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OFFICIAL MASTER'S DEGREE IN THE ELECTRIC POWER INDUSTRY

Master's Thesis

VALUATION OF ELECTRICITY DERIVATIVES

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Abstract

This paper aims to forecast the value of electricity derivatives in the MIBEL system. For that purpose, three different models based on the simulation of future price scenarios are developed.

The project begins with an extensive analysis of prices in spot and forward markets, with the objective of characterizing market's behavior. The obtained results are introduced in the simulation models, building their basis. The models aim to forecast all the possible future price scenarios and their probability. A Brownian motion and a mean reversion model have been used to simulate the spot market. A Monte Carlo algorithm forecasted the forward curve evolution.

Finally, the models are applied to two practical cases. The first one consists on valuating a collar for a consumer. The second one, focuses in the valuation of a combined-cycle gas turbine asset. The calculated results are compared with the ones obtained with the Black-Scholes formula.

As a conclusion, the Monte Carlo model obtains the best trade-off between simplicity and accuracy.

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CHAPTER 1: INTRODUCTION

This chapter will focus on contextualizing the present paper and its objective. The aim is that the reader identifies and understands the main issues this research wants to address and the motivation behind it.

Therefore, a brief introduction will be given under the section Motivation, which will provide a wide picture of the role of the electricity derivatives in electricity markets. Afterwards, the main objectives of the Thesis will be provided, followed by the precise definition of the scope of the research.

1. Motivation

Companies participating in the electricity markets are exposed to many sources of risks, as fuel prices, inflows, the regulatory framework, etc. One of the main sources of uncertainty is the price of the electricity which, due to its physical features, is very volatile. To hedge the risk from random prices, agents can trade in the forward markets. In these markets, we can find both physical and financial contracts. The latter, which are known as electricity derivatives, do not imply the physical delivery of the underlying commodity.[1] Financial derivatives can be used by utilities as a risk management instrument, which allows an efficient allocation of the market risks.

Generally speaking, in a market there are two different procedures to trade electricity derivatives:

- Through bilateral or over-the-counter (OTC) contracts. In these cases, the contract is traded directly between the counterparties, with no intermediates. Thus, there is no organized market. However, these contracts usually comply with the framework established by the ISDA (International Swaps and Derivatives Association).
- Through an organized exchange market. There are standardized contracts that are cleared and settle through a settlement agent or clearing house. This reduces the participants' risk and increases liquidity with respect to the previous type.

In the context of the Iberian Electricity Market (MIBEL), which englobes the Spanish and the Portuguese markets under a regional organization, the organized exchange market is operated by OMIP (Operador del Mercado Ibérico de Energía).[2] OMIP manages the trading on the derivatives market while the entity OMIClear acts as Clearing House and Central Counterparty. Additionally, in order to increase liquidity, OMIP provides a trading platform to act as meeting point for agents who want to fix an OTC.[3]

As it has been explained, electricity derivatives may be used to hedge a company's risk, but it is not their only application. These products can also be used by traders to speculate in the market and by companies to diversify their investment portfolio.

Hence, financial derivatives play an important role in the electricity industry. A correct utilization of these instruments provides considerable benefits to an entity. However, for this correct utilization, it is necessary to have an accurate estimation of their value.

2. Objectives

The object of this project is to determine the value of electricity derivatives in the MIBEL system. The valuation consists on the estimation of the premium that an agent will be willing to pay to a counterparty for the optionality provided by a product of specific characteristics (underlying product and its volatility, time delivery, etc.). These characteristics are defined by the end consumer's needs and his risk aversion profile, since the electricity derivative is created to transfer part of the risks inherent in their activity to other agents or to completely mitigate them. Therefore, the objective of the valuation is to forecast the value that such derivative will provide with a certain confidence level or percentile, in order to determine the value to be paid or demanded for it.

This main objective will be broken down into four main points to be achieved during this research:

1) Understand the needs of the electricity derivatives products by the characterization in the spot and forward markets. The characterization will be focus on:
    a. Statistical analysis
    b. Volatility of prices
    c. Shaping of curves of prices
    d. Liquidity
2) Comprehension of the financial products and its applications:
    a. Forwards
    b. Options
    c. Swaps
3) Development of models to compute the value of electricity derivatives by
    a. Brownian motion models
    b. Mean reversion models
    c. Monte Carlo simulations
    d. Black-Scholes method
4) Application of the developed models to:
    a. Case 1. Retail business: calculation of a cap and a floor
    b. Case 2. Generation business: extrinsic value of a generation asset

3. Scope

Corresponding with the four main objectives, this project has four clearly differentiated sections with diverse methodology:

→ **Section 1** corresponds with the objective of developing an extensive characterization of the electricity prices of the spot and forward markets. This section will be mainly developed in the software RStudio. The results extracted from this section will condition the quantitative simulation models of the prices made in the following steps. The main features to be defined are the probability distribution of prices, their volatility and the curve's shaping. Other market parameters, as the liquidity, may be analyzed, if considered of interest.

→ The next section, **Section 2**, corresponds to the objective of acquiring a full comprehension of the types of financial products traded in the electricity markets. It is sought that the reader is able to identify the needs of the main financial products and have a clear knowledge of their functionality.

→ **Section 3** focuses on the development of quantitative simulation models of electricity prices. The existing models will be analyzed both for the spot price (Brownian-Geometric movement, Mean reversion, etc.) and for the forward curve (Monte Carlo). After analyzing the advantages and disadvantages of each model, an algorithm of each type that allows simulations of spot and forward prices will be programmed in the software RStudio.

→ In the last section, **Section 4**, the developed algorithms of the simulation models will be adapted with the conclusions obtained in Section 1, to the observed prices in the MIBEL markets. The main objective is to assess two practical cases, one for the retail business, consisting in valuing a cap for a client, and another for the generation business. In this one, the extrinsic value of the asset will be determined in the sale of its production in the market. The Black-Scholes formula will be used as a benchmark.

These 4 sections will be englobed under the Chapter 3 of the paper. This chapter provides an overall picture of the project purpose and development. Chapter 2 offers the reader a brief summary of the current state of the art of valuating electricity derivatives. In Chapter 4 the results obtained from the simulation models will be analyzed and, finally, Chapter 5 will gather the main conclusions drafted during the project and provide some aspects to enforce in future projects.

CHAPTER 2: State of the art

[4], [5]

The valuation of financial products has been an issue faced by many agents in all markets since their creation. The necessity of obtaining the most accurate value of the products, in order to hedge market's risks, has been the incentive causing the development of several models and theories addressing this issue. Among them, the research carried out by Fischer Black and Myron Scholes must be highlighted.

These authors focused their most famous work in the valuation of the premium of an option. They created what is known as the Black-Scholes formula, published in 1973. This formula, which will be explained further in this paper, is still used nowadays by most market agents to estimate the value of a derivative.

The Black-Scholes formula relies on some well-defined hypothesis. For instance, the underlying must have a lognormal distribution function and there are neither transactions costs nor taxes. As the reader can imagine, electricity markets do not fulfill many of these hypotheses. To overcome these limitations, subsequent models have added several variations to the Black-Scholes formula in order to adapt it to the conditions found in real markets. Despite the modifications, many agents still use the original formula, due to its simplicity and rapidness.

Another approach when valuating electricity derivatives is the simulation of the future scenarios of the market. Under this approach we must differentiate two types of models. On the one hand, we find fundamental simulations. These seek to forecast the prices by fundamental drivers, as the demand or fuel prices. On the other hand, there are quantitative models which are based on the statistical analysis of the historical data. Compared to the fundamentals, the quantitative models require less and more accessible information.

Finally, a differentiation must be made between the simulations aiming to predict the underlying commodity's value, which will be the spot market price, in our case, and the simulations focused on the forecast of the price of the forward market's products. Since these two have very different characteristics, their simulation models will be very different.

CHAPTER 3: Presentation of the problem

The objective of the present paper is to elaborate a model to valuate electricity derivatives, which means estimating the premium the market is willing to pay for the optionality provided by the product. Thus, to valuate electricity derivatives, the market behavior must be studied and, to the maximum possible extent, characterized. This will be sought during the first section of the chapter: 1. Analysis of electricity prices.

The next section, 2. Monthly electricity price simulations, explains the theory behind some approaches that intend to replicate the studied prices. Similarly, section 3. Electricity derivatives will provide the reader a theoretical perspective of the electricity derivatives products and their applications.

Finally, section 4. Valuation of electricity derivatives, will show the outcomes from the price simulation models developed. These models are supported on the conclusions obtained in sections 1 and 2. Additionally, they are applied to two practical cases, that are explained at the end of the chapter.

This chapter will be developed with the support of the software RStudio, which is an integrated development environment (IDE) for the programming language R. It is an open-source focus on statistical computing and graphics.[6]

1. Analysis of electricity prices

During the following chapter, a study of the electricity prices of both the day-ahead (DA) market and the forward market will be developed. Firstly, there will be a statistical analysis of the prices with the aim of finding the best probability distribution that fits them. Then, other features, as the volatility, shaping or liquidity, will be studied.

The final goal of the statistical analysis is to set the bases for the prediction of prices models that will be developed afterwards.

1.1. Statistical analysis of the spot market price

The study will start by analyzing the daily prices of the DA market. The first step will be to fit the price series into a statistical probability distribution. Particularly, it will be focused on a normal and lognormal distribution. Then, the volatility and shaping of the series will be studied. Finally, the shaping results will be checked by a backtest model. To conclude, the probability distribution and shaping of the monthly prices will also be characterized for the monthly prices of the DA market.

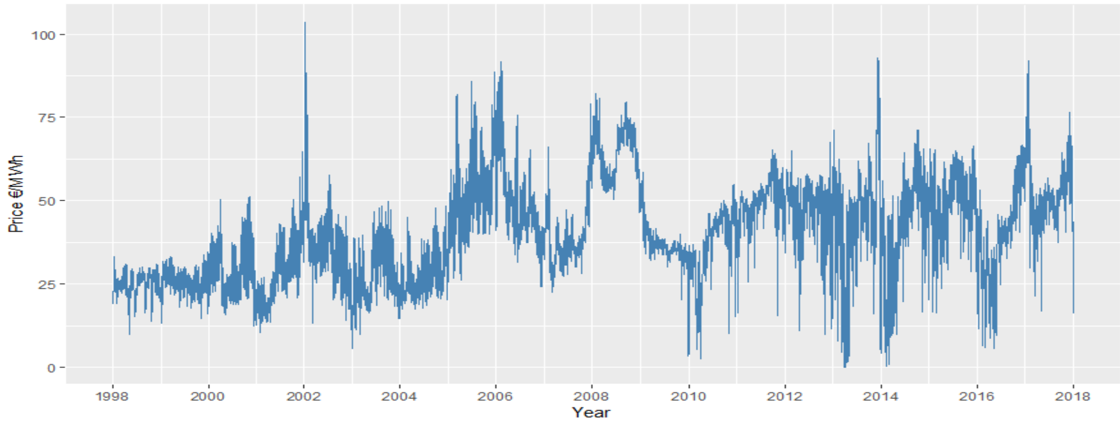

Fig. 1. Day ahead prices since 1998 to 2017

Before starting the analysis, the sample must be prepared. Although there are DA prices since the establishment of the market in 1998, it is considered that not all the results obtained during certain periods of time represent the current behavior of the market nowadays. Thus, we must reduce the time window to include in our study only the results that reflect the current behavior of the DA electricity prices.

On the one hand, the different months of the year have very distinct features which affect the prices. For instance, months with extreme temperatures have higher demands and, therefore, higher prices. Thus, to maintain the reliability of our study, we will always work with a time frame composed of complete years.

On the other hand, when deciding the time scope to be introduced in the study, we must consider that the DA market price is very dependent on certain factors as the regulation established or the presence of renewable source generation capacity in the system. Thus, the first thought is to select the data generated in the market in the closest preceding years. For instance, it seems reasonable to focus on the prices between 2015 and 2017. These years have a very similar regulation and almost the same participation of renewables in the energy mix. Thus, it is reasonable to believe that these prices reflect the current behavior of the market.

However, the prices are also affected by other factors which cannot be assumed similar, even during the closest previous years. Some of these factors, as the hydro inflows or wind resource, are completely random and may be very persistent over time. For instance, prices in 2014 decreased due to a high hydro and wind participation.[7] Other factors, as fuel prices, are mainly governed by international issues, whose effect may remain during a great period of time.

Therefore, if we develop the study as aforementioned, using the prices generated in the period 2015-2017, we will be risking that market behavior in any of these three years was influenced by any of the previously discussed environments. Consequently, we risk that the results of the study are not impartial but rather leaned forward to a specific situation.

To avoid this, we must enlarge the sample considered. Therefore, we pick the time window from 2000 to 2017. The data from these 18 years provide enough information from the market behavior to ensure that the results are neither influenced by sporadic conditions, nor specific regulations or renewables share. The first two years of development of the market are not considered in the study, as they are understood as a learning period, so their results do not reflect a normal behavior.[8] [9]

Once the time window is defined we will have to define the granularity of prices studied. The outcome of the spot market is a program consisting of volume of energy and price per hour. However, it is frequent to study the daily price, meaning the average of the 24 prices of a day. Likewise, the monthly prices will be studied, since having a deep characterization of their behavior will provide high benefits in the development of following sections.

1.1.1. Spot daily prices

During this chapter, the daily prices obtained in the spot market between 2000 and 2017 will be studied. Before starting the analysis, it is convenient to clean the sample. We seek to identify the peak values that lie at an abnormal distance from the rest of the sample. These peaks are denominated outliers and may alter the results. Therefore, we must identify and substitute them with acceptable values.

For this purpose, we use in the software RStudio the function *tsclean*, which belongs to package *fpp2*. This function recognizes the outliers and replaces them with a value calculated with linear interpolation.

In the following chart, the samples before and after treating it are compared. Additionally, the mean value of the cleaned sample throughout the period is also represented.

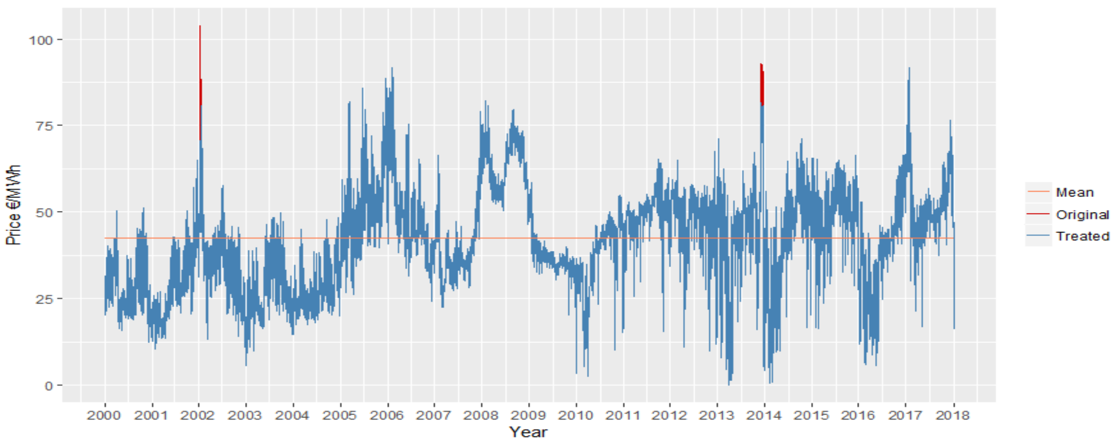

Fig. 2. Day-ahead prices from 2000 to 2017 before and after removing outliers

Therefore, the sample of day-ahead prices introduced in the study is composed by 6574 data, of which 13 have been considered outliers and substituted by linear interpolation.

Once the sample is prepared we can proceed with the statistical analysis. The procedure and results are developed in the following sections.

Statistical distribution model: goodness of fit test

After cleaning the sample, a goodness of fit test will be made. These tests are based on hypothesis testing, which is a method of verifying whether a null hypothesis applies to a sample. Thus, in a goodness of fit test, the null hypothesis consists on whether the sample can be characterized as a defined statistical distribution, which in our case will be a normal or lognormal distribution.

The analytical goodness of fit tests compute a measurement of the discrepancy between the theoretical and the observed distributions. However, there is not a unique way to calculate the "distance" between two different samples. Therefore, depending on the approach chosen, we find several different goodness of fit tests. For the development of this paper we will use the Cramér-Von Mises criterion which calculates the discrepancy as:

$$\omega^2 = \int_{-\infty}^{\infty} [F_n(x) - F'(x)]dF'(x)$$

Being $F'(x)$ the theoretical distribution and $F_n(x)$ the observed distribution.[10]

The results of the test will report a p-value. We set the statistical significance (α) at 0.05. The significance represents the minimum p-value we will accept. Thus, if the p-value is lower than our significance, we will reject the null hypothesis and vice versa.

Other analytical methods, as the Chi-squared test or the Kolmogorov-Smirnov test, have been computed although it is found that, due to the characteristics of our sample, the most trustful results are the ones obtained by the Cramér-Von Mises test. Additionally, for the analysis we also made use of a graphic goodness of fit method, the normal probability plot, which graphically reveals how well the distribution fits our sample.

As it has been mentioned, the goodness of fit test will reveal whether the electricity prices behave with a certain probability distribution. In particular, we will focus in fitting the prices into a normal and a lognormal distribution.

Knowing that μ is the mean of a random continuous variable and σ is its standard deviation, the normal distribution $N(\mu, \sigma)$ is a symmetric bell-shaped distribution. The density function corresponds to the following formula:

$$f(x) = \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{(x-\mu)^2}{2\sigma^2}}$$

A random variable has a lognormal distribution if the logarithms of the data have a normal distribution. The transformation is represented in the following charts:

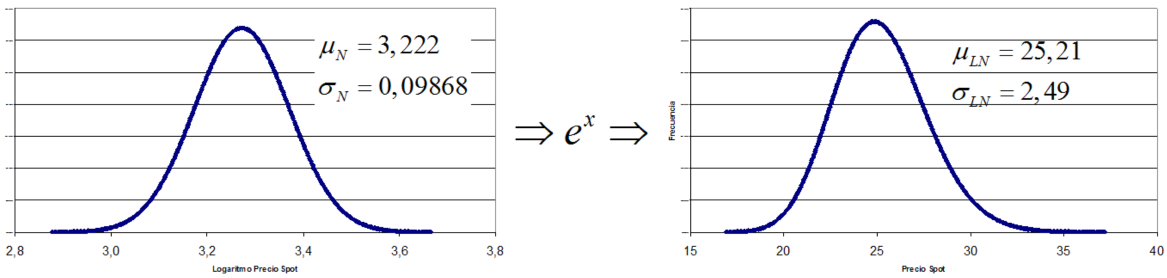

Fig. 3. Distibution of a random variable with lognormal distribution and distribution of its logarithms. Source: Valuation Internal Course Iberdrola

Thus, to decide which distribution fits better the day-ahead prices we will study two different samples: the prices and its logarithms. First, the mean value and deviation of these samples is calculated and then the theoretical normal distribution that corresponds to those characteristics (N(μ, σ)). Plotting the theoretical distribution against the observed we obtain the following images:

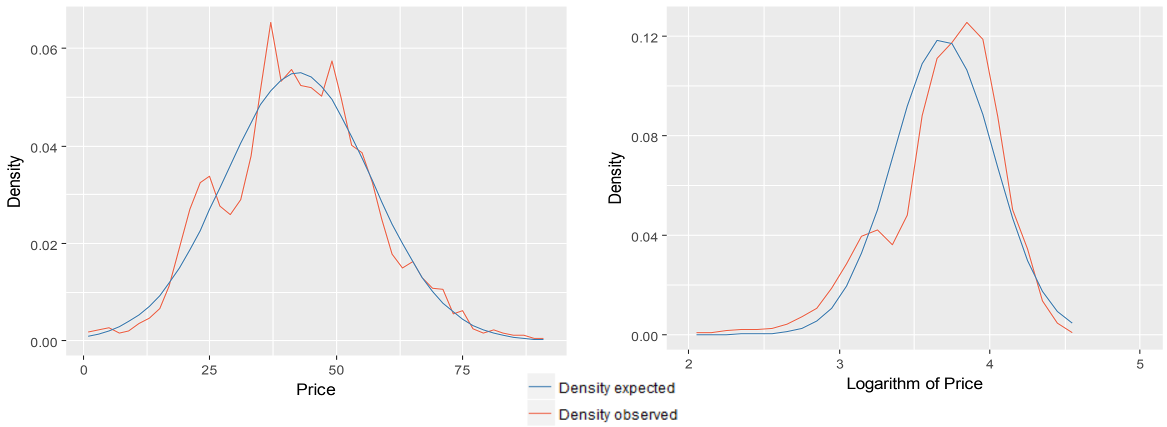

Fig. 4. Theoretical and observed density functions of the normal and lognormal distributions

Afterwards, the Cramér-Von Mises test was ran for the two samples to see which distribution behaves more as normal. The function used was *cvm.test*, included in the *goftest* package of RStudio. The results are showed in the following table:

	Normal	**Lognormal**
Mean (μ)	42,340	3,690
Standard deviation (σ)	14,487	0,333
Cramér-Von Mises ω^2	0,3933	12,334
Cramér-Von Mises p-value	0,07523	$4{,}946 \cdot 10^{-11}$

Table 1. Cramér-Von Mises results for daily spot prices.

