.12 m

- Dynamic viscosity of air (μ): $1.8{\cdot}10^{-5}$ Pa·s
- Friction constant for turbulent flow (φ): 0.015 (approximate value estimated under physiological conditions)
- Average tracheal radius (r): variable over time, with an initial value of 10 mm = 0.01 m
- Mouth/nose pressure (P0): set to 0 Pa to represent atmospheric reference

These values are consistent with the parameters used in mechanical ventilation simulations for adult subjects and fall within the ranges specified in Parameters for Simulation of Adult Subjects During Mechanical Ventilation (Arnal et al., 2018). The peak inspiratory pressure (PIP) and the positive end-expiratory pressure (PEEP) are critical parameters in ventilator settings, as they help ensure adequate ventilation and prevent alveolar collapse.

For the inspiratory flow, a constant flow was considered during the inspiration phase, calculated based on the estimated tidal volume for the patient.

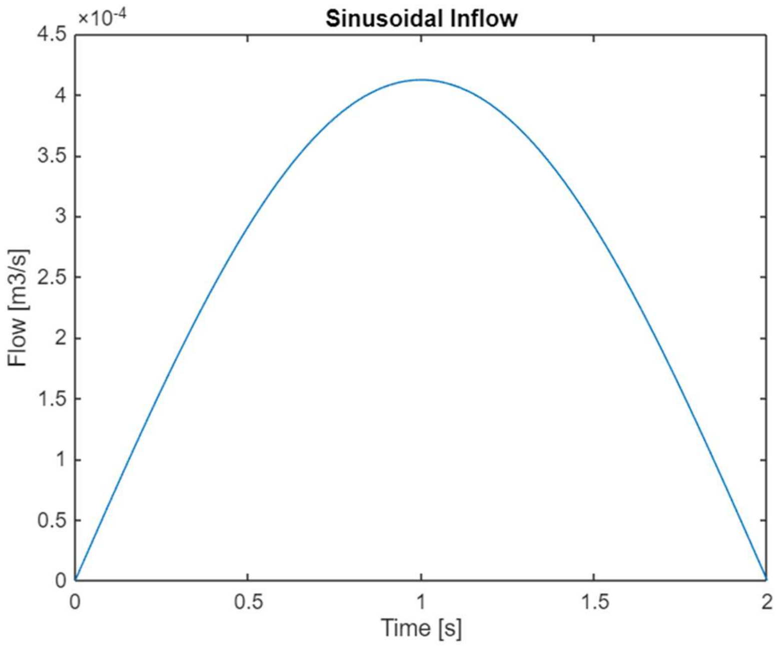

Figure 5. 14 Sinusoidal Inflow

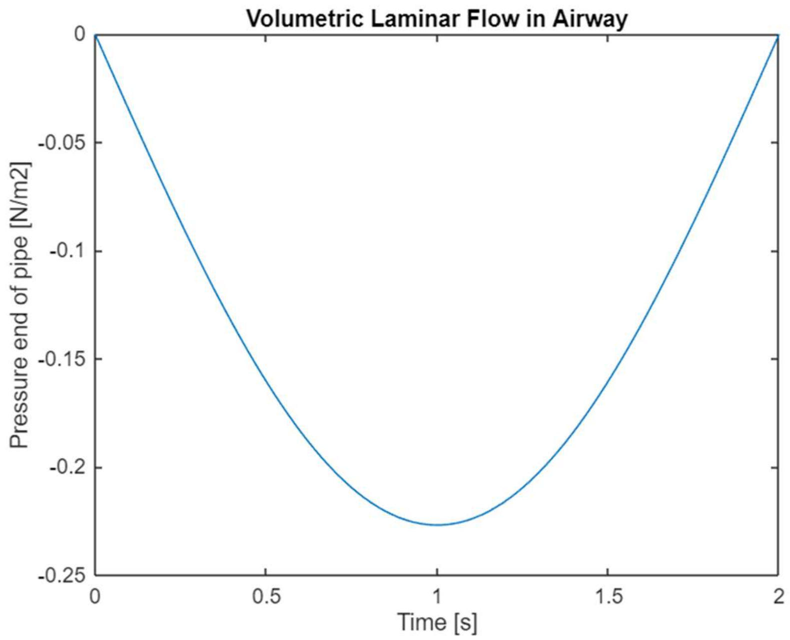

Figure 5. 15 Volumetric Laminar Flow in Airway

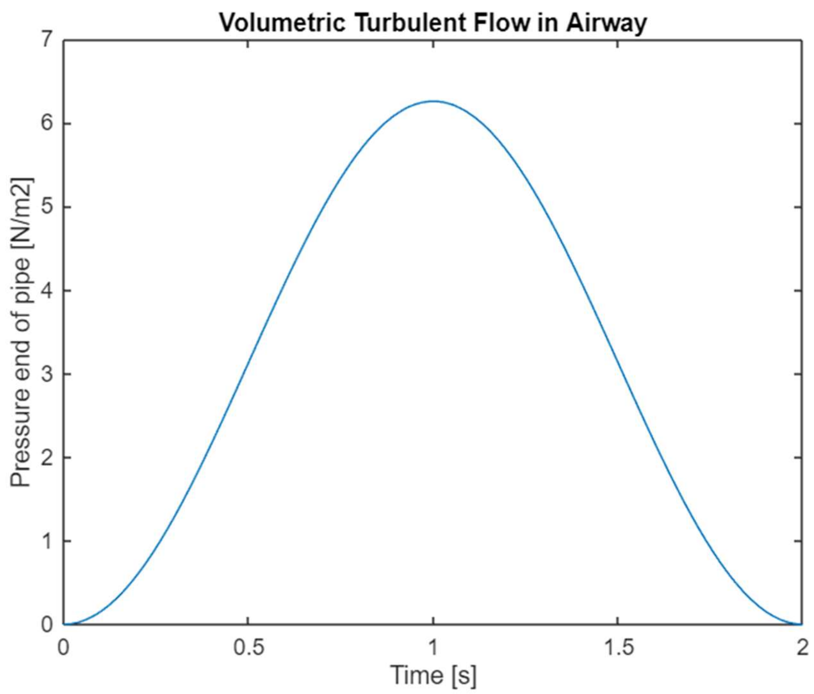

Figure 5. 16 Volumetric Turbulent Flow in Airway

With the input of the sinusoidal flow signal defined in Figure 5. 14 Sinusoidal Inflow, the qualitative results of both laminar and turbulent flow at the end of the trachea can be observed. The resulting pressure function under laminar flow conditions does not align well with the physiological reality of respiratory mechanics. This is because it produces negative pressure values, which are inconsistent with real respiratory behavior: under normal conditions, as the inspiratory flow increases, intrapulmonary pressure also rises, reaching its peak (PIP) at the end of inspiration, and then decreases during expiration.

In contrast, the pressure function derived from the turbulent flow model exhibits a behavior more consistent with the expected respiratory dynamics. Although it remains a simplified model, this function shows a pressure evolution more aligned with pulmonary physiology. Nevertheless, some limitations are identified, particularly in the minimum and maximum pressure values. For example, the model reaches pressure values close to zero, which is physiologically unfeasible, as it would imply collapse of pulmonary structures. This highlights the need to further refine the model by introducing constraints that ensure clinical realism.

To conclude this part of the analysis, the model's behavior will be studied under variations in constant inspiratory flow. For this, a maximum flow rate of 75 L/min and a minimum flow rate of 25 L/min are set. The inflow will be decreased by 5 L/min increments, down to the minimum level, and the relationship between flow variation and pressure variation will be analyzed for both laminar and turbulent flow conditions.

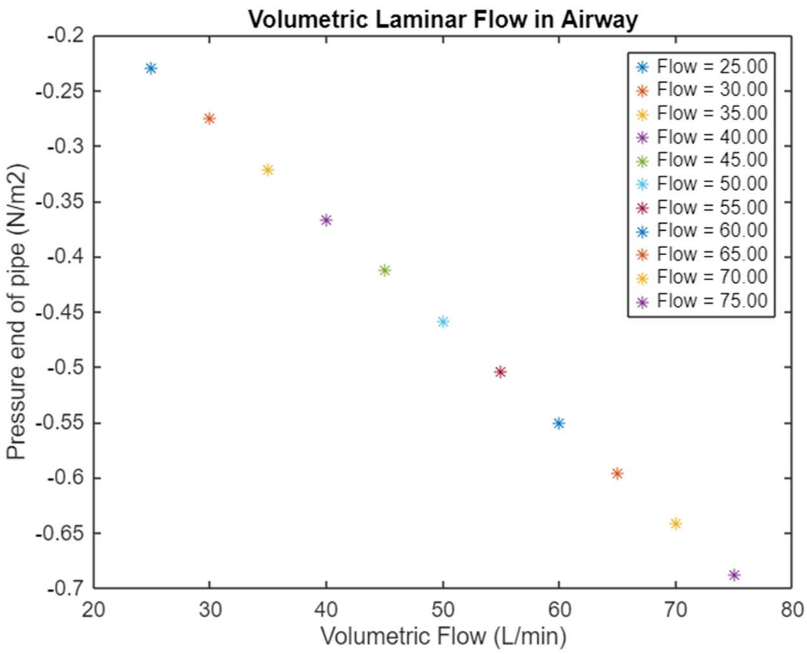

Figure 5. 17 Variation of Volumetric Laminar Inflow in Airway

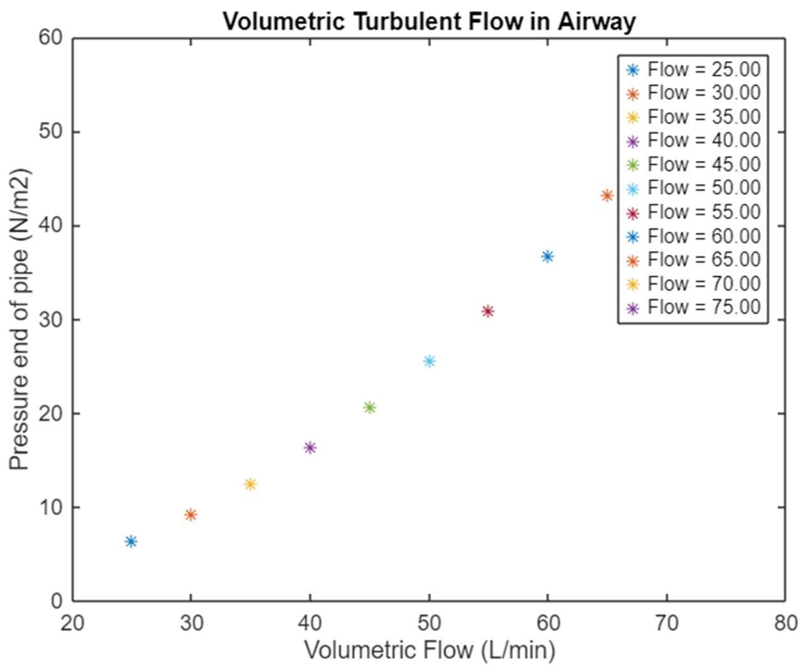

Figure 5. 18 Variation of Volumetric Turbulent Inflow in Airway

It can be observed once again that when respiratory flow decreases, the laminar flow model produces negative pressure values and an approximately linear response. This behavior is highly idealized, since, according to Merritt's pressure–volume model (Equation 4. 8 Pressure-Volume Model defined by Merritt (1978)), the relationship between pressure and volume is inherently nonlinear due to the viscoelastic properties of lung tissue.

In contrast, when modeling inflow as turbulent flow, the system's response to flow variation is significantly more consistent with physiological expectations. In this case, pressure increases in a nonlinear and more pronounced manner as flow increases, more realistically reflecting the dynamic resistance of the airways.

This initial analysis clearly highlights the limitations of simplified models, such as purely laminar flow, and underscores the importance of incorporating dynamic and nonlinear components to more accurately represent the interaction between flow, pressure, and volume in respiratory mechanics.

From this point forward, a detailed study of the two phases of the respiratory cycle, inspiration and expiration, will be conducted, with differentiated modeling to capture their respective physiological characteristics.

5.2 INSPIRATION PHASE

After confirming the limitations of simplified models and verifying the significant presence of turbulent flow in the upper airways, a more physically grounded model is introduced to more realistically represent the behavior of the respiratory system: the model proposed by Merritt. Although originally conceived to describe the pressure curve of a rubber balloon, its nonlinear formulation proves particularly useful for capturing the viscoelastic nature of lung tissue. This analogy is justified by the fact that the expansion and contraction behavior of the lungs in response to pressure stimuli exhibits a volume-dependent response, with progressive stiffness and nonlinear behavior.

