ESCUELA TÉCNICA SUPERIOR DE INGENIERÍA (ICAI)

INGENIERO INDUSTRIAL

# REAL-TIME STATE ESTIMATION OF ACTIVE DISTRIBUTION NETWORKS USING A HYBRID MEASUREMENT MODEL OF REMOTE TERMINAL UNITS AND PHASOR MEASUREMENT UNITS

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# ESTIMACIÓN EN TIEMPO REAL DE REDES DE DISTRIBUCIÓN ACTIVAS USANDO UN MODELO DE MEDIDA HÍBRIDO CON REMOTE TERMINAL UNITS Y PHASOR MEASUREMENT UNITS

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## RESUMEN DEL PROYECTO

**Introducción**

Con el desarrollo de las energías renovables y sistemas de almacenamiento, las redes están experimentando grandes transformaciones. Uno de los ingredientes clave de esta transformación es la capacidad de aprovechar la creciente oferta de los Distributed Energy Resources (DERs). Con el fin de hacerlo, se está llevando a cabo el cambio hacia la gestión activa de redes y el uso de redes de distribución activas (ADNS).

Como las energías renovables muestran producción no constante, las ADNS se caracterizan por una mayor variabilidad y por lo tanto conocer el estado de la red en tiempo real crece en importancia, y con esto, el desarrollo de la estimación de estado en tiempo real (RTSE).

Los principales objetivos de la RTSE son: primero, lograr un control óptimo de tensión para evitar congestiones en la red, minimizar las pérdidas y equilibrar las cargas locales, y segundo, conseguir una detección rápida y ubicación de las faltas en la red, minimizando de esta manera tanto como sea posible el daño asociado.

Las unidades de medida utilizadas en la RTSE son las Remote Teminal Units (RTU) y los Phasor Measurement Units (PMU).

Las RTU son más tradicionales, pueden medir potencias activas y reactivas y magnitudes de tensión y intensidad. Su frecuencia de actualización se encuentra en el rango de 1 a 5 segundos. Las PMUs proporcionan valores precisos sincronizados de fasores de inyecciones de intensidades y tensiones. Ofrecen una mayor precisión que las RTUs. Además, están conectadas al GPS lo que permite un mejor control de ellas. Su frecuencia de actualización es de entre 20 y 100 milisegundos.

El objetivo de este proyecto es ser capaz de estimar el estado dentro de los rangos de control de tensión (unos segundos) y la gestión de faltas (pocos cientos de milisegundos) utilizando un modelo híbrido, es decir, utilizando ambos dispositivos RTUs y PMUs y siendo capaz de estimar a frecuencia PMU teniendo completa observabilidad cuando se tienen los dos tipos de dispositivos. Esto implica que se tienen que crear pseudo-medidas de RTUs a un ritmo PMU para los tiempos en los que las medidas reales RTU no están disponibles.

**Implementación del modelo**

En primer lugar, el estimador de estado Weighted Least Squares fue elegido. Como las RTUs están midiendo potencias, el estimador se convierte en no lineal convirtiéndose el

estimador en un Non Linear Weighted Least Squares (NLWLS). Estando el estado definido en coordenadas polares, ángulo de fase y módulo, la resolución del problema es reducir al mínimo la siguiente ecuación:

$$J(\mathbf{x}) = \sum_{i=1}^{M} \frac{(z_i - h_i(\mathbf{x}))^2}{R_{ii}}$$

Para ello, se requieren iteraciones en cada paso de tiempo, siguiendo los puntos que se muestran a continuación en cada iteración.

- Inicializar el vector de estado, por lo general hecho con un punto de partida con módulo de tensiones igual a 1 y ángulos igual a 0.
- Calcular la función h (x) que liga el estado y la medida, y el Jacobiano H, compuesto por las derivadas parciales de la tensión y potencia.

$$z = h(x) + v \qquad \mathbf{H} = \begin{bmatrix} H_V \\ H_{PQ} \end{bmatrix} = \begin{bmatrix} \frac{\partial V_{ampl}}{\partial \delta} & \frac{\partial V_{ampl}}{\partial V} \\ \frac{\partial \delta}{\partial \delta} & \frac{\partial V}{\partial \delta} \\ \frac{\partial P_{inj}}{\partial \delta} & \frac{\partial P_{inj}}{\partial V} \\ \frac{\partial Q_{inj}}{\partial \delta} & \frac{\partial Q_{inj}}{\partial V} \end{bmatrix}$$

- Calcular la matriz de ganancia G y la función g de la siguiente manera,

$$g(x) = \frac{\partial J(x)}{\partial x} = -H^T(x)R^{-1}[z - h(x)] \qquad G(x^k) = H^T(x^k)R^{-1}H(x^k)$$

- Después, se está en condiciones de calcular el vector de estado del siguiente paso.

$$x^{k+1} = x^k - [G(x^k)]^{-1} g(x^k)$$

- Las iteraciones se detendrán si una de las siguientes condiciones se cumple
  - Condición 1: $J^k$-$J^{k-1}$> 0
  - Condición 2: | $J^k$-$J^{k-1}$ | < ε' donde ε' = $10^{-5}$ p.u.
  - Condición 3: $J^k$ <ε " donde ε " = $10^{-3}$ p.u.

Para la construcción de pseudo-medidas de RTU, se ha implementado una técnica de pronóstico. Además, se decidió utilizar un Single Exponential Smoothing, cuya fórmula es la mostrada a continuación, que era la más adecuada para el perfil de los datos en estudio, y que a su vez permite utilizar el método de intervalos de predicción de una manera sencilla y correcta.

$$F_{t+1} = \alpha Y_t + (1-\alpha)F_t$$

Si la ecuación anterior se utiliza directamente con los datos provenientes de los RTUs, la predicción se hará a un ritmo RTU, en cambio, ya que las previsiones son requeridas a una velocidad de PMU, es necesario la creación de observaciones a esta escala. Esto se realizó por dos métodos.

- Proyección lineal hacia adelante. Se define por la fórmula que se muestra a continuación. Se proyecta una línea que une las dos últimas medidas reales

RTU, dando valores a la siguiente serie de pseudo-medidas hasta que llegue la siguiente medida real de RTU.

$$R_i(t) = R_i(t_{kR-1}) + \left(t - t_{kR-1}\right)\frac{R_i(t_{kR}) - R_i(t_{kR-1})}{t_{kR} - t_{kR-1}}$$

- El uso de las potencias estimadas en el paso anterior como observaciones. Ya que las estimaciones se llevan a cabo en cada paso, teniendo medidas PMU y, o bien medidas reales RTU o pseudo-medidas de RTU, dependiendo del paso de tiempo en el que se esté, se puede calcular la inyección de intensidad en cada nudo utilizando la matriz de admitancias. Por lo tanto las potencias activas y reactivas también pueden ser calculadas.

Con el fin de ser capaz de ejecutar el Estimador de Estado, aparte de la medida, la precisión de esta medida tiene que ser definida. Para las medidas reales esta precisión es dada por el propio dispositivo, sin embargo para las pseudo-medidas, esta tiene que ser calculada. Esto es realizado por medio del método de intervalos de predicción que consiste en dar un límite superior e inferior, asegurando que la observación va a estar dentro de estos límites con un nivel de confianza. Dado que el Single Exponential Smoothing es igual a un Arima (0,1,1), los límites pueden calcularse como:

$W_{t+1}^{LOW} = W_{t+1}^{UPP} = \pm\chi\sqrt{MSE}$ , dónde $MSE = \frac{1}{n}\sum_{t=1}^{n} e_t^{\,2}$ y . $e_t = F_t - Y_t$

Chi es una constante que depende del nivel de confianza escogido.

**<u>Validación del modelo y resultados</u>**

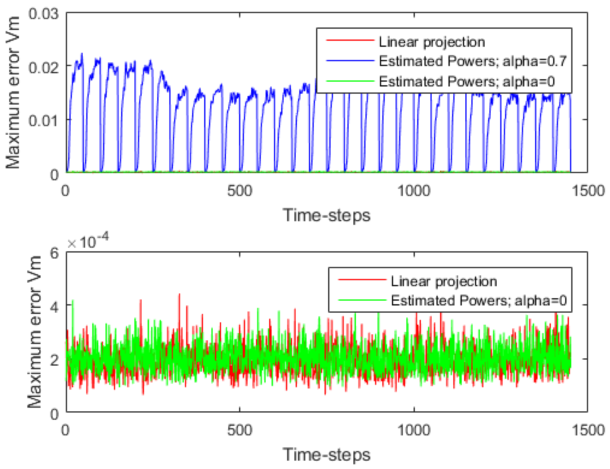
Con el fin de probar el modelo se ha utilizado el alimentador de distribución IEEE 13-bus. Los nudos 2 y 7 se componen de DRS, mientras que todos los demás están formados por cargas. Las RTU están situadas en los nudos 2, 4, 6, 7, 9 y 11, midiendo inyecciones de potencia activa y reactiva, y las PMU están colocadas en los nudos 1, 3, 5, 8, 10, 12 y 13,midiendo fasores de tensión.

Las siguientes suposiciones fueron tomadas:

- Las frecuencias de actualización de las RTUs y las PMUs son de 1 segundo y 20 milisegundos, respectivamente. Por lo tanto habrá 49 pasos de tiempo entre dos medidas de RTU consecutivas donde se crearán pseudo-medidas.
- La desviación estándar de las RTUs es de $10^{-3}$ y de $10^{-4}$ p.u. para las PMUs, respetando de esta manera el hecho de que las PMUs son más precisas.
- La simulación tiene una duración de 1.500 pasos de tiempo de 20 milisegundos cada uno. Por lo tanto se estimará durante 30 segundos en total.
- Las simulaciones son en por unidad con bases de 5 MVA y 15 kV.

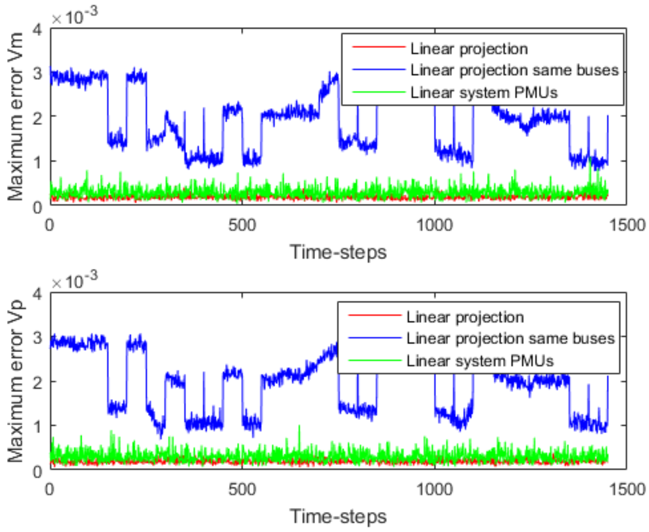
Un problema fue encontrado en el cálculo de los intervalos de predicción. Cuando se calculan los límites, se están usando como error ($e_{t)}$ la diferencia entre el pronóstico y la observación, por lo tanto, no se está considerando el error introducido por la proyección lineal que es en realidad el más grande. Puesto que no podemos conocer los datos reales, este problema no es fácil de resolver y se deja aquí como futuro trabajo. Debido a esto, los resultados se consideran preliminares.

Comparando los dos métodos utilizados para la creación de las observaciones, se vio que el comportamiento de la proyección lineal es mucho mejor que el del método del uso de potencias estimadas. Las potencias estimadas calculadas introducen un error grande en el modelo en cada paso, resultando en un error total mucho mayor. También se probó un tercer método en el que se utiliza el último valor de la medida real de RTU para el siguiente grupo de 49 pseudo-medidas. La comparación del máximo error de la estimación de estado de los tres métodos se muestra en la figura siguiente.




Se puede observar cómo la proyección lineal (rojo) y el uso de la anterior medida real de RTU (verde) muestran una precisión similar y mucho mejor que la del método de las potencias estimadas (azul)

Se probaron otras configuraciones para tener una mejor idea de cómo se comporta el modelo. La comparación se puede observar en la siguiente figura.




En primer lugar, se prueba una configuración con RTUs y PMUs (azul). La precisión disminuye en un orden de magnitud, esto se debe a que la robustez del sistema disminuye, ya que ahora hay algunos nudos en los que no hay unidad de medida.

También se probó una configuración utilizando solo dispositivos PMUs adquiriendo así un sistema lineal (verde). Los PMU aquí miden fasores de voltaje e inyección de intensidad, esto implica que se necesitan menos dispositivos. La precisión se mejora respecto al modelo en el que RTUs y PMUs están situados en los mismos nudos, ya que los PMUs son más precisos que los RTUs y sobre todo que las pseudo-medidas de RTU. Pero en comparación con el modelo original muestran una precisión muy similar debido a la disminución de robustez ya comentado.

**Conclusiones y trabajo futuro**

La proyección lineal funciona mejor que el método de estimación de potencias en el paso anterior.

Dejando el valor de la última medida de RTU para el siguiente grupo de pseudo-medidas también proporciona un resultado interesante.

Aunque hay que tener en cuenta que los resultados son categorizados como preliminares, el modelo propuesto muestra buena precisión, incluso comparable a una configuración utilizando solo dispositivos PMU.

Este trabajo se puede considerar como una primera parte de un proyecto más grande ya que hay varias mejoras y trabajos futuros con los que se puede continuar.

- En primer lugar, resolver el problema descrito y encontrar una manera de explicar el error introducido por la generación de observaciones o encontrar una manera diferente de calcular los intervalos de predicción.
- Cuando esto se haya resuelto, llevar a cabo un análisis de datos erróneos para eliminar los valores atípicos y lograr una mayor precisión.
- Modelar las RTUs para que midan también las tensiones, ya que cuantas más medidas haya disponibles por dispositivo, menor número de ellos serán necesarios para lograr observabilidad. La precisión y robustez disminuirán pero darán lugar a un sistema más económico. Jugando con este balance, un sistema mucho más competente puede ser desarrollado.
- Por último, aplicar el modelo en una red de distribución real siendo capaz de controlarla así en tiempo real, y por qué no, ponerlo en práctica en redes más grandes o de transmisión.

# REAL-TIME STATE ESTIMATION OF ACTIVE DISTRIBUTION NETWORKS USING A HYBRID MEASUREMENT MODEL OF REMOTE TERMINAL UNITS AND PHASOR MEASUREMENT UNITS

## Introduction

With the development of renewables and storage systems, networks are experimenting big transformations. One key ingredient of this transformation is the ability to exploit the increasingly supply of Distributed Energy Resources (DERs). In order to do so we are moving to Active Network Management and the use of Active Distribution Networks (ADNs).

As renewables show non constant production, ADNs are characterized by higher dynamics and therefore being able to know the state at Real Time becomes important, bringing with it, the development of Real Time State Estimation (RTSE) models.

The main goals of RTSE are first, to achieve an optimal voltage control to prevent line congestions, minimize losses and balance local loads, and second, to have a quick fault detection and location, minimizing in this way as much as possible the damage of the network.

The measurement units used in RTSE are the Remote Terminal Units (RTUs) and the Phasor Measurement Units (PMUs).

RTUs are the more traditional ones, they can measure voltage and current magnitudes and active and reactive powers. They're Refresh Rate is in the range of 1 to 5 seconds. PMUs provide precise time-stamped values of injected currents and voltages phasors. They offer better accuracy than RTUs. Moreover, they are connected to the GPS what permits a better control of them. Their Refresh Rate is between 20 and 100 milliseconds.

The goal of this project is to be able to estimate the state within the ranges of voltage control (few seconds) and fault management (few hundreds of milliseconds) using a hybrid model, i.e. using both devices RTUs and PMUs and estimating at a PMU rate having full observability only with the addition of the two kinds of devices. This implies that pseudo-measurements of RTUs have to be created at a PMU rate for the steps where RTU real measurements are not available.

