FACULTAD DE CIENCIAS ECONÓMICAS Y EMPRESARIALES

PERFORMANCE ANALYSIS OF LOW VOLATILITY STRATEGIES IN THE EUROPEAN MARKET IN THE LONG RUN

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ABSTRACT

The former End of Master Project will examine the performance of the STOXX 600 Europe Index from 2001 to 2018 in order to assess whether low volatility stocks are able to overperform the market as a whole in the long run, rejecting the traditional statement in finance which claims a positive relationship between risk and return. The study was carried out combining two different kind of analysis: a *linear regression analysis* to assess whether the variable risk (volatility) has a significant impact over returns and how are they correlated and a total return comparison to evaluate whether low volatility stocks outperform the market as a whole in the long run. The results displayed significant relationships between the variables under extreme market conditions (bull and bear periods). Nevertheless, the correlation during market downturns was higher, which allowed low volatility stocks to outperform the overall market during the selected period due to the power of compounding.

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Introduction

Objective

Since Factor Investing came into scope, theoretical models such as CAPM or Efficient Market Hypothesis, which predict a positive relation between risk and return, have been seriously challenged as empirical evidence from numerous studies has proven a flat or even negative relationship between these two variables. Low volatility stocks for example have historically outperformed the market during bear market periods which is clearly reasonable but yet reject these commonly applied theories. In fact, this has been one of the main justifications of previous research documents addressing low volatility stocks outperformance in the long run.

The subject of this End of Master paper was chosen in order to challenge one of the main arguments of finance: "the higher the risk assumed when investing, the higher the expected return will be". The proven inconsistency of this statement poses opportunities in the Asset Management industry since it may contribute to the optimization of the existing portfolios by reducing their risk exposures while maintaining returns.

This anomaly in the market was coined for the very first time by Robert A. Haugen and and James A. Heins in 1972 and ever since then, many research papers have been written by recognized professionals in the industry to explore this further. However, most of the work done previously has been focused on investigation the US market, whereas less exploration has been done on the European case. Thereby, the main objective of this End of Master Project is to analyze whether this statement is strong enough to make less volatile stocks outperform the market in the long run within the European stock market, being the STOXX 600 Index the sample chosen to represent the market and the benchmark for comparison purposes. Within this sample, 5 different Scenarios (comprised between 2001 and 2018 will be analyzed) in order to assess how the different relationships between these variables according to the selected period affect the performance of the stocks in the long run. Two different Linear Regression Models were designed for each Scenario, to spot these relationships and measure its significance levels in each case.

Structure

The present paper is organized as follows. The next section will provide an overview on the existing literature of the topic, starting from the fundamental theories of portfolio management and seeing how the concept of Factor Investing Strategies started to become increasingly popular and widely applied by many prestigious portfolio managers. The last part of this section will be dedicated to deeply explain the concept of Low Volatility Strategies and which are their benefits and drawbacks.

Section 3 will describe the methodology used for conducting the analysis, beginning with the data collection process which will explain the variables used and why, followed by the database treatment and the calculations made in order to build the different databases to undertake the analysis.

After running the model within all the different scenarios, the obtained results will be presented and interpreted in Section 4. Thereafter, the next section will be dedicated to highlight the limitations of the model and suggest possible further research on the field.

Finally, the last part of the paper will comprise the conclusions of the study undertaken, assessing whether or not the evidence that less volatile stocks outperform significantly the market during downturn periods is strong enough to make them potentially outperform the market in the long-run, according to the different observations of the results obtained through the different analysis made.

Theoretical Framework and Literature Review

Origins of portfolio management

During the 1960's and early 1970's what we know today as Modern Finance started to become increasingly popular. At this point, portfolio management was based in two basic paradigms: Capital Asset Pricing Model (CAPM) that predicted the mean-variance efficiency of the market portfolio and the Efficient Market Hypothesis which sustained that markets are informationally efficient. In this sense, it was generally believed that securities markets were outstandingly efficient and that stock prices fully reflected the information of themselves and the market as a whole. If this would have been true, neither technical analysis or fundamental analysis, which are two techniques used to anticipate future price movements, would have helped investors to achieve greater returns by making a good stock picking rather than just holding a random portfolio of individual stocks or at least not assuming a greater risk.

However, empirical evidence from numerous studies have proven several deficiencies in these models regarding the relationship between risk and realized return since then. This section will explain the fundamentals of the traditional portfolio management theories and how they have influenced in the development of what we know today as Factor Investing.

Modern Portfolio Theory (MPT)

What we know today as Modern Portfolio Theory was born under two main purposes: on one side, showing investors how to choose a portfolio with the maximum expected return for a given amount of risk and, in this same spirit, showing how to choose a portfolio that minimizes risk for a given amount of return. This theory therefore was the origin of risk diversification[1] and explained investors how they may use this technique to minimize risk while maximizing the potential return on their investment.

[1] The rationale behind the diversification concept is that investing through a process of selecting a group of assets which account for a lower collective risk y much safer than investing on the single assets itself.

Harry Markowitz was the first one to spread MPT through his doctoral dissertation titled "Portfolio Selection" published later on by the Journal of Finance in 1952. In this paper, the economist starts by defining the basic guidelines of portfolio construction. HM model, also known as Mean-Variance Model, was based on making the most efficient selection of assets which maximizes expected returns while reducing the standard deviation (risk) of the portfolio.

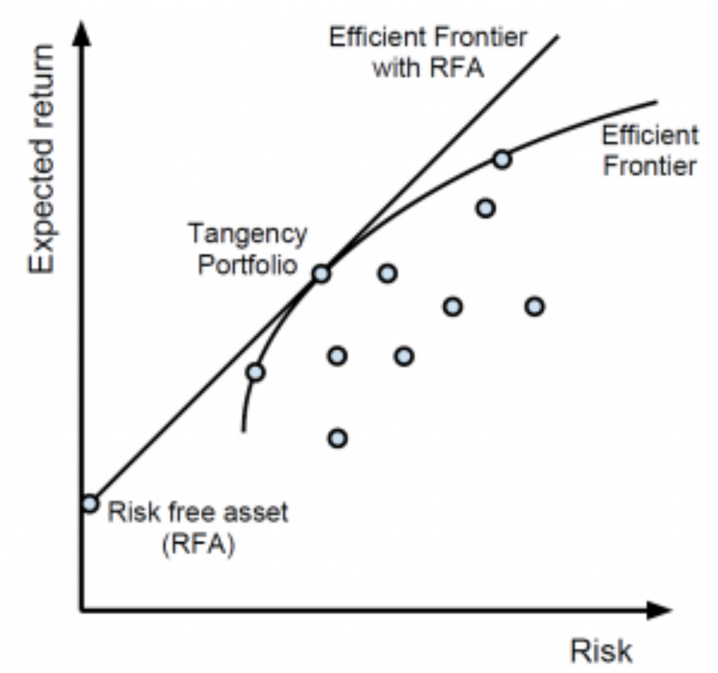
It is important to highlight that his model relies upon a set of simple hypothesis/assumptions that need to be satisfied in order to construct the most efficient portfolio:

1. Investors are rational and risk-averse: this will imply that, having two assets with equal returns, the investor will always choose the less risky one. That is to say, they will be concerned about a positive relation between risk and return and will only assume a greater risk if this will provide them an extra return.
2. An investor has two options: either maximizes his return for a given level of risk o minimizes risk for a given amount of return.
3. We also assume there are no transaction costs for buying or selling securities and no taxes are paid, therefore neither of these issues will determine investors which securities to buy.
4. Market liquidity is infinite, and no one can move the market. Every investor can take positions in any security of any size.
5. Investors are indifferent towards receiving dividends or capital gains.
6. Long term investors and short-term speculators share motivations, target yield and time horizon.
7. Investors share identical beliefs over risk measurement.
8. The only way investors will seek to control risk is by diversification of their assets.
9. Politics and behavioral finance has no influence in the market.
10. The risk of the portfolio is determined by the instability of the returns from the given portfolio.
11. Investors will always select those portfolios which maximize utilization.

Considering all of the assumptions above, Harry Markowitz defined the *Efficient Frontier* which represented the series of optimal portfolios that can be constructed which offer the highest expected return for a given level of risk or imply the lowest risk for a given level of expected return. Thus, those combinations which lie below the efficient frontier are not optimal since there will always be a portfolio with the same amount of risk that will offer higher returns. To keep simplicity, let us consider a portfolio consisting of only two assets. Figure 1 shows many possible combinations of these two assets, those which are closer to the efficient frontier will be more desirable than the other ones in terms of combined risk and return and, of course, each point lying on the efficient frontier line represents an optimal combination of these two assets.

Some years later, James Tobin (1958) introduced a risk-free asset into Markowitz's framework which will represented the minimum rate at which borrows may borrow funds. By introducing this new asset, the number of feasible portfolio combinations increased since the efficient frontier is extended to a tangent line which intercepts with the efficient frontier, starting from the expected return of the risk-free asset.

Figure 1. Efficient Frontier Example (Harry Markowitz)



Source: André Chrisftoffer Andersen Blog *"Modern Portfolio Theory"*

Efficient Market Hypothesis (EMH)

This theory, developed independently by Paul A. Samuelson and Eugene F. Fama during the 1960s sustains the idea that prices fully reflect all the available information in the market. The rationale behind this is the fact that many active market participants are attempting to maximize their profits therefore any kind of relevant information will be directly captured by stock prices until it reaches its fair value again.

The EMH considers three different, but cumulative, forms of the market which will make the hypothesis more or less reliable, depending on the amount of information available:

i. Weak form: based on historical information, it suggests that all new public and private information may not always be available to investors. Therefore, prices just reflect the information from the past and this doesn't allow us to predict the prices of the stock in the future (argues against technical analysis).
ii. Semi-strong form: moves one step forward and suggests all public information about the company is available to investors and directly transferred into its stock price. This will mean that using fundamental analysis or watching the news won't be useful in order to predict future price movements.
iii. Strong form: considers that even non-public information is captured by stock prices. In this sense, not even investors with insider knowledge could take a profit out from it since prices are already trading at their fair value in the market.

In its stronger form, the EMH leaves no space for arbitrage, nevertheless, those market participants who are able to spot anomalies at first place and take the right positions will lead market moves and consequently benefit from further laggard inflows. However, these extra returns will still be proportional to a higher level of risk assumed since taking anticipated positions implies a greater degree of uncertainty, thus entailing greater risks. This premise was strengthened later on by Fisher Black in his publication about "Noise" in 1986. In his paper, Black states that "noise is what makes financial markets possible, but also makes them imperfect". According to his model, noise causes markets to be somewhat inefficient which creates market opportunities for those traders who are able to differentiate between relevant information and pure noise. This supports Robert C. Merton's model[2] which shows how prices are significantly efficient in the long run and pretty inefficient in the short run.

Capital Asset Pricing Model (CAPM)

Using Tobin's portfolio theory as a starting point, William Sharpe developed the Capital Asset Pricing Model (1964) which showed how asset prices adjusted according to the different levels of risk. To do so, Sharpe defined a simple equation for calculating the expected return of an asset given a certain level of risk:

$$\overline{r_a} = r_f + \beta_a(\overline{r_m} - r_f)$$

Where:

r_f = risk-free rate

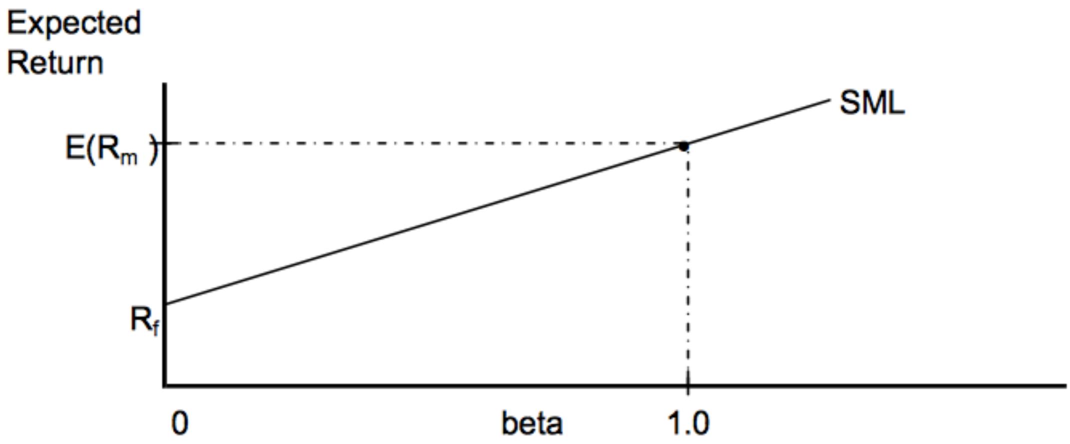
β_a = beta of the security

$\overline{r_m}$ = expected return of the market

[2] Article published in 1971 by the Journal of Economic Theory: "Optimum Consumption and Portfolio Rules in a Continuous-Time Model".

Sharpe assumes in this model that investors are rationale and predicts a positive relationship between risk and return, calculating the expected returns as a sum of the risk-free rate and the risk premium demanded by each investment as a function of its relative risk (β). This equation is used to calculate the cost of capital of an investment and the relationship between the beta and the expected return of the portfolio will determine the Security Market Line (SML) represented graphically in the figure below.

Figure 2. Security Market Line of CAPM Model



Source: Kaplan Financial Knowledge Bank

As we have already seen, CAPM includes a new parameter which wasn't considered in previous models: the beta. Beta is a measure of relative volatility or systematic risk and is used to compare the risk of a portfolio to the risk of the market as a whole. To this effect, a beta coefficient equal to 1 will mean that the portfolio under valuation has the same volatility as the market or, in other words, it will move along with the market. Furthermore, a portfolio with a beta below 1 will mean it's less volatile than the market. That is to say, whenever the market rises, these portfolios will stay behind however when the market falls they will perform better than the market. The same, but opposite dynamic applies for portfolios with betas greater than 1 which means they are more volatile than the market. As a consequence, portfolios with a beta greater than 1 will imply a greater risk assumption which, according to Sharpe, will provide investors a higher expected return. But, is the relationship between the risk premium assumed on an investment directly proportional to the expected returns of a portfolio? And most importantly, shall we take for granted that this relationship is positive?

Although it is true that the CAPM model is the most recognized asset pricing model developed so far and it's still widely used for cost of capital estimations and portfolio performance evaluation, several investigations have proven a number of empirical problems mainly arising from its simplicity. For the purpose of our investigation, we will focus on the beta parameter as an effective risk measure and its actual correlation with the expected returns of the asset/portfolio under observation.

Fama and French models

Nobel Prize Eugene Fama and researcher Ken French have been intensively investigating on the asset pricing field for the last decades. Their first findings were published in 1993 but, since then, they are continually refining their work on the topic. After conducting a deep empirical investigation analyzing the validity and accuracy of the CAPM model on the US market from 1941 to 1990, Fama and French suggested the inclusion of two other factors (size and value) which they believed were significant in order to explain variations in the expected returns of a portfolio. The size effect was included under the belief that small cap stocks generally earn higher returns than those with a large market capitalization. On the other side, the value effect supports the idea that stocks with a low price-to-book ratio perform better than those with a high one. This new model was named the Three-factor model.

However, as I already mentioned above, their models are continuously being tested and allowing them to expand the model even further and include two extra factors, resulting in the latest version of the method we know: The Five-Factor Pricing Model (2015). This renovated model captures not only market risk, size and value but also profitability (stocks with high operating profits perform better) and investment patterns (stocks of companies with a high total asset growth have below average returns), in order to explain variations in the expected returns of a portfolio. Nevertheless, many specialists in the field have recently started to discuss the implications of the inclusion of these two new factors to the original model and suggested alternative factors which they consider more suitable to explain the dependent variable.

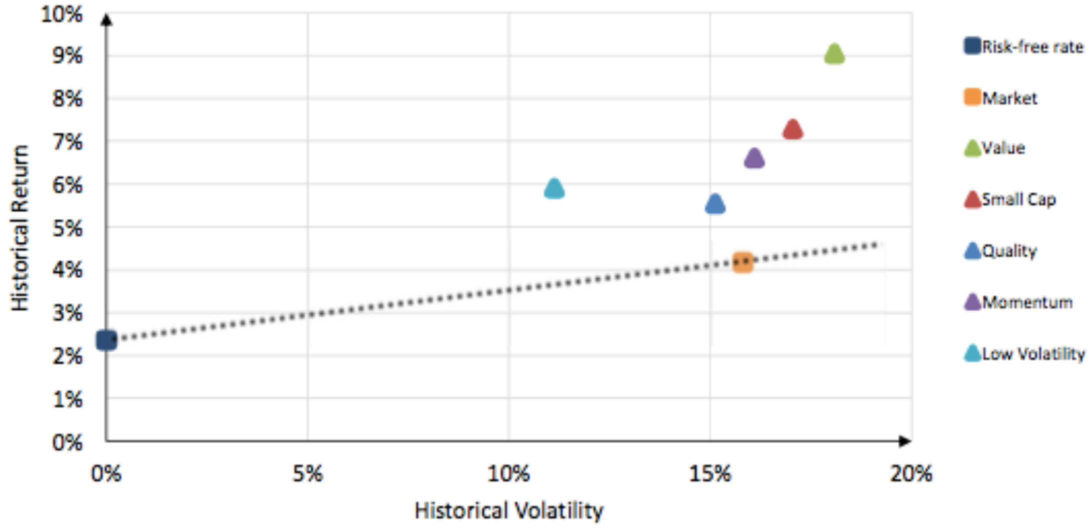
In particular, Pim van Vliet, David Blitz and Matthias Hanauer, three experts on the field currently working for Robeco Asset Management firm, published a research paper in December 2016 explaining a few major concerns of the Five-Factor Model. First of all, they argued that Fama and French failed to provide valuable evidence that a higher market beta exposure is effectively rewarded through higher returns. Additionally, they were particularly surprised regarding the two factors chosen. "The new model still ignores momentum, while this factor is widely accepted within academia and has been around for 20 years" (Pim van Vliet, December 2016). Moreover, they also discussed the fact that a low volatility factor was also omitted although this can be explained as it is difficult to combine it with the risk factor since they contradict each other.