The implementation of this model makes it possible to obtain results that are consistent with expected values in terms of both pressure and volume. In particular, the model adequately reproduces values such as the peak inspiratory pressure (PIP) and the minimum expiratory pressure (PEEP), as well as the radius corresponding to extreme lung volumes. The inspiratory phase, as physiologically expected, shows a progressive increase in pressure until reaching the PIP value, while expiration, assumed to be passive, shows a continuous decrease that stabilizes around the PEEP value.

To perform this simulation, three key components are considered: the turbulent inflow, the pressure formulation based on the Merritt´s model, and the temporal evolution of lung volume and flow. First, the simulation starts from the fundamental definition of respiratory flow:

$$Inflow = \frac{dV}{dt}$$

Figure 5. 19 Volumetric Flow Definition

As the lung is considered a sphere, its volume function is defined as:

$$V = \frac{4}{3} \cdot \pi \cdot r^3$$

Figure 5. 20 Volume Definition

By isolating the radius as a function of time from the integration of the flow, the following expression is obtained:

$$r = \sqrt[3]{\frac{3 \cdot inflow \cdot t}{4 \cdot \pi} + {r_0}^3}$$

Figure 5. 21 Time-Dependent Radio Variation

This final equation is essential for linking Merritt's physical model with the temporal dynamics of the system. It allows for a more accurate estimation of the duration of the inspiratory phase and provides a direct relationship between the evolution of the pulmonary radius and the inflow, thereby ensuring a more faithful representation of lung behavior during mechanical ventilation.

To simulate the inspiratory phase, a constant flow has been defined as the input parameter, since the model behaves as if it were driven by a mechanical ventilator. The flow is modeled as turbulent within the upper airways. From the flow at the end of the trachea, the temporal evolution of the pulmonary radius and the intrapulmonary pressure are calculated using the nonlinear viscoelastic model.

The input parameters used are realistic values typically associated with adult patients. In addition to the physiological values already employed in previous traditional model simulations, the following parameters are included:

- Peak Inspiratory Pressure (PIP): 25 cmH_2O
- Positive End-Expiratory Pressure (PEEP): 7.5 cmH_2O
- Volume at PIP: 80 mL/kg
- Volume at PEEP: 20 mL/kg
- Patient weight: 75 kg
- Inspiration time: 2 seconds
- Inflow: Tidal volume = 4.5 mL

The PIP value used in this model has been set at 25 cmH_2O, which lies within the limits established for lung-protective ventilation (PIP < 30 cmH_2O) (Fernández et al., 2023). In the study by Arnal et al. (2013), plateau pressures (Pplat) observed in patients were slightly lower, but it is important to note that those values correspond to protective ventilation strategies. Although the selected PIP may be slightly elevated compared to values during spontaneous breathing, it likely does not exceed safety thresholds associated with ventilator-induced lung injury (VILI).

On the other hand, the selected PEEP value of 7.5 cmH_2O falls within the interquartile range reported by Arnal et al. (median 8 cmH_2O; IQR: 5–12 cmH_2O). This value represents a baseline level of positive pressure that prevents alveolar collapse at the end of expiration, consistent with both the physiology of the respiratory system and the goal of maintaining residual positive pressure.

The tidal volume used in the model is 4.5 mL, calculated based on the maximum (80 mL/kg) and minimum (20 mL/kg) lung volumes and considering a patient weight of 75 kg. This tidal volume aligns with typical mechanical ventilation settings, which aim to maintain lower-than-normal tidal volumes to reduce the risk of lung injury. According to Arnal et al. and Chatburn, commonly accepted ranges are 6–8 mL/kg.

The inspiration time has been set at 1 seconds, ensuring that the target volume is reached even under low-flow conditions. The time vector for both inspiration and expiration is based on the relationship between inspiratory/expiratory time and the respiratory system time constant, as defined by Chatburn:

$$\tau = R \cdot C$$

Equation 5. 1 Time constant

Where R represents the airway resistance and C the pulmonary compliance. This time constant determines the rate at which the lungs fill or empty in response to a pressure change. In controlled mechanical ventilation, it is considered that the inspiratory time should be at least equal to one full time constant to ensure adequate volume transfer, and preferably up to two time constants to achieve near-complete filling (Chatburn, 2003).

In the study by Arnal et al. (2018), values for the expiratory time constant are reported to vary depending on the patient's pathology. In this model, the adopted inspiratory time covers multiple time constants, ensuring near-complete filling of the modeled lung system.

The selection of a constant inflow represents a simplification of the mechanical ventilation process, in which a predetermined volume of air is delivered to the lungs over a fixed inspiratory time. Since the tidal volume represents the total amount of air that must enter the lungs during one inspiratory cycle, assuming constant flow with magnitude equal to the tidal volume implies that this volume is uniformly distributed over the inspiratory time.

The model is developed to limit the inspiratory time, based on maximum pressure and volume. Once those values are reached, the inspiratory time has been successfully calculated. The actual inspiratory time estimated by the model is stored in the variable `T_vector_insp`.

As shown in Equation 4. 8 Pressure-Volume Model defined by Merritt (1978), it is necessary to determine the value of constant C, which represents the pulmonary compliance. This constant is calculated based on the previously defined data, where `P` is the PIP, `P0` is the PEEP, `r` is the maximum radius, and `r0` is the minimum radius:

$$C = \frac{(P - P_0) \cdot {r_0}^2 \cdot r}{1 - (\frac{r_0}{r})^6}$$

Since the actual duration of the inspiratory phase depends on the initial and final values of pressure and volume, this time differs between inspiration and expiration. The following section presents the development of the model corresponding to the inspiratory phase.

A `while` loop is used for this purpose, as the total simulation time is set beyond the typical inspiratory duration. The objective is to allow the model to autonomously determine the real duration of inspiration based on the evolution of the pulmonary radius r(t), stopping the simulation once the maximum theoretical radius `rmax` is reached.

The operation of the model begins with the calculation of the inflow at the entrance of the lungs, which is determined using the turbulent flow function described in Equation 4. 3 Turbulent Flow Model. For proper implementation in Matlab, a formal reformulation of the original expression was required.

$$P_1 = P_0 + \frac{8 \cdot \varphi \cdot f^2}{\pi \cdot r^4}$$

Next, based on the definition of flow, the variation of the pulmonary radius is calculated from the time vector. This serves as a reference within the model to subsequently apply the Merritt model, which relates pressure and lung volume. The calculation is stopped when the radius reaches the theoretical maximum pulmonary radius to prevent overdistension or collapse.

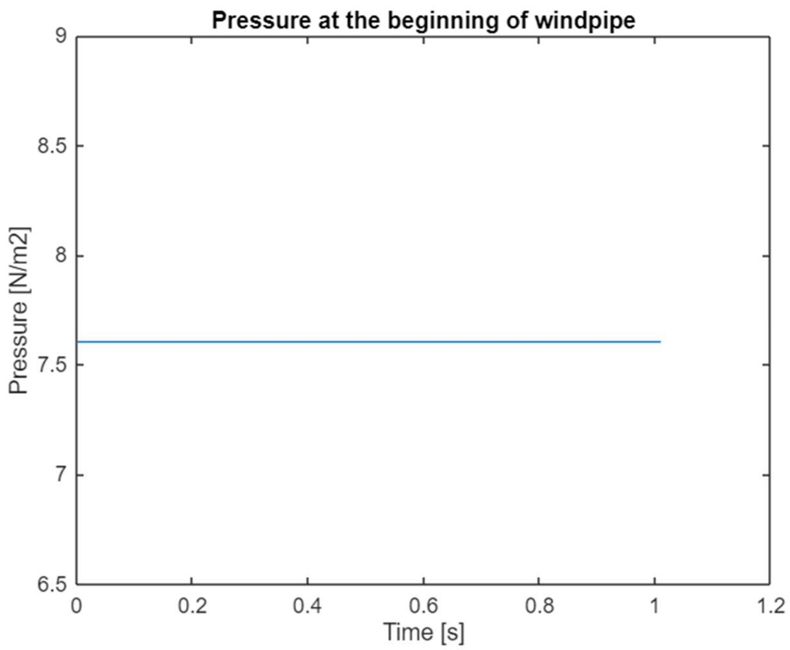

Figure 5. 22 Pressure at the beginning of windpipe

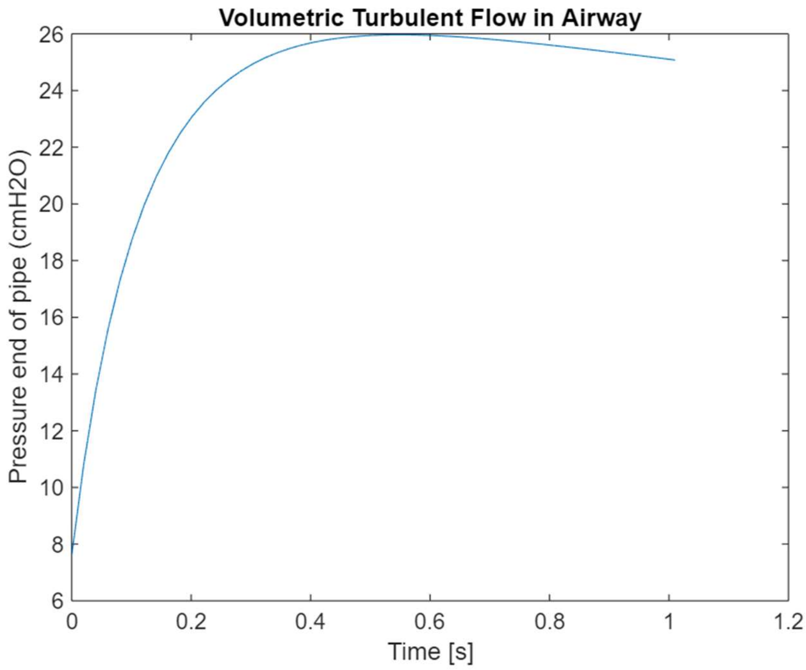

Figure 5. 23 Volumetric Turbulent Flow in Airway Merritt´s Model

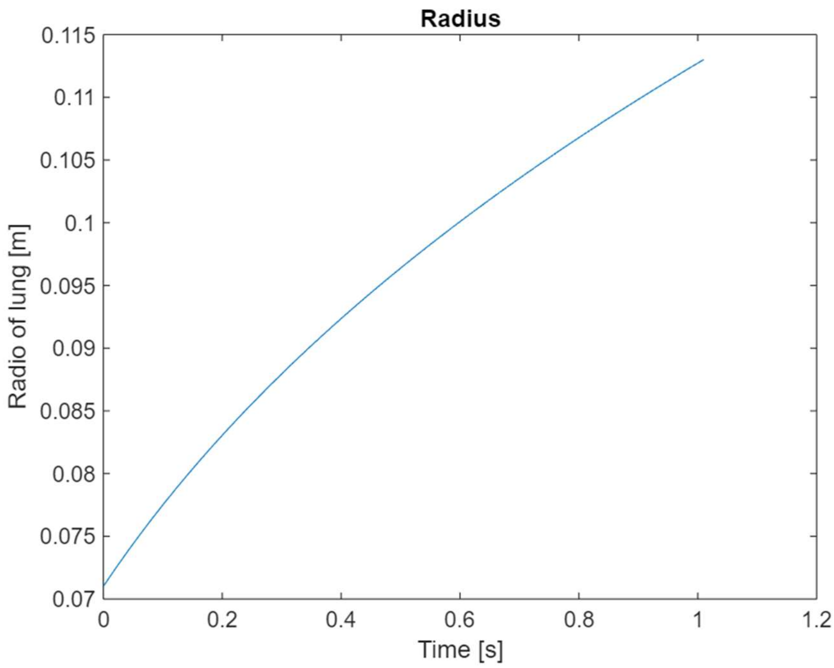

Figure 5. 24 Radius Expansion of the Lungs during Inhalation

As shown in Figure 5. 24 Radius Expansion of the Lungs during Inhalation, the resulting inspiratory time is approximately 1.04 seconds, a value that falls within the physiological range for adults, and which is strongly related to the time constant (Chatburn, 2003; Arnal et al., 2018).

The curve displays a progressive increase in pulmonary radius from its initial value (corresponding to PEEP) to the maximum value associated with the tidal volume. This behavior confirms that the model is capable of coherently estimating effective inspiratory duration based on pressure and volume variation.