## Implementation of the model

First of all, the state estimator was chosen to be a Weighted Least Squares. Since RTUs are measuring powers, the estimator becomes non-linear having in this way a Non Linear Weighted Least Squares (NLWLS). Being the state defined in polar coordinates, angle phase and module, the resolution of the problem is to minimize the following equation:

$$J(\mathbf{x}) = \sum_{i=1}^{M} \frac{(z_i - h_i(\mathbf{x}))^2}{R_{ii}}$$

In order to do so, iterations in each time-step are required following the points shown below in each iteration.

- Initialize the state vector, usually done with a flat start.
- Calculate function h(x) that links the state to the measurement, and the Jacobian H, composed by the partial derivatives of the voltage and power.

$$z = h(x) + v \qquad \mathbf{H} = \begin{bmatrix} H_V \\ H_{PQ} \end{bmatrix} = \begin{bmatrix} \frac{\partial V_{ampl}}{\partial \delta} & \frac{\partial V_{ampl}}{\partial V} \\ \frac{\partial \delta}{\partial \delta} & \frac{\partial \delta}{\partial V} \\ \frac{\partial P_{inj}}{\partial \delta} & \frac{\partial P_{inj}}{\partial V} \\ \frac{\partial Q_{inj}}{\partial \delta} & \frac{\partial Q_{inj}}{\partial V} \end{bmatrix}$$

- Calculate the gain matrix G and function g as follows,

$$g(x) = \frac{\partial J(x)}{\partial x} = -H^T(x)R^{-1}[z - h(x)] \qquad G(x^k) = H^T(x^k)R^{-1}H(x^k)$$

- Then, we are able to calculate the state vector of the next step.

$$x^{k+1} = x^k - [G(x^k)]^{-1} g(x^k)$$

- Iterations will be stopped if one of the following conditions is satisfied
  - Condition 1: $J^k - J^{k-1} > 0$
  - Condition 2: $|J^k - J^{k-1}| < \varepsilon'$ where $\varepsilon' = 10^{-5}$ p.u.
  - Condition 3: $J^k < \varepsilon''$ where $\varepsilon'' = 10^{-3}$ p.u.

For the construction of RTU pseudo-measurements, a forecasting technique was implemented. Moreover it was decided to use a single exponential smoothing forecasting technique, whose formula is given below, that was the most suitable for the profile of data had, and it allowed us to use the prediction intervals method in an easy and correct manner.

$$F_{t+1} = \alpha Y_t + (1-\alpha) F_t$$

If the previous equation is directly used on the incoming RTU data, the forecasting will be done at an RTU rate, instead, as forecasts are wanted at a PMU rate, it is needed the creation of observations at this scale. This was realized by two methods.

- Linear forward projection. It is defined by the formula shown below. It forwards a line connecting the two last RTU real measurements, giving values to the next set of pseudo-measurements until the subsequent real RTU measurement arrives.

$$R_i(t) = R_i(t_{kR-1}) + (t - t_{kR-1}) \frac{R_i(t_{kR}) - R_i(t_{kR-1})}{t_{kR} - t_{kR-1}}$$

- Use of the estimated powers from the previous step as observations. Since estimations are carried out at each step, having PMU measurements and either real RTU measurements or RTU pseudo-measurements, depending on the step, it can be calculated the injection current in each bus using the admittance matrix. Thus the active and reactive powers are also available.

In order to be able to run the State Estimator, apart from the measurement, the accuracy of this measurement has to be defined as well. For the real measurements it is given by the device itself, but for the pseudo-measurements it has to be calculated. This is achieved by the prediction intervals method that consists in giving an upper and a lower bound to the forecast, assuring with a level of confidence that the observation is going to lie inside these limits. Since the Single Exponential Smoothing is equal to an Arima (0,1,1), the bounds can be calculated as:

$W_{t+1}^{LOW} = W_{t+1}^{UPP} = \pm \chi \sqrt{MSE}$, where: $MSE = \frac{1}{n}\sum_{t=1}^{n} e_t^{\,2}$, and $e_t = F_t - Y_t$.

Chi is a constant that depends on the confidence level chosen.

**Validation of the model and results**

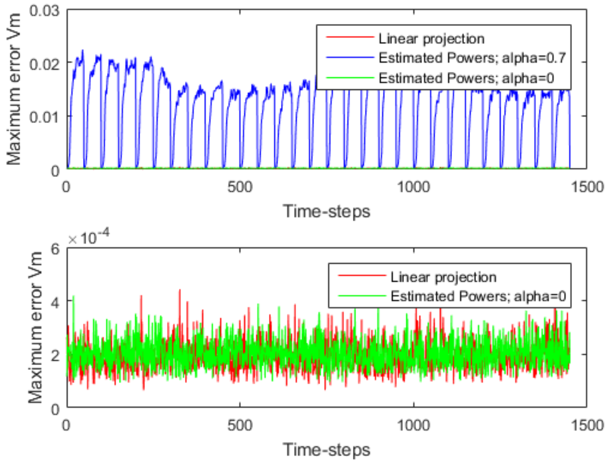
The IEEE 13-bus distribution feeder was used in order to test our model. Buses 2 and 7 are composed of Distributed Energy Resources whereas all the others are formed by loads. RTUs are placed in nodes 2, 4, 6, 7, 9 and 11, measuring active and reactive power injections, and PMUs are located in buses 1, 3, 5, 8, 10, 12 and 13, measuring voltage phasors.

The following assumptions were made:

- The Refresh Rates for RTUs and PMUs are 1 second and 20 milliseconds respectively. So there will be 49 steps in between two consecutive RTU measurements where pseudo-measurements will have to be created.
- The standard deviation for RTUs is $10^{-3}$ and $10^{-4}$ p.u. for PMUs, respecting in this way the fact that PMUs are more accurate.
- The simulation runs for 1500 steps of 20 milliseconds each. Thus 30 seconds in total.
- Simulations are in per unit being the base 5 MVA and 15 kV.

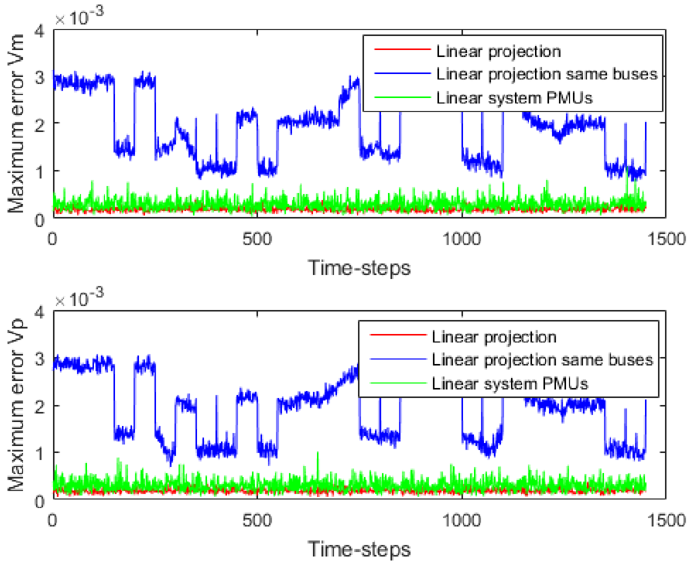
A problem was encountered when calculating the prediction intervals. When calculating the bounds, we are using as error ($e_t$) the one from the forecast to the observations, therefore we are not considering the error introduced by the linear projection which is actually higher than the one being considered. Since we cannot know the real data, this problem is not easy to solve and it is left here as future work. Due to this, results will be categorized as preliminary.

Comparing the two methods used for the creation of observations, it was seen that the linear forward projection behaved much better than the use of estimated powers. The estimated powers calculated introduce a big error into the model at each step resulting in a much bigger total error. It was also tested a third method where the value of the real RTU measurement is used for the next set of 49 pseudo-measurements. The comparison of the state estimation error of the three methods is shown in the figure below.




It can be seen how the linear projection (red) and the use of the previous real RTU measurement (green) show similar accuracy and a much better one than the estimated powers method (blue)

Other configurations were tested to have a better idea of how the model behaves. The comparison can be observed in the following figure.




First, a configuration with RTUs and PMUs placed in the same buses was tested (blue). The accuracy decreases in one order of magnitude, this occurs because the robustness of the system decreases since there are now some buses without measurement unit.

A linear system using only PMUs was also displayed (green). PMUs here measure both voltage and injected current phasors, this implies that less devices are needed. The accuracy is improved to respect the model where RTUs and PMUs are placed in the same buses, since PMUs are more accurate than RTUs and above all than RTU pseudo-measurements. But compared to the original model they show very similar accuracy due to the fact of the robustness.

**Conclusions and future work**

The linear forward projection works better than the previous step estimated powers technique.

Leaving the value of the RTU for the next set of pseudo-measurements is also an interesting result.

Although results are categorized as preliminary, the model proposed shows good accuracy, even comparable to a configuration with only PMU devices.

This project can be considered as a first part of a bigger one since there are more improvements and future work to continue with.

- First of all, solve the described problem and find a way to account for the error introduced by the generation of observations or find a different way of computing the prediction intervals.
- When this is solved, run a bad data analysis to remove outliers and achieve a better accuracy.
- Model RTUs so that they measure also voltages, the more measurements available per device, the less of them will be needed to have observability. Accuracy and robustness will decrease but it will result in a more economic system. Playing with this trade off, a much more competent system can be developed.
- Finally, implement the model in a real distribution network to control it at real time, and why not, implement it in bigger networks or transmission ones.

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# REAL-TIME STATE ESTIMATION OF ACTIVE DISTRIBUTION NETWORKS USING A HYBRID MEASUREMENT MODEL OF REMOTE TERMINAL UNITS AND PHASOR MEASUREMENT UNITS

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# Contents

# 1 Introduction

## 1.1 Evolution of the electricity networks

Current distribution networks are experimenting technological advances that enable big transformations in the electricity system mainly caused by the introduction of renewables, that usually don´t have a constant production, and new storage systems.

One key ingredient of this transformation is the ability to exploit the increasingly supply of Distributed Energy Resources (DERs), such as Distributed Generation (DG), Demand Response (DR) and Distributed Energy Storage Systems (DESS). In order to be able to do so, new technologies and methods in energy grid planning and operation have to be introduced, moving in this way to Active Network Management.

The so called Active Distribution Networks (ADNs) are able to control the different DERs, for both the production and the storage, leading to a flexible topology of the network and the availability to have bidirectional electricity transportation.[12]

## 1.2 Real Time State Estimation

One of the features that has become of big importance with the development of the ADNs is the estimation of the state in real time.

### 1.2.1 Operation functionalities

This Real Time State Estimation (RTSE) has different operation objectives:

- **Optimal voltage control**: Since with the introduction of renewable production, ADNs are characterized by higher dynamics, it is important to be able to control the voltages in the buses having as goal to prevent line congestions, minimize network losses and balance local loads.
- **Fault detection and location**: By knowing the state at Real Time, faults can be detected and arranged rapidly in order to have the less possible damage in the network. The system should also be in charge of the corresponding post-fault management in order to go back to normal operation.[11]

### 1.2.2 Measurement Units: RTUs and PMUs

The main function of these devices is to measure the state of the bus where they are placed and send this information to SCADA. In order to attain full observability, measurement devices have to be placed smartly in the appropriate buses so that the whole system state can be estimated. This is usually done by using the least amount of devices in order to minimize the cost. Although some models have preferred the use of more units for redundancy as explained later in the literature review section.

The two main devices used for RTSE are Remote Terminal Units (RTUs) and Phasor Measurement Units (PMUs).

**Remote Terminal Unit(RTU):** They are the most traditional devices used in state estimation.They usually take measurements of voltage and current magnitudes, active and reactive power flows and injections, of the bus in which they are installed. They are controlled by the SCADA system to which they send the taken measurements. Their Refresh Rate (RR), i.e the time between two consecutive measurements, is usually between 1 and 5 seconds.

**Phasor Measurement Unit (PMU):** It is a new technology whose use in state estimation is currently in initial development and that it is one of the most promising metering devices for the future of power systems [13]. It provides precise time-stamped values of the voltages and injected currents phasors (amplitude and phase angle) for the bus where it is installed. It can also measure the currents that flow in the network branches, which are adjacent to the bus where the PMU is installed. It offers a better accuracy than RTUs, although it is a more expensive technology, and their RR is normally between 20 and 100 milliseconds. The time-stamping of the phasors is achieved due to the fact that the PMU is connected to the GPS.

### 1.2.3 State Estimation structure and steps

State estimation takes place in a few steps. First, RTUs and/or PMUs that are located in specific buses in order to have full observability, measure the corresponding voltages and, in our case, injected currents and/or powers. Then the data is sent to the Phasor Data Concentrator (PDC) via a telecom infrastructure. The incoming data is encapsulated and time-stamped. Right after, the State Estimator (SE) solves a minimization problem and estimates the state(voltage and current phasors, active and reactive powers) for every bus in the network.
The SE can be formulated by using either a Weighted Least Squares (WLS) or a Kalman Filter (KF) algorithm, which will be explained with more detail later on through the project.
Each of the steps described above brings with it some delay. The sum of all these delays is called the total latency, and will result in the total time require to estimate the state.

## 1.3 Motivation and goals of the project

As the above mentioned functionalities' time frames are between few hundred of milliseconds, for fault management, to a few seconds, for voltage control and line congestions in presence of highly-volatile renewables, it is important to be able to estimate the state within these ranges to be able to anticipate them at every step. Since PMU refresh rates are in the order of dozens to hundreds of milliseconds, they look to be more suitable than RTUs.
As was mentioned above, PMUs are expensive devices and they are still not used very often in real ADNs.
The aim of this project is to create a model using both RTUs and PMUs, having full observability with the addition of them but not by having them separately, and to be able to estimate the state at a PMU rate. To do so, virtual RTU pseudo-measurements have to be created at a PMU RR for the buses where RTUs are installed.

In section 2 a review of the literature is provided. Section 3 describes the two main state estimators (WLS and KF) with their algorithms. Section 4 describes how the RTU pseudo-measurements can be constructed using an exponential smoothing forecasting technique and the implementation of the prediction intervals. In section 5 a preliminary validation of the model is presented with the corresponding simulations and results. Section 6 shows different scenarios and modifications done to the original model to have a better understanding of its behavior. Finally section 7 presents the conclusions and future work for this study.

## 2 Review of the literature

A review of the literature was done in order to get ideas and have a better understanding of how the model could be developed, also to try to find out what were the weakness points of other models to be able to make a competitive one.

There have been several models that have added PMUs for redundancy to their original RTU measurement based system, i.e. they attain observability with only the RTUs, so PMUs are only added to have a more accurate measuring and to improve in this way the state estimation. This is the case of papers [5] and [6].

In [1], PMUs and RTUs are also combined, attaining observability with both the two types of devices but not separately. In order to be able to estimate the state in between two consecutive RTU measurements, that in this case is a gap of 4 seconds, it uses the so called pseudo-measurements, or RTU virtual measurements, that are somehow emulating real RTU measurements in order to be able to estimate the state in a shorter range of time. In this paper the pseudo-measurements are calculated using a method called the *Hatchel's Augmented Matrix*.
The model is able to estimate each one second, which is bigger than the PMU RR that wants to be achieved by the model described later in the report.
Another weak point found in this paper is the fact that it is given the same accuracy ($\sigma$) to RTU and PMU devices which is not a correct hypothesis since PMUs have been proved to have better accuracy and performance. This paper is one of the only ones that takes into account the delays that RTUs have when measuring and delivering the data to the PDC. These delays are unknown since RTUs don´t count with a GPS synchronization.