As we can observe, the results obtained in the goodness of fit test for the normal distribution are above the set significance level (α=0.05). This reveals that we cannot reject the null hypothesis: it is acceptable to assume the sample as a variable with normal distribution.

However, the p value obtained when trying to fit the logarithms of the prices into a normal distribution is quite low. Therefore, the test show that the prices cannot be modeled as a lognormal variable, since the p value obtained is much lower our significance level.

Finally, we run a graphical goodness of fit test. The normal probability plot compares the quantiles of a theoretical normal and the observed ones. The results are showed below:

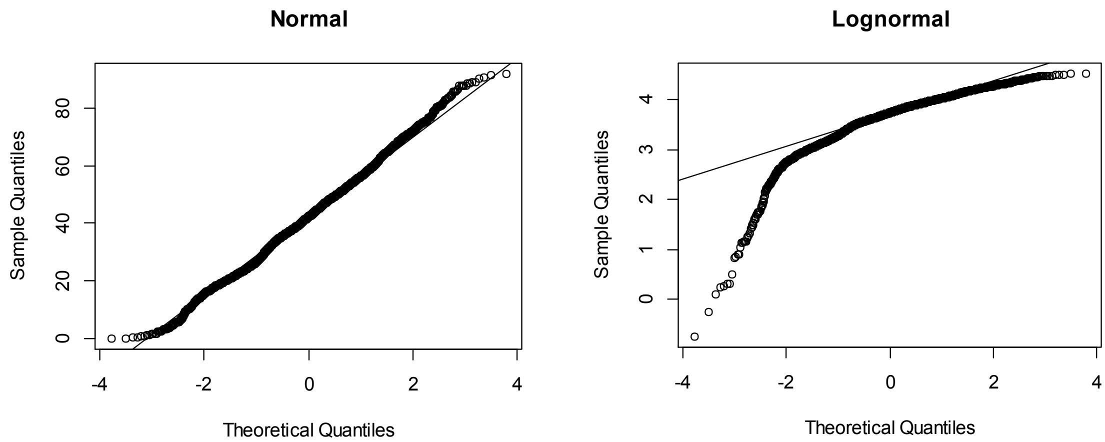

Fig. 5. Normal probability plot for the prices and their logarithms

The representation of the normal probability plot of the real prices show that the sample is light tailed, meaning that there is a higher concentration of data around the mean and less in the extreme values.

On the other hand, the normal probability plot of the logarithms of prices shows that the distribution is left skewed. This means that it is asymmetric, as its left tail is longer than expected. On the image of the right we find an example of a left skewed distribution.

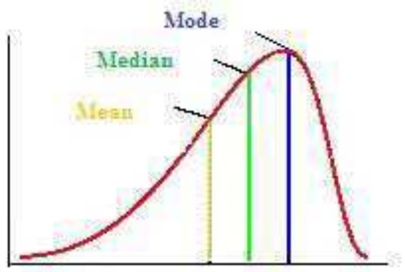

Fig. 6. Left-Skewed distribution function. Source: [11]

As a conclusion, it is worth explaining that fact that the study refutes the hypothesis of prices as a lognormal is remarkable. The logarithmic transformation stabilizes the variance of a sample. The lognormality of prices is a highly extended assumption over the fields of electricity market analysis. In fact many of the valuation models, as the Black-Scholes formula, are based under the assumption that prices behave as a lognormal variable.[12]

Volatility

To make a complete study of the prices from the day-ahead market, it is necessary to define their volatility. This characteristic measures the rate at which a variable evolves, decreasing or increasing from the expected value.[13] It is, therefore, interesting to predict the volatility of the prices, in order to anticipate to a high deviation from the forecasted value, if this was the case.

To compute the volatility, we calculate the performance of each day i by:

$$Performance_i = \log\left(\frac{Price_i}{Price_{i-1}}\right)$$

Then, the monthly volatility is calculated from the standard deviation of the obtained price performances during the pertinent month. Following the next formula, where *i* are the days of the month *j*:

$$\begin{matrix} Monthly \\ Volatility_j \end{matrix} = \sqrt{30\ days} \times \sigma(Performance_i)[12]$$

On the following table and figure, the results obtained since 2000 are shown.

Monthly volatility	Jan.	Feb.	Mar.	Apr.	May	June	July	Aug.	Sept.	Oct.	Nov.	Dec.	Mean
2000	114%	116%	96%	116%	92%	73%	131%	92%	132%	190%	152%	142%	**113%**
2001	161%	154%	71%	81%	84%	109%	125%	77%	113%	123%	136%	85%	**110%**
2002	120%	72%	140%	78%	106%	101%	101%	125%	133%	149%	213%	162%	**125%**
2003	220%	164%	161%	77%	99%	103%	137%	111%	167%	122%	87%	117%	**130%**
2004	95%	85%	111%	78%	97%	132%	99%	89%	124%	128%	88%	108%	**103%**
2005	92%	118%	137%	100%	95%	132%	98%	85%	73%	69%	84%	73%	**96%**
2006	86%	72%	59%	59%	112%	127%	72%	49%	66%	68%	83%	62%	**76%**
2007	55%	78%	53%	54%	56%	71%	53%	41%	42%	38%	49%	62%	**54%**
2008	42%	45%	41%	21%	23%	27%	18%	17%	26%	29%	31%	34%	**30%**
2009	40%	51%	45%	35%	25%	24%	29%	22%	19%	27%	68%	181%	**47%**
2010	163%	163%	247%	237%	89%	71%	21%	28%	39%	144%	129%	131%	**122%**
2011	109%	39%	92%	62%	73%	29%	43%	24%	35%	68%	184%	70%	**69%**
2012	42%	48%	102%	234%	109%	56%	63%	63%	98%	151%	222%	189%	**115%**
2013	139%	289%	Inf%	Inf%	89%	138%	86%	49%	94%	84%	169%	244%	-
2014	421%	543%	392%	257%	79%	138%	72%	48%	47%	90%	170%	122%	**400%**
2015	109%	190%	128%	126%	112%	51%	47%	99%	78%	65%	110%	101%	**101%**
2016	182%	285%	163%	214%	310%	79%	40%	42%	42%	65%	74%	54%	**129%**
2017	51%	95%	120%	143%	55%	37%	27%	36%	41%	44%	58%	100%	**67%**
Mean	**119%**	**145%**	-	-	**95%**	**83%**	**70%**	**61%**	**76%**	**92%**	**117%**	**113%**	**116%**

Table 2. Monthly volatility of prices in the day-ahead market before treatment

In 2013 there are two days with 0 prices, making the volatility infinite. As extremely high values are not representative of the trend of the market, these values are substituted by linear interpolation. Moreover, the outliers of the volatility data are replaced. The following image contrasts the initial values and the treated values.

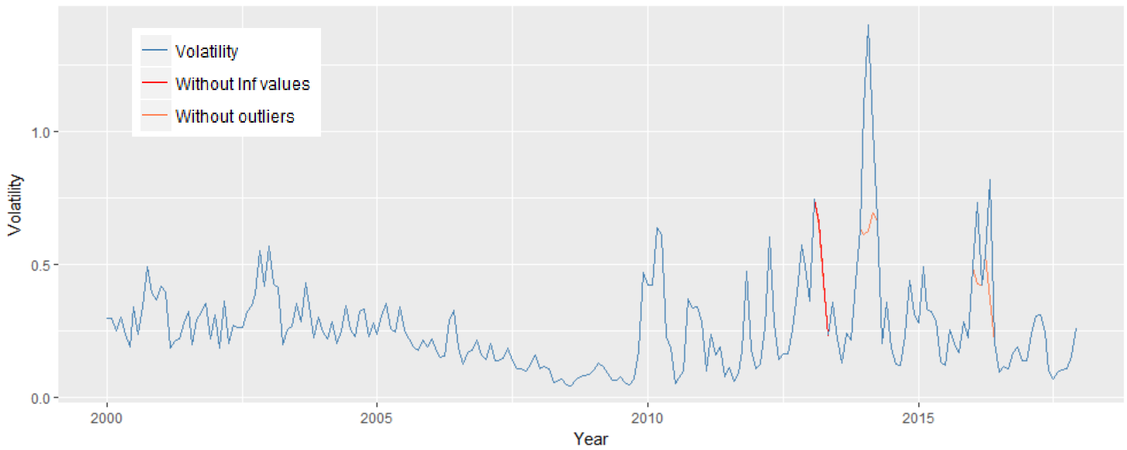

Fig. 7. Monthly volatility of prices in the day-ahead market

Thus, the new mean values for each month are:

Jan.	Feb.	Mar.	Apr.	May	June	July	Aug.	Sept.	Oct.	Nov.	Dec.	Mean
115%	121%	127%	117%	88%	83%	70%	61%	76%	92%	117%	112%	98%

Table 3. Average monthly volatility after treatment.

Daily volatility is set around 18%, which is translated into a monthly volatility of 98%. This is a very high value, as it is common in electricity prices in countries with a liquid market. It is also noticeable that in general terms volatility has grown over time since 2010. The main cause is the increase of installed capacity of renewable energy systems. These units, while bidding at low prices, provide a very intermittent generation. These conditions provoke very fluctuating prices. In this sense, 2013 was a very volatile year as it accounted with a lot of hydro and wind energy, reaching at certain moments the 0 prices. [9]

The final volatility averages are, for this reason, not fully representative of the price trend. The dispersion of the values seen for the same month in different years is too high to consider that future volatility will be the calculated average. This effect is represented below, by the difference between the minimum and maximum values.

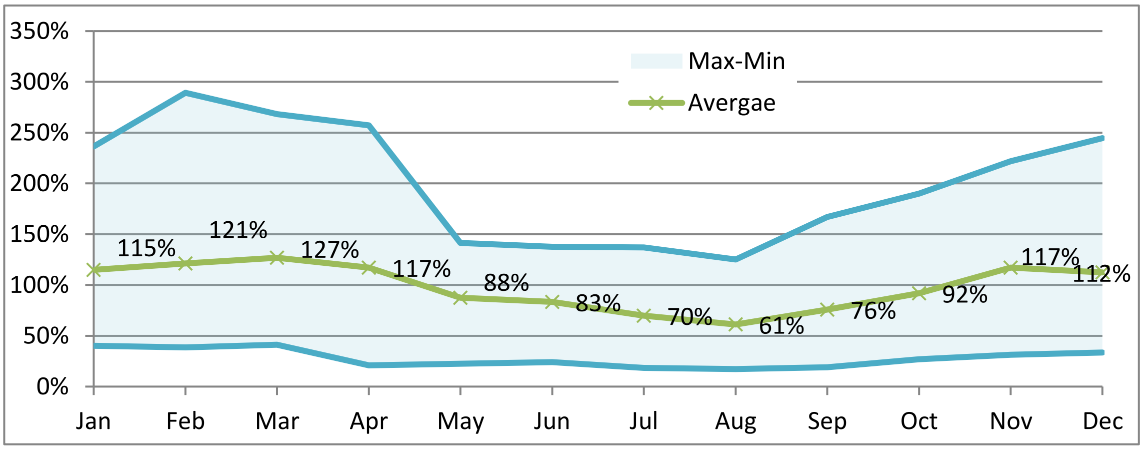

Fig. 8. Average monthly volatility.

However, it is possible to extract some conclusions from this study. For instance, certain months of the year are less volatile than others. Thus, summer months, as july or august, are significantly less volatile than winter and spring months, as march or april. The seasonality of the volatility will be a main characteristic to consider when valuating electricity derivatives.

Shaping

For the characterization of the prices, it is also very useful to define the deviation from the mean. To measure it, a software tool has been designed. This tool calculates the expected difference between the monthly average price and the price of a specific hour.

Therefore, the main objective of this model is to provide these hourly differences so that, together with our best forecast of monthly prices, we can estimate the prices of each hour of a whole year.

The model groups the days into clusters, distinguishing three different days per month: working days, Saturdays and Sundays. This is based under the assumption that all the days of a certain cluster behave similarly. Thus, it is assumed that the prices of all the Sundays of June are the identical. Thanks to this assumption, each month only accounts for three different sets of 24 prices, one per hour of the day, as all the days are classified into a cluster. Therefore, the model calculates 36 different sets repeated during the whole year:

$$Values = 12\ months \times 3\ clusters \times 24\ hours = 864\ values$$

This assumption greatly reduces the amount of data we work with, as we go from 8760 hourly prices in a year to 864 deviations from the monthly averages.

The model forecasts the future deviations with historical data: it computes the mean deviations during a chosen period of time. It has been observed that the most reliable results are reported when introducing the data of the three previous years. However, in the next section the validity of the models' outputs' will be studied.

Backtest of shaping models

The model described in the previous section gives us, together with the forecasted monthly estimations, the expected prices of all the hours of a certain year. However, we do not know the accuracy of the results. Through the backtest process we measure the goodness of the shaping model's results by applying it to past years and comparing the real prices with the model's calculation.

Obviously, the results change depending on the historical data chosen. This means that the results will be strongly affected by the election of the period of time for which the average price difference is calculated. Through the backtest model we will also define the optimum period of time used as an input.

After several trials, a three years-time scope has been elected. Therefore, the model calculates for each hour of 2017 the average difference seen between the spot hourly price and the monthly price from 2014 to 2016. Then, the obtained differences are added to each month's observed average price during 2017. Thus, we have calculated the prices for each hour of 2017.

To find out the accuracy of the model, we compare the results obtained and the actual market results during 2017. An example of the relation between these two is shown in the following chart, which represents the real and estimated prices of January 2017:

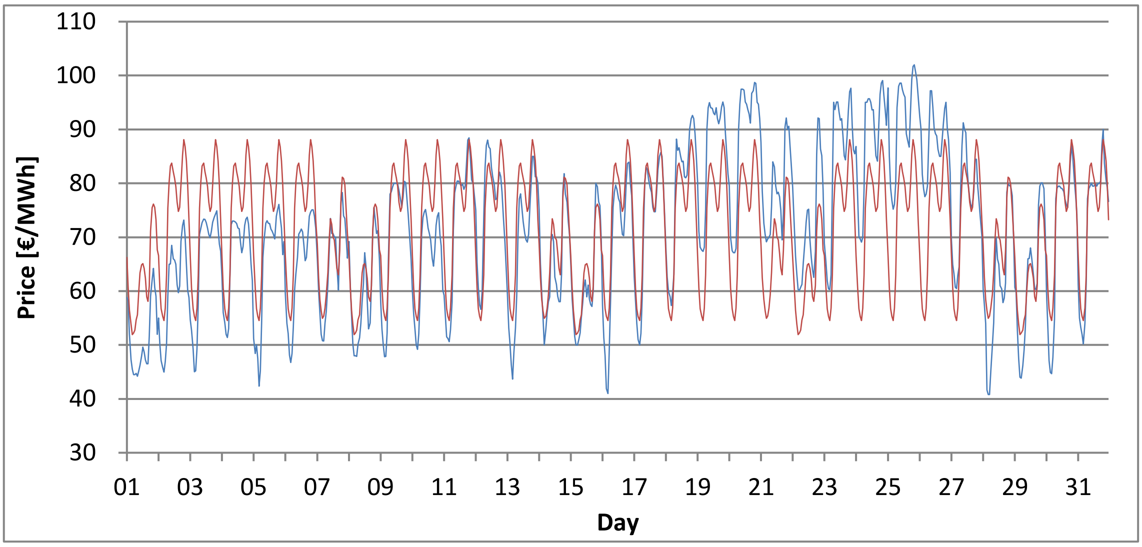

Fig. 9. Calculated (red) and real prices (blue) for all the hours of January 2017

In Fig. 10 the correlation between the two sets of prices is represented. As it can be seen, the coefficient of determination is 0.9164, very close to 1. The mean squared error (MSE) is 11,14, which is translated to a 5,34% of the average of the real prices. The tool is therefore considered very precise.

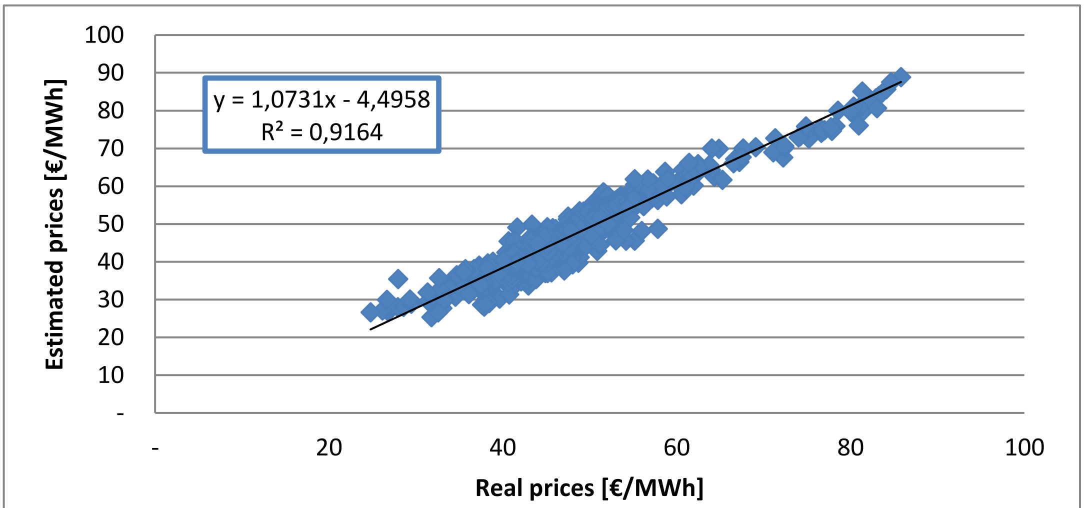

Fig. 10. Correlation between the estimated and the real prices of 2017.

With this section, the analysis of the daily prices is concluded. Their probability distribution has been characterized, followed by a study of its volatility and shaping. On the next section, a similar analysis will be developed to the monthly values.

1.1.2. Monthly prices

During this chapter the monthly spot prices will be studied, in order to conclude the day-ahead market analysis. Monthly prices are calculated as the average of each month's market output. The time window will be maintained from the previous section due to the factors above explained.