In addition, when applying the Merritt model (1978) to calculate the internal lung pressure, it is observed that pressure reaches its peak value (PIP) around the midpoint of the inspiratory phase, slightly exceeding the target pressure defined in the model, before gently decreasing toward a final value of 25 cmH_2O, which corresponds to the upper pressure limit set in the simulation.

This variation may be attributed to numerical approximations and to the fact that pressure is computed using nonlinear functions dependent on pulmonary radius and inspiratory flow. These results validate the use of the turbulent flow model in combination with Merritt's model, reinforcing its ability to realistically simulate the mechanics of the inspiratory phase.

On the other hand, Figure 5. 22 Pressure at the beginning of windpipe shows the evolution of pressure at the end of the trachea and beginning of the lungs. The pressure remains constant, as the inflow to the system is also constant. Since there is no variation in tracheal radius or inflow, the constant pressure output is consistent. This pressure serves as an input to the internal lung pressure model, and its constancy allows a more isolated analysis of the nonlinear tissue behavior represented by the Merritt model.

Finally, it is worth noting that while lung volume varies cubically with radius, the volume progression during inspiration approximates a nearly linear behavior due to the constant inflow, which adds realism to the model without compromising computational simplicity.

5.3 EXPIRATION PHASE

This final part of the model has involved a higher level of complexity compared to the previous phases, as it requires relating several nonlinear functions and establishing appropriate stopping conditions that respect pulmonary physiology. A passive approach is adopted, inspired by the operation of a mechanical ventilator. Expiration is simulated as a process that begins after the cessation of inspiratory flow and progresses spontaneously until the lung volume reaches its minimum value (corresponding to positive end-expiratory pressure, PEEP). To achieve this, a time vector is defined to represent the duration of expiration, which stops when the minimum volume is reached, thereby ensuring that lung collapse is avoided.

The decision to assume passive expiration has a solid physiological basis, as under normal resting conditions, expiration in humans does not require active muscular effort. During this phase, the diaphragm and inspiratory muscles relax, eliminating the negative pressure that drove air into the lungs. As a result, the elastic recoil of lung tissue and the pressure exerted by the thoracic cavity facilitate the natural outflow of air. This phenomenon is closely linked to the viscoelastic properties of the respiratory system, which determines how lung volume decreases when pressure is released.

From a mathematical standpoint, assuming passive expiration allows this phase to be represented using exponentially decreasing functions, which not only aligns with actual physiology but also significantly reduces the complexity of the system of equations. Rather than requiring active modeling of muscle control, it is sufficient to know the initial pressure and volume values at the end of inspiration and to apply a draining function that depends solely on the time constant of the system ($\tau=RC$). This simplification facilitates computational simulation and enables the analysis to focus on the controlled evolution of volume, flow, and pulmonary pressure.

5.3.1 MERRIT´S MODEL

To obtain the evolution of pulmonary pressure during the expiratory phase, the Merritt model, previously defined in Equation 4. 8 Pressure-Volume Model defined by Merritt (1978), is used again. This model was selected over others, even the one proposed by Chatburn (2003), due to its greater accuracy in incorporating the viscoelastic behavior of lung tissue, including memory effects and nonlinearity.

In expiration, the model starts from the physiological condition that lung volume decreases from a maximum value (corresponding to PIP) to a minimum volume (associated with PEEP), without active intervention from the respiratory muscles. To adapt Merritt's model to this phase, the input values are inverted: the initial radius is set as the maximum radius reached at the end of inspiration, and the final radius corresponds to the minimum radius to prevent pulmonary collapse. Similarly, the initial pressure is defined as PIP (maximum pressure), and the final pressure as PEEP.

Since Merritt's model is directly dependent on the geometric configuration of the lungs, it is necessary to recalculate the compliance constant C using these new initial parameters, as the behavior of the tissue changes when the process is reversed. In this simulation, the resulting compliance value for expiration is 0.1622.

The selected approach for expiration is to simulate progressive and iterative behavior based on the temporal progression of the expiratory phase. The model that shows the greatest accuracy regarding pressure variation inside the lungs is Merritt's model. However, since this model is not time-dependent, Equation 4. 10 Volume Model defined by Chatburn (2003) is used to establish the temporal relationship between pressure change and time. While simulating lung mechanics, the outgoing flow (thus negative) is also calculated during this process. Although it was initially intended to compute the variation of pulmonary resistance and tissue compliance, these values are kept constant in this version of the model to maintain simplicity.

Unlike the inspiratory phase, where the calculation is based on a fixed time increment, in this case the iteration is implemented using a `while` loop conditioned by lung volume. The simulation stops when the pulmonary radius reaches the minimum physiologically plausible value (r_0), ensuring that the model does not exceed the structural limits of the respiratory system.

At each iteration, the variables for pressure, flow, compliance, and resistance are dynamically updated, and the input pressure to the system is also calculated. Since the complete code block is extensive, it is included in Annex II in its entirety.

As verified during the inspiratory phase, the Merritt model provides consistent and accurate results, making it equivalent to set the stopping condition based either on pressure or on pulmonary radius: in both cases, the results converge toward the expected values of volume and pressure at the end of expiration.

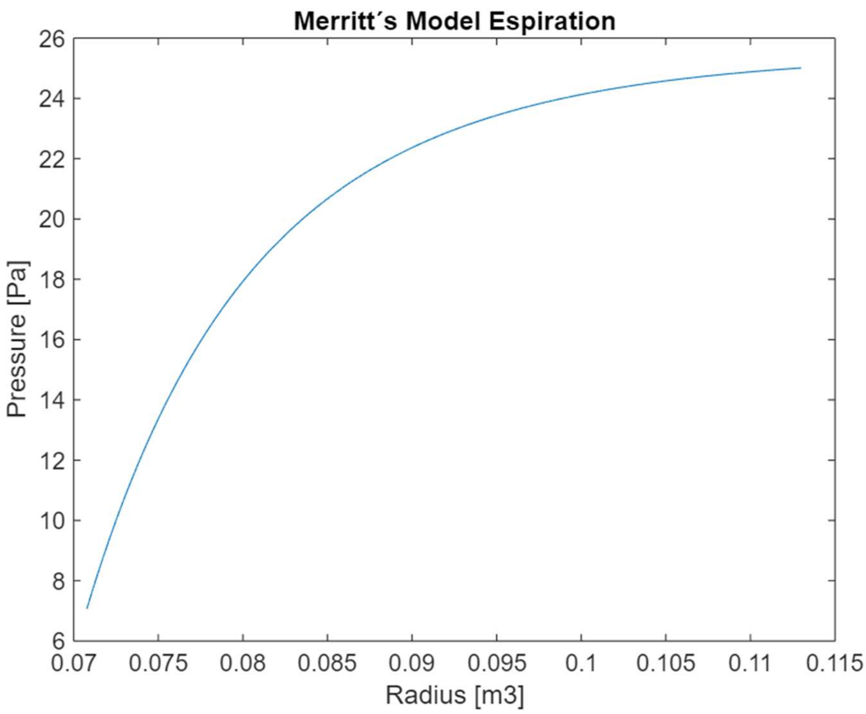

Figure 5. 25 Pressure-Radius Curve During Espiration

It can be observed that the curve obtained exhibits a shape like the one shown in Figure 5. 23 Volumetric Turbulent Flow in Airway Merritt´s Model, as both describe the behavior of the expiratory phase. However, the shape obtained in this simulation differs slightly from the theoretical one. This discrepancy can be explained, first, by the fact that the maximum pressure value (PIP) is not reached exactly at the end of inspiration, but rather around the midpoint of the process.

Results have uncovered certain trends, including an initial steep decay followed by a slow asymptotic relaxation, which would be better described by a power law than by an exponential decay (Dai et al., 2015). This slight discrepancy in curvature is attributed to hysteresis, a mechanical property of lung tissue that reflects the difference between the inspiratory and expiratory pressure curves (Chatburn, 2003). This difference is caused by the viscoelastic nature of the lung parenchyma, which leads to a time-dependent response to deformation.

In addition, although expiration theoretically begins at the PIP, the initial value used in this model corresponds to the final point reached by the inspiratory pressure curve. As a result, the expiratory curve does not start exactly at the PIP achieved during inspiration, but rather at the pressure value corresponding to the end of the inspiratory phase.

While this might appear to be a mismatch, it reflects the final moment of inspiration, which is considered equivalent to the PIP in this model. Since the difference between the computed PIP and the predefined one is minimal, this can be viewed as a modeling error resulting from system simplifications. Nevertheless, the overall shape of the curve remains consistent with the one proposed by Merritt (1978), as shown in Figure 4. 3 Pressure-Radius Curves for Rubber Balloons defined by Merrit

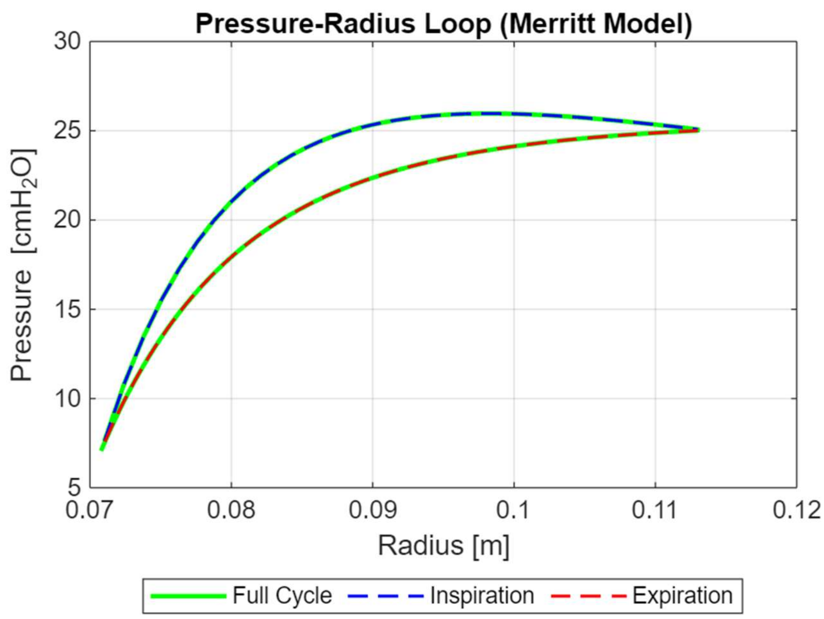

Figure 5. 26 Pressure-Radius Complete Cycle

Despite the reversal of direction between the inspiration and expiration phases, the resulting curve retains the nonlinear shape characteristic of the model proposed by Merritt. This morphology reflects the viscoelastic behavior of lung tissue, which does not respond linearly to pressure stimuli. As the lung expands, tissue stiffness progressively increases, whereas during expiration, the elastic recoil occurs more gradually. This phenomenon results in a pressure curve that rises more steeply at a small radius (high initial compliance) and decreases more gently as the radius diminishes during expiration.

Subsequently, in order to calculate the resistivity and compliance constants of the tissue, it is necessary to theoretically compute the respiratory flow. For this purpose, the expiratory flow function defined by Chatburn is used, described in Equation 4. 11 Flow Model Defined by Chatburn (2003).

This equation represents the theoretical evolution of respiratory flow. In this formulation, flow depends directly on time and on the difference between maximum and minimum pressures, as well as on the values of tissue resistance and compliance.