In [2] and [3] Ankush Sharma et Al. use Holt's method for forecasting the state so that the SE (Kf in [2] and WLS in [3]) gains more accuracy. In [2] RTUs and PMUs have full observability separately which implies the need of a large number of devices. Their measurements are scanned and processed in parallel to be put together afterwards to coordinate the final state estimation. This is what is called a Multi-Agent technique.
In [3] the system is divided into subareas. In the ones where observability is attained with PMUs they are able to estimate at a PMU rate. On the other hand in the areas where they don´t have full observability they are not able to estimate the state and thus the full system state estimation is not possible at a PMU RR.

Paper [4] forecasts the RTU virtual measurements by using the average of the last 15 days' power rate measured at the same time slot as the ones being forecast. It can only estimate the state each 2 seconds (at a RTU RR) and not every 40 ms (PMUS rate).

In [8] and [9] k. Sinha et Al. explain two similar models where observability is also obtained with the sum of RTUs and PMUs, in order to be able to estimate the state between two RTU consecutive measurements, it uses an interpolation technique forwarding the PMU data of the buses that count with this type of device to the ones that don´t.
Less reliability is also given to data coming from buses far from the ones with PMUs installed. The paper also offers a comparison of different configurations of measurement devices to show that the more PMUs are used in the system the better the accuracy will be.

References [10] and [11] show two systems that have been implemented to real distribution networks, in Alliander in the Netherlands and in the EPFL campus in Lausanne (Switzerland) respectively, using only PMUs. The whole architecture and set-up of the system is described. Discrete Kalman Filter is the SE adopted. The results appear to be very good showing the high accuracy of systems that only use PMUs as their measuring devices. Having an error of the order of $10^{-5}$. Latencies are also displayed to prove that the estimation is achieved within the ranges required for the already mentioned operation functionalities. Having results of 55 milliseconds for paper [10] and 61 milliseconds for paper [11].

Also books like *Power System State Estimation* [14], from Abur and Exposito, and *Power System Analysis* [15], from Grainger and Stevenson, were checked and studied for a better understanding in State Estimation. Similarly, Jean-Yves Le Boudec book, *Performance evaluation of computer and communication systems*[18] and S. Makridakis et Al. book, *Forecasting Methods and Applications*[20], were used for getting the required knowledge in statistics used in the development of the model such as forecasting, prediction intervals, distribution and autocorrelation of a set of data etc.

It is also important to highlight that the previous referenced papers as well as others that have been checked have the following characteristics:

- The Refresh Rate given to RTUs is always between 1 and 5 seconds, and 20 to 100 milliseconds for PMUs.
- The accuracy given to the two different types of devices in the form of their standard deviation ($\sigma$) are in the following ranges:
  - RTU:
    - Powers: 0.01 - 0.1 p.u.
    - Voltages: 0.004 - 0.01 p.u.
  - PMU:
    - Voltage magnitude: $1 * 10^{-4}$ - 0.002 p.u.
    - Voltage phase: $1 * 10^{-4}$ - 0.0017 radians

# 3 Implementation of the state estimation

In order to be able to estimate the state, as it has been already mentioned, it is mandatory to have full observability. This means that the number of measurements have to be larger than the number of states estimated that is equal to two times the number of nodes.
Measurements depending on the device can be in form of voltages, injected currents, injected powers and power flows. Usually PMUs measure voltages and/or injected currents and RTUs voltages and/or injected powers. For brevity it will be considered to have a balanced network making in this way reference to the direct sequence only. Measurements then can be written with the following equations:

$$\begin{aligned}\mathbf{z}_V &= [\mathbf{V}_{i,real}, ..., \mathbf{V}_{m,real}, \mathbf{V}_{i,imag}, ..., \mathbf{V}_{m,imag}]^T \\ \mathbf{z}_V &= [\delta_i, ..., \delta_m, \mathbf{E}_i, ..., \mathbf{E}_m]^T\end{aligned} \tag{1}$$

$$\mathbf{z}_I = [\mathbf{I}_{i,real}, ..., \mathbf{I}_{m,real}, \mathbf{I}_{i,imag}, ..., \mathbf{I}_{m,imag}]^T. \tag{2}$$

$$\mathbf{z}_{PQ} = [\mathbf{P}_i, ..., \mathbf{P}_m, \mathbf{Q}_i, ..., \mathbf{Q}_m]^T. \tag{3}$$

Where $z_V$ is the phasor measurement that can be written as real and imaginary part (rectangular notation) or as angle phase and amplitude (polar notation). $z_I$ and $z_{PQ}$ are the measurements of current and power respectively.
The state of a n-buses network can be expressed as:

$$\mathbf{x} = [\mathbf{V}_{i,real}, ..., \mathbf{V}_{n,real}, \mathbf{V}_{i,imag}, ..., \mathbf{V}_{n,imag}]^T \tag{4}$$

$$\mathbf{x} = [\delta_i, ..., \delta_n, \mathbf{E}_i, ..., \mathbf{E}_n]^T \tag{5}$$

So in order to have full observability 2m $\geq$ 2n. Where m is the number of measurements and n the number of nodes. Depending on how many measurements each device is able to provide, we will need more or less of them. Since each measurement has a real and an imaginary, the total number of them is 2m. Same thing occurs with the states, since we have a complex number for each node.

## 3.1 Weighted Least Squares

The WLS is the most common method to solve minimization problems such as the state estimation. It is a static estimator since it infers the system state by only using current time information.
The estimator will output the states of each bus ($x_1$,...,$x_n$) taking as inputs:

- Measurements: $z_V$, $z_I$, and/or $z_{PQ}$
- Accuracy of measurements: given by the noise covariance matrix R. It is assumed that the measurement errors are independent, getting in this way a diagonal matrix.

$$\mathbf{R} = diag(\sigma_1^2, \dots, \sigma_M^2) = \begin{bmatrix} R_{11} & 0 & 0 \\ 0 & \ddots & 0 \\ 0 & 0 & R_{MM} \end{bmatrix} \tag{6}$$

where M is the total amount of measurements (2m)

- Admitance matrix $\bar{Y}$ that defines the structure of the network and links in each bus, the voltage to the respective injected current.

$$\bar{\mathbf{Y}} = \begin{bmatrix} \bar{Y}_{11} & \bar{Y}_{12} & \cdots & \bar{Y}_{1n} \\ \bar{Y}_{21} & \bar{Y}_{22} & \cdots & \bar{Y}_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ \bar{Y}_{n1} & \bar{Y}_{n2} & \cdots & \bar{Y}_{nn} \end{bmatrix} \quad (7)$$

Each term of the matrix could be split into its real and imaginary part as follows:

$$\bar{Y}_{ij} = G_{ij} + \text{i}.B_{ij} \quad (8)$$

where i and j are the bus indexes.

### 3.1.1 Non-linear WLS

In the non-linear WLS, the state is defined in polar coordinatesas shown in equation (5). Usually when RTUs are used, like it is the case of the described model in the report where power injections into the buses are measured, the link between measurements (z) and states (x) is non-linear and it is expressed by the non-linear measurement function $h(x)$.

$$\mathbf{z} = \mathbf{h}(\mathbf{x}) + \mathbf{v} \quad (9)$$

where v represents the measurement noise, assumed to be white and with a normal probability distribution of covariance R.

$$p(\mathbf{v}) \sim N(0, \mathbf{R}) \quad (10)$$

The resolution of the problem is to minimize the following equation:

$$J(\mathbf{x}) = \sum_{i=1}^{M} \frac{(z_i - h_i(\mathbf{x}))^2}{R_{ii}} \quad (11)$$

Then, J(x) is derived and equalized to 0.

$$\mathbf{g}(\mathbf{x}) = \frac{\partial J(x)}{\partial x} = -\mathbf{H}^{\mathbf{T}}(\mathbf{x}) R^{-1}[z - h(x)] = 0 \quad (12)$$

where:

$$\mathbf{H}(\mathbf{x}) = \frac{\partial h(x)}{\partial x} \quad (13)$$

that is the linearized version of h(x) and in the case of having voltages and injected powers yields to:

$$\mathbf{H} = \begin{bmatrix} H_V \\ H_{PQ} \end{bmatrix} = \begin{bmatrix} \frac{\partial V_{ampl}}{\partial \delta} & \frac{\partial V_{ampl}}{\partial V} \\ \frac{\partial \delta}{\partial \delta} & \frac{\partial \delta}{\partial V} \\ \frac{\partial P_{inj}}{\partial \delta} & \frac{\partial P_{inj}}{\partial V} \\ \frac{\partial Q_{inj}}{\partial \delta} & \frac{\partial Q_{inj}}{\partial V} \end{bmatrix} \quad (14)$$

where $H_V$ is the part of the *Jacobian* that is related to the partial derivatives of the magnitude and angle phase of the voltage. They are equal to 1 in the nodes where there is a measurement unit and 0 in the nodes where there is no unit.

On the other hand, the partial derivatives of the powers are a little bit more complex as shown below:

$$\frac{\partial \mathbf{P_i}}{\partial \delta_\mathbf{i}} = \sum_{j=1}^{n} V_i V_j(-G_{ij} sin\delta_{ij} + B_{ij} cos\delta_{ij}) - V_i^2 B_{ii}$$
$$\frac{\partial \mathbf{P_i}}{\partial \delta_\mathbf{j}} = V_i V_j(G_{ij} sin\delta_{ij} - B_{ij} cos\delta_{ij})$$
$$\frac{\partial \mathbf{P_i}}{\partial \mathbf{V_i}} = \sum_{j=1}^{n} V_j(G_{ij} cos\delta_{ij} + B_{ij} sin\delta_{ij}) + V_i G_{ii}$$
$$\frac{\partial \mathbf{P_i}}{\partial \mathbf{V_j}} = V_i(G_{ij} cos\delta_{ij} + B_{ij} sin\delta_{ij})$$
$$\frac{\partial \mathbf{Q_i}}{\partial \delta_\mathbf{i}} = \sum_{j=1}^{n} V_i V_j(G_{ij} cos\delta_{ij} + B_{ij} sin\delta_{ij}) - V_i^2 G_{ii}$$
$$\frac{\partial \mathbf{Q_i}}{\partial \delta_\mathbf{j}} = V_i V_j(-G_{ij} cos\delta_{ij} - B_{ij} sin\delta_{ij})$$
$$\frac{\partial \mathbf{Q_i}}{\partial \mathbf{Vi}} = \sum_{j=1}^{n} V_j(G_{ij} sin\delta_{ij} - B_{ij} cos\delta_{ij}) - V_i B_{ii}$$
$$\frac{\partial \mathbf{Q_i}}{\partial \mathbf{Vj}} = V_i(G_{ij} sin\delta_{ij} - B_{ij} cos\delta_{ij}) \tag{15}$$

where i and j make reference to the different nodes of the network. As it can be seen the partial derivatives have cosines and sines in their development, this is what makes the system non-linear, and iterations will be needed to solve the problem at each time-step.

Going back to equation (12), expanding the non-linear function $\mathbf{g(x)}$ into its Taylor series around the state vector x, and neglecting the higher order terms, it leads to the final equation that gives us the state of iteration k+1 in relation to the previous one:

$$\mathbf{x^{k+1}} = \mathbf{x^k} - [G(x^k)]^{-1} * g(x^k) \tag{16}$$

where G(x) is the so called Gain Matrix and it is equal to:

$$\mathbf{G(x)^k} = \frac{\partial g(x)}{\partial x} = H^T(x^k) * R^{-1} * H(x^k) \tag{17}$$

In order to solve the the non-linear weighted least square problem, an iterative algorithm with the following steps has to be taken in each time-step:

1. Initialize the state vector $x^0$, typically as a flat-start (all bus voltages magnitudes are assumed to be 1 p.u. and angle phases 0);

2. Calculate the nonlinear function $h(x^k)$ and the matrix $H(x^k)$ from equations (9) and (14) respectively;

3. Calculate the Gain Matrix $G(x^k)$ and the function $g(x^k)$ from equations (17) and (12);

4. Calculate the new state vector $x^{k+1}$ with equation (16).

5. Calculate $J^k$ with equation (11) and stop if one of the following conditions is satisfied:
   - Condition 1: $J^k - J^{k-1} > 0$. Since the goal is to minimize J, it should get smaller in each iteration, if it does not, it means that the problem has converged.
   - Condition 2: $|J^k - J^{k-1}| < \varepsilon'$
   - Condition 3: $J^k < \varepsilon''$

   $\varepsilon'$ and $\varepsilon''$ are previously chosen by the user. In our case they were chosen to $10^{-5}$ and $10^{-3}$ respectively.

### 3.1.2 Linear WLS

When measurements come only from PMUs, the SE can be formulated in a linear way. The system state is now defined in rectangular coordinates as in equation (4).
In this case, the link between measurements and states is exact and is expressed through matrix H as follows:

$$\mathbf{z} = \mathbf{H}\mathbf{x} + \mathbf{v} \tag{18}$$

where H is equal to:

$$\mathbf{H} = \begin{bmatrix} H_V \\ H_I \end{bmatrix} \tag{19}$$

$H_V$ is still going to be formed by ones and zeros as explained before.The elements of the matrix $H_I$, since I and V are expressed in rectangular coordinates, are going to be directly the elements of the admittance matrix $\bar{Y}$.

$$\mathbf{H_I} = \begin{bmatrix} [G_{ij}] & [-B_{ij}] \\ [B_{ij}] & [G_{ij}] \end{bmatrix} \tag{20}$$

making the link between the measurements and the states linear since G and B remain constant.
Therefore equation (11) can be rewritten as:

$$J(\mathbf{x}) = \sum_{i=1}^{M} \frac{(z_i - \mathbf{H}\mathbf{x})^2}{R_{ii}} \tag{21}$$

getting directly as a result for the estimated system state:

$$\widehat{\mathbf{x}} = \mathbf{G}^{-1}\mathbf{H}^T\mathbf{R}^{-1}\mathbf{z}. \tag{22}$$

with G being the Gain Matrix shown in equation (17).
It can be observed that now since the algorithm is linear there is no need for iterations, and the state can be estimated in one shot for each time-step.

## 3.2 Kalman Filter

Kalman Filter is a type of dynamic SE. These kind of estimators aim to obtain the system state at a given time by taking into account information available from both measurements and a process model. It first makes a prediction, and then with the actual measurements corrects or filters this prediction, improving in this way the efficiency.
Kalman filter can also be implemented for linear and non-linear system that will depend as in the WLS on the set of measurements used and the composition of the matrix H.

### 3.2.1 Linear KF

The linear KF also called Discrete Kalman Filter (DKF) is defined by a set of two equations:

$$\mathbf{x}_k = \mathbf{A}\mathbf{x}_{k-1} + \mathbf{B}\mathbf{u}_{k-1} + \mathbf{w}_{k-1} \tag{23}$$

$$\mathbf{z}_k = \mathbf{H}\mathbf{x}_k + \mathbf{v}_k \tag{24}$$

where:

- $\mathbf{x}_k$ and $\mathbf{x}_k - 1$ represent the state of the system at time-steps $k$ and $k-1$ respectevely.
- $\mathbf{u}_{k-1}$ represents a set of control variables at time-step $k-1$.
- $\mathbf{A}$ is a matrix that links the system state at time-step $k-1$ with the one at the current time-step $k$, for the case of null control variables and process noise. In principle, matrix $\mathbf{A}$ might change at each time-step but it is usually set equal to the identity matrix $\mathbf{A} = \mathbf{I}$ to reduce complexity.
- $\mathbf{B}$ is a matrix that links the time evolution of the system state with the control variables at time-step $k-1$, for the case of null process noise. It is usually set equal to 0 to make the control variables have no effect on the state of the system.
- The second equation is the same as the one seen for the WLS (equation (18)) where $\mathrm{z}_k$ is the available measurement at the current time-step $k$ and matrix H is the measurements Jacobian.
- $v_k$ and $w_{k-1}$ represent the measurement noise and the process noise respectively. They are assumed to be independent between each other and both white with a normal probability distribution.

$$\begin{aligned} p(\mathbf{w}) &\sim N(0, \mathbf{Q}) \\ p(\mathbf{v}) &\sim N(0, \mathbf{R}) \end{aligned} \tag{25}$$

  $\mathbf{R}$ is the noise covariance matrix already defined in equation (6) for the WLS algorithm; and $\mathbf{Q}$ is the process noise covariance matrix and represents the uncertainties introduced in the computation of the next state.. $\mathbf{Q}$'s values are usually arbitrarily selected and remained constant during the different steps, although they can be computed at each step if the process is known, improving considerably, in this way, the accuracy of the estimation. This process is explained in [16] with detail and will not be discussed here for brevity.