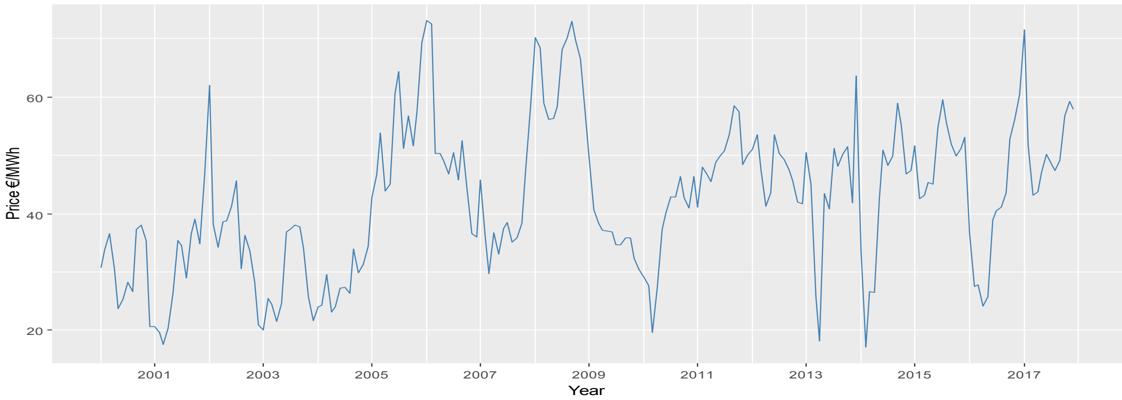

Fig. 11. Monthly spot prices

In this case, there is no need to clean the sample as there are no outliers found when analyzing it through RStudio. Thus, initially the study will try to fit the behavior of a set of 18 x 12 = 216 prices into a probability distribution. Then, the shaping will be characterized.

Statistical distribution

As in the above section, the probability distribution that best fits the sample will be found through a goodness of fit test. Specifically, the analytical Cramér-Von Mises test will be ran through the *cvm.test* function of Rstudio. Then, to obtain a graphic result, the normal probability plot will be done.

As in the previous chapter, it will be analyzed whether the sample behaves with a normal or lognormal distribution. Therefore, two different inputs are created: the prices and its logarithms. These two samples will be analyzed separately to, afterwards, compare the results and find out which distribution defines the prices better.

Obtaining and representing the theoretical normal probability functions (N(μ, σ)) from each sample mean μ and standard deviation σ, we obtain:

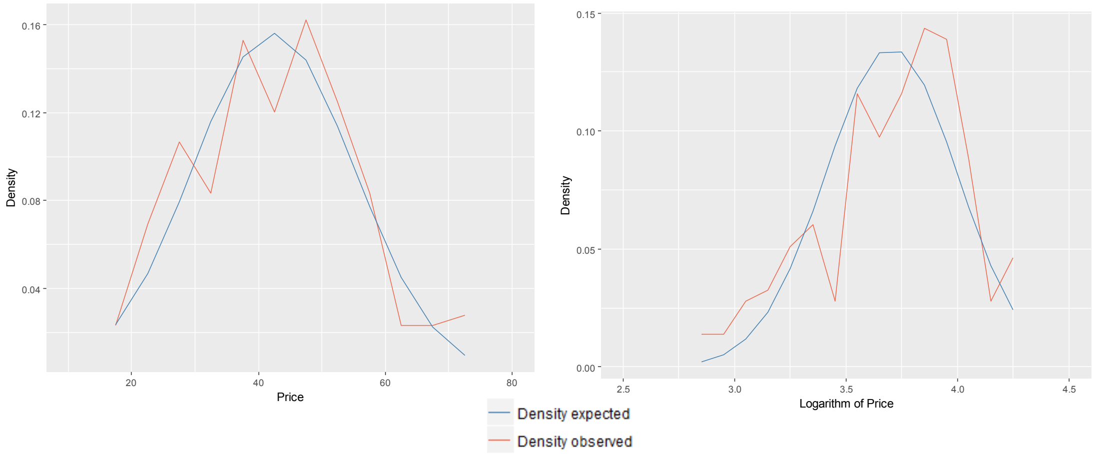

Fig. 12. Theoretical and observed density functions of the normal and lognormal distributions

The results from the Cramér-Von Mises goodness of fit test are:

	Normal	Lognormal
Mean (μ)	42,352	3,703
Standard deviation (σ)	12,700	0,293
Cramér-Von Mises ω^2	0,0633	0,2763
Cramér-Von Mises p-value	0,794	0,108

Table 4. Cramér-Von Mises results for monthly spot prices.

As it can be observed, both results are higher than the established significance level of α=0.05. This indicates that it is acceptable to admit the null hypotheses, which are that each sample behaves as a normal. To obtain a graphic test, the normal probability plots are shown in Fig. 13.

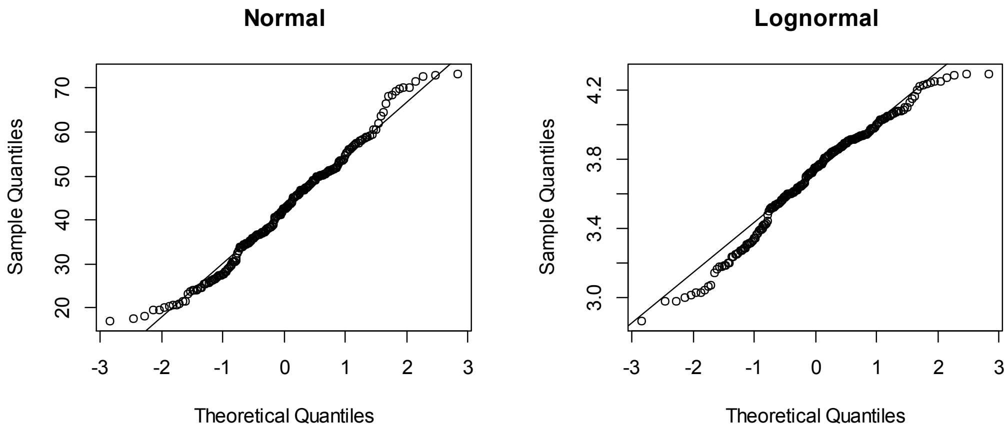

Fig. 13. Normal probability plot for the prices and their logarithms

The conclusions obtained from these graphic tests are equivalent to the ones obtained for the daily prices: the normal variable is light tailed and the lognormal is left-skewed.

The overall results of the study show that the sample behaves as a normal variable. The p-value obtained for this distribution is quite high (0,794), which imply that a small error is made when accepting the hypothesis. However, the p-value for the lognormal distribution is also high (0,108). Thus, it is found that the sample may also be assumed as lognormal variable, although with a higher error than when presuming it as a normal.

As it was mentioned on the previous section, the electricity prices are commonly assumed as lognormal variable to stabilize the variance. Therefore, many highly consolidated models base their calculations on the lognormality of the sample. This is the case of the Black-Sholes models. [5] The possibility of assuming the variable as a lognormal will highly facilitate the future steps of this project.

Shaping

The objective of this section is to characterize the shape of the price curve along the months of a year. This characterization will be developed similarly than when studying the shape of the spot's daily curve. Therefore, when referring to shape, it is meant the expected deviations of each month's price from the year average price. Therefore, an accurate characterization of the shape will permit to calculate the 12 monthly prices just from an estimation of the yearly price.

On the following image the ratio of the month's prices and the year average price is represented for the time window chosen.

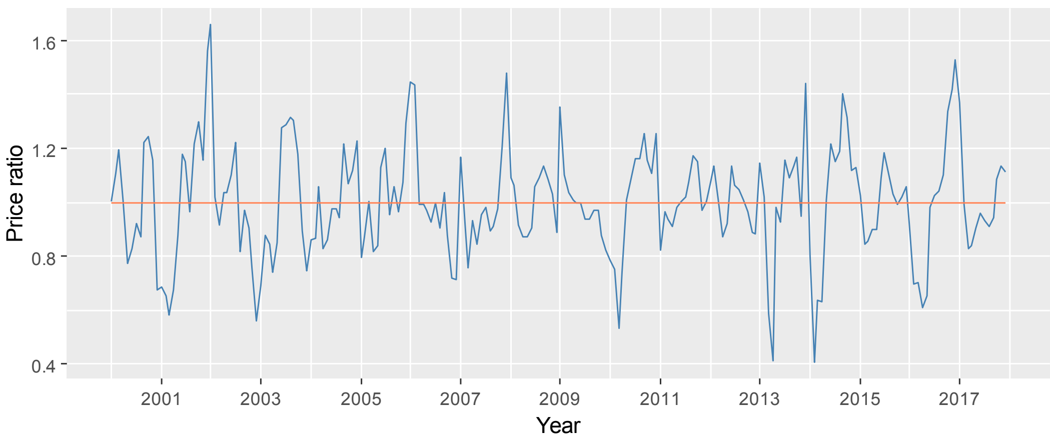

Fig. 14. Monthly price ratio with each year's average price.

Unlike the study of the shaping of the daily prices, in this case the data analyzed is much smaller: 12 prices per year versus 8760. Thus, there is no need to develop a software tool nor to group the months into clusters.

However, the steps followed to compute the shape are very similar. Firstly, the historical data is used to calculate the average deviations of each month. These averages will be assumed as the deviations of the year we want to forecast. It is worth remarking that when calculating the shaping for daily prices, the deviation was calculated as the absolute difference with the mean value (subtract). In this case, the deviation is the proportional difference (division).

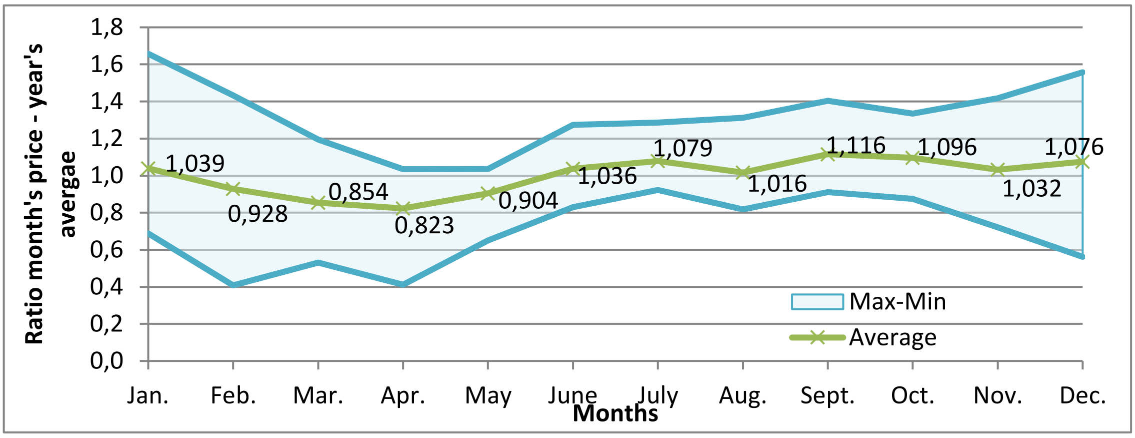

Fig. 15. Minimum, maximum and mean ratios month's price-year's average in the period 2000-2017

The time window of historical data used to calculate the deviations has great influence in the results. In the previous study it was found that the consideration of the data from the three previous year provided accurate results. However, in this case, a wider data frame is considered: all historical values since 2000.

This change is mainly due to the size of our sample: higher number of inputs is needed than just the ones from the three previous years to consider reliable the results. As a proof Fig. 16 plots the correlation between the real 2017's deviations and the calculated deviations' averages from the three previous years and since 2000 are compared.

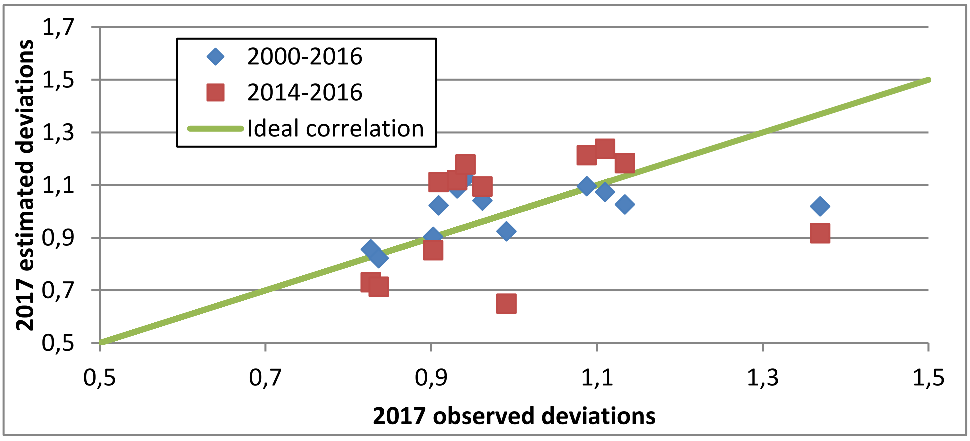

Fig. 16. Correlation with prices in 2017 of shaping with two different time frames.

The figure confirms the idea that a bigger time frame estimates better the shaping of the monthly prices curve for future years.

As this section ends, the study of the spot market concludes. On the following section, we will proceed in a similar way with prices obtained in the forward market.

1.2. Statistical analysis of the forward market price

For the study of the forward prices, the steps followed will be very similar to the ones taken when considering the spot prices. The study will begin by finding the probability distribution that best fits our series. Then, the volatility of the series will be characterized. However, the final step will not consider the shaping of the prices. While this parameter provides big help with spot market prices, as it allows estimating the hourly prices from a monthly price forecast (and similarly with the monthly prices and a yearly forecast), it will not be so revealing when considering forward prices.

Instead, the liquidity of the market will be considered. As it will be explained in the pertinent section, liquidity is a major factor affecting the prices of any market but, in the case of the forward market, it is not granted.

In the forward market there are several different products being traded simultaneously. The products differ from each other depending on the features of the contracts, as the delivery date or the duration. Some of the most common are the M+1 or Q+1, which make reference to the month and the quarter after, respectively. Additionally, depending on the type of load covered we find different products, as base load or peak loads contracts. Thus, we have a wide range of different products.

During the development of the study we will focus on the evolution of the product Y+1 baseload. This product refers to the base energy of the following year. The study was developed for this product for several reasons. Firstly, it is one of the most common products of the forward market, so it is liquid enough. Secondly, it has a period long enough to allow us studying its trend. As it refers to the next year, there is a whole year of evolution of the price of this product. Other products, as the W+1 only have a week of evolution, which does not provide us enough data. Finally, it can be considered as a representative of the evolution of the other products' prices.

As in the previous study, before starting the analysis we must delimit and treat the sample. In this way, the range of data is much shorter as the forward market was established in July 2006.

As it was explained above, it is important to pick complete years, to avoid the influence on the overall results of the seasonality. Thus, the data from the 11 years between 2007 and 2017 is considered. Unlike the spot market, the forward market does not update during weekends, so only the data from working days will be considered. On Fig. 17 these prices are represented.

Fig. 17. Y+1 prices from 2007 to 2017

In the figure we observe that prices frequently suffer a big jump when going from one year to the next one. This is caused by the fact that they are actually valuing two different products. For instance, on December 31st of 2015 the product priced was the energy during 2016 and it was valuated around 47 €/MWh. However, on January 1st of 2016, the product was 2017's energy, and it was valuated around 42 €/MWh. To handle these discontinuities, the prices are normalized by dividing them by each year's average.

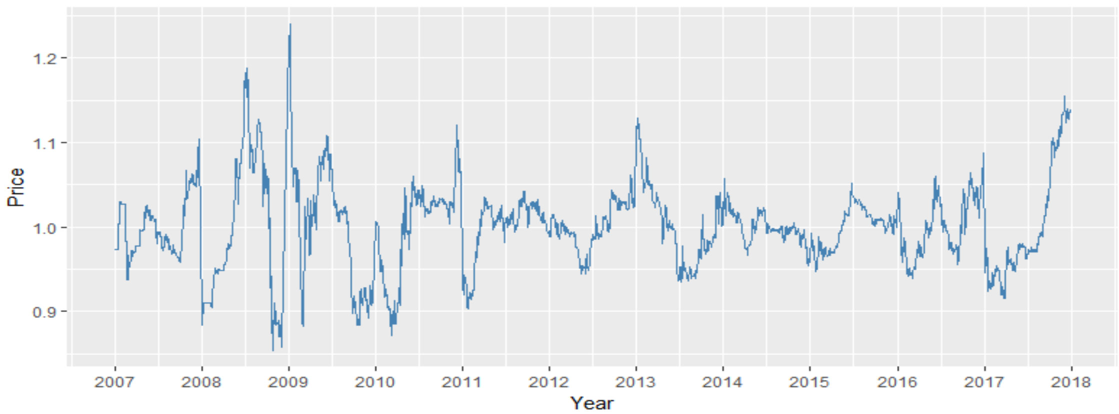

Fig. 18. Y+1 normalized prices

Thus, the sample studied is the formed by the normalized prices. It is composed of 2734 different values.

Statistical distribution model: goodness of fit test

In this chapter, as in previous sections, the normality and lognormality of these prices will be discussed. The procedure will be equivalent to the one developed for the day-ahead market prices.

Firstly, two different samples are created: the prices and its logarithms. Then, their theoretical normal distributions (N(μ,σ)) are represented and contrasted to the sample's histogram:

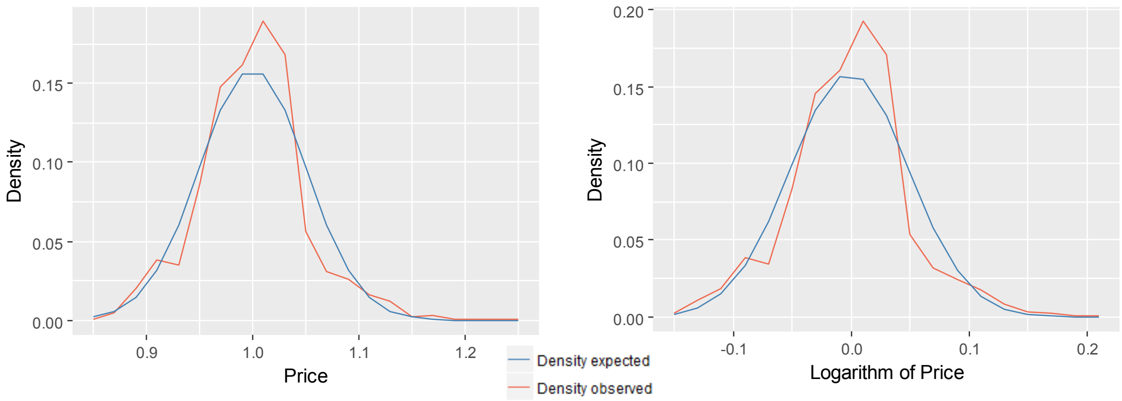

Fig. 19. Theoretical and observed density functions of the normal and lognormal distributions

The Cramér-Von Mises goodness of fit test is ran through RStudio to compute the similarity of the two samples to a normal distribution. The results obtained are:

	Normal	**Lognormal**
Mean (μ)	1,00	- 0,001
Standard deviation (σ)	0,0499	0,0499
Cramér-Von Mises ω^2	2,657	2,713
Cramér-Von Mises p-value	$4{,}328 \cdot 10^{-7}$	$3{,}236 \cdot 10^{-7}$

Table 5. Cramér-Von Mises results for Y+1 2007-2017.

Lastly, the normal probability plot is done for the samples, to obtain an illustrative result of the goodness of fit test. The Fig. 20 shows the two plots.

