UNIVERSIDAD PONTIFICIA COMILLAS DE MADRID
ESCUELA TÉCNICA SUPERIOR DE INGENIERÍA (ICAI)
(Departamento de Organización Industrial)

OPTIMIZACIÓN DE LA EXPLOTACIÓN Y DE LA PREPARACIÓN DE OFERTAS DE UNA EMPRESA DE GENERACIÓN DE ENERGÍA ELÉCTRICA PARA MERCADOS DE CORTO PLAZO

Tesis para la obtención del grado de Doctor

Directores: Prof. Dr. D. Michel Rivier Abbad
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Autor: Ing. D. Álvaro Baíllo Moreno

Madrid 2002

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A METHODOLOGY TO DEVELOP OPTIMAL SCHEDULES AND OFFERING STRATEGIES FOR A GENERATION COMPANY OPERATING IN A SHORT-TERM ELECTRICITY MARKET

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ESTUDIOS DE TERCER CICLO - DOCTORADO– MODELO 15 b)

NOTIFICACIÓN AL INTERESADO DE LA CALIFICACIÓN DE LA LECTURA Y DEFENSA DE LA TESIS

Se hace constar que **Don Álvaro Baillo Moreno**

ha leído y defendido en el día de la fecha su tesis titulada "***OPTIMIZACIÓN DE LA EXPLOTACIÓN Y DE LA PREPARACIÓN DE OFERTAS DE UNA EMPRESA DE GENERACIÓN DE ENERGÍA ELÉCTRICA PARA MERCADOS DE CORTO PLAZO***"

ante el Tribunal constituido por los siguientes Profesores:

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Vocales: Dr. D. Bejamín F. Hobbs,
Dr. D. Francisco J. Prieto Fernández y
Dr. D. José Ignacio Pérez Arriaga
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EL SECRETARIO DEL TRIBUNAL

Destino de este documento una vez cumplimentado
Original: interesado

A Paloma

"Everything that is really great and inspiring is created by the individual who can labor in freedom."

Albert Einstein

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Notation

Indices

Symbol	Interpretation
c	contract;
g	generating unit;
h	hydro unit;
i	agent; firm;
j	component of a vector; agent; firm;
k	scenario; possible realization of an uncertain factor;
l	vertex of the set of possible solutions of a linear program; element of the set of accumulated iterations;
n	hour; hourly auction;
m	stage of a multistage decision process; market mechanism;
o	option;
p	period of time (day);
t	thermal unit;
ν	hour in which a thermal unit remains offline; current iteration of an algorithm;

Sets

Symbol	Interpretation
C^{D}	contracts for differences;
C^{P}	physical contracts;
H	hydro units;
J	segments used to approximate a curve; components of a vector;
K	possible realizations of an uncertain factor;
M	market mechanisms;
M^{E}	market mechanisms in which energy is traded;
M^{R}	market mechanisms in which reserve is traded;
N	hourly auctions;
O^{P}	options that are physically settled;
O^{F}	options that are financially settled;
$\mathrm{O}^{\mathrm{LPP}}$	long put positions that are physically settled;
$\mathrm{O}^{\mathrm{LPF}}$	long put positions that are financially settled;
$\mathrm{O}^{\mathrm{SCP}}$	short call positions that are physically settled;
$\mathrm{O}^{\mathrm{SCF}}$	short call positions that are financially settled;
P	periods (days);
P_{nk}^{m}	constraints affecting the value of option o in the n-th hour if market situation k occurs;
Q^{g}	feasible schedules for generating unit t;
Q_{n}^{m}	feasible outputs for the n-th hourly auction of the m-th market mechanism;
Q_{nk}^{m}	feasible outputs for the n-th hourly auction of the m-th market mechanism if market situation k occurs;
S	supply functions
T	thermal units;
U_{nk}^{m}	constraints affecting the exercise of option o in the n-th hour if market situation k occurs;
V^{O}	optimality cuts;
V^{F}	feasibility cuts;
X	feasible values for the vector of variables $\boldsymbol{x}$;
Λ	vertices of the set of possible solutions of a linear program;

Parameters

Symbol	Interpretation
A, B	matrices of coefficients;
$\boldsymbol{b}$	vector of parameters;
$\boldsymbol{c}$	vector of costs;
$\boldsymbol{e}$	vector of 1's;
f_t	variable fuel cost of thermal unit t, in €/Tcal;
i_n^h	net inflows received by the reservoir corresponding to hydro unit h in hour n, in MWh;
k_g	self-consumption coefficient of generating unit g, in p.u.;
l_t	ramp-rate limit of thermal unit t, in MW/h;
M^q	a large quantity;
M^p	a large price;
o_t	variable O&M cost of thermal unit t, in €/MWh;
p_j	j-th element of a vector of J prices;
p_n^c	exercise price of contract c in hour n, in €/MWh;
p_n^o	exercise price of option o in hour n, in €/MWh;
Q_{nk}^m	volume of energy traded in the n-th hourly auction of the m-th market mechanism if market situation k occurs, in MWh;
$q_{nk}^{m\ l}$	energy sold by the company in the n-th hourly auction of the m-th market mechanism in iteration l if market situation k occurs, in MWh;
q_n^c	amount of energy involved in contract c in hour n, in MWh;
q_n^o	amount of energy involved in option o in hour n, in MWh;
$\underline{q}_g, \overline{q}_g$	minimum and maximum gross output of generating unit g, in MW;
s_t	start-up cost of thermal unit t, in €/startup;
$\underline{w}_h, \overline{w}_h$	minimum and maximum storage level of the reservoir corresponding to hydro unit h, in MWh;
W_{h0}	available energy in the reservoir corresponding to hydro unit h at the beginning of the planning horizon, in MWh;
α_t	linear term of the heat rate function of thermal unit t, in Tcal/MWh;
β_t	intercept of the heat rate function of thermal unit t, in Tcal;
γ	Penalizing coefficient, in €/MWh;
δ_j	slope of the residual demand curve in its j-th segment;
ρ_j	slope of the revenue function in its j-th segment;
π_k	probability of scenario k;
ν_t	minimum number of hours that unit t must remain online, in h;

Symbol	**Interpretation**
η_h	Performance of the pump-turbine cycle of hydro unit h, in p.u.;
θ^l	value of the recourse function in iteration l of Benders' algorithm, in €;
φ^l	value of the feasibility function in iteration l of Benders' algorithm;
$\boldsymbol{\lambda}^l$	value of the dual variables of the optimality subproblem in iteration l of Benders' algorithm; value of the Lagrange multipliers in iteration l;
$\boldsymbol{\mu}^l$	value of the Lagrange multipliers in iteration l;
$\boldsymbol{\sigma}^l$	value of the dual variables of the feasibility problem in iteration l of Benders' algorithm;
σ_n^m	value of the company's market share in the n-th hourly auction of the m-th market mechanism, in € per unit;

Variables

Symbol	Interpretation
$\mathcal{B}$, $\mathcal{B}_i$	benefit, in €; benefit obtained by agent i , in €;
b_{nk}^{h}	energy pumped by hydro unit h in hour n if market situation k occurs, in MWh;
q	quantity;
p	price;
p_{nk}	spot price of electricity in hour n if market situation k occurs, in €/MWh;
p_{nk}^{o}	value of option o in hour n if market situation k occurs, in €/MWh;
p_{nk}^{m}	clearing price for the n-th hourly auction of the m-th market mechanism if market situation k occurs, in €/MWh;
q_{nk}^{g}	net energy produced by generating unit g in hour n if market situation k occurs, in MWh;
q_{nk}^{m}	energy sold by the company g in the n-th hourly auction of the m-th market mechanism if market situation k occurs, in MWh;
r	revenue, in €;
r_{nk}^{g}	amount of reserve provided by generating unit g in hour n if market situation k occurs, in MW;
s_{nk}^{h}	energy spilt from the reservoir corresponding to hydro unit h in hour n if market situation k occurs, in MWh;
t	independent variable;
u_{jnk}^{m}	binary variable indicating that the j-th element of the residual demand curve has been reached in the n-th hourly auction of the m-th market mechanism if market situation k occurs;
u_{nk}^{o}	exercise decision for option o in hour n if market situation k occurs;
u_{nk}^{t}	commitment state of thermal unit t in hour n if market situation k occurs;
$\boldsymbol{v}^{+}$, $\boldsymbol{v}^{-}$	vectors of slack variables;
v_{jnk}^{m}	incremental quantity corresponding to the j-th element of the residual demand curve in the n-th hourly auction of the m-th market mechanism if market situation k occurs, in MWh;
w_{nk}^{h}	energy stored in the reservoir corresponding to hydro unit h at the end of hour n if market situation k occurs, in MWh;
$x_{nkk'}^{m}$	binary variable indicating the relative position of two possible residual demand realizations, k and k', in the n-th hourly auction of the m-th market mechanism;
$\boldsymbol{x}$, $\boldsymbol{y}$	vectors of variables;
y_{nk}^{t}	start-up decision for thermal unit t in hour n if market situation k occurs;
z	approximate value of the dual function, in €;

Symbol	**Interpretation**
z_{nk}^{t}	shut-down decision for thermal unit t in hour n if market situation k occurs;
$\boldsymbol{\sigma}$	vector of dual variables corresponding to the feasibility problem;
$\boldsymbol{\lambda}$	vector of dual variables corresponding to the optimality subproblem; vector of Lagrange multiplers;
$\boldsymbol{\mu}$	vector of Lagrange multiplers;
θ	value of the recourse function, in €;

Functions

Symbol	Interpretation
$c(q)$, $c_i(q)$	cost function, in €; cost function for agent i, in €;
$c'(q)$, $c_i{}'(q)$	marginal cost function, in €/MWh; cost function for agent i, in €/MWh;
$c_{nk}^{t}\left(q^{t}, u^{t}, y^{t}\right)$	operating costs of thermal unit t in hour n if market situation k occurs, in €;
$D(p)$	demand function;
$f(\boldsymbol{x})$	objective function or part of the objective function;
$\boldsymbol{h}(\boldsymbol{x})$	vector of equality constraints $(=\mathbf{0})$;
$\boldsymbol{g}(\boldsymbol{x})$	vector of inequality constraints $(\leq \mathbf{0})$;
$\mathcal{L}(\boldsymbol{x}, \boldsymbol{\lambda}, \boldsymbol{\mu})$	Lagrange function;
$p_{nk}^{m}(q_{nk}^{m})$	inverse residual demand function for the n-th hourly auction of the m-th market mechanism if market situation k occurs, in €/MWh;
$p(t)$, $q(t)$	parametric equations used to represent an offer curve;
$r(q)$	revenue function, in €;
$R(p)$	residual demand function;
$R^{-1}(p)$, $p(q)$	inverse residual demand function;
$r_{nk}^{m}(q_{nk}^{m})$	revenue function for the n-th hourly auction of the m-th market mechanism if market situation k occurs, in €;
$S(p)$	supply function;
$w(\boldsymbol{\lambda}, \boldsymbol{\mu})$	dual function;
$\psi(q, p)$	market distribution function;
$\theta(\boldsymbol{x})$	recourse function;
$\tilde{\theta}(\boldsymbol{x})$	approximation of the recourse function by an outer linearization;
$\varphi(\boldsymbol{x})$	feasibility function;

Acronyms

Symbol	Interpretation
AGC	automatic generation control;
APX	Automated Power Exchange; Amsterdam Power Exchange;
CAISO	California Independent System Operator;
CALPX	California Power Exchange;
CAMMESA	Compañía Administradora del Mercado Mayorista de Electricidad (Marker Operator, Argentina);
CCGT	combined-cycle gas turbine;
CDEC	Centro de Despacho Económico de Cargas (Load Dispatch Center, Chile);
CfDs	contract for differences;
CSF	conjectured supply functions;
CV	conjectural variation;
DCR	demand curve for release;
DCS	demand curve for water in storage;
DP	dynamic programming;
DSP	dual of the subproblem;
DSPF	dual of the feasibility problem;
E&W	England and Wales;
ECNZ	Electricity Corporation of New Zealand;
EEX	European Energy Exchange (Frankfurt);
EMCO/M-Co	Electricity Market Company (New Zealand);
EMF	Energy Modeling Forum;
ESP	Energy Service Provider;
EU	European Union;
GA	genetic algorithms;
GAMS	General Algebraic Modeling Language;
GME	Gestore del Mercato Elettrico (Market Operator, Italy);
GNE	generalized Nash equilibrium;
GRTN	Gestore della Rete di Transmissione Nazionale (ISO and TSO, Italy);
GS	subproblem corresponding to a certain generation unit;
HHI	Hirschman-Herfindahl Index;
IEA	International Energy Agency;
IOU	independent owned utility;

Symbol	Interpretation
IPE	International Petroleum Exchange;
ISO	Independent System Operator;
LP	linear programming;
LPX	Leipzig Power Exchange;
LR	Lagrangian relaxation;
MCP	mixed complementarity problem;
MD	master dual problem;
MDF	market distribution function;
MEM	Mercado Eléctrico Mayorista (Wholesale Electricity Market, Argentina);
MIP	mixed linear-integer programming;
MO	Market Operator;
MP	master problem;
MPEC	mathematical programming with equilibrium constraints;
MS1	subproblem corresponding to the first market mechanism;
MS2	subproblem corresponding to the second market mechanism;
NEMMCO	National Electricity Market Management Company Ltd.
NETA	New Electricity Trading Arrangements;
NGC	National Grid Company;
OASIS	Open Access Same Time Information System;
OFGEM	Office and Gas and Electricity Markets;
OMEL	Compañía Operadora del Mercado Eléctrico (Market Operator, Spain);
OO	ordinal optimization;
OPF	optimal power flow;
OPS	subproblem corresponding to options that are financially settled;
OTC	over the counter;
P	primal problem;
PBC	physical bilateral contract;
PC	personal computer;
PJM	Interconnection formed by Pennsylvania, New Jersey, Maryland, Delaware, Virginia and the District of Columbia;
PWL	Piecewise linear;
PX	Power Exchange;

Symbol	Interpretation
QVI	quasi-variational inequalities;
REE	Red Eléctrica de España (ISO and TSO, Spain);
SC	scheduling coordinator;
SDDP	stochastic dual dynamic programming;
SFE	supply function equilibrium;
SMP	system marginal price;
SP	subproblem;
SPF	feasibility problem;
TSO	Transmisión System Operator;
TSP	two-stage program;
UKPX	UK Power Exchange;
US	United States;
VI	variational inequalities;
VPX	Victoria Power Exchange;

Summary

This thesis addresses the problem of developing an optimal offering strategy for a generation company operating in an electricity spot market, an issue of the maximum relevance due to the process of regulatory reforms that the worldwide power industry has experienced in recent years.

Nevertheless, the idea of constructing optimal offers for an electricity spot market is too general to be developed in a straightforward manner. The variety of spot market designs that can be found throughout the world is so wide that it is impossible to propose a general methodology valid for them all. Therefore, this thesis will commence with an overview of the most relevant spot market designs currently in operation. Based on this description and analysis, the rules that govern the spot market assumed in this thesis will be clearly defined.

The search for an optimal offering strategy requires evaluating the expected benefit of any candidate strategy. In particular, a generation company must be able to estimate the revenues that it expects to obtain in the spot market. This implies modeling the rivals' behavior, given that rivals exert a direct influence on the spot price of electricity. Among the variety of models that have been recently proposed to represent competition in wholesale electricity markets, this thesis should adopt the approach that best adapts to its general objective. As a result of a literature survey, a representation based on residual demand curves and revenue functions will be used to calculate the outcome of the hourly auctions that constitute the spot market of study.

Uncertainty with respect to rivals' behavior is at the root of the development of any offering strategy. It is because of this uncertainty that a generation company gets involved in a decision process more complex than simply choosing a single price for all its output or a specific level of production. However, not every representation of this uncertainty is equally amenable in order to search for an optimal offering strategy. In this thesis, uncertainty about the strategies followed both by rivals and by wholesale buyers in each of the hourly auctions that constitute the spot market will be represented by assuming that the probability distribution of the corresponding residual demand curve has finite support. In other words, it will be assumed that each hourly auction has a limited number of possible outcomes. Given that the spot market of study consists of a sequence of auctions, this approach will yield a representation of the spot market in the form of a multistage stochastic program. It will be shown that this representation is valid not only for the case of a generation company, but also for other agents operating in a spot market such as energy service providers.

The previous multistage stochastic programming framework will be enriched with a detailed model of the company's portfolio. This model will consider each of the company's generation units, including their production costs and technical constraints. It will also take into account the obligations assumed by the company in previous market mechanisms, such as futures or options markets.

Although this thesis focuses on the development of strategies for market mechanisms that operate on a daily basis, the main objective of a generation company is the maximization of its long-term profit. Hence, we should include some sort of guideline in our methodology, so as to orient its results toward this long-term objective. With this purpose, an explicit valuation of the market share obtained by the company in the spot market will be suggested in order to correct the myopic incentive that a generation company has to reduce its sales and increase the spot price of electricity. We will

evaluate the extent to which this parameter correctly represents the long-term objectives of the company from a short-term perspective.

The size of the mathematical program that results when the abovementioned modeling features are put together is unmanageable for current commercial optimizers. In order to improve its numerical tractability, we will have to assume that the relative importance of the spot market mechanisms diminishes as the moment of physical delivery gets nearer. Under this assumption, the problem of choosing an optimal strategy for the spot market will adopt a twofold structure. On the one hand, the problem of developing optimal offers for a specific market mechanism will turn out to be a two-stage stochastic program, taking into account that recourse actions can be adopted in subsequent market mechanisms in order to correct any undesired result. On the other hand, the problem of deciding an optimal weekly unit-commitment schedule will be formulated as a sequence of two-stage stochastic programs. These two aspects of the operation of a generation company in a spot market are mutually consistent.

Both problems require the use of decomposition techniques so that realistic study cases can be formulated and solved under this framework. An analysis of the structure of both problems will be performed in order to identify the decomposition technique that best suits each of them. In the light of this analysis, Benders' decomposition will appear as the most adequate approach to solve the first type of problem, given that it adapts well to its two-stage structure. In contrast, Lagrangian relaxation will be the solution method chosen to address the weekly unit-commitment problem, due to the generalized presence of binary variables. The application of both decomposition techniques will be explained in detail. In particular, the formulation of the Lagrange function will provide an interesting economic interpretation of the Lagrange multipliers and permits a better understanding of the problem.

The adequacy of the methodology developed in this thesis will be confirmed by the results obtained for a collection of realistic numerical examples. A variety of offering strategies will be derived for a generation company participating in a specific session of the Spanish day-ahead market under different circumstances. The sensitivity observed in the solutions proposed by our methodology with respect to a number of relevant factors will confirm its consistency. Additionally, a weekly stochastic unit-commitment schedule will be obtained for the same generation company.

1 Introduction

This introductory chapter presents the general background that motivates this thesis: the regulatory reform that has recently shaken the foundations of the worldwide electric power industry. Due to this reform, generation companies have been forced to adapt to a new way of understanding their business. The traditional regulation, which typically guaranteed generation companies the full recovery of their costs, has been substituted by a competitive framework in which companies' revenues depend on their ability to sell the energy produced by their plants. Among the variety of wholesale market designs that have been implemented throughout the world to introduce competition in the power industry, a relevant number of them include a daily spot market organized as a sequence of auctions. By submitting offers and bids to this spot market, participants are able to define their ultimate position for each trading period. Hence, in many cases, generation companies have had to learn how to express their operational decisions in terms of offering strategies.

This thesis aims to develop an original methodology to construct optimal offers for generation companies operating in electricity spot markets. The achievement of this general objective requires the fulfillment of a number of more specific targets that are identified in this chapter. The organization of this dissertation is also briefly described in order to facilitate its reading.

1.1 The reform of the electric power industry

During the last fifteen years the electric power industry has undergone an unprecedented worldwide process of reforms that has lead to a new way of understanding the supply of electric power. Traditionally, an electric utility assumed the obligation of satisfying the demand for electricity of a certain franchise region. Utilities covered all the phases of supply, including the generation of electricity, its transmission through a high voltage grid toward consumption centers and its distribution to final consumers through medium/low voltage networks. In addition to being in charge of the operation, maintenance and expansion of the generating system and the electric networks, utilities carried out commercial activities such as metering and billing. In general, regulatory authorities guaranteed utilities the full recovery of their costs, as long as their decisions were oriented to cover consumers' demand at the minimum cost.

In the second half of the twentieth century, the gradual expansion of power systems lead to an increase of wholesale electric energy interchanges between neighboring utilities in many regions throughout the world. In some cases, several utilities decided to coordinate the operation of their interconnected control areas in order to increase the efficiency and reliability of their supply. The creation of power pools comprising generating plants, transmission networks and distribution systems of several utilities soon uncovered the conceptual differences that exist between the different activities involved in the supply of electricity. On the one hand, the operation, maintenance and expansion of electric networks are activities that are more efficiently carried out by a single entity. Hence, network businesses can be considered as natural monopolies. On the other hand, the generation of electricity and the commercial aspects of retail supply are activities that can be assumed by a number of competing companies, as long as they are allowed to make use of the existing transmission and distribution facilities.

With the aim of increasing the efficiency of electric supply, a few pioneering countries embarked themselves on the adventure of introducing competition in the power industry. This lead to the creation of wholesale electricity markets, mainly consisting of centralized daily spot markets similar to the preexisting power pools. Indeed, generation companies willing to sell energy in these pioneering spot markets were required to submit information relative not only to their production costs, but also to the technical characteristics and constraints of their units. Based on this information, generation units were dispatched according to the results of complicated optimization procedures. In general, the demand side did not take part in these spot markets. The spot price of electricity for each time period was determined by the variable cost of the last generating unit required to cover the demand for electricity in that period.

Due to the experience gained by regulatory authorities, a second generation of wholesale electricity markets has been conceived that increases the freedom of participants to take their own decisions. A spot market design based on the concept of auction has been developed in which generation companies are required to tender offers and wholesale buyers have to submit bids in order to perform energy transactions. The agents are expected to internalize any particular aspect of their operation in the price of their offers and bids. The spot price of electricity in each time period and each location is obtained as a result of the clearing of these auctions.

In this context, generation companies and wholesale buyers assume a more intense risk exposure than in the traditional framework. They are both subject to the uncertain spot price of electricity, which conditions their revenues and costs, respectively. In order to hedge against this new risk, participants typically carry out part of their transactions through mechanisms other than the spot market, such as bilateral contracts or futures markets. The usage of electricity derivative products is also frequent in mature power markets, reaching transaction volumes that exceed several times those observed in the spot market.

This ongoing process of reforms has triggered a formidable effort from researchers, observers and practitioners oriented to the analysis, representation and estimation of the strategic behavior of participants. The particular features of electricity as a product, together with its central role in the economy of both developing and developed countries undoubtedly increase the relevance of these studies. Although great advances have already been achieved, there are still many controversial open issues, particularly with respect to the design of wholesale electricity markets.

The generation business has been greatly affected by these reforms. Competition has been introduced in this activity in a straightforward manner, in contrast with the typical gradual process followed by regulatory authorities when liberalizing the business of retail supply. As a consequence, generation companies have had to adapt themselves to a new way of making their decisions. Their revenues and the operation of their generation portfolio are now conditioned by the success of their strategies in a variety of market mechanisms. In particular, the spot market is frequently seen as the most relevant trading arena, given that the spot price of electricity is used as a reference for other transactions.

This thesis adopts the perspective of a generation company operating in a wholesale electricity market. The transition from a framework in which companies were guaranteed the full recovery of their fixed and variable costs to a situation in which revenues are uncertain and depend on the companies' strategies has created an urgent need for the development of new decision-support tools. These new methodologies still contemplate the time hierarchy that has traditionally structured the management of generation systems into long, medium and short-term decisions. Their most relevant novelty lies in that revenues are incorporated into the companies' objective function, so that decisions are oriented to the maximization of their profits.

In particular, this thesis focuses on the problem of developing optimal offering strategies for an electricity spot market. This is a task that generation companies must address on a daily basis and that significantly conditions the revenues they obtain for the energy they produce. The development of an offering strategy for the spot market currently constitutes one of the main short-term decisions of a generation company, given that it yields the ultimate position assumed by the company in the wholesale electricity market and the final schedule of its generating units.

In addition to offering a general overview of the process of reforms that has recently revolutionized the power industry, this chapter specifies the objectives that this thesis aims to achieve and justifies their relevance. It also describes the organization of this dissertation so as to facilitate its reading and provide a guideline to better understand the analysis, proposals and conclusions that constitute this work.

1.2 Scope and objectives of this thesis

As has been mentioned, this thesis aims to propose a methodology to develop optimal offering strategies for generation companies operating in electricity spot markets. Such an objective is of vital importance in the current context of regulatory reforms but is too vague to be addressed in a straightforward manner. This section provides a description of the steps that must be covered in order to achieve this target.

The process of reforms experienced by the power industry in recent years has lead to the implementation of an increasing number of regional wholesale electricity markets. Although the authorities that have promoted this regulatory change in each part of the world share a general body of ideas and objectives, individual power markets have peculiarities that make it impossible to find two identical designs. The characteristics of both the generation portfolio and the transmission network of each power system are at the root of these distinctive features. Additionally, each regulatory authority adds its particular flavor when defining the details of the market rules[1]. From the perspective of the research community, this variety of situations complicates the identification of a general methodology to address the problems faced by generation companies in the new framework, particularly in the spot market, thus leading to a certain degree of confusion in the literature. Therefore, the first objective of any work devoted to the development of a methodology that aims to solve some of these problems is to provide a clear definition of the rules that dictate how energy is traded in the market mechanisms of study. However, these rules should be defined with the minimum possible degree of detail in order to avoid an unnecessary loss of generality. In this thesis, a spot market design based on a sequence of daily market mechanisms will be considered. Each of these market mechanisms will be constituted by hourly uniform-price auctions. This general framework is valid for a relevant number of real spot markets currently in operation but is obviously incompatible with recent pay-as-bid proposals. Nevertheless we will argue that the general approach followed in this thesis can be applied to address any wholesale market design.

Due to the introduction of competition, generation companies are plunged into the treacherous waters of uncertainty. Indeed, the benefit of a generation company operating in a wholesale electricity market is subject to a variety of uncertain factors. Some of these, such as the growth of demand, the volume of hydro inflows or the evolution of fuel prices, were already present and well studied before the reforms. However, the main source of uncertainty is new and stems from the strategic behavior of rivals, given that their decisions exert a great influence both on the energy sold by the company and on the price of electricity. As a consequence, in order to evaluate the expected benefits of a certain strategy, a generation company must be able to estimate the behavior of its competitors. This requirement has seduced a great number of researchers that have suggested a variety of approaches to model competition in wholesale electricity markets. The difficulty inherent to these competition models is the

[1] It is common knowledge that the devil is in the details. Hence, these slight differences may play a central role in the outcome of the reforms.

dramatic loss of computational tractability they suffer when they are required to represent complex and realistic situations. Hence, choosing the right modeling assumptions for each situation is a challenge. This leads to the second objective of this thesis: among the variety of competition models suggested in the literature the most adequate modeling approach for the purposes of this thesis must be chosen. A literature survey is of course implicit in this objective.

In addition to evaluating the influence that rivals' decisions exert on the company's revenues, a methodology intended to provide optimal offering strategies for an electricity spot market must include an explicit representation of the uncertainty faced by the company. This requirement is by no means easy to satisfy. Indeed, the majority of methods proposed in the literature to represent this uncertainty present serious limitations concerning their applicability to real study cases. Another objective of this thesis is then to incorporate uncertainty into its developments in a way that permits addressing realistic situations. The accomplishment of this objective will imply the adoption of a specific mathematical approach to evaluate and optimize offering strategies for the spot market. It will be shown that a multistage stochastic programming framework is particularly suitable for this type of problem

The previous two objectives focus on the representation of the spot market from the perspective of a generation company. Even though this aspect is particularly attractive due to its novelty, other facets of the generation business must be considered when developing an offering strategy for the spot market. The portfolio of a generation company dramatically determines the strategies that it can actually adopt. The core of the company's portfolio is constituted by its generating units, whose specific characteristics have to be taken into account. The operation of these units can be represented in a manner similar to the one typical in traditional short-term operation-planning tools such as economic-dispatch or unit-commitment models. Other relevant elements of the company's portfolio are the open positions that it may have assumed in market mechanisms other than the spot market, such as long-term bilateral contracts or electricity derivatives. The consideration of these positions is of paramount importance in order to correctly evaluate the real influence that the spot price of electricity has on the company's revenues. Consequently, the methodology proposed in this thesis has also the objective of including a full representation of the company's portfolio.

Numerical tractability is a major bottleneck for a methodology that aims to consider the influence of rivals' behavior, include an explicit representation of uncertainty and model in detail the company's portfolio. The size of the mathematical program that results when the problem of developing an optimal offering strategy is formulated in this manner is unmanageable for the existing commercial optimizers. The use of decomposition techniques is required to address the numerical resolution of such a problem. Therefore, one additional objective of this thesis is to derive a decomposition scheme that succeeds in overcoming this difficulty.

In order to illustrate the potential of the methodology developed in this thesis, numerical examples must be provided. Two types of study cases are particularly appropriate to prove the validity of the proposed approach. On the one hand, small problems are more suitable to show the features of the suggested mathematical formulation and to evaluate the performance of decomposition techniques. On the other hand, the solution of a series of large realistic problems can be used to confirm the adequacy of the developments of this thesis. This is the last objective that this thesis aims to cover.

To summarize, the following six steps must be taken in order to develop a procedure that provides optimal offering strategies for a generation company operating in an electricity spot market:

i) Define the rules that dictate how electricity is traded in the market mechanisms of interest, trying to reach a tradeoff between generality and specificity.

ii) Review the models of competition proposed in the literature for the case of wholesale electricity markets and identify the most appropriate one, given the purposes of this thesis.

iii) Suggest a consistent approach to represent the uncertainty faced by a generation company in an electricity spot market.

iv) Take explicitly into consideration the company's portfolio including not only its generation assets but also its open positions in market mechanisms other than the spot market.

v) Devise a decomposition scheme that permits the numerical resolution of realistic problems.

vi) Solve a collection of numerical examples so as to illustrate the features of the proposed methodology.

1.3 Organization of this document

The sequence of six stages that have to be covered in order to satisfy the general objective of this thesis provides a consistent structure for the organization of this dissertation. Indeed, each of the chapters of this document corresponds to one of these steps. In this manner, the accomplishment of these partial objectives can be more clearly justified.

Chapter 2 defines the electricity marketplace considered in this thesis. In order to do so, a general overview of the different designs adopted throughout the world to establish wholesale electricity markets is provided. In the light of this panoramic perspective, the rules of the market mechanisms that are relevant for the developments of this thesis are specified. In particular, a spot market design based on a sequence of market mechanisms constituted as sets of hourly auctions is assumed.

Chapter 3 is oriented to the identification of the most adequate approach to represent competition in the context of this thesis. It includes a survey of the variety of lines of research that have been suggested in the literature in recent years. Special attention is given to modeling proposals aiming to represent the influence of the spot market on the short-term operation of generation companies. The analysis of their advantages and shortcomings reveals the specific features that the competition model considered in this thesis should include. A number of modeling challenges are identified whose simultaneous accomplishment would constitute a significant advance with respect to previous developments.

Chapter 4 provides a comprehensive description of the methodology proposed in this thesis. It shows how the problem of deciding an optimal offering strategy for a generation company operating in an electricity spot market can be formulated as a multistage stochastic program. This approach permits an exhaustive representation of the spot market operation including the explicit consideration of uncertainty. This chapter also explains in detail the manner in which the company's portfolio is represented, taking into account not only its generation assets, but also the obligations assumed in previous market mechanisms. The combination of all these aspects results in a large-scale mathematical program whose formulation covers the modeling challenges identified in chapter 3, thus constituting the conceptual core of this thesis.

Chapter 5 is devoted to the definition of a solution approach that provides numerical results for the proposed mathematical model. The analysis begins with the identification of certain minor simplifications in the representation of the spot market that permit the otherwise unmanageable numerical resolution of the problem. Two perspectives are adopted to develop a short-term strategy for a generation company operating in a spot market. On the one hand, a weekly multistage stochastic programming approach can be used to obtain optimal unit-commitment schedules as well as guidelines to distribute available hydro reserves during the week. On the other hand, the problem of developing an optimal offering strategy for each spot market mechanism is formulated as a two-stage stochastic program. A detailed analysis of the structure of both mathematical programs permits the identification of the decomposition technique that best suits each of them. In the light of this analysis, Lagrangian relaxation is seen as the most appropriate decomposition technique for the weekly problem, given the generalized presence of binary variables. In contrast, Benders' decomposition adapts well to the two-stage structure of the second type of problem. In this manner, both decomposition techniques provide a powerful framework to make short-term strategic decisions in the context of an electricity spot market.

Chapter 6 presents a battery of numerical examples in order to illustrate the performance of the proposed solution strategy. The first part of the chapter describes the results obtained when applying Benders' decomposition to a variety of realistic study cases in which a fictitious but representative generation company decides its offering strategy for a certain session of the Spanish day-ahead market. An analysis of the sensitivity of the solutions with respect to different relevant factors is included. The second part of the chapter concentrates on the solution of a numerical example of the weekly multistage stochastic program using Lagrangian relaxation. This chapter constitutes the ultimate confirmation of the flexibility and adequacy of the methodology proposed in this thesis.

Chapter 7 summarizes the conclusions that can be derived from the analysis, developments and results included in this thesis. It also enumerates the main original contributions that this work has yielded. Finally, a number of future lines of study that stem from the research conducted in this thesis are suggested.

2 The electricity marketplace

The variety of approaches adopted by regulatory authorities and governments throughout the world to introduce competition in the wholesale supply of electricity renders it impossible to identify a general framework for this thesis that includes them all. However, in order to achieve the proposed objectives, a particular setting must be defined. This chapter specifies the marketplace attributes considered for the developments of this thesis. The institutions, agents and market mechanisms to be mentioned henceforth are defined under the light of several illustrative real cases. Special attention is paid to the rules governing the mechanisms that constitute the spot market.

Even though this chapter may not be considered an original contribution of this thesis, it certainly aims to provide the background to fully understand the problems addressed further on and the hypotheses under which the proposed solutions hold.

2.1 Introduction

The speed at which reforms are introduced in the power industry of an increasing number of countries and the variety of adopted designs justify the need for a formal definition of the particular wholesale electricity market that will be considered as a framework for the developments of this thesis. Several authors have previously addressed the laborious task of suggesting a full characterization for the existing market designs [Kahn '95, Chao '99]. It is beyond the scope of this thesis to offer a complete and up-to-date review of the situation [Wolak '97, IEA '01]. However, in order to illustrate the hypotheses assumed, reference will be made to the solutions implemented in several real cases and to the opinion of experts in the organization of wholesale electricity markets.

This chapter starts by recalling Schweppe's pioneering definition of the spot price of electricity, a starting point and a benchmark for alternative market designs. In section 3 we will specify the hypotheses relative to the infrastructure of the conceptual power system that will be analyzed in this thesis. Section 4 introduces the institutions that are relevant for the operation of the considered wholesale electricity market and describes the roles they play. Section 5 identifies and characterizes the market participants whose strategies will be examined hereafter. Section 6 is devoted to the definition of the rules that will govern the wholesale market mechanisms through which electricity is traded. Section 7 summarizes the assumptions made throughout the chapter and provides additional details about the marketplace considered in this thesis. Finally, section 8 highlights the most relevant conclusions of the chapter.

2.2 Schweppe's electricity spot market

Due to the special features that characterize electricity as a commodity, the pioneering theories for the design of electricity spot markets[1] were founded on the physical laws that rule power flows through transmission networks (Kirchhoff laws). The electricity spot pricing theory developed in [Schweppe '88] defines the optimal spot price of electricity at a certain node of the transmission network as the incremental cost of supplying one additional energy unit at that node when the system is dispatched in such a way that the net social benefit is maximized. These spot prices account for electricity variable generation costs (including energy and ancillary services), but also for transmission losses and congestions. They constitute the correct economic signals for both producers and consumers that induce them to behave efficiently. However, in practice, calculating Schweppe's electricity spot prices requires the existence of a centralized authority that solves a non-linear and non-convex optimization problem based on detailed information about the elements that constitute the power system. Participants may not be satisfied by the outcome of this obscure market mechanism whose results are extremely sensitive to input data. Lack of transparency may also deter the entrance of new participants, thus limiting the degree of liquidity that can be attained.

An interesting property of Schweppe's spot prices is that, under the hypothesis of a loss-less and uncongested network, the spot price is the same for all the network nodes and equal both to the short-term marginal costs of generation and to the value of consumers' short-term marginal utility. This single-node assumption significantly

[1] In a spot market for a certain commodity, agreements imply immediate physical delivery. Organized spot markets usually provide reference prices for other transactions referred to the same commodity.

simplifies the underlying optimization of Schweppe's spot market definition and paves the way toward simpler market designs similar to those established for other commodities, characterized by their transparency and high degree of decentralization in decision-making. However, this casts a shadow of insecurity over an industry that provides an essential commodity and where reliability of supply has always been a totem.

The conflict between these two trends —tight control by a central authority or decentralized decision-making— and the question of which of the two yields a higher degree of efficiency is at the root of the debate on how wholesale electricity markets should be designed. Although some authors clearly advocate for one of the two alternative frameworks [Hogan '01], in most cases the implementation of hybrid formulas performs correctly.

2.3 Infrastructure of the power system

It is obvious that the choice of an adequate design for a wholesale electricity market is heavily conditioned by the infrastructure of the corresponding power system, particularly that of the transmission network. Throughout most of this thesis, it is assumed that the transmission system does not impose significant constraints for the physical realization of energy transfers between the agents that participate in the wholesale electricity market. It is also assumed that transmission losses account for a low percentage of the overall costs.

This hypothesis is quite restrictive. As a rule of thumb, as the area covered by the market increases, the limitations due to limited transmission capacity are more important[2]. Thus, transmission constraints usually have a minor impact for the analysis of an electricity market restricted to the territory of Spain or England and Wales. On the contrary, they play a major role in Scandinavia, Australia, New Zealand, Argentina, Chile, the Northeastern and Western coasts of the U.S., and a future hypothetic European electricity market. This presumption simplifies the analysis, which would otherwise become cumbersome, and facilitates the achievement of the objectives of this thesis. Nevertheless, we suggest possible solutions to adapt the developments of this work to those cases in which the influence of the transmission network cannot be neglected. One of the advantages of the proposed approach is that it is flexible enough to incorporate new modeling features different from those adopted in principle.

The existing generation mix has also proven to be a determining factor for the performance of wholesale electricity markets. Power systems with a large hydro component but with irregular inflows (e.g. Colombia) require innovative market mechanisms so that participants can hedge against unacceptable risk exposure due to extreme price instability [Vázquez '01]. Hydro systems may also be constituted by a large number of independent reservoirs located in a reduced set of river basins (e.g. Brazil), yielding higher benefits when coordinately operated than if independently administered. Conversely, if generation is mainly gas-fired, electricity prices will be driven by the fluctuations of international gas prices. In this thesis, however, the generation system is assumed to include a variety of generation technologies (nuclear, coal, oil, gas, hydro), so that the proposed approach can be easily adapted to particular cases such as the abovementioned situations.

[2] The shape of the considered area is also important: Chile and New Zealand are two good examples.

2.4 Institutions

Organizing a wholesale electricity market requires creating new institutions and redefining the roles played by preexisting ones. In this section, the main institutions that will be mentioned in subsequent chapters are defined.

2.4.1 Regulatory authorities

Every process of reform is launched with the contribution of one or several regulatory authorities (regulatory commissions, regional administrations, national governments or supra-national institutions as in the case of the European Union). These authorities determine the new rules under which the corresponding wholesale electricity market will operate, supervise the reform process and introduce further modifications if the results diverge from the objectives that motivated the reform (typically, the improvement of the industry's efficiency standards).

Regulatory authorities guarantee that the agents participating in the wholesale electricity market observe the established rules. In particular, great concern has been expressed about the possible exercise of market power[3] by large generation companies, which would increase the price of electricity well above marginal costs. This possibility exists in most real cases, particularly in situations of scarce generation or transmission capacity, and is well documented[4]. However, market participants are not expected to keep the exercise of market power effective in the long run, as this would attract new entrants and increase the threat of adverse regulatory measures [Acutt '01]. Moreover, it has been shown that the operating profits of a generating unit controlled by a company that uses it to exercise market power are less than the operating profits of an identical generating unit controlled by a small fringe company, an effect known as “the curse of market power” [Lien '00]. Consequently, in this thesis the exercise of market power will not be considered a long-term sustainable strategy. On the contrary, generation companies will be assumed to value their market share as a measure of the strategic position they aim to defend[5].

> The restructuring and privatization process initiated in England and Wales (E&W) at the beginning of the 1990s with the Pooling and Settlement Agreement gave birth to the Pool, a mandatory wholesale electricity spot market with daily sessions. Every day, the National Grid Company (NGC), obtained a day-ahead estimate of the system's demand, scheduled generation to meet this estimate and determined Pool prices. Due to the high prices exhibited by the Pool, after several years of operation, the two major generation companies were forced twice to divest part of their assets. Their market share was further reduced due to new entry in the form of combined-cycle gas turbines (CCGTs). Eventually, after more than a decade of controversial operation [Green '99] the regulatory commission, OFGEM, reviewed the design of the Pool and decided to replace it with more transparent and simple trading arrangements, namely the New Electricity Trading Arrangements (NETA) [OFGEM '99].

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[3] Market power is defined as the ability of a firm to raise prices profitably above competitive levels for a significant period of time.

[4] [Borenstein '99] provides a thorough explanation of the concept of market power and its implications for wholesale electricity markets. Other relevant works oriented to the analysis of market power are mentioned in Chapter 3.

[5] However, under this assumption, barriers to entry are a relevant reason of concern [Viscusi '98].

2.4.2 Independent system operator (ISO)

The correct performance of a bulk power system requires the existence of one institution that guarantees that the wholesale supply of electricity is carried out under adequate conditions of quality, security and reliability. With the introduction of competition, this institution is also usually required to be independent from any agent having interests in the wholesale electricity market. We will refer to this entity as the independent system operator (ISO). In some cases, the ISO is also the owner of the high voltage transmission facilities, although this has a negligible effect for the purposes of this thesis.

In order to comply with its responsibilities, the ISO frequently assumes several functions that can be characterized according to the time frame they refer to. In the long term (years), investments in transmission capacity expansion have to be evaluated. In the medium term (months), the maintenance of network elements must be planned. In the short term (one day to a week), scarce transmission capacity must be allocated and the procurement of ancillary services has to be scheduled. In real time (every second), the operation of the power system has to be monitored and the instruments required to keep the system's variables between their limits must be coordinated. Thus, the activity of the ISO increases as the moment of physical delivery gets closer in time.

> The independence of the system operator was not considered in Chile's pioneering reform, where a system operator owned by the generators manages the mandatory load dispatch center (Centro de Despacho Económico de Cargas, CDEC). The fact that its decisions are based on complicated and non-transparent procedures has been the cause of increasing disputes between generators [Rudnick '97].

2.4.3 Market authority

In nearly all cases, the introduction of competition in the power industry has been accompanied by the creation of a series of official market mechanisms through which wholesale transactions can be performed. Three organizational models can be identified according to the authority that assumes the role of administering these market mechanisms.

In the first model, market mechanisms are centralized and managed by the ISO. Under this framework (sometimes designated by the term *pool* or *poolco*), the ISO exerts a tight control over the wholesale electricity market, which probably results in a more efficient short-term operation and a smoother transition to competition. In some cases participation is mandatory. This reduces the flexibility of market participants to make their own decisions. In the long term, the lack of transparency that characterizes the ISO's decision making may discourage new entrants and benefit incumbent generators. Additionally, demand-side responsiveness may be inhibited under the excessive protection of the ISO.

> In Chile, CDEC is responsible both for meeting the demand requirements at the least possible cost and for preserving the security of the system. Similarly, in Argentina, the same company that operates the transmission system, CAMMESA, runs the official wholesale electricity market (Mercado Eléctrico Mayorista, MEM) [Rudnick '97, Gómez '00].

In Australia, an interstate market has been progressively implemented. The Victoria Power Exchange (VPX) began to operate in July 1994, while New South Wales established a wholesale market for electricity in May 1996. Finally, in 1998, Australia's National Electricity Market was created, comprising the interconnected power system formed by the Australian Capital Territory, New South Wales, South Australia and Victoria [Outhred '00]. This mandatory pool is operated by NEMMCO, which is also responsible for the security of the interconnected power system and the coordination of power system planning. [NEMMCO '99].

In some regions of the North-Eastern U.S. (New England, New York and the PJM interconnection formed by Pennsylvania, New Jersey, Maryland, Delaware, Virginia and the district of Columbia), centrally dispatched power pools had been in operation for many years before competition was introduced. In each of these regions a single market institution, the Independent System Operator (ISO), has been established, being basically an extension of the preexisting pool. Recently, an initiative to create a Regional Networked Market including Allegheny Power, New England, New York and PJM has been launched.

In the second model, the main wholesale market mechanisms are also centralized, but managed by an entity different from the ISO, usually known as the market operator (MO), and organized in the form of a power exchange (PX). These market mechanisms are usually based on simple rules, leaving to the agents the full responsibility of deciding the strategy they want to follow in order to sell or purchase electricity. Participants have to learn how to reach their objectives under the new scheme, a process that may require several years, particularly in the case of energy buyers. A careful coordination must be established between the MO and the ISO in order to guarantee that the transactions performed through the PX are physically feasible and comply with reliability and security criteria.

The California electricity market was organized around two different institutions: the California ISO (CAISO) and the California Power Exchange (CALPX). CAISO was made responsible for operating the transmission network, guaranteeing the real-time balance between generation and demand and scheduling the procurement of ancillary services. CALPX was created as a voluntary energy exchange where agents could trade energy (the major Californian utilities were forced to operate only through the CALPX during the first four years).

The core institution of The Nordic Power Market (Norway, Sweden, Finland and Denmark) is a non-mandatory Power Exchange, the NordPool, which operates a series of market mechanisms. NordPool also offers clearing services for contracts traded in over-the-counter (OTC) and bilateral markets[6]. About 25 % of the Nordic power generation is traded through the NordPool, while bilateral and OTC contracts account for the rest. The volume of financial contracts is estimated to be five times the volume of physical transactions. The liquidity and high level of activity achieved by the Nordic Power Market, where a careful coordination is required between the existing transmission system operators, is a success.

During the decade of the 1990s, the electricity industry in New Zealand underwent a series of reforms in order to improve its performance, eliminate cross-subsidies and introduce customer choice [EMPG '01]. The progressive privatization of generation assets owned by the previous monopoly (Electricity Corporation of New Zealand, ECNZ) was accompanied by the establishment in 1988 of an independent state-owned transmission system operator (Transpower) and the creation in October 1996 of a wholesale electricity market operated by the Electricity Market Company (EMCO or M-Co).

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[6] Over-the-counter transactions, can be carried out on whatever terms and with whatever provisions are permitted by law and acceptable to the two counter parties. There are no official rules or restrictions governing the hours or conditions of trading. Trading conventions are developed mostly by market participants. There is no official code prescribing what constitutes good market practice. On the other hand, in organized exchanges, trading takes place publicly in a centralized location. Hours, trading practices and other matters are regulated by the particular exchange and products are standardized. There are margin payments, daily marking to market, and cash settlements through a central clearinghouse.

In Spain, the enactment of law 54/1997 in November 1997 established a fully competitive framework for the generation of electricity. At the same time, a transient process was defined for the liberalization of retail supply. Two institutions are at the heart of the Spanish wholesale electricity market design. On the one hand, the market operator (Compañía Operadora del Mercado Eléctrico, OMEL) is in charge of the series of voluntary short-term market mechanisms through which the majority of physical transactions take place (the volume of bilateral contracts is still low). On the other hand, Red Eléctrica de España (REE), is the owner of the high-voltage transmission network and has been the system operator since 1984.

Italy implemented the EU Directive 96/92/CE[7] with the passing of legislative decree no. 79 in March 1999. The reform introduces new institutions, such as an independent state-owned transmission system operator (Gestore della Rete di Transmissione Nazionale, GRTN) which is responsible for the management of the national transmission grid and for ensuring safety, reliability and the least-cost operation of the electric system while providing a non-discriminatory access to the grid. This TSO has established a market operator (Gestore del Mercato Elettrico, GME) that will manage a national wholesale exchange system.

The third alternative relies on pure bilateral trading and does not require the existence of a centralized market authority. However, the absence of centralized market mechanisms has evident limitations such as large transaction costs, insufficient liquidity, lack of reliable information about market performance and the difficulty to reach the level of coordination required to ensure a secure operation of the power system. Indeed, in those cases where no official market mechanisms have been defined, unofficial power exchanges typically surge driven by private initiative.

Under NETA, the E&W system operator, NGC, is no longer responsible for the operation of the main market mechanisms. On the contrary, NGC is in charge of a short-term balancing mechanism designed to ensure the physical balance of the system. In addition to this, NGC must procure ancillary services such as reserve, frequency control and voltage support, as well as energy to overcome transmission constraints. The main market mechanisms are expected to evolve driven by private initiative in response to the requirements of market participants. Three power exchanges have already been established —UK Power Exchange (UKPX), UK Automated Power Exchange (APX) and the International Petroleum Exchange (IPE)— that operate market mechanisms and offer clearing and settlement services for OTC transactions. In this context, OTC trading has become the preferred way to make long-term transactions [OFGEM '01].

In Germany, after the aggressive liberalization launched under the Act of 29 April 1998, no official electricity market institution has been established, neither a system or grid operator nor an organized exchange with a market operator. The absence of an official electric market authority complicates the access to reliable information about trading volumes, which must be estimated based on surveys [Strecker '00]. This has evident disadvantages such as high transaction costs and lack of transparency and liquidity. Thus, a natural standardization of contracts is in progress. In addition, private power exchanges have surged such as the Frankfurt-based European Energy Exchange (EEX) or the Leipzig Power Exchange (LPX). Some observers also see network access as a significant barrier to entry [IEA '01].

Although this variety of market institutions may appear somewhat confusing, from the point of view of the agents that participate in a wholesale electricity market, whether market mechanisms are operated by the ISO or by another entity is irrelevant. Other factors, such as the number of market participants, their relative size, the existing generation mix and, above all, the rules that dictate how electricity must be

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[7] The basic principles that rule the regulatory reform process of the electricity industry in the European Union were established by Directive 96/92/EC [EU '96]. In it, the development of an internal market for electricity among the EU member states is promoted. The Directive dictates rules that guarantee a non-discriminatory access to the European transmission networks and requires the unbundling of electricity services. It became effective in February 1997. Member nations were called to open at least 26% of their national markets to competition by February 1999, 30% by year 2000 and 35% by 2003.

traded, determine to a greater extent the outcome of the reform. In particular, this thesis assumes a non-mandatory power exchange (PX) similar to the ones currently operating in Spain or Scandinavia. This PX comprises several market mechanisms. The entity responsible for the operation of the PX is referred to as the market operator (MO).

2.5 Market participants

The introduction of competition in a power industry frequently requires previous structural changes. In some occasions these changes lead to the formation of new companies (separation of vertically integrated utilities, privatization processes, divestitures, etc). Typically, additional companies emerge to cover the new needs created by the market. The resulting structure is a key factor for the correct performance of the wholesale electricity market. In this section we introduce the agents that play an important role throughout the rest of this thesis. Their activity and objectives are specified.

2.5.1 The wholesale supply side

Generation companies —the owners of electricity generation facilities— are the most relevant players in wholesale supply. They may come from the vertical disintegration of state-owned or privately owned utilities (in which case they typically present a large size relative to the market under consideration). They may also be the result of a process of privatization or divestitures (the regulatory authority can then enforce a maximum size for the resulting companies). They can further be new entrants whose size is small but can progressively increase. Finally, generation companies whose plants are located in neighboring power systems can also participate, though their influence is limited by the transmission capacity of the interconnection.

The aim of a generation company is to maximize its long-term profit through the operation of its generating plants. This implies recovering both fixed and variable costs which are basically the same as in the traditional regulatory framework, given that no significant technological changes have been introduced since the development of CCGTs [Wood '96]. In contrast, revenues are now driven by uncertain market forces through new market mechanisms, leading to a greater degree of risk exposure.

The mixture of generation technologies owned by a certain company conditions its strategy in the wholesale electricity market. Generation companies with a variety of generating plants (portfolio generators) have greater flexibility to face the risks inherent to the production of electricity (uncertain hydro inflows, fuel prices, demand growth, units' outages, etc.) and those arising from competition (uncertain wholesale electricity prices). Typically, they also have a relative size that allows them to actively participate in a variety of market mechanisms. On the contrary, generation companies that own a particular type of plants or whose size is small will have to concentrate on certain market segments and contractual forms. A good characterization of generation companies (and, in general, of the agents that are likely to participate in a wholesale electricity market) is provided in [OFGEM '99].

This thesis focuses on portfolio generators. In this manner, the developments of this work can be easily adapted to a generation company with a more specific profile. Chapter 3 presents a survey of modeling approaches that have been proposed to represent the strategic behavior of generation companies in wholesale electricity markets.

2.5.2 The wholesale demand side

The characterization of wholesale energy buyers in a particular regulatory framework depends on the freedom of customers to choose a retail supplier. In many cases, the regulatory reform includes a transition period before the liberalization process is completed. During this period, the amount of energy consumption required for a consumer to freely choose its retail supplier gradually decreases until the smallest consumers are finally incorporated. In the meantime, regulated tariffs are maintained and distribution companies still assume the obligation to serve regulated consumers. This may lead to controversial situations in which regulated tariffs are insufficient to cover the cost of purchasing electricity in the wholesale market. In this thesis, it is assumed that all customers are free to choose their retail supplier and that regulated tariffs have been eliminated.

Large industrial consumers may be interested in purchasing their energy directly through wholesale market mechanisms (either in the form of bilateral contracts with generation companies or participating in the PX). However, the main role in wholesale demand is typically played by energy services providers (ESPs). ESPs are companies that act as intermediaries between wholesale electricity suppliers (generation companies) and consumers. In addition, they offer technical advice, commercial services, risk-management solutions, etc. It is a business characterized by a significant degree of risk exposure, due to the consumers' habit of paying stable prices and to the dramatic volatility observed in wholesale electricity prices. ESPs are thus expected to develop creative solutions in order to maximize their profit in this unfriendly environment.

2.5.3 Other agents

Other companies not directly involved in the production or retail supply of electric energy may intervene in wholesale electricity markets. Arbitrageurs, for instance, specialize in identifying market opportunities such as price differentials between neighboring power systems that may justify an energy transfer or forward prices that are excessively low given their estimation of future spot prices, etc. Although the volume of energy traded by arbitrageurs is expected to increase as further degrees of liberalization are attained, they are not essential for the operation of a theoretic wholesale electricity market and are not considered in this thesis.

2.6 Market mechanisms

In previous sections we have identified the market institutions and the agents that take part in the electricity marketplace considered in this thesis. It is now time to specify the market mechanisms that this marketplace consists of and their governing rules. As before, the diversity of regulatory approaches that have been adopted throughout the world will illustrate the discussion.

2.6.1 Physical products

Wholesale electricity markets are conceived as a means to trade electric energy. However, the supply of electric energy implies the existence of a number of generating units, the access to a transmission network and the supervision and control of certain state variables such as the system's frequency of bus voltage magnitudes. Indeed, a variety of physical products can be traded in wholesale electricity markets in addition to electric energy, such as ancillary services, generation capacity or physical transmission rights.

The provision of ancillary services[8] can be carried out through market mechanisms, typically conducted by the ISO, even if energy markets are not under its control [Singh '99a, Singh '99b]. The amount of ancillary services demanded by a power system depends on a wide range of parameters such as the accuracy with which the system's load is estimated, the magnitude of this load, the reliability of the synchronized generators, the existence of interconnections with neighboring power systems and others. The ISO usually determines the requirement of ancillary services as an inelastic quantity[9] due to the perception of short-term security of supply as an obligation. In this thesis we only consider active power reserves. No distinction is made according to their speed of response[10].

Some regulatory frameworks consider generation capacity as another product that must be remunerated so as to promote the large investments that capacity expansion requires, thus increasing the long-term security of supply [Chuang '00]. There is no general consensus on the convenience of these payments[11]. In this thesis we assume that capacity payments do not affect the short-term decision making of generation companies. However, an interesting approach based on the concept of financial options has been recently suggested that combines a natural hedging procedure for energy buyers against price spikes with a secure long-term payment to generators [Vázquez '01]. At the same time it diminishes the incentive these may have to induce price spikes. The influence of this kind of contracts on generators' behavior in short-term electricity markets is evaluated in this thesis.

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[8] Among ancillary services, active power reserves have attracted much attention due to their importance and complex nature. These reserves are used to control the system's frequency and to restore the balance between electricity production and consumption in real-time operation.

[9] The elasticity of demand is defined as the percentage variation of demand divided by the percentage variation of price. If demand is inelastic, it means that it remains the same irrespective of the prices offered by suppliers.