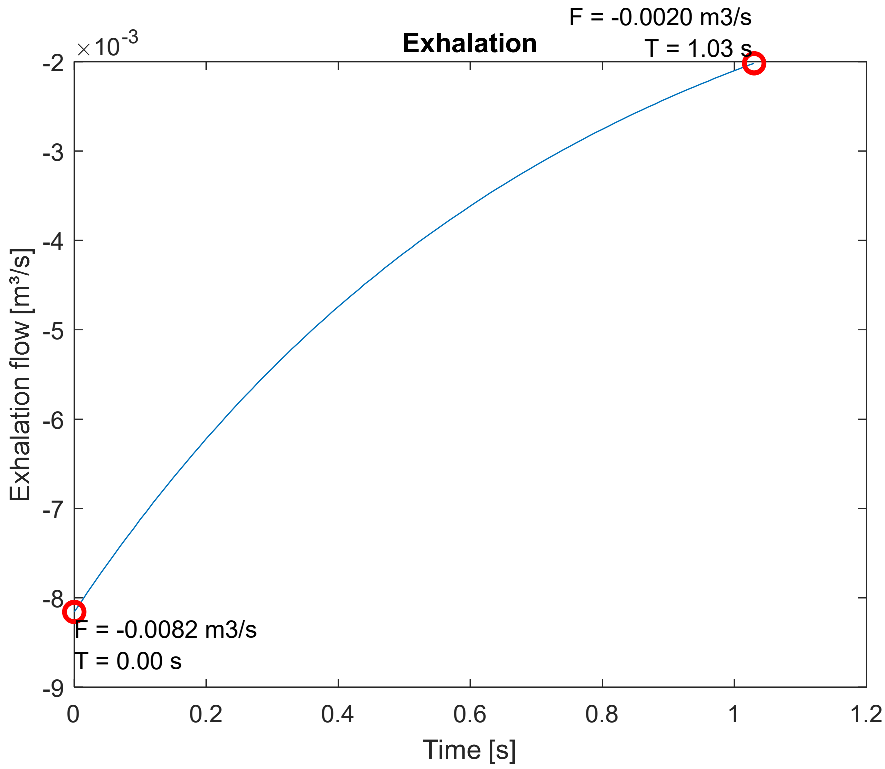

Figure 5. 27 Expiration Flow Graph

This function allows the estimation of the theoretical maximum flow that would occur during passive expiration, based on a high initial pressure (similar to the PIP value) and a lower final pressure (PEEP). As shown in the graph, the behavior of expiratory flow is again nonlinear, which is consistent with the evolution of volume and pressure described by the Merritt model, even though the flow formulation itself is based on Chatburn's approach (Chatburn, 2003).

The flow values are negative, as expected during expiration, since they represent the movement of air out of the respiratory system. Moreover, it is confirmed that despite integrating elements from different nonlinear models, the extreme flow values match the thresholds previously defined for lung volume during inspiration. In this case, a maximum volume of 80 ml/kg and a minimum of 20 ml/kg have been established for an individual of 75 kg.

5.3.2 CHATBURN MODEL

Next, Chatburn's equations are used again to obtain the values of the compliance (C) and resistance (R) constants. Although these functions do not offer the same level of precision as the Merritt model, their use is necessary to determine the volume, and flow variables as a function of time, and to estimate the duration of expiration.

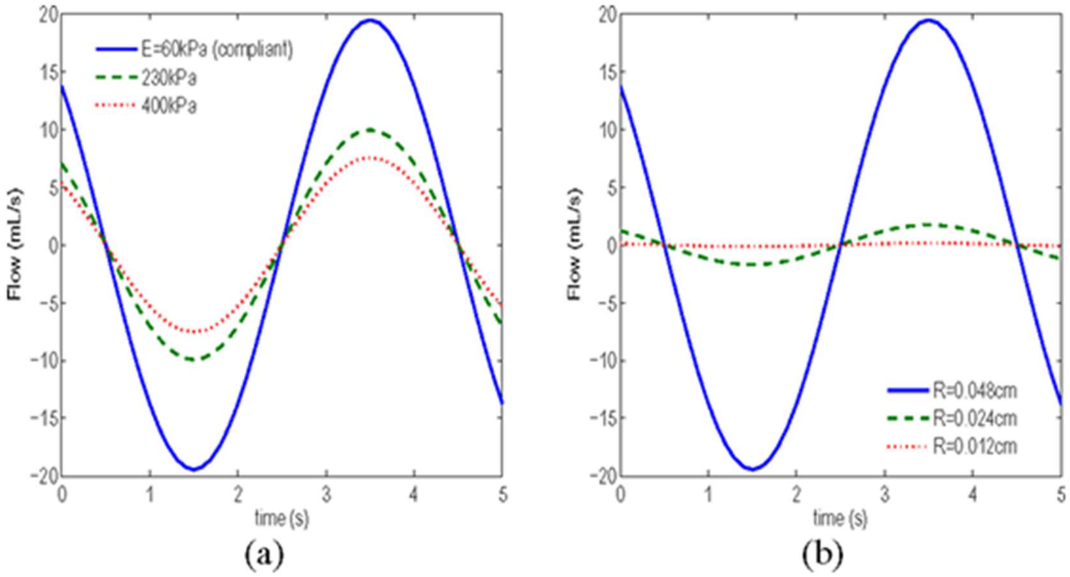

Figure 5. 28 Flow Variations at Alveolar Level caused by (a) Changes in Elastance (E) and (b) Changes in Resistance (R). Adapted Ionescu et al. (2009)

Figure 5.28 shows how the simulated expiratory flow is affected by variations in the parameters of resistance (R) and compliance ($C = 1/E$). As previously explained, these parameters do not behave as constants in the real respiratory system. Several studies have shown that both R and C can vary significantly during the respiratory cycle, depending on the lung condition, respiratory rate, and the phase of the cycle (Ionescu et al., 2009; Chatburn, 2003).

Morphological changes such as thickening of bronchial walls, loss of tissue elasticity, or alveolar collapse in diseases such as COPD or ARDS alter resistance and compliance nonlinearly. These effects can be observed in dynamic models that account for such variations, as illustrated in Figure 5. 28 Flow Variations at Alveolar Level caused by (a) Changes in Elastance (E) and (b) Changes in Resistance (R). Adapted Ionescu et al. (2009), which shows the behavior of flow under different conditions of tissue stiffness (E) and airway resistance (R). However, in this model, despite acknowledging the existence of such variability, it was not implemented, since it was necessary to predefine R and C values to carry out the simulations. As a result, the output is more simplified and less precise.

These equations define the time constant (Equation 5. 1 Time constant), which represents the time required for lung volume to decrease by approximately 63% of the total volume (Chatburn, 2003). Under physiological conditions, it is assumed that this time constant remains constant, as it serves as a reference for pulmonary volume decrease.

According to Chatburn (2003), compliance represents the elastic load of the respiratory system. On the other hand, respiratory system resistance refers to the change in pressure as a function of flow change.

In the context of respiratory mechanics, compliance and resistance are the two fundamental parameters that determine the system's response to an applied pressure. Compliance is associated with the lung's ability to expand in response to a pressure change, whereas resistance refers to the opposition to airflow through the airways. Both concepts are related to system load.

This highlights that both elastance (the inverse of compliance) and resistance determine the effort required to induce ventilation. Since load is expressed in pressure units, the model considers that the product in Equation Equation 5. 1 Time constant governs not only the volume dynamics, but also the flow behavior during passive expiration. This perspective provides a unified interpretation of how both parameters affect lung emptying time and the efficiency of the ventilatory system.

Their values are calculated using the functions defined by Chatburn:

$$R = \frac{PIP - PEEP}{Flow} \cdot e^{\frac{-t}{RC}}$$

Equation 5. 2 Resistance at Respiratory Airways

$$C = \frac{Volume}{PIP - PEEP} \cdot e^{\frac{t}{RC}}$$

Equation 5. 3 Compliance

Since this is a nonlinear model, the different functions used to describe expiration may represent the same physiological phenomenon but can lead to different values for the system constants, depending on the formulation employed. In this case, the parameters that are assumed to be known with certainty at the start of expiration are the lung volume and initial flow, so the resistance (R) and compliance (C) constants are determined based on these values.

The approach does not begin by analyzing the variation of volume or flow over time to directly fit R and C. Instead, the goal is for the model itself to naturally reproduce the dynamic evolution of these variables, along with the associated time vector.

The resulting values are:

- R = $2.1031 \cdot 10^{5}$ Pa/m^3,
- C = $3.5228 \cdot 10^{-6}$ m^3/Pa,
- and RC = 0.7301.

The resulting time constant falls within the range defined by Arnal et al. (2018) during simulations of mechanical ventilation in adult subjects.

The expiration time vector is calculated based on the variation of lung radius. Once the model reaches the minimum physiological radius, the simulation stops. In each iteration of the while loop, the time vector is incremented by one hundredth of a second, to ensure that the computational load remains manageable.

The total expiration time is found to be 1.02 seconds, which is considered valid, as it falls between one and two time constants, and does not reach alveolar collapse.

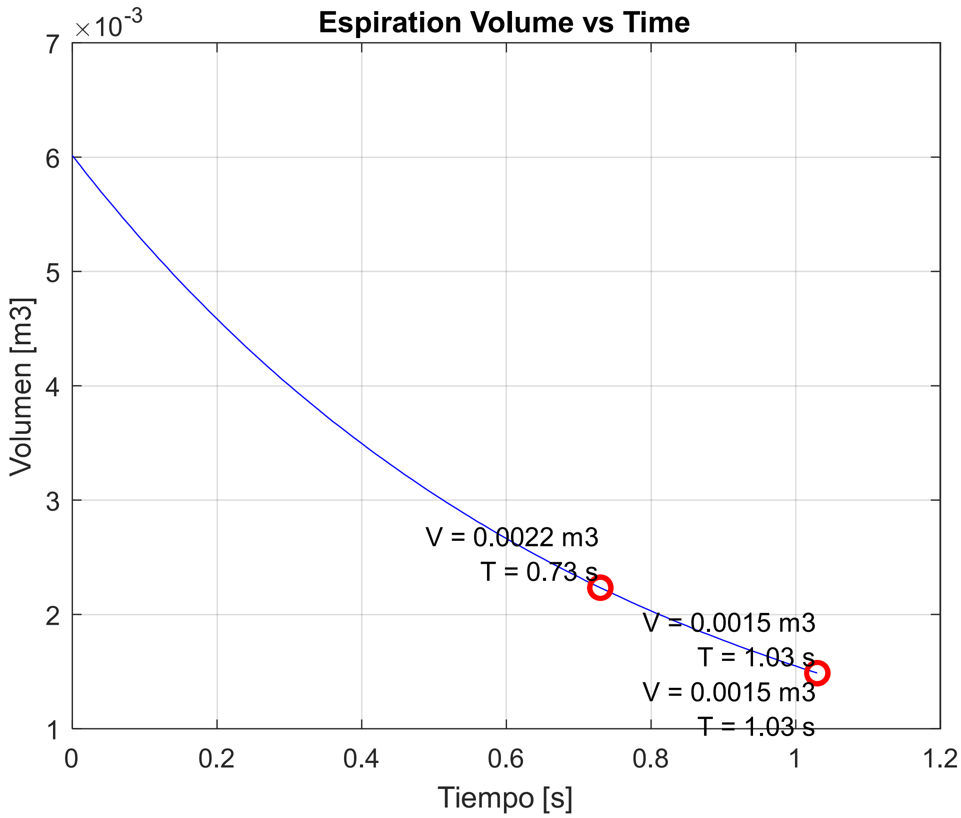

Figure 5. 29 Volume Variation with Time Constants

As shown in the figure above, it can be verified that after one time constant, the lung volume decreases by approximately 63%, in accordance with the definition provided by Chatburn (2003) in Figure 3. 2. Representation of pulmonary volume and pressure behavior during the processes of inspiration and expiration (Chatburn 2003) As indicated in the figure, the expiration time in this model lies between one and two-time constants for completing the exhalation of the tidal volume in the lungs.

Having established the duration of expiration, the following section presents the variation of pressure and flow during the expiratory phase.

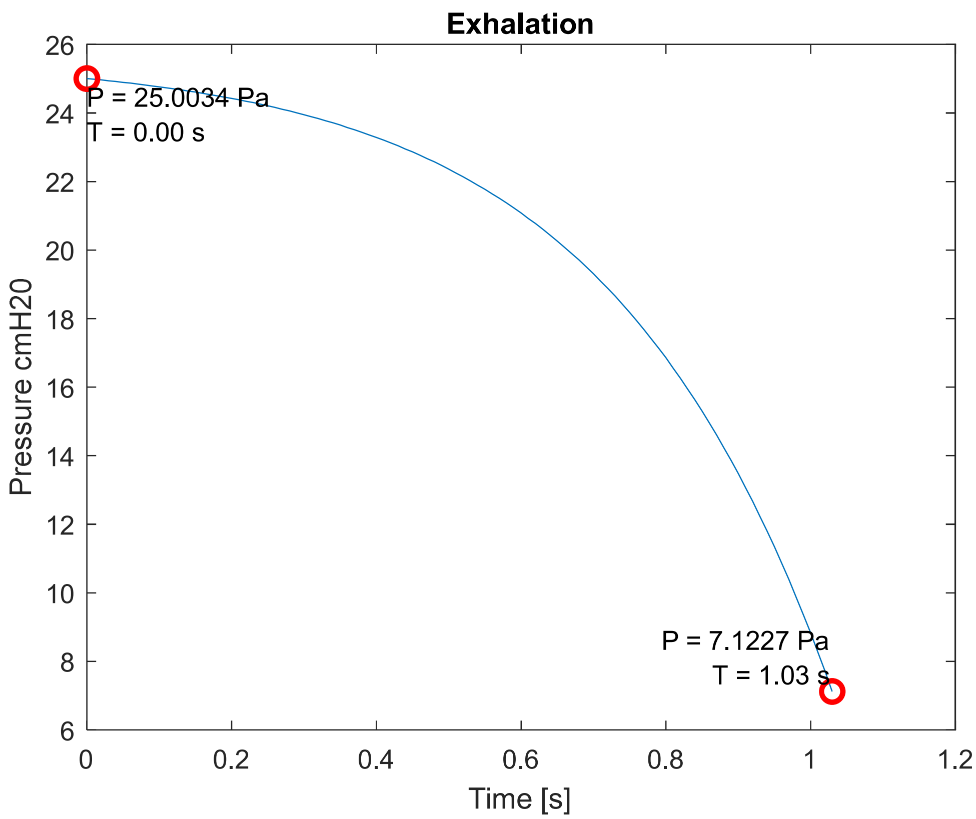

Figure 5. 30 Exhalation Pressure Curve

Finally, the figure above shows the evolution of pressure over time during the expiratory phase. When simulating pulmonary mechanics, it is important to note that the model is based on nonlinear functions, which, as previously discussed, introduce a certain degree of inaccuracy.