Factor Investing Strategies

Factor investing was introduced during the 1970s when academic researchers began to query the main assumptions of the Capital-Asset-Pricing-Model developed by William Sharpe in 1964. As we already mentioned before, the CAPM model assumes that investors are rationale and that the relationship between risk and return is positive which could make sense at that time but may exceed simplicity now that access to information and technology has evolved so significantly.

Factor investing is an investment process which aims to invest strategically in certain factors which have been proven to outperform financial markets over a long period of time. We refer to factors as certain investment characteristics which help to explain the risk and return of a security. Over the last decades, academic studies have demonstrated that several factors have systematically generated higher risk-adjusted returns as represented in Figure 3.

Figure 3. Historical return and volatility of factors (1998-2015)



Source: Investment Innovation Trends: Factor-Based Investing

The first ones to address some CAPM model deficiencies were Haugen and Heins (H&H) through a working paper in 1972 (further reviewed in 1975) in which they documented a negative relationship between risk and return in both the U.S Stock Market and the U.S Bond Market. The purpose of this paper was to analyze critically the risk-return function, reveal its inherent weaknesses and design an alternative test to examine it. The results of their empirical study showed how, in the long run[3], those portfolios with lower variances (less risk) within monthly returns delivered higher average returns than their riskier equivalents. Haugen finally developed further this idea into low-volatility investing which became one of the building blocks of factor investing.

An additional two factors were identified through Fama and French (1993) studies which proved that small cap and value stocks perform significantly better than expected through the traditional Asset Pricing Models. This evidence supported the idea that not the all the performance of stocks can be explained through risk but also by other factors.

[3] The authors decided to take a 46-year time period (1926 to 1971) and split it into nine shorter time periods of 5/6-year periods to reveal the importance of the bull-bear market problem. By doing so, they were able to address the importance of the time-period selection when testing empirical evidence and prove the overall effect when this factor is minimized by selecting a very long period of time.

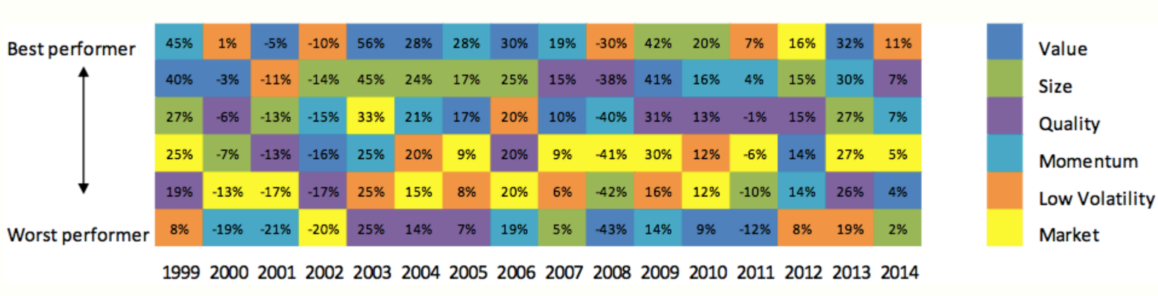
Jegadeesh and Titman (1993) coined another building block of Factor Investing which became known as the momentum effect. Through their studies, they proved that stocks which have performed better in the past are likely to outperform also in the future. The objective of momentum strategies is to benefit from trends in the last 3/6/12 months assuming that they will continue in the same direction at least in the short-term.

Moreover, two additional factors have been identified that provide a premium according to the MSCI organization: quality and dividend yield. The first one aims to capture the excess return on stocks which have low debt, stable earnings growth and strong balance sheets. Dividend yield of course focuses on the choice of stocks which have higher-than-average dividend yields in order to benefit from this extra yield reward.

It is a given fact that Factor investing Strategies have become increasingly popular ever since the seventies. However, there is currently an open debate on whether the topic is here to stay or if it is just a hype. In order to assess this, we should consider that Factor Investing has been built over many years of academic research and it has been proved to work. Furthermore, new market anomalies are still being discovered which will increase the scope of interest over the next years therefore we can assure that Factor Investing has still space to continue developing. Finally, many relevant institutions and professional investors are already allocating part of their portfolios in Factor Investing Strategies which evidences its contribution to portfolio management.

Taking a look at an investigation published by Snaja Centineo and Santo Cenitneo in the Serbian Journal of Management (2016) over Factor-Based Investing we can see how the different factors have worked over the last two decades. Looking at their results, we can see from the table below that there is no clear winner which outperforms the market persistently in the long run. However, we identify the fact that factors perform better or worse according to the different underlying market conditions. For example, we observe how momentum, size and value have performed better during bull markets whereas low volatility and quality factors have been the winners during bear markets. The difficulty comes when trying to time those factors, as Glushkow (2015) highlighted. The continuous rebalancing of factor allocation has been proved not to deliver significantly greater returns compared to a static factor exposure due to the complexity of guessing market timing.

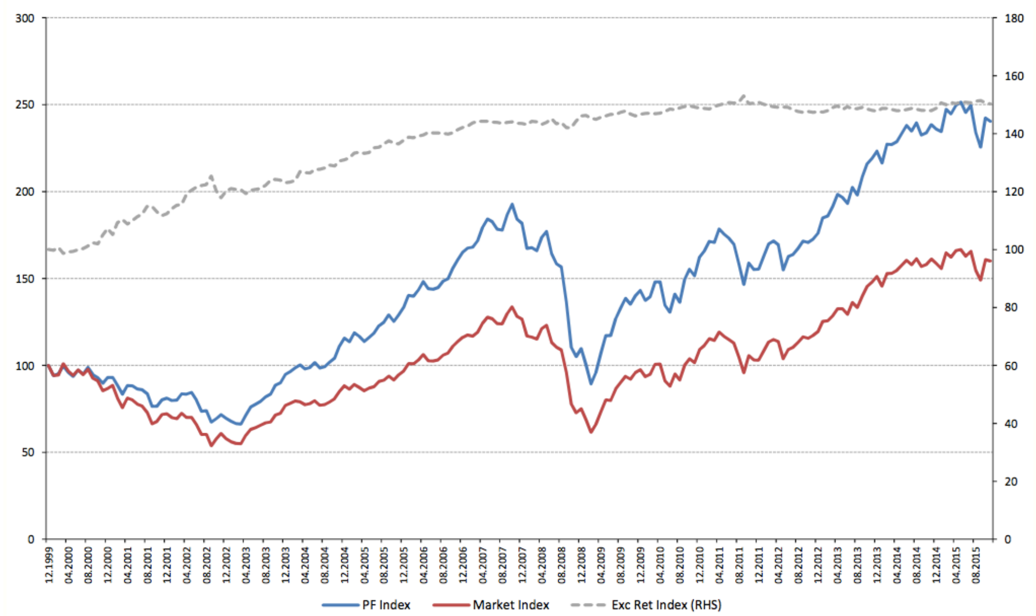
Figure 4. Yearly performance of different factors from 1999-2014



Source: Investment Innovation Trends: Factor-Based Investing

For the purpose of the former paper, this table draws important hints regarding risk and return relationships over the different periods. For instance, whenever momentum strategies are included amongst the top performers of the year, we expect to see a strong positive relationship between these variables. On the other hand, the periods where low volatility strategies overperform are expected to deliver a negative relationship between risk and return. Nevertheless, a well-diversified portfolio including static equal weighted factor exposures may consistently outperform the market due to low correlations between the different factors, as shown in the Figure below where the blue line represents the equally weighted factors portfolio performance and the red line stands for the market performance.

Figure 5. Performance of equal-weighted factor portfolio vs. market



Source: Investment Innovation Trends: Factor-Based Investing

Smart beta Approach

Although there is no universally accepted definition for Smart Beta concept it's usually understood as an alternative notion to Factor Investing Strategies. Smart Beta seeks to enhance returns while improving diversification and reducing risk. Therefore, it is a combination of what we know as active investment as it aims to outperform a certain Index (in other words: they try to capture *alpha*[4]), but it is also passive as its implementation is transparent and set under perfectly defined rules. The basis of the concept if to achieve greater returns than the market by using a number of relatively passive investment strategies assuming no more risk than the one that will be taken by investing in a low-cost total indexed fund (which has a beta equal to one by definition). These strategies often come in the form of an ETF or an Index Fund and they have become increasingly popular among investors over the last years.

To this effect, Smart Beta Strategies are related to multifactor asset pricing models such as Fama and French 3-factor and 5-factor models (1993;2015). The technique used by the portfolio managers implementing Smart Beta Strategies consists in tilting the portfolio towards a specific factor for example value versus growth, smaller versus larger companies or low versus high volatility stocks. Some portfolio managers also blend several factors in order to increase diversification and reduce the beta level of the portfolio while still capturing alpha.

For the purpose of this study, we are going to focus on how the Low Volatility Strategy can be played through these instruments. The strategy relies on the idea that high-beta (riskier) portfolios do not necessarily deliver higher returns than low-beta ones, in opposition to the CAPM Model implications which assured a positive relationship between risk and returns. Having said this, investors who want to bet for those assets with lower betas can do it through these portfolio strategies. One way to do this is simply buying long those stocks with the lowest volatility and at the same time selling short those stocks with the highest volatility. Alternatively, suppose a low-beta portfolio with a beta of 0,5 but that delivers the same return as the market (which by definition has a beta of 1). An investor may have the opportunity to buy a low-beta portfolio on margin, which will double the risk and the return of the portfolio, obtaining double the return of the market by assuming only the risk of the market (beta equal to 1).

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[4] Jensen's Index (1970), most commonly referred to as "alpha" is a risk-adjusted measure which reflects the average return of a certain portfolio above or below its benchmark.

Since their origin, “smart beta” indexes have increased interest in creating low volatility indexes to invest in to benefit from this anomaly in the market. All of these indexes and the underlying methodologies can be categorized into two different groups: heuristic and optimization-based.

Heuristic approaches are relatively simple since they are just ranking-based indexes. On the other hand, *optimization-based indexes* are built through a more complex process, but they are also usually more flexible, more accurate in the long run and reduce unintended exposures to certain styles, countries or sectors. The rationale behind this method is to find the optimal portfolio with the lowest volatility by changing the weights of the different assets according to their volatilities and their correlations. Hence, the quality of the final index will be determined by the accuracy of the estimations (volatilities and correlations between the different assets) and the precision of the weights assigned.

Nonetheless, low volatility indexes are mostly built through heuristic approaches which rank the universe of stocks based on estimations upon their volatility and then select a subset within those members and assign weightings to the different stocks, applying different weighting schemes. These weightings are usually determined by factors such as market capitalization or by the inverse of volatility or variance. Additionally, constraints may be applied to these ranking-based approaches in order to assure reasonable liquidity levels, limit stock weightings or control sector and country exposures, amongst others. In this sense, using heuristic approaches is simple, transparent and have a flexible weighting scheme which makes them very practical and accessible. However, they have many limitations which could handicap the returns of the Index in the long run.

In first place, ranking-based approaches are generally based on individual volatilities of the different stocks hence ignoring the correlation between them and their returns which can have a relevant impact on the strategy, especially when correlations between the stock performance and their volatility is high. Another important limitation of this methodology Is that it may deliver exposure to other (residual) factors therefore failing to provide a pure exposure to low volatility. This implies that part of the risk/return behaviour of these Indexes will be distorted by the movements of these residual factors and this must be considered when analysing their performance.

Low Volatility Strategies

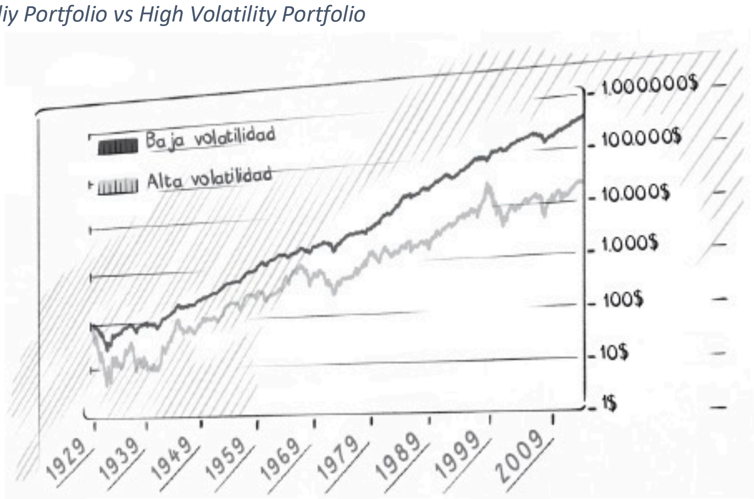
Ever since Robert A. Haugen and and James A. Heins addressed an anomaly in the market in 1972 which implied the flat (or even negative) relationship between risk and return numerous studies after that have confirmed their results. Several explanations have been identified over the last years for low volatility outperformance, mostly behavioral.

The book which motivated the present investigation, *"High Returns from low Risk: A Remarkable Stock Market Paradox"* (Pim & Jan, 2017) analyzed this market anomaly within the US market for the period comprised within January 1929 and December 2016. The hypothesis raised was whether low volatility stocks delivered higher returns than high volatility stocks in the long run. For this purpose, they selected 1000 different stocks from the US market and considered their monthly returns to compute their monthly volatilities. With the data collected, two different portfolios were built:

1. A Low volatility portfolio, comprising the 100 stocks with lower risk (lower volatility)
2. A High volatility portfolio, comprising the 100 stocks with higher risk (higher volatility)

These portfolios were balanced on a quarterly basis due to the fact that volatility of the stocks could vary over time. After conducting their analysis, the authors confirmed that, indeed, low volatile stocks outperformed their higher counterparts in the US market from 1929 to 2016. The figure below shows their main result, where we can see that the Low Volatility Portfolio outperformed the High Volatility one by a factor of 18.

Figure 6. Low Volatiliy Portfolio vs High Volatility Portfolio



Source: *"High Returns from low Risk: A Remarkable Stock Market Paradox"* (Pim & Jan, 2017)

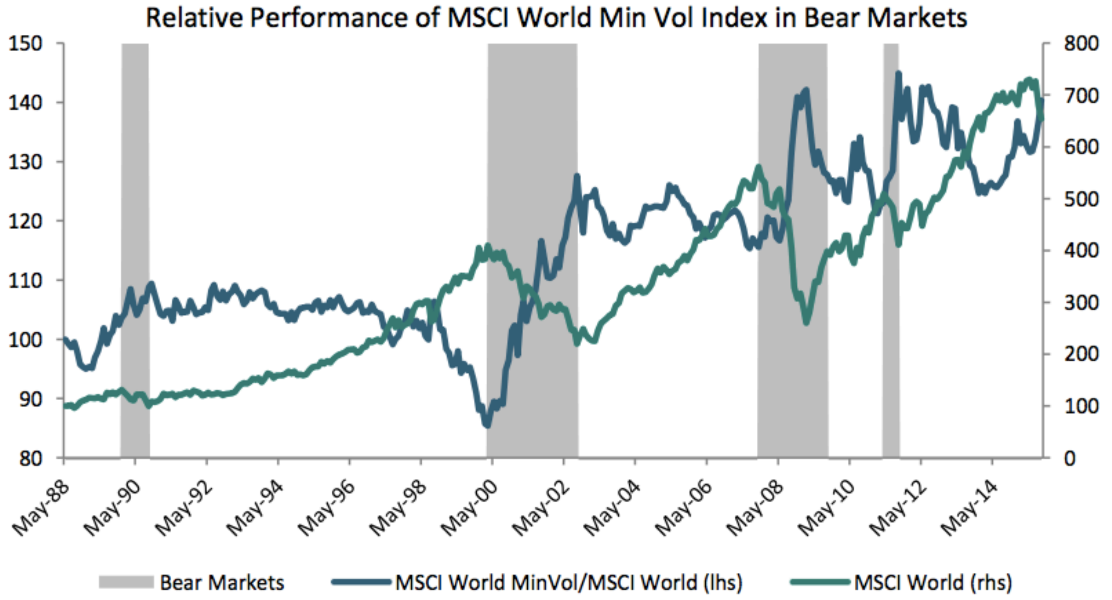
One of the main explanations for this market anomaly has been proven to be the asymmetric behavior of the low/high volatility stocks during the different bear/bull markets. Historically, during bear (declining trend) markets, the dispersion between the betas of low and high volatility portfolios has increased. That is to say, lower volatility stocks have experienced smaller declines than higher volatility, however, during bull markets this dispersion is reduced hence causing low volatility stocks to underperform just slightly.

Due to this effect, low volatility stocks are able to perform better in the long term since their maximum drawdowns are considerably smaller than high volatility ones when there is a market decline, and this will help their overall performance due to the power of compounding[5]. The rationale behind this is that if a stock falls a 10% for example, it won't need a 10% gain to recover its initial value, but a 11%. This effect is larger as the downturn magnitude increases. For instance, if it experiences a decline of 25%, it will need a 33% growth to recover, if it falls 50% it will need a 100% appreciation, and so on.

The MSCI organization published a Research paper in January 2016 where they analyzed the performance of the MSCI World Minimum Volatility Index during different bear markets over the past decades. This investigation considered 4 bear markets, whenever there was a decline of 20% on the MSCI World Index for more than 2 months. Figure 7 shows the results of their investigation, where we can see how the Minimum Volatility Index outperformed the market across all four bear market periods. This better relative performance during turbulent markets provides capital preservation and it is an essential explanation for the low volatility anomaly in the markets.

[5] The compound interest effect is one of the main principles in finance. Albert Einstein, the world's most famous scientist in history described it as "the eighth wonder of the world" and said that "who understands it, earns it, and who doesn't, pays it".

Figure 7. Performance of MSCI World Min Vol vs MSCI World in Bear Markets



Source: MSCI Research Insight: "Constructing Low Volatility Strategies" (*January 2016)*

Regarding behavioural explanations, one of the reasons that can explain this anomaly in the market is the fact that less volatile stocks are usually underpaid by the market due to the irrational preference of investors for more volatile stocks trying to seek higher returns. This implies that less volatile stocks might be mispriced only for the fact that investors haven't considered them for being less risky and, thereby, theoretically, less profitable.