[10] Active power reserves can be categorized, according to the speed of their response, into primary reserve (which includes the effect of the rotating machines' momentum and the action of their speed governors in order to dampen frequency deviations), secondary reserve (which refers to the automatic generation control, AGC, that conducts a number of generators in order to eliminate deviations both in the system's frequency and in the scheduled power interchanges with neighboring systems) and tertiary reserve (which is called upon to face anticipated shortfalls or to replace secondary reserve so that it can be available for future needs). Voltage control is another important ancillary service. Some authors have indicated that faster response reserves (e.g. secondary reserve or AGC) can substitute lower response reserves (e.g. replacement reserves) and have recommended pricing schemes that explicitly take this fact into consideration in order to provide the right incentives for generators to reveal their reserve quality and procurement costs [Oren '01].

[11] Detractors of capacity payments suggest that long-term prices of electricity should allow generators to recover their average costs and provide economic signals for new investment decisions. Those generating units unable to do so would represent bad investment decisions. In practice, this requires sporadic (and unpopular) high prices, which typically occur when generating capacity is scarce, usually casting doubts over possible misbehavior of generators. It is interesting to point out that capacity payments do not eliminate the incentive generators may have to withdraw capacity under scarcity conditions. Additionally, they limit the ability of consumers to decide how much they want to pay for security. Currently, only Spain and some South American countries maintain administered capacity payments.

In those systems in which the structure of the transmission network imposes significant constraints for the physical realization of energy transfers, transmission capacity turns out to be a valuable product. Agents willing to make use of congested transmission lines to drive power from an exporting region to an importing region are typically forced to pay congestion charges that mainly depend on the behavior of generators located in the importing region. Two alternatives exist to provide these agents with a mechanism to hedge against congestion charges: physical and financial transmission rights [Joskow '00]. If energy and transmission markets are perfectly competitive these two types of transmission rights can be considered equivalent [Chao '96]. However, when perfect competition does not hold, both rights can create additional incentives to exercise market power. Physical transmission rights have even worse properties, as their owners can withhold them in order to affect market prices, thus reducing effective transmission capacity. In this thesis it is assumed that no trading takes place either with physical or financial transmission rights.

2.6.2 Time frames supported by market mechanisms

Agents participating in a wholesale electricity market require different market mechanisms to perform their transactions in the way that best suits their business strategy. In particular, the time scope with which the agents may be willing to trade can range from several years in advance to a few minutes prior to physical delivery.

Energy buyers should naturally tend to carry out the majority of their purchases in long-term markets ranging from several years to several weeks in advance. On the one hand, this allows large consumers to respond to high prices, as they may find it too costly to adapt their energy consumption levels to the corresponding prices of electricity on a daily basis. On the other hand, it permits ESPs to hedge against unacceptable risk exposure. Similarly, small generators may find long-term contracts useful to finance their debt and hedge against the volatility of short-term prices. Power exchanges frequently provide mechanisms such as futures markets in which long-term contracts are negotiated in a continuous manner. In parallel, market participants typically develop innovative means of trading with electricity while managing their risk exposure.

In the majority of wholesale electricity markets, there is a market mechanism that is considered as a reference for the rest of transactions and that typically takes place one day prior to physical delivery. There are a number of reasons for this timing, which we summarize as follows.

On the one hand, it is considerably difficult to obtain an accurate estimate for the demand for electricity more than one day in advance, due to its strong dependence on weather. Additionally, generation costs heavily depend on the number and type of available generating units, which can suddenly change due to unexpected failures. Therefore, to guarantee that the main market mechanism correctly represents the costs of providing the actual demand for electricity, it must take place as close as possible to the moment of physical delivery.

On the other hand, the start-up of many thermal generators usually requires several hours and variations in their power output are typically restricted by ramp-rate limits. Market mechanisms running too close to the moment of physical delivery limit the ability of generators to participate and reduce demand-side responsiveness.

As a result of these two sets of conflicting objectives, a good tradeoff is to run the main market mechanism one day in advance (hence the name day-ahead market). In order to guarantee that the exact balance between power generation and demand is ultimately achieved, subsequent short-term market mechanisms such as hour-ahead, real-time or balancing markets are also typically included. This gives the agents the possibility of introducing last-minute adjustments in their positions, prior to physical delivery. Consequently, the day-ahead market does not imply immediate physical delivery and cannot be considered as the spot market. In this thesis, the term *spot market* is used to denote the sequence of short-term mechanisms starting with the day-ahead market and ending with physical delivery. This thesis proposes a methodology for the optimization of the strategies followed by agents participating in such a spot market. Transactions performed in longer-term markets are considered as input data in order to evaluate the overall costs assumed by the agents.

2.6.3 A characterization of spot market mechanisms

As has been indicated, the rules that govern market mechanisms must be carefully designed to guarantee that they foster competition between participants. Several attributes that can be used to establish a characterization of the most relevant spot market designs currently in operation are:

i) The transparency of the rules governing energy trade.

ii) The flexibility of operation conferred to participants.

iii) The degree of decentralization achieved in decision-making.

iv) The degree of integration between spot market mechanisms through which different products are traded: energy, generation capacity, transmission capacity, ancillary services.

v) The particular manner in which spot market mechanisms are cleared, products are priced and financial settlement is reached.

Experts have identified three basic spot market organizational paradigms according to the previous features [Kahn '95, Chao '99]. In order to illustrate the spot market design adopted for the developments of this thesis, a quick review of these three models is provided.

2.6.3.1 The poolco model

In the poolco model the spot market is operated by the ISO. The ISO schedules energy transactions based on complicated optimization procedures, frequently incorporating transmission constraints and considering the provision of ancillary services (integrated dispatch). This has the advantage of guaranteeing that spot market results are technically feasible.

Optimization-based market mechanisms are very similar to the centralized operation-planning models that were used before the reform to obtain minimum-cost generation schedules. In the poolco model, in order to perform this optimization, generators are required to indicate their short-term production costs (including variable costs, start-up costs and no-load costs) and technical constraints (maximum capacity, minimum stable output, ramp-rate limits, etc.) In some cases participation is mandatory, though in others generators have the possibility of self-scheduling with a buying counter party. The demand side plays a secondary role and may even not be allowed to submit buy bids (as in E&W prior to NETA, [Borghetti '01]). Agents willing to hedge against volatile spot prices can enter in bilateral financial contracts.

Spot prices may be calculated ex-ante, based on the results of the day-ahead dispatch and subsequent adjustments, or ex-post, stemming from actual energy transfers. They are typically nodal prices (also known as locational prices) and can be obtained as the dual variables of the scheduling optimization. In this manner, they can include information about transmission losses, congestions and ancillary services, following Schweppe's spot pricing theory. Thus, they constitute economic signals indicating agents which energy transfers are favored by the existing generating and network infrastructures. In some cases they can even incorporate a capacity payment to promote generation capacity expansion.

In Chile, CDEC obtains yearly, monthly, weekly and daily operation schedules based on audited costs, reservoir levels, units' availability and demand forecasts [Rudnick '97]. Due to the existence of large reservoirs with multi-year storage capabilities, a multibusbar multireservoir hydrothermal coordination tool is used to determine the value of water over a four-year scope. In contrast, unit commitment is not important for the dispatch of the Chilean system, which is predominantly hydro. The final real-time dispatch determines the system's hourly marginal cost under a single-bus assumption. Penalty factors are then used to determine the contribution of each generator to marginal losses. Dispatched generators receive ex-post payments based on these hourly prices.

In Argentina, generation companies negotiate long-term contracts limited to their production capacity with distribution companies or large consumers, typically for one year. In parallel, CAMMESA determines three-month seasonal prices based on information provided by distribution, transmission and generation companies [CAMMESA '01]. These are the prices that distribution companies pay for energy purchases exceeding the amount previously contracted with generators. Additionally, the spot market yields hourly spot prices. These are paid either by large consumers who contracted less energy than they actually required or by generators who contracted more energy than they were able to produce. Hourly spot prices are received indistinctly by generators, distributors and large consumers who sell energy in the spot market. The spot market in Argentina is quite similar to the Chilean one [Rudnick '97]. A hierarchical planning process ranging from several years to one day in advance is performed based on the variable costs declared by generators, including an energy price for hydro plants. Thermal plants declare their costs twice a year with a weekly resolution, while the frequency for hydro generators depends on the capacity of their reservoir. Distribution companies submit a seasonal demand forecast. CAMMESA calculates the water value by means of a two-reservoir single-bus dynamic programming model with a three-year scope. A weekly regional hydro dispatch is then performed with a linear programming model. Additionally, the thermal unit-commitment problem is solved taking into account start-up costs of steam and gas turbines and the overhead cost of peaking steam turbines (off-peak cost of committed steam turbines). Finally, the daily thermal economic dispatch is performed with a non-linear model. This model includes an explicit representation of the transmission network, thus yielding nodal prices that take into account both marginal losses and congestions. Market mechanisms also exist for the provision of frequency regulation, spinning reserve, cold reserve and voltage control.

In the E&W Pool, generators were asked to submit sell offers for the following day. Sell offers included start-up costs, no-load costs (cost of operating at the unit's minimum stable output) and four energy prices for different output levels. A mathematical programming model was used to obtain a minimum-cost half-hourly schedule as well as half-hourly system marginal prices (SMP, i.e. the price of the highest accepted offer including average start-up and no-load costs). Energy sales were paid at the Pool Purchasing Price, equal to the SMP plus a capacity payment. If this schedule led to grid constraints, the grid operator required some generators to increase their output, while others were forced to reduce it. Output increments were paid at their offer price plus the capacity payment, whereas decrements received only the capacity payment. Energy purchasers had to pay the Pool Selling Price, equal to the SMP plus an uplift to account for ancillary services, network constraints and transmission losses. The volatility observed in the SMP led to significant use of hedging financial instruments such as contracts for differences[12]. According to OFGEM, "innovative contracting strategies that might have developed if participants had been able to trade bilaterally outside the Pool, have been inhibited by the compulsory trading requirement. Furthermore, mandatory participation has reduced the incentives of the Pool itself to be innovative in the services it offers, since it does not have to compete in order to retain membership" [OFGEM '99].

[12] In a contract for differences (CfDs) two parties agree to exchange the difference between a fixed price and the spot price at a specified moment of time and for a certain amount of energy.

Australia's National Electricity Market consists of a centrally coordinated dispatch process and a spot market. The centrally coordinated dispatch process is based on a security-constrained optimization problem modeled as a linear program [NEMMCO '01]. Generators must indicate a minimum output level (self-dispatch) and a maximum of ten blocks of energy with different prices to indicate how much energy they are ready to sell at each price in the following day. Unlike the E&W Pool, different prices can be submitted for each of the half-hour periods. Generators can also declare their ramp-rate limits. Energy buyers can either indicate a load or specify a set of quantities they are willing to purchase at different prices. Participants may revise their submitted quantities (but not their prices) up to five minutes prior to physical delivery. In the spot market, ex-post half-hourly clearing prices are determined for five-minute time frames and the agents' positions are settled based on the energy actually transferred. Due to the large extension of Australian territory, regional prices are calculated to approximately account for transmission congestions and transmission losses. No capacity payment has been introduced. CfDs between market participants also take place. Two electricity futures contracts are traded in the Sydney Futures Exchange.

In PJM, the ISO operates a voluntary day-ahead market that consists of a security-constrained dispatch based on both bilateral schedules and voluntary offers and bids submitted by the agents. This model is used to determine the corresponding nodal prices (locational marginal prices [PJM '01]). In order to sell energy through this market, generators are required to submit variable-cost-based offers (one price per day). They can also indicate start-up costs and no-load costs or specify a maximum ramp rate. An alternative is to self-schedule with a buying counter party. The day-ahead market is cleared based on the result obtained for the first day of a unit-commitment problem with a one-week time horizon. Agents willing to hedge against high congestion charges can participate in markets for transmission rights.

2.6.3.2 The spot market as a sequence of auctions

This spot market model is the one considered as a framework for this thesis and is therefore treated in more detail.

The auction approach

Instead of relying on complicated optimization techniques, energy transfers are accepted in this spot market model based on a set of rules oriented to yield a solution close enough to the social optimum, while maintaining a reasonable degree of transparency[13]. Many of the concepts underlying the design of such spot markets come from the literature of auction theory [Klemperer '99]. As indicated in [McAfee '87], "an auction is a market institution with an explicit set of rules determining resource allocation and prices on the basis of bids from market participants". Organizing an auction is a practical approach to solve the question of price formation, particularly when information about the costs or benefits associated to the supply of a certain resource are dispersed.

Congestion management after the day-ahead market

Spot markets designed as a sequence of auctions are usually administered by an entity different from the ISO and typically start to operate with a day-ahead session. The day-ahead market clearing process is frequently based on a single-node assumption. Once the day-ahead market clears, the ISO can assess the feasibility of the resulting transactions and allocate scarce transmission capacity. This may invalidate certain energy transfers, while new ones would be required to substitute them. An alternative is to identify network zones interconnected by tie lines that are likely to get

[13] Some observers have referred to these as "the second generation of power exchanges" [Millán '99]

congested. Whenever the day-ahead market clearing process results in congestion between two zones, the spot prices for these zones diverge, in order to reflect that consuming an additional energy unit in each zone has a different incremental cost.

Ancillary services and balancing mechanisms

As mentioned, ancillary services such as active power reserves are frequently traded independently from energy. In many cases, even distinct market mechanisms are established for different types of reserve according to their speed of response. Additional subsequent short-term market mechanisms such as hour-ahead or real-time/balancing markets are provided to give the agents the possibility of adjusting their positions prior to physical delivery.

Offers and bids

Agents willing to transfer energy through these sequential market mechanisms must submit sell offers or buy bids to indicate the prices at which they are willing to perform transactions. The specific format with which offers and bids have to comply in order to be valid is another factor that conditions participants' behavior to a certain extent. In the existing spot markets that have been conceived as sequences of auctions, offers' formats differ notably from one case to another. Some of the attributes that cause these differences to arise are listed below:

i) The most relevant feature is the amount of information that an offer must include. Simple offers (also know as one-part offers) consist only of a quantity (e.g. an amount of MWh for energy markets) and a price per unit of product (e.g. in €/MWh). In contrast, complex or multi-part offers include supplementary cost data (such as start-up costs or no-load costs) and/or information about the generation units' technical constraints (minimum stable output, maximum capacity, ramping-rate limits, etc.) Complex offers require complex auction designs that sacrifice transparency in their intent to achieve higher efficiency levels. Indeed, complex offers are frequently used in conjunction with market mechanisms that are closer to the concept of optimization than to the idea of auction. The obligation to incorporate these complex conditions to offers and bids reduces the flexibility of participants to operate.

ii) In some spot markets offers must be kept invariable throughout the day (as in the E&W Pool), although in the majority of cases offers can vary between trading periods. This gives participants additional degrees of freedom, particularly if simple offers are allowed.

iii) Some systems permit portfolio bidding (as the CALPX, before the crisis) whereas others require the identification of the network zone or node to which the offer is referred (e.g. the NordPool) or even the generating unit that corresponds to each offer (e.g. Spain).

iv) The number of offers that can be submitted to sell the energy produced by each generating unit may be limited (in Spain, each generating unit can offer up to twenty-five blocks of energy at different prices in each hour, whereas in Australia a generator is limited to specifying ten different prices).

v) Although currently uncommon, demand-side bidding may not be allowed.

Pricing

Two pricing schemes can be adopted for these auctions. On the one hand, under the uniform pricing formula, all transactions are valued at the same price, p^*, irrespective of the prices submitted both by winning sellers and buyers. This approach is supported by standard microeconomic theory which clearly states that given the equilibrium price, p^*, the amount of product that the demand side is willing to purchase is equal to the quantity that the supply side is willing to deliver. The benefits of this pricing scheme are shared between sellers and buyers.

On the other hand, it may not be evident that generators should be paid the price of the most expensive accepted MWh (marginal or equilibrium price) rather than the price they actually offered. Uniform pricing, together with the unusually high volatility of electricity spot prices, seem to benefit inframarginal generators in excess, especially when price spikes occur. Advocates of price discrimination argue that the pay-as-bid policy eliminates this possibility and reduces the incentive of large generation companies to artificially increase marginal prices. However, several objections can be formulated against the pay-as-bid rule [Vázquez '00]. Firstly, price spikes take place typically when generation is scarce and should be seen as an economic signal for capacity expansion. It is well documented that generation units with lower short-term marginal costs typically require larger investments for their installation which must be recovered through continuous long-term operation. On the contrary, units with higher incremental costs typically operate during on-peak hours and require price spikes to recover their capital costs. Secondly, the pay-as-bid approach is not an incentive-compatible mechanism, as it induces agents to submit offers that differ from their actual short-term marginal costs in order to recover their long-term average costs. Finally, small generation companies are likely to suffer more than large ones under the pay-as-bid rule, given that their revenues depend solely on their offers and not on the price set by others. This may also deter new entrants. In this thesis, all the sell offers accepted in a certain auction are assumed to receive the same price, which is equal to the price paid by all accepted buy bids.

Final spot prices are typically calculated ex-ante, based on the sell offers and buy bids submitted by participants to each of the market mechanisms. They arise from the aggregation of the sequence of market-clearing prices and include terms referred to energy, ancillary services, transmission losses, congestions or capacity payments, if any. Except for the cases where some sort of zonal pricing is in force, the spot price is uniform.

Non-mandatory spot market

Spot markets designed as a sequence of auctions are typically non-mandatory. Market participants can always choose to participate in OTC markets or enter in longer-term physical bilateral contracts. Therefore, they are endowed with a wider range of possibilities to perform their business, while at the same time they bear the risk of making their own decisions. Generators, for instance, have freedom to decide when to start up and the amount of energy they wish to sell at each price. Whether they recover their start-up and variable costs is their sole responsibility. As in the poolco model, hedging through financial bilateral contracts such as CfDs is frequent, although the variety of contractual forms (options, swaps) is usually wider.

In California, both the day-ahead and hour-ahead markets operated by the CALPX were designed as auctions based on the simple hourly sell offers and buy bids submitted by participants. These were aggregated to form hourly offer and demand curves whose intersection determined the hourly market-clearing price. Agents were not required to identify the particular node of the transmission network to which each offer was referred, so portfolio offering was allowed. Once the day-ahead market cleared, the agents had to identify the nodes corresponding to the accepted offers, so that the CALPX could submit a detailed balanced schedule to the CAISO. Additionally, other entities called Scheduling Coordinators (SCs) could be licensed to submit hourly balanced schedules to the CAISO. Based on this information, the CAISO then obtained an adjusted schedule to avoid congestions. This resulted in different zonal market-clearing prices and network usage charges. In the subsequent hour-ahead market, bids and offers were accepted up to two hours before actual delivery. Markets for ancillary services included AGC/frequency regulation, operating reserves and replacement reserves. Although some observers consider this design as the primary cause for the California's electricity market crisis, similar frameworks have been adopted in other European countries that are performing well.

In the Nordic Power Market, the spot market operated by NordPool, Elspot, is based on 24 hourly auctions for the following day (day-ahead market). Market participants submit simple hourly sell offers and buy bids that lead to hourly aggregate offer curves and aggregate hourly demand curves. The intersection of both curves determines the spot price for each hour. As participants are not required to indicate which generating unit corresponds to each sell offer, portfolio bidding is permitted. However, they must specify their bidding area, so that if the power flow due to the spot market transactions exceeds the existing transmission capacity between bidding areas, slight differences arise between zonal spot prices. Participants must also make an ex-ante notification of any bilateral contracts traded up to that point. An hour-ahead balancing market, Elbas, also exists for Finland and Sweden that allows agents to trade up to two hours prior to delivery and which is in continuous expansion. Additionally, the existing transmission system operators call for real-time bids for upward and downward regulation to keep the balance between generation and demand, thus providing a price for participants' power imbalances.

In Spain, the sequence of market mechanisms devised to promote competition in the provision of energy and ancillary services is very similar to the ones implemented in California and Scandinavia. The day-ahead market consists of 24 hourly auctions based on the complex hourly offers presented by generators and the simple hourly bids submitted by energy buyers. It assumes a single-bus situation and yields a unique reference spot price for each hour of the following day. In spite of the possibility of tendering complex sell offers, the market outcome is essentially the result of matching the aggregate offer and demand curves. Complex conditions include a minimum daily payment required by each generator to operate either as a fixed amount or dependant of its total sale of energy, upward and downward ramp-rate limits and others. All agents are asked to indicate the generating unit that corresponds to each offer (in other words, portfolio offering is not possible). After the day-ahead market clears, a congestion management process is carried out by REE, which results in modifications of the previous schedule. Generators that are forced to decrease their output to alleviate transmission constraints receive no compensation, while those that are required to increase their production receive the price they offered in the day-ahead market. Participants can adjust their physical positions in subsequent sessions of the on-day market until one hour prior to physical delivery. In addition, markets for ancillary services such as frequency control or spinning reserve also take place.

In Italy, the market operator, GME, will manage a national wholesale exchange system which will consist of five market mechanisms: a day-ahead market, an adjustment market, a congestion management procedure, a reserves market and a balancing market. Additionally, an entity owned by the system operator, GRTN, will be in charge of making wholesale purchases of electricity and selling it to the distribution companies to supply franchised consumers. Due to practical difficulties, the startup of this new regime is not expected until the end of year 2002.

New Zealand's can be defined as a hybrid design. Its non-mandatory spot market is based on simple sell offers and buy bids submitted by market participants for each half hour of the following day. A day-ahead session is performed that yields a proposed schedule and forecast half-hourly prices. Agents are then free to change their offers and bids (both volumes and quantities) up to four hours prior to physical delivery. Transpower then obtains a least-cost dispatch based on the latest positions, thus determining nodal prices.

2.6.3.3 Continuous bilateral trading

In the third spot market model, market mechanisms are designed to emulate continuous bilateral trading. Only E&W, after the introduction of NETA, and Germany fall under this category. In both cases, regulatory authorities perceive that the simplicity and flexibility inherent to bilateral trading between agents should yield an increase of liquidity and a reduction of wholesale electricity prices due to fiercer competition. The creation and operation of market mechanisms has been left to private initiative.

> In E&W, the only market mechanism that is still administered by the system operator, NGC, is the Balancing Mechanism, which opens three and a half hours prior to physical delivery. It is conceived as a means to ensure an efficient balancing of the system by the SO, whilst encouraging generators and ESPs to contract ahead for most of their requirements in forward, futures and short-term markets [OFGEM '99]. In the Balancing Mechanism, the SO accepts offers for electricity (generation increases and demand reductions) and bids for electricity (generation reductions and demand increases). Accepted offers are paid at the prices offered and accepted bids pay the prices bid. The largest power exchange by traded volume, UKPX, offers long-term cash- and delivery-settled futures, forward and option contracts. However, most trading is carried out via direct bilateral contracts between participants [OFGEM '01]. Additionally, UKPX and UK APX trade significant volumes of electricity in short-term markets. UKPX provides half-hour contracts which are traded in lots of 0.5 MWh from the start of the day until the opening of the Balancing Mechanism for the corresponding half hour.

2.6.4 Information disclosure

The release of information relative to the transactions performed through the existing wholesale market mechanisms is an issue of great concern for regulatory authorities.

General agreement exists on the convenience of making publicly available the volume of energy transactions and the market clearing price for each market mechanism in every trading period. In the context of the spot market considered in this thesis, any agent can check whether his offers and bids have been adequately administered by simply comparing their prices with the corresponding uniform market-clearing price.

Information related to the technical performance of the existing generation technologies is also provided in some cases to guide generation companies when scheduling their units.

> In Norway and New Zealand forecast and actual reservoir levels are regularly published due to the relative importance of hydro power in both systems. ISOs in the US publish information about the scheduled maintenance of generating units, demand forecasts, power flows, loss factors, etc. through their Open Access Same Time Information Systems (OASIS). Australian NEMMCO provides information about generation schedules and actual generators' output, as well as weekly estimations of supply and demand. In Spain, OMEL publishes the hourly energy sold in the spot market by each type of generation technology with a three-day delay.

On the contrary, information on the transactions carried out by each participant should be treated as confidential information and kept private for a reasonable period of time, to avoid an undesirable surveillance of its behavior by rivals intending to force coordinated bidding strategies. This consideration applies both to the spot market and to the long-term market mechanisms[14].

> Until May 2001, Spanish OMEL informed participants about the hourly sales performed by every generating unit in each of the spot market mechanisms, although these data were not accessible to the general public. After a proposal by the regulatory commission, it was decided to delay the release of this information three months and to make it publicly available.

Finally, the information about offers and bids submitted by participants receives a variety of treatments throughout the world. Revealing the particular offers and bids submitted by each participant is not appropriate because of the aforementioned risk of strategic coordination between agents. Quite the opposite, making offers and bids submitted by all agents public in an aggregate manner reduces the information asymmetries that would otherwise arise due the differences in participants' relative size (it is obvious that a company with a 20 % market share also controls 20 % of the market information).

> In the E&W Pool, although bidding information was made available, the complexity of the market clearing and pricing mechanisms rendered it extremely difficult to establish a link between offers and the system marginal price [OFGEM '99]. In Australia and New Zealand bidding data is published with a 24-hour delay. In Spain, OMEL makes aggregate bidding data publicly available after each spot market session, whereas detailed information about the offers and bids submitted by each participant is published with a three-month delay. In California before the crisis aggregate bidding data was not released until three months had passed. In PJM the delay is extended to six months. In the NordPool individual offers, bids and contracts are treated as confidential information.

In this thesis it is assumed that the MO publishes the following information immediately after the clearing of every market mechanism:

i) Hourly volume of transactions.

ii) Hourly market clearing prices.

iii) Hourly aggregate offer and demand curves.

No specific hypothesis is formulated on the release of information relative to the operation of long-term market mechanisms, which is irrelevant for the purposes of this thesis.

2.7 The electricity marketplace considered in this thesis

This section recapitulates the hypotheses assumed hitherto and provides further details concerning the operation of the power exchange that will be considered for the developments of this thesis.

Table 2.1 summarizes the assumptions made in previous sections regarding the main aspects of the electricity marketplace under consideration.

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[14] However, in [Lien '00] it is argued that if long-term forward positions were published they would serve incumbent generators as a device to transmit potential entrants its commitment to display an aggressive behavior in the spot market, which should lead to a more efficient use of the existing resources.

Infrastructures	Transmission	No significant transmission constraints
	Generation	A variety of generation technologies
Institutions	Regulatory authorities	Market power is not a long-term sustainable strategy
	System operator	Independent. Responsible for the security of the system. Operates the transmission network. Schedules the supply of ancillary services.
	Market operator	Independent. Operates a non-mandatory power exchange.
Market participants	Wholesale supply	Generation companies. Objective: maximize long-term profits.
	Wholesale demand	Energy service providers (ESPs). Intermediaries between generators and consumers.
Market mechanisms	Physical products	Energy.
		Active power reserves.
	Time frames	Long-term mechanisms for futures, options, CfDs, etc.
		Short-term spot market: ▪ Sequence of market mechanisms. ▪ Hourly auctions. ▪ Simple offers and bids. ▪ Uniform pricing.
	Information disclosure	Hourly volume of transactions.
		Hourly market clearing prices.
		Aggregate offer and demand curves.

Table 2.1. The electricity marketplace considered in this thesis.

Given that the general objective of this thesis focuses on the development of optimal offering strategies for generation companies participating in electricity spot markets, it is crucial to provide additional details about the operation of these market mechanisms. The particular PX considered in this thesis includes both long-term market mechanisms and a spot market constituted as a sequence of short-term market mechanisms. Long-term market mechanisms are based on the continuous bilateral trading of both financial and physical contracts between participants. On the contrary, the spot market is designed as a sequence auctions administered by the MO and based on sell offers and buy bids submitted by the agents (Figure 2.1).

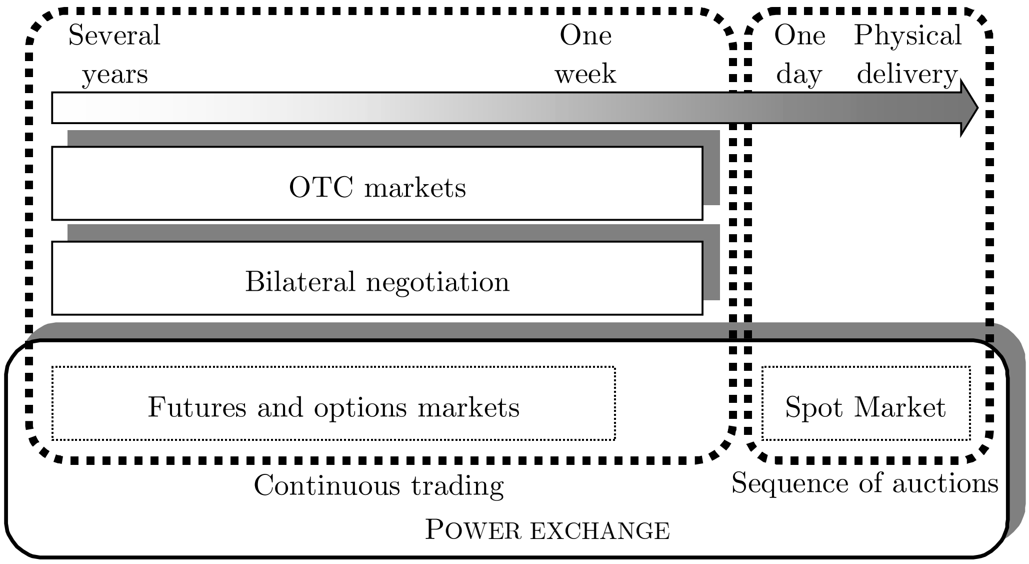

Figure 2.1. Market mechanisms.

2.7.1 Long-term market mechanisms

Two basic long-term contractual forms are considered in this thesis in order to evaluate their influence on the strategies adopted by the agents in the spot market. They are traded indistinctly in the PX or in OTC markets.

A long-term contract c is an agreement to deliver a specific amount of energy q_c at a specified trading period n and a given location for a fixed price p_c per MWh. Contracts can be settled with the physical delivery of the corresponding energy by the selling party and a payment equal to the product $q_c \cdot p_c$ by the buying counter party, in which case they are typically known as *physical* contracts. Alternatively, they can be settled financially, taking the spot price for that trading period, p_n, as a reference. In that case, the selling party receives from the buying counter party a net payment equal to $q_c \cdot (p_c - p_n)$, which is the definition of a contract for differences.

A European call option o gives its holder (long position) the right (but not the obligation) to buy a certain amount of energy q_o at a specific trading period n and a given location for a fixed price p_o per MWh. If the long party decides to exercise its right and the call option is physically settled, the option seller (short position) is forced to provide the amount q_o, receiving a payment equal to $q_o \cdot p_o$ in exchange, but only when the price in the spot market for that trading period, p_n, is higher than the option's strike price p_o. On the contrary, if the option is financially settled, the long party receives a payment equal to $q_o \cdot (p_n - p_o)$ when the spot price is higher than the strike price. Conversely, the holder of a European put option has the right to sell an amount of energy at a fixed price. The short party is forced either to purchase that energy at the strike price (physical settlement) or face a payment equal to $q_o \cdot (p_o - p_n)$ when the spot price is lower than the strike price.

Although a wider variety of contractual forms could be defined, it would add little to the developments of this thesis. Similar approaches to the ones that will be proposed for these two basic contracts can and should be used to evaluate the influence of forward contracts on the optimal offering strategy of an agent in the spot market.

2.7.2 The spot market as a sequence of market mechanisms

As mentioned, a reasonable approximation to Schweppe's definition of the spot price of electricity can be obtained if the optimal scheduling problem is decomposed into a sequence of market mechanisms, allowing sellers to gradually allocate their resources and buyers to gradually carry out their purchases. The spot market considered in this thesis has a structure similar to the ones implemented in Spain or in California before the crisis. It will consist of a day-ahead market, a congestion management procedure, an adjustment market, a reserves market and a balancing mechanism (Figure 2.2). Italy's proposed spot market also follows this design.

Figure 2.2. Sequence of spot market mechanisms.

It is obvious that in such a spot market the main market mechanism is the day-ahead market. This does not imply that the largest volume of transactions will be performed through the spot market, given that long-term bilateral contracting and OTC trading are also allowed. This thesis, however, focuses on the spot market and, particularly, on the day-ahead market. Due to the abovementioned assumption on transmission capacity, the congestion management procedure is ignored hereafter.

2.7.3 Spot market mechanisms as auctions

The spot market considered in this thesis takes the form of a sequence of repetitive sealed-bid[15] multi-unit[16] uniform-price[17] double[18] auctions. More precisely, each market mechanism comprises a set of hourly auctions[19]. For instance, the day-ahead market is constituted by twenty-four hourly auctions that take place one day in advance. The adjustment market has several sessions, each one including auctions for several hours. The balancing mechanism has hourly sessions, each one consisting of a single auction for the following hour. The active power reserve market is held shortly after the day-ahead market and comprises twenty-four hourly auctions[20]. Participants willing to exchange energy through each market mechanism have to submit sell offers and buy bids compliant with the particular format that we specify below.

2.7.4 Offers and bids

In this thesis, the mechanisms integrated in the spot market run based on simple sell offers and buy bids. Participants are allowed to operate as a portfolio in the day-ahead market and the adjustment market, although they are required to submit a detailed schedule after the adjustment market clears. They have no limits on the number of offers or bids they wish to submit and are allowed to tender different ones for each hour. Demand-side bidding is possible in all energy markets, although not in the reserve market, where the ISO acts as a single buyer.

2.7.5 Market clearing, pricing and settlement

Based on the offers and bids submitted by participants, the spot market determines the amount of each product (e.g. energy) that each seller must supply, the quantity that each buyer purchases, the payments that sellers receive and buyers afford and the manner in which these pecuniary transactions are financially settled.

The hourly double auctions that constitute each of the spot market mechanisms are mutually independent. Consequently, every hourly auction has its own clearing, pricing and financial settlement, even when several hours are simultaneously cleared. For instance, although the twenty-four hourly auctions that form the day-ahead market are

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[15] In a sealed-bid auction, each participant ignores the bids submitted by the rest of participants.

[16] In a multi-unit auction, a variable quantity of the resource is allocated, which may have or not a capacity limit.

[17] In a uniform-price auction, all buyers pay the same price and/or all sellers receive the same price. In contrast, if price discrimination is implemented, each buyer pays a different price and/or each seller receives a different price. Although this can be an adequate policy if participants are asymmetric, price discrimination may discourage agents from revealing their actual valuation of the resource. In other words, price discrimination may prevent the auction from being an incentive-compatible mechanism.

[18] According to [McAfee '87], "in a double auction, several buyers and several sellers submit bids simultaneously. The double auction is a stylized representation of organized exchanges such as stock exchanges or commodity markets."

[19] Without loss of generality, the spot market trading period in this thesis will be one hour.

[20] This spot market design almost coincides with the one currently operating in Spain. In this manner, the information relative to the offers and bids submitted to the Spanish spot market can be used for the numerical examples included in this thesis.

cleared at the same time, the results of one of these auctions are based only on the offers and bids submitted by participants for that specific hour.

Every auction follows the same clearing and pricing rules, whether they are part of the day-ahead market, the adjustment market, the balancing mechanism or even the reserve market, whose demand is determined by a single buyer, the ISO.

The clearing process for a certain hourly auction evolves as follows. The set of sell offers submitted for that hour yields an aggregate offer curve, whereas the buy bids form an aggregate demand curve, as represented in Figure 2.3. According to the most basic microeconomic equilibrium theory, the intersection of both curves determines the volume of transactions q^* that should take place. Indeed, sell offers placed below the intersection are ready to supply an amount of energy q^* if the price is p^*, whereas, given that price, buy bids above the intersection are willing to consume the same quantity q^*.

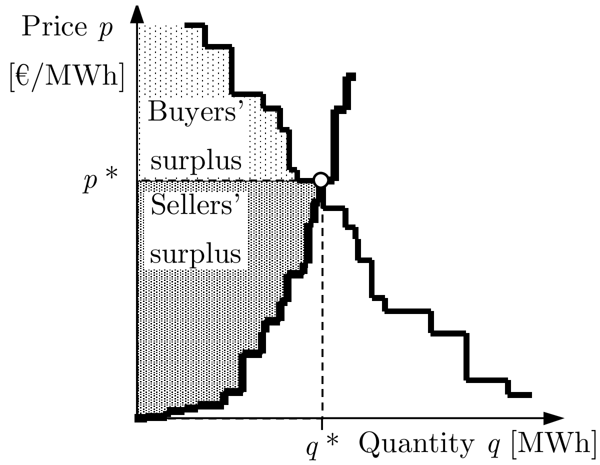

Figure 2.3. Market clearing for a certain hour.

When aggregating offers or bids, the resulting curve can either be defined as a stepwise function (as in Figure 2.3) or can be transformed into a piecewise linear function. The adoption of either of these criteria slightly affects the quantities transferred, as shown in Figure 2.4. In this thesis both representations are indistinctly used, assuming that the number of individual offers and bids is large enough to make the difference between them negligible.

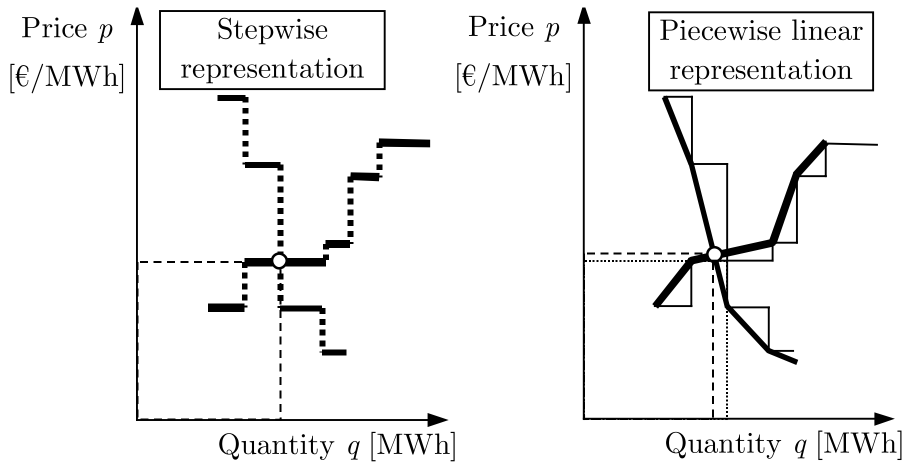

Figure 2.4. Intersection of the aggregate offer and demand curves.

In this thesis a marginal or uniform pricing scheme is adopted for all the hourly auctions that constitute the spot market, as a better approximation to Schweppe's spot pricing theory. The clearing price is determined by the intersection of the aggregate offer and demand curves. In this manner, an offer is fully rejected if and only if its price is strictly greater than the market-clearing price. Similarly, a bid is fully discarded if and only if its price is strictly lower than the market-clearing price[21] (Figure 2.5).

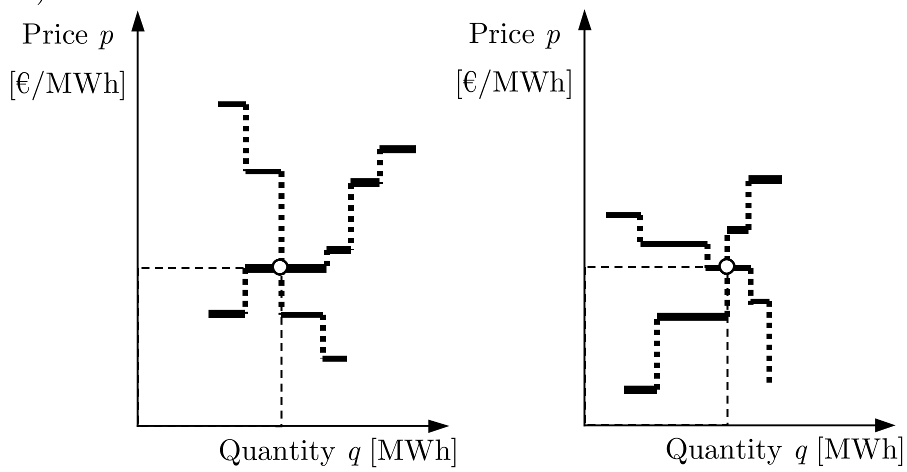

Figure 2.5. Price discovery.

The results of each hourly auction will be firm and will be financially settled accordingly. In other words, every seller with accepted offers is put under the obligation of providing the resultant amount of energy during the corresponding trading period and is granted a payment equal to the product of that quantity and the auction clearing price. Conversely, each buyer with accepted bids is forced both to consume the resultant amount of energy during the corresponding trading period and to face a payment equal to the product of that quantity and the auction clearing price.

2.8 Conclusion

This chapter describes the power system and the wholesale electricity market that constitute the framework for this thesis. It formulates relevant hypotheses concerning the infrastructure of the power system such as the limitations imposed by the transmission network on the physical realization of energy transfers or the technologies comprised in the system's generation mix. Additionally, it identifies the institutions and participants that play a relevant role in the wholesale electricity market, including a characterization of market designs based on the degree of decentralization achieved on decision-making. Most importantly, this chapter defines the time frames supported by the market mechanisms considered in this thesis, as well as the rules that govern them. Particular attention is paid to the design of the spot market, which is critical for the developments of this thesis. The examples that illustrate the discussion confirm the relevance of the regulatory reforms that are affecting the worldwide power industry.

[21] In Spain, market-clearing prices are always determined by the last accepted sell offer and never by the last accepted buy bid. This approach implicitly assumes that the demand side is less active in the spot market than the supply side.

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3 Modeling competition in wholesale electricity markets

Due to the special features of electricity, the power industry has traditionally been subject to close analysis by practitioners, regulatory authorities and different research communities including economists, electrical engineers and experts on operations research. Complex operation planning models that focus on different decision stages of the provision of electricity have systematically been used during decades to minimize the costs of supply prior to the advent of competition.

This modeling tradition has continued after the process of reforms. However, the operation of a deregulated power industry is the result of the interaction of a number of agents with different objectives. Consequently, this interaction must be somehow incorporated into models so that they adequately represent the decision-making process. Concepts from game theory, industrial organization theory, auction theory and other related fields have become frequent in electrical engineering publications during the last decade and great modeling advances have been accomplished due to the impressive research effort that has been made. This chapter presents a survey of the most relevant modeling approaches that have been proposed to represent competition in wholesale electricity markets, in order to justify the particular model chosen for the developments of this thesis. The challenges that this modeling approach entails are also identified.

3.1 Introduction

The main objective of the reforms that have been described in the previous chapter is to improve the efficiency of the power industry. It is believed that competition should force companies to reduce their costs and that this should lead to a reduction of the price of electricity paid by the final consumer. However, the special features of electricity as a commodity and the limited number of generation companies that typically take part in wholesale electricity markets sometimes lead to situations that are far from the ideal of perfect competition. Regulatory authorities have expressed their concern about the consequences of imperfect competition and are willing to identify measures that effectively mitigate these unwanted effects. From a different perspective, generation companies are faced with a new way of understanding their business, which is characterized by a much higher degree of risk exposure due to the uncertain behavior of their competitors and the threat of new entry. The urgent need to answer these and other related questions has triggered an unprecedented research effort devoted to the development of conceptual models that shed light on the possible outcome of these reforms.

A wide range of modeling approaches has been adopted to analyze competition in wholesale electricity markets. Differences between models can be identified in a variety of attributes, such as the hypotheses formulated by the modeler/analyst about the agents' behavior, the specific purpose of the ongoing analysis, the characteristics of the underlying power system, the detail with which the elements of the power system are represented, the organization of the corresponding wholesale electricity market or the technique used to obtain numerical results.

The objective of this chapter is to provide a general framework that justifies the particular model selected to represent competition in this thesis. A succinct characterization of the existing modeling proposals is performed, rather than an extensive review, which is a task that several authors have already successfully addressed [Kahn '98], [Ventosa '01]. Each modeling approach is illustrated with references to representative works.

The problem of modeling the behavior of a finite number of agents that try to maximize their profits in a competitive setting is a ground where a variety of fields of knowledge (microeconomic theory, auction theory, game theory, etc.) get mixed up, providing different perspectives that significantly enrich the analysis. Although many of the existing wholesale electricity markets are based on the interaction of both the supply and the demand side, research efforts have concentrated on modeling competition between generation companies, while treating wholesale energy buyers in an aggregate manner. According to [Ventosa '01], models used to analyze competition between generators can either focus on the behavior of a single generation company or explicitly represent every individual generation company that participates in the market. This particular scheme is the one followed in this chapter to organize the review.

3.2 Models that include an explicit representation of each generation company

Models that fall into this category try to estimate the outcome of a wholesale electricity market as a result of the interaction of a number of agents that are assumed to behave rationally[1]. Each agent develops his[2] strategies based on the characteristics of his generation units, the limits imposed by the transmission system and his perception about the past, present and future behavior of the rest of agents (including both rival generators and demand-side participants). In practice, two main types of models can be identified according to the approach adopted to obtain numerical results: equilibrium models and simulation models [García-González '01].

3.2.1 Equilibrium models

Equilibrium models have been extensively applied to the case of the power industry during the last few years in order to predict its evolution under the new regulatory framework. In particular, they have been used to explore possible market outcomes that may result under an industrial structure with a limited number of relevant agents (oligopoly). The wide range of applications that have been proposed confirms the interest of this approach, although it also has lead to a certain degree of confusion. As mentioned, the development of one of these models requires the combination of several disciplines and each specialist puts the accent on the issues that are more familiar to his or her background.

In an equilibrium[3] model, a problem is formulated in which each agent is assumed to choose his best strategy[4] based on certain conjectures[5] about the behavior of the relevant rest of the world. To search for a solution, the modeler must either enumerate all the possible combinations of strategies or derive the optimality conditions that represent each agent's decision process. The great advantage of deriving optimality conditions is that their interpretation reveals the incentives that drive the agents' decisions. In some cases, under certain simplifications, it is possible to reach a closed-form expression of the solution. In others, sufficient conditions can be found that guarantee that a given set of strategies is an equilibrium. In practice, the possibility of obtaining numerical results for large study cases reinforces the validity of a modeling approach.

Equilibrium models can explicitly take into account the repetitive aspects of wholesale electricity market mechanisms. However, dynamic equilibrium models have not attracted much research interest, mainly because of their inadequacy to incorporate a detailed representation of the wholesale electricity market. Simulation techniques have proven to be an interesting alternative to address dynamic large-scale problems.

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[1] A rational player takes the actions that lead toward his highest expected payoff.

[2] In this thesis, a singular genderless agent is referred to as 'he' and the agent's position as 'his'.

[3] The definition of equilibrium most commonly used is that of Nash: an equilibrium point is such that each player's mixed strategy maximizes his payoff if the strategies of the others are held fixed [Nash '51].

[4] In game theory, a player's strategy is a rule (or function) that associates a player's move with the information available to him at the time when he decides which move to choose. In the context of industrial organization, firms' strategies take the form of quantities, prices or, less frequently, supply functions that express the amount of product that each firm is willing to sell at each price.

[5] Conjectures about each rival can involve assumptions about his actions (which may be correctly estimated or not), his real actions (if the firm's decisions are taken after the rival's move has been made) or his decision rules (which, again, may be correctly estimated or not).

A good analysis of the influence of repetition on the behavior of agents participating in electricity markets can be found in [Rothkopf '99]. From the perspective of auction theory, daily repetition reduces the advantages of progressive auctions with respect to single-shot sealed-bid auctions and diminishes the pernicious effect of selecting winning offers and bids using rules that only yield approximately optimal solutions. On the other hand, repetition increases the threat of tacit collusion between participants. A sociological analysis reveals that the agents that participate in repetitive auctions end up forming a social group and tend to cooperate in order to defend their common interests. From the viewpoint of game theory, the effect of repetition is analyzed under the light of the prisoner's dilemma. When this game is repeated a large but finite number of times the best strategy is to deceive the rival. On the contrary, if the game is played an infinite number of times, the best strategy is to collude.

In [Fabra '99] a dynamic model is used to evaluate the extent to which repetition and the uniform pricing rule can facilitate collusive practices in wholesale electricity markets. Fabra considers a repeated game of capacity-constrained price competition[6] between two firms that interact for an infinite number of periods. Both firms have constant and equal marginal costs, as well as equal production capacity. Demand is modeled as a random variable. The author admits that the model is oversimplified, a necessary toll that must be paid in order to extract relevant conclusions when dynamics are included in an equilibrium model. In particular, the author argues that assuming two symmetrical firms and a single generation technology does not determine the findings of this research, which would still be valid in more complex settings. Its main conclusion is that tacit collusion is more likely to arise in a uniform pricing context. Another relevant factor is the size of the competing generation companies (the larger a company, the less interest it will have in reducing prices to punish other companies for deviating from the collusive equilibrium, whereas the smaller a company, the greater its incentive to break that equilibrium).

In [Barquín '00] a methodology is proposed to calculate water value based on dynamic game theory. A duopoly setting is considered where firms play a repeated Cournot game with uncertain inflows. At every stage each firm decides its thermal and hydro output in order to maximize its present and future profits. By discretizing both the probability space and the admissible reservoir levels, the game is solved via the traditional backward dynamic programming technique. Water value is calculated as the incremental profit obtained due to an additional unit of hydro reserves at the beginning of the game. Although the study case presented by the authors is rather simple, guidelines are given to extend this approach in order to handle more complex situations.

Quite the opposite, static equilibrium models have been profusely used under the assumption that agents decide their strategies simultaneously and are not able to react to their rivals' decisions. Proposals have ranged from the basic Cournot[7] model to more refined versions that include increasingly complex representations of the players' strategies. Particularly, supply function equilibrium (SFE) models have received much attention due to the fact that many of the wholesale electricity market mechanisms are organized as auctions in which each participant submits a supply curve so as to maximize its profits in the context of an uncertain demand curve (see appendix A for a definition of the concept of residual demand curve). Thus, competition in electricity markets can be understood as a game in which players take their strategic decisions in the form of offers that constitute supply functions.

3.2.1.1 Cournot model and extensions

Cournot's oligopoly model has proven to be useful for a diversity of purposes. In particular, it has frequently been used to support market power studies [EMF '99].

[6] The most basic model of price competition is Bertrand's model, which consists of a sequential game in which firms simultaneously choose the prices they want to receive for their production, followed by consumers choosing how much to purchase, with full information about each firm's posted price.

[7] The Cournot model is a simultaneous move game in which the strategy of each firm i is its output level q_i of a homogeneous good. Moreover, each firm correctly estimates the output q_j decided by each of its rivals j. Demand is represented by means of a curve $q(p)$ that indicates the amount of that good q that is consumed at each price p [Daughety '88].

Some of these market power analysis incorporate simple Cournot models merely to illustrate researchers' assertions.

> [Oren '97] presents a theoretical analysis of the possible consequences of relying on a market for financial transmission congestion contracts (TCCs) as a substitute for a design with physical transmission rights. In this context, the holder of a TCC between a certain pair of nodes has the right to receive a payment indexed to their nodal price difference. Using a Cournot model, the author shows that, if generators are assumed to correctly estimate the influence of their decisions on transmission congestions and the unconstrained Cournot equilibrium is not feasible due to limited transmission capacity, a unique equilibrium does not exist. The reason is that generators can offer their production at a number of different prices and perform the same volume of energy transactions (those allowed by the available transmission capacity). In practice, generators in exporting nodes are expected to identify the price at which the energy demanded in the importing region is equal to the existing transmission capacity and offer their production at that price, thus capturing all congestion rents.

Other authors rely on the Cournot model to obtain a numerical evaluation of the potential for market power in a certain electricity industry as an intermediate solution between excessively simple concentration measures, such as the Hirschman-Herfindahl Index (HHI)[8] or the Lerner Index[9], and the obscurity of more sophisticated equilibrium models, such as SFE.

> In [Borenstein '98], Cournot model is used to evaluate the influence of market power on the emerging California electricity market. The authors admit that Cournot model probably overstates the problem of market power due to the inelasticity of the demand curve faced by each of the oligopolistic agents. The SFE alternative, although appraised by the authors, is left aside due to its inherent complexity and the difficulty of expressing generation marginal costs in the form of smooth functions. The transmission network is represented as a radial grid with California importing energy from the three neighboring regions through a set of major flow gates. The three independent owned utilities (IOUs) in California are modeled as Cournot players[10] facing a competitive fringe consisting of both minor in-state producers and out-of-state companies whose sales are constrained by limited importing transmission capacity. Marginal cost functions are constructed for each company including steps due to changes of technology. Fringe firms' supply curves are subtracted from market demand, so as to derive a residual demand curve for the three Cournot firms that is more elastic than the original demand function. The authors recognize the difficulty of modeling the optimal allocation of hydro resources in such a setting and use an approximate technique known as "peak shaving". The algorithm they suggest to calculate Cournot equilibrium is iterative: the profit-maximizing output of each supplier is determined keeping the production of the rest of players fixed and the process is repeated for each player until convergence is reached. The method is used to simulate the monthly operation of the CalPX under a variety of market situations (different demand elasticities, varying hydro reserve levels, divestiture scenarios, etc.) to evaluate the extent to which the IOUs might exert market power.

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[8] The HHI sums the squares of the market shares of all firms in the relevant market to arrive at a statistical measure of concentration.

[9] The Lerner Index attempts to measure market power by subtracting a firm's marginal cost from its price, and then dividing the result by the firm's price.

[10] Authors frequently use the term "Cournot player" when they refer to an agent whose strategic variable is his output. This does not imply that the model under consideration is Cournot's model, which has a precise definition. Indeed, the standard one-leader Stackelberg model can be extended to a multileader situation introducing Cournot players. In [Daughety '88] a case of n firms with m leaders and $n-m$ followers is considered. Each of the followers takes the aggregate output of the leaders as given, makes conjectures about the outputs of all the followers and produces an output. Knowing this, each leader takes the response function of each of the followers, makes conjectures about the outputs of the rest of leaders and computes a best response output to produce. Thus, the followers "play Cournot"

In some cases market analysis disregards the influence of the transmission network and restricts its attention to the behavior of generation companies. Traditional operation planning models that were used in the previous regulatory framework to obtain single-node cost-minimizing hydrothermal schedules have been adapted to the new situation by including a representation of the generators' strategic decisions fall into this category.

> In [Scott '96] a medium-term hydrothermal coordination model based on stochastic dual dynamic programming (SDDP) is used to explore the outcome of New Zealand's wholesale electricity market. The subproblem that is solved at each stage of the SDDP procedure is formulated as a Cournot game in which generation companies maximize their profits taking into account future benefits due to water reserves saved for subsequent stages. Each stage is divided into a number of subperiods that represent different demand levels. Consumers at each subperiod are represented by means of an aggregate demand curve. The influence of forward contracts and options is also accounted for. The equilibrium at each subperiod is approximated by calculating the aggregate output level that makes the average marginal cost equal to the average marginal revenue. This yields an equilibrium price, which is then used to determine the output level of each generation company. The marginal revenue obtained for each of the subperiods of a certain stage can be used to construct a water value curve for that stage (demand curve for release, DCR). A future-profit curve for the water stored in the reservoirs at the end of the current stage (demand for water in storage, DCS) can then be built by aggregating the DCRs of future stages. The authors suggest using the DCS obtained for each week of the year as input data for the short-term scheduling of the generation system. This procedure has been successfully applied to a duopoly case representing New Zealand's power industry under a number of forward contracting scenarios. The relevant conclusion is that a high level of forward contracting and/or demand-side responsiveness reduce the likelihood of strategic gaming in the spot market.

In general, early intents to apply Cournot's oligopoly model to the case of electricity markets did not rely on the special structure of the resulting problem. This resulted in complicated solution procedures or approximate (and inexact) formulations. However, fruitful conclusions were reached from this research that paved the path for future developments.

> In [Ramos '98] and in [Ventosa '99] a set of constraints are introduced in a traditional medium-term linear programming operation planning model to represent the strategic behavior of generation companies as Cournot players. Large-scale problems can be solved under this framework with a conventional LP solver, although, in rigor, the solution is only an approximation of the equilibrium.

Currently, equilibrium models based on Cournot's conjecture are usually formulated as mixed complementarity problems (MCP, [Cottle '92]) —or, alternatively, as a system of variational inequalities, VI—. In this manner, modelers benefit from the existence of powerful commercial solvers capable of solving large-scale MCPs [Billups '97] and from the fact that modeling algebraic languages such as GAMS have been specially adapted to this special kind of problems [Rutherford '95]. Both approaches, MCP and VI, have been successfully applied to perform market power analysis in which the transmission network plays a central role.

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against all other firms, while the leaders "play Cournot" against the other leaders only. No leaders or followers are misjudging what any other player is doing.

[Wei '99] is the first attempt to evaluate the extent to which generators are able to exert market power or the influence of transmission pricing on energy trading under the MCP/VI framework. This paper focuses on the long-term equilibrium reached in a specific power system as a consequence of investment decisions and long-term contracts. Generators are represented as Cournot players that choose their output in order to maximize their profit, taking transmission prices and their rivals' output decisions as fixed. The authors give three arguments to justify the validity of Cournot model for their purposes: its computational convenience, its adequacy to represent the long-term equilibrium reached by generators when deciding their optimal generating capacities and the possibility of extending Cournot analysis to treat more general types of equilibria. The transmission network is represented by the flow conservation equations only, so that phase-flow equations and thermal losses are neglected. Transmission pricing is carried out according to certain rules based on the amount of transmission services demanded by the agents. Consumers are represented by nodal demand functions. The equilibrium is formulated as a set of interrelated profit maximization problems where the strategy of each producer is constrained by those of others producers due to limited transmission capacity, a structure known as Generalized Nash Equilibrium (GNE). The authors transform this set of optimization problems into a single VI problem that can be solved with commercial solvers and provide sufficient conditions for the existence and uniqueness of the equilibrium. The situation of multiple equilibria identified in [Oren '97] when there is no market for physical transmission rights, arises again here. Even in those cases in which the VI problem has a unique solution, multiple equilibria may exist if transmission constraints are such that the GNE does not coincide with the unconstrained Nash equilibrium. However, if transmission prices correctly reflect the cost of congestions then, when the solution to the VI problem is unique, the equilibrium is also unique. A numerical example is solved involving transactions between a number of European countries under two transmission pricing schemes that do not guarantee a unique equilibrium: long-run average-cost pricing and long-run marginal-cost pricing. The results indicate that, due to the exercise of market power, the amount of electric energy supplied in a hypothetic European market might suffer a significant reduction.