Hence, we can define $\widehat{\mathbf{x}}_k^-$ as the "a-priori" state estimate and $\widehat{\mathbf{x}}_k$ as the "a-posteriori" state estimate at step k, getting the "a-priori" and "a-posteriori" estimate errors:

$$\begin{aligned} \mathbf{e}_k^- &= \mathbf{x}_k - \widehat{\mathbf{x}}_k^-. \\ \mathbf{e}_k &= \mathbf{x}_k - \widehat{\mathbf{x}}_k. \end{aligned} \tag{26}$$

and the "a-priori" and "a-posteriori" estimate error covariance:

$$\begin{aligned} \mathbf{P}_k^- &= E[\mathbf{e}_k^- \mathbf{e}_k^{-T}] \\ \mathbf{P}_k &= E[\mathbf{e}_k \mathbf{e}_k^T] \end{aligned} \tag{27}$$

were $\mathbf{E}$ indicates the expected or mean value operator.

The process of the KF occurs in two steps, the time update equation, that is the prediction part, and the measurement update equations where the prediction is corrected once the measurements are obtained.

- Time update equation (prediction): "a-priori" variables are calculated.

$$\widehat{\mathbf{x}}_k^- = A\widehat{\mathbf{x}}_{k-1} + Bu_{k-1} \tag{28}$$

$$\mathbf{P}_k^- = A\mathbf{P}_{k-1}A^T + \mathbf{Q} \tag{29}$$

  where:

  - $\widehat{\mathbf{x}}_{k-1}$ is the "a-posteriori" estimated system state calculated in the previous time-step $k-1$;
  - $\mathbf{P}_{k-1}$ is the "a-posteriori" estimate error covariance matrix calculated in the previous time-step $k-1$.

- Measurement update equations (correction of the estimation): "a-posterior" variables are calculated.

$$\mathbf{K}_k = \mathbf{P}_k^-\mathbf{H}^T(\mathbf{H}\mathbf{P}_k^-\mathbf{H}^T + \mathbf{R})^{-1} \tag{30}$$

$$\widehat{\mathbf{x}}_k = \widehat{\mathbf{x}}_k^- + \mathbf{K}_k(\mathbf{z}_k - \mathbf{H}\widehat{\mathbf{x}}_k^-) \tag{31}$$

$$\mathbf{P}_k = (\mathbf{I} - \mathbf{K}_k\mathbf{H})\mathbf{P}_k^- \tag{32}$$

  where:

  - $\mathbf{K}_k$ is called the "Kalman Gain" and minimizes the "a-posteriori" error covariance $\mathbf{P}_k$ in the current time-step $k$;
  - $\mathbf{z}_k$ is the measurements array in the current time-step $k$ given by (24).
  - The difference $(\mathbf{z}_k - \mathbf{H}\widehat{\mathbf{x}}_k^-)$ is called the measurement residual and it represents the difference between the predicted measurement $\mathbf{H}\widehat{\mathbf{x}}_k^-$ and the actual measurement $\mathbf{z}_k$. It can be noted that when the a priori estimate error $\mathbf{P}_k$ tends to zero, the Kalman Gain also tends to zero. The residual is also weighted less heavily. On the contrary when the measurement covariance $\mathbf{R}$ tends to zero, the residual is weighted more heavily:

$$\lim_{\mathbf{P}_k \to 0} \mathbf{K}_k = \mathbf{H}^{-1} \tag{33}$$

### 3.2.2 Non-linear KF

For solving non-linear state estimation problems, the Extended Kalman Filter (EKF) can be used. The procedure of resolution is the same than with the DKF, but now equations (23) and (24) change to:

$$\mathbf{x}_k = f(x_{k-1}, u_{k-1}, w_{k-1}) \tag{34}$$

$$\mathbf{z}_k = h(x_k, v_k) \tag{35}$$

being **f** and **h** non-linear functions.

Time update equations change to:

$$\widehat{\mathbf{x}}_k^- = f(\widehat{x}_{k-1}, \widehat{u}_{k-1}, 0) \tag{36}$$

$$\mathbf{P}_k^- = A\mathbf{P}_{k-1}A^T + W_k\mathbf{Q}_{k-1}W_k^T \tag{37}$$

Measurement update equations are now equal to:

$$\mathbf{K}_k = \mathbf{P}_k^-\mathbf{H}_k^T(\mathbf{H}_k\mathbf{P}_k^-\mathbf{H}_k^T + V_k\mathbf{R}_kV_k^T)^{-1} \tag{38}$$

$$\widehat{\mathbf{x}}_k = \widehat{\mathbf{x}}_k^- + \mathbf{K}_k(\mathbf{z}_k - h(\widehat{x}_k^-), 0) \tag{39}$$

$$\mathbf{P}_k = (\mathbf{I} - \mathbf{K}_k\mathbf{H}_k)\mathbf{P}_k^- \tag{40}$$

where:

- Noises $v_k$ and $w_{k-1}$ from equations 33 and 34 are approximated to 0.
- $\mathbf{V}_k$ and $\mathbf{W}_k$ are the measurement noise Jacobian matrix and the process noise Jacobian matrix respectively.
- The most important change that has to be noted from the DKF to the EKF is that now, the matrix $\mathbf{H}$ has changed to $\mathbf{H}_k$, which means that it does not remain constant and changes every time-step, similar to what happened in the WLS algorithm.

As proved in reference [16] when only PMUs are used for state estimation, if the calculation of $\mathbf{Q}$ is done at every time-step, the accuracy shown by the DKF is better than the one shown by the linear WLS. It is also true that the latency of the DKF is larger but as it is still in a range of tens of milliseconds it is not an impediment for reaching the operation functionalities of RTSE (voltage control and fault detection).
On the other hand, when also using RTUs, the system becomes non-linear and therefore the algorithms become more complex. Since the WLS is simpler and also shows good performance, it is the one that will be used in the model in question in order to avoid the complexity of the EKF and the corresponding calculations of matrix $\mathbf{Q}$.

# 4 Construction of the RTU virtual pseudo-measurements

Since both PMUs and RTUs are being used in the model, in order to have full observability at a PMU rate, virtual pseudo-measurements have to be constructed to fulfill the lacking income of RTU data in the gap between two consecutive RTU measurements. These pseudo-measurements are created in the same buses where RTUs are placed.

## 4.1 Forecasting and Forward Projection

In order to construct the pseudo-measurements, it was thought about implementing a forecasting technique, so that by relying in previous data, virtual measurements can be created to simulate real future incoming values.

Three exponential smoothing forecasting methods were taking into account for this study:

- **Single exponential smoothing**: It is the simplest of the three methods and it is determined by the next equation:

$$F_{t+1} = \alpha Y_t + (1-\alpha)F_t \tag{41}$$

  where $F_{t+1}$ is the forecast for next step. It is calculated by the weighted sum of the real or observed value $Y_t$ and the forecast $F_t$ in step $t$. $\alpha$ is the smoothing constant or weighting factor, and takes a number between 0 and 1. This model works well when the data do not present a trend and seasonality. Big values of $\alpha$ will translate into trusting more the observation $Y_t$, and low values will mean giving more weight to the previous forecast $F_t$

- **Holt's model**: This method is suitable when the time-series in study presents a clear trend (ascending or descending).

$$\begin{aligned} L_t &= \alpha Y_t + (1-\alpha)(L_{t-1} + b_{t-1}) \\ b_t &= \beta(L_t - L_{t-1}) + (1-\beta)b_{t-1} \\ F_{t+1} &= L_t + b_t \end{aligned} \tag{42}$$

  where F is the forecast value, t is the time step, L is the level or base value, b is the trend, Y is the observed value and $\alpha$ and $\beta$ are smoothing parameters with values between 0 and 1

- **Holt Winter's model**: It is used when both a trend and seasonality are observed in the time-series. It is the most complex method of the three, but it is the one that better works with this kind of data. The formulas for implementing it are the following:

$$\begin{aligned} L_t &= \alpha \frac{Y_t}{S_{t-c}} + (1-\alpha)(L_{t-1} + b_{t-1}) \\ b_t &= \beta(L_t - L_{t-1}) + (1-\beta)b_{t-1} \\ S_t &= \gamma \frac{Y}{L_t} + (1-\gamma)S_{t-c} \\ F_{t+1} &= (L_t + b_t)S_{t-c+m} \end{aligned} \tag{43}$$

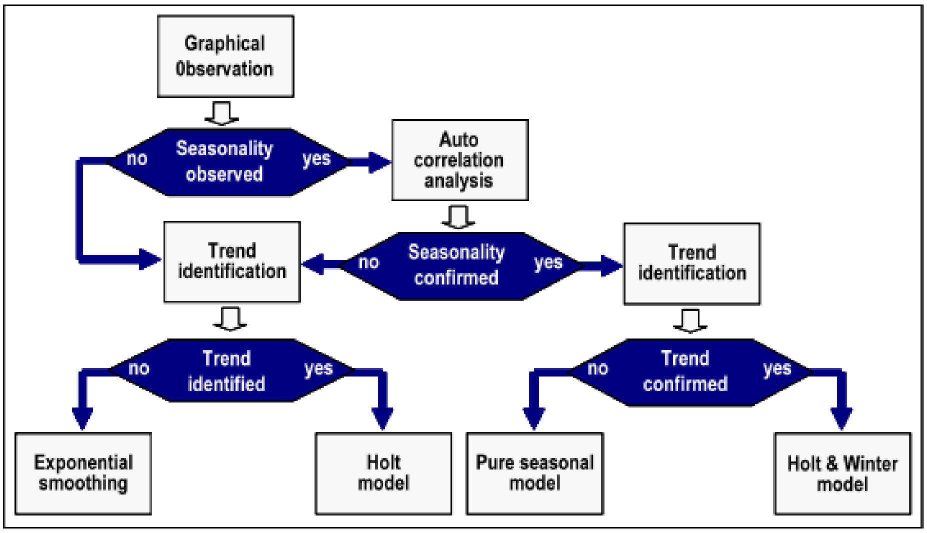


Figure 1: Decision flowchart for forecasting model selection

It can be seen that the equations are very similar to the Holt's model, but now the parameter $S$ is introduced for reflecting the seasonality. $\gamma$ is another parameter between 0 and 1 like $\alpha$ and $\beta$. $c$ is the length of seasonality (e.g., the number of months or quarters in a year) and $m$ is the length of the time-series. Parameters $\alpha$, $\beta$ and $\gamma$, in all the three models, can be optimized to get a more accurate model depending on the data in study. This is usually done by a trial and error method.

In order to choose the most appropriate model for the time-series in question, the flowchart shown in **figure 1** can be followed.
Although forecasting looks like a suitable technique for constructing the pseudo-measurements, it has a big issue that has to be solved. The RR of RTUs is in the order of a few seconds, therefore if the next value of RTUs is forecast, it will be done for the next seconds. Instead, what it is wanted here is to perform a forecast for the next few milliseconds (see **figure 2**). If one of the models explained above is used considering for the Y value (observation) the last measured real RTU, then the next forecast $F_{t+1}$ will be at an RTU RR and not at the PMU desired rate.

In order to fix this setback, the observation value (Y) has to be calculated by a different alternative since we don´t have access to the real values for RTUs at a PMU rate.
Two methods where considered in this study.

- **Linear forward projection technique**. This method creates virtual observations $Y$ at a PMU rate using the two previous RTU real measurements. A straight line is sketched connecting the two previous RTU real measurements and will be used as the upcoming time period's observations until a new RTU measurement arrives. This procedure is defined by the next equation:

$$Y(t) = R(t_{R-1}) + (t - t_{R-1})\frac{R(t_{R-1}) - R(t_{R-2})}{t_{R-1} - t_{R-2}} \tag{44}$$

where $Y(t)$ is the projected value computed for each PMU rate steps in between the two next RTU measurements. $(t_{R-1})$ and $(t_{R-2})$ are the previous two steps where

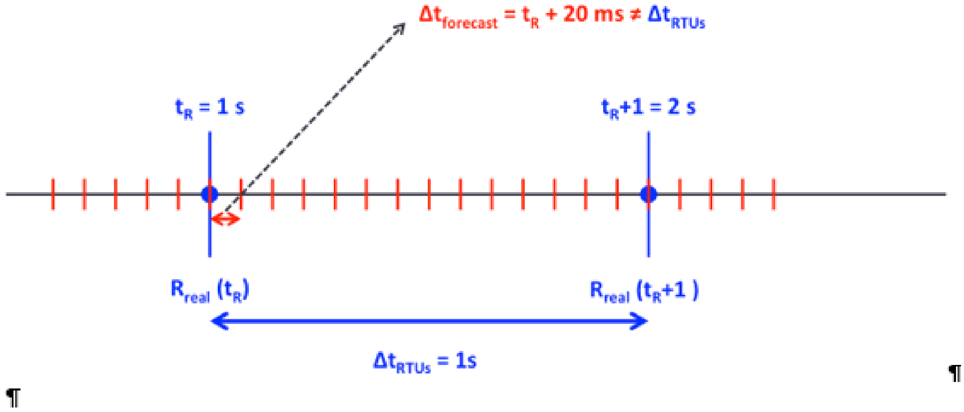


Figure 2: Difference between RR of RTUs and RR of forecast

RTU measurements are available, being R the values measured at the respective time steps.
In this way, virtual values are created at a PMU RR that can be used then in the forecast model as observations $Y_t$ for generating the definitive RTU pseudo-measurements in the buses where RTUs are placed.

- **Use the estimated powers obtained in the previous step**. As we are forecasting the pseudo-measurements for RTUs, the needed observations for such forecasts are the active and reactive powers for the buses where RTUs are installed.
  At a time-step where both an RTU and a PMU real measurement are available, we can run the estimation, getting the state (voltage and phase angle) of every bus. Since we have access to the admittance matrix of the network as stated in (7), the current injections in every bus can be obtained:

$$I = YV \tag{45}$$

  and therefore, the estimated complex power is calculated as:

$$S = VI^* \tag{46}$$

  Getting the active and reactive powers as the real and imaginary parts of $S$ respectively.
  The procedure is the same in the time-steps where only PMUs are available, but here, pseudo-measurements will play the role of RTU real measurements to estimate the new state. The active and reactive power estimations will be then used as the observation $Y_t$ for calculating the forecast for the next step.

## 4.2 Prediction intervals

Once the pseudo-measurements have been constructed as explained in the previous subchapter, the only missing thing to be able to run the State Estimator is their accuracy. If we take a look to the inputs needed for the WLS estimator, the pseudo-measurements will function as the measurements $z_V$, $z_{PQ}$. Their accuracy will be given by their respective standard deviations that will be added to the matrix **R** in equation (6).

Since the standard deviations of the real measurements are given by the devices (RTUs, PMUs), they are not available for the virtual pseudo-measurements. The real values for making a comparison, giving an accuracy in this way, are neither available since the estimation is done at real time. The adopted technique for giving a value to the standard deviations is the so called prediction intervals.