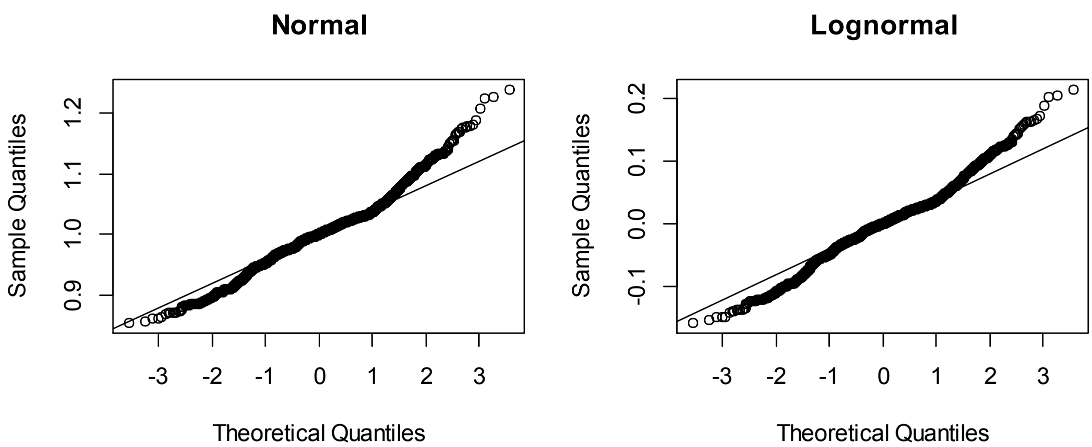

Fig. 20. Normal probability plot for the prices and their logarithms

Both in the results of the two goodness of fit tests as in the comparison of the theoretical and observed distributions, it can be appreciated that the two different samples (prices and their logarithms) fit likewise into a normal. The main reason is that the two samples are quite similar. As shown in Table 5, the price sample has a mean value μ=1 (as it was normalized in the previous section) and the logarithmic sample has μ=0. However, their standards deviations are basically the same (σ≈0,05). The sample of logarithms is similar to the price sample but translated.

As an evidence of this, in the following chart the normalized prices are represented and compared to the sample of logarithms. To facilitate the comparison, the chart also includes the representation of the logarithms translated one unit upwards.

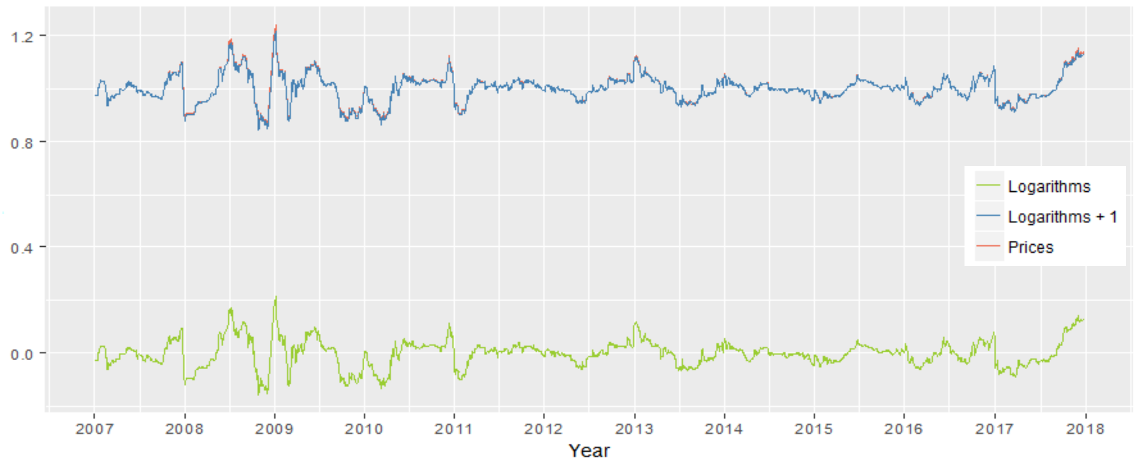

Fig. 21. Y+1 normalized prices and logarithms

On the other hand, the p-values computed in the Cramér-Von Mises test are quite low, indicating that neither of the samples completely adjusts to a normal distribution. Finally, it is concluded that, although accepting an error, it is equivalent whether we assume the prices as a normal or a lognormal variable.

Finally, it is worth mentioning that, although the normalized values for the selected time frame do not behave nor as normal nor a lognormal, when taking the real sample, results are different. As the time frame is closer to the current data, the prices fit better under the discussed distributions. For instance, the results for the 2015-2017 period are:

	Normal	Lognormal
Mean (μ)	45.08	3.806
Standard deviation (σ)	2.876	0.064
Cramér-Von Mises ω^2	0.388	0.266
Cramér-Von Mises p-value	0.0778	0.1696

Table 6. Cramér-Von Mises results for Y+1 [€/MWh] for 2015-2017.

These results present a p-value higher than the significance level for both distributions. However, the tests show that, during this period, the prices statistical distribution corresponds rather to a lognormal than to a normal.

Volatility

The volatility of the forward prices was calculated following the formulas explained when this characteristic was studied for the day-ahead market, in section *1.1.1 Spot daily prices*. The results obtained are shown in the following table and figure:

Monthly volatility	Jan.	Feb.	Mar.	Apr.	May	June	July	Aug.	Sept.	Oct.	Nov.	Dec.	Mean
2007	4,7%	7,2%	2,2%	2,6%	3,5%	3,4%	2,3%	2,4%	1,7%	5,5%	2,8%	4,7%	**3,3%**
2008	28,4%	2,7%	0,8%	1,9%	5,5%	5,5%	7,0%	5,3%	7,6%	10,8%	5,5%	6,6%	**5,2%**
2009	45,0%	9,4%	12,9%	7,9%	5,6%	5,5%	3,5%	1,7%	5,9%	4,8%	3,8%	3,5%	**6,1%**
2010	13,2%	3,4%	7,0%	8,7%	7,4%	4,9%	3,6%	1,7%	1,6%	0,7%	4,9%	6,7%	**4,1%**
2011	16,8%	3,7%	4,0%	2,3%	3,0%	3,7%	2,2%	2,8%	2,3%	2,8%	2,8%	1,5%	**3,2%**
2012	2,9%	2,5%	1,8%	1,5%	2,1%	3,3%	3,6%	1,9%	4,3%	1,9%	1,6%	4,0%	**2,4%**
2013	11,1%	3,9%	1,7%	4,4%	2,6%	2,2%	5,2%	1,9%	3,2%	5,4%	1,6%	8,7%	**3,6%**
2014	7,6%	1,9%	1,7%	1,7%	3,2%	1,9%	1,8%	0,8%	1,9%	1,7%	2,6%	2,7%	**2,1%**
2015	5,6%	3,3%	1,8%	1,3%	1,3%	2,0%	1,0%	1,1%	1,0%	1,1%	1,6%	2,4%	**1,7%**
2016	7,4%	3,7%	2,2%	4,2%	3,0%	4,8%	3,6%	2,0%	3,9%	6,0%	3,3%	3,5%	**3,7%**
2017	15,5%	2,5%	3,2%	3,7%	1,9%	1,3%	1,1%	1,3%	2,6%	3,4%	3,0%	3,6%	**2,7%**
Mean	**10,7%**	**3,6%**	**2,6%**	**3,0%**	**3,2%**	**3,2%**	**2,7%**	**1,9%**	**2,7%**	**3,1%**	**2,8%**	**3,9%**	**3,3%**

Table 7. Forward prices' monthly volatility.

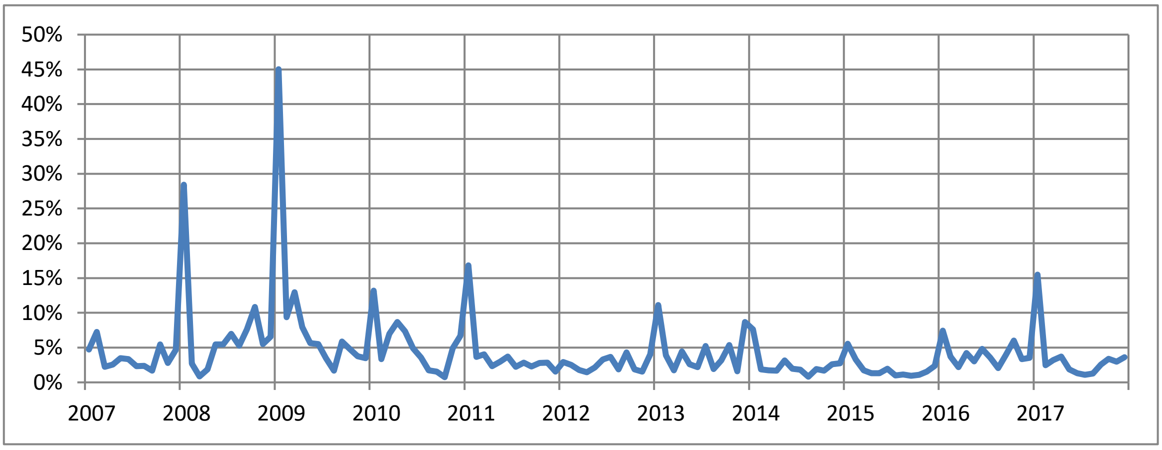

Fig. 22. Monthly volatility of forward prices

As shown, the volatility of January is remarkably higher than for the rest of the months. This is caused by the already explained effect on prices of the product change from one year from another. Thus, these peaks should not be considered when studying the volatility of the forward prices, as they are not representative of the evolution of a product's price over time.

In any case, the calculated mean volatility during these 11 years is over 3,3 %. This value is quite low, especially if we compare it to the spot market one, which was around 116 %. This is understandable as, although the spot market price influences the forward prices, it is not affected by all the daily fluctuations of the spot. The spot rather affects the general trend. Moreover, in this section the volatility of a certain product (Y+1) is studied, not the whole volatility of the forward market. In following sections, a wider view of the volatility of this market will be given.

Liquidity

Liquidity is a key factor in order to have a well-functioning market. It implies that the volumes traded in a certain market are sufficient to ensure that a sole agent's action will not affect the prices. This means that all agents are price-takers. Liquidity, therefore, is a necessary characteristic to have a competitive market. However, in forward electricity markets, a reasonable liquidity level is commonly not achieved. Consequently, prices are extremely affected by certain market actions.

The liquidity of a certain electricity market depends on many different factors, as transparency or costs of participation. Market structure is also one of the main variables: the higher the number of agents participating in the spot market, the higher the needs to trade in the forward market. Likewise, the cross-border contestability highly affects the liquidity, as it opens the doors to participants from other systems.

There is not a unique way to measure liquidity. In the case of forward market's liquidity, several indexes can be used as the volume of transactions or churn factors. The latter is one of the most common indexes and is defined as the ratio of energy traded in the forward markets to total physical consumption. There is not a well-defined churn rate threshold from which the market can be consider liquid, however, it is usually set at a range between 3 and 10 times the demand. [14]

The Fig. 23 represents the energy traded in the OTC market (registered in BME Clearing, in OMI Clear and not registered at all) and in the organized markets of OMIP and EEX, which are several different platforms for forward trading.[15]

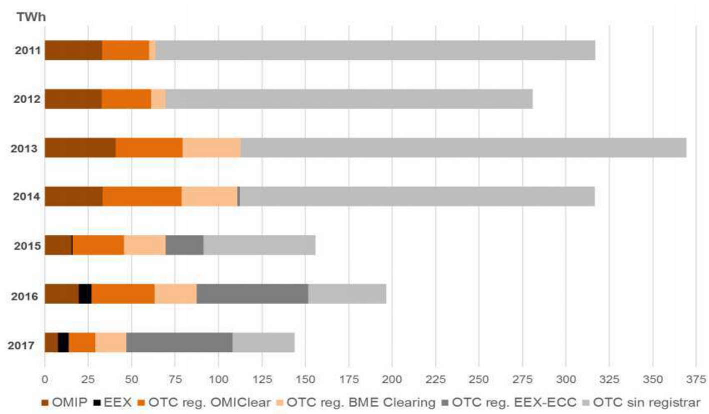

Fig. 23. Trading evolution [TWh] in different platforms. Source: CNMC

It can be seen that majority of long term transactions have been historically effectuated through non-registered OTC contracts. However, their details are not public, so they have low accessibility. Thus, we will consider for this study the contracts closed through OMIP, since, as the public forward market, it publishes openly the results obtained.

Thus, the energy traded in the continuous market of the MIBEL futures market managed by OMIP during 2017 was 7,7 TWh, which was significantly lower than the volume traded during

2016, which reached 19,5 TWh. These figures are translated into a 3,1% and a 9,9% of the total peninsular demand of 2017 (252 TWh) and 2016 (196,5 TWh), respectively.

In general, the volume transacted in OMIP has had a decreasing trend since 2013, which can be explained by several drivers. On the one hand, renewable installed capacity has acquired more weight on the energy mix. These generators are not incentivized to trade in the forward market.[16] On the other hand, the economic crisis reduced overall demand.

The following images represent the volume negotiated in the OTC, OMIP and EEX markets depending on the time lag (in days) between the signature of the contract and the beginning of its exercise. Although it shows the information about a two particular months, December of 2017 and January 2018, the charts can be used to understand the general trend.

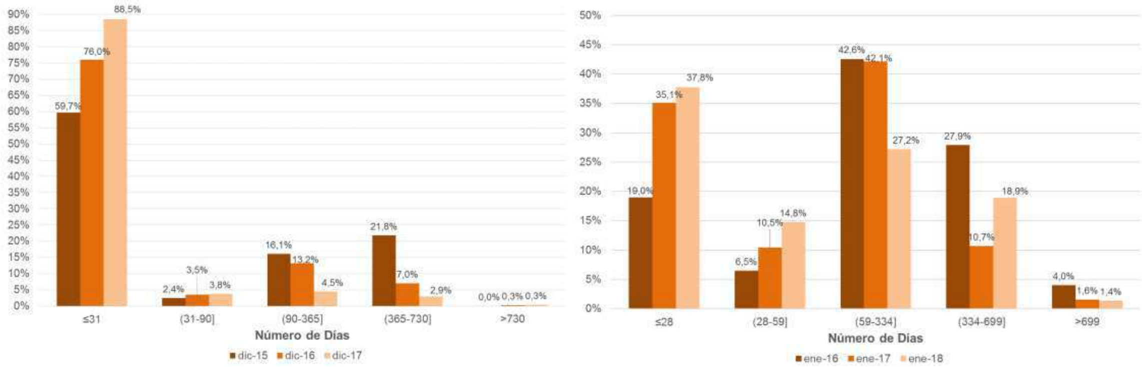

Fig. 24. Volume negotiated in the OTC, OMIP and EEX markets during December 2017 and January 2018 depending on the number of days since the negotiations and the maturity date. Source: CNMC

Thus, the market is focused in contracts with very short maturity, of one month or less. This is usually covered by the weekly and monthly products, as W+1 and M+1. Products for the next year, as the Y+1, are usually less liquid but still concentrate a big percentage of the market activity. However, for longer lags until the maturity there is almost no liquidity.

The following table also resumes the products with higher liquidity in the MIBEL futures market between 2010 and 2015.

Exchange	Countries covered	Product types	Product coverage	Product time-frames	Annual volumes GWh	% of country demand[1]	Key terms
OMIP (MIBEL) Source: MIBEL Contracts (>1 June 2010) (http://www.omip.pt/Downloads/DerivadosdeElectricidade/tabid/104/language/en-US/Default.aspx)	Spain	Financial and physical futures	Baseload	D, WE, M, Q, Y	26,076	10.7%	Participation fee: €10,000 Monthly fee: €125-€833 Standard fee: €0.0025-€0.0075/MWh
			Peakload	Daily	0	0.0%	
				Weekly	0	0.0%	
				Monthly	0	0.0%	
				Quarterly	8	0.0%	
				Yearly	0	0.0%	
		Forwards OTC	Baseload	W, M, Q, Y	216	0.1%	
		Swaps OTC	Baseload	D, WE, W, M, Q, Y	40,765	16.7%	
		Call options	Baseload	M, Q, Y	166	0.1%	
		Put options	Baseload	M, Q, Y	111	0.1%	
		Total			67,341	27.6%	

Table 8. Summary information on MIBEL forward exchange platforms. Source: "*European Electricity Forward Markets and Hedging Products– State of Play and Elements for Monitoring*", ECA 2015

Finally, the Spanish forward market can be compared with other European markets through the following chart:

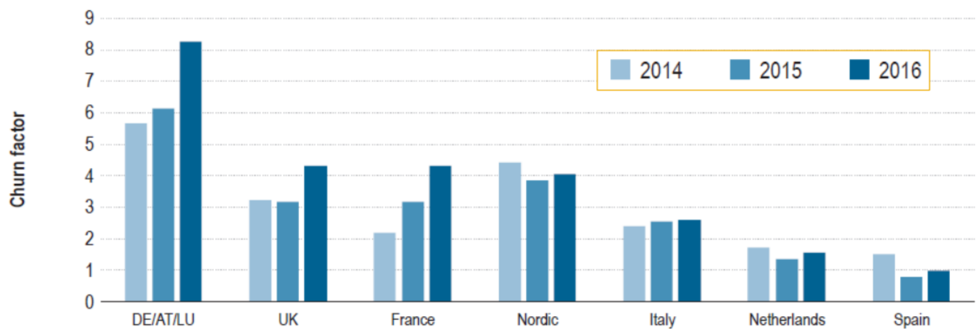

Fig. 25. Churn factors in major European forward markets during 2014 to 2016. Source: ACER

As explained, the churn factors are the ratio of energy traded in the forward markets to total physical consumption. Based on the thresholds established (3-10) it can be concluded that in the Spanish forward market, as in most European countries, the liquidity is quite limited. As an exception we find the German, British, French and Nordic markets.[17]

1.3. Time series study

During this section the prices will be considered and studied as time series, with the purpose detecting and defining any chronological patterns they might have. If there are any, its characterization will facilitate the future forecasts, increasing the accuracy of the results.

For the development of the section the literature "*Forecasting: Principles and Practice*", R. J. Hyndman y G. Athanasopoulos[18] will be used to support the steps, which will be developed in the software RStudio.

The series of prices studied will be the ones considered in the previous sections: day-ahead prices, both daily and monthly, and forward prices for the product Y+1. The analysis will be done following the same steps for the three series. The sought final result is an autoregressive integrated moving average (ARIMA) model to fit the series. The ARIMA models are one of the most common approaches for time series forecasting. They combine autoregressive (AR) and moving average (MA) models. Additionally, they introduce an integration (I), which is usually used to stabilize the mean of a non-stationary variable.

An AR model forecasts the variable using a linear combination of consecutive past values. The order of the model indicates the number of previous data used to predict a certain value. The MA models, instead of using past values, uses a linear combination of past errors from a regression forecast. As in the AR, the MA order references the number of previous data considered in the linear combination. Finally, the integration substitutes the values of the series by their differences with previous values. The order means the number of times the data has been differentiated.

The ARIMA models are commonly expressed as ARIMA(p,d,q), where the numbers p, d and q indicate the orders of the AR, integration and MA, respectively. In case the time series was seasonal, the ARIMA model would be defined as ARIMA(p,d,q)$(P,D,Q)_m$, where the second part references the seasonal component of the model. Thus, P, D and Q are the orders of the AR, integration and MA for the seasonal part. m is the frequency of the time series, which is defined as the number of observations in each seasonal repetition.

The ARIMAs of the series will be determined using the function *arima()* of the package *forecast*. The goodness of fit of the models will be expressed by means of several indexes related with the residuals. These are the differences between the real and the model's values. Firstly, the MAE (mean absolute error) is defined as the average of residuals. The RMSE (root mean squared error) computes the square root of the average of all the obtained squared errors. Thus, if a model minimizes the MAE, it forecasts the median while, if it focuses on the RMSE, it forecasts better the mean. Finally, the MAPE (mean absolute percentage error) will be considered. It is very similar to the MAE although, instead of making the average of the absolute residuals, it considers the residuals in percentage. The MAPE main disadvantage is that it will be infinite if there are any null real values.

When checking the goodness of fit of a model, it must be checked that the residuals are white noise. If they are not, the model is not considering relevant information, leaving it on the residuals. To analyze this, a portmanteau test will be used. In particular, the Ljung-Box test will be ran through the function *checkresiduals()*, from the package *forecast*. It will provide a p-value that, as in the previous section, should be higher than a significance level (α=0.05) in order to accept that the residuals behave as white noise.