The VI approach has been used to evaluate the outcome of a variety of market designs under different assumptions about the strategic behavior of participants and with explicit consideration of the transmission network. Research efforts have been oriented to further analyze the influence of transmission pricing on participants' behavior from the perspective of an integrated European electricity market.

In [Daxhelet '01] a variety of electricity market designs are expressed in the form of VI problems. In this case, both generators and consumers are assumed to have no market power and are represented in terms of supply and demand curves, respectively. The relevant role is played by traders (power marketers) that purchase energy at generation nodes and sell it at demand nodes, making profit of the price difference that they may be able to obtain after paying for transmission services. Competitive traders are unable to exert market power, while oligopolistic traders act as Cournot players both with respect to generators and consumers. Power flows through the transmission network are calculated using the standard DC model. In this context, the authors formulate a number of market designs as VI problems and extract relevant conclusions from their analysis. In particular, the case of a wholesale electricity market with transmission constraints but no market for transmission services previously studied in [Wei '99] is revisited, although in this case the GNE is formulated as a quasi variational inequality problem (QVI). It is shown that, in the presence of transmission constraints, if no market for transmission services exists, the solution of the QVI is indeterminate and multiple generalized equilibria exist. The authors then consider a transmission system operator that maximizes its revenue taking congestion charges as fixed (price taker). Two congestion pricing schemes are explored under this perspective; Chao and Peck's flow gate mechanism [Chao '96] and Schweppe's nodal pricing approach. This research line is continued in [Smeers '01], where the concept of flow gate is extended to include limited transmission capacity that is not physically defined in terms of specific network elements and that may change over time. This allows to represent the limited cross-border transmission capacity between European countries considered by the European Association of Transmission Operators and suggests a dual perspective of the network: traders work with aggregate transmission capacities to perform their energy transactions and ISOs adapt this aggregate information to the physical reality of the transmission system. The authors also analyze how forward energy markets and markets for transmission rights interact with the spot market, in order to further evaluate the ability of traders to hedge against uncertain congestion charges through physical or financial transmission rights.

Similarly, the MCP approach has also proven to be a powerful framework to perform market analysis under a wide range of hypotheses with explicit consideration of a large number of generation units and network nodes. Studies have focused on the comparison of conflicting wholesale market designs such as the bilateral and poolco models.

[Hobbs '01a] successfully combines elements of [Wei '99] and [Daxhelet '01] under an MCP framework, assuming a bilateral market in which the ISO allocates scarce transmission capacity. As in [Wei '99], each generation company decides its output on the belief that its rivals' quantities are fixed (Cournot conjecture) and that transmission prices are not affected by its decisions (Bertrand conjecture). The author sees this latter assumption as a compromise between realism and computability, though overlooking the analysis of [Oren '97], where generators correctly anticipate the influence of their behavior on transmission prices. Power flows are calculated using the linearized DC approximation. Consumers are modeled by means of nodal linear demand functions. The grid owner allocates scarce transmission capacity by maximizing the revenues obtained from participants in payment for transmitting energy between different nodes. In this mechanism the grid owner considers congestion fees and participants' transactions as fixed. This emulates a market for transmission rights in which generators do not exercise market power. In this framework, the cost of transmitting energy between a pair of nodes may not adequately reflect the difference between the prices at which energy is being traded in both nodes. The reason is that congestion charges are determined under the assumption of perfect competition, while generators perform their bilateral transactions acting as Cournot players, a dichotomy that is not easy to justify. An alternative framework similar to that in [Daxhelet '01] is then proposed in which a new type of agent, the arbitrageur, is able to detect price differences that exceed the cost of transmission and try to benefit from it. Arbitrageurs are assumed to behave competitively, ultimately leading to price differences that adequately reflect transport costs. Under this assumption the outcome of the bilateral market coincides with that of a poolco situation with Cournot generators: each generator receives for its production the price of its node, irrespective of the prices at which it sells energy to consumers in other nodes. A small numerical example consisting of a three-network node is solved to illustrate the previous analysis.

From a different perspective, researchers willing to assess the hydrothermal coordination problem of a generation company operating in a wholesale electricity market have also frequently adopted the Cournot framework.

[Bushnell '98], assuming a linear demand curve and firms with linear marginal costs, derives optimality conditions for the hydrothermal coordination problem that lead to a number of remarkable conclusions. Thermal units' should be operated at output levels such that their marginal costs are equal to the company's marginal revenue, as long as their capacity permits it. Hydro units should be scheduled so that the value of hydro resources (water value) is equal to the company's marginal revenue in each planning period. Additionally, hydro reserves should be administered to equalize water value between different planning periods.

In [Ventosa '00a] and in [Rivier '01] a medium-term (one year) operation-planning model for generation companies is formulated and solved using the MCP approach. Each generation company is represented as a Cournot player that schedules the operation of its units with the objective of maximizing its profits. Generating units are modeled with a significant level of detail, something unusual in an equilibrium approach. In particular, the medium-term management of fuel stocks and hydro reserves is optimized taking explicitly into account their chronological evolution. As usual, consumers are aggregated into a linear demand function. No reference is made to the transmission network. Optimality conditions are derived for each firm and interpreted under an operation planning perspective, leading to conclusions similar to those of [Bushnell '98]. Pumped-storage units are also included in this model and their ability to arbitrage between different planning periods is highlighted. This MCP approach is extended in [Ventosa '00b] to account for inflow uncertainty in the context of medium-term hydrothermal coordination. A stochastic dual dynamic programming procedure (SDDP) similar to that of [Scott '96] is proposed in which market equilibrium is computed at every stage as an MCP. Inflow uncertainty is assumed to have finite support and is represented by means of a scenario tree. Firms' profits at each stage include not only present profits, but also future profits which are approximated by a piecewise linear function of the hydro reserves saved for subsequent stages. The SDDP algorithm proceeds backward, so that the slopes of the piecewise linear future-profit function at stage s are obtained from the dual variables of the problem solved at stage $s+1$. This approach has been successfully applied to real-size numerical examples concerning the Spanish wholesale electricity market.

In conclusion, the assumption of generation companies behaving as Cournot players has been extensively used to conduct a diversity of analysis concerning the medium-term outcome of a variety of electricity market designs, including different transmission pricing schemes and additional agents in the form of power marketers or arbitrageurs. The possibility of formulating these models under the MCP/VI framework and benefiting from specific commercial solvers capable of tackling large-scale problems has significantly contributed to the popularity of this approach.

However, a number of drawbacks seem to question the applicability of Cournot model. Firstly, it relies on the demand function to determine equilibrium prices, given that generators' strategies are expressed in terms of quantities and not in the form of offer curves. This seems to substantially deviate from the reality of electricity markets, where a significant number of market mechanisms are based on the offer and bid curves submitted by participants. Secondly, demand in short-term electricity markets is characterized by its inelasticity, which leads to extremely high prices in the Cournot framework. Additionally, Cournot model assumes that each participant knows exactly the decisions taken by his rivals and the shape of the demand function. These shortcomings appear to reinforce the idea that SFE is a better alternative to represent competition in electricity markets [Rudkevich '99].

The fact is that, in many wholesale electricity markets, generation companies have the possibility of increasing prices well above their usual levels. The reasons why they avoid doing so have already been pointed out in Chapter 2, but are difficult to quantify in a model. Nevertheless, although Cournot models may not yield accurate prices, they are generally perceived to adequately reflect the long and medium-term equilibria reached in wholesale electricity markets in terms of quantities. This is based on the intuition that, due to repetitive interaction, generators are able to correctly estimate the energy that their rivals are expected to produce during a time period of about one year[11]. This seems to reconcile Cournot's assumptions with the dynamic aspect of electricity market mechanisms and the uncertainty about rivals' strategy, when considering a medium-term model. Quite the opposite, uncertainty plays a major role in the design of strategies for short-term market mechanisms and must be explicitly considered in related decision-support models, such as the one developed in this thesis.

3.2.1.2 Supply function equilibrium

According to the previous section, a large number of models assume that generation companies both express their decisions in terms of quantities and perceive that their rivals also consider their output as the relevant strategic variable.

In [Klemperer '86] it is shown that, in the absence of uncertainty and given the competitors' strategic variables (quantities or prices), each firm has no preference between expressing its decisions in terms of a quantity or a price, because it faces a unique residual demand. This does not mean that, in the case of a duopoly, the four possible Nash equilibria yield the same outcome: they are all are different and so are

[11] When a coin is thrown it is impossible to know in advance whether the result will be heads or tails. However, if the coin is thrown a large number of times, an accurate prediction of the proportion of heads that will be obtained is straightforward and a measure of the maximum error that can be expected is available.

the corresponding firms' profits. On the contrary, in the presence of uncertainty, it is no longer true that, given the strategic variable of the rival, a firm does not prefer one strategic variable to other. In other words, when a firm faces a range of possible residual demand curves, in general, it expects a bigger profit when expressing its decisions in terms of one of the strategic variables rather than in terms of the other. Two main factors make one firm prefer one strategic variable to other: the slope of its marginal cost curve and the probability distribution estimated for the residual demand curve.

A step forward is to assume that firms are able to choose an intermediate strategy that consists of a supply function. This is the supply function equilibrium approach, which was developed in [Klemperer '89] and has proven to be an extremely attractive line of research for the analysis of equilibrium in wholesale electricity markets.

In [Klemperer '89] a static simultaneous game is proposed in which each firm i expresses its strategic decisions in the form of a twice continuously differentiable supply function, $q_i = S_i(p)$, that indicates the quantity q_i that the firm produces given a price p. In the absence of uncertainty, multiple Nash supply function equilibria exist for the case of a symmetric duopoly and a homogeneous product. When exogenous uncertainty about the demand curve is introduced, the number of supply function equilibria (SFE) is reduced to the set of trajectories that solve a system of differential equations in the region corresponding to possible realizations of the demand curve. A number of interesting results is obtained for the case of a symmetric duopoly and a homogeneous product. All trajectories pass through the origin $(0,0)$ but do not intersect in the rest of their domain. In other words, each equilibrium outcome $(q_0,p_0) \neq (0,0)$ that results from a demand realization is supported by a unique pair of identical supply functions. In particular, if uncertainty has bounded support, some of the trajectories are equivalent, at certain points, to the Cournot or Bertrand outcomes. Moreover, given a realization of the demand curve, the SFE outcome lies, in terms of price, output and profits, between the corresponding Cournot and Bertrand equilibria for that demand curve. The wider the range of possible demand curves, the narrower the set of trajectories that constitute an SFE. The existence of a unique SFE is proved under the particular assumption of linear demand, identical linear marginal cost curves[12] and uncertainty with unbounded support. An analysis of the sensitivity of the solution with respect to the number of symmetric competing firms, the shape of the marginal cost curves and the demand curve is also performed. Results are extended to the case of differentiated products. The explicit consideration of demand uncertainty and of the firms' ability to adopt more flexible strategies than a mere quantity or a price are the two major contributions of this new equilibrium model, whose assumptions and properties are defined by the authors in thorough detail.

Calculating an SFE requires solving a set of differential equations, instead of the typical set of algebraic equations that arises in traditional equilibrium models, where strategic variables take the form of quantities or prices (in fact, the latter can be understood as particularizations of the general SFE problem under certain circumstances). SFE models have thus considerable limitations concerning their numerical tractability. In particular, it is very rare that they include a detailed representation of the generation system under consideration. On the other hand, they provide a more flexible framework to examine a wider range of strategies that simultaneously incorporate quantity decisions and price-setting tactics. The SFE approach was extensively used to predict the performance of the pioneering England & Wales (E&W) Pool, whose revolutionary design did not seem to fit into more conventional oligopoly models. The relative unimportant role played by the transmission network in this particular power system increased the relevance of these studies.

[12] It can be shown that, in the case of identical costs, marginal costs for zero output are irrelevant, given that a translation can be performed in the price domain that makes these marginal costs equal to zero.

In [Green '92] an analysis of the behavior of the duopoly that characterized the E&W electricity market during its first years of operation is performed under the SFE approach, which is seen as the best model to represent competition in the Pool. It is assumed that each company submits a daily smooth supply function, instead of the actual stepwise curve that results from the aggregation of the daily offers corresponding to each of the company's generating units. The authors focus on a static Nash SFE, arguing that such an equilibrium is likely to be reached due to daily repetition and to the short delay with which offers are published. The demand curve faced by generation companies is extremely inelastic (demand-side bidding was almost inexistent) and uncertain due to its variation over time (in the E&W Pool offers were required to be kept invariable throughout the day). A novelty is the introduction of capacity constraints that restrict the number of SFE, eliminating those corresponding to lower equilibrium prices. Interesting conclusions that do not arise in other oligopoly models are reached. For instance, in the case of an asymmetric duopoly it is shown that the large firm finds price increases more profitable and therefore has a greater incentive to submit a steeper supply function. The small firm then faces a less elastic residual demand curve and also tends to deviate from its marginal costs. This could be seen as a case of tacit collusion, previously pointed out by [Bolle '92], where the large generation company suffers the consequences of the curse of market power and indirectly causes an increase of its rivals' profits. The numerical example they provide considers the case of a symmetric n-firm duopoly with quadratic marginal costs and analyzes the influence of demand elasticity and of the number of firms on the resulting SFE for a one-year case based on demand data from three typical days of the E&W Pool. From the whole range of possible SFE the highest-price equilibrium is chosen, which implies an output reduction of more than 10% and a price increase of almost 100% when compared to the competitive outcome. The authors express their concern about these results, which they believe to be caused by an excessive concentration in the generation business due to an inadequate privatization policy. Further research on the influence that long-term contracts and the threat of entry may exert on the behavior of incumbent generation companies is conducted under the SFE approach in [Newbery '97].

The possibility of obtaining reasonable medium-term price estimations with the SFE approach is considerably attractive, particularly when conventional equilibrium models based on the Cournot conjecture have proven to be unreliable in this aspect mainly due to their strong dependence on the elasticity assumed for the demand curve. Indeed, the SFE framework does not require the residual demand curve neither to be elastic nor to be known in advance. Based on the assumption of inelastic demand further advances on the SFE theory have been reported that increase its applicability.

In [Rudkevich '98a] a closed-form expression is obtained that provides the price for an SFE given a demand realization under the assumption of an n-firm symmetric oligopoly with inelastic demand and uniform pricing. Among all the possible SFE, the lowest-price one is considered, in contrast with the highest-price criterion followed in [Green '92]. Convergence problems due to the numerical integration of the SFE system of differential equations are thus overcome. This approach also allows to consider stepwise marginal cost functions, which is more realistic than the convex and differentiable cost functions typical in other SFE models. Consequently, the price at a certain time interval (e.g. one hour) is calculated as a function of the marginal cost curves, the demand for that time interval, the maximum expected demand during the period for which offers are valid (e.g. one day) and the number of identical generation companies under consideration. These theoretical results are applied to the 1995 case of the Pennsylvania power system with a varying number of symmetric firms. Days are classified into ten types according to their demand of energy so that ten different SFE are calculated, one for each day type. Planned capacity outages and a certain degree of uncertainty are also included in the study. Significant price markups[13] are obtained for relatively low values of the Hirschman-Herfindahl Index. As expected, markups increase with the percentage of unavailable capacity and decrease with the number of symmetric firms and the degree of uncertainty about demand. The impact of a change in the payment rules (specifically, the substitution of the uniform pricing rule by the pay-as-bid rule) is explored with a slightly different perspective in [Rudkevich '98b].

[13] The price markup for a certain output level is given by the difference between the actual price and the marginal cost corresponding to that output.

For numerical tractability reasons, researchers have recently focused on the linear SFE model, in which demand is linear[14], marginal costs are *linear* or *affine* and SFE can be obtained in terms of linear or affine supply functions. In fact, this simplification was addressed for the first time in [Klemperer '89], where uniqueness of the equilibrium for the case of a symmetric duopoly with linear demand and linear marginal costs was proved. This line of research has since then witnessed substantial development, mainly due to its ability to handle situations with more than two asymmetric firms.

In [Green '96] the case of an asymmetric n-firm oligopoly with linear marginal costs facing a linear demand curve whose slope remains invariable over time is considered. An SFE expressed in terms of affine supply functions is obtained. Uniqueness is not proved, based on the argument that the assumption of unbounded demand uncertainty is not appropriate for the electricity industry. Consequently, the author restricts the possibilities of the model to giving qualitative, rather than quantitative, predictions for an electricity market. Under this framework, three potential ways of increasing competition are evaluated: forcing incumbents to divest part of their generation assets, breaking them up into a number of smaller companies and encouraging additional entry. The model provides relative measures of the effectiveness of these alternative methods to mitigate market power. Further developments for the linear-SFE paradigm can be found in [Rudkevich '99], where the uniqueness of the affine SFE solution provided in [Green '96] is proved. The possibility of reaching such equilibrium through a repetitive learning process and its robustness with respect to demand uncertainty are assessed.

In [Baldick '00], previous results are extended to the case of affine marginal cost functions and capacity constraints. Solutions for the SFE are provided in the form of piecewise affine non-decreasing supply functions. The case of affine supply functions with no capacity constraints is considered first and a SFE solution is obtained in which the intercept of each firm's supply function is equal to that of its marginal cost function, thus generalizing previous work by [Green '96] and [Rudkevich '99]. The uniqueness of this solution is based on the authors' assumption that offer curves are required to be affine rather than on a formal proof. A step forward is to allow for piecewise linear supply functions so as to avoid negative productions at very low prices and to account for capacity constraints. When a firm finds that the price is too low to produce, it can be eliminated, so that equilibrium can be computed for the rest of firms. This yields a number of different equilibria, each one being valid for a certain range of prices. If the piecewise linear functions that result from different equilibria are non-decreasing then they constitute a valid piecewise linear SFE. Additionally, fringe firms with capacity constraints can be incorporated into the demand function, leading to a piecewise linear modified demand function. In this context, knowing the specific load-duration curve of the system under consideration turns out to be a requisite. The authors propose an ad hoc solution method to overcome this difficulty that provides a reasonable approximation. They use this method to predict the extent to which structural changes in the E&W electricity industry may affect wholesale electricity spot prices.

In [Baldick '01], a comprehensive review of the SFE approach is performed. The authors first revisit the general SFE problem of an asymmetric n-firm oligopoly facing a linear demand curve (no explicit assumption is made relative to the firms' marginal costs) and show the extraordinary complexity of obtaining solutions for the system of differential equations that results. In particular, they highlight the difficulty of discarding infeasible solutions (e.g. equilibria with decreasing supply functions). A number of assumptions typical in SFE analysis such as the requirement that supply functions remain invariable over time are then discussed to shed light on their implications. The stability of the range of equilibria that result in the symmetric n-firm oligopoly with affine marginal costs and no capacity constraints is also discussed. It turns out that, in this symmetric case, every SFE located strictly above the affine equilibrium is unstable, which suggests that those equilibria are unlikely to arise. In particular, it questions the choice of the high-price equilibrium made in [Green '92]. A detailed analysis both from a theoretical and a practical perspective shows that, except for the very special cases of symmetric firms, asymmetric firms with affine marginal costs but no capacity constraints or great variations of demand, it is extremely difficult to find solutions to the general SFE problem that are non-decreasing. However, once the non-decreasing constraints are enforced, the SFE that result are paradoxically strictly increasing, as if these constraints were not binding. Based on these ideas an iterative procedure to calculate feasible SFE solutions is proposed and extensively used to analyze the influence of a variety of factors such as capacity constraints, price caps, bid caps or the time horizon over which offers are required to remain invariable.

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[14] According to [Baldick '00], the precise description would be "affine demand", whereas the term "linear" should be restricted to affine functions with zero intercept.

Some authors have used SFE models to predict the outcome of a given market structure including an explicit representation of the transmission network. Forcing supply functions to be affine typically alleviates the complexity of searching for a solution. As indicated in [Berry '99], this is different from obtaining a linear or affine SFE solution without a priori enforcing it, which is the trend followed by previous authors. Different conceptual approaches have been adopted to obtain numerical solutions for this family of models. In general, no existence or uniqueness conditions are derived.

In [Ferrero '97] the decision process of a group of agents participating in a non-mandatory pool is analyzed from the perspective of game theory. Generation companies are assumed to offer one affine supply curve at each of the nodes in which their units are located. Supply curves are derived from the corresponding marginal cost curves, which are also assumed to be affine, by simply multiplying the slope of the marginal cost function by a constant factor. Transaction costs are calculated based on Schweppe's spot pricing theory, including the influence of transmission constraints. A finite number of offering strategies are defined for each generation company (e.g. offer high, offer low) and a payoff matrix is constructed to identify dominant strategies and predict participants' behavior under a variety of hypotheses, such as coalitions between agents or conjectures about rivals' decisions.

In [Berry '99] an analysis of some of the possible ways in which generation companies may exercise their market power in an electric network is carried out with a SFE model in which supply functions are enforced to be affine a priori. A non-mandatory poolco administered by an ISO is considered that determines nodal prices as a result of a welfare maximization problem with transmission constraints based on the supply and demand functions submitted by participants. Three types of behavior are explored for generation companies: perfect competition, profit maximization by each generator and profit maximization by a monopoly owning all generation. Consumers simply submit their linear marginal utility functions. The problem of determining a generalized Nash equilibrium for this game consists of finding an intercept and a slope for each generator's supply function such that each player is not willing to change its supply functions given his rivals' decisions and the transmission constraints. An iterative procedure is used to search for such a solution in two numerical cases consisting of a two and a four-node network, although its applicability to large problems is significantly limited. A number of interesting policy conclusions are extracted from these studies. The assertion in [Oren '97] that generation companies are likely to capture transmission rents by strategically reducing transmission constraints is partially supported by these results, which raises concern about the long-term distortion that this effect may introduce both in generation and transmission expansion. Regulators are encouraged to base their policy on careful analysis of transmission networks, focusing on flows rather than on transmission constraints.

In [Hobbs '00] an SFE model is proposed in which the strategy of each firm takes the form of a set of nodal affine supply functions. The slope of each firm's nodal supply functions is equal to the slope of the marginal cost function at that node, being the intercept of the supply function the decision variable. The problem is structured in two optimization levels. In the first level, each of the dominant firms decides its nodal supply functions. In the second level the ISO clears the market by solving an OPF problem. Optimality conditions can be obtained for this OPF problem and arranged under a mixed-linear complementarity formulation. These optimality conditions are then introduced into each of the firms' optimal bidding problems, which are thus transformed into a set of mathematical programs with linear complementarity or equilibrium constraints (MPEC). This approach permits the application of sound theoretical developments to assess the existence and uniqueness of an equilibrium and the use of iterative solution methods that can handle large MPEC problems- In this manner, an SFE is obtained that takes into account transmission pricing and network constraints.

The SFE perspective can also be adopted to calculate optimal offering strategies for generators operating in a short-term electricity market. A generation company must conceive its supply curve as the best response to the combined effect of the demand curve and the aggregate supply of its competitors. An SFE can then be understood as the situation in which every generation company opts for its best-response supply curve.

In [Anderson '98] the SFE problem is addressed from the perspective of each individual generator. The short-term problem of calculating the best response of a generator whose costs are expressed as a continuous function and that faces a given residual demand curve is considered first. When uncertainty about the (inelastic) demand is introduced, the authors suggest obtaining the best response for each residual demand realization and constructing a supply curve formed by the combination of these decisions. However, such a curve can only be considered a supply curve if it is monotonically increasing, a property that, in general, cannot be guaranteed. It is then proved that if the competitors' aggregate supply function is convex and differentiable and the firm's costs are convex, then a supply-function response exists that is optimal for any demand realization. This result has the weakness of not being valid when each agent is uncertain about its competitors' aggregate supply function. Based on the concept of best-response supply function, the SFE for a duopoly can be defined as a pair of supply functions such that each one is the best-response supply function for the other. The existence of an SFE between two identical generators with convex costs facing an inelastic demand is proved and an expression for it is derived. Moreover, under certain hypotheses concerning transmission losses, if each generator is located at one end of an unconstrained transmission line then a symmetric SFE can be obtained. The authors also consider the fact that real supply curves are expressed as stepwise functions for which, in general, an optimal response does not exist. A reasonable strategy is then to approximate the uncertain competitors' aggregate supply curve by a convex differentiable supply function, construct an optimal supply-function response and approximate it by a stepwise function. Bounds relative to the accuracy of these approximations for any demand realization are provided.

In spite of the variety of modeling proposals, it is possible to identify a number of attributes that can be used to establish a comparison between different SFE approaches. Some of these attributes refer to the market representation adopted by each author, such as the possibility of considering asymmetric firms and the assumptions made about the shape of the marginal cost curves, the supply functions or the demand curve. Others refer to the model of the generation system (e.g. capacity constraints) or the transmission network (e.g. transmission constraints). Finally, the solution method used by each author and the numerical cases addressed are also two relevant features. Table 3.1 presents a summary of the works that have been reviewed in this section with the aim of illustrating the evolution of this line of research.

	Asymmetric firms	Marginal costs	Demand curve	Supply functions	Capacity constraints	Solution method	Transmission network	Numerical application
[Klemperer '89]	No	$C'(q) \geq 0$ $C''(q) \geq 0$	$D'(p) < 0$ $D''(p) \leq 0$	Twice continuously differentiable	No	Necessary conditions	No	No
[Green '92]	No	Quadratic	Linear	Twice continuously differentiable	Yes	Numerical integration	No	E&W Pool
[Rudkevich '98a]	No	Stepwise	Inelastic	Differentiable	Yes	Closed-form expression	No	Pennsylvania
[Green '96]	Yes	Linear	Linear	Affine	No	Closed-form expression	No	E&W Pool
[Rudkevich '99]	Yes	Linear	Linear	Affine	No	Closed-form expression	No	5-firm case
[Baldick '00]	Yes	Affine	Linear	Piecewise linear	Yes	Heuristics	No	E&W Pool
[Baldick '01]	Yes	Affine	Linear	Piecewise linear non-decreasing	Yes	Heuristics	No	E&W Pool
[Ferrero '97]	Yes	Affine	Inelastic	Affine	Yes	Exhaustive enumeration	Yes	IEEE 30-bus system
[Berry '99]	Yes	Affine	Linear	Affine	Yes	Heuristics	Yes	Four-node case
[Hobbs '00]	Yes	Affine	Linear	Affine	Yes	MPEC	Yes	30-node case
[Anderson '98]	No	$C'(q) \geq 0$ $C''(q) \geq 0$	Inelastic	Differentiable and convex	Yes	Necessary conditions	Yes	Two-node case

Table 3.1. A characterization of SFE models.

In conclusion, the SFE approach presents certain advantages with respect to more traditional models of imperfect competition. In particular, it appears to be an appropriate model to predict medium-term prices of electricity, given that it does not rely on the demand function, as the Cournot model, but on the shape of the equilibrium supply functions decided by the firms. In addition to this, firms' strategies do not need to be modified as demand evolves over time. Quite the opposite, supply functions are specifically conceived to represent the firms' behavior under a variety of demand scenarios[15]. This flexibility, however, is accompanied by significant practical limitations concerning numerical tractability. To date, only under very strong assumptions have SFE problems been solved when applied to real cases. Given that SFE shortcomings are well documented, only the main disadvantages will be cited here. Firstly, in general multiple SFE may exist and it is not clear which of them is more qualified to represent firms' strategic behavior[16]. Secondly, except for very simple versions of the SFE model, existence and uniqueness of a solution are very hard to prove. Thirdly, closed-form expressions of a solution are very rarely obtained. Consequently, numerical methods are needed to solve the system of differential equations, thus increasing the computational requirements of this approach. Moreover, some of this system's solutions may violate the non-decreasing constraint that supply functions must observe. This leads to *ad hoc* solution procedures that usually present convergence problems. Needless to say, transmission constraints are only considered in extremely simplified versions of the SFE model. Nevertheless, research efforts have recently produced encouraging results that may ultimately increase the applicability of this approach.

3.2.1.3 Conjectural Variations[17]

The previous discussion shows that the two more relevant approaches that have been adopted to predict the outcome of electricity markets from the perspective of equilibrium theory appear to have complementary advantages and shortcomings. It is natural then that intermediate solutions arise that try to fill the gap between both trends.

On the one hand, the main deficiency detected in the SFE methodology refers to the difficulty of obtaining a solution even for a single-node asymmetric n-firm case with affine marginal costs and capacity constraints. The fact that multiple SFE may exist and uniqueness is not easy to prove only contributes to aggravate the situation. On the other hand, Cournot's numerical tractability is overshadowed by its inability to predict equilibrium prices. Some authors also highlight that Cournot is a static model in which

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[15] Authors do not seem to agree on the period of time during which offers should be expected to remain invariable. Some refer to start-up costs to indicate that this period should not exceed a number of hours. Others assume that equilibrium supply functions can be maintained during periods ranging from one month to a year. An intermediate approach would be to consider different supply functions for different load levels (e.g. distinguish between on-peak and off-peak hours) and assume they are kept unchanged for no more than a few weeks (so that they reflect changes in fuel prices, inflows, etc.)

[16] The stability analysis performed in [Baldick '01] seems to indicate that higher-price equilibria are less likely to arise due to their inherent instability.

[17] A conjectural variation (CV) is a conjecture by one firm about how other firm will adjust its decision variable with respect to potential adjustments in the first firm's action. CV models, in which each firm's rivals are assumed to react to the firm's output by modifying their own output decisions, have recently received particular attention.

players assume that their rivals will not react to their decisions, thus disregarding the fact that electricity markets are based on the repetitive interaction of participants through a variety of market mechanisms[18]. This has resulted in changes on the conjectures that generators are expected to assume about their competitors' strategic decisions, in terms both of the possibility of future reactions (conjectural variations) and the format of these decisions (more sophisticated than plain quantities).

In [Day '02], after reviewing a diversity of equilibrium models to represent strategic interaction between firms, a model is presented in which each firm assumes that its rivals will react to the market-clearing price by adjusting their quantities. In other words, each firm predicts (perhaps incorrectly) the supply function with which each of its rivals is operating in the wholesale electricity market (conjectured supply functions, CSFs). Yet every firm considers its own output as strategic variable, instead of actually deciding its supply curve (which would be the case of an SFE). The fact that each firm's decisions are restricted by its rivals' actions due to transmission constraints and that every firm anticipates its rivals' reactions yields a type of game that does not fit into Nash's equilibrium definition but into a generalized equilibrium framework. This model is seen as an alternative to Cournot or SFE models, which, in spite of their popularity, have serious limitations. CSFs take the form of affine functions, leading to two different models. In the first one, the slope of the CSFs are assumed to be constant and the intercepts are adjusted so that each CSF correctly reflects both the output decided by the corresponding firm and the market clearing price. In the second one, the intercept is kept constant and the slope is adjusted. Choosing one of the two models and setting the numerical value of the fixed parameters gives the modeler flexibility to represent different degrees of competition. However, existence and uniqueness of the solution can only be readily proved for the case of fixed-slope CSFs. Moreover, the choice of these parameters is not straightforward. With respect to transmission prices, generators act as price takers. The models of the ISO and of the arbitrageurs are similar to those presented in previous proposals [Hobbs '01a]. This leads to an MCP problem that can be treated using specific solvers. Numerical examples based on the E&W case illustrate the potential of this approach.

In [García-Alcalde '02] a smart approach is proposed to correctly reflect the strategic behavior of a number of generation companies participating in a single-node electricity market. They assume that each company makes its decisions based on its perception of the market, which they express in terms of the elasticity of the residual demand faced by the company. In other words, when the company faces a residual demand curve whose elasticity is high (low), the company perceives that its ability to affect the market-clearing price is reduced (increased). This is a generalization of Cournot equilibrium, in which every generation company faces the same residual demand elasticity, given by the elasticity of the aggregate demand curve. The authors also provide a practical method to estimate the long-term elasticity faced by each company. It is based on the assumption that each company took optimal decisions in the past based on an estimation of the elasticity of its residual demand curve. Consequently, the historic behavior of a generation company can be characterized by a single parameter: its perception of its residual demand elasticity. This implicit elasticity does not necessarily coincide with that of the actual residual demand curves faced by the company. Using this method, the authors estimate the long-term elasticities that are implicit in the behavior of the most relevant Spanish generation companies and determine the equilibrium reached by these in a numerical example. The market clearing prices thus obtained correctly reflect the reality of the Spanish wholesale prices of electricity, which are significantly lower than those calculated under Cournot's conjecture.

3.2.2 Simulation models

As indicated, equilibrium models are always based on a formal definition of equilibrium, which is mathematically expressed in the form of a system of algebraic and/or differential equations. This imposes limitations on the representation of

[18] Nevertheless, [Daughety '88] argues that, under a more precise definition of the notion of conjectural variation, the rational and consistent duopoly (oligopoly) equilibrium is, in general, a Cournot equilibrium.

competition between participants in at least two ways. Firstly, there are certain strategic decisions that cannot be easily expressed in terms of equations. Secondly, even if any line of conduct could be characterized through a mathematical model, the resulting set of equations, if it has a solution, is frequently too hard to solve. The fact that power systems are constituted by generation units with complex constraints and by transmission networks that follow Kirchhoff laws only contributes to complicate the situation.

In recent years, the number of simulation models proposed in the technical literature to analyze the operation of wholesale electricity markets has significantly increased. It is the objective of this section to outline some of the modeling approaches that have been adopted, rather than to offer an extensive survey.

Simulation models are an alternative to equilibrium models when the problem under consideration is too complex to be addressed within a formal equilibrium framework. Simulation models typically represent each agent's strategic decision dynamics by a set of sequential rules that can range from scheduling generation units to constructing offer curves that include a reaction to previous offers submitted by competitors. The great advantage of a simulation approach lies in the flexibility it provides to implement almost any kind of strategic behavior. However, this freedom also requires that the assumptions embedded in the simulation be theoretically justified.

In many cases, simulation models are closely related to one of the families of equilibrium models. For example, if in a simulation model companies are assumed to take their decisions in the form of quantities the authors will typically refer to the Cournot equilibrium model in order to support the adequacy of their approach.

It is also frequent that simulation models, although conceptually static, involve iterative solution procedures that emulate repetitive interaction between participants. This is equivalent to the case of equilibrium models in which firms correctly guess the strategic decisions of their competitors, as if they had already been competing for a long time.

In [Otero-Novas '99] a simulation model is presented that considers the profit maximization objective of each generation company while accounting for the technical constraints that affect thermal and hydro generating units. Other relevant factors, such as minimum domestic-fuel consumption rules can also be easily incorporated into this framework. The decisions taken by the generation companies are derived with an iterative procedure. In each iteration, given the results obtained in the previous one, every generation company modifies its strategic position with a two-level decision process. Firstly, each company updates its output for each planning period by means of a profit maximization problem in which market clearing prices are held fixed and a Cournot constraint is included that limits the company's output. Technical constraints are also considered in great detail. Subsequently, the price at which the company offers the output of each generating unit in each planning period is modified according to a descending rule. The authors interpret this framework as a game in which firms first act as Cournot players and then adjust their moves as Bertrand players. New market clearing prices are calculated based on these offers and on the evolution of demand, which is assumed to be inelastic. This approach is applied to the analysis of the one-year operation of the Spanish wholesale electricity market. Numerical results confirm that, when companies consider the influence that the market clearing price exerts on their overall profit rather than considering each generation unit independently, prices tend to increase. This methodology has also been adopted to explore the short-term outcome of a wholesale electricity market [Otero-Novas '00] and to perform risk analysis [Alba '99].

A simulation model based on agents that construct optimal supply functions is proposed in [Day '99] to analyze the potential for Market Power in the E&W Pool. This approach is similar to the SFE scheme, but it provides a more flexible framework that allows considering actual marginal cost data and asymmetric firms. In this model, each generation company assumes that its competitors will keep the same supply functions that they submitted in the previous day. As in [Green '92], uncertainty about the residual demand curve is due to demand variation throughout the day. A procedure to construct nearly optimal supply functions is presented where the price domain is uniformly divided into a number of segments and generation companies decide in which price segments they wish to allocate the energy of their generation plants. This optimization process is based on an exhaustive search, rather than on the solution of a formal mathematical programming problem. After simulating the repetitive interaction of generation companies during several hundreds of days and discarding the supply curves obtained for the first days to eliminate the effect of initial conditions, the authors claim that the resulting average supply functions correctly represent the strategic behavior of the agents. To confirm this assertion, they compare the results of their model for a symmetric case with linear marginal costs to those obtained under the SFE framework, which turn out to be extraordinarily similar. They then apply this model to analyze the behavior of generation companies in the E&W pool in a number of divestment scenarios with different demand elasticity levels and varying degrees of forward contracting. The results support the idea that regulatory authorities frequently understate the potential for market power.

As indicated when discussing dynamic equilibrium models, simulation provides a more flexible framework to explore the influence that the repetitive interaction of participants exerts on the evolution of wholesale electricity markets. Static models seem to neglect the fact that agents base their decisions on the historic information accumulated due to the daily operation of market mechanisms. In other words, agents learn from past experience, they improve their decision making and they adapt to changes of the environment (e.g. competitors' moves, demand variations or uncertain hydro inflows). This suggests that adaptive-agent-based simulation techniques can shed light on features of electricity markets that static models ignore.

[Bower '99] presents a simulation model in which generation companies are represented as autonomous adaptive agents that participate in a repetitive daily market and search for strategies that maximize their profit based on the results obtained in the previous session. Each company expresses its strategic decisions by means of the prices at which it offers the output of its plants. Every day, companies are assumed to pursue two main objectives: a minimum rate of utilization for their generation portfolio and a higher profit than that of the previous day. If a company does not fulfill these goals in a certain session, it changes its strategy in a reasonable manner including a random component, to avoid enforcing a specific behavior to the firms. In this process, the only information available to each generation company consists of its own profits and the hourly output of its generating units. As is frequent in these models, demand side is simply represented by a linear demand curve. This setting allows the authors to test a number of market designs that are relevant for the changes that have recently taken place in the E&W wholesale electricity market. In particular, they compare the market outcome that results under the pay-as-bid rule to the one obtained when uniform pricing is assumed. Additionally, they evaluate the influence of allowing companies to submit different offers for each hour, instead of keeping them unchanged for the whole day. The conclusion is that daily bidding together with uniform pricing yields the lowest prices, whereas hourly bidding under the pay-as-bid rule leads to the highest prices.

Simulation models can also incorporate a representation of the transmission network. In contrast with equilibrium models, where power flow equations are linearized to allow for computational tractability, a simulation framework does not impose, in principle, limitations on the complexity of the power flow model used. This permits a more detailed analysis of the influence of transmission constraints, such as voltage limits, or the effect of transmission losses, an aspect typically ignored in equilibrium models.

In [Weber '98] and in [Weber '99] a simulation tool is described that includes an optimal power flow (OPF) to represent the decision process of the ISO when selecting the sell offers and buy bids that should be accepted in order to maximize social welfare. In this model, both sellers and buyers are assumed to submit nodal linear curves to express the amount of energy they are willing to interchange at different prices in each node. Offers are required to be simple so as to avoid unit-commitment decisions, which would significantly complicate the solution of the OPF. This tool can be used by a generation company to evaluate a variety of offering strategies based on the expected behavior of the rest of agents. However, the number of simulations may become prohibitive if too many offering strategies are explored or an excessive number of market scenarios (competitors' strategies) is considered. Numerical examples illustrate the possibilities of this approach, in particular when the influence of the transmission network must be considered in great detail.

3.2.3 Conclusion

In this section we have described the main modeling approaches that have been recently proposed in the literature in order to represent competition in wholesale electricity markets explicitly considering all the generation companies involved.

We have explored equilibrium models, which typically adopt a static point of view. Cournot models present computational advantages, but show a critical dependence on demand elasticity, a factor that is not easy to estimate. In contrast, SFE models represent more accurately the interaction of participants in the electricity market and the process of price formation, but have important shortcomings concerning numerical tractability. An intermediate solution that has recently been proposed is to introduce conjectural variations in order to represent agents' reactions.

The features of simulation models have also been analyzed. Simulation models provide a more flexible framework to represent firms' behavior but have a less consistent theoretical background.

Neither equilibrium models nor simulation techniques seem the best approach to represent competition for the purposes of this thesis. Indeed, our objective is to develop optimal offering strategies for a generation company operating in a spot market. This requires a detailed representation of the company's portfolio and of the influence of the company's decisions on its own revenues. Hence, a model that focuses on the generation company of study and treats the rest of participants in an aggregate manner seems more appropriate.

3.3 Models that focus on a specific generation company

A generation company willing to optimize its scheduling decisions may not be interested in using an operation planning model that includes an explicit representation of the rest of generation companies, as that requires solving a larger problem and using input data that may be difficult to estimate. The alternative then is to use a model that represents in detail only the generating units owned by the company and considers in an aggregate manner those owned by its competitors (or not considers them at all). The disadvantage of this approach is that price formation cannot be modeled as the result of the interaction of market participants.

According to [Ventosa '01], there are two alternative ways to incorporate electricity prices into a model of this sort. The first one is to assume that the generation company is not able to modify the price of electricity with its decisions. This is a hypothesis that is only realistic for relatively small generation companies, but that significantly simplifies the analysis, as will be highlighted in this section. The second approach is to express the price of electricity as a function of the company's strategic decisions. This can be done by means of the residual demand curve, but other methodologies have also been proposed.

This section presents a survey of models that include an explicit representation of only one generation company with the aim of illustrating the characterization suggested by [Ventosa '01]. Special attention is paid to models that focus on the problem of scheduling generating units for the short term (one day to one week) and those that develop offering strategies for an electricity spot market. A more comprehensive literature survey on the problem of developing optimal offering strategies for competitive electricity markets can be found in [David '00]. Additionally, the new role played by unit-commitment models in a deregulated electricity industry is an issue that several authors discuss in depth in [Hobbs '01b].

3.3.1 Models that assume that prices are exogenously fixed

Assuming that electricity prices are exogenously determined greatly simplifies the problem of determining the short-term profit-maximizing policy that a generation company must follow. In first place, if the price of electricity does not depend on the company's decisions, the company's revenues turn out to be a linear function of its output. Additionally, the problem of optimally scheduling each generating unit owned by a company that operates as a price taker can be considered independent of the rest of the company's generating portfolio, unless some other constraints are present that simultaneously affect the output of several of its generating units (e.g. hydro units located in the same river basin). This allows authors to restrict their attention to the problem of optimally scheduling a single generating unit.

This approach is closely related to the traditional Lagrangian relaxation procedure, where the Lagrangian dual of the unit-commitment problem is formulated by relaxing the complicating constraints that establish a link between the generating units under consideration (see chapter 5 for a more detailed description of the Lagrangian relaxation method). Lagrangian relaxation (LR) leads to an iterative solution procedure that consists of two steps. In the first step, each generating unit is independently scheduled, given the Lagrange multipliers obtained in the previous iteration. A new cut is thus obtained that improves the linear outer approximation of the dual function, which is then used in the second step to search for a better dual solution and update Lagrange multipliers. Assuming exogenous prices is equivalent to assuming that the company's decisions do not affect Lagrange multipliers or, in other words, that the marginal cost of the system ("system's lambda") does not depend on the generation units scheduled by the company. Such a problem can be solved with a single iteration of the LR procedure.

In [Gross '96], the daily scheduling process imbedded in the E&W Pool is formulated as a cost minimization problem and decomposed using the LR approach. The Lagrange multiplier associated to the demand (reserve) constraint is assumed to be equal to the marginal price of energy (reserve). The optimal offering problem for a specific generation company is then expressed in terms of these Lagrange multipliers, which, in general, depend on the company's decisions. However, if perfect competition is assumed, Lagrange multipliers do not vary with the company's decisions and the problem turns out to be extremely easy to solve. In fact, an analytical solution exists which simply consists of revealing the company's true costs and production capacity.

The exogenous-price assumption seems to be a step backward with respect to LR developments. However, this can be acceptable if the company is small enough to admit that its ability to modify electricity prices is negligible with respect to the price uncertainty introduced by its competitors' decisions. Indeed, this has been the choice of a number of authors. Additionally, dynamic programming (DP) appears to be the preferred approach to derive optimal scheduling decisions for a generation unit under uncertain exogenous prices. This is due to the fact that DP was also the method used by many authors to solve the individual generation subproblems in each step of their LR procedures. Thus, researchers have simply applied their DP routines to the new self-commitment problem faced by generation companies acting as price takers in a deregulated electricity market.

In [Rajamaran '01] the self-commitment problem of a generation company is described and solved in the presence of exogenous price uncertainty. The possibility of providing either energy or reserve is considered, as well as the different short-term market mechanisms (day-ahead and real-time) in which the company can participate. The authors highlight that this self-commitment problem does not include the requirement to meet the system's load. The objective function to be maximized is the company's profit, based on the prices of energy and reserve at the nodes where the company's units are located, which are assumed to be exogenously determined and are uncertain. The authors correctly interpret that, in this setting, the scheduling problem for each generating unit can be treated independently, which significantly simplifies the process of obtaining a solution, thus permitting a detailed representation of each unit. A DP approach is suggested in which the state of each unit in each time period must be chosen from a set of discrete states. The exogenous price of energy is modeled by means of a discrete Markov process for which transition probabilities can be obtained from historical data. Reserve prices are assumed to be correlated with energy prices. Fuel prices are expected to remain constant during the time scope of the model (one week). The problem is solved using backward DP and several numerical examples illustrate the possibilities of this approach.

[Valenzuela '01] addresses the problem of constructing energy price scenarios for a similar self-commitment problem. The authors postulate perfect competition and assume that a good estimation of the marginal costs of each generation unit operating in the system is available. The problem then reduces to determining the probability distribution for the marginal generation unit in each time period, including the influence of demand uncertainty and unplanned generation outages. A variety of methods are suggested and compared to approximate these probability distributions.

Given that the case of a small company with no influence on electricity prices is particularly easy to solve, it provides an excellent context to examine certain issues with a level of detail that would be prohibitive otherwise. For example, the intuitive idea that the optimal offering strategy for a generator acting as a price taker is to reveal its marginal costs may not be true if the generator's costs are not convex, as its marginal costs would not be non-decreasing. Even if the generator's marginal costs are increasing, they may not fit into the offer format imposed by the market operator if, for instance, the number of blocks of energy that can be offered at different prices is limited.

In [Neame '01] the problem of constructing the best offer curve for a single generator and a single trading period is addressed. The generator is assumed to act as a price taker facing an uncertain market-clearing price with a known probability distribution. The authors focus on how to express the generator's marginal costs in terms of a stepwise offer curve with a limited number of steps. It is shown that a necessary condition for optimality is that each step of the offer curve has a price equal to the average marginal cost of the energy it comprises. Where to place these steps is, however, a question that may have multiple locally optimal answers. A DP procedure that overcomes this difficulty is proposed for the special case of piecewise linear (not necessarily convex) cost functions.

3.3.2 Models that express the price of electricity as a function of the company's decisions

The special features of electricity as a commodity tend to increase the ability of generation companies to affect electricity prices. It is thus very rare that a generation company can consider the price of electricity as independent of its own decisions. When this is not the case the company can no longer consider the problem of scheduling each of its generating units independently. In addition to this, the company must bear in mind that the price also depends on the strategy followed by the rest of agents. Moreover, the rivals' scheduling decisions depend on the price.

As mentioned, including the rivals' generating units in a model increases the size of the problem that has to be solved and requires estimating their costs and technical parameters. An alternative is to represent the company's competitors by means of their offer curves. These indicate both how rivals' decisions affect the price of electricity and how their scheduling decisions vary with the price of electricity. The question then is how to express their offer curves so as to guarantee that the resulting problem is numerically tractable. A frequent approach is to assume that they take the form of linear or quadratic functions.

In [Wen '01] a method is proposed to develop offering strategies for a generation company participating in a day-ahead energy market and the subsequent spinning reserve market, assuming that all firms submit linear energy and reserve offer curves. The limited capacity of the company's generation units requires a tradeoff solution to maximize its total profit. The authors optimize the parameters of the company's linear offer functions, while those of its competitors are supposed to follow correlated normal distributions. They propose sampling rivals' parameters with the Monte Carlo method and then optimizing the company's parameters with a genetic algorithm. A probability distribution is thus obtained for the company's profits both in the day-ahead and the spinning reserve market.

In [Zhang '00] the daily offering problem of a generation company operating in the New England electricity market is considered by representing offers as quadratic functions. The authors first define a model for the day-ahead market and obtain a closed-form expression both for the market clearing price and for the energy sold by the company. Each of these expressions is a function of the company's quadratic offer curve parameters (which have to be optimized) and of the competitors' parameters (which are assumed to follow discrete probability distributions). They then formulate the company's optimal bidding problem where the above expressions are used to evaluate the company's expected revenues. Production costs and technical constraints are also incorporated. The bidding problem is decomposed using LR, leading to the iterative solution of a set of subproblems, one for each generating unit and an additional one to construct the company's offer curve. The presence of this subproblem, in which the company's revenues are evaluated at each iteration, is an interesting novelty and shows how traditional generation scheduling procedures based on LR can be adapted to the new competitive environment.

Mathematical programming techniques are quite restrictive in terms of their ability to tackle problems that include non-convex functions such as the ones required to accurately model offer curves. In principle, introducing binary variables permits to overcome this difficulty, but their usage must be limited to guarantee that the resulting problem can be handled by the existing mixed linear-integer programming (MIP) solvers.

In [Nowak '00] a mixed linear-integer formulation is adopted to model the market operation and calculate the company's expected revenues. They discretize the price domain into a fixed number of levels. For each hour and each price level, they define three binary variables. The first variable is equal to one for a certain price level only if this level is below the market-clearing price. The second variable is equal to one only if the price level is above the market-clearing price. Finally, the third variable indicates that both prices coincide. The generation company has to decide the optimal quantity to sell and to buy at each price level. The quantities actually accepted by the market operator will depend on the quantity offered by competitors at the same price levels, which are taken as given data. Though results are only reported for a deterministic case, the authors sketch an extension to the stochastic version and suggest the use of a dual decomposition method.

Indeed, some authors explicitly abandon the standard mathematical programming framework, arguing that very good solutions can be obtained using other search methods that do not impose special restrictions on the functions used to represent offer curves.

In [Guan '01] the optimal offering problem of a generation company is formulated explicitly considering the influence that the company's decisions exert on the market-clearing price. The authors regard this as a challenging problem and suggest searching for a good bidding strategy, rather than struggling to obtain the optimal one. This justifies the use of ordinal optimization (OO) as a solution method, which is based on the idea that it is easier to estimate the order of a set of solutions than to calculate their real values. An additional advantage of OO is that it poses no restrictions on the form of the function that defines the company's influence on the market-clearing price. The particular application of OO to the optimal offering problem follows this sequence. First a nominal offer curve is built by optimizing the company's generation schedule given a set of price scenarios. Then a population of alternative offer curves is constructed by perturbing the nominal offer curve. These offer curves are ordered according to a rough estimation of their associated profits and a selection of good enough curves is made. Among these, the best offer curve is identified with a more accurate evaluation method. Further details can be found in [Guan '99b]. An extension to incorporate the reserve market is outlined in [Guan '99a].

In practice, the difficulty of addressing the optimal offering problem with a mathematical programming perspective is caused by the non-concave nature of the company's revenue function. This revenue function is obtained, for each of the spot market auctions, as the product of the company's energy sales, q, and the clearing price, p, which in general depends on the company's sales. This dependence can be expressed through the company's residual demand function, $p(q)$, as is further explained in appendix A. Thus, the company can first estimate the residual demand function for each spot market auction with available historical data and then derive an expected revenue function, $r(q)$, by simply multiplying each output q by the corresponding market clearing price, $p(q)$. The resulting revenue function, although not globally concave, typically has concave sections (i.e. sections where its slope is non-increasing) that can be easily identified. If a binary variable is assigned to each concave section, a mixed linear-integer programming formulation is obtained that can be solved with a commercial MIP optimizer.

[García-González '99] develops a weekly generation scheduling method based on mixed linear-integer programming in which the company faces a series of linear residual demand curves, one for each hour, leading to hourly quadratic revenue functions that are approximated by means of outer linearizations. This approach, together with the use of a commercial MIP optimizer, allows the authors to solve real-size self-commitment problems that explicitly take into account the company's influence on market clearing prices, which is a significant step forward with respect to previous proposals. An alternative representation of the revenue function is suggested in [Baíllo '01], where non-concave revenue functions are expressed in terms of piecewise linear functions. Further developments of this line of research can be found in [García-González '00b].

However, as indicated in [Klemperer '89], a key ingredient that a firm must consider when developing an optimal offering strategy is uncertainty about its residual demand. A firm that knows its residual demand in advance has infinite equally optimal offering strategies. On the contrary, the number of optimal offer curves decreases as uncertainty about the residual demand increases. There are a number of ways in which this uncertainty can be represented and incorporated into an optimal offering procedure. One elegant (although not very practical) alternative is to consider a probability distribution function for the residual demand curve that returns the probability of each offer (q, p) being accepted.

In [Anderson '99] a set of necessary conditions that must be fulfilled by an offer curve to be locally optimal is derived for a generation company facing a market distribution function (MDF). For each offer of the company's supply curve, the MDF returns the probability of it not being fully accepted. The authors base their analysis on the systematic application of optimal control theory to the trajectory followed by the company's supply function. They start by considering a certain offer curve and then perturb it to derive the conditions that must hold for it to be locally optimal. This particular formulation of the problem does not consider intertemporal effects or technical constraints. The same line of reasoning is followed in [Anderson '00] to develop necessary and sufficient conditions for local optimality.

A more convenient approach is to assume that residual-demand uncertainty has finite support. In other words, the number of residual demand curves that the firm may face at each planning period is limited. This number should be high enough to correctly represent the variety of market situations that may arise but not as high as to require a prohibitive computational effort. Several approximate solution procedures have been suggested for the problem of obtaining the optimal offering strategy for a generation company that faces a set of spot market scenarios defined in terms of residual demand curves.

In [Mateo '00], a procedure to obtain a set of residual demand curve scenarios for an auction of ancillary services based on past bidding data is proposed. Given these scenarios, the authors show how to obtain a good approximation for the company's optimal offering strategy using genetic algorithms (GA). A population of offer curves is generated and the individuals are sorted according to their associated expected profit and degree of risk exposure. A new population is then derived from the best individuals of the previous generation and the process continues until no significant improvement is achieved. The promising results obtained with this method suggest future research to enhance its modeling features, by including technical constraints or explicitly considering correlation between hours.

As a matter of fact, when technical constraints are incorporated into a model that includes several spot market scenarios in the form of residual demand curves, the resulting MIP problem cannot be solved with existing state-of-the-art MIP optimizers except for very small study cases.

[Baíllo '00] constitutes an attempt to optimize the offer curve of a generation company taking into consideration several consecutive hours with a number of residual demand scenarios in a single-shot MIP framework. Although the results are encouraging, the size limitations of their proposal make it impractical.

Some authors have decided to decompose the problem into a set of subproblems, one for each scenario. However, the solutions proposed do not provide an upper bound for the loss of optimality caused by this decomposition. This is a shortcoming that the developments of this thesis aim to overcome.

In [García-González '00a] an optimal schedule is obtained for each spot market scenario irrespective of the rest of scenarios under consideration. The set of schedules obtained (one for each scenario) may not be expressed in the form of an offer curve if they are not monotonically increasing with price. In other words, if the output decided by the company for a low price is higher than the output decided for a high price, both decisions are incompatible, as they cannot be simultaneously expressed in the same offer curve. This is not an unlikely result, due to the unpredictable shapes that revenue functions may adopt and to the intertemporal constraints that must be observed when scheduling generation units. The scatterplot obtained in the (q,p) space, constituted by as many offers as market scenarios are considered, can then be used to construct a piecewise linear approximation of the optimal offer curve. No upper bound is provided for the deviation of this solution from the optimal one.

In [Berzal '01] market scenarios are first sorted in a decreasing-likelihood order. An optimal schedule is then obtained for the maximum-likelihood scenario irrespective of the rest of market scenarios. The optimal schedule obtained for the next scenario will be constrained by the schedule obtained for the first one, to guarantee that the company's output decisions are monotonically increasing with price. The algorithm proceeds until a solution for the least likely scenario is obtained that observes the monotonic constraints imposed by all the previous ones. The optimality of this solution cannot be guaranteed, given that the most likely scenario might be extremely restrictive and dramatically condition the solutions obtained for the rest of scenarios.

Given that the objective of this thesis is to provide a consistent procedure to develop optimal strategies for the short-term operation of a generation company, it is interesting to determine some of the relevant attributes that characterize models with similar features, such as the ones that have been reviewed in this section. Some of these features describe the market representation imbedded in the model (e.g. the market mechanisms considered by the model, the manner in which price formation is modeled or how market uncertainty is represented). Others indicate the detail with which the generation system is taken into account. With respect to the transmission network, it is interesting to point out that all the reviewed proposals consider a single-node situation. Finally, the method adopted by each author to obtain numerical results and the format of the solution provided by each model are also two relevant distinguishing features.