In addition, investors tend to overpay for high volatility stocks as they are often famous for having delivered very attractive returns in the past, however, they sometimes ignore the speculative nature of these stocks. This implies low volatile stocks are often underpaid compared to their higher counterparts.

What is more, investors have typically been very confident on their ability to forecast future market fluctuations and therefore go ahead whenever they see clear market opportunities. This generates a certain momentum, which is not compensated by the pessimistic investors since it is more difficult to see them expressing their negative views through short selling, which drives prices of high volatility stocks up, thereby leaving them with lower expected future returns.

Finally, another explanation that has been identified for this market anomaly is the fact that less asset managers cover lower volatility stocks just because most of the time there is less research done by brokers compared to those more volatile which tend to be "in the spotlight". Therefore, it might be easier to find investment opportunities in less volatile stocks since many asset managers are not even considering them.

Methodology

Database description

Variables used and data collection process

As already mentioned at the beginning, the sample chosen for the purpose of the present investigation is the STOXX Europe 600 Index since it gathers information from a large number of companies within the different European countries, thereby significantly reflecting the behavior of the entire scope of interest. The investigation has focused on the European market in order to see if what previous studies on the anomaly under investigation, which have mostly been done over the US market, can be confirmed or not for this market.

This Index is a subset of the STOXX Global 1800[6] Index and derives from the STOXX Europe Total Market Index (TMI). Our benchmark has a fix number of 600 components and represents large, mid and small capitalization companies across 17 countries of the European region: Austria, Belgium, Czech Republic, Denmark, Finland, France, Germany, Ireland, Italy, Luxembourg, Netherlands, Norway, Portugal, Spain, Switzerland and the United Kingdom. All of the STOXX 600 Indexes are reviewed on a quarterly basis, (in March, June, September and December) reselecting the 600 largest companies from the STOXX All Europe Total Market Index[7] (TMI). In addition, stocks must fulfill several criteria in order to be considered:

1. Only the most liquid stocks from each company can be elected.
2. The eligible stocks must have a minimum liquidity of one million euros measured over 3-month average daily trading volume (ADTV).

[6] This Index is an aggregate of the regional 600s from Europe, North America and Asia/Pacific.
[7] This Index represents the Western and Eastern Europe region as a whole and includes approximately 95% of the free float market capitalization of European companies with a variables number of components.

Once the candidate stocks are collected, they are ranked in terms of free-float market capitalization in order to produce the selected list, taking the last trading day of the month preceding the review date as the cut-off date. The first 550 stocks in the ranking list are directly included in the Index, the remaining 50 are chosen from the stock ranked between the 551 and 750. In addition, in order to keep the number of components constant, a deleted stock is replaced by the highest-ranked non-component on the selection list, which will be updated on a monthly basis.

In order to answer the question raised by the former paper, two different kind of analysis were undertaken. The first one consists in a *linear regression analysis* which was carried out using the statistical and econometrical program Gretl in order to assess whether the variable risk (volatility) has a significant impact over returns. To these effects, the following null hypothesis was raised: “there is a positive relationship between the variable volatility (risk measure) and total returns of a stock” which was to be accepted or rejected based on the estimations obtained from the Ordinary Least Squares (OLS) model. The hypothesis was tested over three different time periods in order to observe how the results vary under different market situations decoupled as follows: the first analysis studied the behavior of these stocks from 2001 until today (2018), and the second and third from 2007 and 2012 onwards, respectively. Additionally, two extra scenarios were studied, which considered extreme market conditions (a pure bull and a bear market) to observe the different behaviors of the variables in these particular cases.

The second type of analysis performed was based on ranking-based approach to compare the total returns for the whole period (2001-2018) of the different components of the Index with the total return of the Index itself. This time, another hypothesis was raised which questioned whether “low volatility stocks outperform the market as a whole in the long-run”. This investigation selected the 20 stocks with lower volatility within the index during the whole period (2001-2018) excluding those which weren’t part of the index at some point over the time frame observed and compared their total return against the total return of the market as a whole, represented by a benchmark: the STOXX 600. This same practice was done with the 20 most volatile stocks. Additionally, two virtual portfolios were built, a low volatility one and a high volatility one (both with the same 20 stocks as in the previous analysis) and compared against the whole performance of the Index during this period.

Source of data- Reuters

The sample used to conduct the different analysis was extracted from the Thompson Reuters database. Monthly closing prices were collected for the STOXX 600 and each one of its 600 components as of 01.04.2018 for the period comprised between 01.02.2001 and 01.04.2018. The chosen Index already includes dividend effects since the aim of the investigation is to assess the relationship between the total returns and the volatility of the Index and its different components.

Tools used to perform the analysis

Once obtained the data required for the analysis from Thomson Reuters, the sample was exported to Excel in order to build the final database and make the necessary calculations to perform the different analysis. Five separate databases were built, one per period of study. From now on, we will refer to them as Scenarios 1-5 being:

- Scenario 1: including data from 2001-2018
- Scenario 2: including data from 2007-2018
- Scenario 3: including data from 2012-2018
- Scenario 4: including data from 2007-2011 (Bear Market Case Example)
- Scenario 5: including data from 2012-2015 (Bull Market Case Example

The first step was to remove all the stocks which weren't part of the Index at some point during the selected period, since they were not considered to conduct the analysis for simplifying purposes. After doing so, the sample size for each period was reduced from the initial (600) quantity. The final number of stocks considered in each case are summarized in the following Table.

Table 1. Final Sample sizes for the different Scenarios

	SAMPLE SIZE
SCENARIO 1	389
SCENARIO 2	483
SCENARIO 3	514
SCENARIO 4	483
SCENARIO 5	535

Source: Own elaboration

Taking this data subset, monthly variations were calculated in order to obtain the total returns for each stock. Monthly and Total Stock Returns for the entire period were computed through simple or arithmetic returns as follows:

$$Total\ Stock\ Return = \frac{(P_1 - P_0)}{P_0}$$

Where:

P_0 = Initial Stock Price

P_1 = Final Stock Price

Furthermore, in order to calculate the monthly volatility for each stock, returns from natural logarithmic were calculated by computing the natural logarithms to the division between the final price and initial price in order to equal rise and fall movements in the prices.

$$Natural\ logarithmic\ returns = \ln\left(\frac{P_1}{P_0}\right)$$

Monthly volatilities were then estimated by calculating the standard deviation of these returns for the entire period. Standard deviation is a statistical metric used to measure the dispersion of a set of data from its mean. This measure is commonly used in the financial industry as an approximation to volatility, meaning the higher the standard deviation is for a particular stock, the higher is its volatility and thereby, the higher the risk assumed according to the recognized Financial Principles.

$$SD_{sample} = \sqrt{\frac{\sum|x - \bar{x}|^2}{n-1}}$$

Where:

x = value of the data subset (natural logarithmic returns)

$\bar{x}$ = the mean of the natural logarithmic returns from the sample

n = number of data items within the sample

This same process was repeated for each time period selected to build the five different databases to conduct each of the analysis. Appendix I[8] displays the database used to cover Scenario 1, with a sample size of 389 stocks.

As mentioned at the beginning of the former paper, the investigation process to assess the performance of low volatility strategies in the long run comprised two different kind of analysis were performed using the databases previously built. The first one, consists on a *linear regression analysis* using the statistical and econometrical tool Gretl to assess the significance of the volatility variable (risk) over the independent variable (total returns of a stock) and the relationship between both variables.

[8] The databases built for the remaining Scenarios follow the same format and criteria, only varying in each case the sample size, which changed according to the entry/exit stocks in each period.

In order to do so, the Ordinary Least Squares (OLS) method was used to obtain the necessary information in order to accept or reject the raised null hypothesis: "there is a positive relationship between the variable volatility (risk measure) and total returns of a stock". This method aims to minimize the sum of the squared residuals [9](ε) of the model following the expression below.

$$\sum_{i=1}^{n} e_i^2 = \sum_{i=1}^{n} (y_i - \hat{y}_i)^2$$

The analysis was done over three different time periods in order to observe how the results vary under different market situations. The first analysis studied the relationship from 2001 until today (2018), and the second and third from 2007 and 2012 onwards, respectively. In addition, two more scenarios were tested which considered extreme market conditions: a pure bull market considering the data from 2012 until 2015 and a pure bear market from 2007 to 2011, in order to observe the different behaviors of these variables under these particular conditions.

The model was tested twice in the different scenarios, under two different variables for volatility:

1. Monthly volatilities and total returns
2. Logarithm of the monthly volatilities and total returns

Using the results obtained through these models, the statistical indicators that are going to be used to compare the five different scenarios in both cases are: correlation coefficient, standard deviation, R-squared, p-value and t-statistic which are understood as follows.

- Correlation coefficient: measures the relationship between two variables (x and y). This indicator can take any value between -1 and 1. The closer this measure is to one, it means is there is a positive (linear) relationship between the two variables. On the opposite side, as the correlation coefficient is closer to -1, it means there is a negative relationship between both variables. This measure considers the covariance between both variables and their independent standard deviations as follows:

$$\rho_{xy} = \frac{Cov\ (x; y)}{\sigma_x\ \sigma_y}$$

[9] The sum of the squared differences between the real observed values (y_i) and the estimated values ($\hat{y}_i$).

For the purpose of the former analysis, a correlation coefficient above 0 will indicate a slightly positive relationship between the explanatory variable (volatility) and the explained variable (total returns) and the opposite interpretation whenever the indicator is below 0.

- Standard deviation: is a statistic indicator which measures the dispersion of a dataset relative to its mean. It is calculated as the square root of the variance. The further the different points of the dataset are to the mean, the higher the standard deviation is. In the finance field, the standard deviation is commonly used as an approximation to volatility meaning that, the greater the standard deviation of a security is, the larger will be the price range where it will be moving. Thus, more volatile stocks will have larger standard deviations and less volatile stocks smaller standard deviations.

- R-squared: is a statistical measure which represents the proportion of variability of the dependent variable (total returns in this case) explained by a change in the independent variable (volatility in the former model). R-squared values range from 0 to 1, being 1 commonly understood as 100% and 0 as 0%. An R^2 close to 100% will mean that the entire variance of the dependent variable (total returns) is explained by the explanatory variable (volatility). That is to say, the higher the R^2, the better are variances on the dependent variable explained through your model. In this case, as we know, there are many other variables that affect variations in prices besides risk and since our models just include this factor for simplification purposes the R^2 is expected to be very close to 0.

- Beta: is a coefficient which measures the strength of the effect each individual independent variable has to the dependent variable. The higher the absolute value of this parameter is, the stronger the effect that the variable associated to it will have on the dependent variable. Whenever the correlation between the variables is positive, the β parameter associated to that particular independent variable will also take a positive sign and the opposite will apply in the case of negative correlations.

- P-value: it is a statistical measure which evaluates how well the sample data supports the acceptance of the null hypothesis raised for each variable inside the model. A p-value below 0,05 (common alpha significance level) is determined to have a significant impact on the explained variable, whereas a variable with a p-value above 0,05 will determine that this variable is not statistically significant to explain the changes in the dependent variable.

Model assumptions

It's important to consider the following relevant assumptions that were made in order to build the model.

1. Data selection: only those stocks which were part of the index during the entire period under evaluation were selected for simplifying purposes. This reduces the sample introduced into the model considerably, however the resulting subsets are still large enough to consider the sample representative and deliver significant results.

2. Variables: we assume that the only factor affecting total returns is risk (volatility) in the model. As we know, returns are not only affected by this factor and therefore we should consider this fact when analyzing the results obtained.

Analysis of the results obtained

Linear Regression analysis

The results presented for this analysis correspond to two different models for each scenario. The first model tries to assess the relationship between the monthly volatilities (in %) of each stock and their total returns (in %) obtained over the period selected. On the other hand, the second model tries to define the relationship between the logarithms of the monthly volatilities (in units) and the same total returns (in %). The null hypothesis (H_0) raised for every case was the same one: "there is a positive relationship between the variable volatility (risk measure) and total returns of a stock".

In both cases, the same 5 Scenarios were tested, defined as follows:

- Scenario 1: data from 2001-2018
- Scenario 2: data from 2007-2018
- Scenario 3: data from 2012-2018
- Scenario 4: data from 2007-2011 (Bear Market Case Example)
- Scenario 5: data from 2012-2015 (Bull Market Case Example)

Model 1: Monthly volatilities and Total Returns

The first analysis has been elaborated from the output of 5 different Simple Linear Regression Models, defined by the following expression:

$$Y = \alpha + \beta X + \varepsilon$$

Where:

$Y =$ is the dependent random variable, which represents by the total returns of the stocks

α = is the Y-intercept of the regression line

β = defines the slope of the regression line, which measures the change in Y for every change in X

X = is the independent random variable, represented by the monthly volatilities of the stocks

ε = residuals of the model which represent the differences between the real observed values of Y (y_i) and the estimated values ($\hat{y}_i$)

The main parameters obtained from the models undertaken using monthly volatilities and total returns for the different Scenarios are presented in Table 2.

Table 2. Correlations and Summary of Ordinary Least Squares Model Results

	SCENARIO 1 2001-2018	**SCENARIO 2** 2007-2018	**SCENARIO 3** 2012-2018	**SCENARIO 4** BEAR MARKET[10]	**SCENARIO 5** BULL MARKET[11]
N	389	483	514	483	535
CORRELATION	-0,0528	-0,0436	0,1050	-0,3013	0,0914
R^2	0,002789	0,0019	0,0110	0,0908	0,0084
P-VALUE	0,2988	0,3385	0,0173**	0,0000***[12]	0,0346**
β	-20,8061	-5,3480	11,8002	-5,4157	7,1798
STD DEVIATION	20	5,58	4,9413	0,7815	3,3884

Source: Own elaboration

MAIN INTERPRETATIONS:

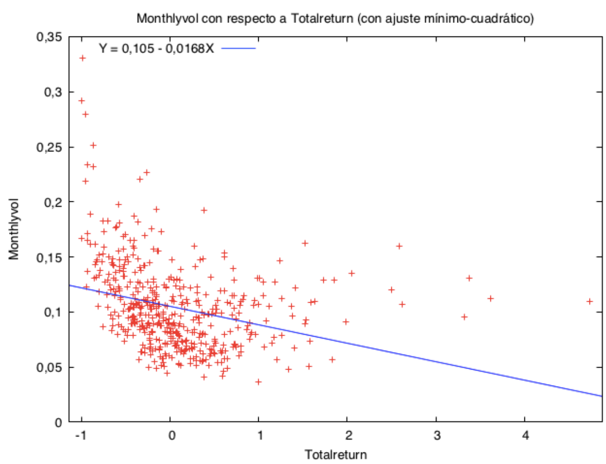
1. **Three** out of the five scenarios under examination have shown a **negative correlation** between the variable monthly volatility and total returns (when risk increases, total returns decrease)
2. However, only **three** scenarios (Scenario 3,4 and 5) have resulted **statistically significant** (P-values < 0,05) to explain the dependent variable.
3. In **Scenario 3**, for every increase in one unit monthly volatility, the total returns of the period will increase by 11,8 units (β = 11,8). Therefore, in this case we **accept the H_0**: "there is a positive relationship between the variable volatility (risk measure) and total returns of a stock". However, the model just **explains 1,10%** of the variations in the total returns according to the results (R^2 = 0,0110) but this is something normal since we know the changes in prices are affected by many other factors besides risk, and we are only considering this variable in the model.

[10] Bear market data was taken from 2007 to 2011.
[11] Bull market data was taken from 2012-2015.
[12] The OLS results displayed precisely 1,36E-11, which suggests the variable monthly volatilities is very significant to explain variations in total returns.

4. In **Scenario 4,** for every increase in one unit of monthly volatility, total returns of the period will decrease by -5,4 units (β = -5,4) therefore in this case we will **reject the null hypothesis** since it's proven that during this period the relationship between risk and return was negative. We can also see from the data obtained that during this period the risk factor had a stronger effect on total returns, **9,08%** of the variations in total returns were explained by the variable volatility (R^2 = 0,0908).
5. In **Scenario 5**, for every increase in one unit of monthly volatility, total returns will increase by 7,18 units (β = 7,18), hence we will **accept the null hypothesis** since the relationship between both variables during the period selected is positive according to the model. However, the model just **explains the 0,8%** of the variations in the total returns (R^2 = 0,0084) due to the existence of many other factors besides risk that may affect the total returns of a stock.

The following Scatter Plot diagrams show the relationship between the two variables (monthly volatilities and total returns) according to the model for the different scenarios studied previously.

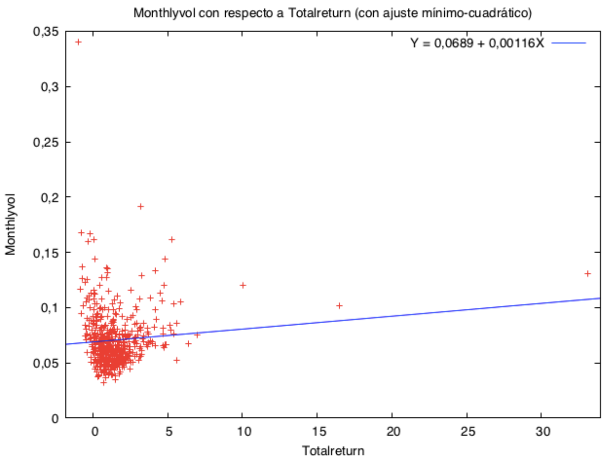
Figure 8. Scatter plot (X-Y) Scenario 4 (BEAR MARKET)



Source: Own elaboration

Figure 8 shows a strong negative relationship between the two variables in Scenario 4 (ρ_{xy} = -0,3013). This makes sense as low volatility stocks are characteristic for preserving capital under turbulent market conditions and high volatility ones are the ones which suffer the most during market downturns.