1.3.1. Daily Spot Prices

The daily spot prices from 2000 to 2017 are represented below. The sample was cleaned from outliers. The frequency of the series is defined at 7, as they are daily values that will have a weekly pattern. Its autocorrelation function (ACF) and partial autocorrelation function (PACF) are also represented. These two graphs are very helpful when considering the ARIMA.

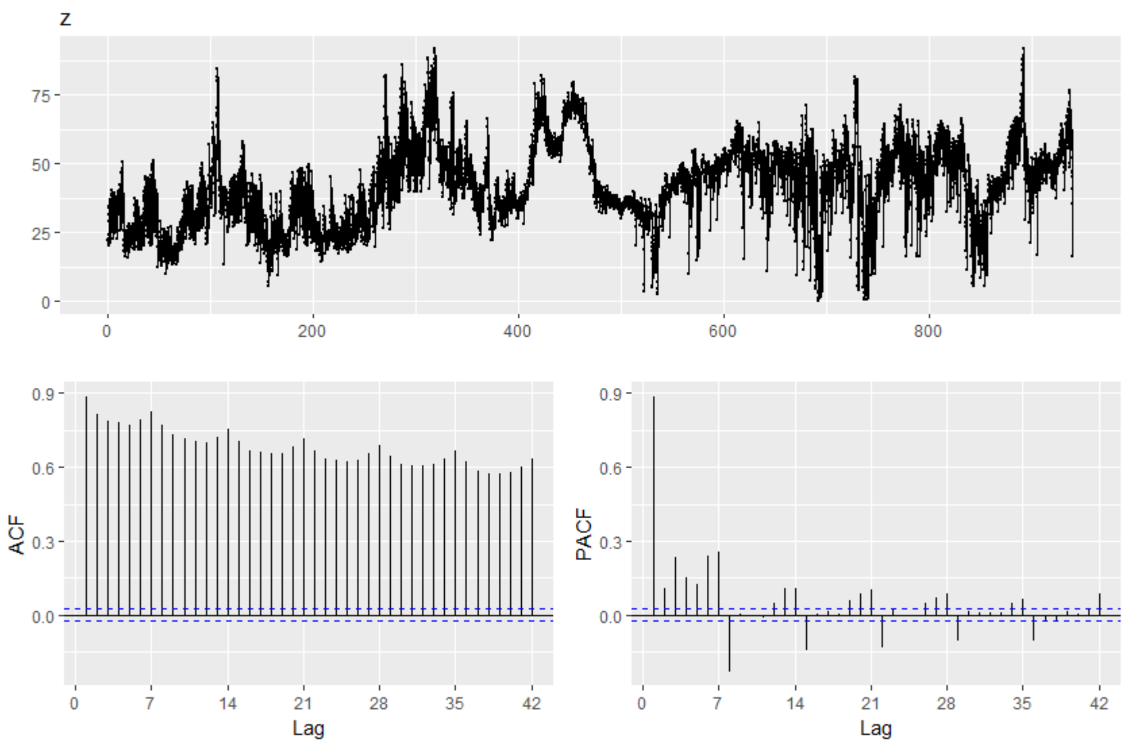

Fig. 26. Spot daily prices and their autocorrelation and partial autocorrelation functions.

The model fitting this series is an ARIMA(3,1,2)(0,1,2)$_7$. Thus, the series is fitted firstly differencing generally and seasonally, and then through a combination of an AR model of order 3 and a second order AM. It also includes a seasonal AM of second order.

Results of ARIMA	
MAE	3.590
MAPE	Inf
RMSE	5.221
Ljung-Box test p-value	0.383

Table 9. Goodness of fit of ARIMA model to daily spot prices

In the Table 9 it can be seen that the errors are quite low. The MAPE index has an infinite value but it should not be consider, as it is caused by some null prices. The p-value of the residual test is higher than our significance level, so the hypothesis of the residuals as white noise can be accepted. On the following image, the real values and the model's output can be compared.

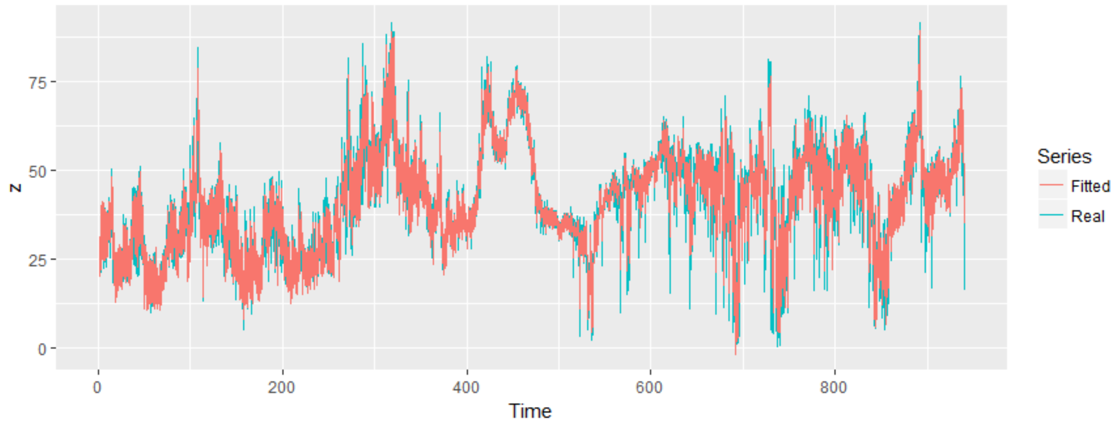

Fig. 27. Real and fitted time series for daily spot prices.

1.3.2. Monthly Spot Prices

The monthly spot prices from 2000 to 2017 are represented below, together with its ACF and PACF. The sample presented no outliers, so there was no need for cleaning it. Now, as we are dealing with monthly values, the frequency of the time series is 12.

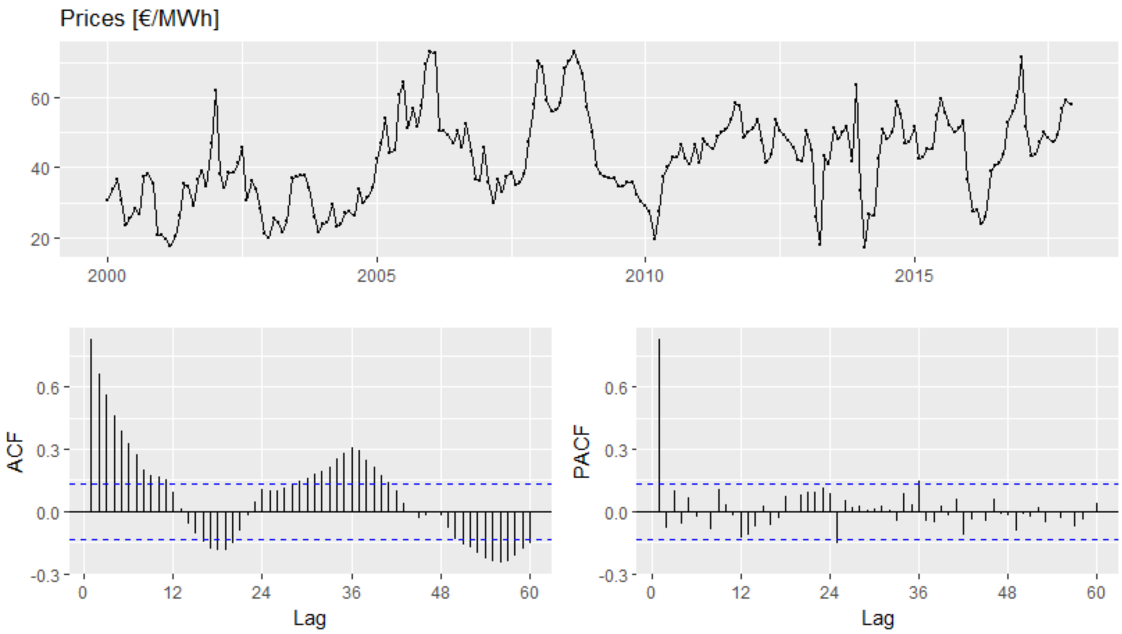

Fig. 28. Monthly prices and their autocorrelation and partial autocorrelation functions.

The most optimal model fitting the series of monthly prices is an ARIMA(0,1,2)(0,0,2)$_{12}$. A general first order differentiation was used to stabilize the mean and make the series stationary. Then it is found that prices can be model through a combination of a MA of order 2, general and seasonal.

Results of ARIMA	
MAE	5.063
MAPE	13.023
RMSE	6.913
Ljung-Box test p-value	0.264

Table 10. Goodness of fit of ARIMA model to monthly spot prices

In the Table 10. Goodness of fit of ARIMA model to monthly spot pricesTable 9 it can be seen that the errors are quite low. The p-value of the residual test is 0.264, which is higher than our significance level, so the residuals behave as white noise. The following chart represents the model's output and the observed values. They are represented slightly lagged to facilitate the comparison.

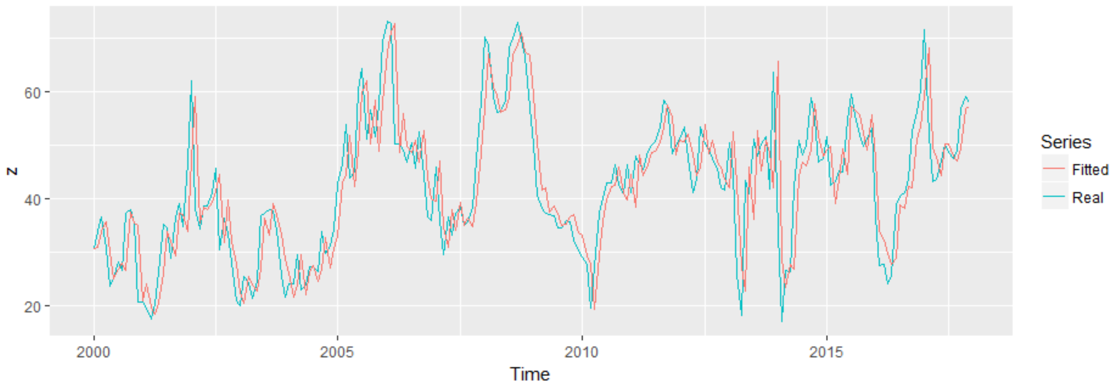

Fig. 29. Real and fitted time series for monthly spot prices

1.3.3. Forward

Finally, an ARIMA model will be designed to fit forward prices. To this purpose, the prices considered will be the ones chosen during the statistical study of the previous section: the Y+1 baseload prices. The time frame, however, will be modified, taking prices from 2015 to 2017, as in Table 6. The frequency of the series is 5, since they are daily prices that only quote on working days. The following image contains the prices, ACF and PACF representation.

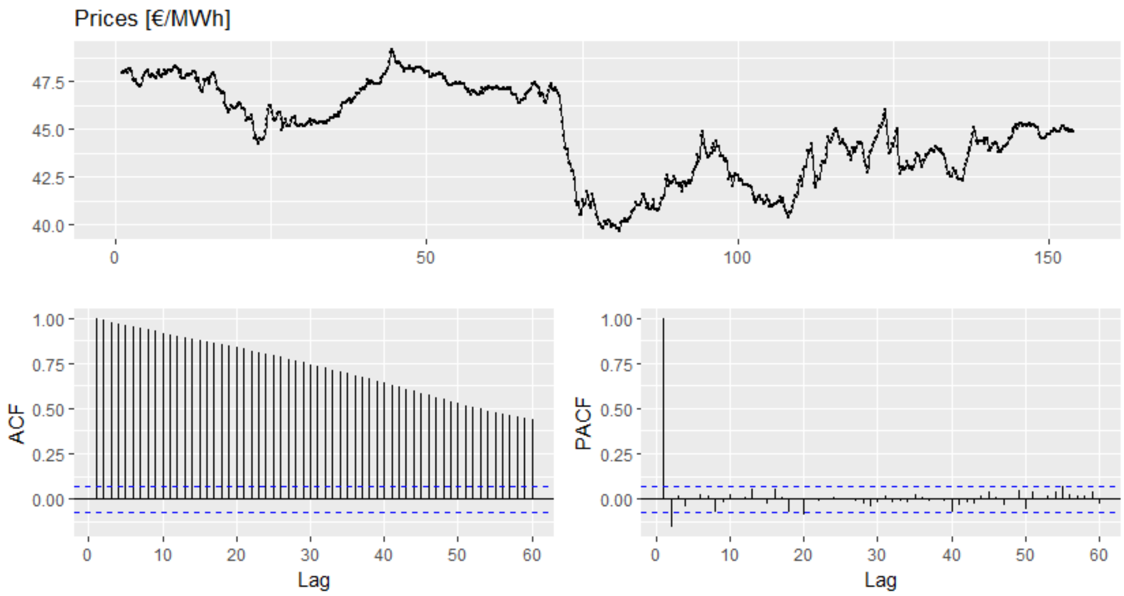

Fig. 30. Y+1 prices and their autocorrelation and partial autocorrelation functions.

The resulting ARIMA model fitting the series is an ARIMA(0,1,1). Thus, the series, previously differentiated, is described by a MA of first order. Compared to the other models, this ARIMA lacks of seasonal component. This means that the data does not shows any pattern for any given frequency, contrary to, for instance, the daily series that has a weekly pattern (frequency 7).

Results of ARIMA	
MAE	0.161
MAPE	0.366
RMSE	0.236
Ljung-Box test p-value	0.265

Table 11. Goodness of fit of ARIMA model to Y+1 forward prices

The goodness of the ARIMA model is shown in the previous table. The errors MAE, MAPE and RMSE are low. Additionally, the p-value for the residuals is above the threshold set by the level of significance, confirming it is acceptable to consider the residuals as white noise.

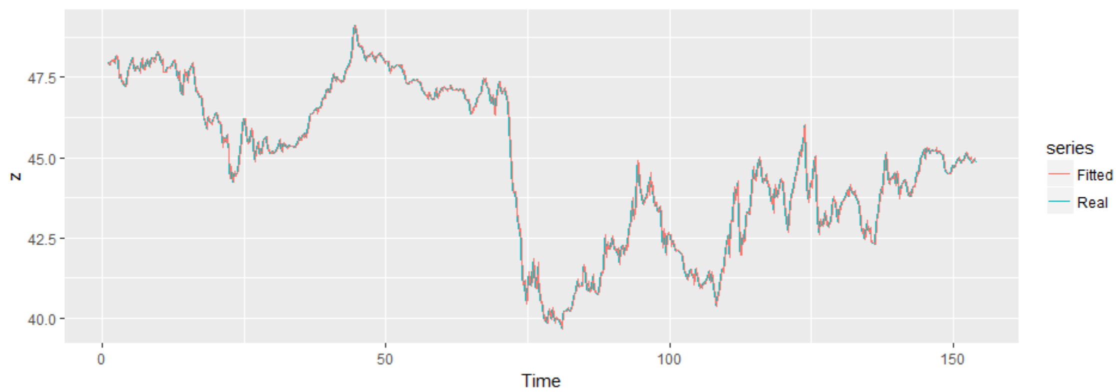

Fig. 31. Real and fitted time series for Y+1 forward prices

As a conclusion, it can be observed that the prices have a strong moving average component. It was remarkable that, due to the features of the product Y+1, it lacked seasonal component.

The findings developed in this section may be use in future projects to simulate the market prices and, afterwards, calculate the value of an electricity derivative. However, due to the scope limitations of this project, they will not be further developed.

2. Monthly electricity price simulations

In the previous chapter the spot and forward prices were analyzed, defining their characteristics. The conclusions obtained will be the bases to develop forecasts of future price values. During this section, the methods to do so will be explained. The final purpose is to have an accurate prediction of the future prices to introduce them in the electricity derivative valuation models.

To simulate the prices evolution, quantitative models will be used. As mentioned in CHAPTER 2: State of the art, these models are based on a statistical analysis of the historical data. Compared to the fundamental models, which compute the prices from explaining variables, as demand, temperature or fuel costs, quantitative models require less information, as they only consider the past values of the prices. Thus, these models are preferable when there is no access to perfect information.

Quantitative models divide the prices in two parts: a deterministic part, which will be known, and a stochastic one. Consequently, the results of these models will be an expected distribution of the prices.

As mentioned, the spot and forward prices will be simulated. Due to their varied characteristics, different models will be developed for each one of them.

Both types of models will use historical and future monthly prices. The reason to choose the monthly aggregation is that in section 1.1.1 Spot daily prices, a shaping model was developed. This tool computed hourly prices from the monthly averages (and historical hourly data). Thus, lower aggregation prices can be calculated by means of an accurate estimation of monthly averages.

2.1. Spot price models

To forecast the future prices from the spot market, two different models will be studied. Each one will be described, detailing their advantages and weaknesses.

It must be recalled that the result of the goodness of fit test for monthly values demonstrated that prices behave with a normal distribution, but they could also be fitted into a lognormal. Thus, applying to prices some transformations typical of lognormal variables is suitable.

2.1.1. Brownian motion model

The Brownian motion is a stochastic process. In particular, it is a Markov process, that is a stochastic process in which the only relevant variable to predict the future state is the current state of the process at the time. Therefore, the expected value of a random variable x_t at time t, depends only on the previous value x_{t-1}. This means that the historic values of the process are irrelevant.

Going further, a Wiener process is a special type of Markov stochastic process. A variable x_t follows a Wiener process if:

$$x_t - x_{t-1} = \Delta x = \xi_t \sqrt{\Delta t}$$

Where ξ_t is a random variable, with independent values for each Δt and that has a normal probability distribution N(0,1). Thus, if x is a variable under a Wiener process, the Δx for steps Δt, are distributed as a normal N(0, Δt).

Having these concepts in mind, the geometric Brownian motion may be described as a stochastic process defined in terms of a Wiener process, as follows:

$$\frac{x_t - x_{t-1}}{x_{t-1}} = \frac{\Delta x_t}{x_{t-1}} = \mu\Delta t + \sigma\Delta z$$

where $\Delta z = \xi_t\sqrt{\Delta t}$ is a Wiener process. μ is the expected increment of x per time step. The term σΔz introduces distortion from the tendency defined by μΔt. Thus, the distortion will be σ times the Wiener process Δz. σ represents the variance or volatility.

If the price value at a certain time t is designated as S_t, it will be defined as:

$$\frac{\Delta S_t}{S_{t-1}} \simeq \ln\frac{S_t}{S_{t-1}} \simeq \mu\Delta t + \sigma\Delta z$$

Recalling that the price behaves as a lognormal, when it is simulated by this model, it is found that the price expected value grows indefinitely with time as:

$$E(S(t)) = S_0 e^{\mu t}$$

The variance also increases indefinitely over time, growing therefore, the uncertainty with σ and time:

$$Var[ln(S(t))] = \sigma^2 t$$

Thus, although the geometric Brownian motion model is a very simple approach to forecast future values, it presents several limitations. For instance, if the simulations are computed over a long period of time, the prices estimated will be within a wide range, increasing the uncertainty. However, real prices in electricity markets do not grow indefinitely with time. In fact, in the long and mid-term, the average of prices tends to mean value, and the uncertainty does not tend to infinity.

Therefore, the Brownian model provides good results for forecasting in the short-term products with low uncertainty. However, for longer terms, this model must be modified as explained in the next section.