The purpose of a prediction interval is to satisfy, at any forward step $t$, the following equation with a certain confidence $\eta$:

$$F_t - W_t^{LOW} \leq Y_t \leq F_t + W_t^{UPP} \tag{47}$$

with:

- $F_t$: forecast or predicted value at time-step $t$
- $Y_t$: Observation at time-step $t$
- $W_t^{LOW}$: lower-bound of the prediction interval at time-step $t$
- $W_t^{UPP}$: upper-bound of the prediction interval at time-step $t$

As it is said in reference [19] there are two main types of model for determining the best way of computing prediction intervals correctly: (i) ARMA/ARIMA models and (ii) adaptive neural-fuzzy inference system. The second category can be adapted to different domains because they learn, in a certain way, the behavior of the variable to be predicted. However as the forecast models, that are taken into consideration here, are equivalent to ARIMA models (Single exponential smoothing = ARIMA (0,1,1); Holt's = ARIMA(0,2,2); Holt Winter's = SARIMA (0,1,m+1)(0,1,0)m) we will only focus on the first category.
It is important highlighting that the fact that the exponential smoothing techniques are equivalent to ARIMA models, is one of the main issues why this kind of forecasting is used. In this way, the calculation of prediction intervals can be done in a formal and simple manner using the below explained procedure.

For the ARIMA models with Gaussian innovation (Normal distribution), the upper and lower bounds of the predicted intervals can be calculated as follows [18]:

$$W_{t+1}^{LOW} = W_{t+1}^{UPP} = \chi\sqrt{\sigma_t^2} \tag{48}$$

where $\chi$ is a coefficient depending on the desired confidence level for the prediction interval($\eta$). It is determined by fix tables. In this study it will be considered a confidence level of 95% to what it corresponds a value of $\chi$ equal to 1.96. $\sigma_t^2$ is the variance of the error in study, in this case the forecast error.

It was also found out that the following equation coming from reference [20] is equivalent to (48) as described below.

$$F_{t+1} = \chi\sqrt{MSE} \tag{49}$$

where MSE is the Mean Square Error and it is calculated by

$$MSE = \frac{1}{n}\sum_{t=1}^{n} e_t^2 \tag{50}$$

being $e_t$ the error of the forecast estated as:

$$e_t = F_t - Y_t \tag{51}$$

If equation (50)is compared to the definition of the variance:

$$\sigma^2 = var = \frac{1}{n}\sum_{t=1}^{n}(x-\mu)^2 \tag{52}$$

where x in this case is going to be the error of the forecast that is the variable under study, and $\mu$ is equal to 0 since the distribution of the errors is Gaussian. Then one can confirm that $\sigma^2$ is the same as MSE and therefore equations (49) and (48) are equivalent.

It can be stated then, that the above described ARIMA model can be used for the time-series in question (forecast errors) if it is proved that it follows a normal distribution and does not show auto-correlation.
The first one can be proved by visually comparing the time series with a random set of data that shows a normal distribution. It can be easily implemented in MATLAB with the command *normplot()*. It is important to remark that this proof only has validation with large time-series.
Lack of auto-correlation can be easily proved with an ACF plot. In order to do so, the residuals of the forecast error ($e_t - e_{t-1}$) can be calculated at each time step and implement the function *autocorr()* of it in MATLAB. It has also being noticed when researching about correlation analysis, that this function works better with small sets of data since the definition equation, in which is based on, depends of the size of the data-set. When larger the size is, bigger will the probability be for the data to be correlated.

The final wanted standard deviations that will be implemented in the R matrix from equation (6) that is introduced in the estimator, will be provided by $\chi\sqrt{MSE}$.

It can be concluded then that for every time-step where RTU measurements are not available, i.e. at a PMU rate in between each 2 RTU measurements, virtual pseudo-measurements will be created composed of: (i) a value coming from the forecasting technique and the addition of one of the two required explained complementary methods (linear forward projection or estimated power injections), and (ii) an accuracy of this pseudo-measurement, computed with the prediction intervals method as stated before.

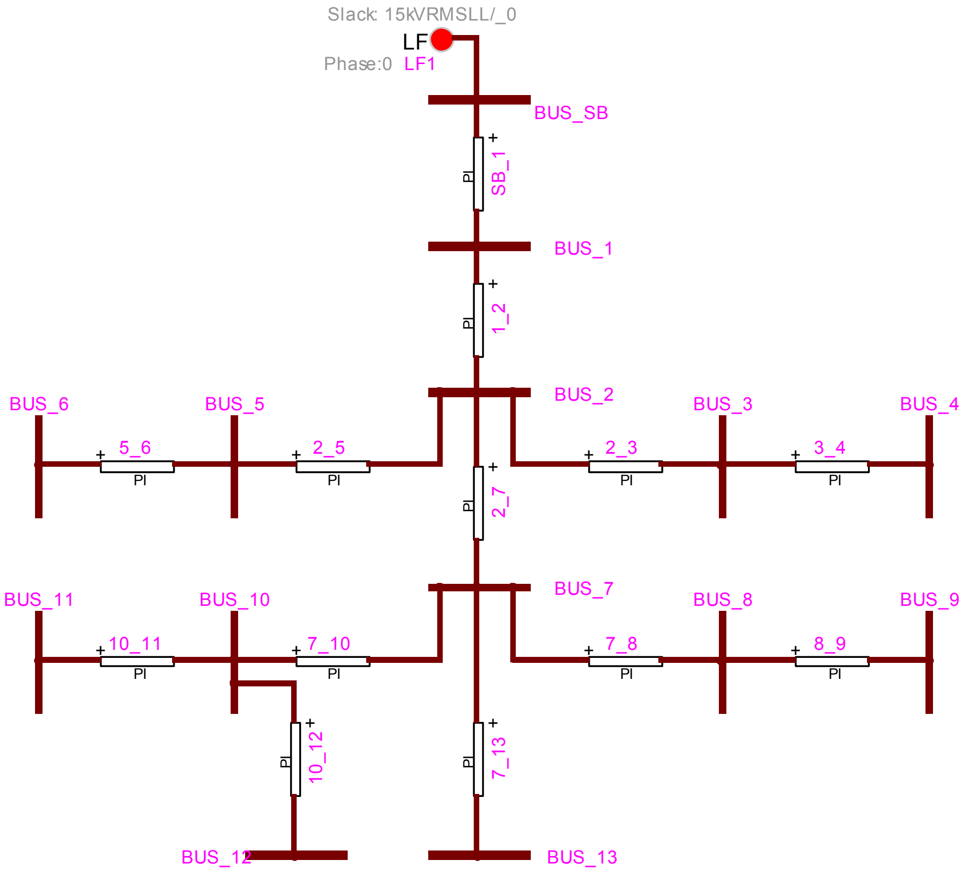


Figure 3: IEEE 13-bus distribution test feeder

# 5 Validation of the model

## 5.1 Description of the selected feeder and configuration

In order to validate the model and run the corresponding simulations the IEEE 13-bus distribution test feeder is used. Its configuration is shown in **figure 3**.

As it can be seen it is composed of 13 buses plus a slack bus connected to bus number one via the so called short-circuit impedance. This bus is added for the sake of the state estimation computation. Buses 2 and 7 are generating active power and buses 3, 4, 5, 6, 8, 9, 10 ,11, 12 and 13 are loads. A short network was used in order to prevent complexity in the first tests.

RTUs measuring only injected active and reactive power were located in nodes 2, 4, 6, 7, 9 and 11 and PMUs measuring voltage phasors in nodes 1, 3, 5, 8, 10, 12 and 13. Full observability is achieved since $2m \geq 2n$ (number of measurements is greater than or equal the number of states). And as it was already described in the introduction, we do not attain observability when considering only the installed RTUs or PMUs. So in order to be able to estimate the state at a PMU rate, RTU pseudo-measurements are needed.

## 5.2 Assumptions

The following assumptions were taken for the implementation of the model and resolution of the problem proposed:

- The refresh rate of RTUs and PMUs are set to 1 second and 20 milliseconds respectively. This means that there will be 49 steps in between two consecutive RTU measurements where the pseudo-measurements will have to be created.
- The standard deviation of RTU and PMU devices are set to $10^{-3}$ and $10^{-4}$ respectively. Respecting in this way the fact that PMUs are more accurate than RTUs and also staying within the range that was observed in the previously made literature review. As discussed before the standard deviation for the pseudo-measurements will have to be calculated.
- It is assumed that RTU and PMU measurements arrive at the same time to the PDC. As it was explained in the introduction, there are some delays that need to be accounted for until the measurement reaches the SE, and they are not the same for RTUs and PMUs. Moreover, PMU delays can be known due to the fact that they are connected via GPS. On the other hand, RTU delays are unknown because of the lack of the GPS connection. These delays are different for each step. This setback will be overcome assuming that we are able to synchronize the RTUs so that the measurements arrive at the same time as the PMU ones, although this would be very difficult to achieve in reality.
- A Non-Linear Weighted Least Squares (NLWLS) state estimator will be used for the resolution of this problem because of the reasons that were already explained in section 3.
- The model will be run for 1500 steps of 20 milliseconds each, so 30 seconds in total.
- The simulations are all run in p.u., the power base is chosen to be 5 MW and the voltage base equal to 15 kV.

## 5.3 Forecasting and creation of the pseudo-measurements

Steps described in section 4 will be followed for the construction of the pseudo-measurements.

### 5.3.1 Choice of the forecasting model

Since the variables forecast are active and reactive powers, in order to know which exponential smoothing forecast technique is more suitable, we will look at the power injection of one of the buses. For instance the active power injection in bus 6, which is displayed in **figure 4** in p.u. As it can be observed, neither a trend or a seasonality can be identified in the time-series shown. Same conclusion is derived by looking at other buses active and reactive powers. Therefore by being guided by the flowchart represented in **figure 1**,the forecasting model chosen is the **Single Exponential Smoothing** whose formula is given in equation (41).

### 5.3.2 Choice of $\alpha$

In order to validate the prediction intervals technique, as it was already explained, we have to show that the error of the forecast follows a normal distribution and the data is uncorrelated. Since the forecasts are initialized each 50 steps (1 second) where a new RTU measurement arrive, we have sets of data of only 49 steps. This number is too small to evaluate whether they follow a normal distribution or not, so this cannot be proved

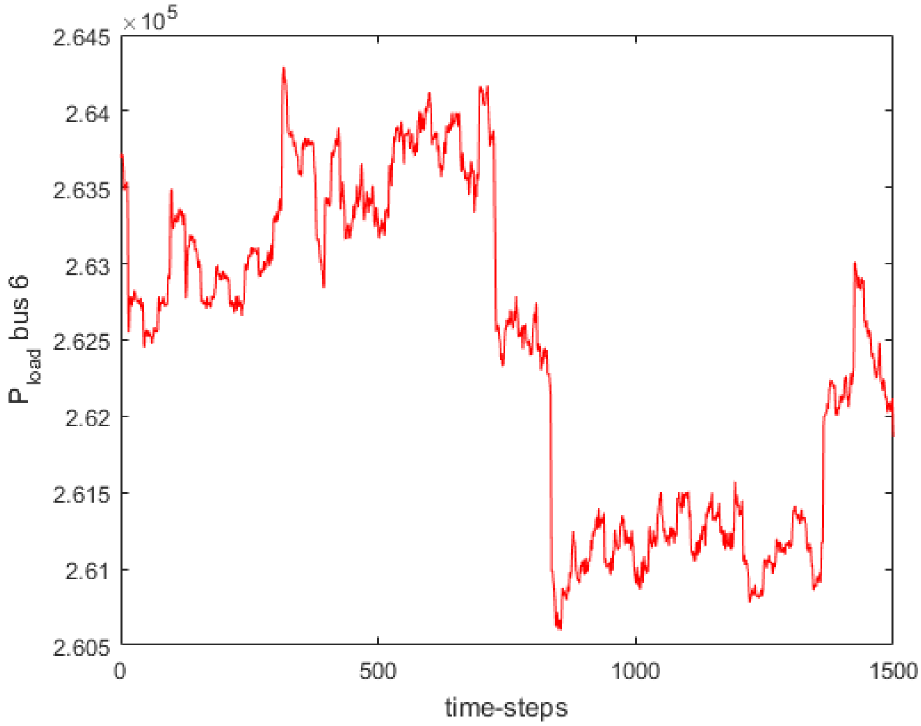


Figure 4: Active power injection in bus 6

and it will be considered as if they did. On the other hand the auto-correlation analysis can be done. In **figure 5** an ACF plot from time-steps 201 to 249 of bus 2 is shown, $\alpha$ is set to 0.7. $\sqrt{49} = 7$ points are displayed. If they lie in between the two limits, then this would mean that the data do not show auto-correlation.

**Figure 6** shows the same analysis but with an $\alpha$ equal to 0.2. As it can be seen the data is correlated.

The smaller $\alpha$ is taken the more correlated the data is, this means that if we put too much weight on the previous forecast when calculating the ones for the next steps, the errors will start showing some correlation between them and therefore the prediction intervals will lose reliability.

According to this, $\alpha$ was set to 0.7 for all the simulations.

### 5.3.3 Linear Forward Projection

The first of the two methods considered for generating the observations $Y_t$ that are then used in the forecasting, is analyzed here with its corresponding calculation of the prediction intervals and the final accuracy this method gives to the estimation.

**acquisition of observations:** The different values of $Y_t$ are calculated with equation (44) as already explained. In figure 7 it is shown the resulting calculated curve together with the real data for the active power injection in p.u. of bus 7 in order to have an idea of how it looks like.

It can be observed that it follows quite good the curve, although when having big changes the accuracy of the model drops, resulting in bigger errors. The red curve will give us at each step of 20 ms the value of Y that will be implemented in the single exponential smoothing formula.