	Markets considered	Price formation	Market uncertainty	Network constraints	Individual generation units	Intertemporal constraints	Solution method	Solution format
[Gross '96]	E&W Pool	Exogenous	No	No	Yes	Yes	Closed-form solution	Offers
[Rajamaran '01]	Energy and reserve	Exogenous	Yes	No	Yes	Yes	DP	Schedule
[Valenzuela '01]	Energy	Exogenous	Yes	No	Yes	Yes	DP	Schedule
[Neame '01]	Energy	Exogenous	Yes	No	Yes	Yes	DP	Offers
[Wen '01]	Energy and reserve	Linear offer functions	Yes	No	Yes	No	GA	Offers
[Zhang '00]	Energy	Function of the company's offers	Yes	No	Yes	Yes	LR	Offers
[Nowak '00]	Energy	Function of the company's output	No	No	Yes	Yes	MIP	Schedule
[Guan '01]	Energy and reserve	Function of the company's offers	Yes	No	Yes	Yes	OO	Offers
[García-González '99]	Energy	Residual demand	No	No	Yes	Yes	MIP	Schedule
[Anderson '99]	Energy	Market distribution function	Yes	No	No	No	Necessary conditions	Offers
[Mateo '00]	Reserve	Residual demand	Yes	No	No	No	GA	Offers
[Baíllo '00]	Energy	Residual demand	Yes	No	Yes	No	MIP	Offers
[García-González '00a]	Energy	Residual demand	Yes	No	Yes	Yes	MIP + heuristics	Offers
[Berzal '01]	Energy	Residual demand	Yes	No	Yes	Yes	MIP + heuristics	Offers

Table 3.2. A comparison of models oriented to develop optimal short-term strategies.

Section 3.5 provides an overview of the particular modeling approach adopted in this thesis in the light of the advantages and shortcomings of other proposals identified in the previous survey.

3.4 The demand side

Although pioneering electricity spot market designs did not usually contemplate the possibility of demand-side bidding, current electricity spot market mechanisms are frequently based on the double-auction concept, where transactions are accepted based on the sell offers and buy bids submitted by participants.

Nevertheless, demand-side responsiveness is still perceived to be low. In particular, models oriented to represent firms' interaction in wholesale electricity markets typically treat the demand side in an aggregate manner, by means of a demand curve that expresses the amount of energy that wholesale buyers are ready to purchase at each price. Moreover, the number of models that have been developed to obtain optimal buy bids for an energy service provider that purchases electricity in the wholesale market and then sells it to final consumers is insignificant when compared to the number of optimal offering procedures proposed for generation companies[19].

In subsequent chapters it will be shown that the model proposed in this thesis can be used to obtain optimal offers for a generation company or optimal bids for a wholesale buyer indistinctly. The main differences between both types of agent can be found in their portfolio, rather than in their perception of the market mechanisms.

3.5 The model considered in this thesis: an overview

Both the interest exhibited by the research community about the problem of developing optimal offering strategies for generation companies and the multiple difficulties that this problem entails confirm the relevance of this thesis and of other parallel ones that have been developed throughout the world in recent years. Additionally, the plethora of methodologies proposed in the literature requires that the particular model adopted to represent competition in this thesis be precisely defined. This section provides a general overview of the model chosen for this thesis in the light of the ideas previously presented in this chapter. It also specifies the particular challenges that this modeling approach involves. Finally, it serves as an introduction to the following chapter, in which the methodology proposed in this thesis to address the problem under consideration is presented.

3.5.1 A model based on residual demand curves

The residual-demand/revenue-function approach described in [García-González '99] and in [Baíllo '01] is the most sensible choice to model competition in an electricity spot market for the purposes of this thesis (see Appendix A for further details). The residual demand curve represents the relevant rest of the world with minimum information requirements. Yet, real residual demand curves yield revenue functions that are non-concave and, as a consequence, not easy to treat in an optimization

[19] [Borghetti '01] is one of the exceptions to this general trend. It provides an extension of the approach proposed in [Gross '96] to include demand-side bidding in a LR-based day-ahead market.

context. Some authors avoid this difficulty by considering linear or quadratic offer curves [Wen '01, Zhang '00].

An important remark is that a model only based on residual demand curves fails to completely represent the strategic behavior of rivals. This is due to the static perspective that a residual demand curve provides about other agents. The residual demand curve informs only about the strategy followed by the rest of agents in a specific situation. It does not reveal how rivals would react if the circumstances suffered a significant change. A sequence of several spot market sessions with similar residual demand curves may indicate that the spot market is in steady state (equilibrium). In this context, an agent can use his expected residual demand curve to evaluate how the market clearing price changes with slight modifications of his strategy. On the contrary, if the agent introduces a great variation in his strategy, the expected residual demand curve will only be valid for the current spot market session, but will be completely useless for the next sessions, given that the rivals' reaction is almost unpredictable. In other words, the residual demand curve may only provide a short-term estimate of the consequences of a certain strategy. Therefore, the results provided by a model based on residual demand curves must be handled cautiously, particularly if they lead to sudden changes in the market conditions. These undesired effects can be dampened by including strategic constraints that guide the model toward a medium-term equilibrium target.

3.5.2 A model that explicitly considers uncertainty

The residual demand curves that a company will face in future auctions are not known in advance. Therefore, the company must make use of historic data to estimate them. This requires that there is public access to historic aggregate offer and demand curves, which is one of the assumptions made in chapter 2[20].

For the sake of simplicity, some models consider only the expected residual demand curve [Nowak '00, García-González '99, Baíllo '01]. However, any rival can modify his behavior from one day to another and induce significant changes in the company's residual demand curve. Hence, it is more convenient to explicitly consider the probability distribution of each of the company's future residual demand curves. In this thesis it is assumed that these probability distributions have finite support. In other words, the number of residual demand curves that can occur in a certain auction is limited. A simple method is described in Appendix B that selects a number of past residual demand curves in order to build a discrete probability distribution for each of the spot market sessions to come. This method takes into consideration the possible correlation that may exist between the residual demand curves that arise in different auctions, even if they are part of different market mechanisms (e.g. the correlation that may exist between the residual demand curve of one of the day-ahead market auctions and the residual demand curve of one of the adjustment market auctions). In practice, the idea is to define spot market scenarios that consist of a collection of residual demand curves, one for each hourly auction of each market mechanism.

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[20] Information about the sell offers and buy bids submitted to each of the hourly auctions that constitute the sequence of spot market mechanisms are assumed to be available some time after that auction is held

The influence of all market scenarios is simultaneously considered in our spot market model. This poses significant computational requirements, as we explain in subsequent chapters. To avoid this, other authors have proposed heuristic decomposition methods that cause optimality losses that are difficult to estimate [García-González '00a, Berzal '01].

3.5.3 A model that decides offering strategies

In this thesis, the decisions taken by the generation company are expressed in the form of offers and not in the form of a generation schedule. As a matter of fact, a generation company operating in wholesale electricity markets is not usually given the possibility of actually deciding the precise output of its generating units. Quite the opposite, the schedule of its units is the result of a sequence of market mechanisms in which the company takes part.

3.5.4 A model that takes the company's portfolio into account

Nevertheless, it is essential that a model intended to provide optimal short-term strategies for a generation company take into account the characteristics of the generation units owned by the company. However, the combination of a full spot market model with a detailed representation of the generating units is not easy to handle. Some proposals simply ignore the intertemporal constraints that affect the generating units, such as the logic of startups and shutdowns, the limits imposed by thermal units' maximum ramp rates, the evolution of hydro reserves, etc. [Anderson '00, Anderson '99, Mateo '00]. This permits considering each hourly auction as an independent trading period, which significantly simplifies the solution of the problem. On the contrary, the approach developed in this thesis includes an explicit representation of these constraints, so that all the auctions that constitute the spot market must be simultaneously considered. The representation of the generation system is not extremely detailed but rather keeps a balance with the spot market representation, so that both aspects receive the attention they deserve.

The resulting model, which is formally described in the next chapter, also considers the company's position with respect to long-term contracts as well as other guidelines that orient the company's strategy in the spot market towards its long-term profit-maximization objective.

3.5.5 A model that can be adapted to the case of an energy service provider

From the perspective of a wholesale energy buyer such as an energy service provider (ESP), the residual demand approach can be easily adapted to obtain a residual offer model that indicates the price that the ESP must pay to purchase different amounts of electricity in a double auction. Chapter 4 provides guidelines for the representation of the ESP's portfolio of customers and forward wholesale contracts as a first step of the process of developing a complete optimal bidding procedure.

3.5.6 Summary of modeling challenges

To summarize, in order to meet the modeling challenges assumed in this thesis, the methodology that we develop in subsequent chapters should:

i) model each of the auctions that constitute the spot market from the perspective of a generation company by means of a number of possible residual demand realizations, together with the corresponding revenue functions, whatever shape they may have;

ii) represent the uncertainty faced by the generation company in the form of a discrete probability distribution using spot market scenarios;

iii) express the optimal strategy for the generation company in terms of offers that can be submitted to the main market mechanisms that constitute the spot market;

iv) simultaneously consider the influence of every spot market scenario when deciding this optimal strategy;

v) incorporate a detailed representation of the company's generating units, including the explicit formulation of intertemporal technical constraints;

vi) allow for the consideration of other elements of the company's portfolio, such as physical or financial long and medium-term contracts;

vii) take into account other long-term strategic objectives, such as the position that the company wishes to defend in the wholesale electricity market;

viii) be applicable to the case of an ESP with minor modifications;

Each of the reviewed proposals meets some of the previous challenges but none covers them all simultaneously. Figure 3.1 illustrates the gap that this thesis aims to fill. As can be seen, our purpose is not only to formulate the problem, but to solve it making use of mathematical programming techniques that yield the best solution possible, while at the same time providing an upper bound for the loss of optimality that may be incurred in those cases where the size of the problem renders it impossible to guarantee optimality.

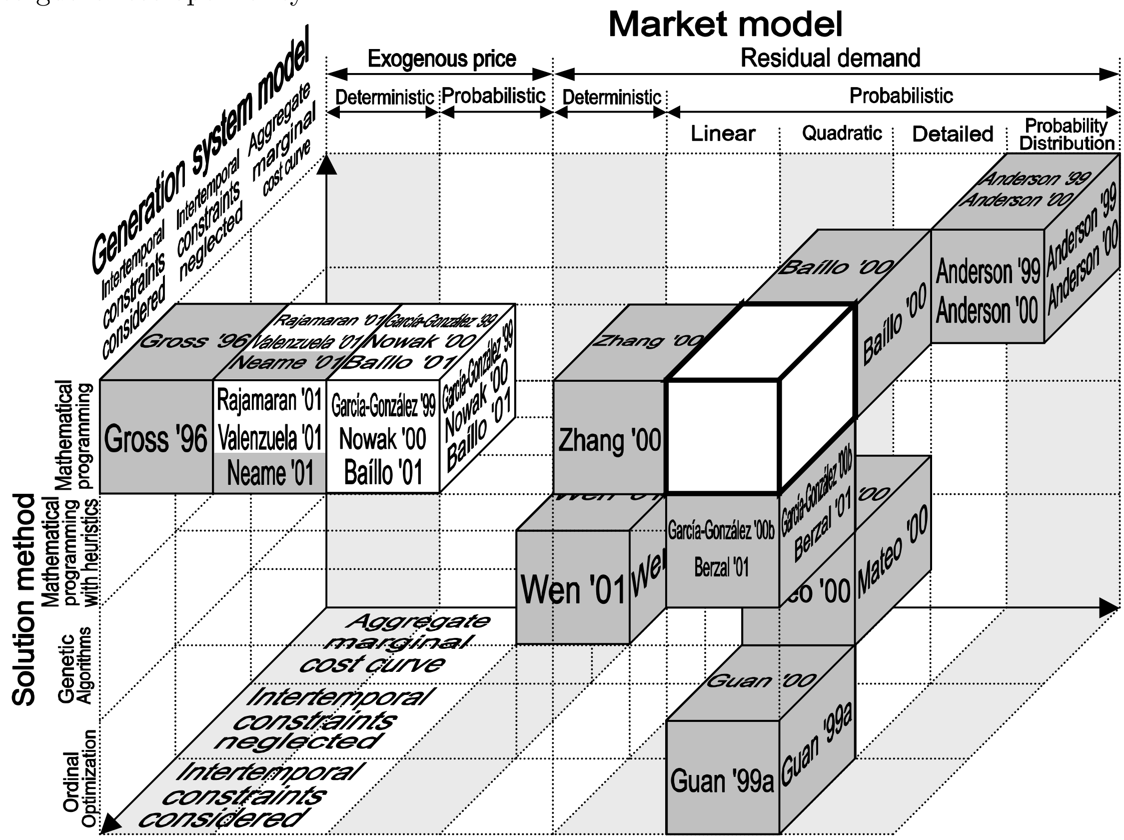

Figure 3.1. Models oriented to develop optimal short-term strategies: a graphic characterization.

3.6 Conclusion

This chapter justifies the modeling approach adopted to represent competition in the wholesale electricity market assumed in this thesis. This model has been chosen based on the analysis of the general modeling trends that have been followed in recent years. The impressive advances registered in this research field gives an idea of the attention that this matter has attracted during the last decade.

Models that evaluate the interaction of agents in wholesale electricity markets have persistently stemmed from the game-theory concept of equilibrium. Some of these models represent equilibrium in terms of variational inequalities or, alternatively, in the form of a complementarity problem, providing a framework to analyze realistic cases that include a detailed representation of the generation system and the transmission network. This line of research has also rendered theoretical results relative to the design of electricity markets or to the medium-term operation of hydrothermal systems in the new regulatory framework. Other models explore in detail the implications of a market design in which agents use offer curves to express their strategic decisions and conclude that, in general, multiple supply function equilibria exist, which creates a new source of uncertainty for regulators. The contribution of simulation models has been significant as well, on account of their flexibility to incorporate more complex assumptions than those allowed by formal equilibrium models.

Researchers have also adopted the viewpoint of a specific generation company to develop decision-support tools explicitly designed to face the challenges posed by the new regulatory framework. Although proposals have been made for the whole range of time scopes that are relevant in the decision process of a generation company, this chapter has focused on short-term models. It has been shown that ignoring the ability of generation companies to modify the price of electricity greatly simplifies the analysis but is unrealistic in most cases. A variety of methodologies have been proposed in the literature to take this effect into account though at the same time incurring in other simplifications that limit the relevance of their results. This thesis aims to take a step forward in this direction by adopting a modeling approach that simultaneously considers all the important ingredients that are involved in the development of short-term optimal strategies for generation companies operating in an electricity spot market.

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4 A model to develop optimal offers for an electricity spot market

After laying the foundations of this thesis in chapters 2 and 3, this chapter presents the main structure of the building. Two lines of development define the approach adopted to formulate the problem under consideration. On the one hand, we define a spot market model that conceives the problem as a multistage stochastic program. This approach adapts well to the case of a spot market designed as a sequence of market mechanisms: each decision stage corresponds to one of the spot market mechanisms and stochasticity is due to the uncertain residual demand curves that the company faces. On the other hand, we suggest a detailed representation of the company's portfolio, including not only its generation plants, but also its position in other market mechanisms and its main long-term strategic objectives.

Although some of the developments of this chapter are subject to the specific institutional assumptions formulated in previous chapters, the conceptual approach is general and can be easily adapted to other spot market designs or different assumptions. In particular, we show how this methodology, originally conceived for the case of a generation company, is also valid for the case of an energy service provider.

4.1 Introduction

This chapter can be considered the conceptual core of this thesis. It describes in detail the formulation proposed to represent the problem faced by a generation company in an electricity spot market.

We suggest an original approach based on the theory of multistage stochastic programming to model the decision process of a generation company in a spot market designed as a sequence of market mechanisms. In such a spot market, the strategy of a generation company is implemented through several decision stages, each one corresponding to one market mechanism. Stochasticity is due to the fact that the actions taken by rivals are not known in advance.

The influence of the rest of agents on the company's results is modeled by means of residual demand curves. In this manner, several sets of residual demand curves define the different spot market situations that the company may face. The development of offering strategies in this framework is reduced to deciding vectors of outputs, one for each possible residual demand realization.

The model presented in this chapter also includes a detailed representation of the company's portfolio. The generation units owned by the company are represented as is usual in unit-commitment and economic-dispatch models. Particular attention is paid to the influence of the position adopted by the company with respect to long-term contracts and options. Although our work focuses on generation companies, we also justify its validity for the case of an energy service provider.

This modeling approach not only covers the challenges identified in the previous chapter, but also yields a problem structure that is amenable to a solution strategy based on primal and dual decomposition methods, as is discussed in subsequent chapters.

4.2 Modeling the spot market

Chapter 3 has emphasized the immense research effort that has been devoted to the analysis of competition in wholesale electricity markets in order to represent the strategic behavior of the agents involved. The convenience of the residual demand model for the purposes of this thesis has also been justified. This section presents the details of the specific model used in this thesis to represent competition in the electricity spot market from the perspective of a particular company[1].

We begin by describing the residual demand model and by showing its connection with the company's revenues. We then analyze the uncertainty faced by the company in the spot market and propose a representation based on discrete probability distributions. This entails important implications with respect to the way in which offering strategies are represented. Moreover, it leads to a multistage stochastic

[1] Appendix A provides some basic concepts about the residual demand model and includes a detailed description of how the residual demand model can be implemented in a mixed-integer programming framework. Its reading is very recommended at this point.

programming conception of the problem, which is one of the main contributions of this thesis.

4.2.1 Residual demand curves and revenue functions

An agent's residual demand function indicates the amount of product q that he is able to sell when the market-clearing price is p:

$$q(p) = R(p). \tag{4.1}$$

The idea of residual demand typically arises in the context of microeconomic theory but can also be applied to the case of a multiunit double auction. In the case of an auction, however, it is more convenient to express the clearing price as a function of the agent's sales:

$$p(q) = R^{-1}(q). \tag{4.2}$$

The residual demand curve condenses the static information about the rest of the world that the agent must take into account when evaluating his strategies for the auction. In particular, the agent can calculate the revenue he expects to obtain in the auction as a function of his own sales with the following expression:

$$r(q) = pq = R^{-1}(q)q. \tag{4.3}$$

Hence, both the auction-clearing price and the revenue obtained by the agent can be expressed as a function of the agent's sales, as illustrated in Figure 4.1.

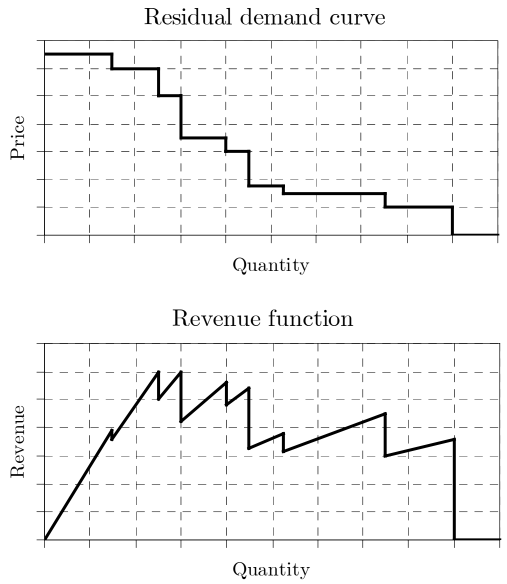


Figure 4.1. A residual demand curve and the corresponding revenue function.

In the mathematical programming models developed in this thesis, these functions are implemented by means of the piecewise linear approximation method described in Appendix A. This method considers a vector of J different prices, $(p_1,\dots,p_J)$, and identifies each residual demand curve by the corresponding vector of J quantities, $(q_1,\dots,q_J)$. The auction-clearing price, p, can then be calculated as a function of the agent's sales, q, with the following expression:

$$
\begin{aligned}
&p = p_1 + \sum_{j<J} \delta_j v_j, \\
&q = \sum_{j<J} v_j, \\
&v_j \le u_j \left(q_{j+1} - q_j\right), \quad j < J, \\
&u_{j+1} \le u_j, \quad j < J - 1, \\
&u_j \in \{0,1\}, v_j \in \mathbb{R}^+, j < J,
\end{aligned}
\tag{4.4}
$$

where δ_j is the slope of the residual demand curve in its j-th segment (the one delimited by p_j and p_{j+1}), u_j is a binary variable that indicates that the quantity q sold by the agent has reached this segment and v_j is the incremental quantity corresponding to this segment (see Appendix A for details). Similarly, the agent's revenue, r, can be obtained as a function of the agent's sales, q:

$$
\begin{aligned}
&r = r + \sum_{j<J} \rho_j v_j, \\
&q = \sum_{j} v_j, \\
&v_j \le u_j \left(q_{j+1} - q_j\right), \quad j < J, \\
&u_{j+1} \le u_j, \quad j < J - 1, \\
&u_j \in \{0,1\}, v_j \in \mathbb{R}^+, j < J.
\end{aligned}
\tag{4.5}
$$

where ρ_j is the slope of the revenue function in its j-th segment.

In this manner, the first of the challenges proposed in chapter 3 is met: every auction can be represented by means of a residual demand curve and a revenue function, whatever shape they may have, making use of these piecewise linear (PWL) approximations.

4.2.2 Sources of uncertainty in a multiunit double auction

Although an offering strategy based only on the expected residual demand curve is not a bad approximation, it is by definition incomplete. In the context of a multiunit double auction, it is because of uncertainty that an agent constructs an offer curve instead of deciding a unique quantity (or price).

We consider only two sources of uncertainty for an agent selling a good in a multiunit double auction. On one hand, the agent does not know the bids that the buyers participating in the auction have submitted. On the other hand, the offers tendered by the rest of sellers are also uncertain.

The residual demand curve is a stepwise function whose steps indistinctly refer to buy bids and to rivals' sell offers. Hence, the residual demand curve is uncertain. In the next sections we describe the model used to represent this uncertainty and analyze the influence that this model exerts on the development of optimal offering strategies.

4.2.3 Choosing the right model to represent uncertainty

In order to represent the uncertainty faced by an agent in a multiunit double auction, the probability distribution for the residual demand curve has to be estimated. Two main factors condition the form in which this probability distribution must be expressed.

The first determinant factor is the historic data that is available to estimate this probability distribution. We have assumed that the aggregate offer and bid curves submitted to a certain auction are revealed shortly after this auction has taken place. Hence, each agent can construct the residual demand curve that he faced in every past auction and use it as input data to estimate the probability distribution of future residual demand curves.

The second aspect that we must consider is the use that this probability distribution is going to receive. This thesis aims to develop optimal offering strategies using mathematical programming techniques. It is evident that there are certain forms of expressing uncertainty that are more convenient than others for a mathematical programming approach.

This section discusses a method to represent residual demand uncertainty and shows how it fails to satisfy the requirements of this thesis, even though it has been successfully applied for other research purposes. The idea is to confirm the relevance of the abovementioned factors when choosing a model to represent uncertainty.

Let us assume that the residual-demand probability distribution for a certain auction is expressed by means of a *market distribution function* $\psi(q,p)$ that indicates the probability with which a certain offer (q,p) will not be completely accepted [Anderson '99]. In other words, $\psi(q,p)$ expresses the probability with which the agent's last accepted offer will be located between the origin and (q,p). Consequently, $\psi(q_2,p_2)-\psi(q_1,p_1)$ provides the probability with which the agent's last accepted offer will lie between (q_1,p_1) and (q_2,p_2), as shown in Figure 4.2.

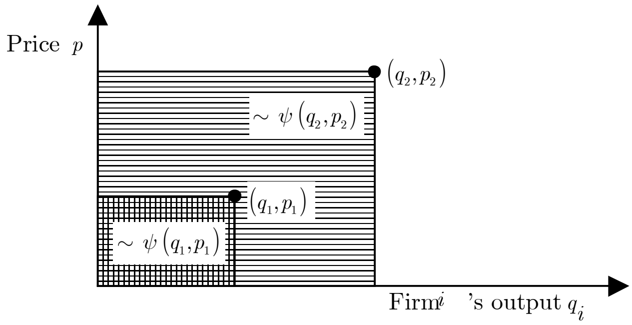


Figure 4.2. The concept of market distribution function.

In conclusion, the increment of $\psi(q,p)$ approximates the probability with which the agent's last accepted offer will be located in a region around (q,p). This idea can be used to evaluate the expected revenue of a certain offering strategy. For instance, let us consider an offer curve expressed in the form of two parametric equations, $\{(q,p) | q=q(t), p=p(t), t\in[0,T]\}$. The expected revenue is calculated as follows:

$$\mathbb{E}r=\int_0^T q(t)\,p(t)\,d\psi. \tag{4.6}$$

A more convenient formulation is obtained after some algebra:

$$\mathbb{E}r = \int_0^T q(t)\,p(t)\,d\psi\big(q(t),p(t)\big) = \int_0^T q(t)\,p(t)\big[\psi_q q'(t) + \psi_p p'(t)\big]\,dt. \tag{4.7}$$

Although the approach proposed in [Anderson '99] and in [Anderson '00] goes well beyond this point, a number of observations can already be done:

i) This expression of the expected revenue does not seem appropriate to search for an optimal offering strategy in a mathematical programming context. The reason is that the decisions that have to be optimized are expressed in terms of functions, $q = q(t)$ and $p = p(t)$, which leads to a functional optimization problem [Pflug '01]. There is not a general approach to obtain a numerical solution for such a problem. Nevertheless, necessary and sufficient conditions for the local optimality of a certain trajectory can be found in [Anderson '00].

ii) The information provided by past residual demand curves is much richer than the one imbedded in the function $\psi(q,p)$. To illustrate this, let us consider two offers, (q_1,p_1) and (q_2,p_2) such that $(q_1-q_2)\cdot(p_1-p_2)<0$ (Figure 4.3). As has been shown, $\psi(q_i,p_i)$, informs us about the probability with which the offer (q_i,p_i), will not be completely accepted. This probability can be estimated from past residual demand curves by simply evaluating the percentage of auctions in which this offer would not have been completely accepted. What $\psi(q,p)$ does not provide is the percentage of cases in which offers (q_1,p_1) and (q_2,p_2) would both have been accepted, or both rejected, or one of them accepted and the other one rejected, which is an information that can actually be obtained from past residual demand curves. Figure 4.3 illustrates this idea with a probability distribution based on two past residual demand curves. Both (q_1,p_1) and (q_2,p_2) have a 50 % probability of not being fully accepted. However, if (q_2,p_2) were accepted, (q_1,p_1) would not be accepted and vice versa, which is an information that can only be derived using past residual demand curves.

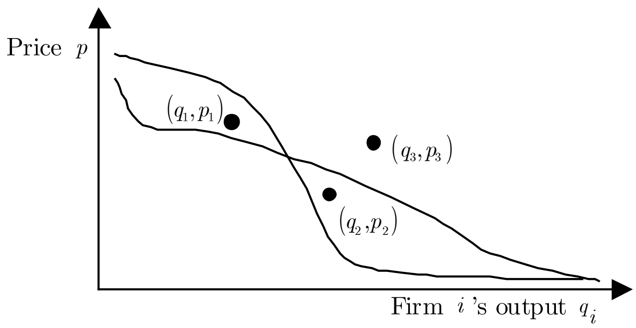


Figure 4.3. The influence of considering an inadequate probability distribution.

This is a minor drawback if only one auction is being considered, given that offers (q_1,p_1) and (q_2,p_2) can never be simultaneously submitted (the offer curve must be increasing). Indeed, $\psi(q,p)$ does provide the probability with which offers (q_2,p_2) and (q_3,p_3) are simultaneously rejected (whenever (q_2,p_2) is rejected) or accepted (whenever (q_3,p_3) is accepted) or even one accepted and one rejected ($\psi(q_3,p_3)-\psi(q_2,p_2)$).

However, when several auctions are simultaneously considered (as in this thesis), the shortcomings of the market distribution function approach are more critical. Let

$\psi^n(q,p)$ be the market distribution function for the n-th auction. As before, $\psi^n(q_i,p_i)$ indicates the probability with which the offer (q_i,p_i) will not be completely accepted in the n-th auction. However, the probability with which this offer will not be completely accepted in two different auctions, n and n', may not be equal to the product $\psi^n(q_i,p_i)\cdot\psi^{n'}(q_i,p_i)$ if a certain degree of correlation exists between both auctions. This correlation can be easily estimated using historic residual demand data for both auctions, but is not provided by the market distribution functions $\psi^n(\bullet)$ and $\psi^{n'}(\bullet)$. As a result, when two or more auctions that may be correlated are simultaneously considered, the market distribution function is not an adequate approach to represent uncertainty.

4.2.4 Probability distributions with finite support

According to the suggested PWL approximation, the residual demand curve whose probability distribution is being sought can be seen as a random vector of J quantities corresponding to a predefined vector of J prices. Let us assume that this random vector has *finite support*. In other words, this vector has a finite number of possible realizations that will be denoted by $k=1,\ldots,K$ and whose probabilities are $\pi_k, k=1,\ldots,K$. The set of all random vectors will be denoted by K[2].

Using a probability distribution that consists of K possible residual-demand realizations is the most adequate approach to represent uncertainty for our purposes. On one hand, it takes full advantage of the available historic information by constructing these future possible realizations from past residual demand curves. Figure 4.4 shows an example in which $K=9$ possible residual demand realizations have been obtained from historic data in order to estimate the probability distribution for the residual demand curve of a fictitious generation company in a particular hourly auction of the Spanish day-ahead market.

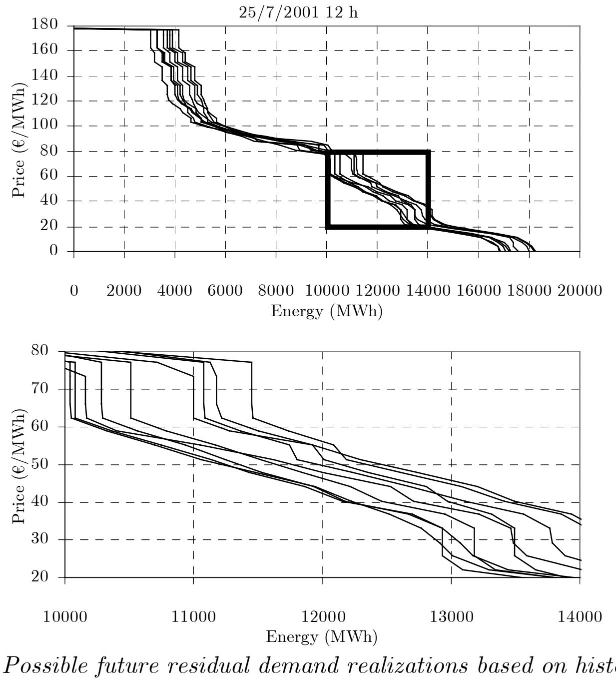


Figure 4.4. Possible future residual demand realizations based on historic data.

[2] This notation convention is kept throughout the thesis. Sets are denoted by regular uppercase characters (e.g. K is the set of spot market scenarios). Italics uppercase characters denote the number of elements of each set (e.g. K is the number of spot market scenarios). Finally, italics lowercase characters denote generic elements of a set (e.g. k is a generic spot market scenario).

On the other hand, assuming a finite number of realizations is a natural approach in mathematical programming, in particular in the field of multistage stochastic programming [Birge '97]. As will be justified, the problem addressed in this thesis can be formulated as a multistage stochastic program.

4.2.5 Implications for the development of optimal offering strategies

The finite support assumption for the residual demand curve has important implications regarding the form in which the agent's offering strategies are expressed. Such a discretization of the residual-demand probability distribution can be seen as a concentration of probability in certain regions of the (q, p) space while the rest is given zero probability. This is obviously a simplification, but provides a very convenient framework for the construction of optimal offer curves. Specifically, the shape of the agent's offer curve between two consecutive residual demand realizations turns out to be irrelevant, given that the probability density in the region between both residual demand curves is null (Figure 4.5).

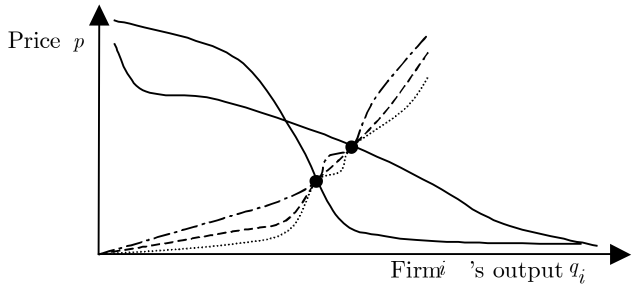


Figure 4.5. The shape of the offer curve between two consecutive residual demand realizations is irrelevant

The offer curve that the agent has to submit can be simply constituted by as many offers as residual demand realizations are considered. An offer curve is then completely defined by a set of pairs, $(q_k, p_k), k = 1,\ldots, K$, that satisfy the following conditions:

i) Each possible residual demand realization k must have one (and only one) corresponding pair (q_k, p_k) that must be located in the residual demand curve. This condition can also be expressed by saying that each quantity q_k yields a price p_k as a result of the k-th residual demand realization: $p_k = R_k^{-1}(q_k)$. These pairs $(q_k, p_k), k = 1,\ldots, K$, can be seen as the set of K offers submitted by the agent to the auction, given the K possible residual demand realizations.

ii) The resulting set of pairs or offers must constitute an offer curve that is non-decreasing. Figure 4.6 depicts two sets of three offers such that only the second set yields a valid offer curve.

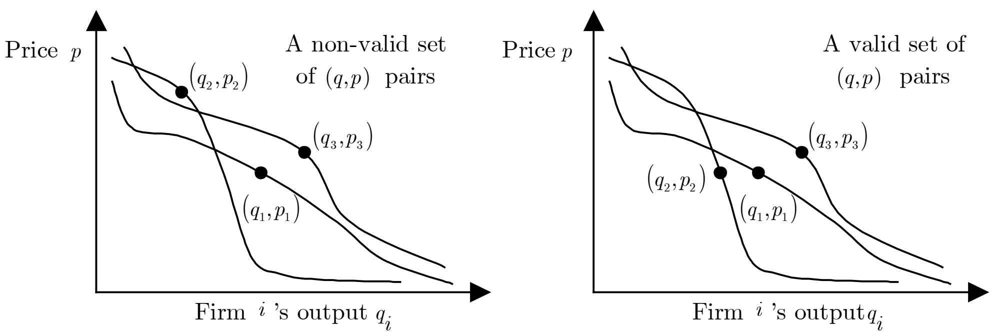


Figure 4.6. (q,p) pairs must constitute an increasing offer curve.

This can be guaranteed by introducing the following constraints:

$$(q_k - q_{k'}) \cdot (p_k - p_{k'}) \geq 0, \quad \forall k, k' > k \tag{4.8}$$

Constraints (4.8) are non-linear, so they cannot be used in a mixed linear-integer framework. Nevertheless, an equivalent MIP formulation exists:

$$\left.\begin{aligned} &q_k - q_{k'} \geq -x_{kk'} M^q, \\ &q_{k'} - q_k \geq -(1 - x_{kk'}) M^q, \\ &p_k - p_{k'} \geq -x_{kk'} M^p, \\ &p_{k'} - p_k \geq -(1 - x_{kk'}) M^p, \end{aligned}\right. \quad \forall k, k' > k, \tag{4.9}$$

where $x_{kk'}$ is a binary variable, M^q is a big quantity and M^p is a big price. When $x_{kk'} = 0$, the first and third constraints are in force, whereas when $x_{kk'} = 1$, the second and fourth constraints exert their influence. This set of constraints is compatible with the case of residual demand curves that intersect, such as the ones depicted in Figure 4.6.

It is worth to note that these constraints create a link between the decisions adopted by the agent for the different residual demand realizations. If he were allowed to submit a non-increasing offer curve then he would be able to decide an independent offer for each residual demand realization and the problem would be significantly easier to solve. In particular, the following two comments describe undesired effects caused by the set of complicating constraints (4.9).

iii) If uncertainty about the residual demand curve and the revenue function were neglected and only the expected residual demand curve and the expected revenue function were considered, it would not be necessary to calculate the auction-clearing price, given that the agent is interested in his revenue rather than in the price. Of course, his revenue depends on the price, but that dependence is already modeled in the revenue function. Therefore, in a deterministic approach the price-evaluation equation is not required and it suffices with the revenue-evaluation equation [Baíllo '01]. On the contrary, when uncertainty is modeled via a number of possible residual demand realizations, the price p_k that is obtained for each demand realization, k, has to be evaluated in order to ensure that the resulting offer curve is non-decreasing.

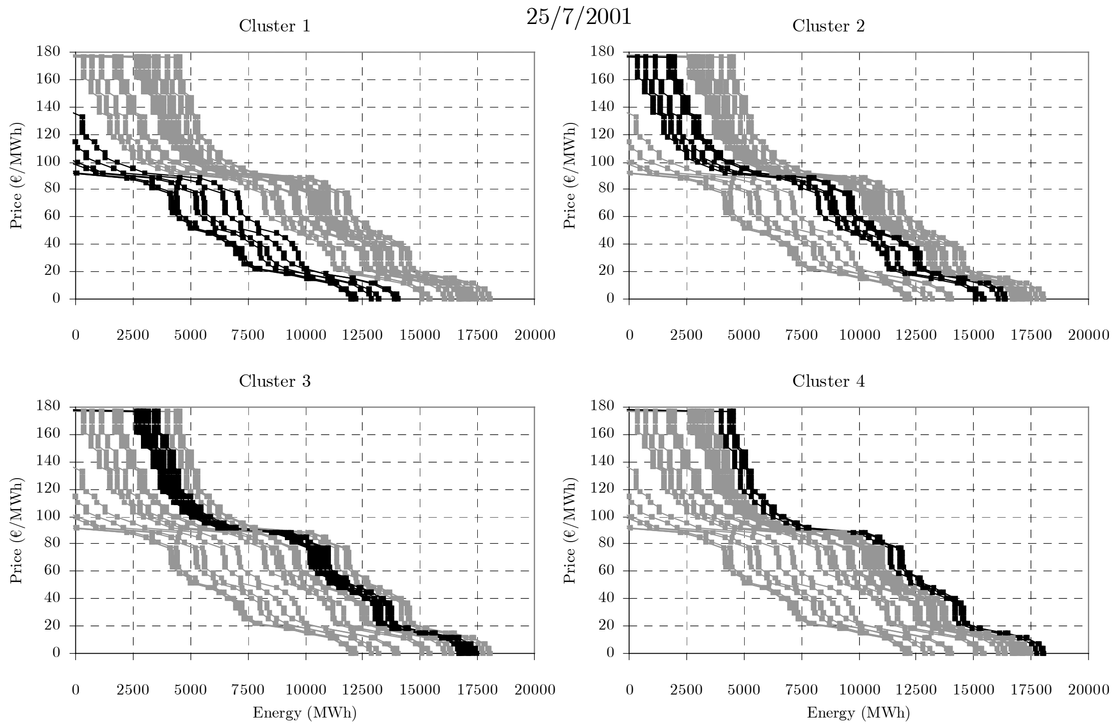
iv) The binary variables u_{jk} that guarantee that the PWL approximation of the k-th revenue function is correctly evaluated, have to be defined for every segment j. This is another disadvantage with respect to the deterministic case, in which binary variables are only required to separate concave sections (see Appendix A). Indeed, when several residual demand realizations and the corresponding revenue functions are handled, the presence of the non-decreasing constraints (4.9) establishes a link between the offers decided for these different realizations that requires the usage of a binary variable for each linear segment of the revenue function.

To summarize, as a result of representing uncertainty by means of a finite number, K, of possible residual demand realizations, the problem of obtaining an optimal offering strategy is equivalent to the problem of deciding K offers $(q_k, p_k), k = 1, \ldots, K$, that must constitute a non-decreasing offer curve.

4.2.6 Correlation between different auctions

The analysis performed hitherto has focused on the case of a single auction that can be part of any of the market mechanisms integrated in the spot market (see chapter 2). However, the correlation that may exist between the residual demand curves that a certain generation company faces in the series of auctions that constitute the spot market should not be neglected. We classify the sources of this correlation into two different categories.

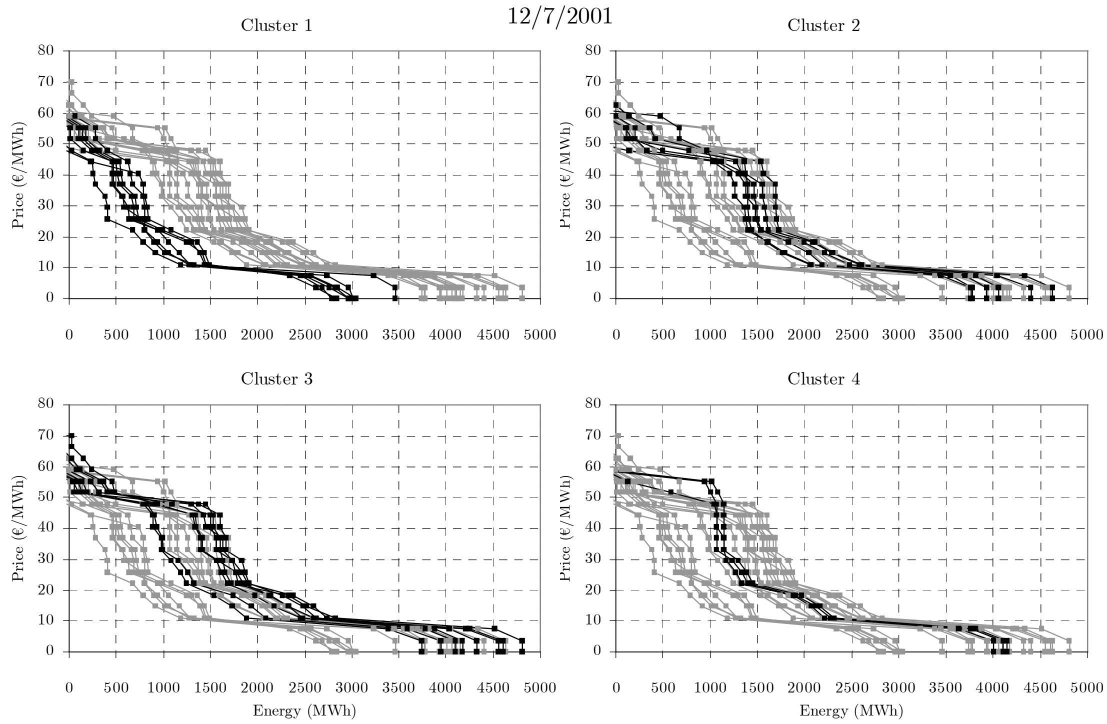
On one hand, the residual demand curves that arise in the sequence of twenty-four hourly auctions of a certain market mechanism (e.g. the day-ahead market) frequently present similarities. The analysis of these similarities can be easily performed if the residual demand curves faced by a company in a particular session of one of these mechanisms are represented with the PWL approximation suggested in previous sections, i.e. as vectors of J quantities that correspond to a vector of J prices. To illustrate this, Figure 4.7 shows an example in which the twenty-four residual demand curves faced by a fictitious generation company in a particular session of the Spanish day-ahead market have been represented as 50-component vectors and have been classified into four types according to their shape. This classification has been carried out with the K-means clustering algorithm [Hartigan '75]. As can be seen, the residual demand curves that belong to the same cluster are extraordinarily similar, which seems to confirm the correlation that exists between the residual demand curves arising in different hourly auctions of the same market mechanism. This intuitive result could be corroborated with further statistical analysis, but that is an issue that goes well beyond the scope of this thesis.



Date	Day	Hour	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24
25/04/2001	Wednesday	Cluster	2	1	1	1	1	1	1	1	2	2	3	4	4	4	3	3	3	3	3	3	2	2	3	2

Figure 4.7. Four types of residual demand curves for a company in the Spanish day-ahead market.

On the other hand, the residual demand curves that arise in auctions of different market mechanisms are also likely to be correlated. For example, it would not be rare that a certain type of residual demand realization in the n-th hourly auction of the day-ahead market were frequently accompanied by a particular type of residual demand realization in the n-th hourly auction of a subsequent market mechanism, such as the adjustment market. Continuing with the example of Figure 4.7, Figure 4.8 shows the shape of the twenty-four residual demand curves faced by the same fictitious company in the first session of the Spanish adjustment market (on-day market). These curves have been classified into four types according to the classification previously performed for the day-ahead market residual demand curves. That is, if the residual demand curve faced by the company in the first hourly auction of the day-ahead market belongs to cluster 2 then the residual demand curve faced by the company in the first hourly auction of the adjustment market is forced to belong to cluster 2. As can be seen, the classification performed for the day-ahead market residual demand curves yields a classification for the adjustment market residual demand curves that seems reasonable (cluster 3 is the only group in which the residual demand curves of the adjustment market present evident differences).



Date	Day	Hour	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24
25/04/2001	Wednesday	Cluster	2	1	1	1	1	1	1	1	2	2	3	4	4	4	3	3	3	3	3	3	2	2	3	2

Figure 4.8. Corresponding types of residual demand curves in the 1st session of the adjustment market.

In conclusion, a relevant level of correlation may exist between the residual demand curves faced by a certain company in the different auctions that constitute the spot market. Hence, to avoid missing possible correlations, a generation company operating in such a spot market should not construct an independent set of scenarios for each hourly auction. What is more, it should not construct an independent set of scenarios for each market mechanism.

In this thesis, the uncertainty faced by a generation company in the spot market will be represented in terms of a discrete probability distribution consisting of a finite number, K, of possible spot market situations. Each market situation, k, consists of a set of residual demand curves, one for each auction of each market mechanism. The probability of the k-th market situation, π_k, indicates the probability with which the residual demand curves that it comprises coincide in the same spot market session. This representation of uncertainty allows us to consider the correlation that may exist between the residual demand curves of different auctions and market mechanisms.

4.2.7 A multistage stochastic programming problem

In the spot market considered in this thesis, we require participants to submit offers and bids for the auctions of each market mechanism after the clearing of the previous market mechanism. For example, a generation company decides its offers for the hourly auctions of the adjustment market after the clearing of the day-ahead market. Similarly, the company decides its offers for the reserve market after the clearing of the adjustment market. Subsequently, the company introduces last-minute changes to its generation schedule through the balancing mechanism.

In each market mechanism, the generation company has the possibility of taking recourse actions to correct any undesired results obtained in previous market mechanisms. Figure 4.9 illustrates the decision process, which obviously has the structure of a multistage stochastic program with recourse [Birge '97]:

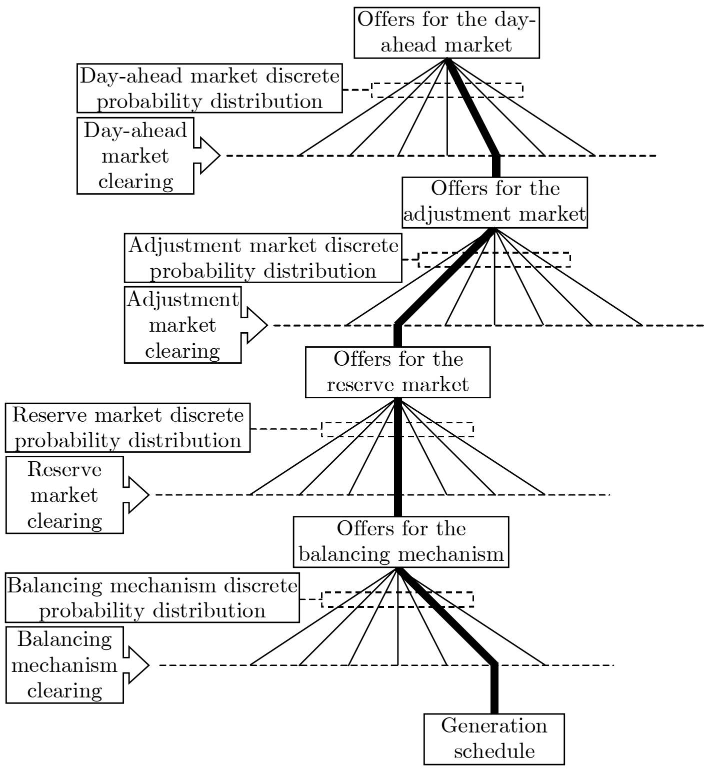


Figure 4.9. The decision process of a generation company in the spot market.

Given this structure, a multistage stochastic programming framework can be adopted in order to address the problem of deciding optimal strategies for an agent that participates in such an electricity spot market. To our knowledge, this approach has not yet been suggested in the literature and can be considered an original contribution of this thesis.

The electricity industry is not at all unfamiliar with the theory of stochastic programming. Stochastic programming techniques have traditionally been used to tackle a variety of problems such as the long-term capacity-expansion problem in the presence of uncertain demand and uncertain fuel prices, the mid-term hydrothermal coordination problem in the presence of uncertain inflows or the short-term unit-commitment problem in the presence of uncertain demand [Wallace '02]. However, with the advent of competition, rivals have become one of the most relevant sources of uncertainty. The next section shows how the K market situations used to represent spot market uncertainty must be structured in order to reflect this sequential decision process.

4.2.8 A representation based on spot-market scenario trees

Due to the abovementioned sequential process, two different market situations, k, k', may be undistinguishable during the first τ decision stages. This is the case of the pair of scenarios k, k' depicted in Figure 4.10, which share τ nodes of the scenario tree corresponding to the first τ stages. Each market situation, k, is thus equivalent to one of these scenarios and the number of market situations, K, is equal to the number of terminal nodes.

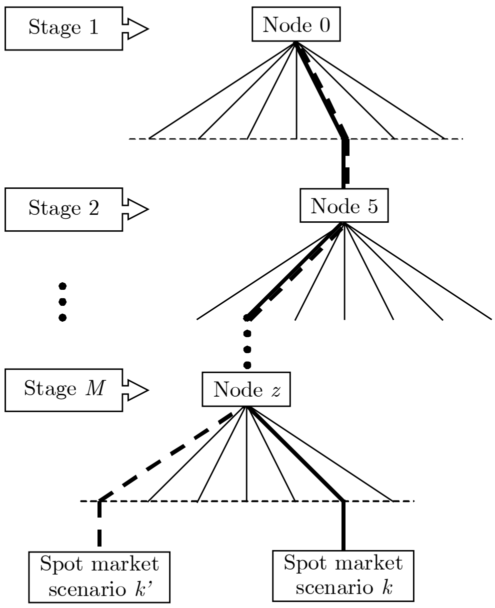


Figure 4.10. Two scenarios in a scenario tree.

The information that is available in scenarios k and k' until stage τ is the same. Hence, the decisions taken for both scenarios until stage τ must also be the same (non-anticipativity). This can be guaranteed by introducing a set of constraints that force these decisions to be equal. An alternative is to formulate the problem in terms of the nodes of the scenario tree, rather than in terms of the scenarios. The probability of each node is the sum of the probabilities of the scenarios sharing that node.

This approach fulfills the requirements of the second challenge proposed in chapter 3, given that the uncertainty faced by the generation company is represented in the form of spot market scenarios. Moreover, it provides multistage stochastic programming structure that can be exploited to solve the problem [Birge '97]. In order to further illustrate the features of this approach three different cases are explored.

4.2.8.1 The case of a single auction

When a single auction is considered, each scenario k refers to one of the possible residual demand realizations that constitute the discrete probability distribution for that auction. The problem is then reduced to deciding the quantity, q_k, that should be sold if the k-th residual demand realization occurs[3]. This quantity yields a price, $p_k(q_k)$, and a revenue for the agent, $r_k(q_k)$. Figure 4.11 depicts this case.

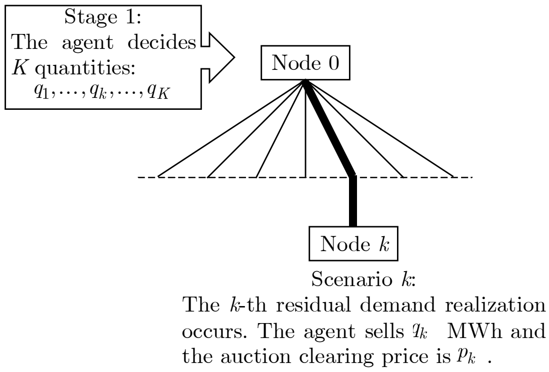


Figure 4.11. The case of a single auction.

For example, let us consider the auction corresponding to hour 1 of the Spanish day-ahead market in its session of July 25th. Assume that the probability distribution for the residual demand curve of a certain agent is defined with the two residual demand scenarios shown in Figure 4.12. The problem then reduces to deciding two quantities, q_1 and q_2. These quantities yield two possible prices, $p_1(q_1)$ and $p_2(q_2)$, with equal probabilities, $\pi_1 = \pi_2 = 0.5$.

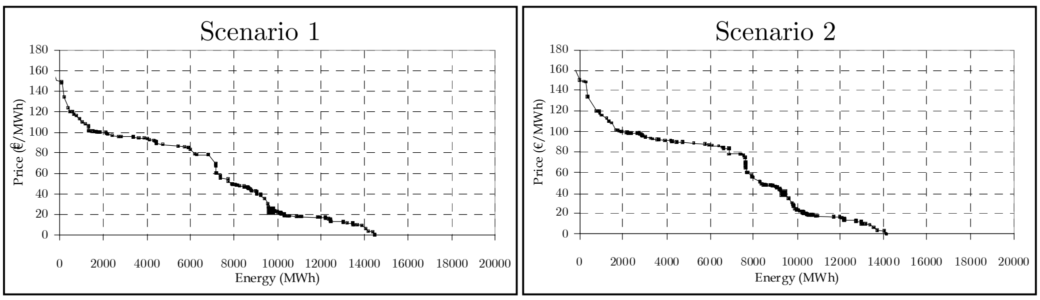


Figure 4.12. Two residual demand scenarios for one auction.

The expected revenue of such an offering strategy is:

$$\mathbb{E}r = \sum_{k \in \mathrm{K}} \pi_k r_k(q_k) = 0.5 \cdot r_1(q_1) + 0.5 \cdot r_2(q_2) = 0.5 \cdot q_1 p_1(q_1) + 0.5 \cdot q_2 p_2(q_2) \qquad (4.10)$$

---

[3] As has been indicated in previous chapters, different products such as energy or active power reserves can be traded in an electricity spot market. In this thesis, the term "quantity" refers to an amount of any of these products. If more precision is required, explicit reference will be made to "energy" or "reserve".

As has been indicated, each pair $\left(q_k, p_k\left(q_k\right)\right)$ can be understood as a point of the agent's offer curve, $q = S\left(p\right)$. Therefore, the strategy followed by the agent in the auction is completely defined by the vector of quantities $\left(q_1, q_2, \ldots, q_K\right)$ and the corresponding residual demand realizations.

In order to guarantee that the quantities decided by the agent are admissible, they must constitute a non-decreasing offer curve. This is enforced by the set of constraints (4.9), which applied to the previous example would take the following form:

$$\begin{aligned} & q_1 - q_2 \geq -x_{12} M^q, \\ & q_2 - q_1 \geq -\left(1 - x_{12}\right) M^q, \\ & p_1\left(q_1\right) - p_2\left(q_1\right) \geq -x_{12} M^p, \\ & p_2\left(q_1\right) - p_1\left(q_1\right) \geq -\left(1 - x_{12}\right) M^p, \end{aligned} \tag{4.11}$$

It must be emphasized that this set of constraints creates a link between the different residual demand scenarios that would not exist otherwise. If the quantities decided for the different scenarios were not required to form a non-decreasing offer curve, each scenario would be independent of the rest and the problem would be significantly easier to solve. This highlights the relevance of the third and fourth challenges proposed in chapter 3: all the spot market scenarios must be simultaneously considered in order to develop offering strategies that can be considered valid.

4.2.8.2 The case of several auctions that are simultaneously cleared

Let us consider now the case of N auctions that are simultaneously cleared (e.g. the 24 auctions that constitute the Spanish day-ahead market). Each market scenario, k, consists of N residual demand curves, one for each auction. In order to distinguish the quantities decided for each of the auctions, the subscript n is introduced. For example, q_{nk} indicates the amount of product that the agent sells in the n-th auction if the market situation k occurs. The corresponding clearing price is $p_{nk}\left(q_{nk}\right)$ and the agent's revenue is $r_{nk}\left(q_{nk}\right)$. The overall expected revenue is estimated as follows:

$$\mathbb{E}r = \sum_{k \in \mathrm{K}} \pi_k \sum_{n \in \mathrm{N}} r_{nk}\left(q_{nk}\right). \tag{4.12}$$

Figure 4.13 illustrates this case. As before, the agent's strategy is completely defined by a vector of quantities. There is no possibility of taking a recourse action.

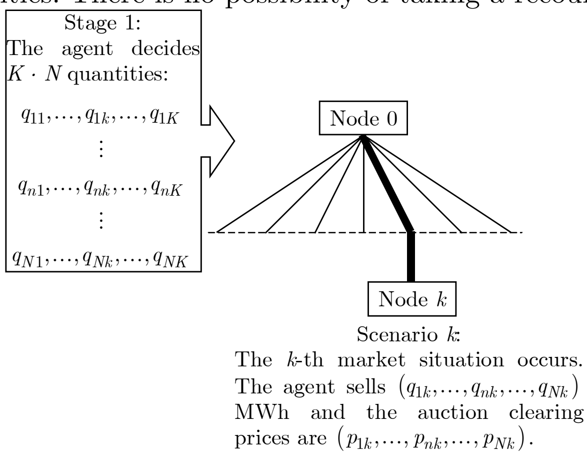


Figure 4.13. The case of several simultaneous auctions.