In this case, the values of resistance (R), compliance (C), and expiratory flow have been obtained using the equations proposed by Chatburn, which are derived from measurements taken by mechanical ventilators. However, these measurements are not entirely accurate, as they may be affected by various clinical and technical factors, such as patient–ventilator interaction, the presence of auto-PEEP, leaks in the respiratory circuit, or the accuracy of the ventilator's sensors.

In this context, it is essential to distinguish between the set PEEP (the value programmed by the clinician on the ventilator) and the total PEEP (the pressure measured at the end of expiration, which may include auto-PEEP).

As shown in the graph, the model initiates expiration with a pressure lower than the final inspiratory pressure (PIP). This difference may be attributed to the nonlinear behavior of the model, which does not enforce a perfectly continuous transition between phases. Therefore, it is expected that some deviations may occur between the theoretical values and the simulation results.

Furthermore, at the end of expiration, the pressure falls below the initially defined PEEP value. This discrepancy may be due to numerical implementation limitations or to the inherent physiological variability of the respiratory system. Factors such as partial airway collapse, internal pressure redistribution, or dynamic resistances not explicitly included in the model could contribute to this behavior.

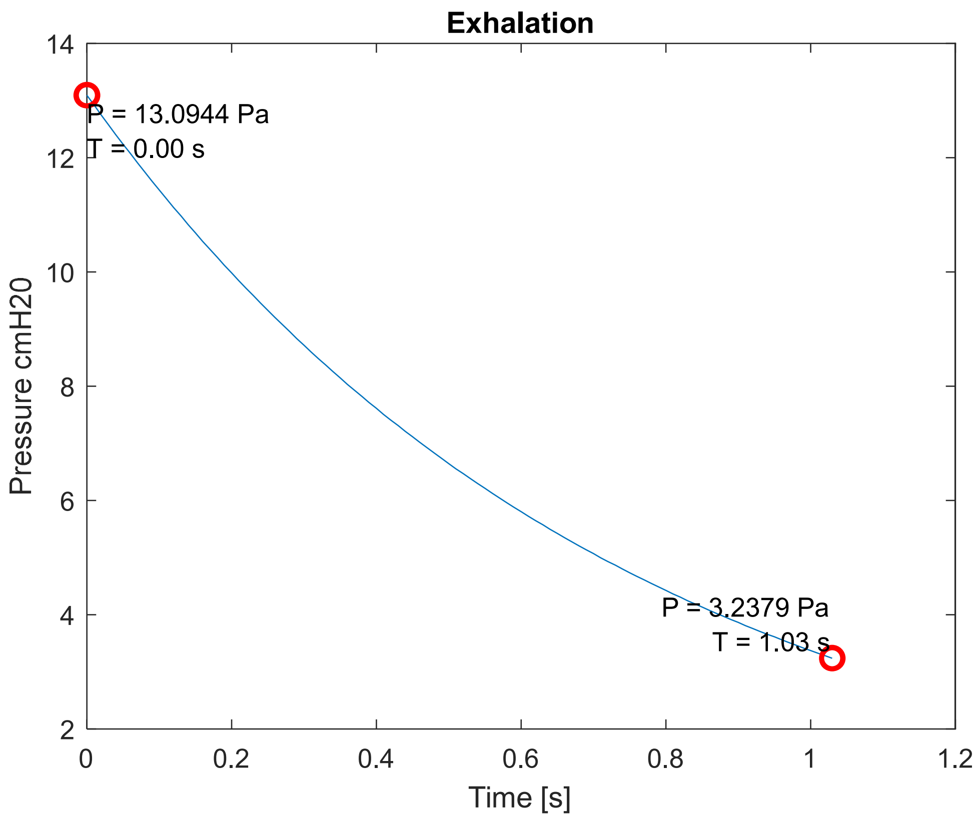

Figure 5. 31 Exhalation Pressure Curve Chatburn´s Model

When applying the Chatburn model for pressure definition using the same input values, such as time, compliance (C), and resistance (R), a significant difference is observed compared to the results obtained with the Merritt model.

According to the Merritt (1978), the initial pressure is approximately 25 cmH_2O, gradually decreasing to 7 cmH_2O over an interval of 1.04 seconds. In contrast, the Chatburn-based model (2003) begins with a notably lower initial pressure of 13 cmH_2O, which decreases to 3.2 cmH_2O within the same time frame.

Both curves appropriately represent the dynamics of passive pulmonary emptying and demonstrate the direct influence of the initial pressure value on the total pressure range observed during expiration. However, the results from the Merritt model are more consistent with the defined PIP and PEEP values and fall within the physiological ranges reported by Arnal et al. (2018, p. 162) for adult patients.

The pressure function used in the Chatburn model corresponds to the following expression:

$$P = \frac{3 \cdot (PIP - PEEP) \cdot C}{4 \cdot \pi} \cdot e^{\frac{-t}{RC}}$$

Equation 5. 4 Chatburn´s Model for Pressure Variation.

Despite these deviations, the model based on Merritt's approximation (1978) allows for the simulation of respiratory variable evolution during passive expiration with a high degree of fidelity. The integration of the functions proposed by Chatburn, combined with the pressure–volume relationship from Merritt's model, provides a solid foundation for representing the elastic recoil of the lungs.

Although this is a simplified approach, it proves suitable for analyzing pulmonary emptying under normal physiological conditions, and it serves as a robust starting point for future extensions aimed at simulating pathological states or personalizing ventilatory parameters based on the patient's individual profile.

5.4 CONSTANT PRESSURE MODEL

To verify the validity of the model developed with constant flow, a new model based on constant pressure has been implemented. This model allows for a comparison between both approaches and, at the same time, provides a more versatile framework, as it can be adapted to different operating conditions. Additionally, the constant pressure model makes it possible to simulate clinical scenarios in which a fixed input pressure is applied, which is common in certain modes of mechanical ventilation. This enables the study of system behavior when the flow is not constant but instead varies depending on the resistance and compliance of the lung tissue.

Therefore, the development of this second model not only offers a way to validate the results obtained with the first but also broadens the potential applications of the system in more realistic environments. Since some ventilators operate under pressure control and others under volume or flow control, having a model that can adapt to both scenarios facilitates its use in the design and testing of new ventilation strategies. In this way, a more complete, flexible, and useful system is achieved for future developments in both simulation and clinical applications.

5.4.1 IMPLEMENTATION

In this version of the model, a constant input pressure is established with a predefined PEEP value, which is the same PEEP used in the constant flow model: 7.5 cmH_2O. The development of this model follows the same approach used in the exhalation phase, where the time vector that defines the model is incremented in hundredths of a second.

Depending on the specific time instant, the flow is calculated at that moment and considered constant during that interval. With this approach, it is possible to gradually compute the variation in lung volume, respiratory flow, and pressure over time.

Once the flow is determined for a given time point, the corresponding lung radius is calculated. To do this, the same volume variation function used in the constant flow model is reused. The development of the variable radius function is as follows:

In this version of the model, a constant input pressure is established with a predefined PEEP value, which is the same PEEP used in the constant flow model: 7.5 cmH_2O. The development of this model follows a logic similar to that applied during the exhalation phase, since the system progresses through small time increments (hundredths of a second), allowing the time vector that defines the model's behavior to be progressively constructed.

At each time step, the value of the resulting flow is calculated and considered constant during that interval. This flow depends directly on the imposed pressure and the physical characteristics of the system. With this approach, it is possible to determine, step by step, the evolution of lung volume, respiratory flow, and the pressure generated inside the lungs, providing a detailed view of the system's dynamic behavior under a fixed pressure condition.

Once the flow has been determined for a specific time instant, the corresponding lung radius is calculated. To do this, the same volume variation function previously used in the constant flow model is reused, adapting it to the pressure-driven input. The development starts from the expression of flow as the derivative of volume:

$$Flow = \frac{dV}{dt} = 4 \cdot \pi \cdot r^2 \frac{dr}{dt}$$

Equation 5. 5 Flow´s definition

From this relationship, the value of the radius is derived as a function of time and accumulated flow:

$$r = \sqrt[3]{\frac{3 \cdot flow \cdot t + 4 \cdot \pi \cdot {r_0}^3}{4 \cdot \pi}}$$

Equation 5. 6 Radio Variation

This expression allows the new radius value to be calculated at each time step, based on the instantaneous flow and the initial radius. Using the obtained radius values, the equation described by Merritt (1978) is applied again to estimate the pressure variation inside the lungs, just as in the inspiratory model with constant flow. In this way, consistency with the physical approach of the original model is maintained, and compatibility between both implementations is ensured.

The resulting values for incoming flow, pressure variation inside the lungs, and lung volume variation are similar to those previously obtained.

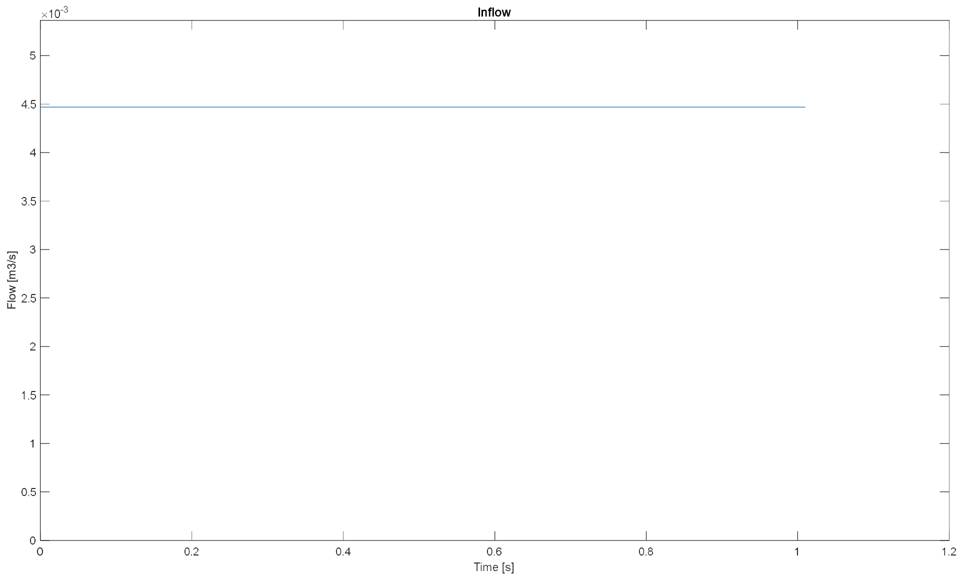

Figure 5. 32 Flow Variation with Constant Pressure Model

As shown in the figure, the flow value obtained from the variation in lung volume is very similar to that defined in the constant flow model. In that model, a constant inflow of 0.0045 m^3/s, was imposed, corresponding to the desired tidal volume. In this case, although the flow is calculated iteratively as a function of time, it remains constant around a value of 0.0044 m^3/s.

This stability in the flow is because, during the model development, a constant value is assumed at each time step, which results in a nearly flat curve. This match between both approaches validates the behavior of the new model, as it correctly reproduces the expected results based on the imposed conditions.

Moreover, obtaining a flow value so close to the one previously defined reinforces the consistency of the formulation and demonstrates that the pressure-controlled model is capable of simulating physiological conditions similar to those achieved with the constant flow model. Therefore, the implementation is confirmed to be consistent with the physical principles of the system and suitable for use in future simulations under different input conditions.