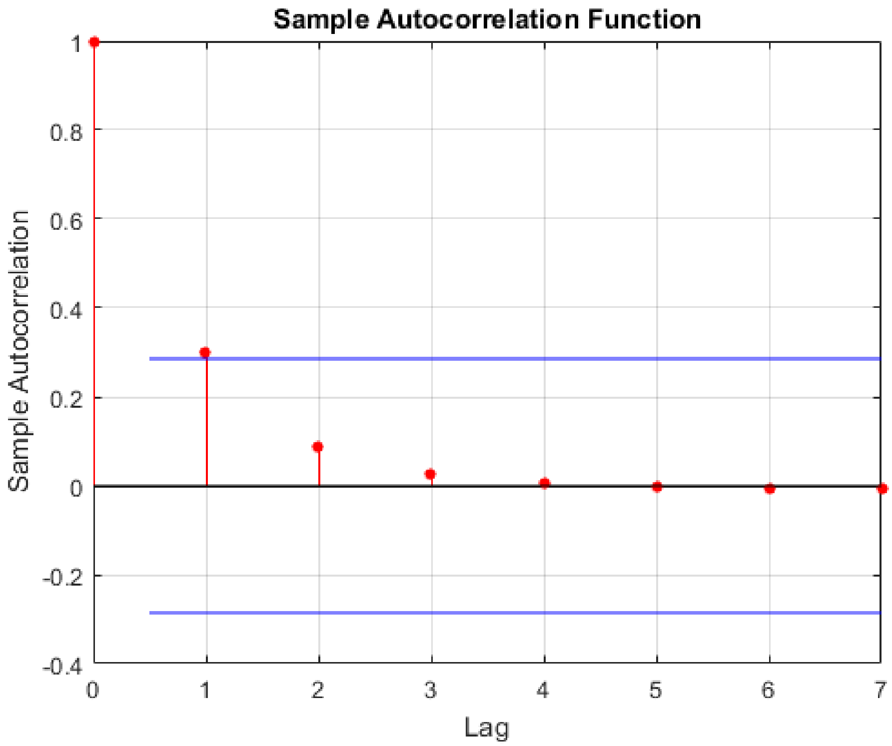


Figure 5: ACF plot with $\alpha = 0.7$

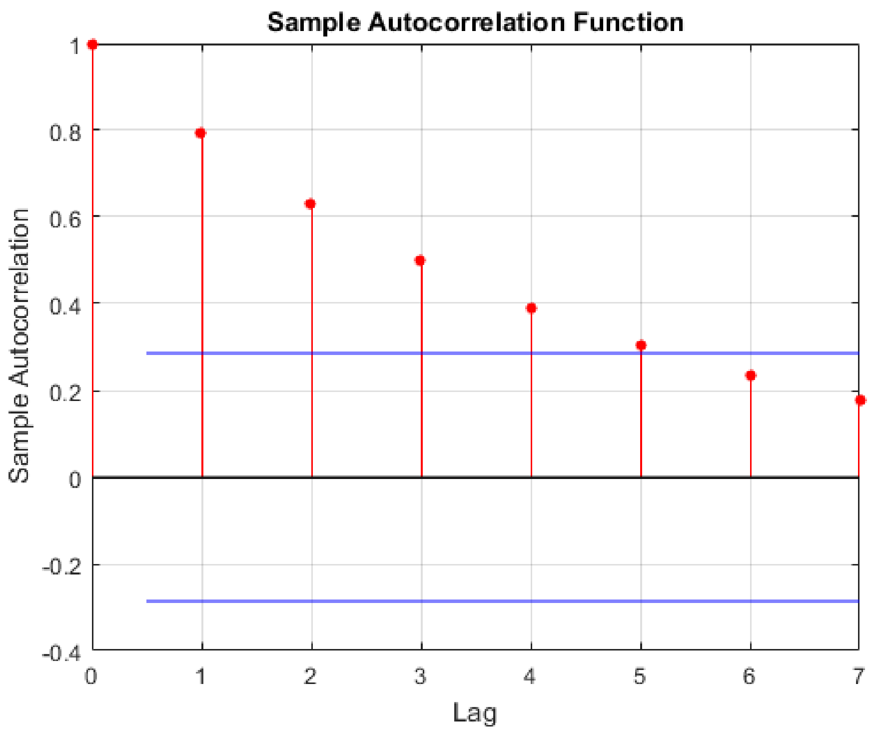


Figure 6: ACF plot with $\alpha = 0.2$

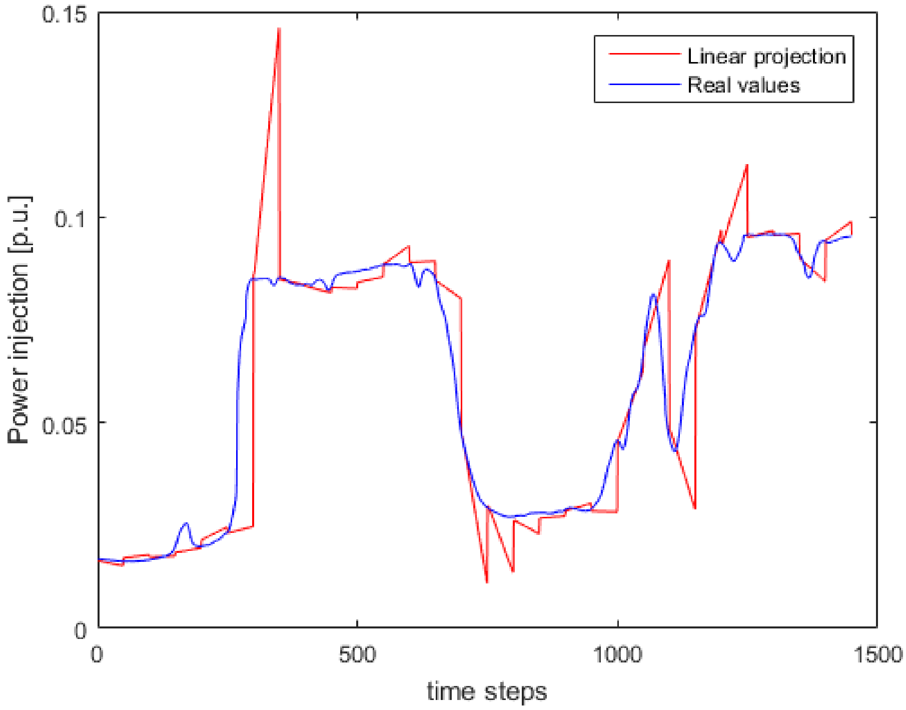


Figure 7: Linear forward projection comparison with real values of power injection

**Prediction intervals; observed problem:** When calculating the prediction intervals, an error in the approach of the resolution of the problem was noted.

First of all, the computation of the prediction intervals is done as explained in section 4.2, the lower and upper bound that will serve as the standard deviations of the pseudo-measurements are calculated with equation (48). These bounds are displayed in **figure 8** for each time-step also for the active power injection in p.u. of bus 7. It can be seen how the bounds are initialized each 50 steps, 1 second, because a new RTU measurement arrives. Also, it can be seen how the bound increases slightly the farther we are from the real measurement, this occurs because the error of the forecast increases at each time step in between two real measurements.

But what is more important to highlight from this figure, is the fact that the bounds are very small, in most of the steps they are smaller than the standard deviation of the device itself, this meaning that we are giving more accuracy to the pseudo-measurement than to the real measurements.

Furthermore, the computation of the whole standard deviation ($10^{-3}$ each second and the prediction interval bounds each 20 ms in between two consecutive seconds) together with the total error of the pseudo-measurements / measurements depending on the step is shown in **figure 9**. It can be seen that the standard deviation computed for the pseudo-measurement do not cover the error, therefore these standard deviations are not correct. This occurs because the error of the forecast used for the calculation of the prediction intervals (equation (51)) is the error between the forecast and the linear projection, thus we are not accounting for the error introduced by the linear projection in this calculation which, in addition, results to be much higher than the one of the forecast with respect to the linear projection, as shown in **figure 10**. The addition of both errors would be equal to the total

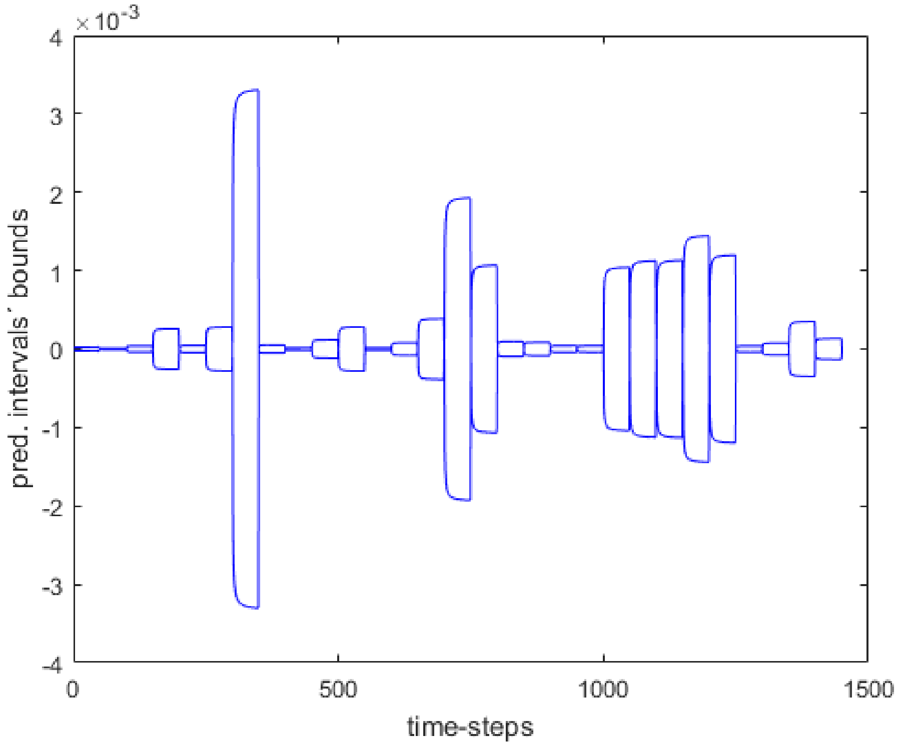


Figure 8: Upper and lower bounds of the prediction intervals

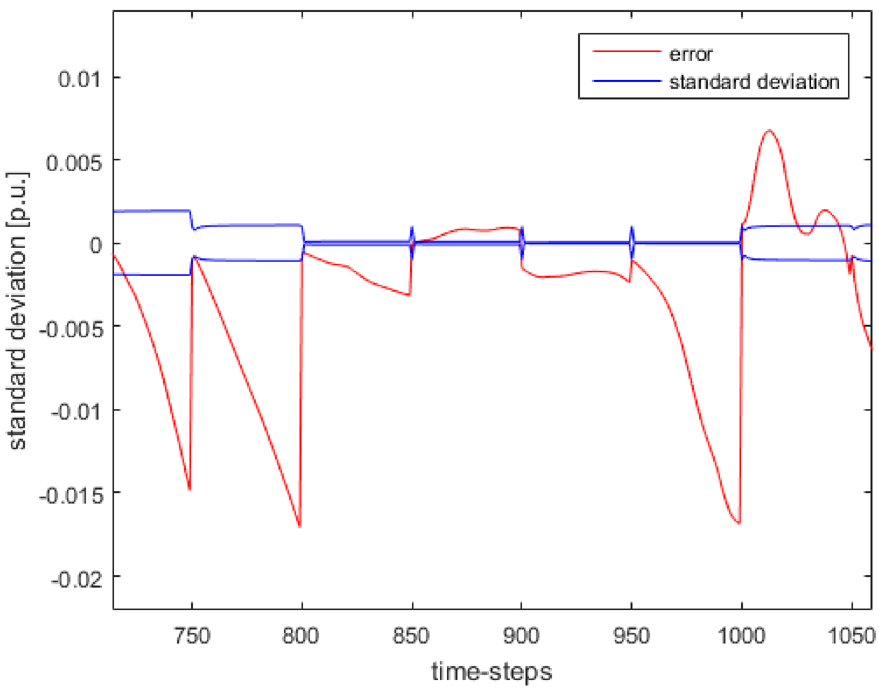


Figure 9: Standard deviation and error of the measurements for a set of time-steps

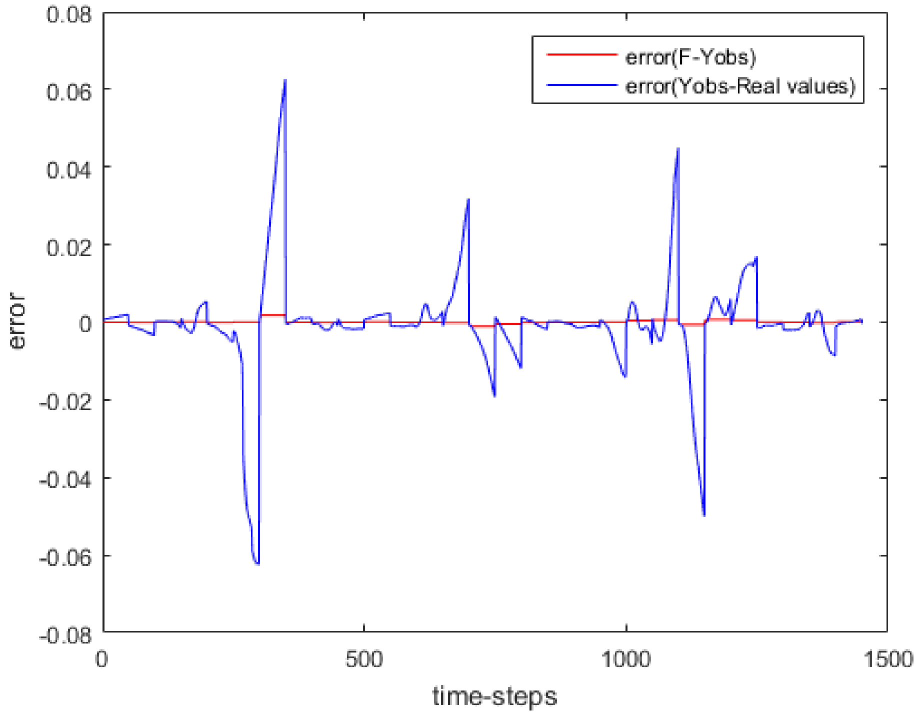


Figure 10: Comparison of the error of the forecast with respect to the linear projection and the error of this last one with respect to the real values

one that should be used for the equation (51) and the respective calculation of the prediction intervals' bounds. To do so, a method to account for the error of the linear projection has to be found. For a matter of time this is not covered in this report and it is left as future work.

This can also explain the autocorrelation observed with some values of $\alpha$. The forecast is going to be a similar curve to the linear projection one. Since we are only taking into consideration the error of the forecast to the linear projection, data can appear to be correlated. This should not happen when the error also includes the linear projection one.

**State Estimation results:** As we have this just described error in the model, the results are categorized as preliminary and have to be checked and recomputed when the problem is fixed. **Figure 11** shows the states (module and phase) of the bus with the maximum error estimated in each time-step. It can be seen that the error is quite small, around $2 * 10^{-4}$, and therefore the technique used shows to be quite accurate.

### 5.3.4 Estimated powers in each step as observations

**acquisition of observations:** In this case, the values of $Y_t$ come from the estimated active and reactive powers of the previous step. Same bus, number 7, as in the linear projection section will be used for the simulations of active power in p.u. for a matter of comparison. To show the behavior of this technique, **figure 12** shows

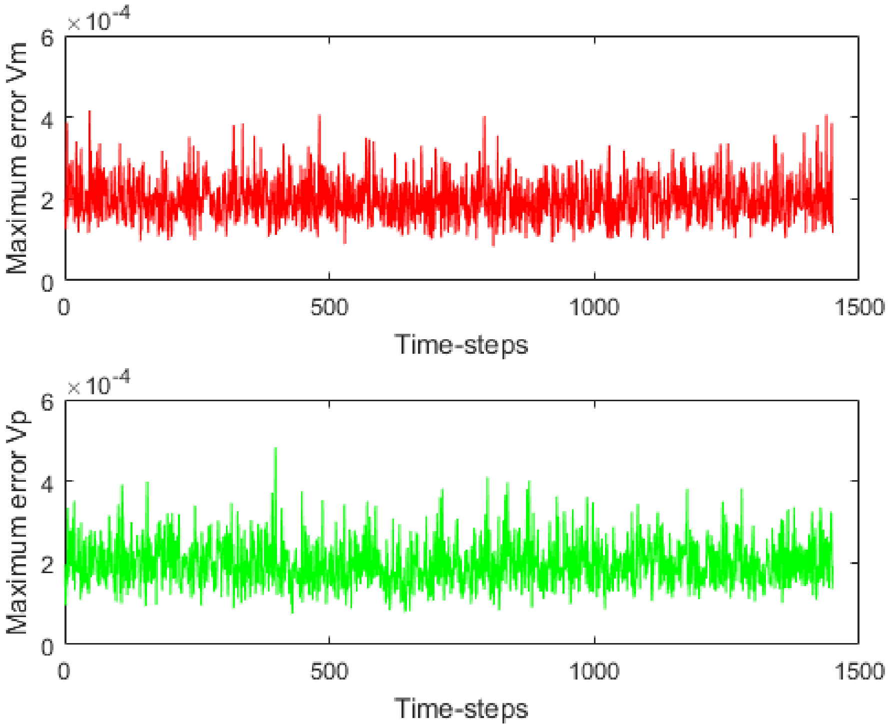


Figure 11: Results of the state estimation maximum error for the linear projection technique

the resulting curve formed by pseudo-measurements and real RTU measurements depending on the time-step, compared to the real data.

As it can be observed, the pseudo-measurements are quite far from the real values, they are not able to follow the line. It shows much worse behavior than the linear projection technique displayed in **figure 7**. This has been found to be like that due to the fact that in the buses where we do not have RTUs, although the state estimation error is very small for the voltage amplitude and phase, when calculating the estimated powers as it is shown in equations (45) and (46), the error increases a lot due to the multiplication of the admittance matrix by the voltages. This can be observed in **figure 13** where the voltage estimated has a small error, around $3*10^{-3}$p.u. when following the curve of the real voltage. This difference is increased to around 2 p.u, when looking at the power's curve.