For example, let us consider 24 auctions of the Spanish day-ahead market corresponding to its session of July 25th. Assume that the day-ahead-market probability distribution for a certain agent is defined with the two scenarios shown in Figure 4.12. The problem would then reduce to deciding two series of 24 quantities. This yields two possible price series with equal probabilities, $\pi_1 = \pi_2 = 0.5$.

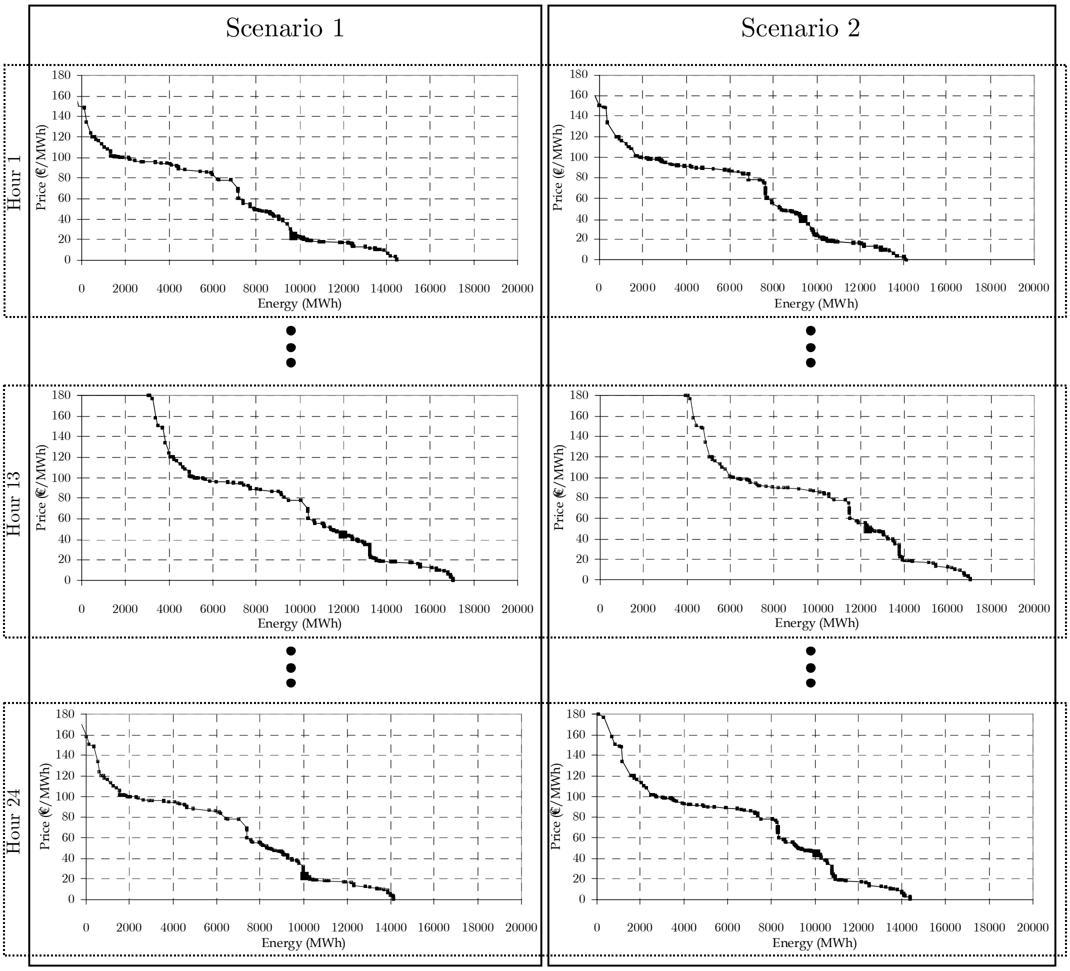


Figure 4.14. Two day-ahead market scenarios.

Again, the two quantities decided for the n -th hour, q_{n1} and q_{n2} , must constitute a non-decreasing offer curve and must observe the following set of constraints:

$$
\begin{aligned}
& q_{n1} - q_{n2} \geq -x_{n12} M^q, \\
& q_{n2} - q_{n1} \geq -\left(1 - x_{n12}\right) M^q, \\
& p_{n1}\left(q_{n1}\right) - p_{n2}\left(q_{n2}\right) \geq -x_{n12} M^p, \\
& p_{n2}\left(q_{n2}\right) - p_{n1}\left(q_{n1}\right) \geq -\left(1 - x_{n12}\right) M^p,
\end{aligned}
\qquad (4.13)
$$

4.2.8.3 The case of several market mechanisms

Consider M market mechanisms, and assume that each market mechanism, m , consists of N_m hourly auctions. Every market scenario, k , is defined by $\sum_{m \in \mathrm{M}} N_m$ residual demand curves, one for each hourly auction. Without loss of generality, assume that the N_m hourly auctions of market mechanism m are simultaneously cleared. Hence, each market mechanism corresponds to a decision stage.

In each stage, m, the agent has to decide $K \cdot N_m$ quantities. However, if scenarios k and k' are indistinguishable for the agent until the clearing of market mechanism m, the decisions taken for both of them will be the same (non-anticipativity). In practice, if the scenario tree has L_m nodes in each stage m, the problem is reduced to deciding $\sum_{m\in \mathrm{M}} L_m$ quantities.

We introduce the superscript m to distinguish the vectors of quantities decided by the agent for each of the market mechanisms. For example, q_{nk}^m indicates the amount of product sold by the agent in the n-th hourly auction of the m-th market mechanism if market situation k occurs. The corresponding auction-clearing price is $p_{nk}^m\left(q_{nk}^m\right)$ and the agent's revenue is $r_{nk}^m\left(q_{nk}^m\right)$. The overall expected revenue is given by:

$$\mathbb{E}r = \sum_{k\in \mathrm{K}} \pi_k \sum_{m\in \mathrm{M}} \sum_{n\in \mathrm{N}} r_{nk}^m\left(q_{nk}^m\right). \tag{4.14}$$

As shown in Figure 4.15, in market mechanism $m+1$ the agent can take a recourse action to correct the result obtained after the previous m stages. If the problem of deciding the strategy for market mechanism $m+1$ is never infeasible whatever decisions are taken in the previous stages, the recourse is said to be complete.

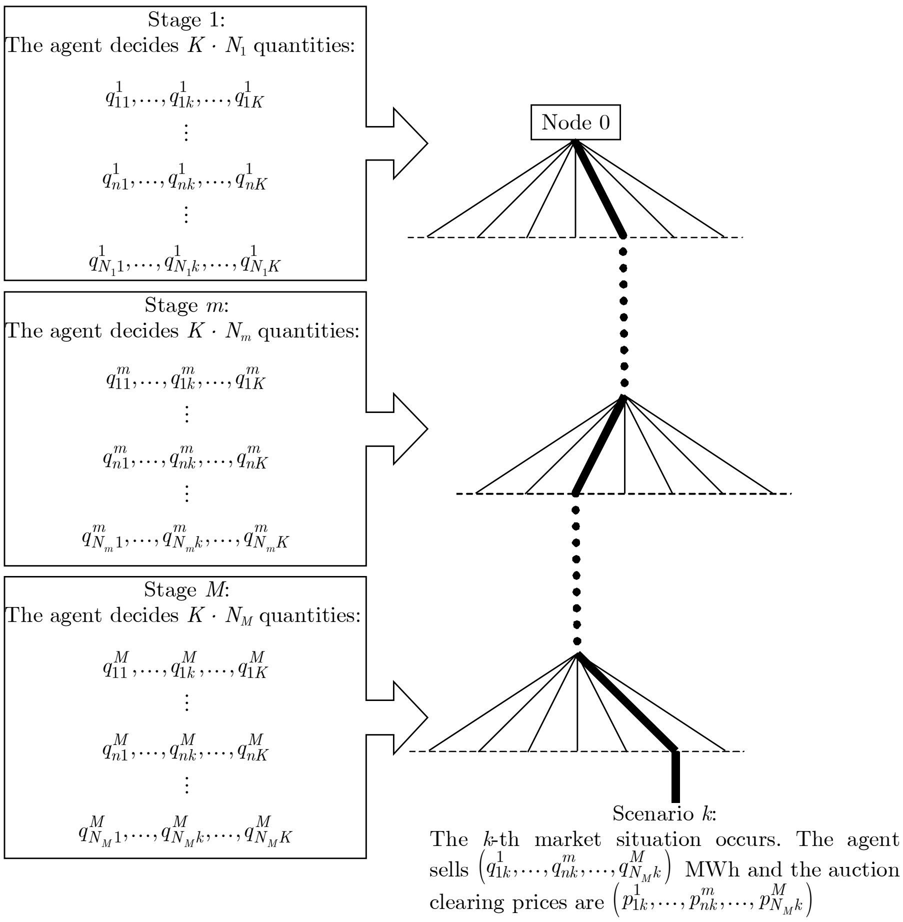


Figure 4.15. The case of several market mechanisms.

For example, let us consider the 24 hourly auctions of the Spanish day-ahead market session and 24 hourly auctions of the Spanish on-day market session that took place on July 25th 2001. Assume that the probability distribution of this sequence of two market mechanisms for a certain agent is defined with the two scenarios depicted in Figure 4.16. The agent must decide two series of 24 quantities for the day-ahead market and two series of 24 quantities for the on-day market. This yields two possible price series for both the day-ahead market and the on-day market with equal probabilities, $\pi_1 = \pi_2 = 0.5$.

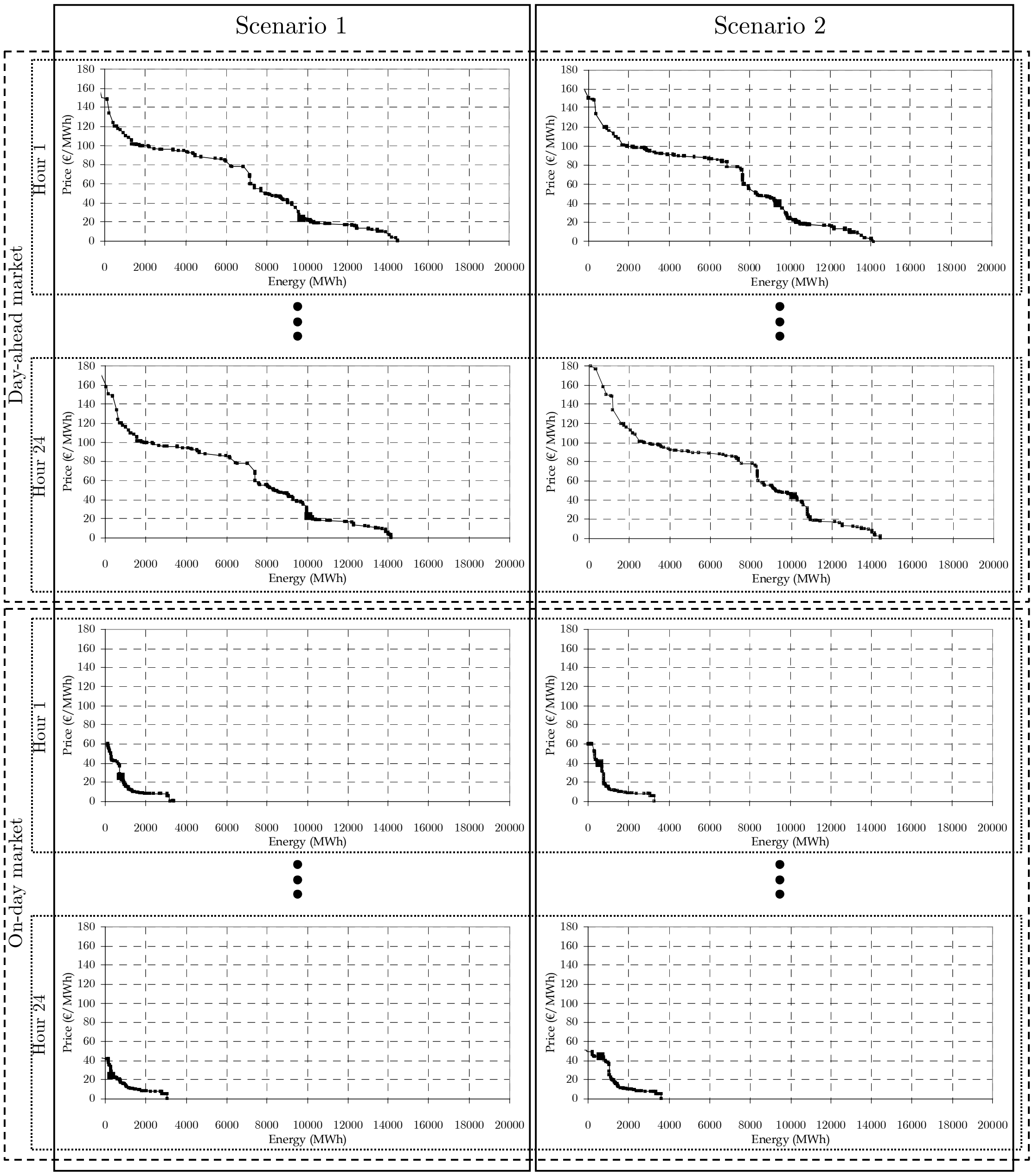


Figure 4.16. Two spot market scenarios.

A compact representation of the previous scenario tree is provided in Figure 4.17. It is important to notice that this scenario structure implies that two possible outcomes are being considered for the day-ahead market, but only one for the on-day market. The reason is that, once one of the two possible realizations of the day-ahead market occurs, the tree suggests only one possible realization for the on-day market.

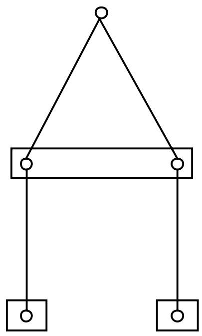


Figure 4.17. A compact representation of the two-stage two-scenario tree.

In this example, the two quantities decided for the n-th hourly auction of the first market mechanism, q_{n1}^{1} and q_{n2}^{1}, must constitute a non-decreasing offer curve and must satisfy the following set of constraints:

$$\begin{aligned} & q_{n1}^{1} - q_{n2}^{1} \geq -x_{n12}^{1} M^{q}, \\ & q_{n2}^{1} - q_{n1}^{1} \geq -\left(1 - x_{n12}^{1}\right) M^{q}, \\ & p_{n1}^{1}\left(q_{n1}^{1}\right) - p_{n2}^{1}\left(q_{n2}^{1}\right) \geq -x_{n12}^{1} M^{p}, \\ & p_{n2}^{1}\left(q_{n2}^{1}\right) - p_{n1}^{1}\left(q_{n1}^{1}\right) \geq -\left(1 - x_{n12}^{1}\right) M^{p}, \end{aligned} \tag{4.15}$$

On the contrary, the two quantities decided for the n-th hourly auction of the second market mechanism, q_{n1}^{2} and q_{n2}^{2}, are not required to constitute a non-decreasing offer curve, given that it is being assumed that only one of the two decisions will actually be made. In other words, only those nodes of the scenario tree that share their ancestor node are linked by the non-decreasing constraints.

This multistage stochastic representation can be extended in order to consider several spot market sessions. After one spot market session has finished, the company has to decide a generation schedule in order to meet the obligations assumed in the different market mechanisms that constitute that spot market session. Then a new spot market session begins in which the company must decide a new offering strategy. When several spot market sessions are considered, we will use the index p to denote a particular spot market session. Hence, q_{nk}^{mp} is the quantity that the company sells in the n-th hourly auction of the m-th market mechanism corresponding to the p-th spot market session if the k-th scenario occurs.

4.2.9 The influence of zonal pricing

We have assumed that the transmission network of the power system of study does not impose significant limitations to electricity trading. However, in order to provide a more general perspective, let us discuss in this section how the residual demand model could be generalized if the spot price of electricity varied from one zone to another due to congestions in certain flow gates. Our aim is to suggest guidelines for such a generalization, leaving its full development as a future line of research.

The first step toward the generalization of the residual demand model is to consider a residual demand curve for each zone. This can be obtained by aggregating the demand curve and the rivals' offer curve for that zone. It is important to specify that the aggregate demand curve of a certain zone is constituted by the bids of consumers located in that zone. Similarly, the aggregate offer curve of a certain zone is due to the offers of generating facilities located in that zone. The residual demand of each zone can be satisfied either with energy generated by the company in that zone or with imports from other zones. These imports are due to sales of both the company and its rivals in other zones.

The generalized model must incorporate a representation of the physical laws that determine how energy flows through the transmission network. A linearized representation would be the most appropriate one, given that transmission capacity constraints are more frequent than voltage constraints. This would require one flow equation for each flowgate (or line) and one balance equation for each zone (or node). It would also introduce the transmission capacity constraints and additional variables such as node phase angles and line flows.

The final step would be to represent the behavior of the market operator, which is oriented to the maximization of the net social benefit. In the single-node case, this behavior is simply modeled by determining the market-clearing price as the intersection of the aggregate offer curve and the aggregate demand curve. In contrast, when transmission capacity constraints are taken into account, this solution may not be feasible. Hence, in order to obtain the first-order optimality conditions that represent the behavior of the market operator, the corresponding Lagrange function must be explicitly formulated and derived. These optimality conditions would be included in the model in the form of a set of equality constraints and a set of Lagrange multipliers. Complementarity slackness conditions might be modeled by means of binary variables.

As mentioned, the formal development of this generalized residual demand model exceeds the scope of this thesis. Nevertheless, it is a line of research that can yield fruitful results both from a theoretical and from a practical perspective.

4.2.10 Building the scenario tree

The construction of a scenario tree such as the ones described in previous sections is not an easy task. Appendix B explains in detail the simple and practical approach adopted to build the scenario trees used for the numerical examples presented in this thesis. The development of a state-of-the-art spot-market scenario-tree construction method would constitute the subject of a doctoral research itself.

Nevertheless, the idea of representing the spot market as a multistage stochastic program constitutes one of the major contributions of this thesis. It provides a consistent framework for the evaluation of the expected revenue associated to a certain short-term strategy expressed in terms of a vector of quantities. Additionally, this model can be easily formulated as a multistage stochastic programming problem in order to search for optimal strategies.

4.3 Modeling the portfolio of a generation company

The previous section has shown that the strategy of an agent in the spot market can be expressed in terms of a vector of quantities. Both the agent's expected revenue and the expected clearing prices can be easily estimated based on this vector of quantities.

Additionally, this vector of quantities yields a set of hourly expected net sales in the spot market. These hourly sales, together with other sales performed in market mechanisms other than the spot market, lead to a supply (or consumption) obligation for the agent in each hour. In the case of a generation company, after the clearing of all the spot market mechanisms, the company must determine its net accumulated sales —not only in the spot market but also in other long-term market mechanisms— so as to derive a production schedule for its generating units. Furthermore, the company must take into account the cost of this production schedule and the constraints that limit the operation of its generating units when deciding its strategy for the spot market.

From the viewpoint of the spot market, the *portfolio* of a generation company is constituted both by its positions in long-term market mechanisms and by the generating units it owns. This section evaluates how the portfolio of a generation company conditions its strategy in the spot market. The proposed approach meets the requirements of the fifth, sixth and seventh challenges defined in chapter 3.

4.3.1 Generating units

A generation system can be formed by a combination of thermal units, hydro plants and pumped-storage stations [Wood '96]. Their representation in a mathematical programming model typically depends on the time scope considered for the analysis.

In this thesis, we adopt a mixed linear-integer modeling approach similar to the one usual in unit-commitment and economic-dispatch models. Equations are formulated for each particular market situation, k, in order to guarantee that a feasible schedule can be obtained irrespective of the spot market outcome. This fulfills the requirements of the fifth challenge proposed in chapter 3. Given that the degree of detail with which generating units can be modeled is somewhat limited by the computational requirements of the spot market model, certain simplifications are assumed.

4.3.1.1 Thermal units

A thermal unit, t, has a fuel consumption characteristic that can be expressed as a non-linear and non-convex function of its power output, due to the presence of several valves and turbines in the gas and/or steam cycle. A usual approximation is to express these costs as a piecewise linear function of the unit's output. Additional costs due to start-ups also have to be accounted for. Although start-up costs depend on the time the unit has been switched off, we neglect this effect. The production costs of generating unit t in hour n for a certain market situation k are thus calculated as follows:

$$c_{nk}^{t} = s_t y_{nk}^{t} + o_t q_{nk}^{t} + f_t \left(\beta_t u_{nk}^{t} + \alpha_t \frac{q_{nk}^{t}}{k_t} \right), \quad t \in \mathrm{T}, n \in \mathrm{N}, k \in \mathrm{K}\,, \tag{4.16}$$

$$y_{nk}^{t} - z_{nk}^{t} = u_{nk}^{t} - u_{n-1k}^{t}, \quad t \in \mathrm{T}, n \in \mathrm{N}, k \in \mathrm{K}\,, \tag{4.17}$$

$$0 \le y_{nk}^{t} \le 1, \quad t \in \mathrm{T}, n \in \mathrm{N}, k \in \mathrm{K}\,, \tag{4.18}$$

$$0 \le z_{nk}^{t} \le 1, \quad t \in \mathrm{T}, n \in \mathrm{N}, k \in \mathrm{K}\,, \tag{4.19}$$

where

c_{nk}^{t} are the production costs of unit t in hour n and situation k, in €,

s_t is the start-up cost of unit t, in €/startup,

y_{nk}^t is the start-up decision for unit t in hour n and situation k,

z_{nk}^t is the shut-down decision for unit t in hour n and situation k,

o_t are the variable O&M costs of unit t, in €/MWh,

q_{nk}^t is the net energy produced by unit t, in MWh,

f_t is the fuel cost, in €/Tcal,

β_t is the independent term of the heat rate function for unit t, in Tcal,

u_{nk}^t is the commitment state (on/off) for unit t in hour n and situation k, $u_{nk}^t \in \{0,1\}$,

α_t is the linear term of the heat rate function for unit t, in Tcal/MWh,

k_t is the self consumption coefficient of thermal unit t, in p.u.

Thermal units also have a maximum capacity, a minimum stable output, and ramp rate limits:

$$\underline{q}_t k_t u_{nk}^t \le q_{nk}^t + r_{nk}^t \le \overline{q}_t k_t u_{nk}^t, \quad t \in \mathrm{T}, n \in \mathrm{N}, k \in \mathrm{K}\,, \tag{4.20}$$

$$-l_t \le q_{nk}^t - q_{n-1k}^t \le l_t, \quad t \in \mathrm{T}, n \in \mathrm{N}, k \in \mathrm{K}\,, \tag{4.21}$$

where

r_{nk}^t is the amount of reserve provided by unit t, in MW,

$\underline{q}_t$ is the minimum gross stable output for unit t, in MW,

$\overline{q}_t$ is the maximum gross capacity for unit t, in MW,

l_t is the ramp rate limit for unit t, in MW/h.

Additional constraints can be considered, such as minimum up and down times, to control the frequency of startups. The following formulation, suggested in [Nowak '99], can be adopted to guarantee a minimum up time of ν_t hours for unit t:

$$u_{n+\nu k}^t + u_{n-1k}^t - u_{nk}^t \ge 0, \quad t \in \mathrm{T}, n \in \mathrm{N}, k \in \mathrm{K}, \nu = 1, \ldots, \min\{N - n, \nu_t - 1\}\,. \tag{4.22}$$

In [Arroyo '00] a variety of constraints for thermal units formulated in terms of mixed linear-integer expressions can also be found.

4.3.1.2 Hydro units

A hydraulic system is formed by a complex network of rivers, dams, channels, and hydroelectric plants. Its administration requires taking into consideration the mutual influence of all these elements. However, the explicit consideration of the configuration of the hydro network requires a modeling effort that would deviate us from our main objectives. Hence, our model manages hydro reserves in an aggregate manner by integrating several hydro plants into a single equivalent hydro unit. The detail of the hydro network can be considered in a subsequent decision stage in order to derive a more precise hydro schedule.

A hydro unit, h, transforms a water flow into electric energy. The energy/flow conversion rate depends on the net head of the upstream reservoir and on the actual water flow through the turbine [Ni '99]. In spite of this, we neglect these dependences and consider a constant energy/flow ratio. This is equivalent to modeling hydro reserves in terms of stored energy, expressed in MWh. Some hydro units can also operate in pumping mode, driving water from the downstream dam back to the upstream reservoir. The state of each reservoir can thus be evaluated in a straightforward manner:

$$w_{nk}^{h} = w_{n-1k}^{h} - \frac{q_{nk}^{h}}{k_h} + i_n^h - s_{nk}^{h} + \eta_h b_{nk}^{h}, \quad h \in \mathrm{H}, n \in \mathrm{N}, k \in \mathrm{K}\,, \tag{4.23}$$

where

w_{nk}^{h} is the energy stored in reservoir h at the end of hour n in market situation k, in MWh,

q_{nk}^{h} is the net energy produced by unit h in hour n and situation k, in MWh,

k_h is the self consumption coefficient of hydro unit h, in p.u.

i_n^h are the net inflows received by reservoir h in hour n, in MWh,

s_{nk}^{h} is the energy spilt from reservoir h due to an excess of reserve in hour n and situation k, in MWh,

b_{nk}^{h} is the energy pumped by unit h in hour n and situation k, in MWh.

η_h is the performance of the pump-turbine cycle for unit h, in p.u.,

Hydro units have upper and lower bounds for their operation variables and reservoirs have minimum and maximum levels:

$$0 \le q_{nk}^{h} + r_{nk}^{h} \le k_h \overline{q}_h, \quad h \in \mathrm{H}, n \in \mathrm{N}, k \in \mathrm{K}\,, \tag{4.24}$$

$$0 \le b_{nk}^{h} \le \overline{b}_h, \quad h \in \mathrm{H}, n \in \mathrm{N}, k \in \mathrm{K}\,, \tag{4.25}$$

$$0 \le s_{nk}^{h}, \quad h \in \mathrm{H}, n \in \mathrm{N}, k \in \mathrm{K}\,, \tag{4.26}$$

$$\underline{w}_h \le w_{nk}^{h} \le \overline{w}_h, \quad h \in \mathrm{H}, n \in \mathrm{N}, k \in \mathrm{K}\,. \tag{4.27}$$

In this thesis we assume that a certain amount of energy, W_{h0}, is available in each reservoir h at the beginning of the planning horizon. A medium-term hydrothermal model can determine this energy. An alternative approach would be to assign a value to the energy that is left unused at the end of the planning horizon, according to the dual variables returned by the medium-term model.

4.3.1.3 The balance between the net sales in the spot market and the generation schedule

Let us assume that the generation company operates only in the spot market. After the clearing of all the spot market mechanisms, $m = 1,\dots,M$, the company has to produce with its generating units the net energy it has sold for each hour n. This yields the following energy balance equation:

$$\sum_{m \in \mathrm{M}^{\mathrm{E}}} q_{nk}^{m} = \sum_{t \in \mathrm{T}} q_{nk}^{t} + \sum_{h \in \mathrm{H}} q_{nk}^{h} - b_{nk}^{h}, \quad n \in \mathrm{N}, k \in \mathrm{K}\,, \tag{4.28}$$

where M^{E} is the set of spot market mechanisms in which energy is traded.

The company also has the obligation of providing the reserve it has sold for each hour n. A reserve balance equation must then be formulated:

$$\sum_{m \in \mathrm{M}^{\mathrm{R}}} q_{nk}^{m} = \sum_{t \in \mathrm{T}} r_{nk}^{t} + \sum_{h \in \mathrm{H}} r_{nk}^{h}, \quad n \in \mathrm{N}, k \in \mathrm{K}, \tag{4.29}$$

where M^{R} is the set of spot market mechanisms in which reserve is traded.

It is clear that each market situation k implies a different generation schedule. Equation (4.28) is the first version of the energy balance equation that the generation company must observe to guarantee a consistent operation. This equation will be gradually enhanced in order to reflect all the alternative mechanisms that the company can make use of to perform its sales.

4.3.1.4 The influence of the generation schedule on the company's benefit

We can already formulate a preliminary expression of the company's short-term expected benefit, assuming that the company operates only in the spot market. The company's expected benefit is given by the difference between the net expected revenues obtained from its sales in the spot market auctions and the generating costs of thermal units[4]:

$$\mathbb{E}\mathcal{B} = \sum_{k \in \mathrm{K}} \pi_k \left[\sum_{m \in \mathrm{M}} \sum_{n \in \mathrm{N}} r_{nk}^{m}\left(q_{nk}^{m}\right) - \sum_{t \in \mathrm{T}} \sum_{n \in \mathrm{N}} c_{nk}^{t}\left(q_{nk}^{t}, y_{nk}^{t}, u_{nk}^{t}\right) \right]. \tag{4.30}$$

Additional terms will be gradually added to this expression of the company's expected benefit until its complete formulation is attained.

4.3.2 Forward contracts

As shown in chapter 2, there are a variety of alternative ways of trading with electricity in addition to the spot market. Almost always, generation companies have the possibility of selling part of their production through long-term contracts[5]. These contracts, in their most basic form, consist of an agreement to exchange a certain amount of energy for a fixed price at certain hours (for instance, the peak hours of a specific month) and at a certain node of the network. Before entering into one of these contracts, the company must certainly evaluate the benefits that it expects to obtain from it. In the short-term, however, the company must evaluate the influence that its portfolio of long-term contracts exerts on the benefits it expects to obtain in the spot market. From this perspective, the particular form in which these contracts are settled plays an important role.

The settlement of these contracts can be either physical or financial. In the first case, the generation company assumes the obligation of providing the specified amount of energy at the hours and network node arranged and receives in exchange the fixed price agreed. The second type of contract does not imply a physical energy transaction but rather a cash flow in which one party pays the other the difference between the agreed price and the spot price for that particular hour. These two contractual forms

[4] The costs of providing reserve are neglected.

[5] The particular form in which these contracts are negotiated (OTC, bilaterally or through an organized market) is irrelevant here.

present certain differences regarding their representation in a short-term model that must be analyzed.

4.3.2.1 Physical settlement

If a long-term contract implies the obligation of physically supplying a certain amount of energy during a number of hours, the generation company must take this into account when scheduling its generating units. This type of contacts are commonly known as *physical bilateral contracts* (PBCs).

Let C^{P} be the set of PBCs signed by the generation company and let c be one of these contracts. The amount of energy that the company has agreed to serve in hour n as a result of that contract is q_n^c MWh and the price that the company will be paid is p_n^c €/MWh. The influence that these contracts exert on the company's energy balance equation is expressed as follows:

$$\sum_{m\in\mathrm{M}^{\mathrm{E}}} q_{nk}^m + \sum_{c\in\mathrm{C}^{\mathrm{P}}} q_n^c = \sum_{t\in\mathrm{T}} q_{nk}^t + \sum_{h\in\mathrm{H}} q_{nk}^h - b_{nk}^h, \quad n\in\mathrm{N}, k\in\mathrm{K}. \tag{4.31}$$

In addition to this, the company's revenue increases in $\sum_{c\in\mathrm{C}^{\mathrm{P}}} p_n^c q_n^c$ €. However, this is a constant term and cannot be modified by introducing changes in the strategy followed by the company in the spot market. The formulation of the company's expected benefit with this additional term is:

$$\mathbb{E}\mathcal{B} = \sum_{k\in\mathrm{K}} \pi_k \left[\sum_{m\in\mathrm{M}} \sum_{n\in\mathrm{N}} r_{nk}^m - \sum_{t\in\mathrm{T}} \sum_{n\in\mathrm{N}} c_{nk}^t \right] + \sum_{c\in\mathrm{C}^{\mathrm{P}}} \sum_{n\in\mathrm{N}} p_n^c q_n^c \,. \tag{4.32}$$

where $r_{nk}^m = r_{nk}^m\left(q_{nk}^m\right)$ and $c_{nk}^t = c_{nk}^t\left(q_{nk}^t, y_{nk}^t, u_{nk}^t\right)$.

4.3.2.2 Financial settlement

A generation company enters into a long-term contract with the objective of securing part of its revenues, instead of being fully exposed to the uncertainty of the spot price. This can be achieved by means of a physical bilateral contract but also with a contract for differences (CfDs). This is a financial contract in which one party pays the other the difference between a fixed price and the spot price for a certain hour.

Let C^{D} be the set of contracts for differences signed by the generation company. A contract for differences, c, for a quantity q_n^c and a price p_n^c does not affect the company's energy balance equation. On the contrary, if market situation k occurs, the net revenue obtained by the company in hour n due to that contract for differences is given by $\left(p_n^c - p_{nk}\right) q_n^c$, where p_{nk} is the spot price of electricity in hour n [6]. This leads to the following expression for the company's expected benefit:

[6] The spot price of electricity in hour n, p_{nk}, is obtained as a weighted combination of the clearing prices that have resulted in the spot market auctions for hour n. A CfDs can also be indexed to the price of one of the spot market mechanisms, p_{nk}^m.

$$\mathbb{E}\mathcal{B} = \sum_{k \in \mathrm{K}} \pi_k \left[\sum_{m \in \mathrm{M}} \sum_{n \in \mathrm{N}} r_{nk}^m + \sum_{c \in \mathrm{C}^{\mathrm{D}}} \sum_{n \in \mathrm{N}} \left(p_n^c - p_{nk} \right) \cdot q_n^c - \sum_{t \in \mathrm{T}} \sum_{n \in \mathrm{N}} c_{nk}^t \right]. \tag{4.33}$$

4.3.2.3 Equivalence of both types of contracts

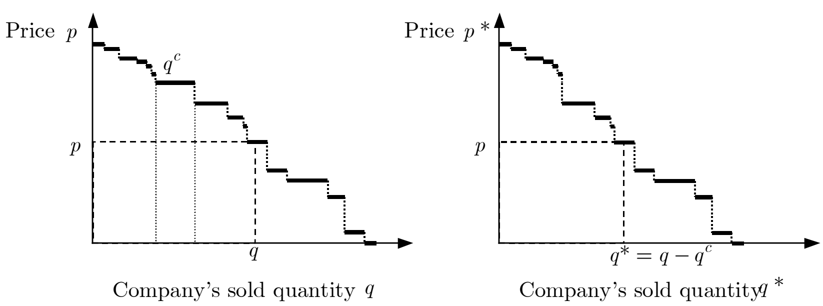
In order to show the equivalence between PBCs and CfDs, let us consider a single market mechanism with a unique auction. Let us also assume that the discrete probability distribution for that auction consists of a unique residual demand curve. Under these assumptions, the clearing price is denoted by $p(q)$, where q is the energy sold by the company in that auction. Let us also express the company's generation costs as a function of its production in the form $c(q)$. If the company enters into a CfDs for a quantity q^c and a price p^c the company's benefit is given by:

$$\mathcal{B} = p(q) \cdot q + \left(p^c - p(q) \right) \cdot q^c - c(q) = p(q) \cdot \left(q - q^c \right) + p^c \cdot q^c - c(q). \tag{4.34}$$

Let us define now the change of variable $q^* = q - q^c$. The company's benefit is then expressed as:

$$\mathcal{B} = p\left(q* + q^c \right) \cdot q* + p^c \cdot q^c - c\left(q* + q^c \right), \tag{4.35}$$

which is similar to the benefit that the company would obtain if it entered into a PBC for a quantity q^c and a price p^c and its sales in the auction were $q*$. The only inconsistency seems to arise from the value of the market clearing price, which is a function of both the energy sold by the company in the spot market, $q*$, and the energy sold by the company through the PBC, q^c, instead of being a function exclusively of the energy sold by the company in the spot market. However, we must take into account that the residual demand function faced by the company changes when the company enters into a PBC for q^c MWh. The reason is that its counter party reduces its net purchase in the auction in exactly q^c MWh. This is equivalent to a horizontal translation of the residual demand curve. A new residual demand curve $p*(q*)$ is obtained such that $p*(q*) = p\left(q* + q^c \right)$, as illustrated in Figure 4.18. Consequently, we can consider that both types of contracts are equivalent, except for the fact that CfDs do not include the obligation to supply energy, which is certainly an advantage.



Residual demand curve with no physical bilateral contract

Residual demand curve with a physical bilateral contract for q^c MWh

Figure 4.18. The influence of a bilateral physical contract on the residual demand.

4.3.3 Options

Generation companies frequently have the possibility of trading with options[7]. A generation company will typically assume long put positions (i.e. it will purchase the right to sell energy at a fixed price) and/or short call positions (i.e. it will sell the right to buy energy at a fixed price). In this section we focus on the positions adopted by the company that have not been cancelled (i.e. they are still open) when the corresponding spot market session takes place. From the perspective of the spot market, the option price paid by the company when it assumes a long position or the option price received in the case of a short position is not relevant. What matters is the influence that these positions exert on the company's strategy in the spot market.

When evaluating the effect of long-term contracts, we distinguished between PBCs and CfDs. This distinction seems unnecessary in the case of options, given that agents are not likely to be interested in trading with options that are physically settled. However, the formulation of both types of options is an interesting exercise that illustrates the potential of the MIP modeling approach. The equivalence between physical and financial options will be explored when formulating the Lagrangian function in chapter 5.

4.3.3.1 Long put position

A generation company may be interested in assuming a long put position in order to guarantee a minimum revenue for the energy produced by its units. It is not so clear, however, which type of agent would be interested in assuming a short put position.

Let $\mathrm{O}^{\mathrm{LPP}}$ be the set of long put positions assumed by the generation company that are physically settled, i.e. if the company decides to exercise its right to sell, it will actually have to produce the corresponding energy. Consider the case of one of these options, o, for q_n^o MWh with an exercise price of p_n^o €/MWh. The decision of exercising this option will be represented by a binary variable u_{nk}^o. If in scenario k the company decides to exercise option o in hour n, then $u_{nk}^o = 1$. In other case, $u_{nk}^o = 0$.

The influence that the company's long put positions exert on its own benefit is evaluated in the following expression:

$$\mathbb{E}\mathcal{B} = \sum_{k \in \mathrm{K}} \pi_k \left[\sum_{m \in \mathrm{M}} \sum_{n \in \mathrm{N}} r_{nk}^m + \sum_{o \in \mathrm{O}^{\mathrm{LPP}}} \sum_{n \in \mathrm{N}} u_{nk}^o \cdot p_n^o \cdot q_n^o - \sum_{t \in \mathrm{T}} \sum_{n \in \mathrm{N}} c_{nk}^t \right]. \tag{4.36}$$

The energy balance equation is then:

$$\sum_{m \in \mathrm{M}^{\mathrm{E}}} q_{nk}^m + \sum_{o \in \mathrm{O}^{\mathrm{LPP}}} u_{nk}^o q_n^o = \sum_{t \in \mathrm{T}} q_{nk}^t + \sum_{h \in \mathrm{H}} q_{nk}^h - b_{nk}^h, \quad n \in \mathrm{N}, k \in \mathrm{K}. \tag{4.37}$$

It should be noticed that there is no need to enforce the value of the binary variable u_{nk}^o. The optimization procedure will decide the right value when maximizing the company's expected benefit.

[7] Again, the particular form in which these options are negotiated is irrelevant here.

Consider now the set of long put positions assumed by the generation company that are financially settled, $\mathrm{O}^{\mathrm{LPF}}$. If the company decides to exercise one of these options, o, the company receives a payment equal to $\left(p_n^o - p_{nk}\right) q_n^o$ €, where p_{nk} is the spot price of electricity in hour n. This leads to the following expression for the company's expected benefit:

$$\mathbb{E}\mathcal{B} = \sum_{k \in \mathrm{K}} \pi_k \left[\sum_{m \in \mathrm{M}} \sum_{n \in \mathrm{N}} r_{nk}^m + \sum_{o \in \mathrm{O}^{\mathrm{LPF}}} \sum_{n \in \mathrm{N}} u_{nk}^o \cdot \left(p_n^o - p_{nk}\right) \cdot q_n^o - \sum_{t \in \mathrm{T}} \sum_{n \in \mathrm{N}} c_{nk}^t \right]. \quad (4.38)$$

The term $u_{nk}^o \cdot \left(p_n^o - p_{nk}\right) \cdot q_n^o$ is non-linear (u_{nk}^o is a variable and so is p_{nk}). An equivalent mixed linear-integer expression must be derived if state-of-the-art MIP optimizers are to be used to solve this problem. For that purpose a new variable, p_{nk}^o, is introduced such that $p_{nk}^o = \max\left\{p_n^o, p_{nk}\right\}$. The new mixed linear-integer expression for the company's expected benefit is:

$$\mathbb{E}\mathcal{B} = \sum_{k \in \mathrm{K}} \pi_k \left[\sum_{m \in \mathrm{M}} \sum_{n \in \mathrm{N}} r_{nk}^m + \sum_{o \in \mathrm{O}^{\mathrm{LPF}}} \sum_{n \in \mathrm{N}} \left(p_{nk}^o - p_{nk}\right) \cdot q_n^o - \sum_{t \in \mathrm{T}} \sum_{n \in \mathrm{N}} c_{nk}^t \right]. \quad (4.39)$$

To guarantee that $p_{nk}^o = \max\left\{p_n^o, p_{nk}\right\}$, the following constraints must be introduced:

$$\begin{aligned} & p_{nk} - p_{nk}^o \geq -M^p u_{nk}^o, \quad o \in \mathrm{O}^{\mathrm{LPF}}, n \in \mathrm{N}, k \in \mathrm{K}, \\ & p_n^o - p_{nk}^o \geq -M^p \left(1 - u_{nk}^o\right), \quad o \in \mathrm{O}^{\mathrm{LPF}}, n \in \mathrm{N}, k \in \mathrm{K}, \\ & u_{nk}^o \in \{0,1\}, \quad o \in \mathrm{O}^{\mathrm{LPF}}, n \in \mathrm{N}, k \in \mathrm{K}, \end{aligned} \quad (4.40)$$

where M^p is a large price. To illustrate how this formulation works let us assume that $u_{nk}^o = 0$. In that case constraints (4.40) reduce to:

$$\begin{aligned} & p_{nk} - p_{nk}^o \geq 0, \quad o \in \mathrm{O}^{\mathrm{LPF}}, n \in \mathrm{N}, k \in \mathrm{K}, \\ & p_n^o - p_{nk}^o \geq -M^p, \quad o \in \mathrm{O}^{\mathrm{LPF}}, n \in \mathrm{N}, k \in \mathrm{K}. \end{aligned} \quad (4.41)$$

Given that the company's objective is the maximization of its expected benefit, the variable p_{nk}^o will take the maximum possible value, which in this case is p_{nk}. Therefore, $u_{nk}^o = 0$ implies that $p_{nk}^o = p_{nk}$ and the result is that the company does not exercise the option. Conversely, if $u_{nk}^o = 1$ constraints (4.40) reduce to:

$$\begin{aligned} & p_{nk} - p_{nk}^o \geq -M^p, \quad o \in \mathrm{O}^{\mathrm{LPF}}, n \in \mathrm{N}, k \in \mathrm{K}, \\ & p_n^o - p_{nk}^o \geq 0, \quad o \in \mathrm{O}^{\mathrm{LPF}}, n \in \mathrm{N}, k \in \mathrm{K}. \end{aligned} \quad (4.42)$$

In this case the maximum value for p_{nk}^o is p_n^o. Hence, the company obtains a benefit equal to $\left(p_n^o - p_{nk}\right) q_n^o$ € from its long put position.

4.3.3.2 Short call position

A generation company that assumes a short call position is limiting the maximum price that it will receive for its production but it receives a payment in exchange: the price of the option. Its counter party can be an energy service provider willing to hedge against the risk of facing price spikes. This can be seen as a form of capacity payment, as suggested in [Vázquez '01].

Let $\mathrm{O}^{\mathrm{SCP}}$ be the set of short call positions assumed by the company that are physically settled. With each of these options, o, the company sells to the other party the right to buy q_n^o MWh with an exercise price of p_n^o €/MWh. We represent the counter-party's exercise decision by a binary variable, u_{nk}^o. If in scenario k the counter party decides to exercise option o in hour n, then $u_{nk}^o = 1$. In other case, $u_{nk}^o = 0$. The formulation of the company's expected benefit and the expression of the energy balance equation are equivalent to those suggested for the case of the long put position, (4.36) and (4.37):

$$\mathbb{E}\mathcal{B} = \sum_{k \in \mathrm{K}} \pi_k \left[\sum_{m \in \mathrm{M}} \sum_{n \in \mathrm{N}} r_{nk}^m + \sum_{o \in \mathrm{O}^{\mathrm{SCP}}} \sum_{n \in \mathrm{N}} u_{nk}^o \cdot p_n^o \cdot q_n^o - \sum_{t \in \mathrm{T}} \sum_{n \in \mathrm{N}} c_{nk}^t \right]. \tag{4.43}$$

$$\sum_{m \in \mathrm{M}} q_{nk}^m + \sum_{o \in \mathrm{O}^{\mathrm{SCP}}} u_{nk}^o q_n^o = \sum_{t \in \mathrm{T}} q_{nk}^t + \sum_{h \in \mathrm{H}} q_{nk}^h - b_{nk}^h, \quad n \in \mathrm{N}, k \in \mathrm{K}. \tag{4.44}$$

However, it is obvious that the option exercise decision corresponds to the counter party and is not oriented to the maximization of the generation company's profit. As a result, a set of constraints must be introduced to guarantee that the binary variables correctly represent the decisions taken by the counter party:

$$\begin{aligned} & p_n^o - p_{nk} \geq -M^p u_{nk}^o, \quad o \in \mathrm{O}^{\mathrm{SCP}}, n \in \mathrm{N}, k \in \mathrm{K}, \\ & p_{nk} - p_n^o \geq -M^p \left(1 - u_{nk}^o\right), \quad o \in \mathrm{O}^{\mathrm{SCP}}, n \in \mathrm{N}, k \in \mathrm{K}. \end{aligned} \tag{4.45}$$

It is easy to check that $p_n^o > p_{nk}$ implies $u_{nk}^o = 0$, so the option is not exercised. Conversely, if $p_n^o < p_{nk}$ then $u_{nk}^o = 1$ and the option is exercised.

Consider now the set of short call positions assumed by the company that are financially settled, $\mathrm{O}^{\mathrm{SCF}}$. The expression of the company's expected benefit that must be used is given by an expression equivalent to the one suggested for financial long put positions, (4.39):

$$\mathbb{E}\mathcal{B} = \sum_{k \in \mathrm{K}} \pi_k \left[\sum_{m \in \mathrm{M}} \sum_{n \in \mathrm{N}} r_{nk}^m + \sum_{o \in \mathrm{O}^{\mathrm{SCF}}} \sum_{n \in \mathrm{N}} \left(p_{nk}^o - p_{nk}\right) \cdot q_n^o - \sum_{t \in \mathrm{T}} \sum_{n \in \mathrm{N}} c_{nk}^t \right]. \tag{4.46}$$

In this case the expression for the auxiliary price is $p_{nk}^o = \min\left\{p_n^o, p_{nk}\right\}$. Hence, the following constraints have to be introduced:

$$\begin{aligned} & p_{nk}^o \leq p_{nk}, \quad o \in \mathrm{O}^{\mathrm{SCF}}, n \in \mathrm{N}, k \in \mathrm{K}, \\ & p_{nk}^o \leq p_n^o, \quad o \in \mathrm{O}^{\mathrm{SCF}}, n \in \mathrm{N}, k \in \mathrm{K}, \end{aligned} \tag{4.47}$$

and no binary variables are required.

4.3.4 The value of the company's market share

The expressions of the company's expected benefit that we have formulated in previous sections focus only on the short term. However, the company's objective is (or should be) the maximization of its long-term benefit. This includes aspects such as the strategy followed by the company in order to defend its market position against new entrants. As stated in [Viscusi '98], one pricing strategy is for an incumbent firm always to set the price that maximizes its current profit. Typically, setting such a high price will cause the fringe to invest in capacity and expand. This can be called myopic pricing. The polar opposite case is that the incumbent sets a price that prevents all fringe expansion (limit pricing). Myopic pricing gives higher profits today, while limit pricing gives higher profits in the future. Pricing at a level to exclude from the market less efficient competitors is, of course, what competition is supposed to do. Pricing to exclude equally or more efficient competitors is known as predatory pricing and constitutes an intent to acquire the monopoly position.

Let us evaluate the influence that the slope of the residual demand curve, $p(q_i)$, exerts on the short-term strategy of a certain firm, i, whose costs are given by $c_i(q_i)$. Firm i's short-term benefit is given by:

$$\mathcal{B}_i = p(q_i)\, q_i - c_i(q_i)\,. \tag{4.48}$$

The output q_i that maximizes the firm's short-term profit must fulfill the following first-order optimality condition:

$$\frac{\partial \mathcal{B}_i}{\partial q_i} = p(q_i) + \frac{\partial p(q_i)}{\partial q_i} q_i - c_i'(q_i) = 0 \Rightarrow p(q_i) - c_i'(q_i) = -\frac{\partial p(q_i)}{\partial q_i} q_i\,. \tag{4.49}$$

Hence, firm i's optimal strategy is to produce a quantity such that the difference between the resulting price, $p(q_i)$, and its marginal costs, $c_i'(q_i)$, is equal to $\left|\frac{\partial p(q_i)}{\partial q_i} q_i\right|$. This difference is usually known as price markup.

Consider a spot market auction corresponding to an on-peak hour. In on-peak hours the slope of the residual demand curve that a generation company faces is typically very steep and the company's output is high, which suggests a large price markup. Consequently, if no long-term guidelines were included, the model would blindly tend to reduce the company's production so as to cause a increase of the clearing price and maximize the company's current profit. However, this also allows competitors to sell more quantity at a higher price. Taking into account that the price of electricity usually behaves as a mean-reverting process, if the firm repeatedly gives up its position during on-peak hours, competitors will find it profitable to increase their market shares and, in the long run, prices will return to the original level. If this happens, the company will have lost its market position. The possibility of suffering from market-power mitigation measures applied by the regulator is another adverse consequence of this myopic behavior.

In order to avoid these undesirable effects, the company should take into consideration the future value of its current market position when designing its short-term strategy. This can be simply done by including an additional term in the benefit expression:

$$\mathcal{B}_i = p(q_i)\, q_i - c_i(q_i) + \sigma \frac{q_i}{Q}, \tag{4.50}$$

where σ is the value of the company's market share, expressed in € per unit and Q is the total trading volume expected for that particular auction. The value of σ can be obtained from a medium-term strategic model that includes a minimum-market share constraint. In subsequent chapters we will illustrate the relevance of this additional term with numerical examples. A more specific expression of the company's expected benefit, including the future value of the company's current market share is:

$$\mathbb{E}\mathcal{B} = \sum_{k \in \mathrm{K}} \pi_k \left[\sum_{m \in \mathrm{M}} \sum_{n \in \mathrm{N}} r_{nk}^m + \sigma_n^m \frac{q_{nk}^m}{Q_{nk}^m} - \sum_{t \in \mathrm{T}} \sum_{n \in \mathrm{N}} c_{nk}^t \right]. \tag{4.51}$$

where σ_n^m is the value of the company's market share in the n-th hourly auction of the m-th market mechanism expressed in € per unit and Q_{nk}^m is the total trading volume expected for that particular auction if market situation k occurs.

4.4 The case of an energy service provider

4.4.1 The short-term problem of an energy service provider

The activity of an energy service provider (ESP) in a wholesale electricity market presents an important conceptual difference with respect to the business of a generation company. A generating unit can be seen as an *option*, given that its owner is typically free to decide whether it produces electricity or it remains offline. Of course, in the long term, the unit must produce in order to be profitable, since the price paid for such an option —the cost of the plant— is extremely high. On the contrary, an ESP does not require large investments to operate. The core of its portfolio is constituted by *obligations* in the form of retail contracts with its customers[8].

An ESP must buy in the wholesale market the energy purchased by its customers according to these retail contracts. The ESP can perform these wholesale purchases either through medium- and long-term contracts or through the spot market. Hence, in the short-term, an ESP is faced with a portfolio of wholesale long positions (purchases) and retail short positions (sales) that must be balanced with purchases in the spot market. The problem is then to decide the bids that must be submitted to the spot market in order to buy the required amounts of energy at the minimum possible cost. This problem presents certain symmetries with respect to that of a generation company and can be addressed following the ideas presented in previous sections. Nevertheless, it must be emphasized that an ESP is subject to a more significant risk exposure in the spot market than a generation company, given that an ESP is forced to balance its position, whereas the generation company can decide the amount of energy that it wishes to produce. In order to reduce this risk exposure, an ESP should perform the majority of its purchases through medium- and long-term wholesale contracts.

[8] Some of these contracts can be options, but it is rare that customers decide to expose themselves to the risk of the spot market.

4.4.2 The portfolio of an energy service provider

As indicated, the portfolio with which an ESP goes to the spot market is typically integrated by wholesale long positions and retail short positions. In order to develop an optimal strategy for the spot market, we must take into account the influence of these contracts. This influence can be represented in a mathematical programming problem following the same logic that has been used when modeling the portfolio of a generation company.

For instance, an ESP can enter in a physical bilateral contract with a generation company and purchase a certain amount of energy that the generation company will supply in a specific moment of the future at a certain node of the network. In this manner, the ESP secures part of its purchases and reduces its exposure in the spot market. The influence that such a contract exerts on the strategy followed by the ESP in the spot market can be evaluated following the ideas presented in section 4.3.2.1. The conclusion is the same if the ESP enters in a contract for differences.

Trading with options in the wholesale market is also another way in which an ESP can reduce its risk exposure. For example, an ESP can assume a long call position for a certain amount of energy where the counter party is a generation company that assumes a short call position. In this manner, the ESP guarantees the maximum price that it will have to pay for that energy. On the contrary, it is not likely that an ESP be interested in assuming a short put position, given that this would only contribute to increase its risk exposure.

The variety of retail contracts that an ESP can sign with its customers is extremely wide. Nevertheless, the range of contractual forms that can be formulated with mixed linear-integer expressions is also very broad. The analysis of a particular case would surely shed light over this matter and will be suggested as a future line of research.

In conclusion, the portfolio of an ESP consists mainly of wholesale and retail contracts. The ESP must consider the influence of these contracts when developing its bidding strategy. It is expected that this influence can be evaluated in a mathematical programming framework using mixed linear-integer expressions.

4.4.3 The spot market from the perspective of an ESP

When deciding its strategy for the spot market, an ESP must evaluate the change that its purchase decisions will induce in the spot price. This can be evaluated in each of the spot-market auctions by means of a *residual offer curve*. Such a curve is obtained by subtracting the bid curves of the rest of wholesale buyers from the aggregate offer curve. The residual offer curve leads to a cost function, $c(q)$, that indicates the cost incurred by the ESP when purchasing q MWh in that particular auction. These cost functions present the distinctive shape shown in Figure 4.19. They are non-decreasing piecewise linear curves. A representation based on PWL approximations identical to the one suggested for residual demand curves can be used to implement these curves in a MIP model.

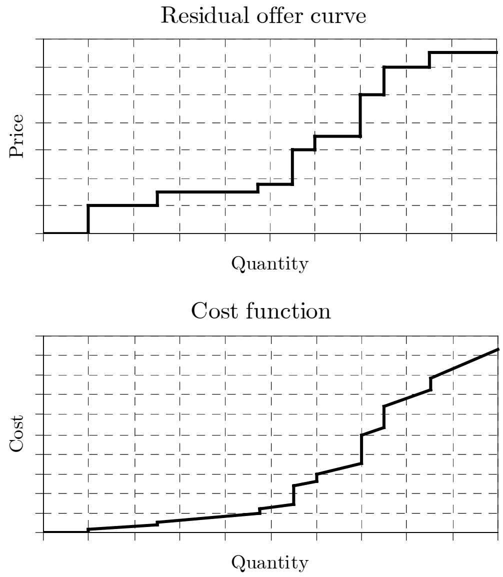


Figure 4.19. A residual offer curve and the corresponding cost function.

In conclusion, the problem of finding the optimal strategy of an ESP in the spot market presents similarities with the case of a generation company that can be exploited in order to follow the same line of attack. Specifically, we have shown that the portfolio of an ESP is constituted by wholesale and retail contracts that should adapt well to a mixed linear-integer framework. Additionally, the perspective that an ESP has of the spot market is symmetrical to the one of a generation company, which permits the usage of a similar representation. Consequently, the modeling approach suggested in this chapter, originally conceived for the case of a generation company, also meets the eighth challenge, which requires that the model can be easily adapted to the case of an ESP.

4.5 Problem statement

This section summarizes the mixed linear-integer formulation that results from the analysis of the problem faced by a generation company operating in a spot market. The formulation of such a problem is itself a contribution, as it has required the full comprehension of the real problem and is the first step toward its solution. Indeed, it has been shown that the eight challenges identified in chapter 3 are fulfilled by this modeling approach.

4.5.1 Objective function

The objective function that must guide the generation company when designing its strategy for the spot market is its expected benefit. In previous sections we have identified a variety of elements that have an influence on the company's expected benefit, such as the revenues obtained by the company in the different spot market mechanisms, the production costs of its generating units, the positions assumed by the company with respect to options and long-term contracts or the future value of the company's current market share. The following equation integrates the expressions that we have suggested to evaluate these contributions in a mixed linear-integer framework.