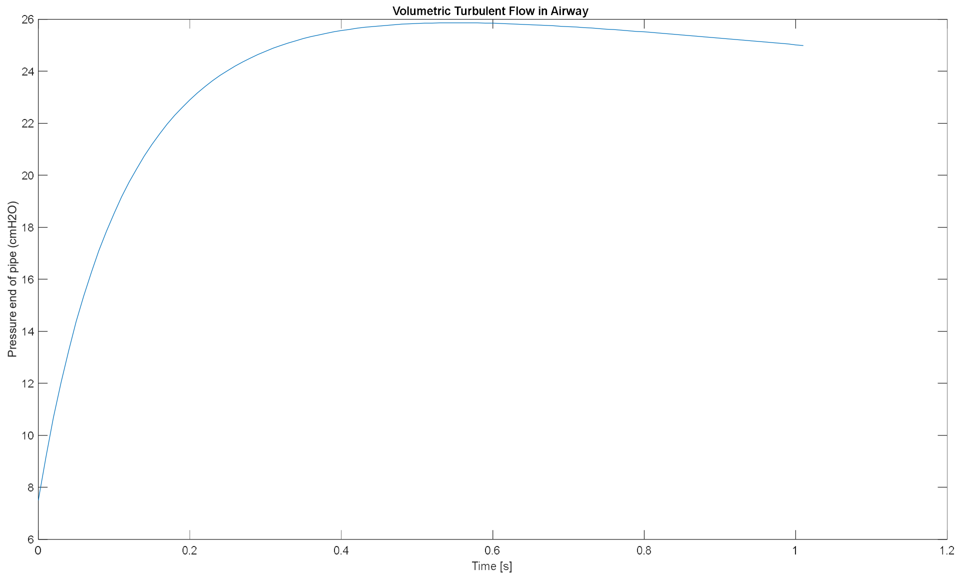

Figure 5. 33 Pressure Variation from Merrit´s Model

This figure shows the variation in lung pressure over time, obtained using the Merritt (1978) model applied to the case of constant input pressure. The maximum pressure value reached in this model is 25.86 cmH_2O, which is slightly lower than the value obtained in the constant flow model, where a peak of 25.96 cmH_2O was reached.

As observed, the difference between both models is minimal, which further confirms the validity of this second implementation. The curve exhibits a behavior like that of the previous model, reaching its maximum value (PIP) approximately halfway through the total inspiration time. In this case, the inspiration lasts 1.01 seconds, which is also slightly shorter than the duration recorded in the constant flow model.

These small variations are due to model limitations and the way the flow is calculated based on the imposed pressure, which introduces certain numerical deviations. However, the overall behavior of the curve, as well as the pressure value achieved, closely aligns with expectations, reinforcing the consistency of the model under different input conditions.

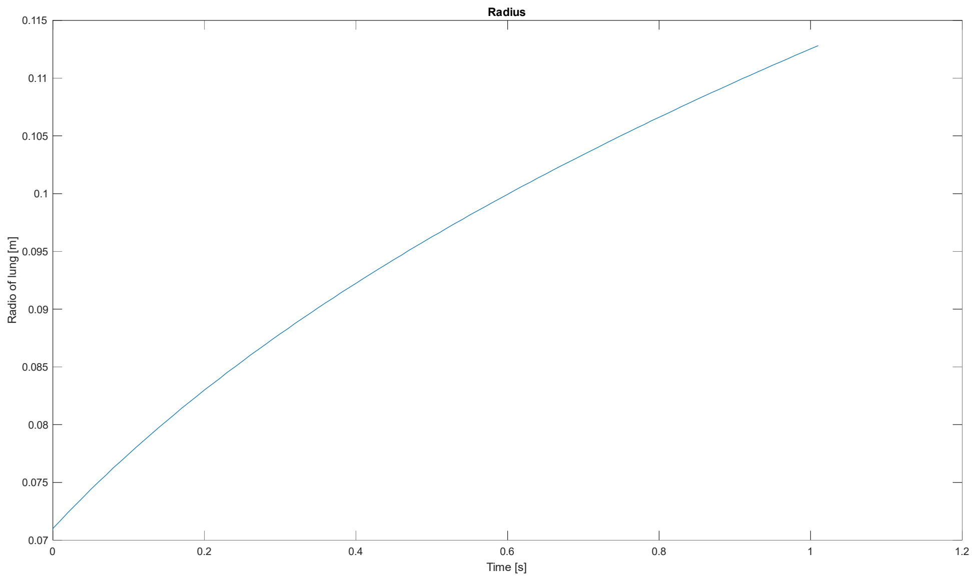

Figure 5. 34 Radio Variation in Constant Pressure Model

As can be seen, the evolution of the lung radius in this model presents a curve that is almost identical to the one obtained in the constant flow model. The shape of the curve is smooth and increasing, with a decreasing slope coming to the end of the inspiration phase, indicating a progressive growth of lung volume in response to the flow generated by the constant pressure.

The minimum and maximum radius values are the same in both models, since they were defined identically as boundary conditions within the system. This consistency not only validates the coherence between both approaches but also demonstrates that the geometric behavior of the model remains stable regardless of whether the input condition is flow or pressure.

Finally, the variation values for pressure, flow, and radius are shown for both the constant flow model and the constant pressure model. As can be visually observed, regardless of the selected input condition, the results obtained during inspiration, and consequently the final values that will define expiration, are very similar in both cases. This confirms the validity of the model under different boundary conditions.

As mentioned earlier, the greatest difference is observed in the input flow graph. In the constant flow model, the flow value is fixed based on an assumed inspiration duration of 1 second. However, in the constant pressure model, the flow is calculated at each discrete time step (dt), resulting in a very similar value (0.0044 m^3/s).

The total inspiration time in the constant pressure model is 1.01 seconds, which suggests that the estimated flow in this case may be considered more accurate, as it dynamically adapts to the system's behavior. Despite these small differences, the rest of the variables show an almost identical evolution, reinforcing the robustness of the model and confirming that both approaches can produce consistent results.

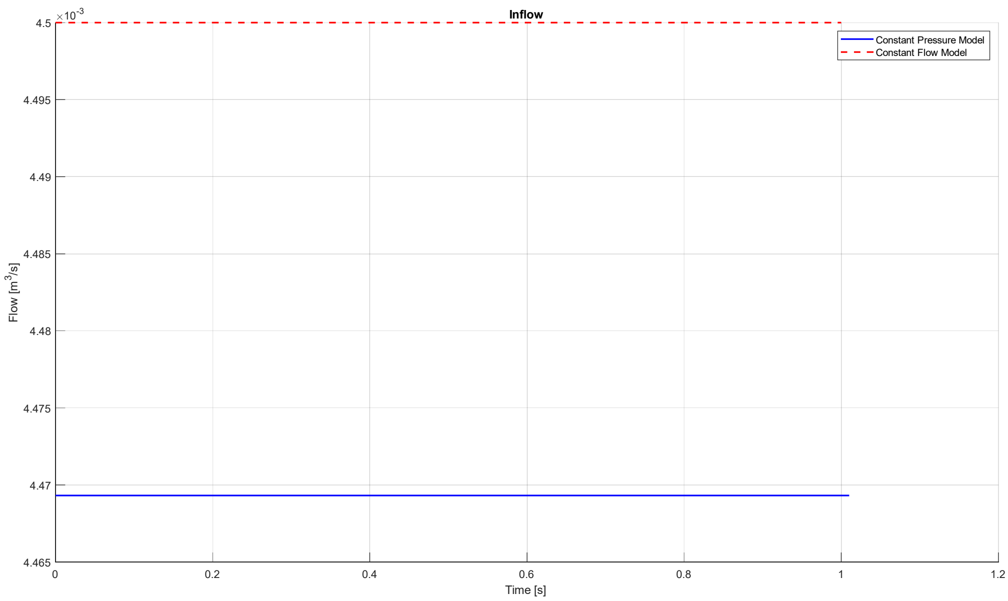

Figure 5. 35 Flow Variation of the Two Models

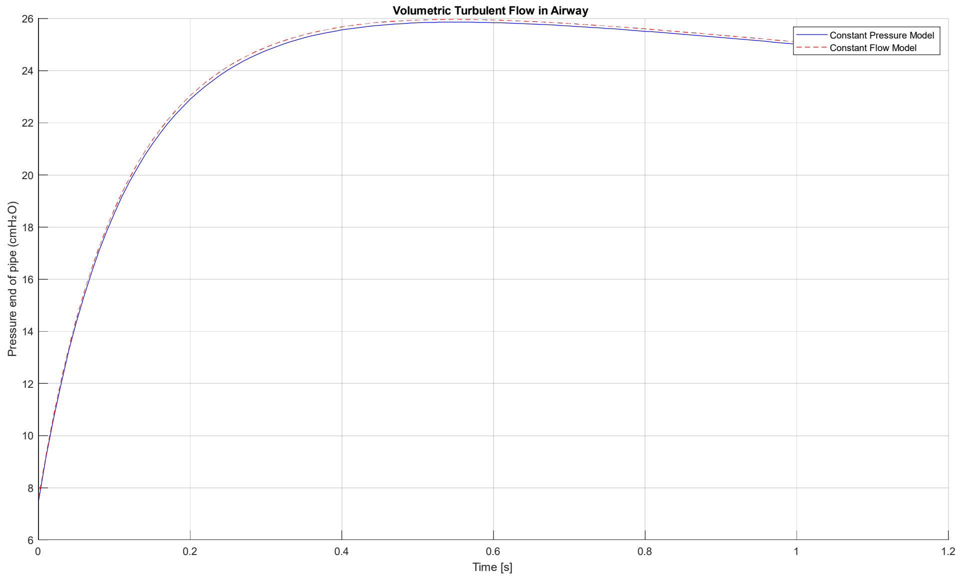

Figure 5. 366 Pressure Variation of the Two Models

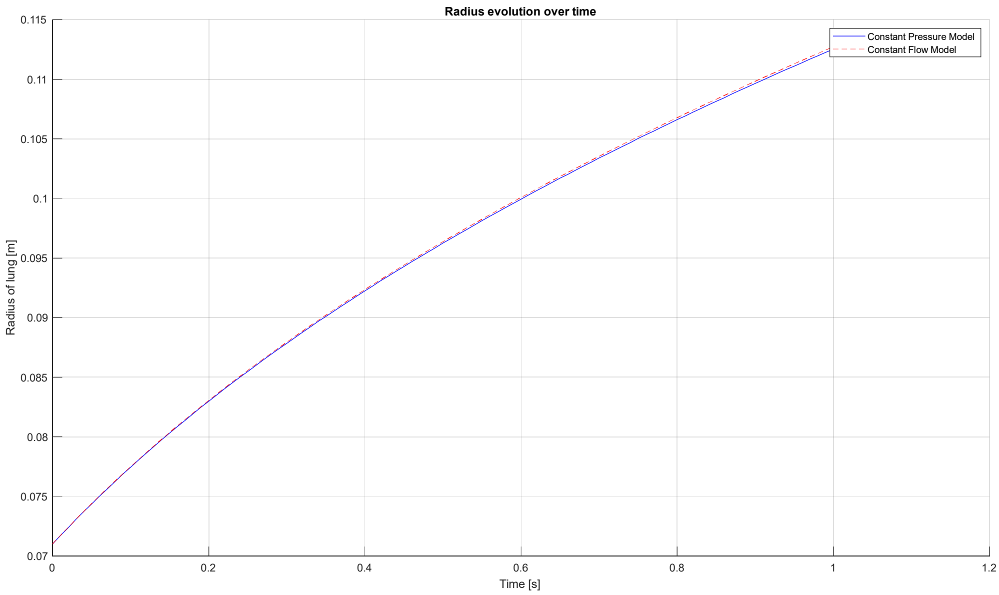

Figure 5. 377 Flow Variation of the Two Models

Chapter 6. RESULTS ANALYSIS

This chapter aims to carry out a critical evaluation of the developed mathematical model, identifying its main limitations and proposing possible directions for improvement or extension.

To this end, a set of graphs is presented to illustrate the complete respiratory cycle, focusing on the three fundamental variables that define pulmonary mechanics: pulmonary pressure, respiratory flow, and lung volume. These representations enable a comprehensive analysis of the system's dynamic behavior and help validate the consistency of the model with physiological expectations.