So at each step when calculating the forecast we are introducing such a big error resulting in the already commented **figure 12**.

The error of the forecast will decrease when reducing $\alpha$. This occurs because the smaller $\alpha$ is, the less weight we put on the previous faulty power estimation, and more on the previous forecast. Since the forecast every 50 steps is equalized to the real RTU measurement, when having a small value of $\alpha$, we will be putting more weight in this value.

In addition, when we have a value of $\alpha$ equal to zero, we are maintaining the real value measured by the RTU for the next 49 steps, which is also an interesting result that has to be considered and presents the less error. It does not use the estimated powers in its calculation because it is canceled with the null value of $\alpha$. Its behavior

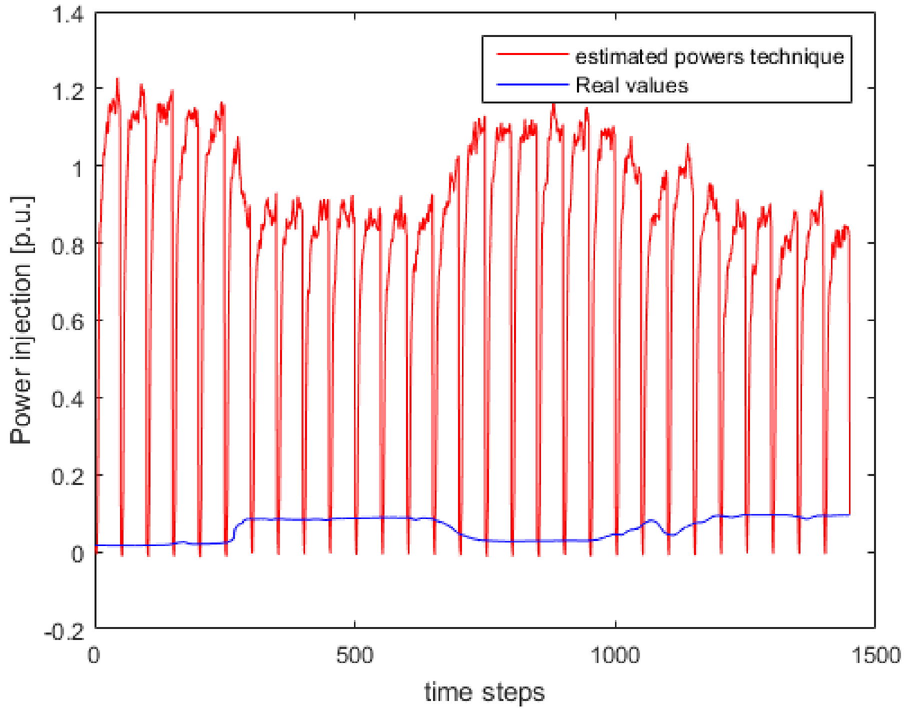


Figure 12: Estimated powers technique comparison with real values of power injection

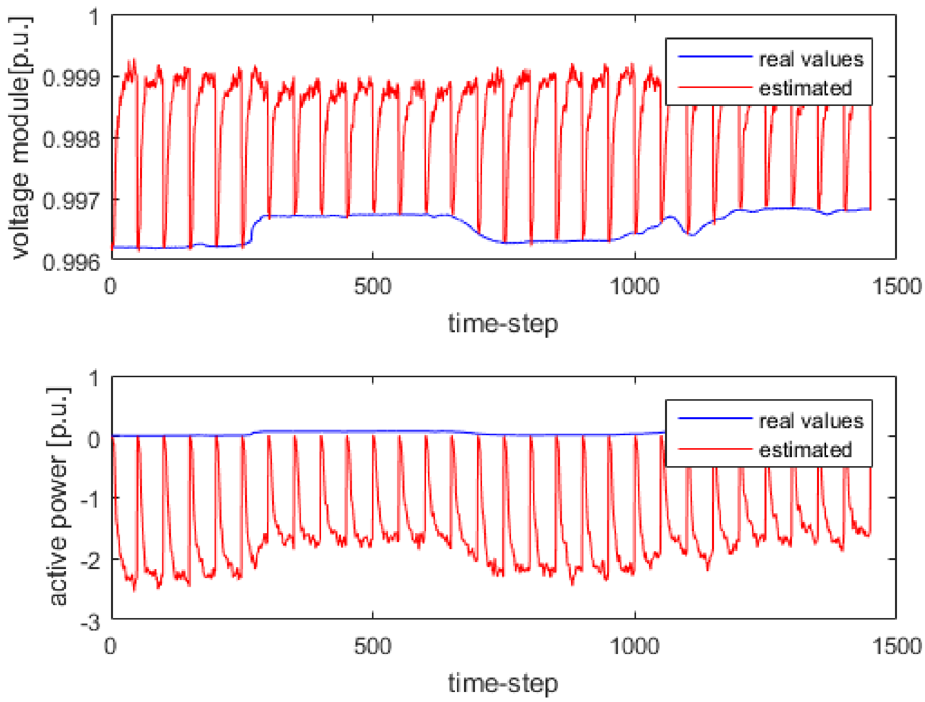


Figure 13: voltage and power estimation errors

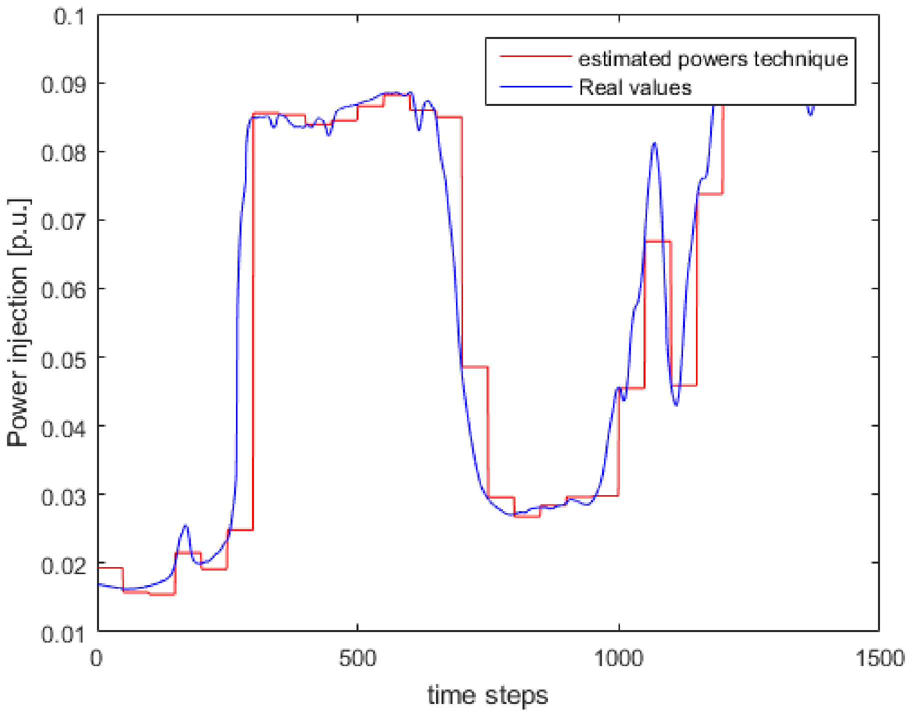


Figure 14: RTU measurement used for the next set of pseudo-measurements

is represented in **figure 14**.

**Prediction intervals:** They show the same problem as estated before since we cannot account for the error introduced by the construction of the observations, in this case formed by the injection power estimated in the previous step. The result is shown in **figure 15** where it can be seen that the error is not covered by the standard deviation in most of the steps.

In the case of $\alpha$ set to zero, we do not use observations and prediction intervals cannot be constructed. In this case the standard deviation of the pseudo-measurements is chosen to a value of three times the standard deviation of the RTU device ($3*10^{-3}$). This is done to take account for the error of the pseudo-measurement somehow, although it requires a deeper study to be able to establish this factor or a method to calculate the standard deviation in a proper manner. The result is shown in **figure 16** where it can be seen that the standard deviation still does not cover the error in some steps.

**State estimation results:** The results for the maximum error of the state estimation are shown in **figure 17** for the case with $\alpha$=0.7 and in **figure 18** for $\alpha$=0. It can be concluded from these figures that the maximum error of the use of this technique is much larger than the one when using the linear forward projection technique. This is due to the already commented big error that the estimation of powers introduce to the forecast model. On the other hand when using $\alpha$ equal to 0, that is the same as using the value of the real RTU measurement for the next set of pseudo-measurements, the power's estimation is not used and thus the results obtained are better, it can be seen that they are very similar to the ones obtained with the linear projection, around $2 * 10^{-4}$.

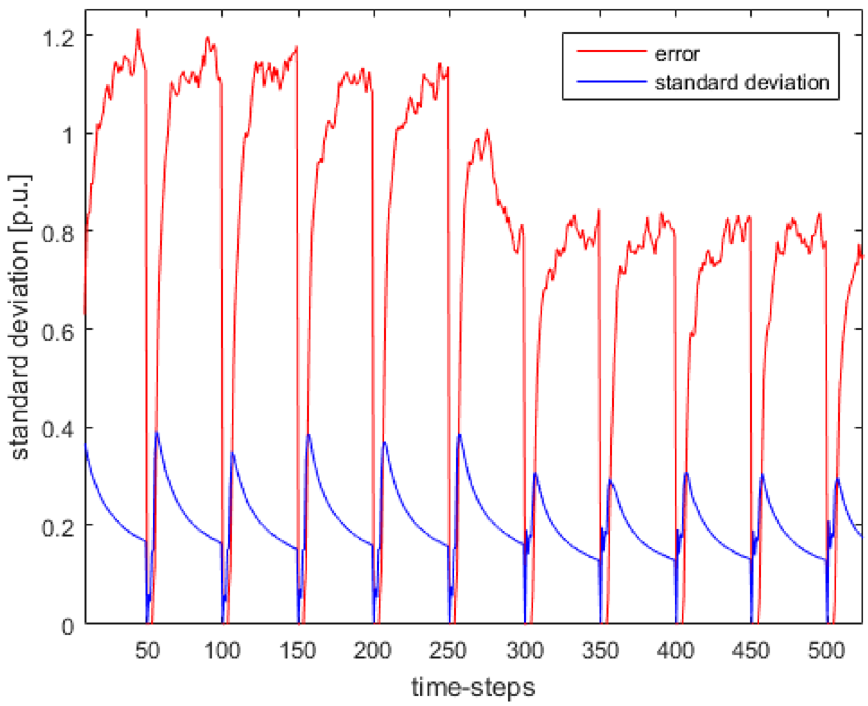


Figure 15: Standard deviation and error for a set of measurements and pseudo-measurements with $\alpha = 0.7$

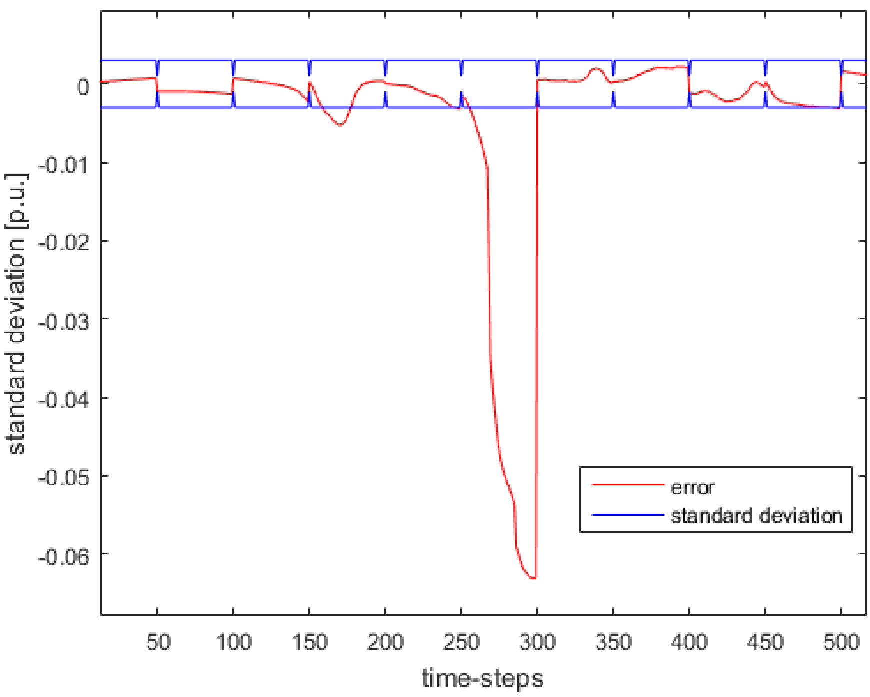


Figure 16: Standard deviation and error for a set of measurements and pseudo-measurements with $\alpha = 0$

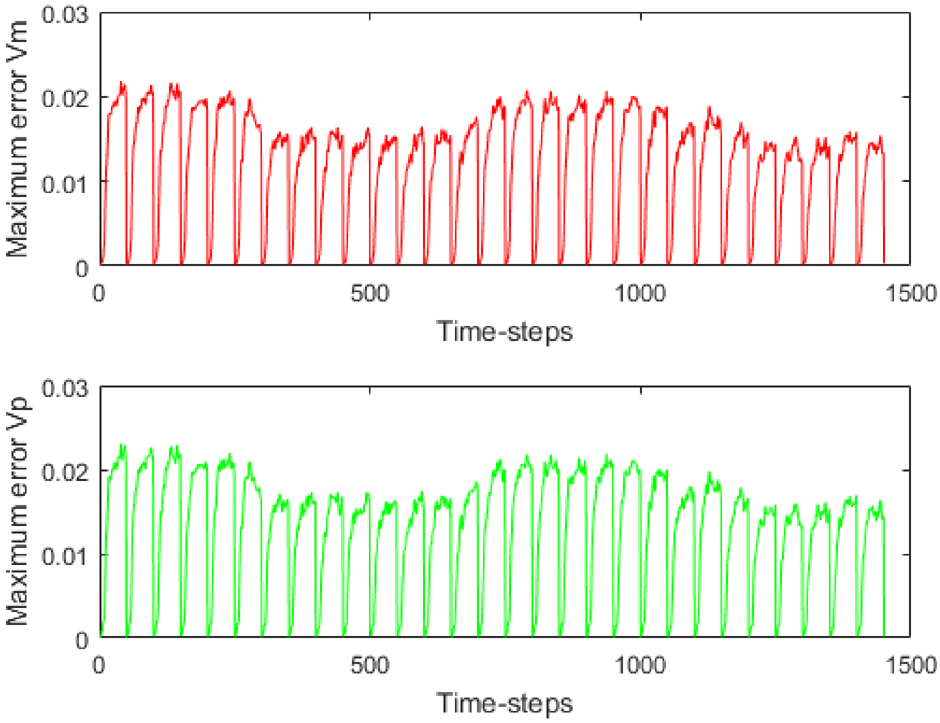


Figure 17: Results of the state estimation maximum error for the power estimation of previous step technique for $\alpha$=0.7