Other terms can be added, provided that they are formulated as mixed linear-integer expressions.

$$\begin{aligned}\mathbb{E}\mathcal{B} = \sum_{k\in\mathrm{K}} \pi_k \Bigg[& \sum_{m\in\mathrm{M}} \sum_{n\in\mathrm{N}} r_{nk}^m \left(q_{nk}^m\right) + \sigma_n^m \frac{q_{nk}^m}{Q_{nk}^m} \\ & + \sum_{c\in\mathrm{C}^{\mathrm{D}}} \sum_{n\in\mathrm{N}} \left(p_n^c - p_{nk}\right) \cdot q_n^c \\ & + \sum_{o\in\mathrm{O}^{\mathrm{P}}} \sum_{n\in\mathrm{N}} u_{nk}^o \cdot p_n^o \cdot q_n^o + \sum_{o\in\mathrm{O}^{\mathrm{F}}} \sum_{n\in\mathrm{N}} \left(p_{nk}^o - p_{nk}\right) \cdot q_n^o \\ & - \sum_{t\in\mathrm{T}} \sum_{n\in\mathrm{N}} c_{nk}^t \left(q_{nk}^t, y_{nk}^t, u_{nk}^t\right) \Bigg] \\ & + \sum_{c\in\mathrm{C}^{\mathrm{P}}} \sum_{n\in\mathrm{N}} p_n^c q_n^c . \end{aligned} \tag{4.52}$$

where O^{P} is the set of options that are physically settled and O^{F} is the set of options that are financially settled. A number of terms of the previous objective function are constants and will not be altered by the strategy followed by the company in the spot market. Hence, they could be suppressed when implementing the model.

4.5.2 Constraints

4.5.2.1 Energy balance equation

In our model, the most relevant constraint is the energy balance equation. This constraint establishes a link between different aspects of the company's operation. In particular, it indicates the amount of energy that the generation company must produce in order to meet the obligations assumed due to its net sales in the wholesale electricity market. The following formulation of the energy balance equation includes all the elements analyzed in previous sections. Other contributions can be incorporated in the form of mixed linear-integer expressions.

$$\sum_{m\in\mathrm{M}^{\mathrm{E}}} q_{nk}^m + \sum_{c\in\mathrm{C}^{\mathrm{P}}} q_n^c + \sum_{o\in\mathrm{O}^{\mathrm{P}}} u_{nk}^o q_n^o = \sum_{t\in\mathrm{T}} q_{nk}^t + \sum_{h\in\mathrm{H}} q_{nk}^h - b_{nk}^h, \quad n\in\mathrm{N}, k\in\mathrm{K}. \tag{4.53}$$

4.5.2.2 Reserve balance equation

The reserve balance equation evaluates the amount of reserve that the generation company must provide with its units as a result of its sales in the reserve market mechanisms:

$$\sum_{m\in\mathrm{M}^{\mathrm{R}}} q_{nk}^m = \sum_{t\in\mathrm{T}} r_{nk}^t + \sum_{h\in\mathrm{H}} r_{nk}^h, \quad n\in\mathrm{N}, k\in\mathrm{K}. \tag{4.54}$$

4.5.2.3 Spot market

In this chapter we have used a compact notation to represent both the spot market clearing prices and the revenues obtained by the company in each of the spot market mechanisms. However, in practice, a number of constraints have to be included in order to correctly represent the operation of the spot market. In particular, we approximate residual demand curves and revenue functions by means of piecewise linear functions, according to the following expressions:

$$\begin{aligned}
&p_{nk}^{m} = p_{1nk}^{m} + \sum_{j<J} \delta_{jnk}^{m} v_{jnk}^{m}, \\
&r_{nk}^{m} = r_{1nk}^{m} + \sum_{j<J} \rho_{jnk}^{m} v_{jnk}^{m}, \\
&q_{nk}^{m} = \sum_{j<J} v_{jnk}^{m}, \qquad\qquad\qquad\qquad n \in \mathrm{N}, k \in \mathrm{K}, m \in \mathrm{M}, \\
&v_{jnk}^{m} \leq u_{jnk}^{m} \left(q_{j+1nk}^{m} - q_{jnk}^{m} \right), \quad j < J, \\
&u_{j+1nk}^{m} \leq u_{jnk}^{m}, \quad j < J-1, \\
&u_{jnk}^{m} \in \{0,1\}, v_{jnk}^{m} \in \mathbb{R}^{+}, j < J,
\end{aligned} \tag{4.55}$$

To guarantee that the decisions suggested by the model constitute admissible offering strategies for the spot market, a set of non-decreasing constraints has to be included for each pair of residual demand realizations considered in every spot market auction:

$$\begin{aligned}
&q_{nk}^{m} - q_{nk'}^{m} \geq -x_{nkk'}^{m} M^{q}, \\
&q_{nk'}^{m} - q_{nk}^{m} \geq -\left(1 - x_{nkk'}^{m}\right) M^{q}, \\
&p_{nk}^{m}\left(q_{nk}^{m}\right) - p_{nk'}^{m}\left(q_{nk'}^{m}\right) \geq -x_{nkk'}^{m} M^{p}, \qquad n \in \mathrm{N}; k, k' \in \mathrm{K}, k' > k; m \in \mathrm{M}, \\
&p_{nk'}^{m}\left(q_{nk'}^{m}\right) - p_{nk}^{m}\left(q_{nk}^{m}\right) \geq -\left(1 - x_{nkk'}^{m}\right) M^{p}, \\
&x_{nkk'}^{m} \in \{0,1\},
\end{aligned} \tag{4.56}$$

4.5.2.4 Generating units

In this model, the constraints that take into account the technical features of each generating unit are quite similar to the ones traditionally used in unit-commitment or economic-dispatch models to derive short-term minimum-cost generation schedules. The following equations are used to represent thermal units:

$$c_{nk}^{t} = s_t y_{nk}^{t} + o_t q_{nk}^{t} + f_t \left(\beta_t u_{nk}^{t} + \alpha_t \frac{q_{nk}^{t}}{k_t} \right), \quad t \in \mathrm{T}, n \in \mathrm{N}, k \in \mathrm{K}\,, \tag{4.57}$$

$$y_{nk}^{t} - z_{nk}^{t} = u_{nk}^{t} - u_{n-1k}^{t}, \quad t \in \mathrm{T}, n \in \mathrm{N}, k \in \mathrm{K}\,, \tag{4.58}$$

$$0 \leq y_{nk}^{t} \leq 1, \quad t \in \mathrm{T}, n \in \mathrm{N}, k \in \mathrm{K}\,, \tag{4.59}$$

$$0 \leq z_{nk}^{t} \leq 1, \quad t \in \mathrm{T}, n \in \mathrm{N}, k \in \mathrm{K}\,, \tag{4.60}$$

$$\underline{q}_t k_t u_{nk}^{t} \leq q_{nk}^{t} + r_{nk}^{t} \leq \overline{q}_t k_t u_{nk}^{t}, \quad t \in \mathrm{T}, n \in \mathrm{N}, k \in \mathrm{K}\,, \tag{4.61}$$

$$-l_t \leq q_{nk}^{t} - q_{n-1k}^{t} \leq l_t, \quad t \in \mathrm{T}, n \in \mathrm{N}, k \in \mathrm{K}\,, \tag{4.62}$$

$$u_{n+\nu k}^{t} + u_{n-1k}^{t} - u_{nk}^{t} \geq 0, \quad t \in \mathrm{T}, n \in \mathrm{N}, k \in \mathrm{K}, \nu = 1, \ldots, \min\left\{N - n, \nu_t - 1\right\}. \tag{4.63}$$

Another set of equations refers to hydro units:

$$w_{nk}^{h} = w_{n-1k}^{h} - \frac{q_{nk}^{h}}{k_h} + i_{nk}^{h} - s_{nk}^{h} + \eta_h b_{nk}^{h}, \quad h \in \mathrm{H}, n \in \mathrm{N}, k \in \mathrm{K}\,, \tag{4.64}$$

$$0 \leq q_{nk}^{h} + r_{nk}^{h} \leq k_h \overline{q}_h, \quad h \in \mathrm{H}, n \in \mathrm{N}, k \in \mathrm{K}\,, \tag{4.65}$$

$$0 \leq b_{nk}^{h} \leq \bar{b}_{h}, \quad h \in \mathrm{H}, n \in \mathrm{N}, k \in \mathrm{K}\,, \tag{4.66}$$

$$0 \leq s_{nk}^{h}, \quad h \in \mathrm{H}, n \in \mathrm{N}, k \in \mathrm{K}\,, \tag{4.67}$$

$$\underline{w}_{h} \leq w_{nk}^{h} \leq \overline{w}_{h}, \quad h \in \mathrm{H}, n \in \mathrm{N}, k \in \mathrm{K}\,. \tag{4.68}$$

Many other constraints can be introduced, as long as they are expressed in terms of mixed linear-integer equations.

4.5.2.5 Options and long-term contracts

In order to guarantee that the influence of the long put and short call positions assumed by the company is correctly evaluated, several constraints have to be introduced. Specifically, long put positions that are financially settled require the following set of constraints:

$$\begin{aligned} & p_{nk} - p_{nk}^{o} \geq -M^{p} u_{nk}^{o}, \quad o \in \mathrm{O}^{\mathrm{LPF}}, n \in \mathrm{N}, k \in \mathrm{K}, \\ & p_{n}^{o} - p_{nk}^{o} \geq -M^{p} \left(1 - u_{nk}^{o}\right), \quad o \in \mathrm{O}^{\mathrm{LPF}}, n \in \mathrm{N}, k \in \mathrm{K}, \\ & u_{nk}^{o} \in \{0,1\}, \quad o \in \mathrm{O}^{\mathrm{LPF}}, n \in \mathrm{N}, k \in \mathrm{K}, \end{aligned} \tag{4.69}$$

Short call positions that are physically settled must be accompanied by another set of constraints:

$$\begin{aligned} & p_{n}^{o} - p_{nk} \geq -M^{p} u_{nk}^{o}, \quad o \in \mathrm{O}^{\mathrm{SCP}}, n \in \mathrm{N}, k \in \mathrm{K}, \\ & p_{nk} - p_{n}^{o} \geq -M^{p} \left(1 - u_{nk}^{o}\right), \quad o \in \mathrm{O}^{\mathrm{SCP}}, n \in \mathrm{N}, k \in \mathrm{K}. \end{aligned} \tag{4.70}$$

Finally, short call positions that are financially settled entail a different set of constraints:

$$\begin{aligned} & p_{nk}^{o} \leq p_{nk}, \quad o \in \mathrm{O}^{\mathrm{SCF}}, n \in \mathrm{N}, k \in \mathrm{K}, \\ & p_{nk}^{o} \leq p_{n}^{o}, \quad o \in \mathrm{O}^{\mathrm{SCF}}, n \in \mathrm{N}, k \in \mathrm{K}, \end{aligned} \tag{4.71}$$

4.5.2.6 Other long-term strategic objectives

We have justified that a company should not blindly fall into the trap of short-term profits but rather identify long-term targets that guide the company's daily operation. In particular, we have suggested including an additional term into the objective function that represents the future return that the company's current market share will yield.

4.6 Conclusion

In this chapter we have formulated the problem of optimizing the strategy of a company in the electricity spot market as a multistage stochastic program. This approach reflects the decision process that actually takes place in the spot market considered in this thesis, which is designed as a sequence of market mechanisms. Moreover, the proposed formulation meets the eight modeling challenges established in chapter 3. Let us examine this in further detail:

- A methodology based on piecewise linear approximations has been suggested to represent both the residual demand curves and the revenue functions of all the spot market auctions (challenge 1).
- The scenario-tree structure used to represent spot-market uncertainty, in addition to adapting well to a multistage stochastic programming framework, takes into consideration the correlation that may exist between different spot market auctions (challenge 2).
- To guarantee that the decisions suggested by the model constitute admissible offering strategies for the spot market (challenge 3), a set of non-decreasing constraints has been defined. This requires that all the spot market scenarios be simultaneously considered (challenge 4).
- The operation of the generating units has been modeled in detail with a formulation similar to the one typically used in unit-commitment and economic-dispatch models (challenge 5).
- A variety of other elements of the generation company's portfolio can be incorporated into the model (challenge 6), following the guidelines suggested for long-term contracts and options.
- A novel approach has been proposed to guarantee that the model takes into account the company's objective of defending a certain market position (challenge 7).
- The applicability of this model to the case of an energy service provider has been justified (challenge 8).

In this manner, one of the main objectives of this thesis has been fulfilled. Indeed, the model presented in this chapter constitutes an original approach to address the problem faced by a generation company in an electricity spot market. It takes into account the aspects that have a relevant influence on the company's operation, including the strategy that its rivals are expected to follow and the company's own portfolio. It also presents a structure that suggests the usage of decomposition techniques to obtain numerical results. In the following chapter, a detailed analysis of this structure will lead to a solution procedure based on two decomposition techniques, Lagrangian relaxation and Benders' decomposition, that turn out to be complementary.

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5 The development of a solution strategy

The general problem faced by a generation company in an electricity spot market has been formulated as a multistage stochastic program. However, this formulation must be particularized to address more specific problems in order to obtain numerical results for real-size study cases. In this chapter two particularizations of this general approach are suggested. On one hand, the problem of developing an offering strategy for one spot market mechanism is formulated as a two-stage program, assuming that the relative importance of the subsequent market mechanisms diminishes as the moment of physical delivery gets nearer. On the other hand, the weekly stochastic unit-commitment problem is formulated as a sequence of two-stage stochastic programs, one for each day of the week.

Both mathematical programs turn out to be large-scale programs whose numerical resolution requires a careful analysis of their structure and properties in order to identify adequate solution strategies. In particular, Benders' decomposition is considered the most convenient approach to solve the two-stage optimal offering problems. In contrast, Lagrangian relaxation is more appropriate to address the weekly stochastic unit-commitment problem. The combination of both techniques provides a powerful framework to handle the short-term operation of a generation company participating in an electricity spot market.

5.1 Introduction

The adoption of a multistage stochastic programming approach to represent the problem faced by a generation company in a spot market is one of the main contributions of this thesis. However, the formulation of such a problem would be completely useless if it were impossible to obtain numerical results for it.

This chapter constitutes an effort to develop a solution strategy that can be used to solve real-size problems. It begins by reducing the multistage stochastic program into a sequence of two-stage programs, an approximation that greatly simplifies the search for a solution.

Each two-stage program represents the process of developing an offering strategy by a generation company for a particular market mechanism. It models not only the uncertainty of the current market mechanism, but also the influence of the subsequent market mechanism and the daily generation schedule that results from the sequence of sales performed by the company.

In accordance with the previous idea, this chapter also suggests representing the weekly stochastic problem of the generation company as a sequence of two-stage programs. This provides a framework to make unit-commitment decisions and to manage hydro resources on a weekly basis explicitly considering the uncertainty of the spot market.

The formidable size of the mathematical programs that result when the real short-term problems faced by a generation company are formulated in this manner makes it impossible to derive numerical solutions for them without making use of some sort of decomposition technique.

An analysis of the structure of the suggested two-stage program leads to the usage of two alternative decomposition techniques for its numerical resolution. On the one hand, Benders' decomposition adapts well to the two-stage structure of the problem and to the presence of complicating variables. An original ad hoc method is proposed to accelerate the convergence of Benders' algorithm when solving this particular type of problem that reinforces the adequacy of this approach. On the other hand, Lagrangian relaxation is a technique frequently used in the context of short-term generation scheduling, where the presence of coupling constraints complicates the search for a solution. Its application to the two-stage problem provides some meaningful economic interpretations, but its advantages are outnumbered by its shortcomings. In this manner, Benders' algorithm is the right choice to solve the two-stage program.

In contrast, when it comes to addressing the weekly stochastic unit-commitment problem, Benders' decomposition cannot be used due to the generalized presence of binary variables. In this context, Lagrangian relaxation turns out to be the most appropriate technique [Nowak '99]. In this manner, the short-term problems faced by a generation company in a spot market can be addressed with a combination of these two decomposition approaches.

5.2 A sequence of two-stage stochastic programs

5.2.1 The relative importance of market mechanisms

In chapter 4, we have formulated the problem faced by a generation company in an electricity spot market as a multistage stochastic program. According to this approach, in the first stage the company decides its offers for the day-ahead market. After the clearing of the auctions that constitute the day-ahead market, the company must decide its offers for the adjustment market based on the results obtained in the day-ahead market. Hence, the decisions taken by the company for the adjustment market can be seen as recourse actions oriented to correct previous undesired results. The process goes on until the balancing mechanism is cleared. A generation schedule results from this sequence of market mechanisms.

In practice, the volumes traded in the sequence of market mechanisms diminish as the moment of physical delivery gets nearer. For example, in the Spanish spot market, the volume of energy traded in the adjustment market is usually between 10 % and 20 % of the volume traded in the day-ahead market. Similarly, the reserve market is less relevant than the adjustment market and so forth. This suggests separating the company's multistage decision process into a sequence of two-stage decision processes. Specifically, when deciding the offers for the day-ahead market, the generation company might consider only the adjustment market and neglect the influence that the reserve market and the balancing mechanism have on its final generation schedule.

Furthermore, the company might also neglect the influence of the uncertainty faced in the adjustment market by simply reducing its discrete probability distribution to the set of expected hourly residual demand curves. In other words, the company can assume that each possible realization of the day-ahead market is accompanied by a single possible realization of the adjustment market. This results in the simplified decision process depicted in Figure 5.1. It must be noticed that the expected outcome for the adjustment market typically depends on the day-ahead market results, so each possible day-ahead market realization will be accompanied by a different expected situation for the adjustment market. While the decisions taken for the first market mechanism take the form of an offer curve, the decisions for the second market mechanism are expressed as expected sales. Hence, the quantities decided for the second market mechanism are not required to constitute a non-decreasing offer curve.

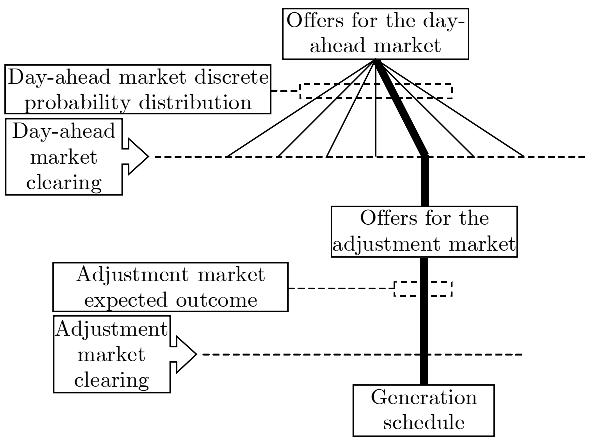

Figure 5.1. A simplified version of the decision process from the perspective of the day-ahead market.

Similar two-stage decision processes result for each of the spot market mechanisms, as represented in Figure 5.2. In this manner, when a company decides its offering strategy for one of the spot market mechanisms, it should solve one of these two-stage programs, instead of solving a unique multistage stochastic program for the whole spot market session. This simplification significantly reduces the computational effort required to solve the problem faced by the generation company in the spot market. Nevertheless, the size of these two-stage programs still requires the use of advanced mathematical programming techniques in order to obtain numerical results.

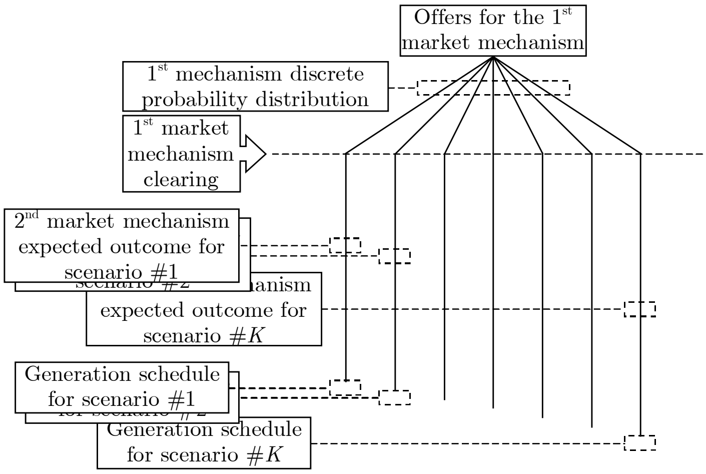

Figure 5.2. The scenario structure for each of the two-stage stochastic programs.

5.2.2 A compact formulation for a two-stage program

Let us formulate the two-stage program corresponding to a set of two consecutive market mechanisms $\mathrm{M} = \{1,2\}$. According to the general formulation developed in chapter 4, the company's objective function for this particular problem is expressed as:

$$
\begin{aligned}
\mathbb{E}\mathcal{B} = \sum_{k \in \mathrm{K}} \pi_k \Bigg[& \sum_{n \in \mathrm{N}} p_{nk}^1\left(q_{nk}^1\right) \cdot q_{nk}^1 && \text{Revenues 1st market mechanism} \\
& + \sigma_n^1 \frac{q_{nk}^1}{Q_{nk}^1} && \text{Market-share value 1st mkt. mec.} \\
& + \sum_{n \in \mathrm{N}} p_{nk}^2\left(q_{nk}^2\right) \cdot q_{nk}^2 && \text{Revenues 2nd market mechanism} \\
& + \sigma_n^2 \frac{q_{nk}^2}{Q_{nk}^2} && \text{Market-share value 2nd mkt. mec.} \\
& + \sum_{c \in \mathrm{C}^{\mathrm{P}}} \sum_{n \in \mathrm{N}} p_n^c q_n^c && \text{Physical bilateral contracts} \\
& + \sum_{c \in \mathrm{C}^{\mathrm{D}}} \sum_{n \in \mathrm{N}} \left(p_n^c - p_{nk}^1\left(q_{nk}^1\right)\right) \cdot q_n^c && \text{Contracts for differences} \\
& + \sum_{o \in \mathrm{O}^{\mathrm{P}}} \sum_{n \in \mathrm{N}} u_{nk}^o \cdot p_n^o \cdot q_n^o && \text{Physical options} \\
& + \sum_{o \in \mathrm{O}^{\mathrm{F}}} \sum_{n \in \mathrm{N}} \left(p_{nk}^o - p_{nk}^1\left(q_{nk}^1\right)\right) \cdot q_n^o && \text{Financial options} \\
& - \sum_{t \in \mathrm{T}} \sum_{n \in \mathrm{N}} c_{nk}^t\left(q_{nk}^t, y_{nk}^t, u_{nk}^t\right) \Bigg] && \text{Generation costs}
\end{aligned}
\tag{5.1}
$$

where the revenue obtained by the company in each of the two spot market mechanisms is expressed in terms of the corresponding clearing prices, instead of using the hourly revenue functions. Additionally, both CfDs and options are indexed to the clearing prices of the first market mechanism. Hence, the revenues obtained by the company due to these products depend only on the energy sold by the company through the first market mechanism.

Assuming that both market mechanisms are devoted to energy trading, the energy balance equation takes the following form:

$$q_{nk}^1 + q_{nk}^2 + \sum_{c \in \mathrm{C^P}} q_n^c + \sum_{o \in \mathrm{O^P}} u_{nk}^o q_n^o = \sum_{t \in \mathrm{T}} q_{nk}^t + \sum_{h \in \mathrm{H}} q_{nk}^h - b_{nk}^h, n \in \mathrm{N}, k \in \mathrm{K}\,. \quad (5.2)$$

A relevant observation is that, if the market mechanism of interest is not the day-ahead market, the sales performed in previous market mechanisms of the current spot market session should be treated as supply obligations assumed by the company, as suggested for physical contracts.

The objective function, (5.1), together with constraints (5.2) constitute the nucleus of the two-stage program. The rest of constraints would be formulated as indicated in chapter 4 and add little to the developments of this chapter. Hence, they will rather be expressed in a compact manner. For example, the quantities offered by the company for the first market mechanism in each hour n must satisfy constraints (4.55) and (4.56), which are condensed in equation (5.3):

$$\left\{q_{nk}^1, k \in \mathrm{K}\right\} \in \mathrm{Q}_n^1, \quad n \in \mathrm{N}\,. \quad (5.3)$$

Equation (5.3) establishes a link between the quantities decided for each of the possible outcomes of the first market mechanism. In contrast, it has been assumed that, once the first mechanism has been cleared, the company decides a unique quantity for the second market mechanism, based on its expected outcome. Hence, non-decreasing constraints (4.56) do not affect the second-stage decisions taken by the company. Only constraints (4.55) must be formulated for these recourse actions. We express this by means of the following equation:

$$q_{nk}^2 \in \mathrm{Q}_{nk}^2, \quad n \in \mathrm{N}, k \in \mathrm{K}\,. \quad (5.4)$$

It is worth noticing that equations (5.3) and (5.4) both include integrality constraints due to the binary variables that guarantee both that non-concave revenue functions are correctly evaluated and that the resulting offer curves are non-decreasing.

The schedule decided for each thermal generating unit, t, in each scenario k consists of a set of N production levels $\left\{q_{nk}^t, n \in \mathrm{N}\right\}$ that must comply with the unit's technical constraints, (4.57) to (4.63). Similarly, the schedule of each hydro unit, h, for each scenario k must observe constraints (4.64) to (4.68). The limitations imposed by these technical constraints are comprised in equations (5.5) and (5.6):

$$\left\{q_{nk}^t, y_{nk}^t, u_{nk}^t, n \in \mathrm{N}\right\} \in \mathrm{Q}^t, \quad t \in \mathrm{T}, k \in \mathrm{K}\,, \quad (5.5)$$

$$\left\{q_{nk}^h, b_{nk}^h, n \in \mathrm{N}\right\} \in \mathrm{Q}^h, \quad h \in \mathrm{H}, k \in \mathrm{K}\,. \quad (5.6)$$

Finally, the binary variables used to represent the exercise of options and the auxiliary variables with which the current value of each option is evaluated are subject to constraints (4.69) to (4.71), which are condensed in equations (5.7) and (5.8):

$$p_{nk}^{o} \in \mathrm{P}_{nk}^{o}, \quad o \in \mathrm{O}^{\mathrm{F}}, n \in \mathrm{N}, k \in \mathrm{K}\,, \tag{5.7}$$

$$u_{nk}^{o} \in \mathrm{U}_{nk}^{o}, \quad o \in \mathrm{O}^{\mathrm{P}}, n \in \mathrm{N}, k \in \mathrm{K}\,. \tag{5.8}$$

This compact formulation will be used in subsequent sections in order to analyze the adequacy of two different solution strategies, given that it facilitates the application of decomposition methods.

5.2.3 A numerical example

In this chapter, a number of small numerical examples are provided in order to illustrate the methodologies proposed. All these examples are based on the same input data so that the different solution approaches can be compared[1]. They address the two-stage problem faced by a fictitious generation company in the Spanish day-ahead market on Wednesday October 24th 2001. Uncertainty in the day-ahead market is represented by means of two scenarios. Residual demand curves and revenue functions are approximated by means of piecewise linear curves with six linear segments. No long-term contracts are considered. Commitment decisions are taken as input data. Figure 5.3 indicates the hourly values given to the coefficient that represents the value of market share in the day-ahead market. For example, a value of 2 M€ per unit in a certain hour means that a future profit of 2 M€ would be obtained if a 100 % of market share were reached in that hour.

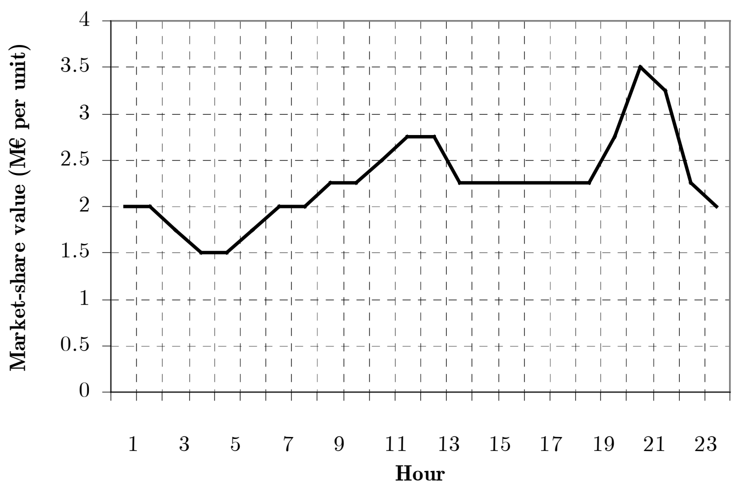

Figure 5.3. Hourly market-share value for the two-scenario case.

The resulting problem has 13249 equations and 13771 variables, 474 of which are binary. It has been formulated in the algebraic modeling language GAMS [Brooke '96]. 6.5 s are required to solve it with the commercial solver CPLEX 7.5 in a Pentium III 1 GHz 256 MB PC. The solution obtained is 30.112293 M€. Approximately 26 M€

[1] The details of the input data used for the numerical examples of this thesis can be found in appendix C.

correspond to the value of the current market share[2]. The revenues obtained in the day-ahead market are 8 M€, whereas those obtained in the adjustment market account for -0.4 M€. Finally, the variable costs are 3.5 M€.

Figure 5.4 shows the amount of energy offered by the company for the two scenarios in each of the hourly auctions that constitute the day-ahead market and the adjustment market. It also depicts the clearing prices that result.

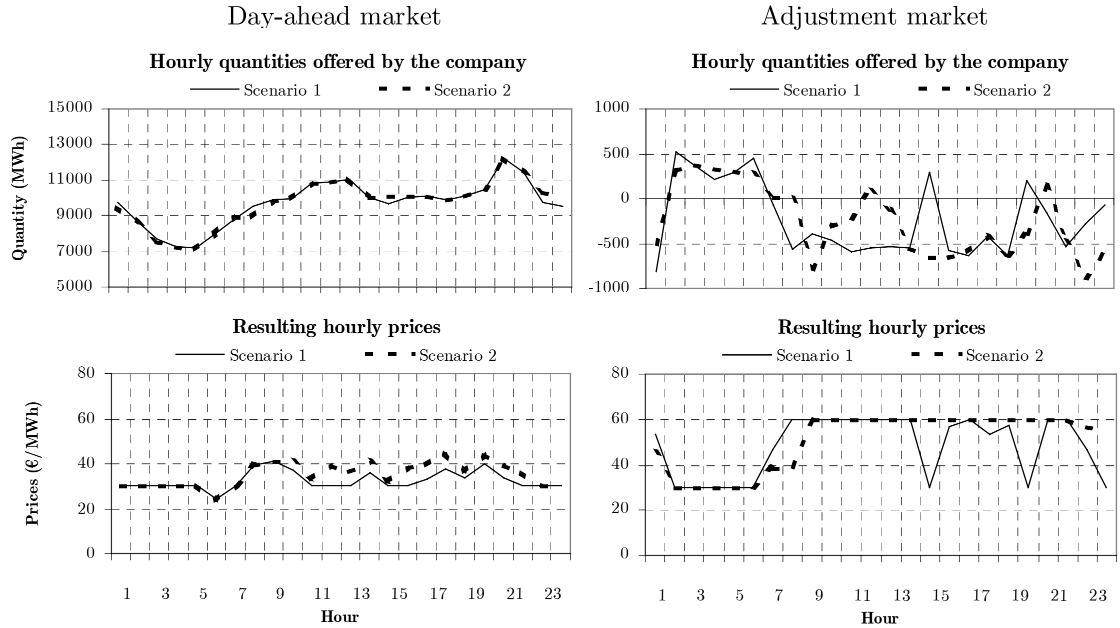

Figure 5.4. Hourly offers for the day-ahead market and the adjustment market.

The aggregation of the offers decided for both market mechanisms yields two generation schedules, one for each scenario. These schedules consist of a production program and a pumping profile, as can be seen in Figure 5.5.

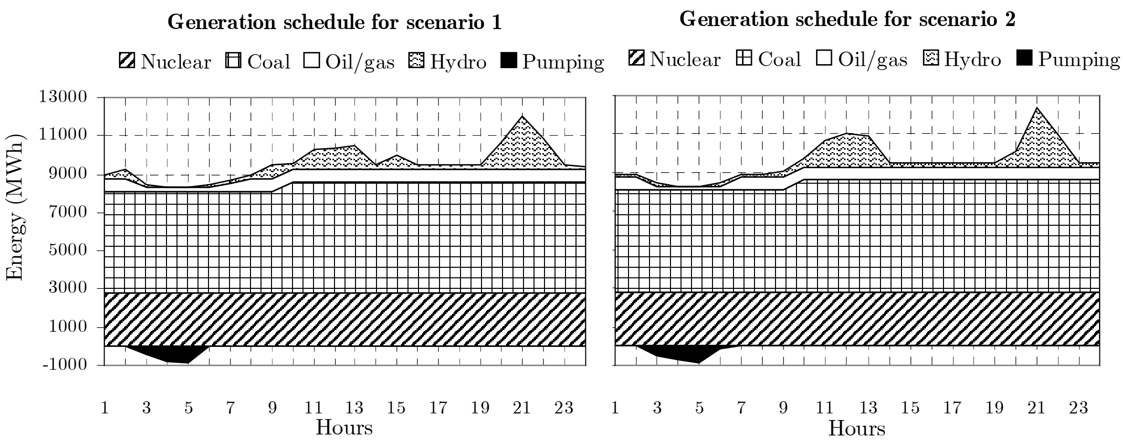

Figure 5.5. Hourly generation and pumping schedules for the units owned by the generation company.

[2] The value of the company's market share in the day-ahead market has been adjusted to obtain clearing prices that are similar to those observed in the Spanish spot market.

Figure 5.6 illustrates the manner in which the company's influence on the market-clearing price is represented. It depicts the offers decided by the model for the two residual demand scenarios considered in the 21st hourly auction of the day-ahead market. It also shows the corresponding revenues that the company would obtain in both scenarios. The same magnitudes are represented in Figure 5.7 for the adjustment market.

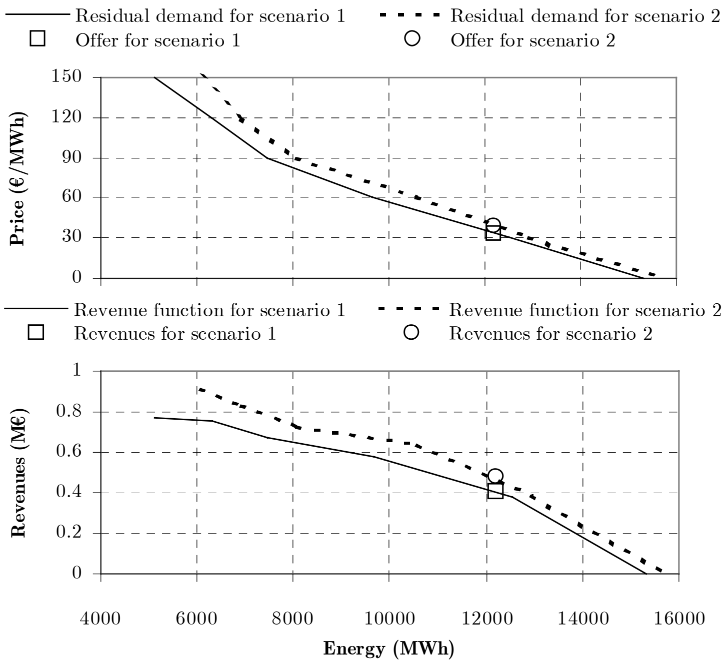

Figure 5.6. Detail of the offers decided for the 21st hourly auction of the day-ahead market.

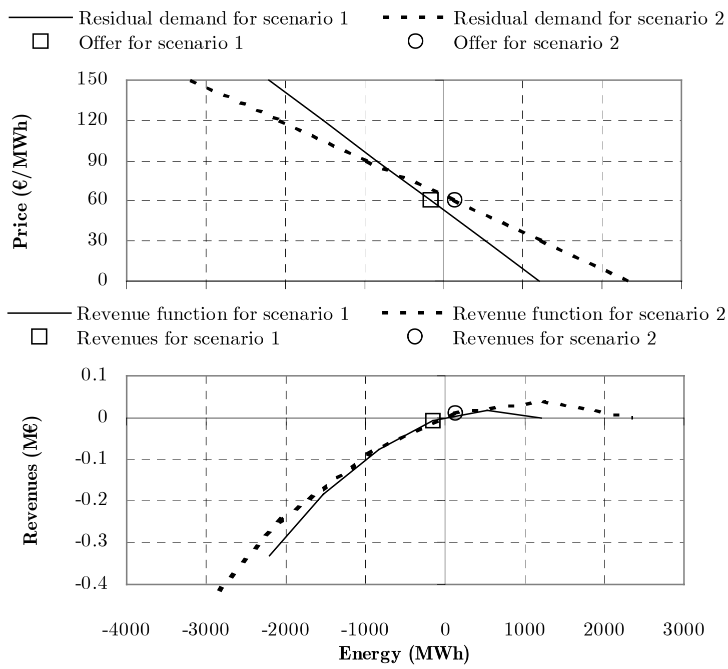

Figure 5.7. Detail of the offers decided for the 21st hourly auction of the adjustment market.

This numerical example serves as a benchmark to test the solution strategies that are developed in this chapter. Due to its small size (only two scenarios are considered and six linear segments are used to approximate each curve) CPLEX is able to solve it in a straightforward manner, without making use of the elaborate decomposition techniques that are presented below.

5.2.4 The weekly stochastic unit-commitment problem

Assuming that the relative importance of the market mechanisms that constitute a certain spot market session diminishes as the moment of physical delivery gets nearer has the advantage of reducing the problem faced by a generation company in a specific spot market session to a sequence of two-stage stochastic programs.

However, spot market sessions are repeated on a daily basis and it cannot be assumed that the results of tomorrow's day-ahead market are negligible if compared with those of today's day-ahead market. This has important implications that have already been pointed out. In first place, some medium-term strategic guideline must be considered when deciding the offers for the current spot market session, so as to avoid myopic decisions that may trigger undesirable reactions of rivals in subsequent spot market sessions. The value of the company's market share in the spot market is the parameter used in this thesis to orient short-term decisions toward long-term objectives. Secondly, the startup and shutdown of thermal units are decisions that should not be taken with a time scope of only one day. The reason is that it takes several days to recover the cost of starting up certain thermal plants. Hence, when deciding the offering strategy for a specific spot market session, the generation company should already have a weekly unit-commitment schedule. Finally, water reserves have a future value and cannot be used arbitrarily. A limited amount of hydro energy should be assigned to each day, in order to avoid an inefficient use of this valuable resource. This assignment can also be done with a one-week perspective.

Although several unit-commitment models that take into account the influence of the company's decisions on the spot price of electricity have recently been proposed, their deterministic approach is inconsistent with the stochastic perspective adopted in this thesis [García-González '00, Baíllo '01]. Assuming that the two most relevant market mechanisms are the day-ahead market and the adjustment market, the weekly multistage stochastic program can be seen as a sequence of two-stage programs. This yields the scenario structure represented in Figure 5.8. A simplified procedure to build such a scenario tree can be found in appendix B.

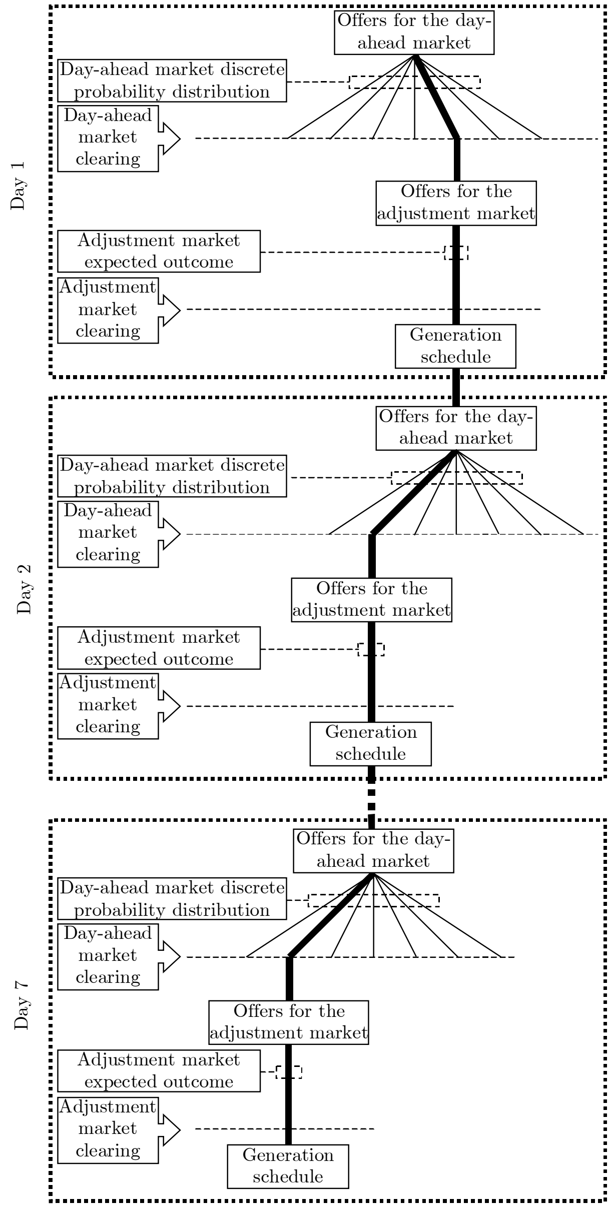

Figure 5.8. The decision process from the perspective of a whole week.

It is important to emphasize that the decisions suggested by this weekly multistage stochastic model should be updated every day, after the uncertainty of the current spot market session has been unveiled. It must also be noticed that these decisions are flexible, in the sense that they depend on the realization of uncertainty. In other words, the model does not provide a rigid unit-commitment schedule, but rather a collection of schedules adapted to the different spot market scenarios that may arise during the week. For instance, if three spot market situations are considered for the first day and it is Monday, three different startup strategies will be obtained, each one corresponding to one particular market outcome. In order to implement these decisions, it suffices to develop an offering strategy that assumes a different unit-commitment schedule for each spot market scenario.

In this manner, the problems that a generation company participating in a spot market faces in the short term (one day to a week) can be addressed with two types of closely related mathematical models. On the one hand, the problem of deciding an offering strategy for each particular market mechanism can be tackled as a two-stage stochastic program using the formulation suggested in 5.2.2. On the other hand, the problem of deciding the startup and shutdown of the thermal units and of assigning the hydro resources that should be used each day can be addressed with a weekly stochastic unit-commitment model structured as a sequence of two-stage programs.

5.3 Analysis of the problem structure

According to the ideas presented in the previous section, we formulate the problems faced by a generation company in the spot market as a sequence of large-scale mixed linear-integer two-stage stochastic programs. As indicated in [Geoffrion '70], large-scale mathematical programs are characterized not only by their size, but also by their structure. Almost always, a large-scale program has a distinctive structure that can be exploited in order to obtain numerical solutions. Solution strategies have already been proposed in the literature for the most common structures, so the first step to solve a large-scale program is to determine whether it belongs to one of these standard types (it is not infrequent that a given problem falls simultaneously into two or more of these general categories). In this section, we analyze two structural aspects of the mathematical programs formulated in this chapter: the presence of complicating variables and the presence of complicating constraints. Both aspects are explored with the aim of identifying a suitable solution strategy. The analysis is valid both for the two-stage program and for the weekly stochastic unit-commitment model.

5.3.1 Complicating variables

A certain problem includes *complicating variables* if the problem that is obtained after fixing the value of certain variables is significantly easier to solve. In each of our problems, binary variables can be considered as complicating variables, given that when their value is fixed each problem turns out to be a linear program.

Binary variables appear in our spot market model to guarantee that the revenue functions of each market mechanism, m, are correctly evaluated (u_{jnk}^{m}) and to ensure that the resulting offer curves are strictly increasing ($x_{nkk'}^{m}$). We also use binary variables to model the exercise of the options included in the company's portfolio (u_{nk}^{o}). Finally, the on/off state of each thermal unit t in each scenario k and each hour n is also represented with a binary variable (u_{nk}^{t}). Figure 5.9 shows the matrix structure of the mathematical programs proposed in this thesis, highlighting the presence of binary variables.

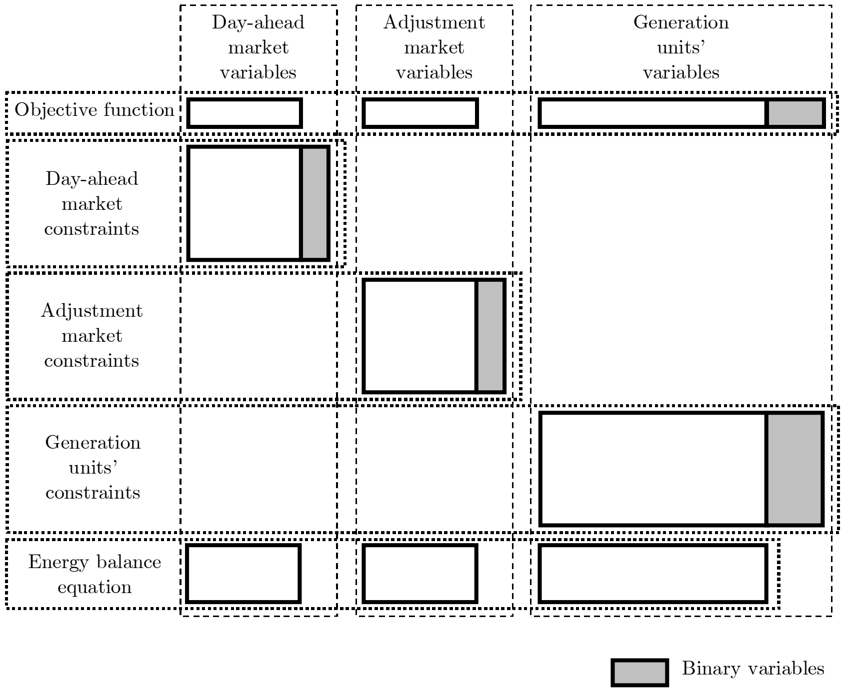

Figure 5.9. The matrix structure of the mathematical programs.

In [Benders '62], a partitioning procedure is proposed that exploits the presence of complicating variables. It separates the problem into two parts. The first part comprises the complicating variables, whereas the second part is obtained by fixing the complicating variables in the original problem and must have a convex structure. It can be proved that the first part, together with additional information about the second part in the form of a finite number of linear constraints, constitutes a problem whose solution is identical to that of the original problem. Benders suggested an algorithm that sequentially obtains these linear constraints. In every iteration, new values are given to the complicating variables by solving a problem constituted by the first part of the problem and the linear constraints already obtained (*master problem*). These new values of the complicating variables yield a new version of the second part of the problem (*subproblem*), which is then solved to derive a new linear constraint. After a finite number of iterations, the linear constraints obtained condense enough information about the subproblem to guarantee that the solution of the master problem coincides with that of the original problem.

A more detailed review of Benders' algorithm is performed in a subsequent section, in which we explain the application of this decomposition technique to the two-stage program that addresses the problem of developing optimal offers for a certain market mechanism. We justify that the binary variables corresponding to the second-stage decisions can be omitted under certain mild assumptions. This yields the matrix structure depicted in Figure 5.10. As can be seen, Benders' decomposition not only benefits from the presence of complicating variables, but also adapts well to the two-stage structure of the problem.

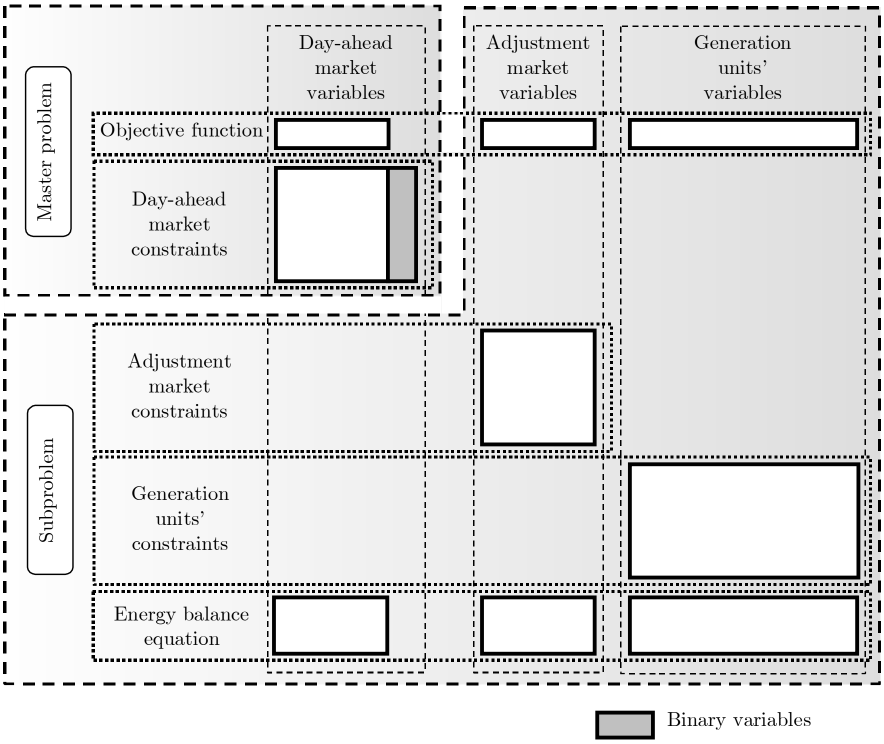

Figure 5.10. Benders' decomposition exploits the presence of complicating variables.

5.3.2 Complicating constraints

A large-scale program is said to have *complicating constraints* if by relaxing a number of its constraints a new problem is obtained that is significantly easier to solve. Complicating constraints are also frequently referred to as *coupling constraints* because when they are relaxed the original large-scale problem naturally decomposes into a set of smaller problems. In the mathematical programs developed in this thesis, the main set of complicating constraints corresponds to the energy balance equation. This equation establishes a link between the decisions taken by the company in the different market mechanisms and the schedule of the company's generating units in each hour n and each scenario k, as shown in Figure 5.9. If the complicating constraints are relaxed, the problem naturally decomposes into the following smaller subproblems:

i) One subproblem for each hourly auction of the spot market, including decisions relative to the exercise of options.

ii) One subproblem for each generating unit for the whole time scope of the model.

The effect of relaxing the energy balance equation is illustrated in Figure 5.11.

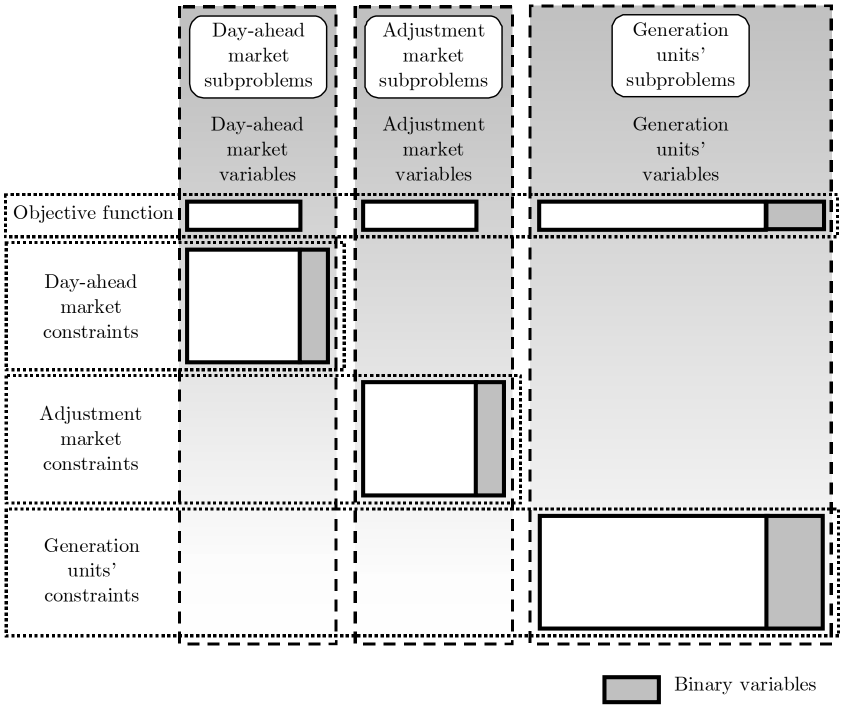

Figure 5.11. The relaxation of complicating constraints yields smaller subproblems.

Needless to say, the solution provided by the problem or problems that result when one or several constraints of the original problem are eliminated does not coincide with the solution of the original problem. However, there are solution strategies that are particularly suitable for problems with complicating constraints. Specifically, dual decomposition methods, such as Lagrangian relaxation, have been extensively used to deal with the traditional weekly unit-commitment problem, in which the complicating constraints are those guaranteeing that the generation units cover the demand for energy and reserve. Unlike Benders' decomposition, Lagrangian relaxation does not imply special requirements with respect to the presence of binary variables. This makes it particularly suitable to address the weekly stochastic unit-commitment problem. A detailed description of how Lagrangian relaxation can be applied to address both the two-stage program and the weekly stochastic unit-commitment problem is provided in subsequent sections.

5.4 A Benders' decomposition approach to solve the two-stage program

A variety of authors have revisited Benders' approach from different perspectives, thus increasing the relevance and applicability of this technique. Benders proposed his decomposition algorithm to address large-scale mathematical programs that included complicating variables [Benders '62].

In [Van Slyke '69], Benders' algorithm was "reinvented" to tackle the L-shaped linear programs that typically appear in optimal control theory and stochastic programming (hence the name "L-shaped method") This method has become a standard way of addressing two-stage linear recourse problems [Birge '97]. In [Pereira '91], a nested version of Benders' decomposition was applied to the multi-stage stochastic hydrothermal coordination problem under the name "stochastic dual dynamic programming".

In this section, we begin by providing a brief and intuitive description of Benders' algorithm based on the previous references. The use of this technique to solve each of the abovementioned two-stage problems requires the adoption of a number of assumptions that are justified in detail. In this manner, a solution strategy based on Benders' method is finally presented that can be used to obtain optimal offers for a generation company operating in an electricity spot market.

5.4.1 Benders' decomposition algorithm

Consider the following non-linear two-stage program, TSP:

$$
\text{(TSP)} \quad
\begin{aligned}
\max_{\boldsymbol{x},\boldsymbol{y}} \quad & f(\boldsymbol{x}) + \boldsymbol{c}^{\mathrm{t}}\boldsymbol{y} \\
\text{s.t.} \quad & \boldsymbol{x} \in \mathrm{X}, \\
& A_1\boldsymbol{x} + B_1\boldsymbol{y} = \boldsymbol{b}_1, \\
& A_2\boldsymbol{x} + B_2\boldsymbol{y} \le \boldsymbol{b}_2, \\
& B_3\boldsymbol{y} = \boldsymbol{b}_3, \\
& B_4\boldsymbol{y} \le \boldsymbol{b}_4,
\end{aligned}
\qquad (5.9)
$$

where $f(\bullet)$ is a function that can be non-linear and non-concave and X is a set that can be non-convex (e.g. it can be defined as a number of integrality constraints affecting the vector of variables $\boldsymbol{x}$). From the perspective of [Benders '62], $\boldsymbol{x}$ comprises the complicating variables (e.g. integer or binary variables). According to [Van Slyke '69], $\boldsymbol{x}$ represents the vector of first-stage decisions that must be taken before the realization of a number of uncertain factors, while $\boldsymbol{y}$ refers to the second-stage decisions that are taken as recourse actions once the value of those uncertain factors is known.

Problem TSP can be reformulated as follows:

$$
\text{(TSP')} \quad
\begin{aligned}
\max_{\boldsymbol{x}} \quad & f(\boldsymbol{x}) + \theta(\boldsymbol{x}) \\
\text{s.t.} \quad & \boldsymbol{x} \in \mathrm{X},
\end{aligned}
\qquad (5.10)
$$

where $\theta(\boldsymbol{x})$ is a function that can be evaluated solving the following linear program, SP:

$$
\text{(SP)} \quad
\begin{aligned}
\theta(\boldsymbol{x}) = \max_{\boldsymbol{y}} \quad & \boldsymbol{c}^{\mathrm{t}}\boldsymbol{y} \\
\text{s.t.} \quad & B_1\boldsymbol{y} = \boldsymbol{b}_1 - A_1\boldsymbol{x}, \\
& B_2\boldsymbol{y} \le \boldsymbol{b}_2 - A_2\boldsymbol{x}, \\
& B_3\boldsymbol{y} = \boldsymbol{b}_3, \\
& B_4\boldsymbol{y} \le \boldsymbol{b}_4.
\end{aligned}
\qquad (5.11)
$$

In the context of two-stage stochastic programming, $\theta(\boldsymbol{x})$ is usually referred to as *recourse function* and evaluates the benefit obtained due to the recourse actions, $\boldsymbol{y}$, that can be taken based on the first stage decisions, $\boldsymbol{x}$, and after the realization of uncertainty. The recourse function is concave because SP is a linear program. If the algebraic expression of the recourse function were known in advance, problem TSP would be reduced to solving problem TSP'. Sorrowfully, the only general method to evaluate the recourse function is to solve problem SP, which is referred to as *subproblem.* Benders' algorithm is based on the approximation of the recourse function by means of an outer linearization. This is done by iteratively solving problem SP and the following problem, which is an approximation of TSP':

$$\text{(MP)}\quad \begin{aligned} \underset{\boldsymbol{x}}{\text{Max}} \quad & f(\boldsymbol{x}) + \tilde{\theta}(\boldsymbol{x}) \\ \text{s.t.} \quad & \boldsymbol{x} \in \text{X}, \end{aligned} \tag{5.12}$$

where $\tilde{\theta}(\boldsymbol{x})$ is an outer linearization of the recourse function $\theta(\boldsymbol{x})$. Problem MP is usually referred to as *master problem.* After a finite number of iterations, the outer linearization obtained for the recourse function is good enough to guarantee that the solution of MP coincides with that of TSP.