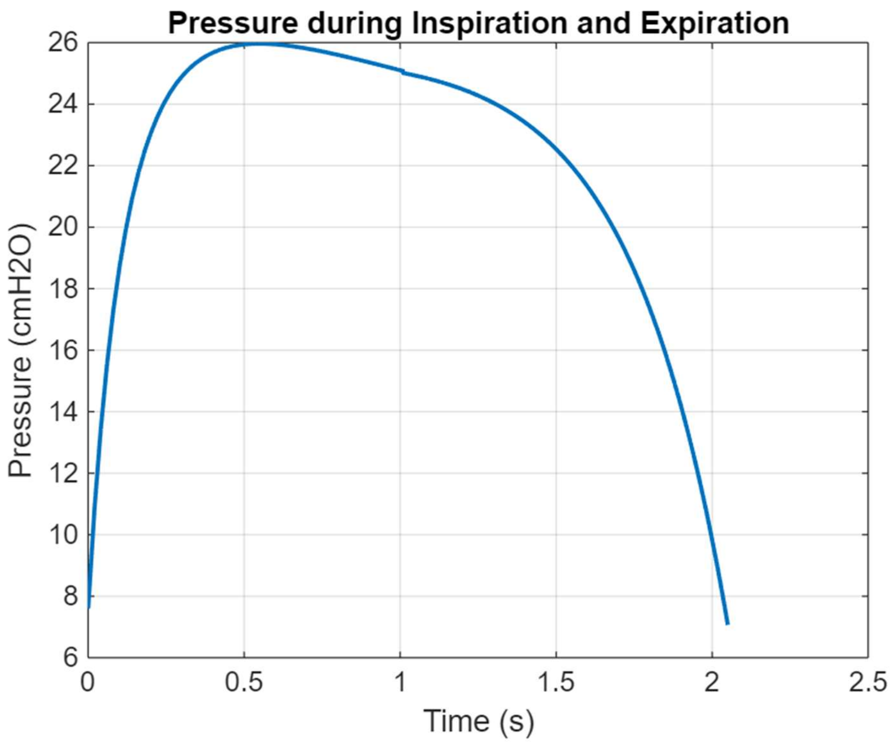

Figure 6. 1 Variation of Pressure in Complete Respiratory Cycle

The figure shows the complete pressure variation cycle throughout a simulated breath. It can be observed that the peak inspiratory pressure (PIP) reaches a maximum value of 26 cmH_2O approximately halfway through the inspiratory phase, before stabilizing around 25 cmH_2O, which was manually defined as the PIP in the model. During expiration, the initial point is taken as the last pressure value from inspiration, although this does not exactly match the theoretical PIP, introducing a slight discontinuity between phases.

Additionally, a rapid pressure rise is observed at the beginning of inspiration. This behavior indicates low pulmonary compliance at the start of the cycle, meaning the system requires higher pressure to initiate alveolar filling. This phenomenon is consistent with the real physiology of the lung, where the pressure–volume curve is nonlinear and exhibits lower distensibility during the early stages of inspiration.

The positive end-expiratory pressure (PEEP) was set at 7.5 cmH_2O, from which the inspiration begins. However, in the simulated expiration, the pressure is observed to decrease to a minimum value of 7 cmH_2O, indicating a discrepancy from the reference value. This difference may be attributed to various factors, such as numerical limitations of the model, the presence of nonlinearities in the equations used, and the viscoelastic nature of lung tissue. The exponential pressure decay following the peak is typical of passive expiration governed by the time constant.

Both the inspiratory and expiratory phases have been modeled using the formulation proposed by Merritt for rubber balloons (1978). Specifically, there is a notable similarity between the pressure curve generated by the model and the theoretical curve shown in **¡Error! No se encuentra el origen de la referencia.**, from Merritt's original article. This similarity is partly due to the fact that the developed model seeks to replicate the behavior of real mechanical ventilators, using reference values such as those described in Parameters for Simulation of Adult Subjects During Mechanical Ventilation (Arnal et al., 2018).

However, these values may be subject to inaccuracies derived from patient positioning, the dynamic response of the ventilator, or the specific characteristics of the respiratory circuit. Moreover, the equations used are nonlinear, which adds complexity to predicting the system's exact behavior.

A significant difference from Merritt's original model is that, in his formulation, pressure starts at 0 Pa when the radius is zero, which is not physiologically realistic in the pulmonary context, where there is always a residual volume and pressure to prevent alveolar collapse. Nonetheless, the overall behavior of the radius during both inspiration and expiration resembles that of the balloon, exhibiting an asymmetric pattern characteristic of viscoelastic materials. This asymmetry, or hysteresis, is indicative of internal energy loss, a typical feature of lung tissue due to its elastic and viscous properties.

Finally, although the expiratory pressure in the model is shown as positive, it could theoretically be considered negative depending on the reference system used. In this work, it was decided to keep it positive to maintain consistency throughout the model. However, if represented with a negative sign, the resulting curve would be symmetrical and consistent with the theory proposed by Merritt.

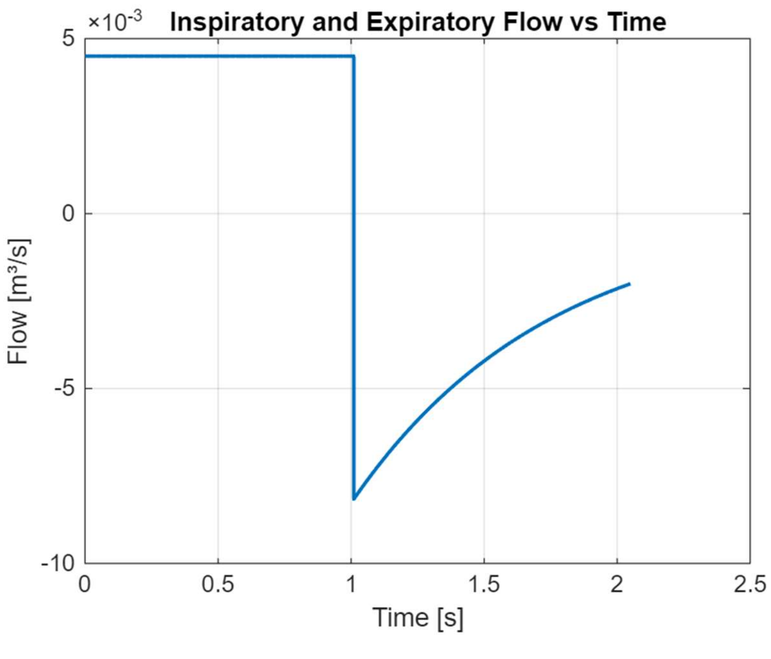

Figure 6. 2 Flow Variation during Complete Expiratory Cycle

The model was designed to apply a constant flow during the inspiratory phase. This flow was defined based on the classical respiratory flow formulation as the ratio between the change in volume and the change in time. Given that the inspiratory time was estimated at approximately 1 second and a tidal volume of 0.0045 m^3 was considered, a constant inspiratory flow of 0.0045 m^3/s was imposed.

In contrast, expiration was modeled as a passive process, allowing the simulation of the actual expiratory flow generated by the relaxation of the pulmonary system. The graph shows a sharp transition at the start of expiration, which corresponds to the moment the imposed flow ends, and the autonomous emptying of the lung begins. The expiratory flow is represented with a negative sign, as it denotes outward air movement.

In absolute terms, the peak expiratory flow recorded is lower than the inspiratory flow, reaching an approximate maximum of 0.0022 m^3/s. This behavior is characteristic of an exponential discharge, typical of passive pulmonary emptying. The rate of this decline is governed by the time constant. If the values of resistance (R) or compliance (C) were higher, the emptying would be slower, and thus the flow would decrease more gradually.

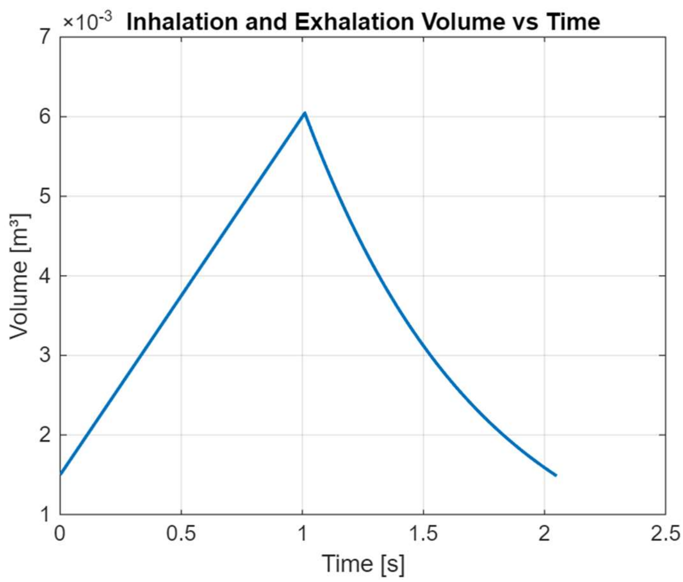

Figure 6. 3 Volume Curve during Complete Expiratory Cycle

The figure illustrates the evolution of lung volume throughout a complete respiratory cycle, including both inspiration and expiration. During the inspiratory phase, which lasts 1.02 seconds, the volume increases linearly, which is consistent with the application of a constant inflow. This linearity is expected, since with a constant flow, the accumulated volume is directly proportional to the elapsed time.

A maximum volume of 6×10^{-3} m^3 is reached, equivalent to 6 liters. This value corresponds to the maximum tidal volume calculated from the model's initial conditions (80 ml/kg for a 75 kg patient).

During the expiratory phase (starting at 1 second), the volume decreases exponentially, reflecting the behavior of passive emptying in the respiratory system. This exponential decay results directly from the dynamics described by the pulmonary time constant, where R represents resistance and C compliance.

It can also be observed that the volume does not return completely to zero, as the lung never fully empties. A residual (minimum) volume remains, consistent with the PEEP value and the functional residual capacity, which prevents alveolar collapse.

The combined behavior of linear increase during inspiration and exponential decrease during expiration, adequately reproduces the physiological dynamics of the respiratory system and validates the model under normal mechanical ventilation conditions.

The three presented graphs allow for an integrated evaluation of the behavior of the main respiratory variables: pressure, flow, and volume, over a full inspiration–expiration cycle. During the inspiratory phase, the model responds to a constant flow input, generating a linear increase in volume and an ascending pressure curve that reaches a peak (PIP) before stabilizing. This behavior correctly reflects the progressive accumulation of volume and pressure in a system with elastic and resistive properties.

In the expiratory phase, an exponential decline is observed in both flow and volume, consistent with the passive emptying of the lungs governed by the time constant. Despite not being determined by an exponential function, pressure also decreases in nonlinear fashion that resembles exponential decay, remaining within physiologically coherent ranges. The fact that the expiratory flow is negative reinforces its interpretation as outward airflow.

Overall, the model successfully reproduces the natural asymmetry of the respiratory cycle: an active, rapid, and controlled inspiration followed by passive, slower expiration governed by tissue properties. The shapes of the curves, their temporal continuity, and the correspondence between the peak and minimum values confirm that the system realistically captures pulmonary dynamics under normal conditions, respecting both the physiology and mechanical limitations of the human respiratory system.

6.1.1.1 Limitations of Chatburn´s Model

Despite the model's ability to accurately represent the inspiratory and expiratory phases under normal physiological conditions, there are a number of inherent limitations stemming from the simplifications made in the Chatburn (2003) model, which are important to acknowledge.

The first limitation concerns the simplification of actual pulmonary mechanics. The model is based on the equation of motion for the respiratory system, which considers only the elastic (compliance) and resistive (resistance) components. However, this formulation does not account for the viscoelastic behavior of lung tissue, which includes phenomena such as hysteresis, time-dependent relaxation, and dependence on respiratory frequency. These behaviors cannot be accurately described using linear models.

Another limitation is the exclusion of muscular effort. In the developed model, it is assumed that the input pressure is generated solely by the ventilator, disregarding any contribution from the patient. Therefore, the model is limited to passive and fully controlled conditions with no patient interaction.

Thirdly, there is a dependence on measurements obtained from ventilators, which have been previously discussed. The input variables are based on values recorded by mechanical ventilators, which may be subject to inaccuracies due to factors such as patient–ventilator interaction, the presence of autoPEEP, circuit leaks, or sensor sensitivity. These potential inaccuracies may affect the final values produced by the model.

6.1.1.2 Improvements of the Model

Despite having developed a model that improves upon the classical formulation proposed by Chatburn (2003) through the incorporation of Merritt's pressure-volume model for rubber balloons (1978), which allows for the inclusion of viscoelastic properties equivalent to those present in lung tissue, there are still several relevant aspects that could be enhanced to increase the system's accuracy and versatility.