Figure 9. Scatter plot (X-Y) Scenario 5 (BULL MARKET)

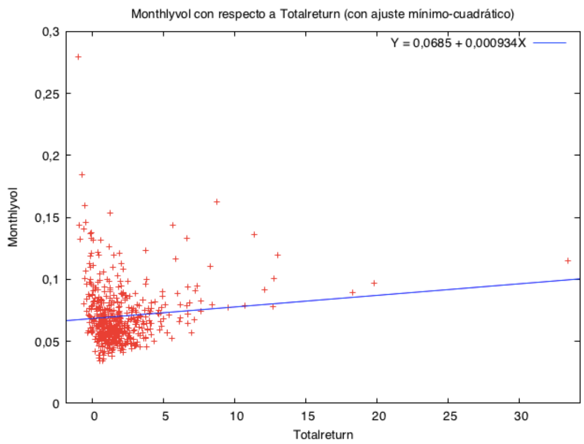


Source: Own elaboration

Figure 9 shows how the data from the bull market Scenario is distributed. We can see there is also a positive relationship between the variables, (positive correlation 0,0914). We observe how the relationship between the variables is much stronger during extreme market conditions (bear and bull periods) than over a longer period of time which covers different stages of the economic cycle.

Figure 10. Scatter plot (X-Y) Scenario 3 (2012-2018)

Figure 10 represents how the data for Scenario 3 is distributed. We can see how the line of best fit shows a positive correlation between the variables, as already represented in Table 1 through the correlation coefficient (ρ_{xy} = 0,1050).



Source: Own elaboration

Another interesting fact is that the correlation between risk and return is stronger *(ρ_{xy}= -0,30 vs +0,09)* during bear markets (Scenario 4) compared to bull markets (Scenario 5) and there is less dispersion of the data from its mean *(lower standard deviation: 0,7815 vs 3,3884).* Consequently, the bear case model is statistically more significant than the bull market one (R^2 = 0,0908 vs 0,0084). Despite this, monthly volatility has a smaller impact on total returns during bear markets compared to bull markets (-5,4% vs +7,18% total return change for every 1% increase in volatility).

Model 2: Logarithm of monthly volatilities and Total Returns

The previous Simple Linear Regression Models were repeated, this time substituting the independent random variable (X) monthly volatilities with the logarithms of the monthly volatilities. The new models followed the hereafter expression:

$$Y = \alpha + \beta X + \varepsilon$$

Where:

$Y =$ is the dependent random variable, which represents by the total returns of the stocks

α = is the Y-intercept of the regression line

β = defines the slope of the regression line, (measures the % change in Y for every % change in X)

X = is the independent random variable, represented by the logarithms of the monthly volatilities of the stocks

ε = residuals of the model which represent the differences between the real observed values of Y (y_i) and the estimated values ($\hat{y}_i$)

The main parameters obtained from the linear regression model performed using the logarithms of the monthly volatilities and the total returns for the different Scenarios are presented in Table 3.

Table 3. Correlations and Summary of Ordinary Least Squares Model Results (Log model)

	SCENARIO 1	**SCENARIO 2**	**SCENARIO 3**	**SCENARIO 4**	**SCENARIO 5**
	2001-2018	2007-2018	2012-2018	BEAR MARKET[13]	BULL MARKET[14]
N	389	483	514	483	535
CORRELATION	-0,0098	-0,0242	0,0136	-0,2835	0,1138
R^2	0,0001	0,0005	0,0002	0,0804	0,0129
P-VALUE	0,8470	0,5953	0,7589	0,0000***[15]	0,0085***
β	-0,8410	-0,6377	0,2902	-1,2958	1,7602
STD DEVIATION	4,3557	1,1997	0,9453	0,1999	0,6659

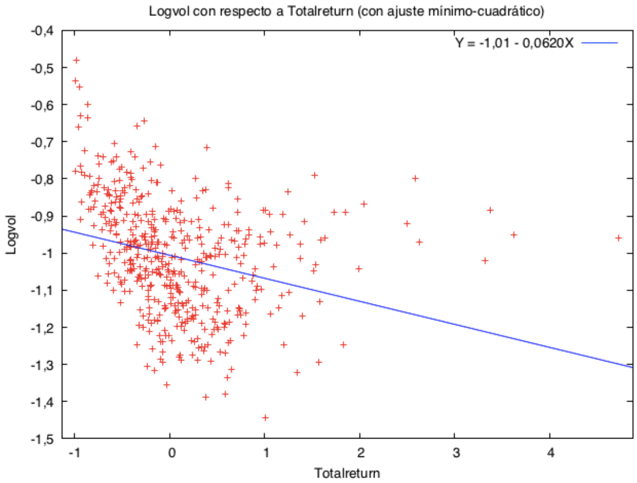
Source: Own elaboration

MAIN INTERPRETATIONS:

1. Once again, the results of the linear regression analysis prove a **negative correlation** between the variables logarithm of the monthly volatilities and total returns in **three** out of the five scenarios that were tested. This means that we will only accept the null hypothesis (H_0) "there is a positive relationship between the variable volatility (risk measure) and total returns of a stock" in two of the cases (Scenario 3 and Scenario 5) where the relationship is positive according to the correlation coefficient (ρ_{xy} = 0,0136 and 0,1138, respectively).
2. However, only the **two** scenarios of extreme market conditions (Scenarios 4 and 5) have resulted **statistically significant** (P-values < 0,05) in order to explain the dependent variable.

[13] Bear market data was taken from 2007 to 2011.
[14] Bull market data was taken from 2012-2015.
[15] The OLS results displayed precisely 2,23E-10.

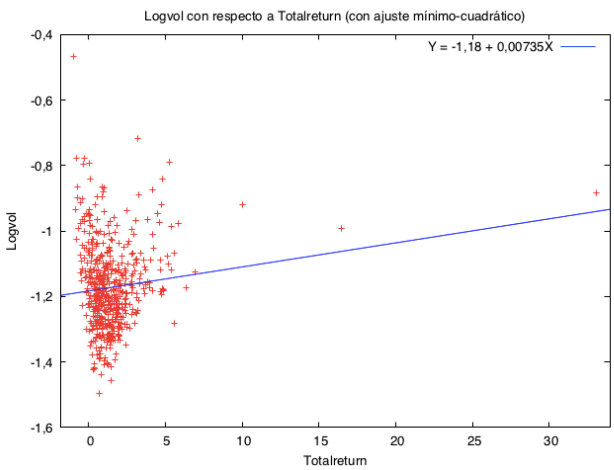
Figure 11. Scatter plot (X-Y) Scenario 4 (BEAR MARKET) Log Model



Source: Own elaboration

3. **Scenario 4** results, which consider a bear market period (from 2007 to 2011), show a **strong negative relationship** between the variables, as it is represented in the dispersion graph to the left (Figure 11). Considering the outputs from the model presented in Table 3, we will **reject the null hypothesis** given that there is no positive relationship amongst these two variables within the data of the selected period as the results show how a 1% increase in monthly volatility will make returns decrease by 1,3% (β = -1,2958). In this model, the variable logarithm of the **monthly volatility explains 8,04%** of the variations in the total returns (R^2= 0,0804) which is quite high considering that there are many other factors affecting this variable apart from risk.

Figure 12. Scatter plot (X-Y) Scenario 5 (BULL MARKET) Log Model



Source: Own elaboration

4. On the opposite side, **Scenario 5** which simulated a pure bull market case (considering data from 2012-2015) proved a very **strong positive relationship** between the two variables and hence we will **accept the null hypothesis** under this particular market condition as it is graphically displayed in the dispersion graph next to these lines (Figure 12). The outputs from the model state a beta of 1,76 which means that for every 1% increase in monthly volatility, total returns in this period increased by 1,76%. In this case, the variable logarithm of the monthly volatility **explains 1,29%** of the variations in the total returns (R^2= 0,0129).

5. Even so, it has been observed that, once again, **the correlation between these variables is stronger during bear markets** than bull markets (ρ_{xy}= -0,28 vs +0,11) and that the variable **volatility is more significant** in order to explain changes in the total returns **during market downturns** (R^2=0,08 vs 0,01). Nevertheless, the betas of both Scenarios show that changes in volatility haver a greater effect on total returns during bull markets (β=-1,30 vs 1,76).

Total Return comparison (Ranking-based approach)

The aim of second analysis performed was to compare the performance of the individual stocks with the performance of the Index during the full period (2001-2018). For the defined purpose, 3 different tests were undertaken. In first place, two portfolios were built (Low Vol and High Vol Portfolios). The Low Vol Portfolio was built through a ranking-based approach, selecting the 20 stocks with lower monthly volatility out of the 389 comprised in the Index during this period. Table 4 displays the component stocks of the portfolio together with the total returns obtained along the period and their monthly volatilities, sorted out from least to most volatile.

Table 4. Summary Total Returns and Monthly Volatilities Low Vol Portfolio

	Company	Total return	Monthly vol
1	NESTLE 'R'	248,37%	3,85%
2	COFINIMMO	165,63%	3,88%
3	SWISSCOM 'R'	137,84%	4,12%
4	NOVARTIS 'R'	88,05%	4,37%
5	DIAGEO	550,83%	4,55%
6	SSE	458,00%	4,62%
7	RECKITT BENCKISER GROUP	980,14%	4,78%
8	COLRUYT	516,93%	4,88%
9	AIR LIQUIDE	377,85%	4,89%
10	DANONE	197,25%	4,99%
11	ESSILOR INTL.	740,72%	5,02%
12	RELX	251,48%	5,15%
13	RELX	205,43%	5,16%
14	SWEDISH MATCH	1631,08%	5,18%
15	GLAXOSMITHKLINE	68,12%	5,21%
16	UNITED UTILITIES GROUP	308,75%	5,30%
17	TOTAL	181,39%	5,33%
18	PENNON GROUP	567,75%	5,34%
19	BUNZL	613,54%	5,35%
20	BRITISH AMERICAN TOBACCO	1721,13%	5,35%
STOXX EUROPE 600 - TOT RETURN INDEX		**80,13%**	**4,68%**
Total Low Vol Portfolio		**502,82%**	**2,86%**

Source: Own elaboration

The results show that only one stock from the 20 selected didn't outperform the Index (Glaxosmithkline: 68,12% total return vs 80,13%) considering the full period under examination (from 2001 to 2018). Another interesting observation from these results is that all of the stocks with a smaller monthly volatility than the market[16] (< 4,68%) overperformed the Index.

On the other hand, the High Vol Portfolio was built through the same procedure but selecting the 20 stocks with higher monthly volatility from the data subset. Table 5 presents the total returns over the period chosen of the selected stocks for this portfolio together with their monthly volatilities, sorted out again from least to most volatile.

Table 5. Summary Total Returns and Monthly Volatilities High Vol Portfolio

	Company	Total return	Monthly vol
1	BTG	-40,84%	14,46%
2	BOLIDEN	393,63%	14,56%
3	UBISOFT ENTM.	557,76%	14,89%
4	ARCELORMITTAL	294,17%	14,93%
5	SUBSEA 7	18,70%	15,01%
6	INMOBILIARIA COLONIAL	-87,13%	15,14%
7	FREENET (XET)	435,59%	15,20%
8	ALSTOM	-77,76%	15,24%
9	BARRATT DEVELOPMENTS	326,19%	15,35%
10	VESTAS WINDSYSTEMS	12,14%	15,60%
11	ABB LTD N	1,21%	15,65%
12	OC OERLIKON	-60,08%	15,81%
13	1&1 DRILLISCH	1353,77%	16,07%
14	ALTRAN TECHNOLOGIES	-79,85%	16,45%
15	AGEAS (EX-FORTIS)	-74,26%	16,56%
16	PROSIEBENSAT 1 (XET) MEDIA	72,60%	17,06%
17	ASHTEAD GROUP	2460,71%	18,68%
18	BANK OF IRELAND GROUP	-92,74%	19,91%
19	AIB GROUP	-99,70%	19,95%
20	MARINE HARVEST	-97,56%	22,39%
STOXX EUROPE 600 - TOT RETURN INDEX		**80,13%**	**4,68%**
Total HIgh Vol Portfolio		**260,83%**	**3,35%**

Source: Own elaboration

[16] The Index was considered as an approximation (sample) to the whole European market (population).

Moreover, we see how the assumption of a positive relationship between risk and return is not fulfilled for the data selected as those stocks included in the High Vol Portfolio (Table 5) didn't obtain greater returns than the ones in the Low Vol Portfolio (Table 4). Indeed, we can assure that, on average, the least volatile stocks outperformed their higher volatility counterparts, suggesting a negative correlation between these two variables, considering risk as the only factor affecting the performance of the stocks *(ceteris paribus).*

In order to explore this anomaly and assess the reasons why this has happened the following 2 graphs (Figure 13 and 14) show the evolution of the different components of both portfolios and the Index along the period from 2001 to 2018. The grey shadowed areas refer to market downturn periods. These graphs provide several interesting outcomes:

1. In first place, if we compare the performance of the different stocks within the grey areas (bear market periods) higher volatility stocks suffer greater drops during market downturns. This was identified as one of the main factors for low volatility stocks overperformance for the entire period (due to the compounding power effect explained previously). The fact that low volatility stocks have smaller maximum drawdowns during market turbulences allows them to preserve capital and therefore, have a better performance during long periods of time.

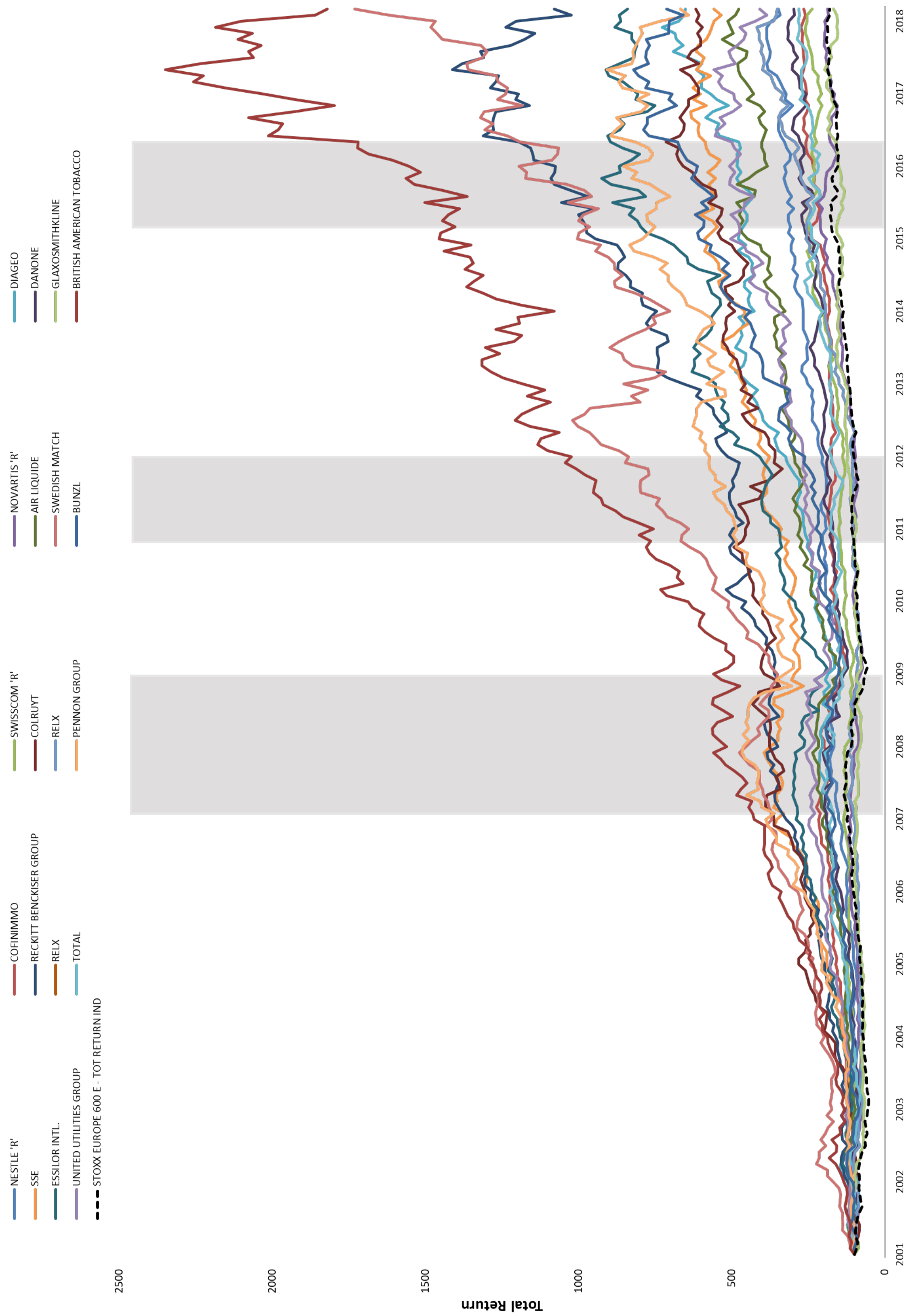
2. Another interesting observation is that higher risk premiums are not compensated with greater returns in the long run, hence an investor willing to invest for a long period of time (a full economic cycle at least) will obtain the same (or even greater) returns investing in low rather than high volatility stocks while assuming lower risks.

3. In addition, investing in high volatility stocks will only be interesting from a risk point of view during bull markets. As shown in the graphs, these stocks performed better during buoyant market periods like for example 2005-2007 or 2012-2015 periods.

4. In both cases, investing in the market as a whole (taking the STOXX 600 Europe Index as an approximation to the market) is less profitable in the long run although this statement shouldn't be taken for granted due to constraints of the composition of the Index (liquidity, capitalization, etc) and diversification effect (600 components vs single stock).