In order to explain how Benders' algorithm constructs the outer linearization that approximates the recourse function in the master problem, we divide the discussion into three parts. The first part refers to the linear constraints that are added to the master problem in order to avoid first-stage decisions for which no feasible recourse action exists. The second part is dedicated to the linear constraints that approximate the recourse function. Finally, the third part presents the organization of the algorithm.

5.4.1.1 Feasibility [Van Slyke '69]

Assume a certain value for the vector of first-stage decisions, $\overline{\boldsymbol{x}}$. In order to evaluate the value of the recourse function, $\theta(\overline{\boldsymbol{x}})$, problem SP must be solved. However, there is no guarantee that a feasible solution can be obtained for problem SP given the value of the first-stage decisions, $\overline{\boldsymbol{x}}$. In order to determine whether a feasible recourse action exists, the following problem can be solved:

$$\text{(SPF)}\quad \begin{aligned} \varphi(\overline{\boldsymbol{x}}) = \underset{\substack{\boldsymbol{y},\\ \boldsymbol{v}_1^+, \boldsymbol{v}_1^-, \boldsymbol{v}_2^-}}{\text{Min}} \quad & \boldsymbol{e}^{\text{t}}\boldsymbol{v}_1^+ + \boldsymbol{e}^{\text{t}}\boldsymbol{v}_1^- + \boldsymbol{e}^{\text{t}}\boldsymbol{v}_2^- \\ \text{s.t.} \quad & B_1\boldsymbol{y} + \boldsymbol{v}_1^+ - \boldsymbol{v}_1^- = \boldsymbol{b}_1 - A_1\overline{\boldsymbol{x}}, \\ & B_2\boldsymbol{y} - \boldsymbol{v}_2^- \leq \boldsymbol{b}_2 - A_2\overline{\boldsymbol{x}}, \\ & B_3\boldsymbol{y} = \boldsymbol{b}_3, \\ & B_4\boldsymbol{y} \leq \boldsymbol{b}_4, \\ & \boldsymbol{v}_1^+ \geq \boldsymbol{0}, \boldsymbol{v}_1^- \geq \boldsymbol{0}, \boldsymbol{v}_2^- \geq \boldsymbol{0}. \end{aligned} \tag{5.13}$$

where $\boldsymbol{e}$ is a vector of 1's. We can interpret $\boldsymbol{v}_1^+$, $\boldsymbol{v}_1^-$ and $\boldsymbol{v}_2^-$ as three vectors of slack variables that measure the infeasibilities produced by the vector of first-stage decisions, $\overline{\boldsymbol{x}}$. These infeasibilities are evaluated in the objective function, $\varphi(\overline{\boldsymbol{x}})$. Hence, given the first-stage decisions, $\overline{\boldsymbol{x}}$, a feasible second-stage decision exists if and only if the solution of problem SPF ,$\varphi(\overline{\boldsymbol{x}})$, is equal to zero. On the contrary, $\varphi(\overline{\boldsymbol{x}}) > 0$ implies that no feasible recourse actions can be taken. In that case, a constraint must be added to problem MP in order to avoid the first-stage decision $\overline{\boldsymbol{x}}$.

To obtain the formulation of such a constraint let us consider the dual problem of problem SPF:

$$
\text{(DSPF)}\qquad
\begin{aligned}
\varphi(\overline{\boldsymbol{x}}) = \underset{\substack{\boldsymbol{\sigma}_1,\boldsymbol{\sigma}_2,\\ \boldsymbol{\sigma}_3,\boldsymbol{\sigma}_4}}{\text{Max}} \quad & {\boldsymbol{\sigma}_1}^{\mathrm{t}}\left[\boldsymbol{b}_1 - A_1\overline{\boldsymbol{x}}\right] + {\boldsymbol{\sigma}_2}^{\mathrm{t}}\left[\boldsymbol{b}_2 - A_2\overline{\boldsymbol{x}}\right] + {\boldsymbol{\sigma}_3}^{\mathrm{t}}\boldsymbol{b}_3 + {\boldsymbol{\sigma}_4}^{\mathrm{t}}\boldsymbol{b}_4 \\
\text{s.t.} \quad & {\boldsymbol{\sigma}_1}^{\mathrm{t}}B_1 + {\boldsymbol{\sigma}_2}^{\mathrm{t}}B_2 + {\boldsymbol{\sigma}_3}^{\mathrm{t}}B_3 + {\boldsymbol{\sigma}_4}^{\mathrm{t}}B_4 = \boldsymbol{0}, \\
& -\boldsymbol{e} \le \boldsymbol{\sigma}_1 \le \boldsymbol{e}, \\
& -\boldsymbol{e} \le \boldsymbol{\sigma}_2 \le \boldsymbol{0}, \\
& \boldsymbol{\sigma}_4 \le \boldsymbol{0}.
\end{aligned}
\qquad (5.14)
$$

Given the vector of first-stage decisions, $\overline{\boldsymbol{x}}$, a feasible recourse action can be taken if and only if the solution of DSPF, $\varphi(\overline{\boldsymbol{x}})$, is equal to zero. On the contrary, $\varphi(\overline{\boldsymbol{x}}) > 0$ implies that no feasible second-stage decision exists. The constraint that must be added to problem MP in order to avoid the solution $\overline{\boldsymbol{x}}$ in subsequent iterations is:

$$
{\boldsymbol{\sigma}_1}^{*\mathrm{t}}\left[\boldsymbol{b}_1 - A_1\boldsymbol{x}\right] + {\boldsymbol{\sigma}_2}^{*\mathrm{t}}\left[\boldsymbol{b}_2 - A_2\boldsymbol{x}\right] + {\boldsymbol{\sigma}_3}^{*\mathrm{t}}\boldsymbol{b}_3 + {\boldsymbol{\sigma}_4}^{*\mathrm{t}}\boldsymbol{b}_4 \le 0, \qquad (5.15)
$$

where ${\boldsymbol{\sigma}_1}^*$, ${\boldsymbol{\sigma}_2}^*$, ${\boldsymbol{\sigma}_3}^*$ and ${\boldsymbol{\sigma}_4}^*$ are the vectors of dual variables obtained in the current iteration after solving problem SPF. This constraint is usually known as *feasibility cut* [Van Slyke '69; Birge '97]. After some algebra a more convenient expression for such a constraint can be obtained:

$$
\begin{aligned}
& {\boldsymbol{\sigma}_1}^{*\mathrm{t}}\left[\boldsymbol{b}_1 - A_1\boldsymbol{x}\right] + {\boldsymbol{\sigma}_2}^{*\mathrm{t}}\left[\boldsymbol{b}_2 - A_2\boldsymbol{x}\right] + {\boldsymbol{\sigma}_3}^{*\mathrm{t}}\boldsymbol{b}_3 + {\boldsymbol{\sigma}_4}^{*\mathrm{t}}\boldsymbol{b}_4 \le 0 \Rightarrow \\
& \Rightarrow {\boldsymbol{\sigma}_1}^{*\mathrm{t}}\left[\boldsymbol{b}_1 - A_1\overline{\boldsymbol{x}} + A_1\overline{\boldsymbol{x}} - A_1\boldsymbol{x}\right] + {\boldsymbol{\sigma}_2}^{*\mathrm{t}}\left[\boldsymbol{b}_2 - A_2\overline{\boldsymbol{x}} + A_2\overline{\boldsymbol{x}} - A_2\boldsymbol{x}\right] + \\
& \quad + {\boldsymbol{\sigma}_3}^{*\mathrm{t}}\boldsymbol{b}_3 + {\boldsymbol{\sigma}_4}^{*\mathrm{t}}\boldsymbol{b}_4 \le 0 \Rightarrow \\
& \Rightarrow \varphi(\overline{\boldsymbol{x}}) + {\boldsymbol{\sigma}_1}^{*\mathrm{t}}A_1(\overline{\boldsymbol{x}} - \boldsymbol{x}) + {\boldsymbol{\sigma}_2}^{*\mathrm{t}}A_2(\overline{\boldsymbol{x}} - \boldsymbol{x}) \le 0
\end{aligned}
\qquad (5.16)
$$

5.4.1.2 *Optimality [Pereira '91]*

Let us assume now that, given the vector of first-stage decisions, $\overline{\boldsymbol{x}}$, the solution obtained for problem SPF is $\varphi(\overline{\boldsymbol{x}}) = 0$, i.e., a feasible recourse action exists. In order to determine the best possible recourse action and its benefit, the following version of the subproblem has to be solved:

$$
\text{(SP)}\qquad
\begin{aligned}
\theta(\overline{\boldsymbol{x}}) = \underset{\boldsymbol{y}}{\text{Max}} \quad & \boldsymbol{c}^{\mathrm{t}}\boldsymbol{y} \\
\text{s.t.} \quad & B_1\boldsymbol{y} = \boldsymbol{b}_1 - A_1\overline{\boldsymbol{x}}, \\
& B_2\boldsymbol{y} \le \boldsymbol{b}_2 - A_2\overline{\boldsymbol{x}}, \\
& B_3\boldsymbol{y} = \boldsymbol{b}_3, \\
& B_4\boldsymbol{y} \le \boldsymbol{b}_4.
\end{aligned}
\qquad (5.17)
$$

Let us consider the dual problem of problem SP:

$$
\text{(DSP)}\qquad
\begin{aligned}
\theta(\overline{\boldsymbol{x}}) = \underset{\substack{\boldsymbol{\lambda}_1,\boldsymbol{\lambda}_2,\\ \boldsymbol{\lambda}_3,\boldsymbol{\lambda}_4}}{\text{Min}} \quad & {\boldsymbol{\lambda}_1}^{\mathrm{t}}\left[\boldsymbol{b}_1 - A_1\overline{\boldsymbol{x}}\right] + {\boldsymbol{\lambda}_2}^{\mathrm{t}}\left[\boldsymbol{b}_2 - A_2\overline{\boldsymbol{x}}\right] + {\boldsymbol{\lambda}_3}^{\mathrm{t}}\boldsymbol{b}_3 + {\boldsymbol{\lambda}_4}^{\mathrm{t}}\boldsymbol{b}_4 \\
\text{s.t.} \quad & {\boldsymbol{\lambda}_1}^{\mathrm{t}}B_1 + {\boldsymbol{\lambda}_2}^{\mathrm{t}}B_2 + {\boldsymbol{\lambda}_3}^{\mathrm{t}}B_3 + {\boldsymbol{\lambda}_4}^{\mathrm{t}}B_4 = \boldsymbol{0}, \\
& \boldsymbol{\lambda}_2 \ge \boldsymbol{0}, \\
& \boldsymbol{\lambda}_4 \ge \boldsymbol{0}.
\end{aligned}
\qquad (5.18)
$$

As can be seen, $\overline{\boldsymbol{x}}$ only appears in the objective function of DSP. Hence, the set of possible solutions for DSP does not depend on the value of the first-stage decisions. Let $\boldsymbol{\lambda} = (\boldsymbol{\lambda}_1, \boldsymbol{\lambda}_2, \boldsymbol{\lambda}_3, \boldsymbol{\lambda}_4)$ be the vector of variables of problem DSP and let $\Lambda = \left\{\boldsymbol{\lambda}^1, \boldsymbol{\lambda}^2, \ldots, \boldsymbol{\lambda}^l, \ldots, \boldsymbol{\lambda}^L\right\}$ be the set of vertices of the constraint set. The solution of problem DSP is obtained in (at least) one of these vertices. As a result, problem DSP can be solved by enumeration:

$$
\text{(DSP')} \qquad
\begin{aligned}
\theta(\overline{\boldsymbol{x}}) = \text{Max} \quad & \theta \\
\text{s.t.} \quad & \theta \leq \boldsymbol{\lambda}_1^{1\,\text{t}}\left[\boldsymbol{b}_1 - A_1\overline{\boldsymbol{x}}\right] + \boldsymbol{\lambda}_2^{1\,\text{t}}\left[\boldsymbol{b}_2 - A_2\overline{\boldsymbol{x}}\right] + \boldsymbol{\lambda}_3^{1\,\text{t}}\boldsymbol{b}_3 + \boldsymbol{\lambda}_4^{1\,\text{t}}\boldsymbol{b}_4, \\
& \vdots \\
& \theta \leq \boldsymbol{\lambda}_1^{l\,\text{t}}\left[\boldsymbol{b}_1 - A_1\overline{\boldsymbol{x}}\right] + \boldsymbol{\lambda}_2^{l\,\text{t}}\left[\boldsymbol{b}_2 - A_2\overline{\boldsymbol{x}}\right] + \boldsymbol{\lambda}_3^{l\,\text{t}}\boldsymbol{b}_3 + \boldsymbol{\lambda}_4^{l\,\text{t}}\boldsymbol{b}_4, \\
& \vdots \\
& \theta \leq \boldsymbol{\lambda}_1^{L\,\text{t}}\left[\boldsymbol{b}_1 - A_1\overline{\boldsymbol{x}}\right] + \boldsymbol{\lambda}_2^{L\,\text{t}}\left[\boldsymbol{b}_2 - A_2\overline{\boldsymbol{x}}\right] + \boldsymbol{\lambda}_3^{L\,\text{t}}\boldsymbol{b}_3 + \boldsymbol{\lambda}_4^{L\,\text{t}}\boldsymbol{b}_4.
\end{aligned}
\qquad (5.19)
$$

According to this expression, the recourse function $\theta(\boldsymbol{x})$ is a piecewise linear function of the first-stage decisions, $\boldsymbol{x}$. This function is defined by as many hyperplanes as vertices has the set of possible solutions of the dual problem, DSP. In general, it is not easy to calculate all the vertices of the set Λ. Hence, an alternative is to obtain these vertices sequentially. Given a new first-stage decision, problem SP can be solved to obtain the dual variables associated to its constraints. These dual variables define one of the vertices of the set of possible solutions of the dual problem.

Assume that the vector of dual variables $\boldsymbol{\lambda}^l$ corresponding to the l-th vertex of problem DSP has been obtained by solving problem SP for a given value of the first-stage decisions, $\overline{\boldsymbol{x}}$. The following condition holds:

$$
\theta(\overline{\boldsymbol{x}}) = \boldsymbol{\lambda}_1^{l\,\text{t}}\left[\boldsymbol{b}_1 - A_1\overline{\boldsymbol{x}}\right] + \boldsymbol{\lambda}_2^{l\,\text{t}}\left[\boldsymbol{b}_2 - A_2\overline{\boldsymbol{x}}\right] + \boldsymbol{\lambda}_3^{l\,\text{t}}\boldsymbol{b}_3 + \boldsymbol{\lambda}_4^{l\,\text{t}}\boldsymbol{b}_4. \qquad (5.20)
$$

Consequently, the recourse function must observe this constraint for any value of $\boldsymbol{x}$:

$$
\theta(\boldsymbol{x}) \leq \boldsymbol{\lambda}_1^{l\,\text{t}}\left[\boldsymbol{b}_1 - A_1\boldsymbol{x}\right] + \boldsymbol{\lambda}_2^{l\,\text{t}}\left[\boldsymbol{b}_2 - A_2\boldsymbol{x}\right] + \boldsymbol{\lambda}_3^{l\,\text{t}}\boldsymbol{b}_3 + \boldsymbol{\lambda}_4^{l\,\text{t}}\boldsymbol{b}_4. \qquad (5.21)
$$

which is typically known as an *optimality cut*. Such a constraint must be included in the master problem in order to improve the approximation of the recourse function. After a sufficient number of optimality cuts has been generated, the approximation of the recourse function is good enough to guarantee that the solution provided by the master problem coincides with that of the original problem.

A more convenient expression for the optimality cuts is obtained after some algebra:

$$
\begin{aligned}
& \theta(\boldsymbol{x}) \leq \boldsymbol{\lambda}_1^{l\,\text{t}}\left[\boldsymbol{b}_1 - A_1\boldsymbol{x}\right] + \boldsymbol{\lambda}_2^{l\,\text{t}}\left[\boldsymbol{b}_2 - A_2\boldsymbol{x}\right] + \boldsymbol{\lambda}_3^{l\,\text{t}}\boldsymbol{b}_3 + \boldsymbol{\lambda}_4^{l\,\text{t}}\boldsymbol{b}_4 \Rightarrow \\
& \Rightarrow \theta(\boldsymbol{x}) \leq \boldsymbol{\lambda}_1^{l\,\text{t}}\left[\boldsymbol{b}_1 - A_1\overline{\boldsymbol{x}} + A_1\overline{\boldsymbol{x}} + A_1\boldsymbol{x}\right] + \boldsymbol{\lambda}_2^{l\,\text{t}}\left[\boldsymbol{b}_2 - A_2\overline{\boldsymbol{x}} + A_2\overline{\boldsymbol{x}} - A_2\boldsymbol{x}\right] \\
& \qquad + \boldsymbol{\lambda}_3^{l\,\text{t}}\boldsymbol{b}_3 + \boldsymbol{\lambda}_4^{l\,\text{t}}\boldsymbol{b}_4 \Rightarrow \\
& \Rightarrow \theta(\boldsymbol{x}) \leq \theta(\overline{\boldsymbol{x}}) + \boldsymbol{\lambda}_1^{l\,\text{t}}A_1(\overline{\boldsymbol{x}} - \boldsymbol{x}) + \boldsymbol{\lambda}_2^{l\,\text{t}}A_2(\overline{\boldsymbol{x}} - \boldsymbol{x}),
\end{aligned}
\qquad (5.22)
$$

which is the expression of the region delimited by a hyperplane that is tangent to the recourse function in $\overline{\boldsymbol{x}}$.

5.4.1.3 The algorithm

Both optimality and feasibility cuts are linear constraints conceived to improve the piecewise linear representation of the subproblem that is included in the master problem. This permits the selection of better first-stage decisions without having to solve the full two-stage problem, TSP. Benders suggested the following algorithm to iteratively obtain these linear constraints:

Step 1: Set $\nu = 0, \mathrm{V}^{\mathrm{O}} = \varnothing, \mathrm{V}^{\mathrm{F}} = \varnothing$.

Step 2: Set $\nu = \nu + 1$. If $\nu = 1$ set the value of θ to zero. Solve problem MP formulated as:

$$
(\mathrm{MP}) \quad \begin{aligned}
\max_{\boldsymbol{x},\theta} \quad & f(\boldsymbol{x}) + \theta \\
\text{s.t.} \quad & \boldsymbol{x} \in \mathrm{X}, \\
& \theta \le \theta\left(\boldsymbol{x}^l\right) + \boldsymbol{\lambda}_1^{l\,\mathrm{t}} A_1 \left(\boldsymbol{x}^l - \boldsymbol{x}\right) + \boldsymbol{\lambda}_2^{l\,\mathrm{t}} A_2 \left(\boldsymbol{x}^l - \boldsymbol{x}\right), \quad l \in \mathrm{V}^{\mathrm{O}}, \\
& \varphi\left(\boldsymbol{x}^l\right) + \boldsymbol{\sigma}_1^{l\,\mathrm{t}} A_1 \left(\boldsymbol{x}^l - \boldsymbol{x}\right) + \boldsymbol{\sigma}_2^{l\,\mathrm{t}} A_2 \left(\boldsymbol{x}^l - \boldsymbol{x}\right) \le 0, \quad l \in \mathrm{V}^{\mathrm{F}}.
\end{aligned} \tag{5.23}
$$

A new value for the first-stage decisions, $\boldsymbol{x}^\nu$, is obtained.

Step 3: Solve problem SPF to obtain $\varphi\left(\boldsymbol{x}^\nu\right)$ and the dual variables $\boldsymbol{\sigma}_1^\nu$, $\boldsymbol{\sigma}_2^\nu$, $\boldsymbol{\sigma}_3^\nu$ and $\boldsymbol{\sigma}_4^\nu$. If $\varphi\left(\boldsymbol{x}^\nu\right) > 0$, add the constraint $\varphi\left(\boldsymbol{x}^\nu\right) + \boldsymbol{\sigma}_1^{\nu\,\mathrm{t}} A_1 \left(\boldsymbol{x}^\nu - \boldsymbol{x}\right) + \boldsymbol{\sigma}_2^{\nu\,\mathrm{t}} A_2 \left(\boldsymbol{x}^\nu - \boldsymbol{x}\right) \le 0$ to problem MP, add ν to the set V^{F} and go to step 2.

Step 4: Solve problem SP to obtain $\theta\left(\boldsymbol{x}^\nu\right)$ and the dual variables $\boldsymbol{\lambda}_1^\nu$, $\boldsymbol{\lambda}_2^\nu$, $\boldsymbol{\lambda}_3^\nu$ and $\boldsymbol{\lambda}_4^\nu$. Add the constraint $\theta \le \theta\left(\boldsymbol{x}^\nu\right) + \boldsymbol{\lambda}_1^{\nu\,\mathrm{t}} A_1 \left(\boldsymbol{x}^\nu - \boldsymbol{x}\right) + \boldsymbol{\lambda}_2^{\nu\,\mathrm{t}} A_2 \left(\boldsymbol{x}^\nu - \boldsymbol{x}\right)$ to problem MP. Add ν to the set V^{O}.

Step 5: Check for convergence. If false, go to step 2.

5.4.2 Application to the two-stage program

In this section we show how Benders' algorithm can be applied to the two-stage program defined in 5.2.2. The objective is to formulate the three problems required to use this algorithm: the master problem, the subproblem and the feasibility problem.

5.4.2.1 The master problem

The master problem comprises the decisions referring to the strategy of the company in the first market mechanism. Given that the long-term positions of the company are indexed to the price of this market mechanism, their influence is also evaluated in the master problem. Additionally, optimality cuts and feasibility cuts must be incorporated. The following formulation results:

$$
\text{(MP)}\quad
\begin{aligned}
&\underset{\substack{q^1_{nk},\theta,\\ u^o_{nk},p^o_{nk}}}{\text{Max}} \sum_{k\in\mathrm{K}} \pi_k \Bigg[\sum_{n\in\mathrm{N}} p^1_{nk}\left(q^1_{nk}\right)\cdot q^1_{nk} + \sigma^1_n \frac{q^1_{nk}}{Q^1_{nk}} \\
&\qquad + \sum_{c\in\mathrm{C^P}} \sum_{n\in\mathrm{N}} p^c_n q^c_n + \sum_{c\in\mathrm{C^D}} \sum_{n\in\mathrm{N}} \left(p^c_n - p^1_{nk}\left(q^1_{nk}\right)\right)\cdot q^c_n \\
&\qquad + \sum_{o\in\mathrm{O^P}} \sum_{n\in\mathrm{N}} u^o_{nk}\cdot p^o_n \cdot q^o_n + \sum_{o\in\mathrm{O^F}} \sum_{n\in\mathrm{N}} \left(p^o_{nk} - p^1_{nk}\left(q^1_{nk}\right)\right)\cdot q^o_n \Bigg] + \theta \\
&\text{s.t.}\quad \varphi^l + \sum_{k\in\mathrm{K}} \sum_{n\in\mathrm{N}} \sigma_{nk}{}^l \left(q_{nk} - q_{nk}{}^l\right) \le 0, \quad l\in\mathrm{V^F} \\
&\qquad \theta - \theta^l - \sum_{k\in\mathrm{K}} \sum_{n\in\mathrm{N}} \lambda_{nk}{}^l \left(q_{nk} - q_{nk}{}^l\right) \le 0, \quad l\in\mathrm{V^O}, \\
&\qquad \left\{q^1_{nk}, k\in\mathrm{K}\right\} \in \mathrm{Q}^1_n, \quad n\in\mathrm{N}, \\
&\qquad p^o_{nk} \in \mathrm{P}^o_{nk}, \quad o\in\mathrm{O^F}, n\in\mathrm{N}, k\in\mathrm{K}, \\
&\qquad u^o_{nk} \in \mathrm{U}^o_{nk}, \quad o\in\mathrm{O^P}, n\in\mathrm{N}, k\in\mathrm{K}.
\end{aligned}
\tag{5.24}
$$

5.4.2.2 The subproblem

The subproblem evaluates the revenues obtained by the company in the second market mechanism as well as the cost of its final generation schedule. For this reason, the subproblem includes the energy balance equation, in which the first-stage decisions are taken into account.

$$
\text{(SP)}\quad
\begin{aligned}
&\theta^\nu = \underset{\substack{q^2_{nk},q^t_{nk},\\ q^h_{nk},b^h_{nk}}}{\text{Max}} \sum_{k\in\mathrm{K}} \pi_k \left[\sum_{n\in\mathrm{N}} p^2_{nk}\left(q^2_{nk}\right)\cdot q^2_{nk} + \sigma^2_n \frac{q^2_{nk}}{Q^2_{nk}} - \sum_{t\in\mathrm{T}} \sum_{n\in\mathrm{N}} c^t_{nk}\left(q^t_{nk}, y^t_{nk}, u^t_{nk}\right) \right] \\
&\text{s.t.} \sum_{t\in\mathrm{T}} q^t_{nk} + \sum_{h\in\mathrm{H}} q^h_{nk} - b^h_{nk} - q^2_{nk} = \\
&\qquad = q^1_{nk}{}^\nu + \sum_{c\in\mathrm{C^P}} q^c_n + \sum_{o\in\mathrm{O^P}} u^o_{nk}{}^\nu q^o_n, \quad n\in\mathrm{N}, k\in\mathrm{K}, \\
&\qquad q^2_{nk} \in \mathrm{Q}^2_{nk}, \quad n\in\mathrm{N}, k\in\mathrm{K}, \\
&\qquad \left\{q^t_{nk}, y^t_{nk}, u^t_{nk}, n\in\mathrm{N}\right\} \in \mathrm{Q}^t, \quad t\in\mathrm{T}, k\in\mathrm{K}, \\
&\qquad \left\{q^h_{nk}, b^h_{nk}, n\in\mathrm{N}\right\} \in \mathrm{Q}^h, \quad h\in\mathrm{H}, k\in\mathrm{K}.
\end{aligned}
\tag{5.25}
$$

This formulation of the subproblem includes two types of binary variables. On the one hand, a set of binary variables guarantees that the revenue functions of the second market mechanism are correctly evaluated. On the other hand, binary variables are used to represent the commitment state of thermal units. This is an important drawback, given that Benders' algorithm requires the subproblem to be convex. We will show that a convex formulation can be obtained for the subproblem under certain mild assumptions.

5.4.2.3 The feasibility problem

The feasibility problem is similar to the subproblem, except for the slack variables and the objective function. The comments made about the presence of binary variables in the subproblem are relevant also in this case.

$$\varphi^{\nu} = \underset{v_{nk}^{+}, v_{nk}^{-}}{\text{Min}} \quad \sum_{k} \sum_{n} v_{nk}^{+} + v_{nk}^{-}$$

$$\text{(FSP)} \quad \begin{aligned} &\text{s.t.} \sum_{t \in \mathrm{T}} q_{nk}^{t} + \sum_{h \in \mathrm{H}} q_{nk}^{h} - b_{nk}^{h} - q_{nk}^{2} + v_{nk}^{+} - v_{nk}^{-} = \\ &\quad = q_{nk}^{1}{}^{\nu} + \sum_{c \in \mathrm{C}^{\mathrm{P}}} q_{n}^{c} + \sum_{o \in \mathrm{O}^{\mathrm{P}}} u_{nk}^{o}{}^{\nu} q_{n}^{o}, \quad n \in \mathrm{N}, k \in \mathrm{K}, \\ &\quad q_{nk}^{2} \in \mathrm{Q}_{nk}^{2}, \quad n \in \mathrm{N}, k \in \mathrm{K}, \\ &\quad \left\{ q_{nk}^{t}, y_{nk}^{t}, u_{nk}^{t}, n \in \mathrm{N} \right\} \in \mathrm{Q}^{t}, \quad t \in \mathrm{T},, k \in \mathrm{K} \\ &\quad \left\{ q_{nk}^{h}, b_{nk}^{h}, n \in \mathrm{N} \right\} \in \mathrm{Q}^{h}, \quad h \in \mathrm{H}, k \in \mathrm{K}. \end{aligned} \tag{5.26}$$

5.4.3 The presence of binary variables in the subproblem

As has been indicated, although Benders' decomposition adapts well to the two-stage structure of the problem, the presence of binary variables in the second stage invalidates this approach. Hence, it is worth reconsidering whether these binary variables are essential for a correct representation of the problem.

In particular, let us consider the two-stage problem corresponding to the day-ahead market illustrated in Figure 5.1. Binary variables are required in the second stage in order to represent the following aspects:

i) According to the simplifications introduced in section 5.2, each possible day-ahead market realization is assumed to be accompanied by a unique possible adjustment market outcome. The correct evaluation of the company's revenues in the adjustment market scenarios requires binary variables to account for possible non-concavities.

ii) Thermal units' commitment states in each hour and each scenario are represented by means of binary variables.

We can justify the elimination of these binary variables under certain mild assumptions.

On the one hand, if the revenue functions used to represent the adjustment market were forced to be concave, no binary variables would be required. In particular, if the residual demand curves faced by the generation company in the adjustment market were assumed to be linear (instead of piecewise linear) the revenue function would turn out to be parabolic and concave, as illustrated in Figure 5.12. In the numerical example solved in section 5.2.3 the residual demand functions were assumed to be linear, as can be seen in Figure 5.7. This linear approximation of the second-stage residual demand curves eliminates the need for binary variables.

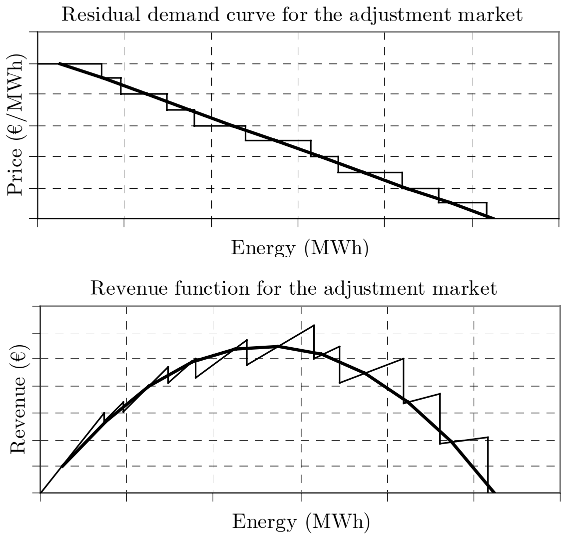

Figure 5.12. Linearization of the adjustment-market residual demand curve.

On the other hand, it has been indicated that thermal units' commitment decisions are usually taken with a one-week perspective rather than with a time scope of only one day. Hence, when deciding an offering strategy for the market mechanisms of a specific spot market session, the binary variables that represent the commitment state of the thermal units should be considered as input data. After the uncertainty of that spot market session has been unveiled, the weekly unit-commitment schedule can be updated to account for any unexpected change in the market conditions.

In conclusion, by adopting certain reasonable assumptions, second-stage binary variables can be eliminated. This permits the use of Benders' decomposition to obtain numerical solutions for these problems, as is shown in the next numerical example.

5.4.4 Numerical example

The numerical example solved in section 5.2.3 can be solved using Benders decomposition, given that neither the adjustment market nor the generating units require binary variables for their representation.

The suggested Benders' algorithm has been implemented in GAMS language. The master problem for this numerical example has 1297 equations (in the first iteration) and 764 variables, 474 of which are binary. The subproblem has 11953 equations and 13009 variables. The feasibility problem has 11953 equations and 13105 variables, due to the presence of slack variables (two slack variables per hour and per scenario make a total of 96). Table 5.1 compares the attributes of these problems with those of the full problem solved in section 5.2.3.

	Full problem	Master problem	Subproblem
Obj. function	1	1	1
Equations	13248	1296	11952
Variables	13770	763	13008
Binary variables	474	474	0

Table 5.1. Attributes of the problems that result from Benders' decomposition.

88 iterations of Benders' algorithm are required to solve this two-stage program. Table 5.2 shows the evolution of the algorithm during the first fifteen iterations. It can be seen that in six iterations the offers decided by the master problem for the day-ahead market yield an infeasible subproblem. The feasibility cuts generated by the algorithm eliminate all infeasibilities after the seventh iteration.

Iteration	Master	Subproblem	Infeasibility	Obj. function
IT1	35.44591		-0.954437	
IT2	35.441504		-0.4785	
IT3	35.438018	-8.780276		26.657742
IT4	39.140633		-1.868153	
IT5	39.134687		-0.242236	
IT6	39.133743		-0.324065	
IT7	39.132685		-0.000092	
IT8	39.132684	-1.509484		20.469494
IT9	32.955894	-3.811005		30.023687
IT10	30.216797	-3.619947		29.961037
IT11	30.201207	-3.851254		30.065004
IT12	30.168791	-3.64353		30.036351
IT13	30.150968	-4.149272		30.023969
IT14	30.14834	-4.058577		30.053459
IT15	30.146364	-4.008166		30.083179

Table 5.2. Evolution of Benders' algorithm during the first 15 iterations.

The right column of Table 5.2 evaluates the objective function of the original problem, given the values obtained for the variables in each iteration of Benders' algorithm. It can be seen that the master problem always provides an upper bound for the objective function of the original problem. When the objective function of the master problem coincides with the value obtained for the objective function of the original problem the algorithm stops. Figure 5.13 compares the evolution of the objective function of the original problem with that of the master problem.

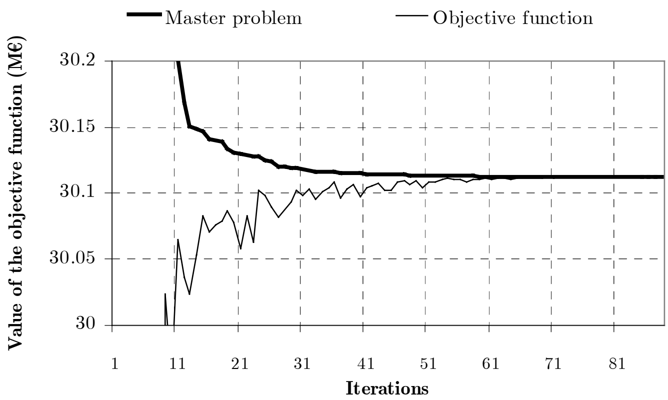

Figure 5.13. Evolution of the objective function in Benders' algorithm.

The evolution of the objective function of the subproblem is depicted in Figure 5.14. It can be seen that, while the objective function of the master problem evolves monotonically, the objective function of the subproblem oscillates around its final value.

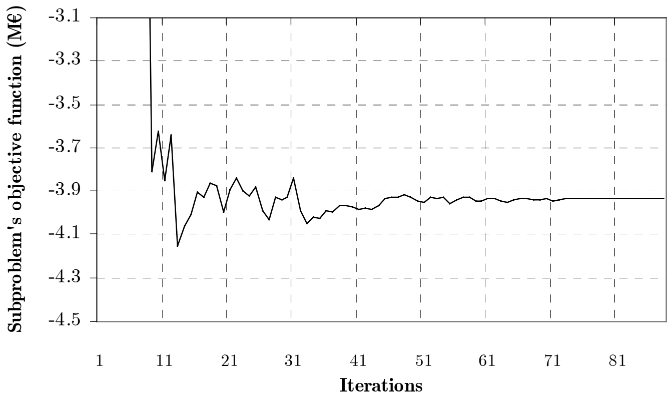

Figure 5.14. Evolution of the subproblem's objective function in Benders' algorithm.

An interesting subproduct of Benders' decomposition is the value of the dual variables corresponding to the constraints that establish a link between the master problem and the subproblem. In this numerical case, the link between the master problem and the subproblem is created by the energy balance equation. Figure 5.15 represents the values obtained for the corresponding dual variables. These values can be interpreted as the marginal costs of the offering decisions adopted for each of the hourly auctions of the day-ahead market. These costs are due to the energy that the company must produce to comply with its obligations, but also to the recourse actions taken in the adjustment market.

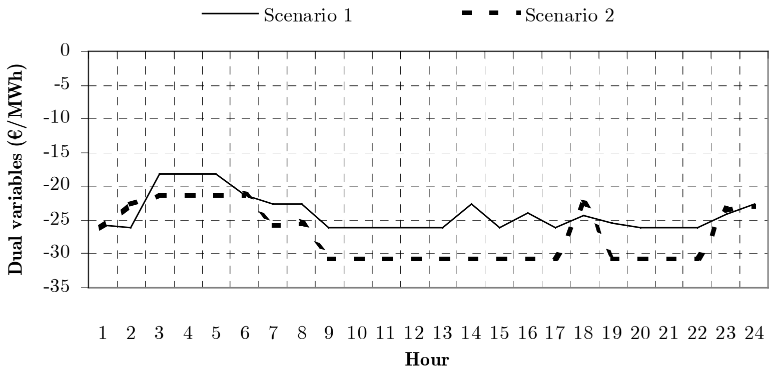

Figure 5.15. Dual variables corresponding to the energy balance equation.

In this particular case, Benders' decomposition cannot compete with the speed of CPLEX solving the original two-stage program (267 s. were required to solve the problem using Benders' algorithm, whereas CPLEX solves the original problem in only only 6 s. as indicated in section 5.2.3). However, the solution of this small example illustrates the possibilities of Benders' algorithm. Moreover, given that the solution obtained with both approaches coincides, we have reasons to assume that our implementation of Benders' algorithm is correct.

5.4.5 Separability of the master problem

Even though Benders' decomposition separates the two-stage program into a master problem and a subproblem, in practice, the master problem is still a large-scale problem. This is a great disadvantage, given that the master problem must be solved in each iteration of Benders' algorithm. In particular, it is the presence of binary variables what complicates the resolution of the master problem (for real-size problems, a commercial optimizer, such as CPLEX 7.5, is frequently unable to find an integer solution). In this section we show that by separating the master problem into a number of smaller problems a feasible integer solution can be easily obtained.

Assume that, when solving problem FSP in iteration l, the infeasibility is discriminated by hours and scenarios according to the equation:

$$\varphi_{nk}{}^{l} = v_{nk}^{+}{}^{l} + v_{nk}^{-}{}^{l}. \tag{5.27}$$

This yields the following expression for the feasibility cuts:

$$\sum_{k\in \mathrm{K}} \sum_{n\in \mathrm{N}} \varphi_{nk}{}^{l} + \sigma_{nk}{}^{l}\left(q_{nk} - q_{nk}{}^{l}\right) \le 0, \quad l \in \mathrm{V^F}. \tag{5.28}$$

It is interesting to notice that if each feasibility cut is separated into a set of hourly cuts, a more restrictive condition is imposed to the master problem:

$$\begin{aligned} &\sum_{k\in \mathrm{K}} \varphi_{nk}{}^{l} + \sigma_{nk}{}^{l}\left(q_{nk} - q_{nk}{}^{l}\right) \le 0, \quad n\in \mathrm{N}, l \in \mathrm{V^F} \Rightarrow \\ &\Rightarrow \sum_{k\in \mathrm{K}} \sum_{n\in \mathrm{N}} \varphi_{nk}{}^{l} + \sigma_{nk}{}^{l}\left(q_{nk} - q_{nk}{}^{l}\right) \le 0, \quad l \in \mathrm{V^F}. \end{aligned} \tag{5.29}$$

Similarly, we can separate the recourse function into as many contributions as scenarios and hours are being considered:

$$\theta^{l} = \sum_{k\in \mathrm{K}} \sum_{n\in \mathrm{N}} \theta_{nk}{}^{l} \tag{5.30}$$

$$\theta = \sum_{k\in \mathrm{K}} \sum_{n\in \mathrm{N}} \theta_{nk} \tag{5.31}$$

The following expression is then obtained for the optimality cuts:

$$\sum_{k\in \mathrm{K}} \sum_{n\in \mathrm{N}} \theta_{nk} - \theta_{nk}{}^{l} - \lambda_{nk}{}^{l}\left(q_{nk} - q_{nk}{}^{l}\right) \le 0, \quad l \in \mathrm{V^O}. \tag{5.32}$$

As before, if each optimality cut is separated into hourly cuts, the resulting constraints are more restrictive:

$$\begin{aligned} &\sum_{k\in \mathrm{K}} \theta_{nk} - \theta_{nk}{}^{l} - \lambda_{nk}{}^{l}\left(q_{nk} - q_{nk}{}^{l}\right) \le 0, \quad n\in \mathrm{N}, l \in \mathrm{V^O} \Rightarrow \\ &\Rightarrow \theta - \theta^{l} - \sum_{k\in \mathrm{K}} \sum_{n\in \mathrm{N}} \lambda_{nk}{}^{l}\left(q_{nk} - q_{nk}{}^{l}\right) \le 0, \quad l \in \mathrm{V^O}. \end{aligned} \tag{5.33}$$

Consequently, if the following set of hourly problems is solved, the solution obtained constitutes a feasible solution for problem MP. However, it is likely that the solution obtained will not be optimal for problem MP, given that more restrictive conditions are being imposed:

$$
\begin{aligned}
&\underset{\substack{q_{nk}^1,\theta_{nk},\\ u_{nk}^o,p_{nk}^o}}{\text{Max}} \sum_{k\in\mathrm{K}} \pi_k \Bigg[p_{nk}^1\left(q_{nk}^1\right)\cdot q_{nk}^1 + \sigma_n^1 \frac{q_{nk}^1}{Q_{nk}^1} \\
&\qquad + \sum_{c\in\mathrm{C}^\mathrm{P}} \sum_{n\in\mathrm{N}} p_n^c q_n^c + \sum_{c\in\mathrm{C}^\mathrm{D}} \sum_{n\in\mathrm{N}} \left(p_n^c - p_{nk}^1\left(q_{nk}^1\right)\right)\cdot q_n^c \\
&\qquad + \sum_{o\in\mathrm{O}^\mathrm{P}} \sum_{n\in\mathrm{N}} u_{nk}^o \cdot p_n^o \cdot q_n^o + \sum_{o\in\mathrm{O}^\mathrm{F}} \sum_{n\in\mathrm{N}} \left(p_{nk}^o - p_{nk}^1\left(q_{nk}^1\right)\right)\cdot q_n^o \Bigg] \\
&\quad + \sum_{k\in\mathrm{K}} \theta_{nk} \\
(\mathrm{MP}_n)\ \text{s.t.}\ & \sum_{k\in\mathrm{K}} {\varphi_{nk}}^l + {\sigma_{nk}}^l\left(q_{nk} - {q_{nk}}^l\right) \le 0, \quad l\in\mathrm{V}^\mathrm{F} \qquad (5.34)\\
& \sum_{k\in\mathrm{K}} \theta_{kn} - {\theta_{nk}}^l - {\lambda_{nk}}^l\left(q_{nk} - {q_{nk}}^l\right) \le 0, \quad l\in\mathrm{V}^\mathrm{O}, \\
& \left\{q_{nk}^1, k\in\mathrm{K}\right\} \in \mathrm{Q}_n^1, \\
& p_{nk}^o \in \mathrm{P}_{nk}^o, \quad o\in\mathrm{O}^\mathrm{F}, k\in\mathrm{K}, \\
& u_{nk}^o \in \mathrm{U}_{nk}^o, \quad o\in\mathrm{O}^\mathrm{P}, k\in\mathrm{K}.
\end{aligned}
$$

In practice, the separation of the optimality cuts into hourly cuts only produces a loss of optimality in the solution of the master problem. In contrast, the separation of the feasibility cuts into hourly cuts may cause the master problem to be infeasible. This suggests avoiding the use of feasibility cuts. A simple approach is to adopt the following formulation for the subproblem, in which the energy balance constraint has been relaxed with the introduction of two slack variables. The use of these slack variables is penalized in the objective function.

$$
\begin{aligned}
&\theta^\nu = \underset{\substack{q_{nk}^2,q_{nk}^t,\\ q_{nk}^h,b_{nk}^h}}{\text{Max}} \sum_{k\in\mathrm{K}} \pi_k \Bigg[\sum_{n\in\mathrm{N}} p_{nk}^2\left(q_{nk}^2\right)\cdot q_{nk}^2 + \sigma_n^2 \frac{q_{nk}^2}{Q_{nk}^2} - \gamma\left(v_{nk}^+ + v_{nk}^-\right) \\
&\qquad - \sum_{t\in\mathrm{T}} \sum_{n\in\mathrm{N}} c_{nk}^t\left(q_{nk}^t, y_{nk}^t, u_{nk}^t\right) \Bigg] \\
&\text{s.t.} \sum_{t\in\mathrm{T}} q_{nk}^t + \sum_{h\in\mathrm{H}} q_{nk}^h - b_{nk}^h - q_{nk}^2 + v_{nk}^+ - v_{nk}^- = \\
(\mathrm{SP'})\quad & = {q_{nk}^1}^\nu + \sum_{c\in\mathrm{C}^\mathrm{P}} q_n^c + \sum_{o\in\mathrm{O}^\mathrm{P}} {u_{nk}^o}^\nu q_n^o, \quad n\in\mathrm{N}, k\in\mathrm{K}, \qquad (5.35)\\
& q_{nk}^2 \in \mathrm{Q}_{nk}^2, \quad n\in\mathrm{N}, k\in\mathrm{K}, \\
& \left\{q_{nk}^t, y_{nk}^t, u_{nk}^t, n\in\mathrm{N}\right\} \in \mathrm{Q}^t, \quad t\in\mathrm{T}, k\in\mathrm{K} \\
& \left\{q_{nk}^h, b_{nk}^h, n\in\mathrm{N}\right\} \in \mathrm{Q}^h, \quad h\in\mathrm{H}, k\in\mathrm{K}.
\end{aligned}
$$

where γ is a penalizing coefficient.

In conclusion, two simplifications facilitate the numerical resolution of the two-stage problem with Benders' decomposition. On the one hand, slack variables can be introduced into the subproblem in order to avoid the use of feasibility cuts. On the other hand, the master problem can be separated into a set of hourly problems so as to obtain a feasible initial solution. This reduces the effort required by the branch and bound algorithm to search for the optimal solution of the master problem. The idea of separating the master problem into hourly problems is an original contribution of this thesis.

5.4.6 Numerical example

Let us solve the two-stage problem proposed in section 5.2.3 making use of the two techniques presented in the previous section. In first place we introduce slack variables into the subproblem to avoid infeasibilities. The value given to the penalizing coefficient is $\gamma = 1$ M€/GWh.

In this case the algorithm converges in 83 iterations and 190 s., i.e., five iterations less than with the standard Benders' decomposition and a reduction of almost 30 % in the execution time. This suggests that introducing slack variables, although being less elegant, is an effective strategy. Table 5.3 shows the evolution of the algorithm during the first twenty iterations. As can be seen, the presence of slack variables prevents the infeasibility of the subproblem.

Iteration	Master	Subproblem	Total
IT1	35.44591	-9.315766	26.130144
IT2	35.056869	-6.201827	28.855042
IT3	32.578624	-2.869617	29.709007
IT4	30.672184	-4.425674	29.973338
IT5	30.432099	-3.679893	30.044709
IT6	30.177776	-4.141789	30.029525
IT7	30.165454	-4.10417	30.001585
IT8	30.159941	-4.108961	30.061101
IT9	30.155745	-3.916013	30.061797
IT10	30.147818	-3.932412	30.087031
IT11	30.13999	-4.038316	30.080739
IT12	30.137461	-4.02202	30.083379
IT13	30.136322	-3.994939	30.074618
IT14	30.136129	-3.968863	30.069797
IT15	30.135819	-4.04927	30.077806
IT16	30.135631	-3.879207	30.088226
IT17	30.128751	-4.037095	30.058724
IT18	30.128484	-3.908015	30.069284
IT19	30.128232	-3.967812	30.08499
IT20	30.128086	-3.933326	30.102728

Table 5.3. Evolution of Benders' algorithm with slack variables in the subproblem.

We now solve the problem separating the master problem into twenty-four hourly problems. As indicated, this is likely to yield a suboptimal solution. However, it is a quick manner of obtaining a feasible solution. In large-size problems, such as the ones addressed in chapter 6, the branch-and-bound algorithm implemented in CPLEX is unable to find a feasible solution for the original problem due to the size of the branch-and-bound binary tree. This strategy turns out to be very useful in such situations.

Table 5.4 presents the evolution of the algorithm when the master problem is separated into hourly problems. The best feasible solution is obtained in seven iterations. It can be seen that, given the more restrictive nature of the master problem, the objective function of the master problem intersects with the objective function of the original problem, which is something that never happens in the standard Benders' algorithm. The solution obtained, 30.059836 M€, is suboptimal by only 0.174%. This suggests that the use of this technique, at least in a first stage of the solution process, is highly recommendable.

Iteration	Master	Subproblem	Total
IT1	35.44591	-9.315766	26.130144
IT2	32.965207	-3.883533	29.263376
IT3	30.345947	-4.132596	29.850918
IT4	30.081702	-3.864007	30.055584
IT5	29.94533	-3.863922	30.022586
IT6	29.849264	-3.849681	30.059836
IT7	29.824912	-3.849681	30.059836

Table 5.4. Evolution of Benders' algorithm when the master problem is separated into hourly problems.

5.4.7 Advantages and disadvantages of Benders' decomposition for the two-stage program

The application of Benders' decomposition to solve the two-stage program presents several advantages that we list below:

- It decomposes the original problem into two smaller problems that are easier to solve.
- It adapts well to the two-stage structure of the problem.
- Even if the optimal solution cannot be obtained, it is easy to reach a "reasonably good" feasible solution by separating the master problem into a number of hourly problems.

Nevertheless, this technique has also two shortcomings that must be clearly identified:

- The master problem, although having a smaller size than the original problem, is still large enough to complicate the search for the solution.
- No binary variables can be included in the subproblem. This limits the accuracy with which the second market mechanism can be modeled. It also prevents the use of this method to solve the stochastic weekly unit-commitment problem.

In conclusion, Benders' decomposition seems a suitable approach to solve the abovementioned two-stage program, as long as no binary variables are included in the subproblem. The adequacy of this technique is further illustrated in chapter 6, where more realistic numerical examples are solved.

5.5 A Lagrangian relaxation approach to solve the two-stage program

A large-scale problem with complicating constraints is particularly amenable for a *dual decomposition* solution strategy. Dual decomposition methods are based on the *dualization* of complicating constraints. In this manner, a Lagrangian dual problem is obtained whose objective function can be iteratively approximated by means of an *outer linearization*. In each iteration, one of the cutting planes that constitute the outer linearization of the dual objective function is obtained by solving a set of smaller problems. After a finite number of iterations, the approximation provided by this outer linearization is good enough to yield the solution of the dual problem. This approach is commonly known as Lagrangian relaxation [Geoffrion '74] and is frequently applied to large-scale stochastic programs [Nowak '99]. In this context, two strategies are typically adopted: *scenario decomposition* (dualization of the non-anticipativity constraints that establish a link between different scenarios) or *component decomposition* (dualization of the coupling constraints that link elements of the problem that would otherwise be independent, e.g. generation units) [Römisch '01].

If the original (primal) problem is convex, its solution coincides with that of the dual problem. Hence, the solution provided by the dual problem satisfies the complicating constraints that have been dualized. On the contrary, if the primal problem is non-convex (e.g. it includes integrality conditions) it cannot be guaranteed that the solution provided by the dual problem will be primal feasible, i.e., in general, the dual solution will not meet the dualized complicating constraints. If the primal problem is a maximization problem, the dual solution constitutes an upper bound of the primal solution and the difference between both solutions is the so-called *duality gap*.

The two-stage problem formulated in this chapter includes binary variables, which means that its dualization will not provide a primal feasible solution. Nevertheless, Lagrangian relaxation, when applied to non-convex problems, is usually accompanied by heuristic postprocessing methods that introduce minor modifications in the dual solution in order to obtain a similar solution that is primal feasible. This approach has been extensively used in recent years to address the unit-commitment problem [Sheblé '94, Sen '98].

This section shows how Lagrangian relaxation can be applied to the two-stage program that represents the problem of developing an offering strategy for one of the spot market mechanisms (the matrix structure of one of this mathematical programs is depicted in Figure 5.16, to illustrate the presence of complicating constraints). This section also serves as an introduction to the following section, in which Lagrangian relaxation is applied to the weekly stochastic unit-commitment problem. We begin by presenting some basic concepts of Lagrangian duality and by formulating the Lagrangian dual problem of the two-stage program. A solution method is then explained that is based on the outer linearization of the dual function.

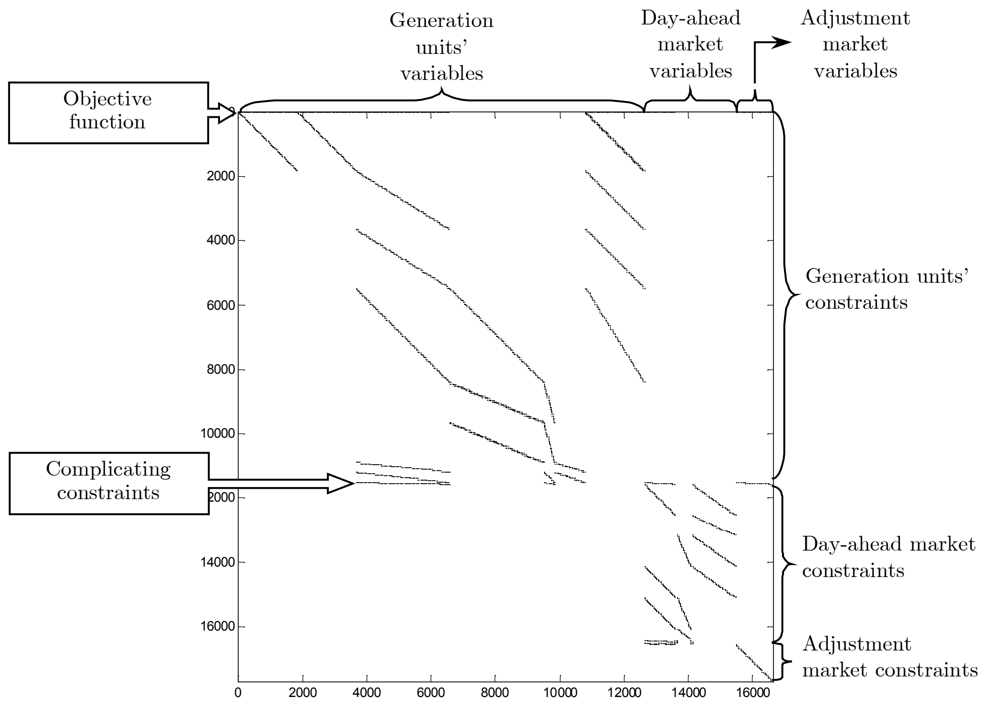

Figure 5.16. Matrix structure of one real two-stage program.

5.5.1 The Lagrangian dual problem

Let us consider the following general expression of a nonlinear programming problem P, which we will refer to as *primal problem* [Bazaraa '93]:

$$(\mathrm{P})\quad \begin{aligned} &\operatorname*{Max}_{\boldsymbol{x}} && f(\boldsymbol{x}) \\ &\text{s.t.} && \boldsymbol{h}(\boldsymbol{x}) = \boldsymbol{0}, \\ & && \boldsymbol{g}(\boldsymbol{x}) \le \boldsymbol{0}, \\ & && \boldsymbol{x} \in \mathrm{X} \end{aligned} \tag{5.36}$$

One of the most popular definitions of the concept of *dual problem* is the one based on Lagrangian duality. The *Lagrangian dual problem* D of problem P is defined as follows:

$$(\mathrm{D})\quad \begin{aligned} &\operatorname*{Min}_{\boldsymbol{\lambda},\boldsymbol{\mu}} && w(\boldsymbol{\lambda},\boldsymbol{\mu}) \\ &\text{s.t.} && \boldsymbol{\mu} \ge \boldsymbol{0}. \end{aligned} \tag{5.37}$$

where $w(\boldsymbol{\lambda},\boldsymbol{\mu}) \triangleq \operatorname*{Sup}_{\boldsymbol{x}}\left\{ f(\boldsymbol{x}) - \boldsymbol{\lambda}^{\mathrm{t}}\boldsymbol{h}(\boldsymbol{x}) - \boldsymbol{\mu}^{\mathrm{t}}\boldsymbol{g}(\boldsymbol{x}), \boldsymbol{x} \in \mathrm{X} \right\}$ is the *Lagrangian dual function* and $\boldsymbol{\lambda}$ and $\boldsymbol{\mu}$ are called *Lagrange multipliers.*

It is worth noticing that several Lagrangian dual problems can be defined for the same primal problem, depending on which constraints are included in $\boldsymbol{g}(\boldsymbol{x}) \le \boldsymbol{0}$ and $\boldsymbol{h}(\boldsymbol{x}) = \boldsymbol{0}$ and which constraints are comprised in the definition of the set X.