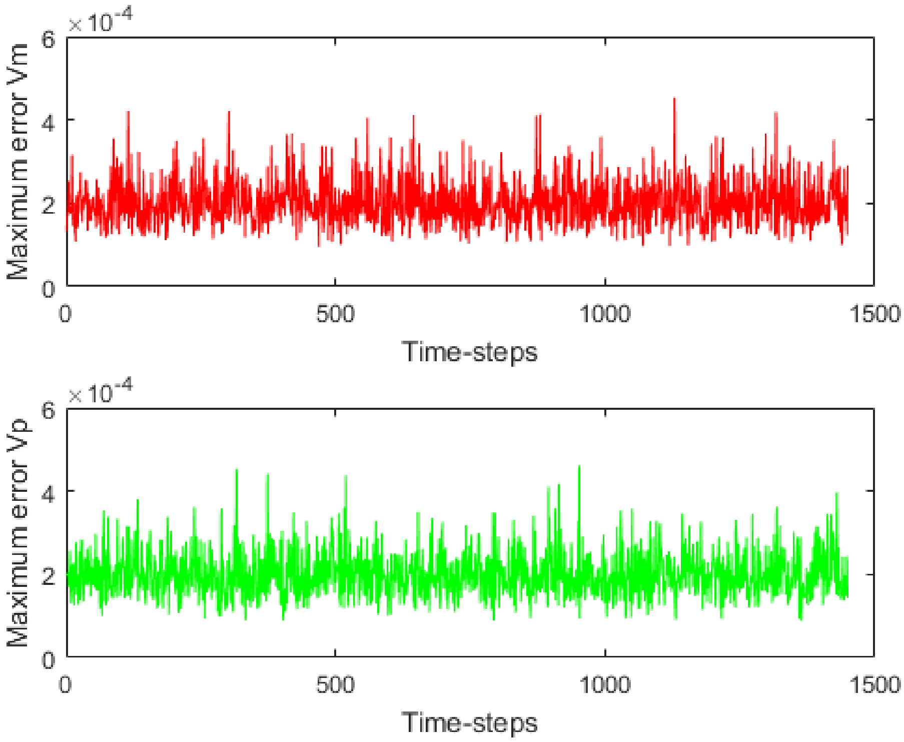


Figure 18: Results of the state estimation maximum error for the power estimation of previous step technique for $\alpha$=0

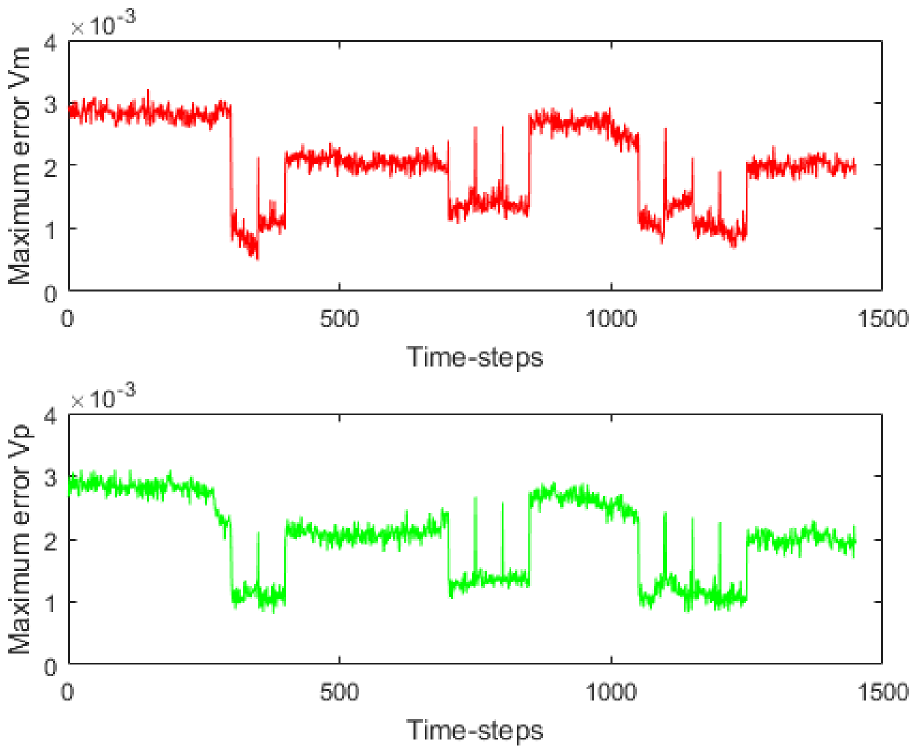


Figure 19: State estimation maximum error, RTUs and PMUs located in same buses

# 6 Sensitivity and performance of the model

In order to have a better understanding of the behavior of the model, several simulations with different configurations were tested so that to compare the accuracy of the estimator. Although as it was stated before, we categorize these results as preliminary, since we are not able to account for the error introduced by the implementation of the observations in a proper manner, neither with the lineal projection technique nor with the power estimation one.
For these comparisons the model with the linear forward projection will be utilized since it showed better results.

In first place the model used and described in the previous section where we had PMUs and RTUs installed in different nodes is compared to a configuration where we situate both of them in the same buses, i.e. we have measurements of voltage module and phase, coming from PMUs, and active and reactive power, coming from RTUs, in nodes 1, 3, 5, 8, 10, 12 and 13. The maximum error of the estimation is shown in **figure 19**.
As it can be observed the error is augmented in approximately one order of magnitude, this occurs due to the fact that now we have all the measurements focused in the same buses, therefore there are some buses without devices having a less robust system and thus decreasing the accuracy.

A configuration with only PMU devices is now tested. They are placed in buses 1, 3, 5, 8, 10, 12 and 13, same buses as before, but now these PMUs will be configured to measure both voltages and injection current phasors. As it was already covered through section 3 of the report, when having these two types of measurements the system becomes linear.

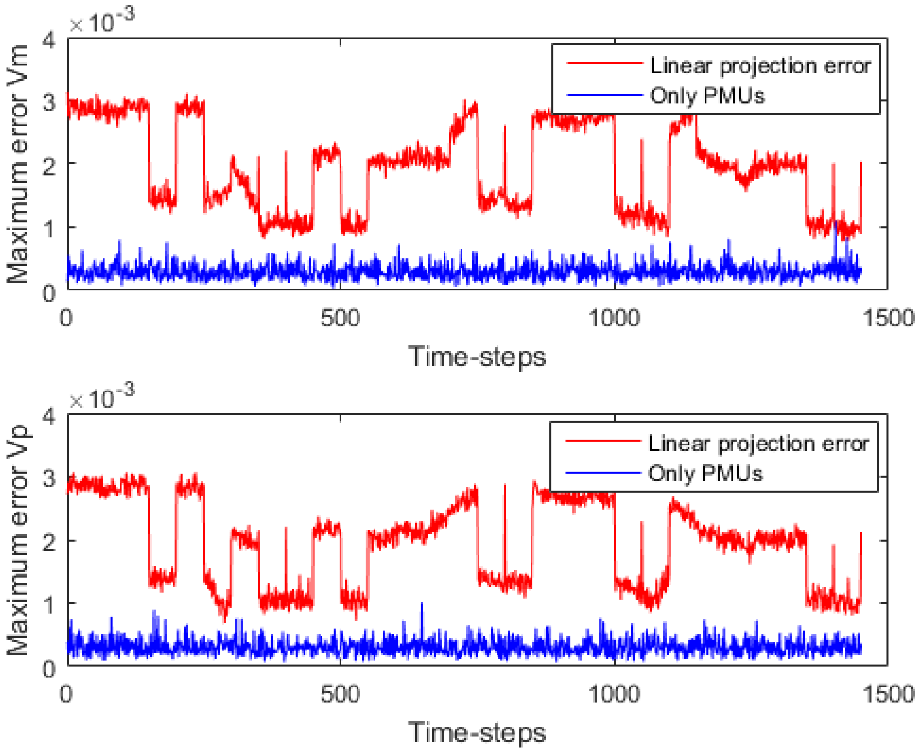


Figure 20: Comparison of maximum error between a linear system with only PMUs and a hybrid system with RTUs located in same buses as PMUs

A linear WLS was implemented for this case. The maximum error for the estimation for comparison is displayed **in figure 20**.

It can be seen that the error compared to the corresponding hybrid configuration where PMUs and RTUs are gathered in the same buses has diminished one order of magnitude, this is due to the better accuracy PMUs show compared to RTUs and above all compared to the pseudo-measurements of RTUs.

On the other hand when comparing this error to the one for the principal model discussed in section 5, they are approximately the same as can be seen in **figure 21**. This can be justified with the fact that when having a measurement unit in each different bus the robustness of the system increases and so does its accuracy.

It is also important to highlight that this linear configuration, since it is measuring at a rate of 20 ms, it is able to fulfill the real time operation functionalities such as the fault detection and location that, as it was already said, it is in the range of a few hundreds of milliseconds. On the other hand since the linear projection technique only depends on the previous two RTU real measurements, it will have a maximum time of reaction of one second and will not be as efficient when detecting faults, leaving the system defenseless for a short period of time until the next RTU real measurement arrives.

In addition, we can also observe that more devices are needed for the hybrid model than the linear one, which translates in more expenses.

One more configuration has been tested. RTUs and PMUs are set in the same buses as in the original configuration, but in this case RTUs are virtually programmed to measure every 20 milliseconds, in this way, RTUs pseudo-measurements are not needed since the system is fully observable at a PMU RR.

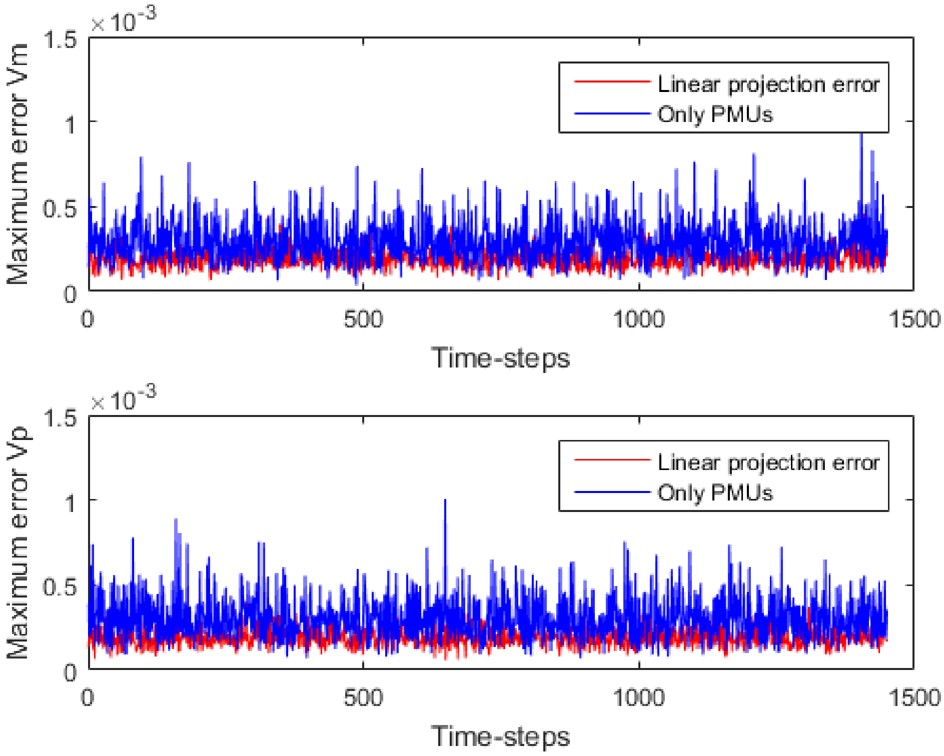


Figure 21: Comparison of maximum error between a linear system with only PMUs and a hybrid system with RTUs located in different buses than PMUs

The results of the comparison of the maximum error of the estimation are shown in **figure 22**.It can be seen that the error is practically the same between the two configurations, this means that the linear projection technique works very good and it's able to construct the state without practically introducing errors to the output of the estimator.

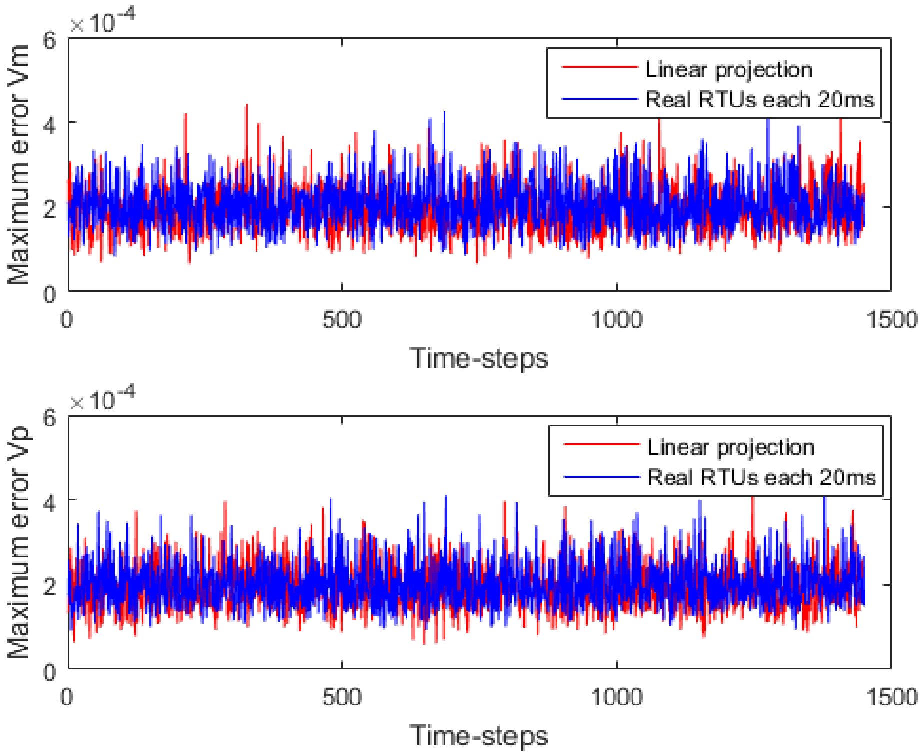


Figure 22: Comparison of maximum error between two hybrid systems, one using pseudo-measurements and the other one having RTU measurements each 20 milliseconds

# 7 Conclusions and future work

During this project, a hybrid state estimator model has been implemented, capable of estimating the state at a PMU rate. In order to do so, the construction of RTU pseudo-measurements was required. This was done by the use of exponential smoothing forecasting and prediction intervals techniques. In this way, each 20 milliseconds, data from PMUs and from RTU pseudo-measurements were used by the state estimator, NLWLS in this case, to estimate the states.

Two different methods for constructing the pseudo-measurements were studied, the linear forward projection and the use of the previous step estimated powers. It was concluded that although estimated voltages are very accurate, when calculating the corresponding powers, big errors appear and therefore it is better not to use them in the following steps. Methods like the linear projection or leaving the value of the last RTU measurement for the following set of pseudo-measurement showed a better accuracy and behavior.

Several configurations were compared to see the efficiency and accuracy of the proposed model. Results, although they are tagged as preliminary due to an error found that gives us some uncertainty, show that the proposed model is competent and shows very good accuracy.

We can classify this report as a first part of a bigger project, since there are some improvements that can be made and are left here as future work for a matter of time.
First of all, the error that has already been described has to be solved, it is important to be able to calculate the standard deviations of the pseudo-measurements correctly. In order to do so, if exponential smoothing is used, a way of accounting for the error introduced by the linear projection has to be found. Other alternative is to find a different way, from exponential smoothing, of creating the pseudo-measurement that also allows us to use the prediction intervals technique.
Once this is done, a bad data analysis can be run to remove outliers from the state estimation and achieve a better performance.

Another important feature to be implemented is the modeling of the RTUs so that they measure both voltages and active and reactive power injections. In this way more measurements are available per device and therefore less of them will be needed. Accuracy and robustness will decrease but it will result in a more economic system. Playing with this trade off, a much more competent system can be developed.
Also, finding a model with pseudo-measurements sensitive to big changes in the grid and able to detect faults, as PMUs do, would be of course an interesting result.

Finally, the model can be implemented in a real distribution network to have a Real Time supervision and control of it. It can also be tested in bigger distribution and transmission networks than the one used.

Lausanne, June the $10^{th}$ 2016

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