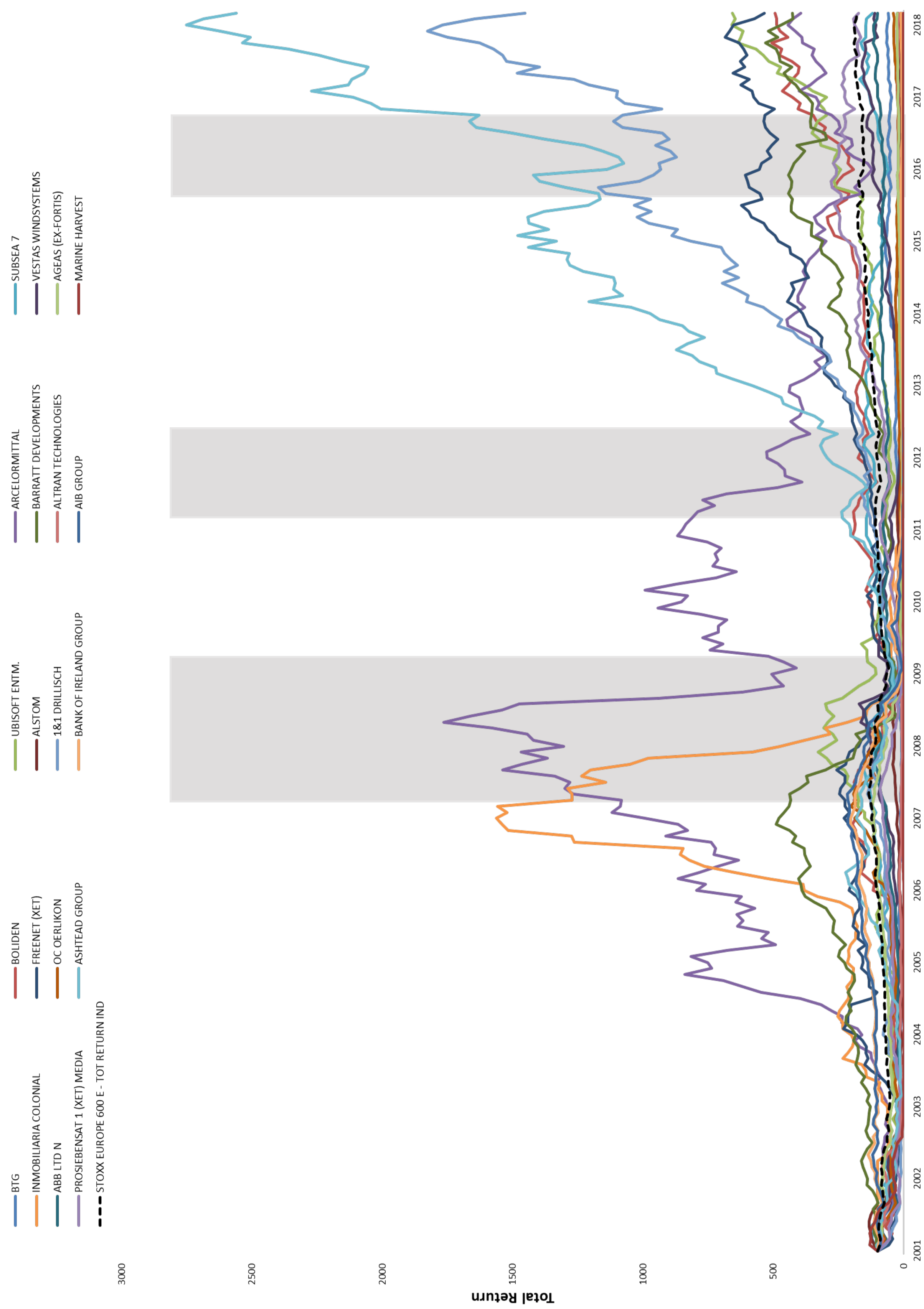
5. From the lowest volatile selection, only 1 out of the 20 stocks underperformed the market considering the entire period while within the most volatile selection, only 7 out of the 20 stocks outperformed the STOXX 600 Index during the period.

Figure 13. Total Return 20 least volatile stocks vs. STOXX 600 Total Return (2001-2018)



Source: Own elaboration

Figure 14. Total Return 20 most volatile stocks vs. STOXX 600 Total Return (2001-2018)



Source: Own elaboration

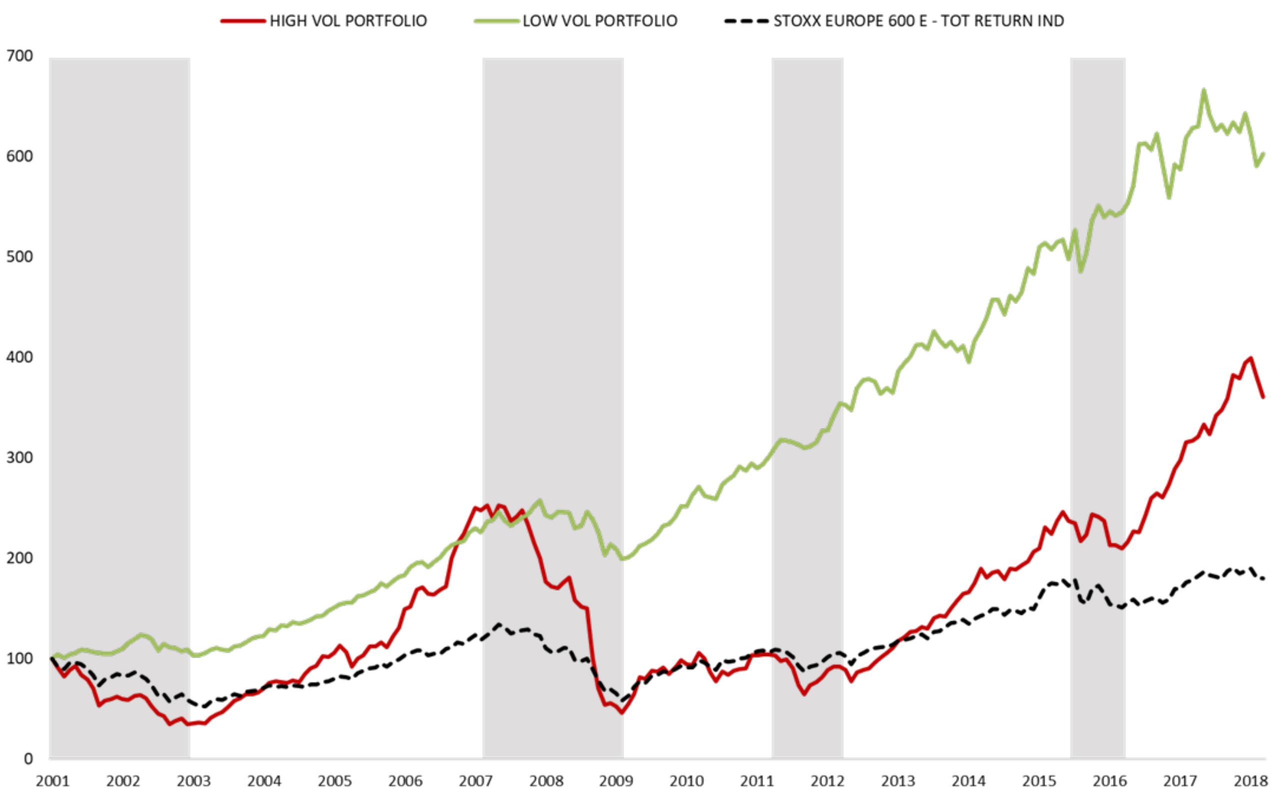
The third test conducted to complete the analysis was a comparison between the total returns and the risk[17] of the two portfolios previously built (Low Vol and High Vol) and the total returns and the risk of the market in the whole period (2001-2018). To make these calculations, equal weights were assigned to the individual components of the portfolios (each stock weighted 5%). Table 6 summarizes the main results obtained and Figure 15 displays the evolution of the 3 alternatives along the period.

Table 6. Summary Total Returns and Monthly Volatilities Portfolios vs Market

	Total return	Monthly vol
STOXX EUROPE 600 - TOT RETURN INDEX	80,13%	4,68%
Total Low Vol Portfolio	502,82%	2,86%
Total HIgh Vol Portfolio	260,83%	3,35%

Source: Own elaboration

Figure 15. Total Return Low & High Volatility Portfolios vs. STOXX 600 Total Return (2001-2018)



Source: Own elaboration

[17] The portfolio risks were calculated considering the correlation amongst the different components and assuming equal weights for each one (5%). To serve as an example, Appendix III shows the calculations done to reach to the monthly volatility of the Low Vol Portfolio.

The results show how a portfolio built with low volatility stocks (green line) outperforms the market as a whole in the long run (black dotted line) and a portfolio built with high volatile stocks (red line). Furthermore, it is observed once again that the positive relationship between risk and returns is only fulfilled during positive market trends (bull markets) such as 2005-2007 where we can see the High Vol Portfolio clearly shining over its counterparts. Nevertheless, we can also see it dropping significantly during market turbulent periods (displayed as grey shadowed areas in Figure 15). These bigger drawdowns are the main explanation for a worse performance of these strategies in the long run.

To sum up, based on the results obtained, low volatility strategies work better in the long run (as long as we consider a complete economic cycle) thanks to their preservation of capital characteristic which allow minimum drawdown behaviours during turbulent market conditions.

Limitations of the model and possible further research

This section describes several limitations of the former study have been identified once analyzed the obtained results that can serve as recommendations for further research on this topic. Regarding the data selection process, for simplifying purposes, only those stocks which had been part of the Index during the entire period under evaluation were considered. This excluded those stocks which had entered/exited the Index at some intermediate point along the period and thereby doesn’t fully reflect the performance of the entire Index.

Additionally, the period chosen to perform the analysis was short compared to previous studies over the topic which covered longer time frames. Related to this issue, the results were conditioned by the fact that the chosen period included several turbulent trading cycles (1 recession, 2 booms, 3 crisis, and 4 recovery periods). This favored low volatility stocks over their counterparts, therefore the results of the former analysis should be confirmed through a longer time period in order to make sure they are not biased.

Moreover, the investigation didn't consider other factors that also have an impact on returns besides risk like for example market capitalization or liquidity. Towards future research on the topic, it will be interesting to introduce new variables to the model which could capture this effect instead of assuming that the entire return variations are a consequence of the risk factor. This limitation was evidenced by the OLS estimation outcomes since they delivered low significance levels (low R^2) and in some cases, distorted P-values which manifested low reliability on the model.

Regarding the portfolios built in order to perform the second analysis, several limitations were also identified. Firstly, no constraints besides volatility were applied in order to select he portfolio components. This may result in undesired exposures to certain countries or sectors that could result in a biased and less optimized portfolio in the long run. Secondly, in order to build a completely optimized portfolio, besides considering the volatility of the individual stocks correlation between the individual stock performance and with the entire market itself should be considered, especially when the correlations between the stock performance and the total returns is high.

Additionally, no weighting criteria were applied when building the portfolios, in which each component had an equal weight (5%) without considering important factors that may determine their performance such as size, industry or liquidity. Finally, some of the 20 stocks selected to build the Low Vol Portfolio registered a higher monthly volatility (>4,68%). along the period than the Index itself.

Conclusions

The objective of this End of Master Project was to analyze whether the fact that low volatility stocks outperformed the overall market significantly during market downturns was a strong enough reason to make these participants outperform the market as a whole in the long run within the European market, assuming the STOXX 600 Europe as a representative sample of the population for comparison purposes.

Having conducted the complete analysis from different perspectives we may draw a set of conclusions according to the data obtained. First of all, the results show how there is a significant negative relationship between the variables risk (volatility) and return under bear market conditions. In the opposite way, during bull market (upward trend) conditions, the relationship is strongly positive. An interesting fact is also that the correlation between these two variables is considerably stronger during bear markets. This implies that whenever the market as whole falls, the downward movement for high volatile stocks is stronger, allowing lower stocks to outperform in the long run. However, looking at the entire period under examination (2001-2018) there is no clear relationship between the variables in the long run, probably because the observed variable is being affected additionally by many other factors besides risk.

Moreover, it has been observed that investing in high volatility stocks only delivers considerably higher returns during bull markets but and, that in the long run (considering a full economic cycle) they underperform their lower riskier counterparts due to the compounding effect and the greater maximum drawdowns during market turbulences.

To sum up, with the results obtained, we can say that, for the European market[18] case from 2001 to 2018, the fact that low volatility stocks preserve capital during downturn periods (which makes them overperform the market under these scenarios) was evidence enough for these participants to deliver greater returns than the market as a whole and their more volatile counterparts.

[18] Considering the STOXX 600 European Index as a representative simple for the whole European market.

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Appendix I

Database Example

Table 7. Database Scenario 1 (n = 389)

	Company	Monthly vol	Log vol	Total returns	Mean returns
1	NESTLE 'R'	3,85%	-0,79	248,37%	0,68%
2	COFINIMMO	3,88%	-1,00	165,63%	0,55%
3	SWISSCOM 'R'	4,12%	-1,05	137,84%	0,51%
4	NOVARTIS 'R'	4,37%	-1,11	88,05%	0,40%
5	DIAGEO	4,55%	-1,02	550,83%	1,02%
6	SSE	4,62%	-0,81	458,00%	0,95%
7	RECKITT BENCKISER GROUP	4,78%	-1,25	980,14%	1,28%
8	COLRUYT	4,88%	-1,09	516,93%	1,01%
9	AIR LIQUIDE	4,89%	-1,17	377,85%	0,88%
10	DANONE	4,99%	-1,14	197,25%	0,65%
11	ESSILOR INTL.	5,02%	-1,01	740,72%	1,17%
12	RELX	5,15%	-1,11	251,48%	0,75%
13	RELX	5,16%	-0,91	205,43%	0,68%
14	SWEDISH MATCH	5,18%	-0,78	1631,08%	1,53%
15	GLAXOSMITHKLINE	5,21%	-1,00	68,12%	0,39%
16	UNITED UTILITIES GROUP	5,30%	-0,94	308,75%	0,83%
17	TOTAL	5,33%	-0,70	181,39%	0,65%
18	PENNON GROUP	5,34%	-0,88	567,75%	1,07%
19	BUNZL	5,35%	-1,31	613,54%	1,10%
20	BRITISH AMERICAN TOBACCO	5,35%	-1,10	1721,13%	1,56%
21	SEVERN TRENT	5,40%	-1,00	532,00%	1,04%
22	IMPERIAL BRANDS	5,44%	-0,82	905,82%	1,28%
23	GBL NEW	5,48%	-0,78	190,67%	0,67%
24	LUNDBERGFORETAGEN 'B'	5,53%	-1,05	1115,37%	1,37%
25	RUBIS	5,55%	-1,09	2149,59%	1,68%
26	UNILEVER DR	5,57%	-0,91	314,79%	0,85%
27	HEINEKEN HLDG.	5,58%	-0,98	325,67%	0,86%
28	ENI	5,59%	-0,83	168,16%	0,64%
29	L'OREAL	5,60%	-0,73	206,97%	0,70%
30	SANOFI	5,65%	-0,91	88,62%	0,47%
31	UNILEVER (UK)	5,66%	-1,12	526,30%	1,06%
32	ABERTIS INFRAESTRUCTURAS	5,69%	-1,11	651,04%	1,14%
33	CHOC.LINDT & SPRUENGLI	5,72%	-1,20	743,86%	1,20%
34	HEINEKEN	5,74%	-1,21	192,05%	0,68%
35	EMS-CHEMIE 'N'	5,75%	-1,19	806,21%	1,24%
36	ROCHE HOLDING	5,77%	-1,08	132,57%	0,58%
37	PERNOD-RICARD	5,82%	-0,95	689,41%	1,18%
38	SMITHS GROUP	5,89%	-1,09	290,19%	0,84%

39	BEIERSDORF (XET)	5,90%	-0,97	221,01%	0,74%
40	ROYAL DUTCH SHELL A	5,94%	-1,08	90,92%	0,49%
41	RED ELECTRICA	5,94%	-0,95	962,24%	1,33%
42	ENEL	5,95%	-1,15	115,33%	0,55%
43	KERRY GROUP 'A'	5,98%	-1,05	606,59%	1,13%
44	HALMA	5,99%	-1,08	1317,74%	1,48%
45	SMITH & NEPHEW	6,00%	-0,91	454,58%	1,02%
46	HENKEL PREF. (XET)	6,01%	-0,70	479,91%	1,04%
47	ASTRAZENECA	6,16%	-1,02	222,64%	0,76%
48	UNIBAIL-RODAMCO SE REIT	6,17%	-0,93	656,01%	1,18%
49	MORRISON(WM)SPMKTS.	6,18%	-0,81	87,53%	0,50%
50	COLOPLAST 'B'	6,23%	-1,16	1730,75%	1,62%
51	SAINSBURY J	6,24%	-1,12	35,57%	0,34%
52	ASSOCIATED BRIT.FOODS	6,25%	-1,08	619,76%	1,16%
53	EDP ENERGIAS DE PORTUGAL	6,26%	-1,12	125,39%	0,59%
54	BIC	6,31%	-1,06	229,28%	0,78%
55	SCA 'B'	6,33%	-1,04	1132,64%	1,43%
56	SCHINDLER 'P'	6,38%	-0,92	1082,93%	1,41%
57	VINCI	6,41%	-1,04	946,77%	1,35%
58	ATLANTIA	6,41%	-0,98	672,89%	1,20%
59	SVENSKA HANDBKN.'A'	6,44%	-1,23	327,60%	0,91%
60	TESCO	6,45%	-1,03	28,08%	0,33%
61	HENNES & MAURITZ 'B'	6,46%	-1,08	112,11%	0,57%
62	CENTRICA	6,46%	-1,05	31,54%	0,34%
63	VISCOFAN	6,55%	-1,20	1218,26%	1,48%
64	SAMPO 'A'	6,56%	-1,09	1052,02%	1,41%
65	HSBC HOLDINGS	6,58%	-1,05	72,60%	0,48%
66	KONE 'B'	6,60%	-0,84	3898,53%	2,03%
67	GREAT PORTLAND ESTATES	6,60%	-1,17	370,37%	0,97%
68	DASSAULT AVIATION	6,62%	-1,03	878,22%	1,34%
69	GEBERIT 'R'	6,65%	-1,08	1395,32%	1,54%
70	ACKERMANS & VAN HAAREN	6,68%	-1,01	464,04%	1,07%
71	VODAFONE GROUP	6,69%	-1,17	71,11%	0,48%
72	BP	6,69%	-1,27	81,75%	0,51%
73	CASTELLUM	6,73%	-1,17	1215,77%	1,49%
74	SPIRAX-SARCO ENGR.	6,74%	-1,09	2401,44%	1,80%
75	LAND SECURITIES GROUP	6,76%	-0,84	127,57%	0,62%
76	THALES	6,78%	-1,06	244,74%	0,83%
77	BRITISH LAND	6,81%	-1,27	202,77%	0,77%
78	BOLLORE	6,83%	-0,91	1091,82%	1,45%
79	IBERDROLA	6,89%	-1,07	241,68%	0,84%
80	TELEFONICA	6,89%	-1,14	-8,61%	0,19%
81	PARGESA 'B'	6,91%	-1,16	113,29%	0,60%
82	BARRY CALLEBAUT	6,93%	-1,17	981,76%	1,40%
83	FORTUM	6,94%	-1,19	1517,55%	1,60%