2.1.2. Mean reversion model

To address the drawbacks of the Brownian motion model, the mean reversion model introduces a value to which the prices tend in the long term, $S_{LT} = e^{\mu}$. The process is defined as:

$$\frac{\Delta S_t}{S_{t-1}} = \alpha(\mu - \ln(S_{t-1}))\Delta t + \sigma\Delta z$$

Where α is the speed to which the prices reverse to the long-term value. As the price S behaves as a lognormal, we find that the expected value and variance are:

$$E[\ln(S(t))] = \mu - \frac{\sigma^2}{4\alpha}(1 - e^{-2\alpha t}) + (\ln(S_0) - \mu)e^{-\alpha t}$$

$$Var[\ln(S(t))] = \frac{\sigma^2}{2\alpha}(1 - e^{-2\alpha t})$$

The mean reversion model provides better results for volatile products and longer periods of study, as it establishes a price value in the long term, delimiting the uncertainty and avoiding its infinite growth over time.

Many other corrections may be introduced to these formulas to correct their failures and provide better results. For instance, the long-term price value and volatility are considered constants. They may be substituted by time dependent values, to consider the seasonal patterns or long-term trends. However, for the sake of simplicity the study will focus on these two models as representatives of the spot forecast methods.

2.2. Forward curve models

When considering spot markets, the output is one price: the price of electricity in that time (hour, day …). However, forward markets consider several products. One of the main distinctive features of these products is the delivery time. The forward price curve provides the price of these products as a function of their delivery time. It goes from close delivery times (as W+1) to higher ones (as Y+2).

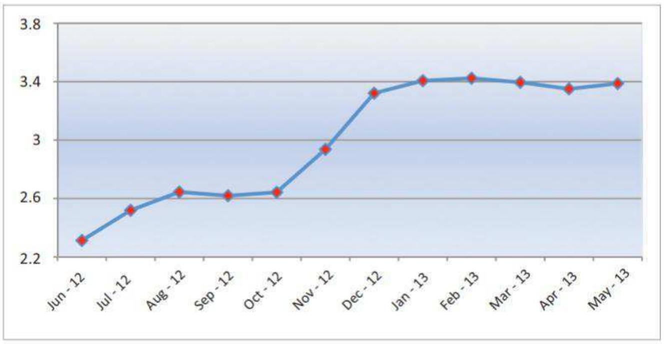

Fig. 32. A commodity's forward curve for several future months. [19]

The values of the forward curve may be considered as the market's best forecast of the value of a product in the considered delivery time. However, other aspects must be considered, as liquidity, which may cause that the forward market is not reflective of the future scenario.

For each working day, the market gives a different value to future electricity. This is reflected by a different value of the forward products depending on their delivery dates. This variation in the quotations produces a new different forward curve per day. The forward curve models must compute the evolution over time of the whole curve.

2.2.1. Monte Carlo model

The objective of the Monte Carlo method is to compute the forecast by creating several scenarios according with the products' volatility. Per scenario i, the forward price F at time t of a product with delivery time in month T is:

$$F_i(t,T) = F(0,T) \cdot e^{k_i(T-t)-\frac{1}{2}\cdot\sigma(T-t)^2}$$

The Monte Carlo method starts computing the volatility σ(T-t). It is seen that the volatility of the products is higher as the lag until the delivery time T is smaller.

Computing the volatility of the different products as seen in section 1.1.1 Spot daily prices: Volatility and plotting them as a function of the time until delivery (T-t), we find that they may be adjusted with an exponential trend line. In the following chart, the annualized volatility of electricity products with times until delivery (T-t) from 1 to 12 months and their trend line is shown:

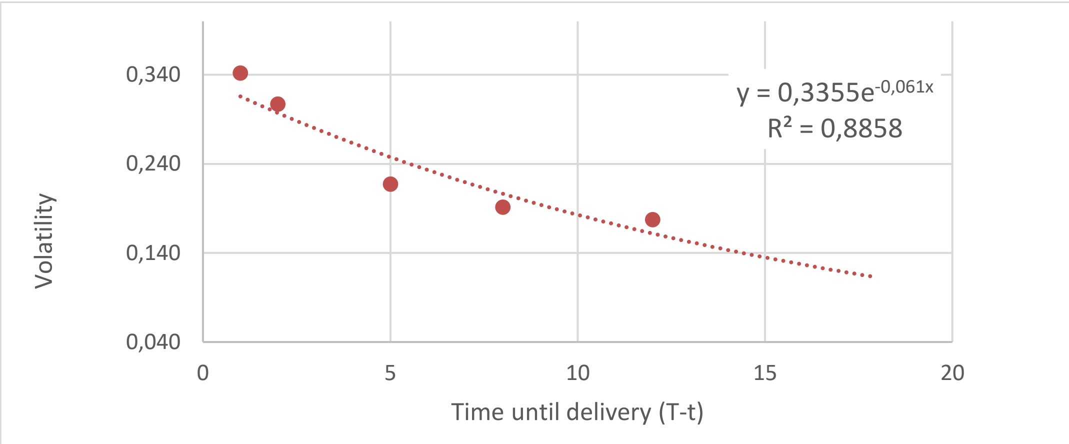

Fig. 33. Calculated volatility of forward products

Therefore, the volatility is considered as:

$$\sigma(T-t) = \sigma \cdot e^{-\alpha\cdot(T-t)}$$

The Monte Carlo's approach is based on the creation of several scenarios to reflect all the possible future situations and their probability. To create these scenarios, the computed curve σ(T-t) will be the multiplied by i random values to create several scenarios:

$$k_i(T-t) = \xi_i \cdot \sigma(T-t)$$

Where ξ is a random variable with normal probability distribution N(0,1).

The Monte Carlo forward models provides a simple method to compute the expected forward curve and its distribution.

3. Electricity derivatives

[4], [20]

Before continuing with the project, the main types of electricity derivatives will be explained. The comprehension of the characteristics of these products is essential in order to develop an accurate valuation.

Firstly, a derivative is a financial product between two or more counterparts whose value depends on the future value of an underlying asset. This underlying asset includes any kind of good, as a commodity, stocks or a currency. During this project, the underlying considered is the electricity price.

There are four main types of derivative products: futures, forward, options and swaps. Their definition, main characteristics will be detailed below.

3.1. Futures and forwards

Future and forward contracts are agreements between a buyer and a seller to exchange an asset for a certain price in a certain time in the future.

The difference among them is that, while forwards are private bilateral contracts, futures are transactions closed in a market. Thus, futures have public prices. Although forward contracts are bilateral, it is frequent to close them through a public clearing house, to reduce credit risk.

To increase liquidity and reduce transaction costs, futures are usually standardized contracts (quantity, delivery times, duration…). Forwards, however, are more flexible, as they are directly design by the counterparts with the aim supplying their needs. In contrast, they imply higher transaction costs and credits risk.

Although both types of contracts can be physical or financial, it is frequent that forwards imply the physical delivery, while futures are mainly financial.

3.2. Options

An option is a contract that provides the right to buy or sell a certain quantity of an asset for a given price during a certain period of time. Unlike, futures and forward contracts, options trade the right to sell or buy, but not the obligation. This right is acquired by a certain amount of money, called the premium. The price at which the asset is bought or sold is the strike price.

There are several types of options attending different design parameters. For instance, attending on whether the option gives the right to buy or to sell the asset. Thus, call options give the right to buy the asset, while put options give the right to buy it.

Regarding the delivery dates, European options must be distinguished from American options. The former gives the right to exercise the option only on the expiration date. In contrast, the American options can be exercised throughout a period, anytime until the expiration date.

A common hedging strategy is the denominated collar. A collar consists in the contracting of both a call and a put option, with the objective of hedging against adverse prices while eliminating the premium. To do so, each option's premiums are compensated. Thus, the agent is subject to two different strike prices, which are then translated into a cap and a floor price.

In the same way to the collar, the combination of several options allows agents to design products that adapt to their needs.

3.3. Swaps

A swap transaction is the simultaneous purchase and sale of an obligation or a similar underlying asset, of equivalent capital, in which the exchange of financial agreements provides the counterparties more favorable conditions than without the contract.

In electricity markets, swaps frequently consist in the purchase or sale of the electricity at a fixed price. Thus, in essence, swaps are a combination of electricity forwards with multiple settlement dates and constant price.

Swaps are commonly used by agents in electricity markets to obtain price certainty in short to mid-term.

4. Valuation of electricity derivatives

4.1. Valuation models

During this section, it will be described how models have been created from the formulas presented in 2 Monthly electricity price simulations.

Both the models for spot prices (Brownian and Mean reversion) as for the forward (Monte Carlo) provide as a result several scenarios of expected monthly prices. Thanks to the shaping model developed in 1.1.1 Spot daily prices, it is possible to covert the monthly prices resulting from the forecasting models to hourly prices with high accuracy.

Additionally, the Black-Scholes model will be explained. This model's output will be used as a benchmark, to compare the results obtained in the developed models for the practical cases.

4.1.1. Brownian motion model

As it was explained, the Brownian model was determined by:

$$\ln \frac{S_t}{S_{t-1}} \simeq \mu \Delta t + \sigma \Delta z$$

In this formula, μ is the expected growth per Δt and σΔz, the distortion. Indeed, σ may be defined as the volatility of the product. Several methods are found to compute this volatility.

One approach is to calculate the forecast of future monthly prices, with the monthly volatility calculated from daily prices, as developed in section 1.1.1 Spot daily prices. The introduced monthly volatility will be the one calculated in Table 3. Average monthly volatility after treatment., which is the average of each months' volatility since 2000.

As it was found when analyzing daily prices, some months as January are more volatile than others, as July. Therefore, the main advantage of this first approach is that it is able to reflect in the output this feature, by introducing in the model an average volatility per month. However, the results obtained are the ones shown in the next image, setting S_0=50 €/MWh, μ=0 and with a scope of 24 months.

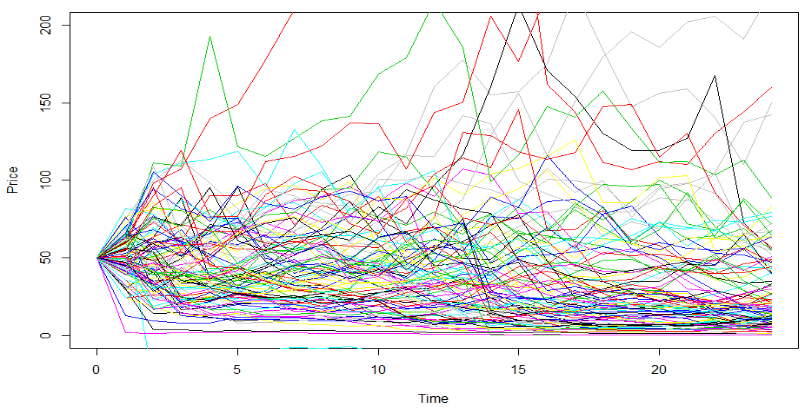

Fig. 34. Result Brownian model with average monthly volatility σ, S_0=50 €/MWh, μ=0 and t=24 months.

The prices' volatility is too high and does not reflect the real market behavior. This is caused by the computation of volatilities from daily prices. These prices suffer higher volatilities than what is actually transferred to monthly prices. Therefore, this approach must be rejected.

If forecasting monthly prices, the introduced volatility must be calculated only from monthly values. With this purpose, the formulas presented in section 1.1.1 Spot daily prices will be used as:

$$Perf_i = \log\left(\frac{Monthly\ price_i}{Monthly\ price_{i-1}}\right) \qquad \underset{Volatility}{Monthly}_j = \frac{1}{\sqrt{12\ Months}} \times \sigma\left(Perf_{i-13,i-1}\right)$$

This time, the calculated volatility does not contain a monthly characterization, as it was computed from the standard deviation of the last twelve monthly prices. Thus, we do not lose any information by doing the average value of all the volatilities, calculated from 2000 to 2017. The result is σ = 0.1739. Fixing the same inputs as before (S_0=50 €/MWh, μ=0 and with a scope of 24 months), the result is:

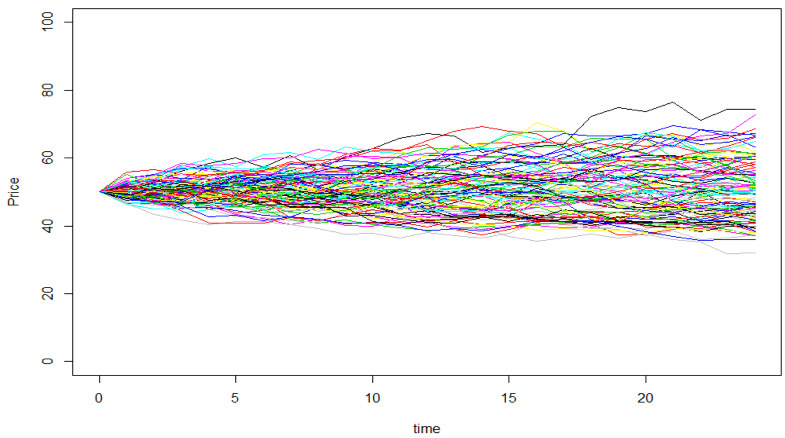

Fig. 35. Result Brownian model with σ=0.1739, S_0=50 €/MWh, μ=0 and t=24 months.

As it can be seen, the results provided reflect better the real market behavior. However, if the time frame studied is too high, the results will be within a wide range, introducing too much uncertainty. This can be seen in the next chart, where the time studied has been 20 years (24 months). On the right chart, the drift has been established at μ=1%, meaning that the trend is for prices to grow 1% of S_0, 0.5 €/MWh, per month. The image intends to show the model's sensitivity to μ.

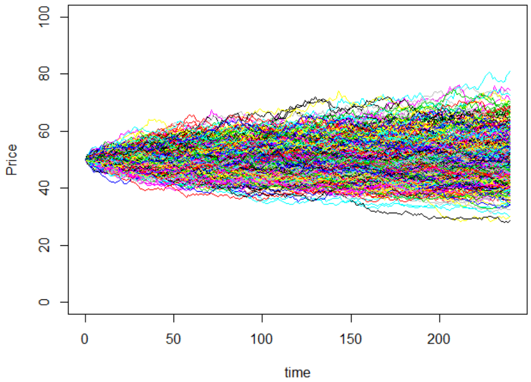

Fig. 36. Result Brownian model with σ=0.1739, S_0=50 €/MWh, μ=0 and t=240 months (20 years)

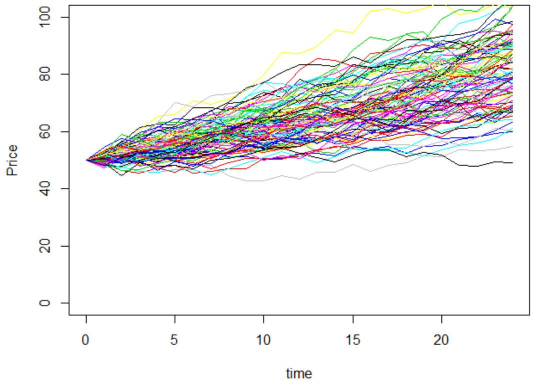

Fig. 37. Result Brownian model with σ=0.1739, S_0=50 €/MWh, μ=1% and t=24 months

4.1.2. Mean reversion model

As already explained, the mean reversion model intended to solve some of the limitations of the Brownian model. It is defined by:

$$\ln\frac{S_t}{S_{t-1}} \simeq \frac{\Delta S_t}{S_{t-1}} = \alpha(\mu - \ln(S_{t-1}))\Delta t + \sigma\Delta z$$

The model introduces a long-term value ($S_{LT}=e^{\mu}$) to which prices tend at a speed of α. The lower the value of α, the closer the model is to a pure Brownian motion model. The sensitivity of the model to α may be seen in the following figures.

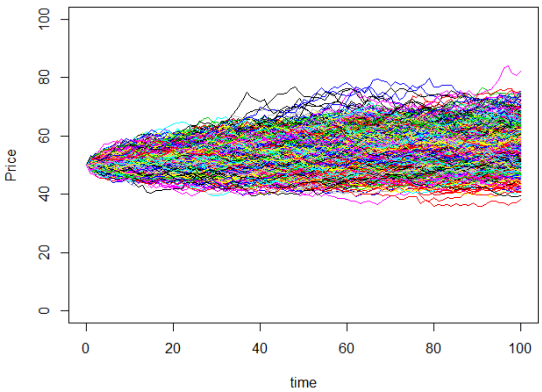

Fig. 38. Result mean reversion model with σ=0.1739, S_0=50 €/MWh, e^{μ}=60 €/MWh, α=1 and t=100 months

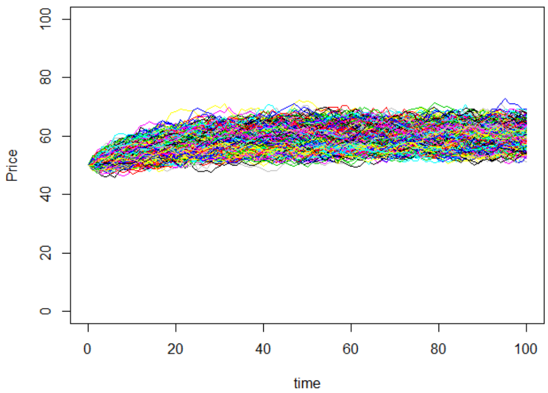

Fig. 39. Result mean reversion model with σ=0.1739, S_0=50 €/MWh, e^{μ}=60 €/MWh, α=6 and t=100 months

4.1.3. Monte Carlo model

The Monte Carlo model intends to forecast, not only a certain price, but the future forward curves. The formula describing this process is the one that follows:

$$F_i(t,T) = F(0,T)\cdot e^{k_i(T-t)-\frac{1}{2}\cdot\sigma(T-t)^2}$$

where σ is the volatility and k_i the variable differencing among scenarios. These two variables are computed by:

$$\sigma(T-t) = \sigma \cdot e^{-\alpha \cdot (T-t)} \quad \text{and} \quad k_i(T-t) = \xi_i \cdot \sigma(T-t)$$

Being σ and α parameters deduced from historical values of the volatility.

Introducing in the model the forward curve of 12 months ahead and the volatility deducted from the products that reflect these prices, the results obtained are:

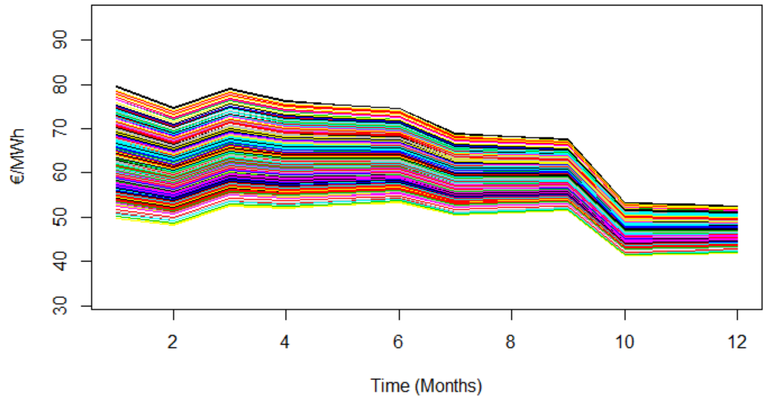

Fig. 40. Results Monte Carlo simulation for 12 months and historical standard deviations

The results show that the fact that products with closer delivery times are more volatile cause that we have a wider range of prices, meaning higher uncertainty, for values of the forward curve closer to t=0 than for those with further deliveries.

4.1.4. Black-Scholes

[5]

In 1973, Fischer Black and Myron Scholes published the Black-Scholes formula to valuate European options. Their work caused great influence in the way in which market participants price and hedge options.