First, the volume and flow functions used during expiration, which are based on Chatburn's model (2003), rely solely on elastic (compliance) and resistive (resistance) parameters, without considering viscoelastic effects or hysteresis phenomena. This limits the model's ability to accurately reproduce the real behavior of lung tissue, particularly under pathological conditions.

Furthermore, the developed system assumes a constant inflow during inspiration, which restricts the model to flow-controlled ventilation modes. Incorporating alternative input functions, such as variation of flow entrance or pressure ramp, which would allow the simulation of other clinical modes, such as pressure-controlled ventilation, thereby expanding the model's applicability.

Another possible improvement is the inclusion of patient muscular effort (Pmus) in the equation of motion, which would enable the simulation of spontaneous or assisted breathing and allow for a more in-depth study of patient–ventilator interaction.

In addition, it would be beneficial to allow resistance and compliance parameters to vary dynamically as a function of volume, pressure, or time, since these values are not constant in real physiology and reflect phenomena such as alveolar overdistension or progressive recruitment of ventilatory units.

At a more advanced level, a significant extension would be the incorporation of models using fractional-order differential equations, which can more faithfully represent the viscoelastic behavior of the respiratory system.

Finally, a natural evolution of the model would be the development of an automated control system capable of estimating and adjusting output parameters based on initial conditions or clinical variations. This would transform the model into an interactive tool, adaptable to different patient profiles and useful in both educational and clinical settings.

Chapter 7. CONCLUSIONS AND FUTURE WORK

One of the main achievements of the project has been the improved accuracy in estimating respiratory pressure compared to the classical model by Chatburn (2003), resulting in more consistent values with actual lung physiology and within clinically established ranges. This was made possible by integrating several complementary approaches: physical modeling, nonlinear mathematical formulations, and numerical simulation using Matlab.

The developed system allows for the simulation of a complete pulmonary breathing cycle, clearly distinguishing the active inspiration phase, controlled by the ventilator, from the passive expiration phase, modeled as an elastic discharge of the respiratory system. During inspiration, a constant flow input has been implemented, generating pressure and volume evolution calculated over time. During expiration, the exponential equations proposed by Chatburn (2003) have been applied to represent lung emptying, with results adjusted using Merritt's nonlinear pressure-volume model.

To validate the constant flow model and provide a more versatile solution, a constant pressure model was also developed. This model differs from the previous one only during the inspiration phase, as the expiration phase results from the evolution of the parameters reached during the first stage.

The modular structure of the model, which separates the two phases, enables independent analysis and facilitates future extensions. In addition, the generated graphs have been validated and shown to qualitatively and quantitatively reproduce physiological phenomena such as the temporal asymmetric between inspiration and expiration, lung tissue hysteresis, and the realistic progression of the respiratory cycle.

An important point to note is that the time vector during inspiration is estimated to last 1 second, as this duration is supported by validated data. However, in practice, the model reaches the peak inspiratory pressure (PIP) before completing the full second. If a shorter inspiration time is desired, it would be sufficient to adjust the termination condition by setting the maximum pressure as the limit instead of the maximum volume.

In summary, the main objective of the project has been achieved: the development of an autonomous model capable of simulating full respiratory mechanics with adjustable parameters and behavior consistent with human physiology. The resulting system lays a solid foundation for future enhancements, such as the inclusion of fractional viscoelasticity, the simulation of spontaneous breathing, or the incorporation of a dynamic control system to adapt the model to different clinical profiles or ventilatory scenarios.

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APPENDIX I: SDG ALIGNMENT

This project aligns primarily with the Sustainable Development Goals (SDGs): Goal 3 – Good Health and Well-being, Goal 4 – Quality Education, and Goal 9 – Industry, Innovation and Infrastructure.

It aligns most closely with Goal 3, as this goal aims to improve citizens' quality of life and promote their overall well-being. Specifically, it contributes to SDG 3 in the following ways:

- Target 3.2: Reduce neonatal and child mortality. The modeling of pulmonary mechanics can contribute to the design of safer and more personalized ventilation strategies, which is especially relevant in neonatal units, where patients are particularly vulnerable to ventilator-induced lung injury (VILI).
- Target 3.8: Access to essential healthcare services. The development of mathematical models that accurately simulate respiratory dynamics in ventilated patients can support the design of more effective control systems in mechanical ventilators, potentially improving healthcare delivery in intensive care units (ICUs).
- Target 3.B: Support research and development of health technologies. This work, based on the simulation of pulmonary mechanics, represents a step forward in understanding the physiology of respiratory processes, particularly in clinical contexts such as acute respiratory distress syndrome (ARDS), thus aligning with the need to strengthen biomedical R&D.

Regarding SDG 4, the project contributes to:

- Target 4.7: which aims to ensure that all learners acquire the theoretical and practical knowledge necessary to promote sustainable development, including scientific and technological education.

Concerning SDG 9, it contributes particularly to:

- Target 9.5, which promotes scientific research and technological capacity in high-value-added industrial sectors such as the biomedical field. The implementation of the model in a computational environment (Matlab) and its orientation toward clinical applications reinforce technological innovation in the healthcare sector.

Finally, the project also aligns with SDG 17- Partnerships for the Goals:

- Target 17.6: Enhance international cooperation on science, technology, and innovation. The collaboration with a faculty member from the University of San Diego has enabled the exchange of academic knowledge and expertise, fostering a cross-border partnership that strengthens the research process and contributes to global scientific progress.

APPENDIX II: MODEL IMPLEMENTATION CODE

The following code corresponds to the calculation of inhalation and therefore shows the variation of the pressure at the entrance of the lungs, pulmonary pressure, and pulmonary radius in constant flow model.

```
P0 = [];
P_t = [];
r_inhalation = [];
T_vector_insp = [];
dt = 0.01;
t = 0;
r = radioVariation(r0, inflow, t);
while r <= rmax
    p0 = turbulent(l, radio, phi, inflow, 0);
    P0(end+1) = p0;
    r = radioVariation(r0, inflow, t);
    r_inhalation(end+1) = r;
    p = pressureModelLungs(p0, C, r0, r);
    P_t(end+1) = p;
    T_vector_insp(end+1) = t;
    t = t + dt;
end
```

Code for the calculation of inhalation in constant pressure model

```
Inflow = @(P1, P0, r, l, phi) sqrt(((P1-P0)*pi*r^4)/(8*l*phi));

Inhalation
Flow = [];
P_t = [];
r_inhalation = [];
T_vector_insp = [];

P1 = PEEP;
dt = 0.01;
t = 0;
r = r0;
while r <= rmax
    f = Inflow(P1, 0, radio, l, phi);
    Flow(end+1) = f;
    r = radioVariation(r0, f, t);
    r_inhalation(end+1) = r;
    p = pressureModelLungs(P1, C, r0, r);
    P_t(end+1) = p;
    T_vector_insp(end+1) = t;
    t = t+dt;

end
```

Calculation of the compliance (C) and resistance (R) constants.

```
flow = exhalationFlow(PIP, 0, radio, 1, phi)
C = @(V, PIP, PEEP, t, R, C) (V/(PIP-PEEP)) * exp(t/(R*C));
R = @(F, PIP, PEEP, t, R, C) ((PIP-PEEP)/F) * exp(-t/(R*C));
r_inhalation(end)
volume_vector_meritt = 4/3*pi.*r_inhalation.^3
volume_vector_meritt(end)
R_val = R(flow, PIP, PEEP, 0, 1, 1)
C_val = C(volume_vector_meritt(end),PIP,PEEP,0,1,1)
RC_val = R_val*C_val
```

The following snippet corresponds to the implementation of the loop that calculates the evolution of the pulmonary radius, expiratory pressure, respiratory flow, and the dynamic update of the system properties (resistance and compliance).

```
r_exhalation = @(PIP, PEEP, R, C, t) ((3 * (PIP - PEEP)*C) / (4 * pi) * exp(-t /
(R * C)))^(1/3);
F = @(R, C, PIP, PEEP, t) -1/R * (PIP-PEEP)*exp(-t/(R*C));
ExhalationPressure = @(R, C, VT, t) (VT./C) * exp(-t./(R.*C));

P_final = 0;
P0 = PIP;
increment_pressure = P0;
r = rmax;
r0_exp = rmax;
P0 = PIP;
t=0;
C2 = C_value(PEEP, PIP, r0_exp, r0);

r_vector = [];
P_vector = [];
P2_vector = [];
T_vector = [];
R_vector = [];
C_vector = [];
F_vector = [];
P0_prime_vector = [];


while r >= r0
    dt = 0.01;
    r = r_exhalation(PIP, PEEP, R_val, C_val, t);
    p = pressureModelLungs(P0, C2, r0_exp, r);
    f = F(R_val, C_val, PIP, PEEP, t);
    C_val = C(4/3*pi*r.^3, PIP, PEEP, t, R_val, C_val);
    R_val = R(-f, PIP, PEEP, t, R_val, C_val);
    p2 = ExhalationPressure(R_val, C_val, VT, t);
    P0_prime = turbulent(l, radio, phi, f, 0);

    P_vector(end+1) = p;
    P2_vector(end+1) = p2;
    T_vector(end+1) = t;
    r_vector(end+1) = r;
    F_vector(end+1) = f;
    C_vector(end+1) = C_val;
    R_vector(end+1) = R_val;
    P0_prime_vector(end+1) = P0_prime;

    t = t + dt;
end
```

Code to verify that after one time constant the volume decreases by 63%, after two time constants by 87%, and after three time constants by 95%.

```
V_exhaled_vector = V_vector_prime(1) * 0.37;
[~, idx] = min(abs(V_vector_prime - V_exhaled_vector))


V_exhaled_vector2 = V_vector_prime(1) * 0.13
[~, idx2] = min(abs(V_vector_prime - V_exhaled_vector2))


V_exhaled_vector3 = V_vector_prime(1) * 0.05
[~, idx3] = min(abs(V_vector_prime - V_exhaled_vector3))

vol_val = V_vector_prime(idx);
time_val = T_vector(idx);
vol_val2 = V_vector_prime(idx2);
time_val2 = T_vector(idx2);
vol_val3 = V_vector_prime(idx3);
time_val3 = T_vector(idx3);

figure;
plot(T_vector, V_vector_prime, 'b');
hold on;

% Marcar el punto
plot(time_val, vol_val, 'ro', 'MarkerSize', 8, 'LineWidth', 2);
plot(time_val2, vol_val2, 'ro', 'MarkerSize', 8, 'LineWidth', 2);
plot(time_val3, vol_val3, 'ro', 'MarkerSize', 8, 'LineWidth', 2);
% Mostrar el texto con volumen y tiempo redondeados
str = sprintf('V = %.4f m3\nT = %.2f s', vol_val, time_val);
text(time_val, vol_val, str,
'VerticalAlignment','bottom','HorizontalAlignment','right');
str = sprintf('V = %.4f m3\nT = %.2f s', vol_val2, time_val2);
text(time_val2, vol_val2, str,
'VerticalAlignment','bottom','HorizontalAlignment','right');
str = sprintf('V = %.4f m3\nT = %.2f s', vol_val3, time_val3);
text(time_val3, vol_val3, str,
'VerticalAlignment','top','HorizontalAlignment','right');

xlabel('Tiempo [s]');
ylabel('Volumen [m3]');
title('Espiration Volume vs Time');
grid on;
```

Representation of the complete cycles of pressure, volume, and pulmonary flow.

```
T_total = [T_vector_insp, T_vector + T_vector_insp(end)];
P_total = [P_t, P_vector];

P_total_cmH2O = P_total / 98.0638;

figure;
plot(T_total, P_total_cmH2O, 'LineWidth', 1.5);
xlabel('Time (s)');
ylabel('Pressure (cmH2O)');
title("Pressure during Inspiration and Expiration");
grid on;

T_total_flow = [T_vector_insp, T_vector + T_vector_insp(end)];

F_total = [inflow_vector, F_vector];  % signo negativo para espiración

figure;
plot(T_total_flow, F_total, 'LineWidth', 1.5);
xlabel('Time [s]');
ylabel('Flow [m³/s]');
title('Inspiratory and Expiratory Flow vs Time');
grid on;

T_total_volume = [T_vector_insp, T_vector + T_vector_insp(end)];

V_total = [volume_inhalation, V_vector_prime];

figure;
plot(T_total_volume, V_total, 'LineWidth', 1.5);
xlabel('Time [s]');
ylabel('Volume [m³]');
title('Inhalation and Exhalation Volume vs Time');
grid on;
```