84	INVESTOR 'B'	6,96%	-1,11	366,93%	0,99%
85	CASINO GUICHARD-P	6,96%	-1,08	-22,02%	0,12%
86	ICADE REIT	6,98%	-1,08	553,36%	1,16%
87	WHITBREAD	7,00%	-0,93	911,03%	1,38%
88	DCC	7,02%	-1,08	1325,79%	1,55%
89	INTU PROPERTIES	7,05%	-1,14	15,86%	0,32%
90	BAE SYSTEMS	7,14%	-1,13	323,77%	0,95%
91	VIFOR PHARMA	7,20%	-1,41	1234,38%	1,53%
92	CARREFOUR	7,21%	-1,21	-54,74%	-0,13%
93	CNP ASSURANCES	7,22%	-0,85	390,40%	1,03%
94	REPSOL YPF	7,22%	-0,93	58,50%	0,48%
95	HISCOX DI	7,25%	-1,00	1151,78%	1,49%
96	NEXT	7,25%	-1,09	1029,84%	1,45%
97	KLEPIERRE	7,26%	-1,06	493,47%	1,13%
98	SEB	7,27%	-1,02	1179,70%	1,51%
99	SHAFTESBURY	7,29%	-1,30	534,51%	1,17%
100	ACS ACTIV.CONSTR.Y SERV.	7,29%	-1,04	553,74%	1,18%
101	WOLTERS KLUWER	7,33%	-1,18	202,41%	0,80%
102	SIKA 'B'	7,33%	-1,02	2206,82%	1,80%
103	ROTORK	7,33%	-1,15	1440,08%	1,61%
104	SECURITAS 'B'	7,35%	-1,11	144,53%	0,70%
105	TATE & LYLE	7,38%	-0,96	363,73%	1,02%
106	NOVO NORDISK 'B'	7,38%	-1,13	1172,04%	1,51%
107	SODEXO	7,41%	-1,34	155,62%	0,72%
108	SCHNEIDER ELECTRIC SE	7,44%	-1,07	252,45%	0,89%
109	AXEL SPRINGER (XET)	7,44%	-0,99	261,45%	0,90%
110	WH SMITH	7,44%	-1,05	964,72%	1,44%
111	SOLVAY	7,45%	-0,98	208,60%	0,82%
112	COBHAM	7,46%	-1,10	133,98%	0,69%
113	RECORDATI INDUA.CHIMICA	7,46%	-1,20	1614,40%	1,67%
114	IMERYS	7,47%	-1,01	390,14%	1,05%
115	ENDESA	7,47%	-1,06	360,96%	1,02%
116	DEUTSCHE TELEKOM (XET)	7,48%	-1,04	-16,13%	0,19%
117	SGS 'N'	7,48%	-1,02	708,71%	1,30%
118	MARKS & SPENCER GROUP	7,48%	-1,05	144,36%	0,72%
119	LONZA GROUP	7,49%	-1,24	248,79%	0,88%
120	FRESENIUS MED.CARE (XET)	7,49%	-1,13	258,91%	0,90%
121	BB BIOTECH N	7,51%	-1,23	283,40%	0,93%
122	BASF (XET)	7,51%	-1,25	542,37%	1,19%
123	G4S	7,52%	-0,86	160,00%	0,75%
124	ORION 'B'	7,53%	-0,94	799,08%	1,35%
125	UCB	7,56%	-1,30	126,22%	0,68%
126	GETINGE	7,56%	-1,31	398,39%	1,07%
127	JOHNSON MATTHEY	7,57%	-1,05	380,86%	1,05%
128	SKY	7,58%	-0,95	67,22%	0,54%

129	UDG HEALTHCARE PUBLIC	7,61%	-1,08	1035,30%	1,48%
130	KINGFISHER	7,63%	-0,86	51,58%	0,49%
131	FONCIERE DES REGIONS	7,64%	-0,91	1258,32%	1,57%
132	DORMA KABA HOLD	7,64%	-0,96	151,14%	0,74%
133	ASSA ABLOY 'B'	7,65%	-1,08	354,54%	1,03%
134	ORKLA	7,66%	-0,91	522,98%	1,18%
135	VOPAK	7,66%	-1,02	647,15%	1,27%
136	DERWENT LONDON	7,69%	-1,12	440,88%	1,12%
137	ADIDAS (XET)	7,72%	-1,16	1201,16%	1,55%
138	HAMMERSON	7,80%	-0,82	194,35%	0,82%
139	A2A	7,82%	-1,13	11,69%	0,36%
140	DSM KONINKLIJKE	7,83%	-1,06	667,31%	1,29%
141	CEZ	7,84%	-1,09	959,19%	1,46%
142	ASSICURAZIONI GENERALI	7,85%	-1,00	-31,37%	0,12%
143	NORDEA BANK	7,87%	-0,97	239,99%	0,91%
144	SKF 'B'	7,87%	-1,18	830,03%	1,40%
145	E ON N (XET)	7,88%	-0,98	15,41%	0,38%
146	HANNOVER RUCK. (XET)	7,90%	-1,12	515,63%	1,19%
147	RPC GROUP	7,90%	-1,02	1382,52%	1,64%
148	KESKO 'B'	7,91%	-1,28	1067,72%	1,51%
149	KOMERCNI BANKA	7,93%	-0,91	882,99%	1,43%
150	LVMH	7,95%	-1,18	467,26%	1,16%
151	MERCK KGAA (XET)	7,96%	-1,12	390,80%	1,09%
152	PEARSON	7,97%	-1,26	-6,62%	0,28%
153	TDC	7,97%	-0,96	208,20%	0,86%
154	HUHTAMAKI	7,98%	-1,03	872,52%	1,43%
155	IMI	7,99%	-1,22	696,47%	1,33%
156	INDUSTRIVARDEN 'A'	8,01%	-1,11	263,25%	0,95%
157	AKZO NOBEL	8,03%	-1,10	165,46%	0,80%
158	FUCHS PETROLUB PF. (XET)	8,04%	-1,02	6692,43%	2,40%
159	BT GROUP	8,05%	-0,99	-5,48%	0,29%
160	WPP	8,06%	-1,25	87,97%	0,63%
161	WILLIAM DEMANT HLDG.	8,06%	-1,24	170,77%	0,80%
162	THE SWATCH GROUP 'B'	8,08%	-1,07	169,42%	0,80%
163	AURUBIS (XET)	8,09%	-1,22	861,10%	1,43%
164	AMER SPORTS	8,09%	-1,19	544,58%	1,24%
165	SKANSKA 'B'	8,13%	-1,09	299,91%	1,01%
166	LAFARGEHOLCIM	8,14%	-0,99	-21,04%	0,22%
167	HERMES INTL.	8,15%	-1,14	1063,00%	1,53%
168	CRH (DUB)	8,16%	-0,92	165,93%	0,81%
169	BMW (XET)	8,17%	-1,02	235,36%	0,92%
170	SAGE GROUP	8,18%	-0,91	155,55%	0,79%
171	ACCOR	8,19%	-1,18	153,36%	0,78%
172	VIVENDI	8,20%	-0,98	-37,09%	0,10%
173	REMY COINTREAU	8,21%	-1,10	448,62%	1,16%

174	UMICORE	8,23%	-1,16	1693,76%	1,74%
175	LAGARDERE GROUPE	8,23%	-1,16	18,78%	0,42%
176	BERKELEY GROUP HDG.	8,24%	-1,13	1470,43%	1,69%
177	SANDVIK	8,25%	-1,10	503,62%	1,22%
178	FERROVIAL	8,27%	-1,26	674,92%	1,34%
179	AVIVA	8,27%	-0,90	20,83%	0,43%
180	CHRISTIAN DIOR	8,29%	-1,10	977,94%	1,50%
181	CLOSE BROTHERS GROUP	8,30%	-0,97	186,05%	0,85%
182	MAPFRE	8,32%	-0,90	210,43%	0,90%
183	BAYER (XET)	8,33%	-0,94	190,42%	0,86%
184	ATLAS COPCO 'A'	8,33%	-0,82	1884,67%	1,81%
185	SYDBANK	8,33%	-0,97	843,74%	1,44%
186	JYSKE BANK	8,36%	-0,98	388,18%	1,12%
187	BALOISE-HOLDING AG	8,38%	-1,16	49,90%	0,54%
188	FABEGE	8,39%	-0,95	1476,35%	1,70%
189	BOUYGUES	8,40%	-1,15	77,58%	0,63%
190	RICHEMONT N	8,40%	-1,07	515,11%	1,24%
191	MUENCHENER RUCK. (XET)	8,40%	-1,12	5,09%	0,37%
192	MICHELIN	8,44%	-1,08	390,79%	1,14%
193	NIBE INDUSTRIER 'B'	8,44%	-0,97	4366,09%	2,22%
194	DNB	8,44%	-0,90	628,13%	1,32%
195	PUBLICIS GROUPE	8,52%	-1,22	111,03%	0,73%
196	CARLSBERG 'B'	8,53%	-1,10	175,34%	0,84%
197	HELVETIA HOLDING N	8,57%	-1,12	167,96%	0,84%
198	JERONIMO MARTINS	8,58%	-0,96	952,46%	1,52%
199	SAFRAN	8,58%	-0,96	473,17%	1,22%
200	FRESENIUS (XET)	8,64%	-1,14	585,93%	1,31%
201	DAILY MAIL 'A'	8,69%	-1,10	19,78%	0,47%
202	BUCHER INDUSTRIES	8,72%	-1,18	915,71%	1,51%
203	SCHRODERS	8,77%	-1,02	280,21%	1,03%
204	SHIRE	8,78%	-1,11	195,51%	0,91%
205	ELECTROCOMP.	8,80%	-0,99	148,14%	0,83%
206	BBA AVIATION	8,80%	-1,01	172,14%	0,87%
207	BNP PARIBAS	8,82%	-1,25	136,35%	0,80%
208	TELECOM ITALIA	8,84%	-1,08	-75,81%	-0,30%
209	DSV 'B'	8,84%	-1,17	2131,38%	1,92%
210	TELE2 'B'	8,85%	-1,05	157,15%	0,85%
211	AMBU 'B'	8,86%	-1,24	11728,59%	2,75%
212	STRAUMANN HLDG.	8,88%	-0,96	511,70%	1,27%
213	OMV	8,89%	-0,92	885,20%	1,52%
214	A P MOLLER - MAERSK 'B'	8,89%	-1,13	84,59%	0,69%
215	BHP BILLITON	8,90%	-1,01	861,70%	1,51%
216	SIEMENS (XET)	8,90%	-1,26	69,09%	0,65%
217	EURAZEO	8,90%	-1,10	230,84%	0,98%
218	ELISA	8,91%	-1,09	424,81%	1,19%

219	MEGGITT	8,92%	-0,98	366,07%	1,14%
220	SAINT GOBAIN	8,93%	-1,01	90,27%	0,71%
221	BALFOUR BEATTY	8,97%	-1,08	312,50%	1,09%
222	LEGAL & GENERAL	8,98%	-0,65	270,97%	1,04%
223	VICTREX	8,98%	-1,13	1008,91%	1,57%
224	RWE (XET)	8,98%	-1,03	-7,19%	0,36%
225	SWEDBANK 'A'	9,03%	-1,05	227,25%	0,98%
226	SEB 'A'	9,04%	-1,10	185,08%	0,91%
227	UPM-KYMMENE	9,05%	-0,97	322,89%	1,12%
228	OLD MUTUAL	9,06%	-1,07	169,79%	0,89%
229	ELECTROLUX 'B'	9,07%	-1,21	547,98%	1,33%
230	BANCO SANTANDER	9,08%	-1,08	20,18%	0,50%
231	BBV.ARGENTARIA	9,08%	-1,41	-23,69%	0,28%
232	PHILIPS ELTN.KONINKLIJKE	9,13%	-1,13	25,60%	0,52%
233	UBS GROUP	9,19%	-1,03	-44,04%	0,14%
234	ST.JAMES'S PLACE ORD	9,19%	-1,14	256,36%	1,04%
235	SAAB 'B'	9,20%	-1,07	644,05%	1,40%
236	DANSKE BANK	9,21%	-0,98	180,73%	0,92%
237	WARTSILA	9,24%	-0,90	2706,98%	2,06%
238	BELLWAY	9,26%	-1,10	1206,40%	1,68%
239	MEDIOBANCA BC.FIN	9,26%	-1,03	35,35%	0,58%
240	H LUNDBECK	9,30%	-1,36	128,74%	0,83%
241	BOSKALIS WESTMINSTER	9,31%	-1,13	410,28%	1,22%
242	NEX GROUP	9,31%	-0,80	2573,57%	2,05%
243	NORSK HYDRO	9,34%	-1,04	244,01%	1,03%
244	SPECTRIS	9,34%	-1,05	634,87%	1,41%
245	PRUDENTIAL	9,40%	-0,96	225,22%	1,01%
246	RENTOKIL INITIAL	9,41%	-1,12	102,15%	0,79%
247	HAYS	9,45%	-1,12	10,53%	0,48%
248	HOMESERVE	9,48%	-0,88	1231,63%	1,70%
249	DAIMLER (XET)	9,50%	-1,16	155,04%	0,91%
250	STANDARD CHARTERED	9,50%	-1,10	41,36%	0,62%
251	GLANBIA	9,51%	-1,27	3231,91%	2,17%
252	VOLVO 'B'	9,52%	-1,24	720,52%	1,48%
253	UBM	9,52%	-0,98	189,29%	0,97%
254	DASSAULT SYSTEMES	9,54%	-0,95	284,53%	1,09%
255	SBM OFFSHORE	9,55%	-1,04	51,98%	0,65%
256	ELEKTA 'B'	9,57%	-0,95	4201,71%	2,31%
257	KONECRANES	9,57%	-0,94	750,77%	1,50%
258	BANKINTER 'R'	9,60%	-0,77	145,97%	0,90%
259	AALBERTS INDUSTRIES	9,61%	-1,03	817,38%	1,54%
260	FLUGHAFEN ZURICH	9,62%	-1,07	473,40%	1,30%
261	TELEPERFORMANCE	9,63%	-0,95	444,86%	1,29%
262	STORA ENSO 'R'	9,64%	-0,92	167,26%	0,95%
263	ROLLS-ROYCE HOLDINGS	9,64%	-0,92	641,67%	1,43%

264	ZURICH INSURANCE GROUP	9,65%	-1,32	-10,12%	0,38%
265	ADECCO 'R'	9,69%	-1,13	-11,12%	0,41%
266	DEUTSCHE LUFTHANSA (XET)	9,75%	-1,23	69,12%	0,72%
267	KERING	9,75%	-1,29	192,00%	0,99%
268	SEGRO	9,76%	-1,29	117,90%	0,84%
269	WEIR GROUP	9,79%	-1,09	1249,55%	1,74%
270	EIFFAGE	9,85%	-1,03	1314,11%	1,79%
271	MAN (XET)	9,86%	-1,14	360,21%	1,22%
272	BPER BANCA	9,86%	-0,99	-42,43%	0,21%
273	CREDIT SUISSE GROUP N	9,90%	-0,97	-65,09%	-0,02%
274	3I GROUP	9,90%	-1,24	49,60%	0,68%
275	SAP (XET)	9,95%	-1,02	168,27%	0,99%
276	SONOVA N	9,96%	-1,13	182,83%	0,99%
277	GAM HOLDING	9,97%	-0,86	46,11%	0,68%
278	RSA INSURANCE GROUP	9,99%	-1,23	-23,39%	0,36%
279	AGGREKO	10,01%	-1,10	147,63%	0,93%
280	ALLIANZ (XET)	10,04%	-1,00	-0,77%	0,49%
281	HEIDELBERGCEMENT (XET)	10,16%	-1,26	127,55%	0,91%
282	HEXAGON 'B'	10,16%	-1,05	6559,98%	2,58%
283	KPN KON	10,17%	-1,04	-49,71%	0,16%
284	VOESTALPINE	10,24%	-1,07	924,71%	1,65%
285	RHEINMETALL (XET)	10,27%	-1,09	1402,16%	1,86%
286	SMITH (DS)	10,33%	-1,20	928,29%	1,68%
287	ANTOFAGASTA	10,37%	-1,05	1756,18%	1,97%
288	INTESA SANPAOLO	10,43%	-0,93	12,83%	0,59%
289	GEORG FISCHER 'R'	10,44%	-1,18	327,06%	1,25%
290	NOKIAN RENKAAT	10,46%	-1,08	3214,15%	2,26%
291	SWISS RE	10,49%	-1,25	4,12%	0,53%
292	MAN GROUP	10,50%	-1,00	229,59%	1,12%
293	BOSS (HUGO) (XET)	10,50%	-0,96	505,14%	1,42%
294	DUERR (XET)	10,55%	-1,02	939,42%	1,70%
295	VALEO	10,57%	-0,97	445,99%	1,38%
296	BE SEMICONDUCTOR	10,57%	-1,20	838,39%	1,66%
297	PERSIMMON	10,59%	-1,13	1677,83%	1,96%
298	GEA GROUP (XET)	10,61%	-1,06	215,80%	1,12%
299	WENDEL	10,61%	-0,93	305,07%	1,23%
300	TRAVIS PERKINS	10,62%	-1,34	224,92%	1,14%
301	K + S (XET)	10,63%	-1,14	857,73%	1,65%
302	THYSSENKRUPP (XET)	10,66%	-1,13	64,56%	0,80%
303	INTERMEDIATE CAPITAL GP.	10,67%	-1,01	532,72%	1,46%
304	AVEVA GROUP	10,70%	-1,27	1936,29%	2,04%
305	METSO	10,71%	-1,13	584,16%	1,50%
306	SCHIBSTED A	10,71%	-1,14	475,79%	1,43%
307	INFORMA	10,72%	-1,06	229,30%	1,13%
308	WIENERBERGER	10,74%	-1,05	53,98%	0,78%