The Black-Scholes formula is designed for an environment that meets the following assumptions:

- The price of the underlying commodity behaves as lognormal, with constant μ and σ and follows a geometric Brownian motion.
- The underlying commodity is perfectly divisible
- There are neither transaction costs nor taxes.
- The option does not generate any dividends during the considered period
- There are no arbitrage operations without risk
- There is possibility to continuously change the positions
- The risk-free rate of interest for investors is the same for borrowing and lending.
- In the short-term, this risk-free rate of interest, r, is constant

Depending on whether the option is a call or a put option, the formula consists in:

$$V_C = S \cdot N(d_1) - Ke^{-rT} \cdot N(d_2) \quad \text{or} \quad V_P = Ke^{-rT} \cdot N(-d_2) - S \cdot N(-d_1)$$

where d_1 and d_2 are defined as:

$$d_1 = \frac{\ln\left(\frac{S(0,T)}{X}\right)+\left(r+\frac{\sigma^2}{2}\right)\cdot T}{\sigma\sqrt{T}} \quad \text{and} \quad d_2 = d_1 - \sigma\sqrt{T}$$

The components of the formula are:

- The N(x) formula is the cumulative distribution function for a normal variable.
- X is the strike price of the option, which will depend on the practical case
- S the underlying price, which will be estimated with the value of the forward curve that month.
- σ is the annualized volatility. It will be estimated similarly than in the Monte Carlo method: by exponential interpolation.
- T the lag until the delivery
- r, the risk-free rate of interest, is 0.

These formulas are deduced from the design of a risk-free portfolio, by balancing the position in the spot market with the one adopted with the option.

4.2. Application of the models

Once the different models have been analyzed, it is interesting to apply them to practical cases and study their accuracy. During this section we will describe the different scenarios to address with the models.

Al these practical cases are described from the point of view of a utility that receives a request from a client to provide a service. To do so, the utility will need to forecast the future scenarios and their risk. The client will be a consumer in the first case and, in the second one, a generator.

4.2.1. Case 1: retail

It is frequent that utilities receive the request by consumers to hedge their risk. One of the main ways to do so is protecting the agent against high prices, by fixing a cap by an option. In this case, it will be supposed that an agent wants to establish the cap at a strike price of 75 €/MWh. For selling this contract, the utility claims a premium, usually in terms of €/MWh.

The main objective of the utility is to measure the risk it is receiving from this contract. Thus, the higher the risk, the higher the premium. However, it must ensure that it is correctly calculated because, in case it is over dimensioned, the utility may lose the contract. On the other hand, in case it is under dimensioned, the utility will lose money with it.

Other way to design this hedge is to design a collar, which sets both a cap and a floor. In this case, there will be no premium, as the extra costs assumed by the utility by paying all price above the cap, will be recovered by the profit of demanding to the consumer the floor price every time the market is below it.

The exercise will consist in determining the parameters of the two designs for a cap of 75 €/MWh. This is both the premium and the floor. To do so, the client provides its hourly consumption curve for the whole duration of the contract, which will be 1 year from July 2018 to June 2019.

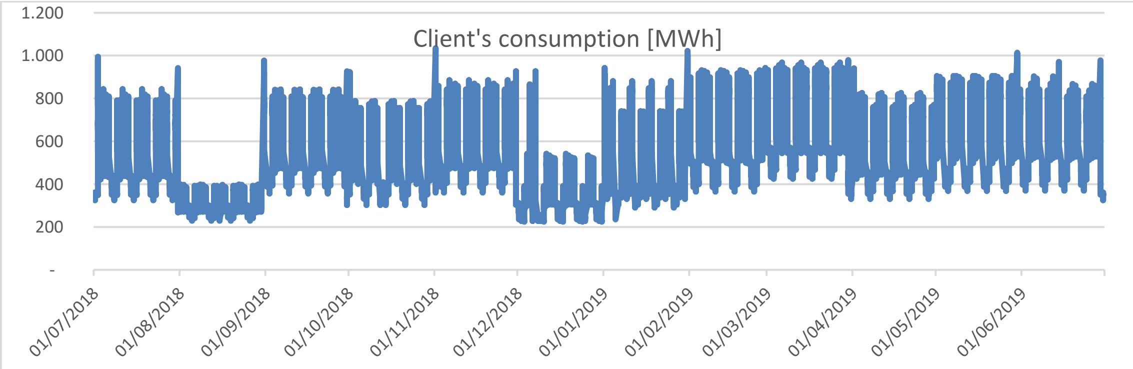

Fig. 41. Hourly consumption of the client for the considered period of time (July 2018 to June 2019)

To calculate the value of the premium or the floor, several scenarios of prices will be developed, with the aim of deducting the distribution of future prices. These scenarios will be created with the simulation models that have been developed and the shaping tool, to compute hourly prices.

Once the hourly prices are computed, the utility's loss in € per hour when installing the cap will be calculated. By dividing the total loss by the total consumption, the prime would be obtained:

$$Premium_i\left[\frac{€}{MWh}\right] = -\frac{Loss_i[€]}{Total\ Consumption_h[MWh]}$$
$$= -\frac{\sum_h (Cap - Price_{h,i})\left[\frac{€}{MWh}\right] \times Consumption_h[MWh]}{Total\ Consumption_h[MWh]}$$

This process will provide us a distribution of premium values, one per scenario i. The premium will be set at a value that ensures the recovery of the mean loss. However, it will be interesting to study the loss distribution. This will be done by analyzing the percentiles and the standard distribution.

To calculate the floor of the collar, a similar process will be followed:

$$Balance_i\left[\frac{€}{MWh}\right] = \frac{Gain_i - Loss_i[€]}{Total\ Consumption_h[MWh]}$$
$$= \frac{\sum_h [(Price_{h,i} - Floor)\ or\ (Cap - Price_{h,i})]\left[\frac{€}{MWh}\right] \times Consumption_h[MWh]}{Total\ Consumption_h[MWh]}$$

A different balance will be obtained per scenario i. In this case, the floor will be set at a value that ensures the balance of 0 in the percentile 15%. This means that, in 85% of the cases, the utility will obtain a gain by hedging the client's risk. To find the floor's value that ensures this recovery, an iterative procedure will be followed.

The percentiles and standard distribution will also be analyzed to study the utility's risk.

4.2.2. Case2: generation

The next practical case consists in the request of a generator's agent to valuate a CCGT unit. Traditionally, generations assets have been mainly valuated over their intrinsic value. This is defined as the value of the unit acquired from dispatching in the market, considering the expected price. To calculate it, forward prices are commonly used as the expected price. However, to correctly address this problem, the extrinsic value must be also considered. [21]

The extrinsic value considers all the possible future scenarios and their probability. Although the most probable scenario is the one considered for the intrinsic value, the dispersion from it and the unit's response must be estimated.

To this purpose, the CCGT will be valuated as an option contract which can be exercise during the valuation period. The strike price is the variable cost of generating. Thus, every time the market price is above the strike price (variable cost), the CCGT will be committed and will generate electricity. The obtained profit is the difference among these prices multiplied by the quantity produced.

The main focus of this exercise is to calculate the extrinsic value of a CCGT of 400 MW during the second semester of 2018. To do so, the variable costs are computed as:

$$Variable\ cost\ [€/MWh] = fuel\ cost + ATR + O\&M + Tax + CO_2$$

The fuel costs, CO_2 costs and the tax, which is defined in terms of €/GJ, are variables that depend of the performance of the CCGT. Thus, if the asset works baseload, it will have higher efficiency, as it reduces the number of startups and shut downs. However, these types of plants are currently working to complement the intermittency of renewables, following an operation named cycling, which reduces their efficiency.

Thus, the variable costs of the plant are calculated under two different efficiency scenarios: η=39% and η=49%.

Costs €/MWh	Fuel (49%)	Fuel (39%)	ATR	O&M	Tax (49%)	Tax (39%)	CO_2 (49%)	CO_2 (39%)	Total costs (49%)	Total costs (39%)
01/07/2018	44,1	55,4	2,48	2,1	4,8	6,0	5,5	6,6	**58,9**	**72,6**
01/08/2018	43,9	55,1	2,48	2,1	4,8	6,0	5,5	6,6	**58,7**	**72,3**
01/09/2018	43,9	55,2	2,48	2,1	4,8	6,0	5,5	6,6	**58,8**	**72,4**
01/10/2018	44,4	55,8	2,48	2,1	4,8	6,0	5,5	6,6	**59,3**	**73,0**
01/11/2018	45,0	56,5	2,48	2,1	4,8	6,0	5,5	6,6	**59,8**	**73,7**
01/12/2018	45,0	56,5	2,48	2,1	4,8	6,0	5,5	6,6	**59,8**	**73,7**

Table 12. CCGT variable costs

To calculate the extrinsic value of the CCGT several scenarios of prices will be developed, with the aim of considering the most probable scenario and the possible deviations. These scenarios will be created with the simulation models that have been developed. These will create monthly prices that will be then converted into hourly prices through the designed shaping tool.

Once the hourly prices are computed, it will be assumed that the plant works at full power every time the price is above its costs. This, however is an approximation adopted for simplicity's sake. In a real valuation, several technical parameters, as the start-up and shut-down limitations, would be considered.

The value obtained with the simulation models will be compared with the output of the Black-Scholes formula. This will be used as calculating the value of a call option whose strike price is the asset's variable cost. Thus, the formula generates the premium a utility requires to compensate the losses of hedging a consumer from higher prices than the strike. This loss is equivalent to the CCGT's gained margin in the hours it is committed.

In the next chapter, the results provided by each model will be analyzed and commented.

CHAPTER 4: Results

1. Valuation for different cases

1.1. Case 1: retail

In this section, the premium and the floor requested by a consumer will be calculated. For each model, the prices scenarios and the percentiles of the results will be provided.

1.1.1. Brownian model

The starting point of the Brownian model that computes the prices between July 2018 and June 2019 will be the monthly value of May 2018, as the price of June is not available. The uncertainty of the extra month between the valuation and the delivery of the contract, must be also introduced in the model.

Thus, S_0 will be 55 €/MWh.[22] The volatility σ will be the one commented in the previous chapter, σ=0,1739. The drift µ will be set at 0% as, when studying the forward curve, it can be seen that prices are expected to increase the first part of the year and then suffer a big decrease in the following months. The results obtained are:

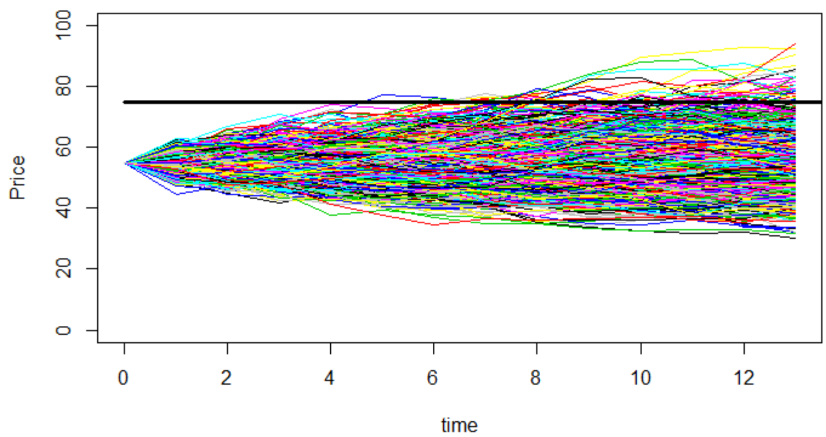

Fig. 42. Result Brownian model with σ=0.1739, S0=50 €/MWh and t=14 months and cap of 75 €/MWh

Computing the hourly prices, the results obtained are the following:

Percentiles of gain (€/MWh) for cap=75 €/MWh										
0%	10%	15%	20%	30%	40%	60%	80%	100%	**Mean**	Standard deviation
-11,45	-0,40	-0,17	-0,09	-0,01	0	0	0	0	**- 0,29**	0,616

Table 13. Results from Brownian motion model for cap= 75 €/MWh

Thus, the requested premium should be 0,29 €/MWh. As it could be deducted from the huge uncertainty obtained with this model, the calculated premiums suffer high variations depending on the percentile. This is reflected both in the high standard deviation and in the high value of the maximum loss. This means that depending on how much the utility wants to hedge it risks, the premium would be fix at very different values.

To compute the floor of the collar, an iterative procedure is followed, obtaining that, for a floor of 47 €/MWh:

Percentiles of gain (€/MWh) for cap=75€/MWh and floor=47€/MWh											
0%	10%	15%	20%	30%	40%	50%	60%	80%	100%	Mean	Standard deviation
-11.46	-0,18	0	0,17	0,42	0,68	1,03	1,43	2,66	13,23	0,60	1,456

Table 14. Brownian model's results for cap and floor case

Therefore, with a floor of 47€/MWh, the utility's mean gain is 0,60 €/MWh. The standard distribution is even higher than in the previous case, about 2,5 times the mean gain. Plotting the results:

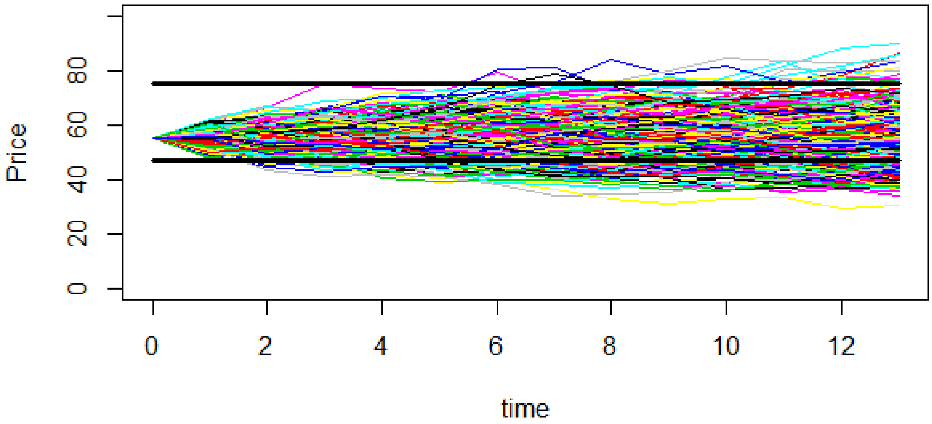

Fig. 43. Mean reversion model with σ=0.1739, S0=55 €/MWh and t=13 months with cap of 75€/MWh and floor of 47 €/MWh.

1.1.2. Mean reversion model

The mean reversion introduces a long-term value. To compute it, the initial value S_0 has been chosen, for the reasons explained above. Thus $S_0 = S_{LT} = 55€/MWh$. The speed to which the prices tend to the long-term value will be set at α=0,5.

This valuation must also consider the time lag between the introduced current price and the delivery of the product. This lag, which corresponds with the month of June 2018, introduces uncertainty that must be considered. The monthly prices calculated are:

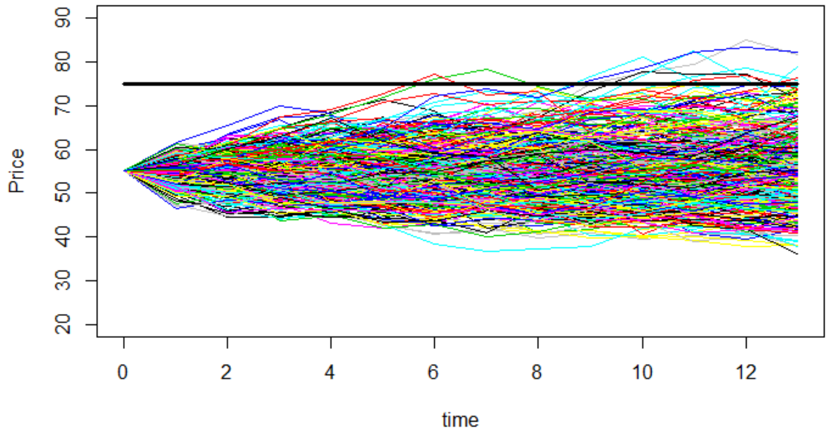

Fig. 44. Results from mean reversion model with σ=0.1739, S_0=55 €/MWh, e^{μ}=55 €/MWh, α=0,5 and t=13 months and cap of 75 €/MWh

Then, the hourly prices per scenario will be computed with the shaping tool. Computing the utility's loss of setting a cap of 75 €/MWh, we obtain:

Percentiles of gain (€/MWh) for cap=75 €/MWh										
0%	10%	15%	20%	30%	40%	60%	80%	100%	**Mean**	Standard deviation
-7,60	-0,13	-0,12	-0,06	0,01	0	0	0	0	**- 0,13**	0,333

Table 15. Results from mean reversion model for cap= 75 €/MWh

Therefore, the premium is estimated at 0,13 €/MWh. In the worst computed case, the loss would be of about 7,60 €/MWh, which is significantly lower than in the previous case. Thus, the utility's risk is lower. Other way to see this is through the standard deviation, which is 0,333. This is around half the previous case.

To offer the client a floor price, instead of this premium, the obtained results are:

Percentiles of gain (€/MWh) for cap=75€/MWh and floor=47€/MWh											
0%	10%	15%	20%	30%	40%	50%	60%	80%	100%	Mean	Standard deviation
-7,60	-0,1	-0,01	0,08	0,24	0,40	0,59	0,84	1,66	8,51	0,92	1,062

Table 16. Brownian model's results for cap and floor case

The mean reversion model provides a floor of 47 €/MWh to compensate the potential loss of the utility, caused by the cap price. With this design of the collar, the utility's mean gain would be 0,92 €/MWh but with a quite high standard distribution. This indicates, again, that the utility is assuming a big risk by signing this contract.

In the following image, the cap and the floor are represented together with the prices.

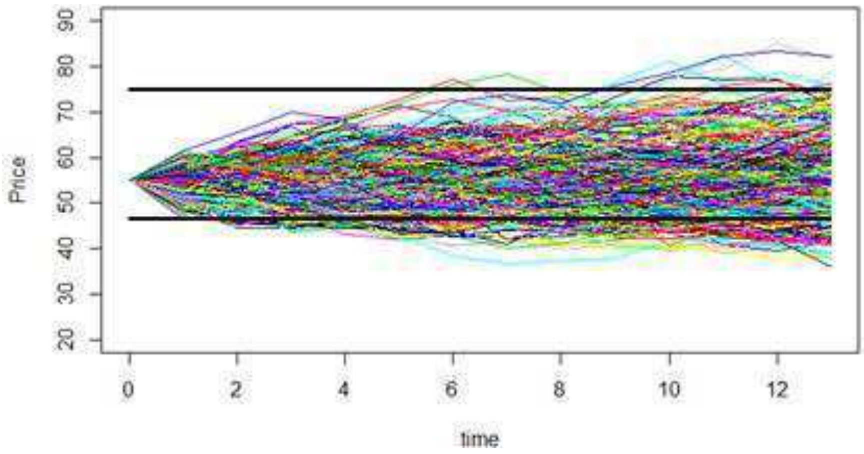

Fig. 45. Results from mean reversion model with σ=0.1739, S0=55 €/MWh, eμ=55 €/MWh, α=0,5 and t=13 months and cap and floor of 75 and 47€/MWh, respectively

1.1.3. Monte Carlo model

The Monte Carlo model creates several scenarios from a random modification in the volatility. The volatility is a time dependent variable calculated as shown in the Fig. 33. Summarizing, the volatility of the curve was expressed as a function of the time until delivery by:

$$\sigma(T-t) = \sigma \cdot e^{-\alpha \cdot (T-t)} = 0{,}3355 \cdot e^{-0{,}061 \cdot (T-t)}$$

The considered prices to calculate this volatility are the ones provided by the market between September of 2017 and the June 8th of 2018. On the other hand, the introduced forward curve is conformed by the market prices of June 8th of 2018:

2018						2019					
July	Aug.	Sept.	Oct.	Nov.	Dec.	Jan.	Feb.	Mar.	Apr.	May	June
63.53	60.55	64.92	63.40	63.40	63.40	59.19	59.19	59.19	47.07	47.07	47.07

Table 17. Forward curve prices (€/MWh)

The results are the represented below together with the cap:

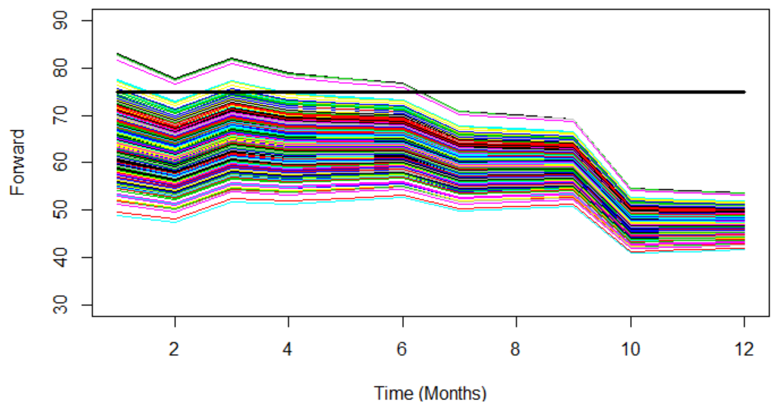

Fig. 46. Results of Monte Carlo simulation for forward prices and cap of 75 €/MWh

Once these monthly values are calculated per simulation, the hourly prices will be estimated with the shaping tool. Then the premium will be calculated, obtaining:

Percentiles of gain (€/MWh) for cap=75 €/MWh											
0%	10%	15%	20%	30%	40%	50%	60%	80%	100%	**Mean**	Standard deviation
-3,92	-0,59	-0,40	-0,24	-0,11	-0,05	-0,02	0	0	0	**- 0,20**	0,546

Table 18. Monte Carlo results for cap with premium case

Therefore, the premium would be established at 0,20 €/MWh. The standard deviation and maximum calculated cost are remarkably lower than in the last cases.