309	RIO TINTO	10,76%	-1,14	550,80%	1,47%
310	SARTORIUS PREF. (XET)	10,86%	-1,04	7815,57%	2,74%
311	FERGUSON	10,87%	-1,09	329,68%	1,26%
312	LLOYDS BANKING GROUP	10,89%	-1,10	-62,16%	0,12%
313	KINNEVIK 'B'	10,96%	-1,12	513,36%	1,49%
314	ORANGE	10,99%	-1,22	-56,56%	0,19%
315	KINGSPAN GROUP	11,01%	-1,23	926,51%	1,74%
316	SOCIETE GENERALE	11,05%	-1,13	26,84%	0,72%
317	DEUTSCHE BANK (XET)	11,09%	-0,93	-78,09%	-0,13%
318	PEUGEOT	11,10%	-1,13	-7,06%	0,58%
319	QIAGEN (XET)	11,13%	-1,00	-16,28%	0,53%
320	TGS-NOPEC GEOPHS.	11,14%	-0,88	717,71%	1,64%
321	ATOS	11,15%	-1,03	18,42%	0,69%
322	UNICREDIT	11,16%	-1,17	-82,98%	-0,25%
323	AXA	11,30%	-1,13	23,82%	0,72%
324	ITV	11,32%	-1,04	21,70%	0,74%
325	EUROFINS SCIENTIFIC	11,35%	-1,02	1266,90%	1,92%
326	PLASTIC OMNIUM	11,35%	-0,92	2713,95%	2,28%
327	INGENICO GROUP	11,37%	-1,02	193,09%	1,15%
328	ERSTE GROUP BANK	11,44%	-0,88	323,36%	1,35%
329	TULLOW OIL	11,44%	-1,05	262,96%	1,29%
330	KONINKLIJKE AHOLD DELHAIZE	11,47%	-0,82	-3,93%	0,55%
331	PORSCHE AML.HLDG. (XET) PREF.	11,60%	-1,20	215,71%	1,23%
332	CONTINENTAL (XET)	11,62%	-1,19	1536,59%	2,01%
333	SCOR SE	11,67%	-1,09	-54,57%	0,23%
334	SAIPEM	11,68%	-1,04	-50,83%	0,31%
335	BARCLAYS	11,70%	-1,29	-24,00%	0,56%
336	CLARIANT	11,74%	-0,88	-41,06%	0,41%
337	SOFTWARE (XET)	11,77%	-0,98	147,66%	1,12%
338	BANCO COMR.PORTUGUES 'R'	11,89%	-1,39	-98,15%	-1,23%
339	UNITED INTERNET (XET)	12,00%	-1,08	4119,24%	2,56%
340	RANDGOLD RESOURCES	12,04%	-1,13	5655,90%	2,74%
341	LOGITECH 'R'	12,05%	-0,84	246,80%	1,33%
342	VOLKSWAGEN PREF. (XET)	12,07%	-1,10	694,85%	1,70%
343	STMICROELECTRONICS (MIL)	12,09%	-1,05	-47,17%	0,41%
344	RANDSTAD	12,13%	-1,05	342,70%	1,46%
345	HOCHTIEF (XET)	12,15%	-1,16	1001,22%	1,90%
346	ANGLO AMERICAN	12,16%	-1,02	139,54%	1,18%
347	TRELLEBORG 'B'	12,18%	-1,19	944,89%	1,93%
348	BANCO BPM	12,20%	-0,95	-91,75%	-0,48%
349	FAURECIA	12,20%	-1,17	109,17%	1,10%
350	HOWDEN JOINERY GP.	12,21%	-0,97	518,18%	1,68%
351	ASML HOLDING	12,26%	-1,27	469,31%	1,59%
352	GN STORE NORD	12,27%	-0,97	47,42%	0,91%
353	FIAT CHRYSLER AUTOS.	12,33%	-0,91	165,69%	1,24%

354	CAPGEMINI	12,38%	-0,94	-31,85%	0,55%
355	AEGON	12,38%	-0,83	-73,87%	0,08%
356	INCHCAPE	12,48%	-1,02	1106,62%	1,97%
357	KBC GROUP	12,58%	-1,04	103,01%	1,12%
358	ING GROEP	12,66%	-1,12	-27,01%	0,61%
359	NOKIA	12,73%	-1,12	-78,24%	0,07%
360	SWISS LIFE HOLDING	13,04%	-1,08	-38,39%	0,62%
361	SOPRA STERIA GROUP	13,11%	-1,21	391,59%	1,58%
362	STOREBRAND	13,15%	-0,95	65,41%	1,08%
363	AIR FRANCE-KLM	13,28%	-1,25	-57,20%	0,47%
364	OUTOKUMPU 'A'	13,31%	-1,25	-55,44%	0,48%
365	ERICSSON 'B'	13,65%	-0,92	-80,50%	0,13%
366	ROYAL BANK OF SCTL.GP.	13,76%	-1,28	-92,10%	-0,38%
367	FASTIGHETS BALDER 'B'	13,93%	-1,04	98,24%	1,28%
368	COMMERZBANK (XET)	14,13%	-0,98	-93,38%	-0,35%
369	TAYLOR WIMPEY	14,42%	-0,81	155,67%	1,45%
370	BTG	14,46%	-1,05	-40,84%	0,77%
371	BOLIDEN	14,56%	-1,14	393,63%	1,85%
372	UBISOFT ENTM.	14,89%	-1,19	557,76%	2,03%
373	ARCELORMITTAL	14,93%	-1,18	294,17%	1,77%
374	SUBSEA 7	15,01%	-1,09	18,70%	1,13%
375	INMOBILIARIA COLONIAL	15,14%	-1,17	-87,13%	0,13%
376	FREENET (XET)	15,20%	-0,99	435,59%	1,99%
377	ALSTOM	15,24%	-0,92	-77,76%	0,38%
378	BARRATT DEVELOPMENTS	15,35%	-1,02	326,19%	1,81%
379	VESTAS WINDSYSTEMS	15,60%	-1,12	12,14%	1,27%
380	ABB LTD N	15,65%	-1,03	1,21%	1,30%
381	OC OERLIKON	15,81%	-1,01	-60,08%	0,76%
382	1&1 DRILLISCH	16,07%	-0,97	1353,77%	2,60%
383	ALTRAN TECHNOLOGIES	16,45%	-1,16	-79,85%	0,48%
384	AGEAS (EX-FORTIS)	16,56%	-0,97	-74,26%	0,48%
385	PROSIEBENSAT 1 (XET) MEDIA	17,06%	-1,09	72,60%	1,73%
386	ASHTEAD GROUP	18,68%	-1,14	2460,71%	3,22%
387	BANK OF IRELAND GROUP	19,91%	-1,09	-92,74%	0,80%
388	AIB GROUP	19,95%	-1,02	-99,70%	-0,89%
389	MARINE HARVEST	22,39%	-1,12	-97,56%	0,42%
	STOXX EUROPE 600 E - TOT RETURN IND	**4,68%**	**-1,33**	**80,13%**	**0,39%**

Source: Own elaboration

Appendix II

Gretl outputs Model 1 (Monthly volatilities and Total Returns):

Scenario 1:

gretl: matriz de correlación

```
corr(Monthlyvol, Totalreturns) = -0,05280793
Bajo la hipótesis nula de no correlación:
 t(387) = -1,04031, con valor p a dos colas 0,2988
```

gretl: modelo 1

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```
Modelo 1: MCO, usando las observaciones 1-389
Variable dependiente: Totalreturns

              Coeficiente  Desv. Típica  Estadístico t  valor p
  -------------------------------------------------------------
  const         7,82882      1,88272         4,158      3,95e-05 ***
  Monthlyvol  -20,8061      20,0000         -1,040      0,2988

Media de la vble. dep.  5,954789   D.T. de la vble. dep.   10,79479
Suma de cuad. residuos  45086,59   D.T. de la regresión    10,79365
R-cuadrado              0,002789   R-cuadrado corregido    0,000212
F(1, 387)               1,082236   Valor p (de F)          0,298847
Log-verosimilitud      -1476,379   Criterio de Akaike      2956,758
Criterio de Schwarz     2964,685   Crit. de Hannan-Quinn   2959,901
```

Scenario 2:

gretl: matriz de correlación

```
corr(Monthlyvol, Totalreturn) = -0,04364062
Bajo la hipótesis nula de no correlación:
 t(481) = -0,958026, con valor p a dos colas 0,3385
```

gretl: modelo 1

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```
Modelo 1: MCO, usando las observaciones 1-483
Variable dependiente: Totalreturn

              Coeficiente   Desv. Típica   Estadístico t   valor p
  ---------------------------------------------------------------
  const         2,54473      0,512790          4,963       9,67e-07 ***
  Monthlyvol   -5,34801      5,58232          -0,9580      0,3385

Media de la vble. dep.  2,076046   D.T. de la vble. dep.   3,377241
Suma de cuad. residuos  5487,106   D.T. de la regresión    3,377529
R-cuadrado              0,001905   R-cuadrado corregido   -0,000171
F(1, 481)               0,917814   Valor p (de F)          0,338531
Log-verosimilitud      -1272,226   Criterio de Akaike      2548,452
Criterio de Schwarz     2556,812   Crit. de Hannan-Quinn   2551,737
```

Scenario 3:

gretl: matriz de correlación

```
corr(Monthlyvol, Totalreturn) = 0,10495476
Bajo la hipótesis nula de no correlación:
 t(512) = 2,38804, con valor p a dos colas 0,0173
```

gretl: modelo 1

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```
Modelo 1: MCO, usando las observaciones 1-514
Variable dependiente: Totalreturn

              Coeficiente   Desv. Típica   Estadístico t   valor p
  ---------------------------------------------------------------
  const         1,22399      0,367033          3,335       0,0009  ***
  Monthlyvol   11,8002       4,94135           2,388       0,0173  **

Media de la vble. dep.  2,054646   D.T. de la vble. dep.   2,667708
Suma de cuad. residuos  3610,634   D.T. de la regresión    2,655564
R-cuadrado              0,011016   R-cuadrado corregido    0,009084
F(1, 512)               5,702755   Valor p (de F)          0,017300
Log-verosimilitud      -1230,334   Criterio de Akaike      2464,668
Criterio de Schwarz     2473,153   Crit. de Hannan-Quinn   2467,994
```

Scenario 4:

gretl: matriz de correlación

```
corr(Monthlyvol, Totalreturn) = -0,30128590
Bajo la hipótesis nula de no correlación:
 t(481) = -6,92971, con valor p a dos colas 0,0000
```

gretl: modelo 1

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```
Modelo 1: MCO, usando las observaciones 1-483
Variable dependiente: Totalreturn

               Coeficiente  Desv. Típica  Estadístico t  valor p
  --------------------------------------------------------------
  const          0,637906    0,0866726        7,360      8,02e-13 ***
  Monthlyvol    -5,41573     0,781524        -6,930      1,36e-11 ***

Media de la vble. dep.  0,075064   D.T. de la vble. dep.   0,696531
Suma de cuad. residuos  212,6181   D.T. de la regresión    0,664856
R-cuadrado              0,090773   R-cuadrado corregido    0,088883
F(1, 481)               48,02092   Valor p (de F)          1,36e-11
Log-verosimilitud      -487,1919   Criterio de Akaike      978,3838
Criterio de Schwarz     986,7439   Crit. de Hannan-Quinn   981,6691
```

Scenario 5:

gretl: matriz de correlación

```
corr(Monthlyvol, Totalreturn) = 0,09139589
Bajo la hipótesis nula de no correlación:
 t(533) = 2,11891, con valor p a dos colas 0,0346
```

gretl: modelo 1

Archivo Editar Contrastes Guardar Gráficos Análisis LaTeX

```
Modelo 1: MCO, usando las observaciones 1-535
Variable dependiente: Totalreturn

              Coeficiente   Desv. Típica   Estadístico t   valor p
  ------------------------------------------------------------------
  const        0,961629      0,254481         3,779        0,0002   ***
  Monthlyvol   7,17978       3,38843          2,119        0,0346   **

Media de la vble. dep.   1,468775   D.T. de la vble. dep.   2,006332
Suma de cuad. residuos   2131,590   D.T. de la regresión    1,999808
R-cuadrado               0,008353   R-cuadrado corregido    0,006493
F(1, 533)                4,489765   Valor p (de F)          0,034560
Log-verosimilitud       -1128,913   Criterio de Akaike      2261,825
Criterio de Schwarz      2270,390   Crit. de Hannan-Quinn   2265,176
```

Gretl outputs Model 2: Logarithm of monthly volatilities and Total Returns

Scenario 1:

gretl: matriz de correlación

```
corr(Totalreturns, Logvol) = -0,00981454
Bajo la hipótesis nula de no correlación:
 t(387) = -0,193084, con valor p a dos colas 0,8470
```

gretl: modelo 1

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```
Modelo 1: MCO, usando las observaciones 1-389
Variable dependiente: Totalreturns

              Coeficiente   Desv. Típica   Estadístico t   valor p
  ------------------------------------------------------------------
  const         5,06001       4,66642         1,084        0,2789
  Logvol       -0,841022      4,35573        -0,1931       0,8470

Media de la vble. dep.   5,954789   D.T. de la vble. dep.    10,79479
Suma de cuad. residuos   45208,31   D.T. de la regresión     10,80821
R-cuadrado               0,000096   R-cuadrado corregido    -0,002487
F(1, 387)                0,037281   Valor p (de F)           0,846994
Log-verosimilitud       -1476,903   Criterio de Akaike       2957,807
Criterio de Schwarz      2965,734   Crit. de Hannan-Quinn    2960,950
```

Scenario 2:

```
gretl: matriz de correlación

corr(Totalreturn, Logvol) = -0,02422847
Bajo la hipótesis nula de no correlación:
 t(481) = -0,531528, con valor p a dos colas 0,5953
```

```
gretl: modelo 1
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Modelo 1: MCO, usando las observaciones 1-483
Variable dependiente: Totalreturn

             Coeficiente   Desv. Típica   Estadístico t   valor p
  ---------------------------------------------------------------
  const        1,38948       1,30080          1,068       0,2860
  Logvol      -0,637650      1,19966         -0,5315      0,5953

Media de la vble. dep.  2,076046   D.T. de la vble. dep.   3,377241
Suma de cuad. residuos  5494,349   D.T. de la regresión    3,379758
R-cuadrado              0,000587   R-cuadrado corregido   -0,001491
F(1, 481)               0,282522   Valor p (de F)          0,595298
Log-verosimilitud      -1272,545   Criterio de Akaike      2549,089
Criterio de Schwarz     2557,449   Crit. de Hannan-Quinn   2552,374
```

Scenario 3:

```
gretl: matriz de correlación

corr(Totalreturn, Logvol) = 0,01356769
Bajo la hipótesis nula de no correlación:
 t(512) = 0,30703, con valor p a dos colas 0,7589
```

gretl: modelo 1

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```
Modelo 1: MCO, usando las observaciones 1-514
Variable dependiente: Totalreturn

             Coeficiente   Desv. Típica   Estadístico t   valor p
  ----------------------------------------------------------------
  const       2,39476       1,11399         2,150         0,0320   **
  Logvol      0,290237      0,945305        0,3070        0,7589

Media de la vble. dep.  2,054646   D.T. de la vble. dep.   2,667708
Suma de cuad. residuos  3650,177   D.T. de la regresión    2,670066
R-cuadrado              0,000184   R-cuadrado corregido   -0,001769
F(1, 512)               0,094267   Valor p (de F)          0,758945
Log-verosimilitud      -1233,134   Criterio de Akaike      2470,267
Criterio de Schwarz     2478,751   Crit. de Hannan-Quinn   2473,592
```

Scenario 4:

gretl: matriz de correlación

```
corr(Totalreturn, Logvol) = -0,28347526
Bajo la hipótesis nula de no correlación:
 t(481) = -6,48303, con valor p a dos colas 0,0000
```

gretl: modelo 1

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```
Modelo 1: MCO, usando las observaciones 1-483
Variable dependiente: Totalreturn

             Coeficiente   Desv. Típica   Estadístico t   valor p
  -----------------------------------------------------------------
  const      -1,23418       0,204228        -6,043        3,03e-09 ***
  Logvol     -1,29577       0,199870        -6,483        2,23e-10 ***

Media de la vble. dep.  0,075064   D.T. de la vble. dep.   0,696531
Suma de cuad. residuos  215,0535   D.T. de la regresión    0,668653
R-cuadrado              0,080358   R-cuadrado corregido    0,078446
F(1, 481)               42,02974   Valor p (de F)          2,23e-10
Log-verosimilitud      -489,9425   Criterio de Akaike      983,8850
Criterio de Schwarz     992,2451   Crit. de Hannan-Quinn   987,1703
```

Scenario 5:

gretl: matriz de correlación

```
corr(Totalreturn, Logvol) = 0,11375255
Bajo la hipótesis nula de no correlación:
 t(533) = 2,64334, con valor p a dos colas 0,0085
```

gretl: modelo 1

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```
Modelo 1: MCO, usando las observaciones 1-535
Variable dependiente: Totalreturn

             Coeficiente   Desv. Típica   Estadístico t   valor p
  ---------------------------------------------------------------
  const        3,53193       0,785262         4,498       8,43e-06 ***
  Logvol       1,76016       0,665886         2,643       0,0085   ***

Media de la vble. dep.   1,468775   D.T. de la vble. dep.    2,006332
Suma de cuad. residuos   2121,732   D.T. de la regresión     1,995178
R-cuadrado               0,012940   R-cuadrado corregido     0,011088
F(1, 533)                6,987242   Valor p (de F)           0,008451
Log-verosimilitud       -1127,672   Criterio de Akaike       2259,345
Criterio de Schwarz      2267,910   Crit. de Hannan-Quinn    2262,696
```