To compute the design of the collar, an iterative procedure is followed, obtaining that, for a floor of 48 €/MWh:

Percentiles of gain (€/MWh) for cap=75€/MWh and floor=48€/MWh											
0%	10%	15%	20%	30%	40%	50%	60%	80%	100%	Mean	Standard deviation
-3,82	-0,25	0	0,21	0,42	0,56	0,68	0,82	1,14	2,82	0,60	0,891

Table 19. Monte Carlo results for cap and flor case

When setting a floor of 48 €/MWh, the average gain per scenario is 0,60 €/MWh with a standard deviation of 0,891€/MWh, which is quite low. This indicates that, under the scenario simulated by the model, the utility is not facing a big risk.

In the following figure the different price scenarios and the cap and floor are represented.

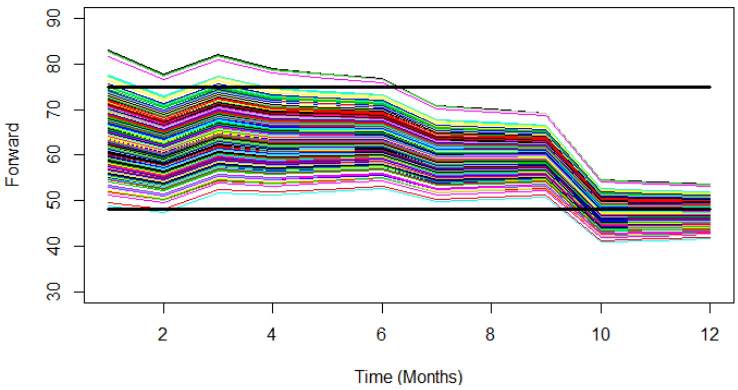

Fig. 47. Forward prices with cap and calculated floor.

1.2.Case 2: generation

For this case, the chosen scenarios are the ones created to calculate the premium and the floor of the previous case. These computed prices for time frame from July 2018 to June 2019. In this case, we will focus in the second semester of 2018 (July to December).

The results obtained from each model are represented in this paper by means the percentiles and average values of three different variables. First, the margin in €/MWh per hour is analyzed. When the price is lower than the variable cost, the margin is 0. Otherwise, it is the difference among those. Then, the hours the CCGT is committed per scenario are calculated. Since the study is done for a semester, the total number of hours it could have been committed would be

4380 hours. Finally, the overall profit per scenario in k€ is analyzed. This is calculated by multiplying the hourly margin by 400 MW.

To conclude the section, the case is computed with the Black-Scholes formula. The output obtained will be used as a benchmark for the hourly margin calculated in the other models.

1.2.1. Brownian model

The results obtained in the Brownian model are the following:

Brownian	Margin [€/MWh]	Hours committed	Profit [k€]
	η=49%		
0%	0,00	0	0
10%	0,01	45	11
20%	0,09	203	158
30%	0,24	667	432
40%	0,57	1.225	1.009
50%	1,00	1.510	1.758
60%	1,58	2.080	2.797
70%	2,30	2.299	4.060
80%	3,36	2.536	5.938
90%	5,17	2.900	9.132
100%	15,30	4.367	27.026
Mean	**2,69**	**1.621**	**4.757**
Standard deviation			**7.930**
	η=39%		
0%	0,00	0	0
10%	0,00	0	0
20%	0,00	0	0
30%	0,00	0	0
40%	0,00	0	0
50%	0,00	0	0
60%	0,00	0	0
70%	0,00	0	0
80%	0,00	21	2
90%	0,10	199	182
100%	4,12	2.630	7.269
Mean	**0,38**	**259**	**678**
Standard deviation			**2.187**

Table 20. Results of the CCGT valuation from the Brownian simulation

1.2.2. Mean reversion model

For the mean reversion model, we obtain:

Mean Reversion	Margin [€/MWh]	Hours committed	Profit [k€]
	η=49%		
0%	0,00	0	0
10%	0,04	150	66
20%	0,19	474	336

30%	0,34	735	606
40%	0,68	1.085	1.194
50%	1,00	1.422	1.771
60%	1,68	1.795	2.964
70%	1,99	2.044	3.510
80%	2,94	2.345	5.188
90%	4,43	2.679	7.830
100%	9,17	3.637	16.204
Mean	**1,76**	**1.462**	**3.115**
Standard deviation			**3.587**
	η=39%		
0%	0,00	0	0
10%	0,00	0	0
20%	0,00	0	0
30%	0,00	0	0
40%	0,00	0	0
50%	0,00	0	0
60%	0,00	0	0
70%	0,00	0	0
80%	0,01	63	18
90%	0,16	315	291
100%	1,33	1.416	2.352
Mean	**0,06**	**87**	**109**
Standard deviation			**359**

Table 21. Results of the CCGT valuation from the mean reversion simulation

1.2.3. Monte Carlo model

Finally, the Monte Carlo simulation results are:

Mean Reversion	Margin [€/MWh]	Hours committed	Profit [k€]
	η=49%		
0%	0,27	523	472
10%	1,97	2.120	3.477
20%	2,79	2.341	4.924
30%	3,60	2.606	6.360
40%	4,46	2.773	7.882
50%	5,47	3.010	9.661
60%	6,43	3.246	11.353
70%	7,71	3.486	13.621
80%	9,12	3.685	16.110
90%	11,09	3.915	19.586
100%	20,26	4.357	35.779
Mean	**6,10**	**2.998**	**10.771**
Standard deviation			**6.399**
	η=39%		

0%	0,00	0	0
10%	0,00	0	0
20%	0,00	0	0
30%	0,00	43	1
40%	0,03	152	51
50%	0,08	172	135
60%	0,14	318	250
70%	0,34	780	609
80%	0,73	1.351	1.294
90%	1,57	1.982	2.781
100%	7,69	3.515	13.589
Mean	**0,50**	**619**	**877**
Standard deviation			**1.773**

Table 22. Results of the CCGT valuation from the Monte Carlo simulation

1.2.4. Black-Scholes model

As it was explained in 4.2.2 Case2: generation, the extrinsic value of a CCGT can be computed by means of the Black-Scholes formula, considering the asset as a call option. Thus, the calculated premium or value of the call should be coincident with the gained hourly margin.

As the studied period is the second semester of 2018, the values introduced in the formula for the price and volatility are those of the forward curve of those months. The formula provides a premium per month. The result is calculated as the average for the whole semester.

Black-Scholes	Margin [€/MWh]
η=49%	7,07
η=39%	1,16

Table 23. Results of the CCGT valuation from the Black-Scholes formula

On the section below, the obtained results are analyzed.

2. Analysis of the results

To facilitate the analysis, the following table summarizes the results obtained in the Case 1 from each simulation model:

	Cap + Premium			Cap + Floor				
	Mean (Premium)	Standard deviation	Max. loss	Floor	Mean	Standard deviation	Max. gain	Max. loss
Brownian	-0,29	0,616	-11,45	47	0,6	1,456	13,23	-11.46
Mean Reversion	-0,13	0,333	-7,6	47	0,92	1,062	8,51	-7,6
Monte Carlo	-0,20	0,546	-3,92	48	0,6	0,891	2,82	-3,82

Table 24. Results obtained in Case 1. [€/MWh]

As it can be seen, the results provided by the Brownian motion model have, as expected, higher volatility. For instance, in the calculation of the premium for a cap hedge case, the volatility was 0,616, which is high compared to the ones provided by the other two models, which were 0,333 for the mean reversion and 0,546 for the Monte Carlo model. This is caused by what was described on the previous sections: the Brownian simulation introduces a constant volatility, which increases the range of values without limits.

On the other hand, the mean reversion model is the one providing the smallest standard deviation (0,333), which means the smallest uncertainty. This is controlled by the speed α at which the prices tend to in the long-term value. In the previous case, α was estimated attending to the results. If it had been lower, the model would have been more similar to a Brownian model, which means more uncertainty.

Thus, the standard deviation is a measure of the risk assumed by the utility when selling the cap. A high deviation means that the possible future price scenarios are within a wide range. Therefore, the uncertainty is quite high. In addition to the standard deviation, another parameter that shows the utility's risk is the maximum loss computed. This is not necessarily correlated with the standard deviation: the simulation could provide the majority of scenarios within a narrow range of prices and, then, a few scenarios with very high prices. This phenomenon explains why, although the standard deviation of the mean reversion model is lower than the Monte Carlo's, it maximum loss is almost double.

Therefore, if the standard deviation and/or the maximum loss is high, the criteria to set the premium may change from the one followed here: instead of a premium that ensures the recovery of the mean loss, it may be chosen throughout the percentiles, in order to ensure the recovery in a certain percentage of the scenarios. For instance, the premium set at the value of the percentile 15%, ensures the recovery in 85% of the scenarios. This same logic may be followed when establishing a floor.

On the other hand, the results obtained when designing a collar hedge provide higher standard deviation. This indicates that the utility is assuming more risk when signing this configuration in comparison to the cap + premium.

It is remarkable that the calculated premium for the mean reversion model (0,13 €/MWh) is lower than the one estimated by the Brownian and Monte Carlo (0,29 €/MWh and 0,20 €/MWh). The fact that it is lower than the Brownian model is not surprising, due to the already mentioned high volatility. However, it is also higher that the Monte Carlo's results. This is mainly caused by the introduction of the forward curve in the latter, which guides the result to reflect high prices in certain time frames.

It is remarkable that the calculated premium for the Monte Carlo model (0,20 €/MWh) is higher than the one estimated by the mean reversion model (0,17 €/MWh and 0,12 €/MWh). This is mainly caused by the introduction of the forward curve in the Monte Carlo model, which guides the result to reflect high prices in certain time frames.

The spot models do not have that information, so they do not provide such high values. They have the starting point at the current price (55 €/MWh), which is significantly lower than what the forward market expects for the following months curve (around 65 €/MWh). Thus, the estimated loss when establishing a cap is lower than in the Monte Carlo model.

This same effect can be appreciated in the results of the floor calculation: while the spot models calculate a floor of 47 €/MWh, the Monte Carlo results in 48 €/MWh. The second simulation computes higher prices, estimating a higher floor to balance the effect of the cap established.

These conclusions are in accordance with the results obtained in the valuation of the CCGT, which are gathered in the following table:

	Brownian		Mean Reversion		Monte Carlo		Black-Scholes	
η (%)	39%	49%	39%	49%	39%	49%	39%	49%
Margin (€/MWh)	0,38	2,69	0,06	1,76	0,5	6,1	1,16	7,07
Hours (h)	259	1.621	87	1.462	619	2.998		
Profit (k€)	678	4.757	109	3.115	877	10.771		
Standard deviation (k€)	2.187	7.930	359	3.587	1.773	6.399		

Table 25. Summary of the results for the valuation of a CCGT

The standard deviation of the Brownian simulations, in comparison to the computed profit is remarkably high, as expected.

Attending, for instance, the scenario with higher efficiency (η=49%), the CCGT's profit mean value is 4.757 k€ for the Brownian model, 3.115 k€ for the mean reversion simulation and 10.771 k€ in the Monte Carlo model. As explained, the scenarios created by the latter provide higher price values, so the CCGTs are committed more hours and, therefore, have higher profit.

On the other hand, the margin calculated by the Black-Scholes formula (7,07 €/MWh) is similar to the Monte Carlo's (6,1 €/MWh), yet higher. However, this indicates that the simulations performed by the forward model are reliable, while those of the spot models do not adapt to the real market behavior.

CHAPTER 5: Conclusions

To finalize the present project, a brief summary of the developed conclusions will be provided during this chapter. Then, under the section Future works, some of the lines of the project to be further developed will be explained. These include the areas of improvement and the ones that were beyond the scope of this project and could not be developed due to time limitation.

1. Conclusions

During this project, three different models to valuate electricity derivatives have been designed. The objective is the comparison of each model's accuracy and limitations.

To this purpose, the study started with an extensive analysis of the prices in the spot and forward markets. One of the main findings of the project is that, in contrast with the general assumption, prices do not always behave as a lognormal variable. In fact, spot prices fit better as a normal variable. This is remarkable, as many valuation models are developed from the hypothesis of lognormality.

However, monthly prices do distribute as a lognormal, so the previously mentioned models could be used for this aggregation of prices without assuming a big error. Likewise, during certain time frames, forward prices also have a lognormal distribution. Particularly, when considering the most recent years. This suggest that the forward market is leading towards adopting such behavior.

An observation must be made regarding forward markets: prices are highly affected by the liquidity and, under certain situations, these may reflect a constraint situation, rather than the optimum market behavior. Although, in Spain we do not find the lowest figures, they are still quite behind to other European markets.

Another remarkable finding was that the shaping of the curve of hourly prices from the monthly average is easily characterized. This implies that, if a model with accurate monthly forecasts is designed, a precise estimation of hourly prices will be available.

The analysis of prices concluded with a brief time series analysis. This study showed that, contrary to the spot series considered, forward prices do not have a seasonal pattern and consist, mainly, in a moving average. On the other hand, monthly spot prices also consisted in a moving average, but general and seasonal. Finally, spot daily prices evidenced a more complex ARIMA. This could be caused either by the higher volatility of these series or by the big data frame introduced in the model. These deductions could be used in future simulation models.

After this first section, the theory behind monthly price simulations and electricity derivatives products was explained, providing the reader a comprehensive sight of the topic. The reviewed models were the ones that were afterwards developed and tested for two practical cases. The results provided were depicted in the previous section.

The two models developed for the spot price simulations provided a simple and fast approach. However, they are limited. For instance, the Brownian model does not allow to introduce any available information from future values, as the one that can be deducted from the forward curve. This makes the model suitable for very short terms, where the future scenario will not present great differences from the current one.

In contrast, the mean reversion simulation requires the long-term value and the speed α. This allows the user to introduce the tendency prices will have in the future. In the previous example, the prices' long-term value was set as the average of the forward curve. However, part of the information of the curve is lost when computing its average. Additionally, the speed α must be estimated, being a parameter that highly affects the output of the simulation. Thus, with a good estimation of the parameter α, this model will provide reliable results for longer scopes than the Brownian. It is, nevertheless, limited to short terms.

Finally, and in contrast to the previous spot simulations, the Monte Carlo model respects all the information from the forward curve. It considers the difference among its monthly prices. Moreover, the Monte Carlo method computes the volatility of the products that conform the forward curve, maintaining the difference between values depending on the lag until the delivery date. Its accuracy was proved by comparing its results with the ones provided by the Black-Scholes formula.

After applying the three models to two practical cases several conclusions could be extracted. On the one hand, it was proved that the models behaved as initially expected. The Brownian introduced high uncertainty and the mean reversion was highly conditioned by the introduced long-term value and speed α. Both of them were limited due to the lack of information about the future. The Monte Carlo model, however, provided reliable results.

Another main finding was the fact that the value of a derivative cannot be set at the mean value provided by the model, without attending the risk assumed by the utility. This is measured by means of the standard deviation and extreme gain or loss. In these cases, the distribution of the scenarios must be studied and, depending on the external conditions, as the agent's risk aversion profile, a premium ensuring the recovery in a certain percentage of the scenarios will be chosen. Thus, independently of the simulation used, there is need of a final analysis by an expert. The results, therefore, will be very much dependent of the analyst's criteria. This final analysis is, however, limited when using the Black-Scholes formula, as it only provides the value of the derivative, omitting the assumed risk.

It can be also concluded that, when fixing a cap hedge, it not indifferent whether it is complemented with a premium or a floor. Thus, when designing a collar, the risk is assuming more risk, as there are two sources of uncertainty: the loss caused by the cap and the recovery by the floor. However, when adopting the first configuration the only source of uncertainty is the loss caused by the cap, while the premium is translated into a fixed payment.

Finally, the Black-Scholes formula, which is the main valuation method used by many agents, presents several drawbacks. On the one hand, it is based on some hypothesis (as the lognormality of the prices) which are frequently not met. A simulation of how the market is actually going to evolve provides more robust results. Additionally, the formula's output is only the derivative's value, omitting other information that might be useful by the analyst, as the risk assumed by the utility.

To conclude, when valuating an electricity derivative, a combination of the Black-Scholes formula and a simulation model provides the most reliable results, as long as the user is able to interpret the results. The simulation model elected is conditioned by the product's characteristics. Specially, by the lag until delivery and its duration. The spot models are simple and provide reliable results for very short terms. The Monte Carlo model, while maintaining its simplicity, is considered the one with most accurate results for longer time periods, as it considers all the available information about the future scenario the user has.

2. Future works

Due to the limitations of time, this project has fixed its scope to the areas of work that I believe are the most interesting and provide a wide view of the faced issue. However, I have faced a trade-off between the development of full detailed studies and the time invested. This section aims to provide the reader some sight about the lines of work where this project can be further developed.

The first section that might be enhanced is the statistical analysis of the prices. The analysis of the present project limited its scope to comparing between the goodness of fit of prices to a normal and a lognormal distribution. However, price may suit better other distributions not considered in this project.

Additionally, it was clear that the different regulations have affected the behavior of market prices. To avoid a high distortion due to temporal effects, I have chosen a wide time frame, but a future project could be focus on the study of prices under the current regulation.

On the other hand, the translation of the conclusions obtained in the statistical study to the simulation models could be enhanced as well. The results showed that, although many times the hypothesis of prices behaving as a lognormal could not be discarded, prices always behave better as a normal. Simulation and valuation models use the lognormality of prices as a basis. On a future work, these could be adapted to a normal variable.

Although a brief time series analysis was here described, its results were vaguely used but to extract some conclusions. This section could be improved with more developed models. The information extracted from them could be applied to simulation models or even generate the simulations themselves.

Concerning the models developed, they present several areas of improvement. The spot models (Brownian and mean reversion) can be further developed to reflect better the real behavior of prices in the markets. This is done by introducing modifications as the one that creates the mean reversion model from the Brownian one.

Moreover, the mean reversion uses two variables that indicate the trend of values in the long term. This is the long-term price and the speed α at which prices tend to the former. In the practical cases these two were estimated but, for a more realistic representation of the market, these should be calculated attending observed parameters.

To check the accuracy of the models, a backtest must be ran. For this, a simulation of the price scenario must be computed, providing the expected prices and distribution. Afterwards, this should be compared with the real market's behavior.

Finally, the practical case of the valuing a CCGT was simplified. On a real basis, the asset's extrinsic value would be estimated by a stirp of options reflecting the prices, instead of just one single option. Moreover, the model does not include any technical limitations, which should also be considered for a better valuation.

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