Appendix III

Monthly Volatility of Low Vol Portfolio Calculation

- First Step was to obtain the correlation matrix (Matrix 1) from the monthly returns:

Correlation matrix

	NESTLE 'R'	COFINIMMO	SWISSCOM 'R'	NOVARTIS 'R'	DIAGEO	SSE	T BENCKISER G	COLRUYT	AIR LIQUIDE
NESTLE 'R'	1	0,20665	0,26662	0,50400	0,46451	0,26023	0,51172	0,14161	0,34071
COFINIMMO	0,20665	1	0,20138	0,12692	0,25596	0,21644	0,21182	0,26487	0,35169
SWISSCOM 'R'	0,26662	0,20138	1	0,21723	0,18872	0,07981	0,14844	0,19746	0,09818
NOVARTIS 'R'	0,50400	0,12692	0,21723	1	0,30781	0,21584	0,30525	0,05526	0,41486
DIAGEO	0,46451	0,25596	0,25596	0,30781	1	0,43889	0,46892	0,07805	0,33871
SSE	0,26023	0,21644	0,21644	0,21584	0,43889	1	0,34162	0,21091	0,26371
RECKITT BENCKISER GROUP	0,51172	0,21182	0,21182	0,30525	0,46892	0,34162	1	0,14263	0,29932
COLRUYT	0,14161	0,26487	0,26487	0,05526	0,07805	0,21091	0,14263	1	0,04426
AIR LIQUIDE	0,34071	0,35169	0,35169	0,41486	0,33871	0,26371	0,29932	0,04426	1
DANONE	0,60169	0,31191	0,31191	0,36479	0,41113	0,30594	0,57146	0,12714	0,48245
ESSILOR INTL.	0,43894	0,11692	0,11692	0,33735	0,40522	0,27989	0,31995	0,04638	0,46169
RELX	0,38311	0,27485	0,27485	0,26634	0,52540	0,28505	0,54490	0,07953	0,36722
RELX	0,38311	0,27485	0,27485	0,26634	0,52540	0,28505	0,54490	0,07953	0,36722
SWEDISH MATCH	0,28631	0,09083	0,09083	0,16645	0,28679	0,27521	0,23718	0,00948	0,25583
GLAXOSMITHKLINE	0,40319	0,14395	0,14395	0,45783	0,49782	0,35751	0,36637	0,05613	0,30497
UNITED UTILITIES GROUP	0,30189	0,20645	0,20645	0,25860	0,44834	0,58237	0,34712	0,15079	0,33693
TOTAL	0,25657	0,14864	0,14864	0,25761	0,31137	0,24838	0,24316	0,02112	0,49433
PENNON GROUP	0,19005	0,19987	0,19987	0,23705	0,33791	0,58595	0,25026	0,18902	0,24861
BUNZL	0,47656	0,25093	0,25093	0,35938	0,54486	0,27248	0,52926	0,10032	0,40225
BRITISH AMERICAN TOBACCO	0,42499	0,30199	0,30199	0,27233	0,47059	0,41269	0,56987	0,13075	0,34213

DANONE	ESSILOR INTL.	RELX	RELX	WEDISH MATC	AXOSMITHKLI	ED UTILITIES GI	TOTAL	ENNON GROU	BUNZL	I AMERICAN TO
0,60169	0,43894	0,38311	0,38311	0,28631	0,40319	0,30189	0,25657	0,19005	0,47656	0,42499
0,31191	0,11692	0,27485	0,27485	0,09083	0,14395	0,20645	0,14864	0,19987	0,25093	0,30199
0,11578	0,12214	0,20575	0,20575	0,06609	0,26047	0,10455	0,26923	0,07699	0,10361	0,15574
0,36479	0,33735	0,26634	0,26634	0,16645	0,45783	0,25860	0,25761	0,23705	0,35938	0,27233
0,41113	0,40522	0,52540	0,52540	0,28679	0,49782	0,44834	0,31137	0,33791	0,54486	0,47059
0,30594	0,27989	0,28505	0,28505	0,27521	0,35751	0,58237	0,24838	0,58595	0,27248	0,41269
0,57146	0,31995	0,54490	0,54490	0,23718	0,36637	0,34712	0,24316	0,25026	0,52926	0,56987
0,12714	0,04638	0,07953	0,07953	0,00948	0,05613	0,15079	0,02112	0,18902	0,10032	0,13075
0,48245	0,46169	0,36722	0,36722	0,25583	0,30497	0,33693	0,49433	0,24861	0,40225	0,34213
1	0,41966	0,45800	0,45800	0,34795	0,35554	0,34495	0,26568	0,22996	0,51171	0,45071
0,41966	1	0,28844	0,28844	0,26794	0,30634	0,32071	0,23700	0,29669	0,33285	0,26779
0,45800	0,28844	1	1,00000	0,14601	0,28238	0,34698	0,39007	0,25434	0,54987	0,45158
0,45800	0,28844	1,00000	1	0,14601	0,28238	0,34698	0,39007	0,25434	0,54987	0,45158
0,34795	0,26794	0,14601	0,14601	1	0,23344	0,27488	0,22455	0,28472	0,24369	0,36767
0,35554	0,30634	0,28238	0,28238	0,23344	1	0,36901	0,23020	0,25929	0,45174	0,32128
0,34495	0,32071	0,34698	0,34698	0,27488	0,36901	1	0,28499	0,68088	0,29791	0,40859
0,26568	0,23700	0,39007	0,39007	0,22455	0,23020	0,28499	1	0,19710	0,30832	0,28326
0,22996	0,29669	0,25434	0,25434	0,28472	0,25929	0,68088	0,19710	1	0,19146	0,37889
0,51171	0,33285	0,54987	0,54987	0,24369	0,45174	0,29791	0,30832	0,19146	1	0,45415
0,45071	0,26779	0,45158	0,45158	0,36767	0,32128	0,40859	0,28326	0,37889	0,45415	1

- Secondly, an intermediate matrix (Matrix 2) was calculated multiplying the standard deviations of the stocks in order to obtain later on the covariance matrix:

Intermediate matrix

	NESTLE 'R'	COFINIMMO	SWISSCOM 'R'	NOVARTIS 'R'	DIAGEO	SSE	T BENCKISER G	COLRUYT	AIR LIQUIDE
NESTLE 'R'	0,001486	0,001496	0,001587	0,001684	0,001753	0,001779	0,001842	0,001880	0,001883
COFINIMMO	0,001496	0,001507	0,001598	0,001696	0,001766	0,001792	0,001855	0,001894	0,001897
SWISSCOM 'R'	0,001587	0,001598	0,001695	0,001799	0,001872	0,001900	0,001967	0,002008	0,002011
NOVARTIS 'R'	0,001684	0,001696	0,001799	0,001909	0,001987	0,002017	0,002088	0,002131	0,002135
DIAGEO	0,001753	0,001766	0,001872	0,001987	0,002069	0,002100	0,002173	0,002218	0,002222
SSE	0,001779	0,001792	0,001900	0,002017	0,002100	0,002131	0,002206	0,002252	0,002255
RECKITT BENCKISER GROUP	0,001842	0,001855	0,001967	0,002088	0,002173	0,002206	0,002284	0,002331	0,002335
COLRUYT	0,001880	0,001894	0,002008	0,002131	0,002218	0,002252	0,002331	0,002379	0,002383
AIR LIQUIDE	0,001883	0,001897	0,002011	0,002135	0,002222	0,002255	0,002335	0,002383	0,002387
DANONE	0,001925	0,001938	0,002056	0,002182	0,002271	0,002305	0,002386	0,002435	0,002440
ESSILOR INTL.	0,001933	0,001947	0,002065	0,002191	0,002281	0,002315	0,002397	0,002446	0,002450
RELX	0,001986	0,002000	0,002121	0,002251	0,002343	0,002378	0,002462	0,002513	0,002517
RELX	0,001989	0,002003	0,002124	0,002255	0,002347	0,002382	0,002466	0,002517	0,002521
SWEDISH MATCH	0,001996	0,002010	0,002132	0,002262	0,002355	0,002390	0,002474	0,002525	0,002530
GLAXOSMITHKLINE	0,002008	0,002022	0,002144	0,002276	0,002369	0,002404	0,002489	0,002540	0,002545
UNITED UTILITIES GROUP	0,002043	0,002057	0,002182	0,002316	0,002410	0,002446	0,002533	0,002585	0,002589
TOTAL	0,002054	0,002069	0,002194	0,002328	0,002423	0,002460	0,002546	0,002599	0,002603
PENNON GROUP	0,002058	0,002073	0,002198	0,002333	0,002428	0,002465	0,002552	0,002604	0,002609
BUNZL	0,002063	0,002078	0,002203	0,002339	0,002434	0,002470	0,002558	0,002610	0,002615
BRITISH AMERICAN TOBACCO	0,002063	0,002078	0,002204	0,002339	0,002434	0,002471	0,002558	0,002611	0,002615

DANONE	ESSILOR INTL.	RELX	RELX	WEDISH MATC	AXOSMITHKLI	ED UTILITIES GI	TOTAL	ENNON GROU	BUNZL	I AMERICAN TO
0,001925	0,001933	0,001986	0,001989	0,001996	0,002008	0,002043	0,002054	0,002058	0,002063	0,002063
0,001938	0,001947	0,002000	0,002003	0,002010	0,002022	0,002057	0,002069	0,002073	0,002078	0,002078
0,002056	0,002065	0,002121	0,002124	0,002132	0,002144	0,002182	0,002194	0,002198	0,002203	0,002204
0,002182	0,002191	0,002251	0,002255	0,002262	0,002276	0,002316	0,002328	0,002333	0,002339	0,002339
0,002271	0,002281	0,002343	0,002347	0,002355	0,002369	0,002410	0,002423	0,002428	0,002434	0,002434
0,002305	0,002315	0,002378	0,002382	0,002390	0,002404	0,002446	0,002460	0,002465	0,002470	0,002471
0,002386	0,002397	0,002462	0,002466	0,002474	0,002489	0,002533	0,002546	0,002552	0,002558	0,002558
0,002435	0,002446	0,002513	0,002517	0,002525	0,002540	0,002585	0,002599	0,002604	0,002610	0,002611
0,002440	0,002450	0,002517	0,002521	0,002530	0,002545	0,002589	0,002603	0,002609	0,002615	0,002615
0,002493	0,002504	0,002572	0,002576	0,002585	0,002601	0,002646	0,002660	0,002666	0,002672	0,002673
0,002504	0,002515	0,002584	0,002588	0,002597	0,002612	0,002658	0,002672	0,002678	0,002684	0,002684
0,002572	0,002584	0,002654	0,002658	0,002667	0,002683	0,002730	0,002745	0,002751	0,002757	0,002758
0,002576	0,002588	0,002658	0,002662	0,002671	0,002687	0,002734	0,002749	0,002755	0,002761	0,002762
0,002585	0,002597	0,002667	0,002671	0,002681	0,002697	0,002744	0,002759	0,002765	0,002771	0,002771
0,002601	0,002612	0,002683	0,002687	0,002697	0,002713	0,002760	0,002775	0,002781	0,002788	0,002788
0,002646	0,002658	0,002730	0,002734	0,002744	0,002760	0,002809	0,002824	0,002830	0,002836	0,002837
0,002660	0,002672	0,002745	0,002749	0,002759	0,002775	0,002824	0,002839	0,00285	0,00285	0,00285
0,002666	0,002678	0,002751	0,002755	0,002765	0,002781	0,002830	0,002845	0,002851	0,002858	0,002858
0,002672	0,002684	0,002757	0,002761	0,002771	0,002788	0,002836	0,002852	0,002858	0,002864	0,002865
0,002673	0,002684	0,002758	0,002762	0,002771	0,002788	0,002837	0,002852	0,002858	0,002865	0,002865

- The covariance matrix was the result of multiplying Matrix 1 and Matrix 2:

Covariance matrix:	NESTLE 'R'	COFINIMMO	SWISSCOM 'R'	NOVARTIS 'R'	DIAGEO	SSE	T BENCKISER G	COLRUYT	AIR LIQUIDE
NESTLE 'R'	0,001486	0,000309	0,000423	0,000849	0,000814	0,000463	0,000943	0,000266	0,000642
COFINIMMO	0,000309	0,001507	0,000322	0,000215	0,000452	0,000388	0,000393	0,000502	0,000667
SWISSCOM 'R'	0,000423	0,000322	0,001695	0,000391	0,000353	0,000152	0,000292	0,000396	0,000197
NOVARTIS 'R'	0,000849	0,000215	0,000391	0,001909	0,000612	0,000435	0,000637	0,000118	0,000886
DIAGEO	0,000814	0,000452	0,000479	0,000612	0,002069	0,000921	0,001019	0,000173	0,000753
SSE	0,000463	0,000388	0,000411	0,000435	0,000921	0,002131	0,000754	0,000475	0,000595
RECKITT BENCKISER GROUP	0,000943	0,000393	0,000417	0,000637	0,001019	0,000754	0,002284	0,000332	0,000699
COLRUYT	0,000266	0,000502	0,000532	0,000118	0,000173	0,000475	0,000332	0,002379	0,000105
AIR LIQUIDE	0,000642	0,000667	0,000707	0,000886	0,000753	0,000595	0,000699	0,000105	0,002387
DANONE	0,001158	0,000605	0,000641	0,000796	0,000934	0,000705	0,001364	0,000310	0,001177
ESSILOR INTL.	0,000848	0,000228	0,000241	0,000739	0,000924	0,000648	0,000767	0,000113	0,001131
RELX	0,000761	0,000550	0,000583	0,000600	0,001231	0,000678	0,001342	0,000200	0,000924
RELX	0,000762	0,000551	0,000584	0,000601	0,001233	0,000679	0,001344	0,000200	0,000926
SWEDISH MATCH	0,000571	0,000183	0,000194	0,000377	0,000675	0,000658	0,000587	0,000024	0,000647
GLAXOSMITHKLINE	0,000809	0,000291	0,000309	0,001042	0,001179	0,000860	0,000912	0,000143	0,000776
UNITED UTILITIES GROUP	0,000617	0,000425	0,000450	0,000599	0,001081	0,001425	0,000879	0,000390	0,000872
TOTAL	0,000527	0,000307	0,000326	0,000600	0,000755	0,000611	0,000619	0,000055	0,001287
PENNON GROUP	0,000391	0,000414	0,000439	0,000553	0,000821	0,001444	0,000639	0,000492	0,000649
BUNZL	0,000983	0,000521	0,000553	0,000840	0,001326	0,000673	0,001354	0,000262	0,001052
BRITISH AMERICAN TOBACCO	0,000877	0,000628	0,000665	0,000637	0,001146	0,001020	0,001458	0,000341	0,000895

DANONE	ESSILOR INTL.	RELX	RELX	WEDISH MATC	AXOSMITHKLI	ED UTILITIES GI	TOTAL	ENNON GROU	BUNZL	I AMERICAN TO
0,001158	0,000848	0,000761	0,000762	0,000571	0,000809	0,000617	0,000527	0,000391	0,000983	0,000877
0,000605	0,000228	0,000550	0,000551	0,000183	0,000291	0,000425	0,000307	0,000414	0,000521	0,000628
0,000238	0,000252	0,000436	0,000437	0,000141	0,000559	0,000228	0,000591	0,000169	0,000228	0,000343
0,000796	0,000739	0,000600	0,000601	0,000377	0,001042	0,000599	0,000600	0,000553	0,000840	0,000637
0,000934	0,000924	0,001231	0,001233	0,000675	0,001179	0,001081	0,000755	0,000821	0,001326	0,001146
0,000705	0,000648	0,000678	0,000679	0,000658	0,000860	0,001425	0,000611	0,001444	0,000673	0,001020
0,001364	0,000767	0,001342	0,001344	0,000587	0,000912	0,000879	0,000619	0,000639	0,001354	0,001458
0,000310	0,000113	0,000200	0,000200	0,000024	0,000143	0,000390	0,000055	0,000492	0,000262	0,000341
0,001177	0,001131	0,000924	0,000926	0,000647	0,000776	0,000872	0,001287	0,000649	0,001052	0,000895
0,002493	0,001051	0,001178	0,001180	0,000900	0,000925	0,000913	0,000707	0,000613	0,001367	0,001205
0,001051	0,002515	0,000745	0,000746	0,000696	0,000800	0,000852	0,000633	0,000794	0,000893	0,000719
0,001178	0,000745	0,002654	0,002658	0,000389	0,000758	0,000947	0,001071	0,000700	0,001516	0,001245
0,001180	0,000746	0,002658	0,002662	0,000390	0,000759	0,000949	0,001072	0,000701	0,001518	0,001247
0,000900	0,000696	0,000389	0,000390	0,002681	0,000630	0,000754	0,000619	0,000787	0,000675	0,001019
0,000925	0,000800	0,000758	0,000759	0,000630	0,002713	0,001019	0,000639	0,000721	0,001259	0,000896
0,000913	0,000852	0,000947	0,000949	0,000754	0,001019	0,002809	0,000805	0,001927	0,000845	0,001159
0,000707	0,000633	0,001071	0,001072	0,000619	0,000639	0,000805	0,002839	0,00056	0,00088	0,00081
0,000613	0,000794	0,000700	0,000701	0,000787	0,000721	0,001927	0,000561	0,002851	0,000547	0,001083
0,001367	0,000893	0,001516	0,001518	0,000675	0,001259	0,000845	0,000879	0,000547	0,002864	0,001301
0,001205	0,000719	0,001245	0,001247	0,001019	0,000896	0,001159	0,000808	0,001083	0,001301	0,002865

- Finally, the variance of the portfolio was calculated considering the above covariances and the weights of each stock in the portfolio (5% each component). The portfolio volatility was calculated as the squared root of the variance to obtain the following result:

Portfolio variance	**0,082%**
Portfolio risk (measured by standard deviation)	**2,86%**