In general, the algebraic expression of the Lagrangian dual function is not known. Hence, in order to solve D, an iterative procedure must be followed. In each iteration, given the current value of the Lagrange multipliers $\boldsymbol{\lambda}$ and $\boldsymbol{\mu}$, the corresponding value of the dual function, $w(\boldsymbol{\lambda},\boldsymbol{\mu})$, is obtained by solving the maximization problem included in its definition:

$$\text{(S)}\quad \begin{array}{ll} \operatorname*{Max}_{\boldsymbol{x}} & f(\boldsymbol{x}) - \boldsymbol{\lambda}^{\mathrm{t}}\boldsymbol{h}(\boldsymbol{x}) - \boldsymbol{\mu}^{\mathrm{t}}\boldsymbol{g}(\boldsymbol{x}), \\ \text{s.t.} & \boldsymbol{x} \in \mathrm{X}. \end{array} \tag{5.38}$$

Problem S is usually referred to as the *Lagrangian dual subproblem* and its structure is very similar to that of the primal problem P. Problem S can be seen as a relaxation of problem P due to the inclusion of constraints $\boldsymbol{h}(\boldsymbol{x}) = \boldsymbol{0}$ and $\boldsymbol{g}(\boldsymbol{x}) \le \boldsymbol{0}$ in its objective function, which explains the term *Lagrangian relaxation* (LR). The objective function of problem S is the *Lagrange function* of problem P, whose definition is:

$$\mathcal{L}(\boldsymbol{x},\boldsymbol{\lambda},\boldsymbol{\mu}) \triangleq f(\boldsymbol{x}) - \boldsymbol{\lambda}^{\mathrm{t}}\boldsymbol{h}(\boldsymbol{x}) - \boldsymbol{\mu}^{\mathrm{t}}\boldsymbol{g}(\boldsymbol{x}), \tag{5.39}$$

The effort required to evaluate the dual function $w(\boldsymbol{\lambda},\boldsymbol{\mu})$ depends on the difficulty of solving problem S. If constraints $\boldsymbol{h}(\boldsymbol{x}) = \boldsymbol{0}$ and $\boldsymbol{g}(\boldsymbol{x}) \le \boldsymbol{0}$ constitute the complicating constraints of problem P, problem S turns out to be significantly easier to solve than problem P. Typically, due to the relaxation (or dualization) of these complicating constraints, problem S naturally decomposes into a number of smaller problems.

An interesting geometric interpretation of the dual problem can be found in [Bazaraa '93 pp. 201-205], where the concept of duality gap is also explained from a geometric perspective.

We now apply the previous definitions to the structure of the two-stage program formulated in section 5.2.2. The primal problem P is expressed in the following form:

$$\operatorname*{Max}_{\substack{q^1_{nk},q^2_{nk},\\ u^o_{nk},p^o_{nk}\\ q^t_{nk},y^t_{nk},u^t_{nk},\\ q^h_{nk},b^h_{nk}}} \quad f \equiv \left\{ \begin{array}{l} \displaystyle\sum_{k\in\mathrm{K}} \pi_k \left[\sum_{n\in\mathrm{N}} p^1_{nk}\left(q^1_{nk}\right)\cdot q^1_{nk} + \sigma^1_n \frac{q^1_{nk}}{Q^1_{nk}} + \sum_{n\in\mathrm{N}} p^2_{nk}\left(q^2_{nk}\right)\cdot q^2_{nk} + \sigma^2_n \frac{q^2_{nk}}{Q^2_{nk}} \right. \\ \displaystyle\qquad + \sum_{c\in\mathrm{C^D}} \sum_{n\in\mathrm{N}} \left(p^c_n - p^1_{nk}\left(q^1_{nk}\right)\right)\cdot q^c_n \\ \displaystyle\qquad + \sum_{o\in\mathrm{O^P}} \sum_{n\in\mathrm{N}} u^o_{nk}\cdot p^o_n \cdot q^o_n + \sum_{o\in\mathrm{O^F}} \sum_{n\in\mathrm{N}} \left(p^o_{nk} - p^1_{nk}\left(q^1_{nk}\right)\right)\cdot q^o_n \\ \displaystyle\qquad \left. - \sum_{t\in\mathrm{T}} \sum_{n\in\mathrm{N}} c^t_{nk}\left(q^t_{nk}, y^t_{nk}, u^t_{nk}\right) \right] \end{array} \right.$$

s.t.

$$\boldsymbol{h} = \boldsymbol{0} \equiv \left\{ q^1_{nk} + q^2_{nk} + \sum_{c\in\mathrm{C^P}} q^c_n + \sum_{o\in\mathrm{O^P}} u^o_{nk} q^o_n - \sum_{t\in\mathrm{T}} q^t_{nk} - \left[\sum_{h\in\mathrm{H}} q^h_{nk} - b^h_{nk}\right] = 0, n\in\mathrm{N}, k\in\mathrm{K}, \right.$$

$$\boldsymbol{x} \in \mathrm{X} \equiv \left\{ \begin{array}{ll} \left\{q^1_{nk}, k\in\mathrm{K}\right\} \in \mathrm{Q}^1_n, & n\in\mathrm{N}, \\ q^2_{nk} \in \mathrm{Q}^2_{nk}, & n\in\mathrm{N}, k\in\mathrm{K}, \\ \left\{q^t_{nk}, y^t_{nk}, u^t_{nk}, n\in\mathrm{N}\right\} \in \mathrm{Q}^t, & t\in\mathrm{T}, k\in\mathrm{K}, \\ \left\{q^h_{nk}, b^h_{nk}, n\in\mathrm{N}\right\} \in \mathrm{Q}^h, & h\in\mathrm{H}, k\in\mathrm{K}, \\ p^o_{nk} \in \mathrm{P}^o_{nk}, & o\in\mathrm{O^F}, n\in\mathrm{N}, k\in\mathrm{K}, \\ u^o_{nk} \in \mathrm{U}^o_{nk}, & o\in\mathrm{O^P}, n\in\mathrm{N}, k\in\mathrm{K}. \end{array} \right.$$

As can be seen, problem P has been formulated with the objective of dualizing the energy balance equation, which has been identified as a set of complicating constraints. The Lagrange function corresponding to this problem is:

$$\begin{aligned}
&\mathcal{L}\left(q_{nk}^{1}, q_{nk}^{2}, u_{nk}^{o}, p_{nk}^{o}, q_{nk}^{t}, y_{nk}^{t}, u_{nk}^{t}, q_{nk}^{h}, b_{nk}^{h}, \lambda_{nk}\right) = \\
&\sum_{k \in \mathrm{K}} \pi_k \left[\sum_{n \in \mathrm{N}} p_{nk}^{1}\left(q_{nk}^{1}\right) \cdot q_{nk}^{1} + \sigma_n^{1} \frac{q_{nk}^{1}}{Q_{nk}^{1}} + \sum_{n \in \mathrm{N}} p_{nk}^{2}\left(q_{nk}^{2}\right) \cdot q_{nk}^{2} + \sigma_n^{2} \frac{q_{nk}^{2}}{Q_{nk}^{2}} \right. \\
&\qquad + \sum_{c \in \mathrm{C}^{\mathrm{D}}} \sum_{n \in \mathrm{N}} \left(p_n^{c} - p_{nk}^{1}\left(q_{nk}^{1}\right)\right) \cdot q_n^{c} \\
&\qquad + \sum_{o \in \mathrm{O}^{\mathrm{P}}} \sum_{n \in \mathrm{N}} u_{nk}^{o} \cdot p_n^{o} \cdot q_n^{o} + \sum_{o \in \mathrm{O}^{\mathrm{F}}} \sum_{n \in \mathrm{N}} \left(p_{nk}^{o} - p_{nk}^{1}\left(q_{nk}^{1}\right)\right) \cdot q_n^{o} \\
&\qquad \left. - \sum_{t \in \mathrm{T}} \sum_{n \in \mathrm{N}} c_{nk}^{t}\left(q_{nk}^{t}, y_{nk}^{t}, u_{nk}^{t}\right) \right] \\
&- \sum_{k \in \mathrm{K}} \sum_{n \in \mathrm{N}} \lambda_{nk} \left[q_{nk}^{1} + q_{nk}^{2} + \sum_{c \in \mathrm{C}^{\mathrm{P}}} q_n^{c} + \sum_{o \in \mathrm{O}^{\mathrm{P}}} u_{nk}^{o} q_n^{o} - \sum_{t \in \mathrm{T}} q_{nk}^{t} - \left[\sum_{h \in \mathrm{H}} q_{nk}^{h} - b_{nk}^{h} \right] \right]
\end{aligned} \tag{5.40}$$

After some algebra, a more convenient formulation of the Lagrange function can be obtained:

$$\begin{aligned}
&\mathcal{L}\left(q_{nk}^{1}, q_{nk}^{2}, u_{nk}^{o}, p_{nk}^{o}, q_{nk}^{t}, y_{nk}^{t}, u_{nk}^{t}, q_{nk}^{h}, b_{nk}^{h}, \lambda_{nk}\right) = \\
&\sum_{k \in \mathrm{K}} \pi_k \left[\sum_{n \in \mathrm{N}} \left[p_{nk}^{1}\left(q_{nk}^{1}\right) + \frac{\sigma_n^{1}}{Q_{nk}^{1}} - \frac{\lambda_{nk}}{\pi_k} \right] \cdot q_{nk}^{1} + \sum_{n \in \mathrm{N}} \left[p_{nk}^{2}\left(q_{nk}^{2}\right) + \frac{\sigma_n^{2}}{Q_{nk}^{1}} - \frac{\lambda_{nk}}{\pi_k} \right] \cdot q_{nk}^{2} \right. \\
&\qquad + \sum_{c \in \mathrm{C}^{\mathrm{P}}} \sum_{n \in \mathrm{N}} \left(p_n^{c} - \frac{\lambda_{nk}}{\pi_k} \right) q_n^{c} + \sum_{c \in \mathrm{C}^{\mathrm{D}}} \sum_{n \in \mathrm{N}} \left(p_n^{c} - p_{nk}^{1}\left(q_{nk}^{1}\right)\right) \cdot q_n^{c} \\
&\qquad + \sum_{o \in \mathrm{O}^{\mathrm{P}}} \sum_{n \in \mathrm{N}} u_{nk}^{o} \cdot \left(p_n^{o} - \frac{\lambda_{nk}}{\pi_k} \right) \cdot q_n^{o} + \sum_{o \in \mathrm{O}^{\mathrm{F}}} \sum_{n \in \mathrm{N}} \left(p_{nk}^{o} - p_{nk}^{1}\left(q_{nk}^{1}\right)\right) \cdot q_n^{o} \\
&\qquad \left. + \sum_{t \in \mathrm{T}} \sum_{n \in \mathrm{N}} \frac{\lambda_{nk}}{\pi_k} q_{nk}^{t} - c_{nk}^{t}\left(q_{nk}^{t}, y_{nk}^{t}, u_{nk}^{t}\right) + \sum_{h \in \mathrm{H}} \sum_{n \in \mathrm{N}} \frac{\lambda_{nk}}{\pi_k} \left(q_{nk}^{h} - b_{nk}^{h}\right) \right].
\end{aligned} \tag{5.41}$$

A straightforward economic interpretation can be suggested for the coefficient $\dfrac{\lambda_{nk}}{\pi_k}$. It can be considered as the marginal cost associated to a local variation of the total quantity sold by the company in hour n and market scenario k. It can also be interpreted as the marginal revenue obtained by the company due to the total energy produced with its generation units in hour n and market scenario k. It should be noticed that, in the Lagrange function, the term that evaluates the influence of physical bilateral contracts turns out to be very similar to the term corresponding to CfDs. Certain similarities can also be spotted between the term corresponding to options that are physically settled and the term that refers to options that are financially settled.

In order to evaluate the dual function, $w(\boldsymbol{\lambda})$, for a given value of the Lagrange multipliers, $\boldsymbol{\lambda}^{\nu}$, the following Lagrangian dual subproblem has to be solved:

$$
\begin{aligned}
w\left(\lambda_{nk}{}^{\nu}\right) = \underset{\substack{q_{nk}^1, q_{nk}^2,\\ u_{nk}^o, p_{nk}^o\\ q_{nk}^t, y_{nk}^t, u_{nk}^t,\\ q_{nk}^h, b_{nk}^h}}{\text{Max}} \quad & \sum_{k\in \mathrm{K}} \pi_k \Bigg[\sum_{n\in \mathrm{N}} \left[p_{nk}^1\left(q_{nk}^1\right) + \frac{\sigma_n^1}{Q_{nk}^1} - \frac{\lambda_{nk}{}^{\nu}}{\pi_k} \right] \cdot q_{nk}^1 \\
& + \sum_{n\in \mathrm{N}} \left[p_{nk}^2\left(q_{nk}^2\right) + \frac{\sigma_n^2}{Q_{nk}^1} - \frac{\lambda_{nk}{}^{\nu}}{\pi_k} \right] \cdot q_{nk}^2 \\
& + \sum_{c\in \mathrm{C}^{\mathrm{P}}} \sum_{n\in \mathrm{N}} \left(p_n^c - \frac{\lambda_{nk}{}^{\nu}}{\pi_k} \right) q_n^c + \sum_{c\in \mathrm{C}^{\mathrm{D}}} \sum_{n\in \mathrm{N}} \left(p_n^c - p_{nk}^1\left(q_{nk}^1\right)\right) \cdot q_n^c \\
& + \sum_{o\in \mathrm{O}^{\mathrm{P}}} \sum_{n\in \mathrm{N}} u_{nk}^o \cdot \left(p_n^o - \frac{\lambda_{nk}{}^{\nu}}{\pi_k} \right) \cdot q_n^o + \sum_{o\in \mathrm{O}^{\mathrm{F}}} \sum_{n\in \mathrm{N}} \left(p_{nk}^o - p_{nk}^1\left(q_{nk}^1\right)\right) \cdot q_n^o \\
& + \sum_{t\in \mathrm{T}} \sum_{n\in \mathrm{N}} \frac{\lambda_{nk}{}^{\nu}}{\pi_k} q_{nk}^t - c_{nk}^t\left(q_{nk}^t, y_{nk}^t, u_{nk}^t\right) + \sum_{h\in \mathrm{H}} \sum_{n\in \mathrm{N}} \frac{\lambda_{nk}{}^{\nu}}{\pi_k} \left(q_{nk}^h - b_{nk}^h \right) \Bigg]
\end{aligned}
$$

$$
\text{s.t. } \boldsymbol{x} \in \mathrm{X} \equiv \begin{cases}
\left\{ q_{nk}^1, k \in \mathrm{K} \right\} \in \mathrm{Q}_n^1, & n \in \mathrm{N}, \\
\left\{ q_{nk}^2, k \in \mathrm{K} \right\} \in \mathrm{Q}_{nk}^2, & n \in \mathrm{N}, \\
\left\{ q_{nk}^t, y_{nk}^t, u_{nk}^t, n \in \mathrm{N} \right\} \in \mathrm{Q}^t, & t \in \mathrm{T}, k \in \mathrm{K}, \\
\left\{ q_{nk}^h, b_{nk}^h, n \in \mathrm{N} \right\} \in \mathrm{Q}^h, & h \in \mathrm{H}, k \in \mathrm{K}, \\
p_{nk}^o \in \mathrm{P}_{nk}^o, & o \in \mathrm{O}^{\mathrm{F}}, n \in \mathrm{N}, k \in \mathrm{K}, \\
u_{nk}^o \in \mathrm{U}_{nk}^o, & o \in \mathrm{O}^{\mathrm{P}}, n \in \mathrm{N}, k \in \mathrm{K}.
\end{cases}
$$

Due to the dualization of the energy balance equation, this Lagrangian dual subproblem naturally decomposes into a number of smaller subproblems. The first type of subproblem, $\mathrm{MS1}_n$, decides the offering strategy for the n-th hourly auction of the first market mechanism and evaluates the influence of the contracts and options that are indexed to the clearing price of that auction, given the current value of the Lagrange multipliers:

$$
(\mathrm{MS1}_n) \quad
\begin{aligned}
w_{\mathrm{MS1}_n}\left(\lambda_{nk}{}^{\nu}\right) = \underset{q_{nk}^1, p_{nk}^o}{\text{Max}} \sum_{k\in \mathrm{K}} \pi_k & \Bigg[\left[p_{nk}^1\left(q_{nk}^1\right) + \frac{\sigma_n^1}{Q_{nk}^1} - \frac{\lambda_{nk}{}^{\nu}}{\pi_k} \right] \cdot q_{nk}^1 \\
& + \sum_{c\in \mathrm{C}^{\mathrm{P}}} \left(p_n^c - \frac{\lambda_{nk}{}^{\nu}}{\pi_k} \right) q_n^c + \sum_{c\in \mathrm{C}^{\mathrm{D}}} \left(p_n^c - p_{nk}^1\left(q_{nk}^1\right)\right) \cdot q_n^c \\
& + \sum_{o\in \mathrm{O}^{\mathrm{F}}} \left(p_{nk}^o - p_{nk}^1\left(q_{nk}^1\right)\right) \cdot q_n^o \Bigg] \\
\text{s.t. } & \left\{ q_{nk}^1, k \in \mathrm{K} \right\} \in \mathrm{Q}_n^1, \\
& p_{nk}^o \in \mathrm{P}_{nk}^o, \quad o \in \mathrm{O}^{\mathrm{F}}, k \in \mathrm{K}.
\end{aligned}
$$

The second type of subproblem, $\mathrm{MS2}_{nk}$, selects the offering strategy for the n-th auction of the second market mechanism in each scenario k, given the value of the Lagrange multiplier:

$$(\text{MS2}_{nk}) \quad \begin{aligned} & w_{\text{MS2}_{nk}}\left(\lambda_{nk}{}^{\nu}\right) = \underset{q_{nk}^{2}}{\text{Max}}\ \pi_k \left[p_{nk}^{2}\left(q_{nk}^{2}\right) + \frac{\sigma_n^2}{Q_{nk}^{1}} - \frac{\lambda_{nk}{}^{\nu}}{\pi_k} \right] \cdot q_{nk}^{2} \\ & \text{s.t. } q_{nk}^{2} \in \mathrm{Q}_{nk}^{2}. \end{aligned}$$

The third type of subproblem decides the exercise of each of the options that are physically settled in each hour n and each scenario k, given the value of the Lagrange multipliers:

$$(\text{OPS}^{\,o}_{\ nk}) \quad \begin{aligned} & w_{\text{OPS}_{nk}^{o}}\left(\lambda_{nk}{}^{\nu}\right) = \underset{u_{nk}^{o}}{\text{Max}}\ \pi_k \left[\sum_{o \in \mathrm{O}^{\mathrm{P}}} \sum_{n \in \mathrm{N}} u_{nk}^{o} \cdot \left(p_n^{o} - \frac{\lambda_{nk}{}^{\nu}}{\pi_k} \right) \cdot q_n^{o} \right] \\ & \text{s.t. } u_{nk}^{o} \in \mathrm{U}_{nk}^{o}, o \in \mathrm{O}^{\mathrm{P}}. \end{aligned}$$

The fourth type of subproblem optimizes the schedule of each thermal unit t , given the value of the Lagrange multipliers:

$$(\text{GS}_t) \quad \begin{aligned} & w_{\text{GS}_t}\left(\lambda_{nk}{}^{\nu}\right) = \underset{q_{nk}^{t}, y_{nk}^{t}, u_{nk}^{t}}{\text{Max}} \sum_{k \in \mathrm{K}} \pi_k \sum_{n \in \mathrm{N}} \frac{\lambda_{nk}{}^{\nu}}{\pi_k} q_{nk}^{t} - c_{nk}^{t}\left(q_{nk}^{t}, y_{nk}^{t}, u_{nk}^{t}\right) \\ & \text{s.t. } \left\{ q_{nk}^{t}, y_{nk}^{t}, u_{nk}^{t}, n \in \mathrm{N} \right\} \in \mathrm{Q}^{t}, \quad k \in \mathrm{K}. \end{aligned}$$

Finally, the fifth type of subproblem optimizes the schedule of each hydro unit h , given the value of the Lagrange multipliers:

$$(\text{GS}_h) \quad \begin{aligned} & w_{\text{GS}_h}\left(\lambda_{nk}{}^{\nu}\right) = \underset{q_{nk}^{h}, b_{nk}^{h}}{\text{Max}} \sum_{k \in \mathrm{K}} \pi_k \sum_{h \in \mathrm{H}} \sum_{n \in \mathrm{N}} \frac{\lambda_{nk}{}^{\nu}}{\pi_k} \left(q_{nk}^{h} - b_{nk}^{h}\right) \\ & \text{s.t. } \left\{ q_{nk}^{h}, b_{nk}^{h}, n \in \mathrm{N} \right\} \in \mathrm{Q}^{h}, \quad k \in \mathrm{K}. \end{aligned}$$

Figure 5.17 illustrates the types of subproblems that Lagrangian relaxations yields when applied to the two-stage problem:

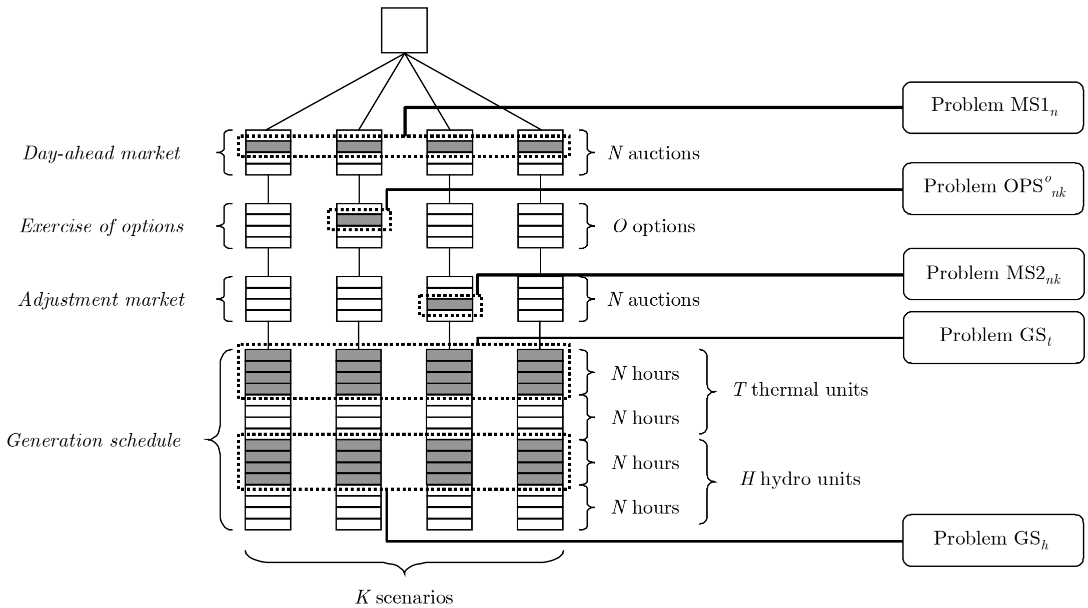

Figure 5.17. A representation of the effect of Lagrangian relaxation on the two-stage problem.

When Lagrangian relaxation is applied to the traditional problem of obtaining the minimum-cost generation schedule that covers a given demand profile, only the subproblems GS_t and GS_h arise. The identification of the presence of subproblems that focus on the optimization of the strategy followed by the generation company in the spot market can be considered an original contribution of this thesis. The structure of these subproblems permits a better understanding of the nature of the general problem of participating in an electricity spot market. This is one of the main advantages of adopting a dual decomposition approach.

5.5.2 An outer-linearization method to solve the dual problem

Let us propose a method to solve the dual problem. Any iterative procedure devised to solve the dual problem must alternatively evaluate the dual function for a certain value of the Lagrange multipliers and then update these multipliers in a descent direction of the dual function. On the one hand, as has been shown, by choosing an adequate version of the dual problem, the subproblem that must be solved to evaluate the dual function for a given value of the Lagrange multipliers is significantly easier to solve than the primal problem. On the other hand, a method is required to update Lagrange multipliers. A good survey of the several methods that have been proposed in the literature for this purpose is carried out in [Jiménez '99a].

In this thesis, Lagrange multipliers are updated based on a sequential approximation of the dual function by means of an outer linearization. The method is based on the definition of the Lagrangian dual function[3]:

$$w(\boldsymbol{\lambda},\boldsymbol{\mu}) \triangleq \operatorname*{Sup}_{\boldsymbol{x}} \left\{ f(\boldsymbol{x}) - \boldsymbol{\lambda}^{\mathrm{t}} \boldsymbol{h}(\boldsymbol{x}) - \boldsymbol{\mu}^{\mathrm{t}} \boldsymbol{g}(\boldsymbol{x}), \boldsymbol{x} \in \mathrm{X} \right\}.$$

If we let $z = w(\boldsymbol{\lambda},\boldsymbol{\mu})$, it is clear that $z \geq f(\boldsymbol{x}) - \boldsymbol{\lambda}^{\mathrm{t}} \boldsymbol{h}(\boldsymbol{x}) - \boldsymbol{\mu}^{\mathrm{t}} \boldsymbol{g}(\boldsymbol{x})$ for any $\boldsymbol{x} \in \mathrm{X}$. The dual problem D can then be restated as follows:

$$\text{(D)} \quad \begin{aligned} \min_{z,\boldsymbol{\lambda},\boldsymbol{\mu}} \quad & z \\ \text{s.t.} \quad & z \geq f(\boldsymbol{x}) - \boldsymbol{\lambda}^{\mathrm{t}} \boldsymbol{h}(\boldsymbol{x}) - \boldsymbol{\mu}^{\mathrm{t}} \boldsymbol{g}(\boldsymbol{x}), \quad \boldsymbol{x} \in \mathrm{X} \\ & \boldsymbol{\lambda} \geq \mathbf{0}. \end{aligned} \tag{5.42}$$

The above problem is a linear program. However, the constraints that limit the variation of z are infinite (they must hold for *any* $\boldsymbol{x} \in \mathrm{X}$) and are not explicitly known. An alternative is to consider an approximate problem, MD, that includes a finite number of these constraints corresponding to a finite number of points $\boldsymbol{x}_1,\ldots,\boldsymbol{x}_L$:

$$\text{(MD)} \quad \begin{aligned} \min_{z,\boldsymbol{\lambda},\boldsymbol{\mu}} \quad & z \\ \text{s.t.} \quad & z \geq f\left(\boldsymbol{x}^l\right) - \boldsymbol{\lambda}^{\mathrm{t}} \boldsymbol{h}\left(\boldsymbol{x}^l\right) - \boldsymbol{\mu}^{\mathrm{t}} \boldsymbol{g}\left(\boldsymbol{x}^l\right), \quad l = 1,\ldots,L, \\ & \boldsymbol{\lambda} \geq \mathbf{0}. \end{aligned} \tag{5.43}$$

This problem is usually known as *master dual problem* and is a linear program too. To check if its solution $(z^*,\boldsymbol{\lambda}^*,\boldsymbol{\mu}^*)$ is also the solution of problem D, the dual function must be evaluated. To do so, we solve the Lagrangian dual subproblem:

[3] This discussion is taken from [Bazaraa '93 pp. 224-227].

$$w\left(\boldsymbol{\lambda}^*,\boldsymbol{\mu}^*\right)=\underset{\boldsymbol{x}}{\text{Max}}\left\{f(\boldsymbol{x})-\boldsymbol{\lambda}^{*\text{t}}\,\boldsymbol{h}(\boldsymbol{x})-\boldsymbol{\mu}^{*\text{t}}\,\boldsymbol{g}(\boldsymbol{x}),\boldsymbol{x}\in\text{X}\right\}, \tag{5.44}$$

If the solution of this problem, $\boldsymbol{x}^*$, is such that $z^*\le f\left(\boldsymbol{x}^*\right)-\boldsymbol{\lambda}^{*\text{t}}\,\boldsymbol{h}\left(\boldsymbol{x}^*\right)-\boldsymbol{\mu}^{*\text{t}}\,\boldsymbol{g}\left(\boldsymbol{x}^*\right)$, it means that an $\boldsymbol{x}^*\in\text{X}$ exists for which the constraint of problem (5.42) does not hold. To avoid this, a new constraint must be added to problem MD in the form $z\ge f\left(\boldsymbol{x}^*\right)-\boldsymbol{\lambda}^{\text{t}}\boldsymbol{h}\left(\boldsymbol{x}^*\right)-\boldsymbol{\mu}^{\text{t}}\boldsymbol{g}\left(\boldsymbol{x}^*\right)$.

It is interesting to notice that when problem (5.44) is solved and it turns out that $z^*\le w\left(\boldsymbol{\lambda}^*,\boldsymbol{\mu}^*\right)$ what actually happens is that the piecewise linear approximation of the dual function, z, is below the dual function. The new constraint that must be added to avoid this undesired result can be reformulated as follows:

$$\begin{aligned}
z &\ge f\left(\boldsymbol{x}^*\right)-\boldsymbol{\lambda}^{\text{t}}\boldsymbol{h}\left(\boldsymbol{x}^*\right)-\boldsymbol{\mu}^{\text{t}}\boldsymbol{g}\left(\boldsymbol{x}^*\right)\\
&=\underbrace{f\left(\boldsymbol{x}^*\right)-\boldsymbol{\lambda}^{*\text{t}}\,\boldsymbol{h}\left(\boldsymbol{x}^*\right)-\boldsymbol{\mu}^{*\text{t}}\,\boldsymbol{g}\left(\boldsymbol{x}^*\right)}_{w(\boldsymbol{\lambda}^*,\boldsymbol{\mu}^*)}+\\
&\quad+\boldsymbol{\lambda}^{*\text{t}}\,\boldsymbol{h}\left(\boldsymbol{x}^*\right)+\boldsymbol{\mu}^{*\text{t}}\,\boldsymbol{g}\left(\boldsymbol{x}^*\right)-\boldsymbol{\lambda}^{\text{t}}\boldsymbol{h}\left(\boldsymbol{x}^*\right)-\boldsymbol{\mu}^{\text{t}}\boldsymbol{g}\left(\boldsymbol{x}^*\right)\\
&=w\left(\boldsymbol{\lambda}^*,\boldsymbol{\mu}^*\right)-\left(\boldsymbol{\lambda}-\boldsymbol{\lambda}^*\right)^{\text{t}}\boldsymbol{h}\left(\boldsymbol{x}^*\right)-\left(\boldsymbol{\mu}-\boldsymbol{\mu}^*\right)^{\text{t}}\boldsymbol{g}\left(\boldsymbol{x}^*\right),
\end{aligned} \tag{5.45}$$

which is the expression of the region delimited by a hyperplane that is tangential to the dual function in $\left(\boldsymbol{\lambda}^*,\boldsymbol{\mu}^*\right)$.

In conclusion, five steps define the iterative process that must be followed to solve the dual problem using an outer linearization to approximate the dual function:

Step 1: Set $\nu=0$.

Step 2: Set $\nu=\nu+1$.
If $\nu=1$, choose initial values for the Lagrange multipliers: $\boldsymbol{\lambda}^1,\boldsymbol{\mu}^1$ and go to step 3.
If $\nu>1$, update the Lagrange multipliers $\boldsymbol{\lambda}^\nu,\boldsymbol{\mu}^\nu$ by solving the master dual:

$$\begin{aligned}
&\underset{z^\nu,\boldsymbol{\lambda}^\nu,\boldsymbol{\mu}^\nu}{\text{Min}} && z^\nu\\
&\text{s.t.} && z^\nu\ge w\left(\boldsymbol{\lambda}^l,\boldsymbol{\mu}^l\right)-\left(\boldsymbol{\lambda}^\nu-\boldsymbol{\lambda}^l\right)^{\text{t}}\boldsymbol{h}\left(\boldsymbol{x}^l\right)-\left(\boldsymbol{\mu}^\nu-\boldsymbol{\mu}^l\right)^{\text{t}}\boldsymbol{g}\left(\boldsymbol{x}^l\right),\quad l=1,\ldots,\nu-1,\\
&&& \boldsymbol{\mu}^\nu\ge\mathbf{0}.
\end{aligned}$$

Step 3: Solve the Lagrangian subproblem to evaluate the Lagrangian dual function:
$w\left(\boldsymbol{\lambda}^\nu,\boldsymbol{\mu}^\nu\right)=\underset{\boldsymbol{x}}{\text{Max}}\quad f(\boldsymbol{x})-\boldsymbol{\lambda}^{\nu\text{t}}\boldsymbol{h}(\boldsymbol{x})-\boldsymbol{\mu}^{\nu\text{t}}\boldsymbol{g}(\boldsymbol{x}),\ \text{s.t. }\boldsymbol{x}\in\text{X}.$

Step 4: Check for convergence: If $z^\nu\le w\left(\boldsymbol{\lambda}^\nu,\boldsymbol{\mu}^\nu\right)$ go to step 2.

Step 5: End.

The evolution of this algorithm is illustrated in Figure 5.18. The reader must notice that in the first iterations the master dual problem is likely to be unbounded due to the reduced number of linear constraints that constitute the piecewise linear approximation of the dual function. To avoid this, several alternatives exist. In [Cerisola '02] a general approach based on Farkas' laws is suggested that obtains the minimum number of constraints required to guarantee that the master dual problem is not unbounded. In [Jiménez '99b], the feasibility region for the Lagrange multipliers is artificially bounded to avoid the oscillations that are typically observed in LR procedures. These artificial boundaries are dynamically updated. Although Cerisola's method is general and does not require the tuning of parameters, if the artificial boundaries suggested by Jiménez are chosen with care, the LR algorithm converges in significantly less time.

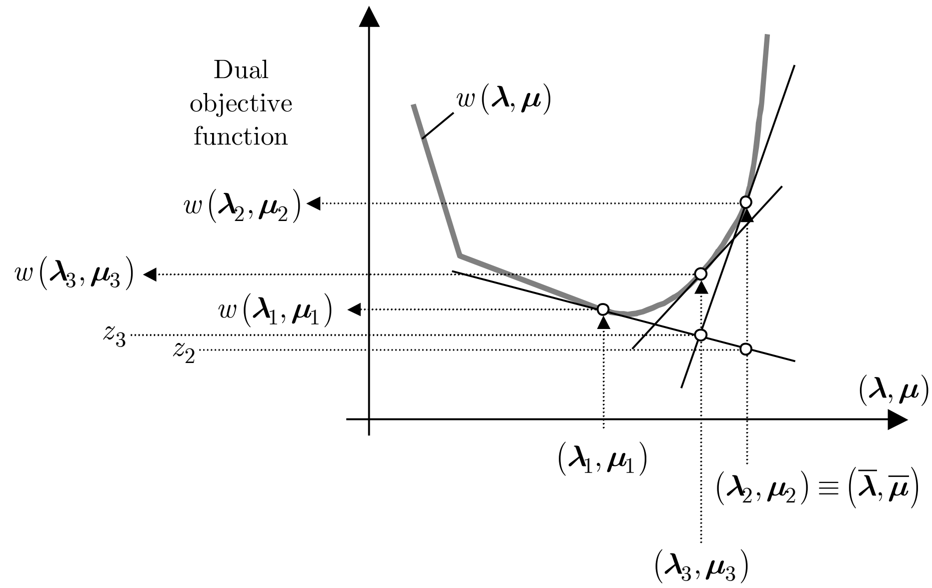

Figure 5.18. The algorithm used to solve the dual problem.

Let us define now the particular iterative process that should be used in order to solve the dual problem of the problem formulated in 5.5.1:

Step 1: Set $\nu = 0$.

Step 2: Set $\nu = \nu + 1$.
If $\nu = 1$, choose initial values for the Lagrange multipliers: $\boldsymbol{\lambda}_{nk}^{1}$ and go to step 3.
If $\nu > 1$, update the Lagrange multipliers $\boldsymbol{\lambda}_{nk}^{\nu}$ by solving the master dual:

$$\min_{z^{\nu},\lambda_{nk}^{\nu}} \; z^{\nu}$$

$$\text{s.t. } z^{\nu} \geq w\left(\lambda_{nk}^{l}\right) - \sum_{k}\sum_{n}\left(\lambda_{nk}^{\nu} - \lambda_{nk}^{l}\right)\left(q_{nk}^{1}{}^{l} + q_{nk}^{2}{}^{l} + \sum_{c\in \mathrm{C}^{\mathrm{P}}} q_{n}^{c} + \sum_{o\in \mathrm{O}^{\mathrm{P}}} u_{nk}^{o}{}^{l}\cdot q_{n}^{o} - \sum_{t\in \mathrm{T}} q_{nk}^{t}{}^{l} - \left[\sum_{h\in \mathrm{H}} q_{nk}^{h}{}^{l} - b_{nk}^{1}{}^{l}\right]\right),$$

$$l = 1,\ldots,\nu - 1,$$

Step 3: To evaluate the Lagrangian dual function, problems $\mathrm{MS1}_n$, $\mathrm{MS2}_{nk}$, OPS_{nk}^{o}, GS_t and GS_h have to be solved. The value of the dual function is calculated as:

$$w\left(\boldsymbol{\lambda}_{nk}^{\nu}\right) = \sum_{n\in \mathrm{N}} w_{\mathrm{MS1}_n}\left(\boldsymbol{\lambda}_{nk}^{\nu}\right) + \sum_{k\in \mathrm{N}}\sum_{n\in \mathrm{N}} w_{\mathrm{MS2}_{nk}}\left(\boldsymbol{\lambda}_{nk}^{\nu}\right) + \sum_{k\in \mathrm{N}}\sum_{n\in \mathrm{N}}\sum_{o\in \mathrm{O}^{\mathrm{P}}} w_{\mathrm{OPS}_{nk}^{o}}\left(\boldsymbol{\lambda}_{nk}^{\nu}\right) + \sum_{t\in \mathrm{T}} w_{\mathrm{GS}_t}\left(\boldsymbol{\lambda}_{nk}^{\nu}\right) + \sum_{h\in \mathrm{H}} w_{\mathrm{GS}_h}\left(\boldsymbol{\lambda}_{nk}^{\nu}\right).$$

Step 4: Check for convergence: If $z^{\nu} \leq w\left(\lambda_{nk}^{\nu-1}\right)$ go to step 2.

Step 5: End.

This algorithm presents no relevant conceptual differences with respect to other applications of the LR method that can be found in the literature. As indicated, the main contribution of our approach is the identification of the new subproblems that arise due to the competitive framework in which the generation company must develop its business.

5.5.3 A numerical example

In this section the two-stage problem proposed in 5.2.3 is solved again using the suggested LR algorithm, implemented in GAMS language. The two techniques that guarantee that the master dual problem is not unbounded are used here in order to determine which of the two is more efficient.

5.5.3.1 An approach based on Farkas' laws

We begin by solving the dual problem using the approach suggested in [Cerisola '02]. This method, based on Farkas' laws, divides the iterative solution of the dual problem into two phases. In the first phase, Lagrange multipliers are bounded between –1 and +1 and the hyperplanes that are tangent to the dual function are forced to pass through the origin. After a number of iterations, the objective function is null, indicating that the hyperplanes already obtained are enough to guarantee that the master dual problem is bounded. Figure 5.19 shows the evolution of this first phase for the considered problem, which required 49 iterations.

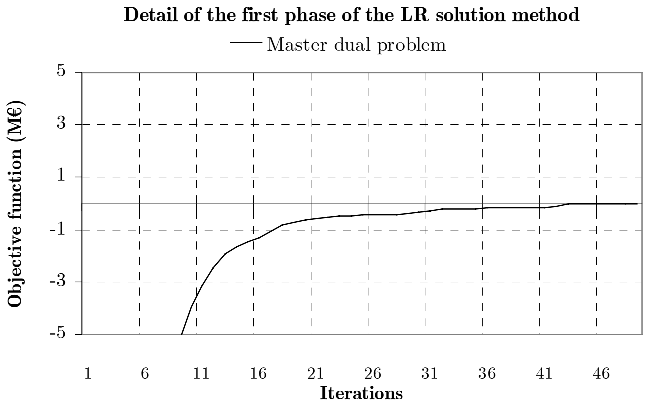

Figure 5.19. Evolution of Cerisola's first phase.

The second phase of the solution process is the standard LR algorithm. Figure 5.20 shows the evolution of this second phase for the considered problem. It can be seen that the objective function of the master dual problem is monotonically increasing, given that no bounds are imposed on the Lagrange multipliers. This second phase required 1502 iterations and about 6 h of CPU time. The solution obtained is 30.112433 M€, which is an upper bound for the solution of the original problem (30.112293 M€). The duality gap is 0.000140 M€.

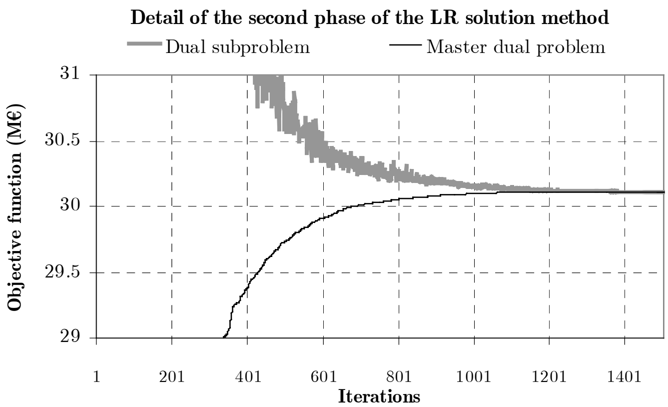

Figure 5.20. Evolution of Cerisola's second phase.

Due to the dualization of the complicating constraints, the hourly balance between the generation schedule and the sales of the company in both market mechanisms is not satisfied. Figure 5.21 represents the value of the hourly infeasibilities that arise due to this dualization. As can be seen, these infeasibilities are rather important if compared with the hourly energy sales of the company (about 10000 MWh). The reason is that the solution of the subproblems varies dramatically with minor changes in the value of the Lagrange multipliers. This is a well-known drawback of Lagrange relaxation and is commonly overcome by introducing postprocessing heuristics that provide a primal feasible solution.

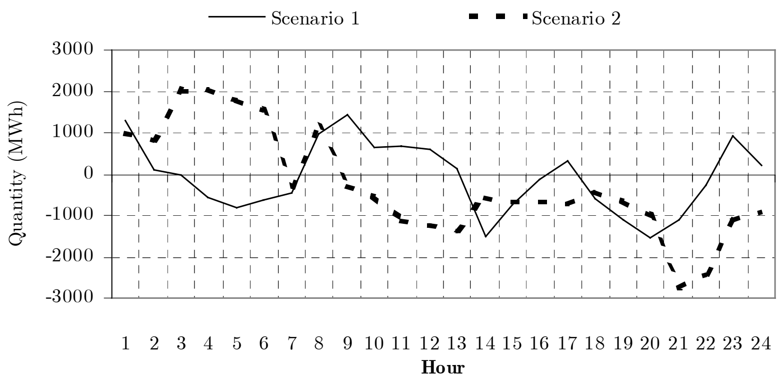

Figure 5.21. Infeasibilities due to the dualization of the energy balance equation.

The value obtained for the Lagrange multipliers is also an interesting information. As has been indicated, in this problem the Lagrange multipliers can be interpreted as the company's marginal revenues in the spot market. Figure 5.22 reflects the value of the Lagrange multipliers for this numerical example.

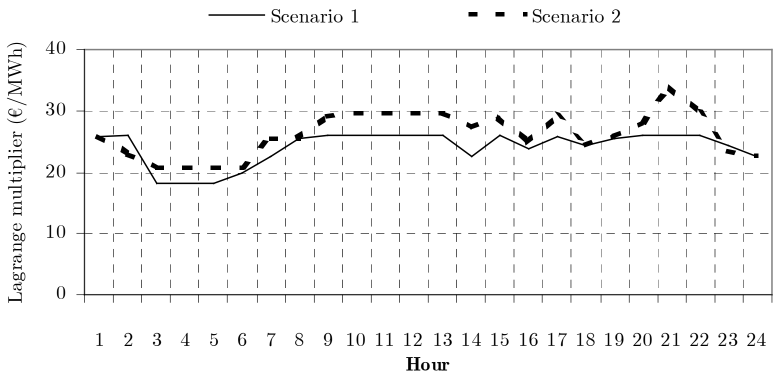

Figure 5.22. Value of the Lagrange multipliers.

5.5.3.2 An approach based on defining a dynamic feasibility region

In [Jiménez '99b], the variation of the Lagrange multipliers in each iteration is artificially limited by imposing upper and lower bounds that are updated in each iteration. We have tried this approach by limiting the variation of the Lagrange multipliers in ± 0.5 €/MWh. As can be seen in Figure 5.23, only 338 iterations of the LR algorithm and about 2 h of CPU time are required to obtain the solution of the dual problem. A good choice of the bounds for the variation of the Lagrange multipliers significantly reduces the oscillation of the master dual problem, thus eliminating one of the main disadvantages of the standard LR algorithm. This approach will be used in the rest of the applications of the LR algorithm included in this thesis.

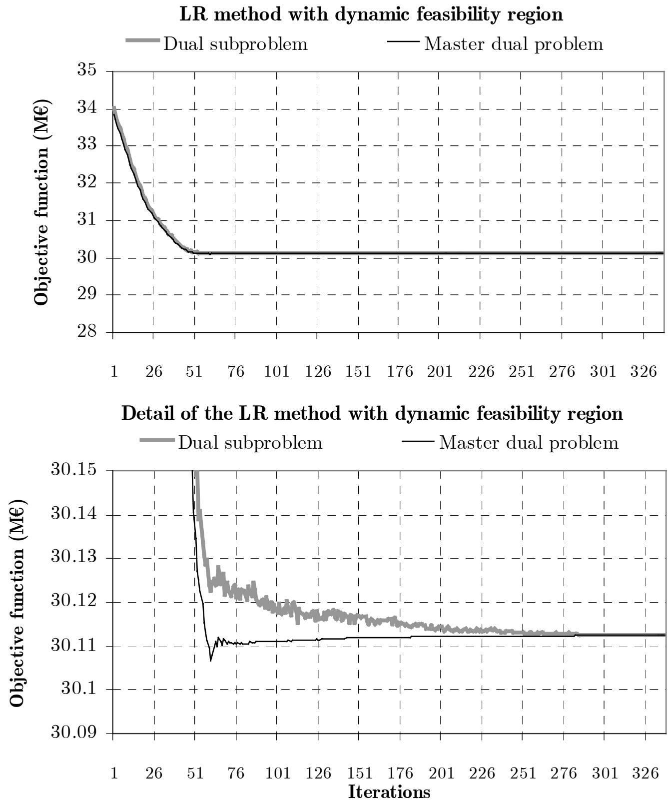

Figure 5.23. Evolution of the LR algorithm with dynamic feasibility region.

5.5.4 Advantages of the LR approach for the two-stage program

Lagrangian relaxation presents both pros and cons when applied to the two stage-program formulated in this chapter. In particular, three relevant advantages are:

- It decomposes the original problem into a number of meaningful subproblems that are significantly easier to solve than the original problem.
- It provides the value of the Lagrange multipliers, which have an interesting economic intepretation.
- It admits the presence of binary variables in whatever part of the problem.

However, two shortcomings limit the applicability of this solution method:

- The solution of the dual problem does not satisfy the dualized constraints.
- The LR algorithm entails a greater computational effort than Benders' algorithm. It requires both a higher number of iterations and more CPU time. Due to the oscillations typical in LR solution processes, intermediate solutions obtained by interrupting the algorithm too soon are of limited interest.

These two disadvantages are enough to outweigh the advantages of the LR solution approach. In particular, the fact that the energy balance equation is not fulfilled implies the risk of not being able to develop a generation schedule that satisfies the obligations assumed by the company in the spot market. Hence, Benders' decomposition is considered a better choice than Lagrangian relaxation to solve the two-stage programs that represent the problem faced by the generation company in each spot market mechanism.

5.6 Lagrangian relaxation and the weekly unit-commitment problem

5.6.1 An overview

As the previous section points out, the main disadvantage of Lagrangian relaxation is that, when the primal problem is not convex, the dual solution it provides is not primal feasible. Nevertheless, a frequent approach to address the traditional unit-commitment problem is to dualize the demand and reserve constraints, which establish a link between the different generation units. In this manner, the dual problem is iteratively solved by calculating the optimal schedule of each generating unit, given the current value of the Lagrange multipliers and subsequently updating these multipliers. The independent schedules thus obtained do not satisfy the demand and reserve constraints but are usually very close to the primal optimal solution. In particular, the numerical values of the binary variables that represent the commitment states of the generation units provide a good approximation of the optimal unit-commitment schedule.

The weekly stochastic unit commitment problem proposed in section 5.2.4 can be addressed with a LR approach by dualizing the complicating constraints. In this case we consider as complicating constraints not only the energy balance equation, but also the non-decreasing constraints that force the offers decided by the model to constitute a valid offer curve. Obviously, the numerical solution obtained with this approach will suggest an offering strategy with infeasible features, such as offer curves that present decreasing offer blocks or imbalances between the expected sales and the expected generation schedule. However, when solving the weekly unit-commitment problem we are still not worried about the shape of our offer curve for each of the spot market auctions. On the other hand, we can assume that the unit-commitment schedule thus obtained is very close to the optimum. Moreover, the dual information provided by this approach is extremely useful to evaluate the state of the spot market.

Figure 5.17 illustrates the types of subproblems that Lagrangian relaxations yields when applied to the weekly problem:

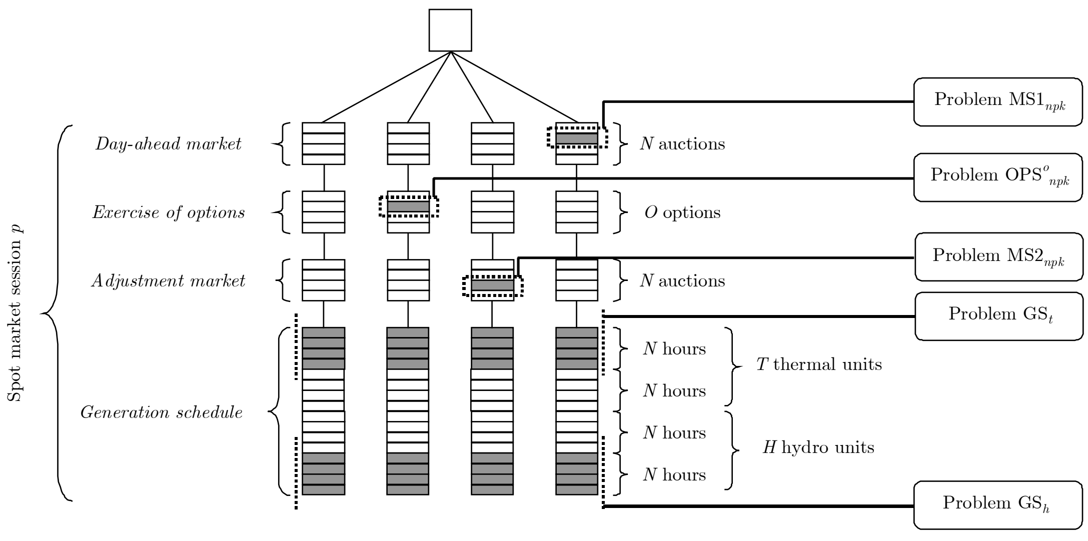

Figure 5.24. A representation of the effect of Lagrangian relaxation on the weekly problem.

5.6.2 Mathematical formulation

Let us formulate the dual problem that results when both the energy balance equation and the non-decreasing constraints are dualized. In this case the primal problem P can be stated as follows:

$$\underset{\substack{q^1_{npk},q^2_{npk},\\ q^t_{npk},u^o_{npk}}}{\text{Max}}\ f \equiv \left\{ \begin{aligned} &\sum_{k\in \mathrm{K}} \pi_k \left[\sum_{p\in P}\sum_{n\in \mathrm{N}} r^1_{npk}\left(q^1_{npk}\right) + \sigma^1_{np}\frac{q^1_{npk}}{Q^1_{npk}} + \sum_{n\in \mathrm{N}} r^2_{npk}\left(q^2_{npk}\right) + \sigma^2_n \frac{q^2_{npk}}{Q^2_{npk}} \right.\\ &\qquad + \sum_{c\in \mathrm{C^D}}\sum_{n\in \mathrm{N}}\left(p^c_{np} - p^1_{npk}\left(q^1_{npk}\right)\right)\cdot q^c_{np}\\ &\qquad + \sum_{o\in \mathrm{O^P}}\sum_{n\in \mathrm{N}} u^o_{npk}\cdot p^o_{np}\cdot q^o_{np} + \sum_{o\in \mathrm{O^F}}\sum_{n\in \mathrm{N}}\left(p^o_{npk} - p^1_{npk}\left(q^1_{npk}\right)\right)\cdot q^o_{np}\\ &\qquad \left. -\sum_{t\in \mathrm{T}}\sum_{n\in \mathrm{N}} c^t_{npk}\left(q^t_{npk}, y^t_{npk}, u^t_{npk}\right)\right] \end{aligned}\right.$$

s.t.

$$\boldsymbol{h} = \boldsymbol{0} \equiv \left\{ \begin{aligned} &q^1_{npk} + q^2_{npk} + \sum_{c\in \mathrm{C^P}} q^c_{np} + \sum_{o\in \mathrm{O^P}} u^o_{npk} q^o_{np}\\ &-\sum_{t\in \mathrm{T}} q^t_{npk} - \left[\sum_{h\in \mathrm{H}} q^h_{npk} - b^h_{npk}\right] = 0, \end{aligned}\right. \qquad n\in \mathrm{N}, p\in \mathrm{P}, k\in \mathrm{K},$$

$$\boldsymbol{g} \le \boldsymbol{0} \equiv \left\{ \begin{aligned} &q^1_{npk} - q^1_{npk'} + x^1_{npkk'}M^q \ge 0,\\ &q^1_{npk'} - q^1_{npk} + \left(1 - x^1_{npkk'}\right)M^q \ge 0,\\ &p^1_{npk}\left(q^1_{npk}\right) - p^1_{npk'}\left(q^1_{npk'}\right) + x^1_{npkk'}M^p \ge 0,\\ &p^1_{npk'}\left(q^1_{npk'}\right) - p^1_{npk}\left(q^1_{npk}\right) + \left(1 - x^1_{npkk'}\right)M^p \ge 0, \end{aligned}\right. \qquad n\in \mathrm{N}, p\in \mathrm{P}, k, k'\in \mathrm{K},$$

$$\boldsymbol{x} \in \mathrm{X} \equiv \left\{ \begin{aligned} &q^1_{npk} \in \mathrm{Q}^1_{npk}, && n\in \mathrm{N}, p\in \mathrm{P}, k\in \mathrm{K},\\ &q^2_{npk} \in \mathrm{Q}^2_{npk}, && n\in \mathrm{N}, p\in \mathrm{P}, k\in \mathrm{K},\\ &\left\{q^t_{npk}, y^t_{npk}, u^t_{npk}, n\in \mathrm{N}, p\in \mathrm{P}\right\} \in \mathrm{Q}^t, && t\in \mathrm{T}, k\in \mathrm{K},\\ &\left\{q^h_{npk}, b^h_{npk}, n\in \mathrm{N}, p\in \mathrm{P}\right\} \in \mathrm{Q}^h, && h\in \mathrm{H}, k\in \mathrm{K},\\ &p^o_{npk} \in \mathrm{P}^o_{npk}, && o\in \mathrm{O^F}, n\in \mathrm{N}, p\in \mathrm{P}, k\in \mathrm{K},\\ &u^o_{npk} \in \mathrm{U}^o_{npk}, && o\in \mathrm{O^P}, n\in \mathrm{N}, p\in \mathrm{P}, k\in \mathrm{K}. \end{aligned}\right. \tag{5.46}$$

where the non-decreasing constraints are now identified with the generic set of inequality constraints $\boldsymbol{g}(\boldsymbol{x}) \le \boldsymbol{0}$. In this manner we dualize both the energy balance equation and the non-decreasing constraints.

The Lagrange function corresponding to this problem is:

$$\begin{aligned}
&\mathcal{L}\left(q^1_{npk}, q^2_{npk}, u^o_{npk}, p^o_{npk}, q^t_{npk}, q^h_{npk}, b^h_{npk}, x^1_{npkk'}, \lambda_{npk}, \mu^{q+}_{npkk'}, \mu^{q-}_{npkk'}, \mu^{p+}_{npkk'}, \mu^{p-}_{npkk'}\right) = \\
&\sum_{k\in \mathrm{K}} \pi_k \left[\sum_{p\in \mathrm{P}} \sum_{n\in \mathrm{N}} p^1_{npk}\left(q^1_{npk}\right)\cdot q^1_{npk} + \sigma^1_{np}\frac{q^1_{npk}}{Q^1_{npk}} + \sum_{n\in \mathrm{N}} p^2_{npk}\left(q^2_{npk}\right)\cdot q^2_{npk} + \sigma^2_{np}\frac{q^2_{npk}}{Q^2_{npk}} \right.\\
&\qquad + \sum_{c\in \mathrm{C}^{\mathrm{D}}} \sum_{n\in \mathrm{N}} \left(p^c_{np} - p^1_{npk}\left(q^1_{npk}\right)\right)\cdot q^c_{np} \\
&\qquad + \sum_{o\in \mathrm{O}^{\mathrm{P}}} \sum_{n\in \mathrm{N}} u^o_{npk}\cdot p^o_{np}\cdot q^o_{np} + \sum_{o\in \mathrm{O}^{\mathrm{F}}} \sum_{n\in \mathrm{N}} \left(p^o_{npk} - p^1_{npk}\left(q^1_{npk}\right)\right)\cdot q^o_{np} \\
&\qquad \left. - \sum_{t\in \mathrm{T}} \sum_{n\in \mathrm{N}} c^t_{npk}\left(q^t_{npk}\right)\right] \\
&- \sum_{k\in \mathrm{K}} \sum_{p\in \mathrm{P}} \sum_{n\in \mathrm{N}} \lambda_{npk}\left[q^1_{npk} + q^2_{npk} + \sum_{c\in \mathrm{C}^{\mathrm{P}}} q^c_{np} + \sum_{o\in \mathrm{O}^{\mathrm{P}}} u^o_{npk} q^o_{np} - \sum_{t\in \mathrm{T}} q^t_{npk} - \left[\sum_{h\in \mathrm{H}} q^h_{npk} - b^h_{npk}\right]\right] \\
&+ \sum_{k\in \mathrm{K}} \sum_{k'>k} \sum_{p\in \mathrm{P}} \sum_{n\in \mathrm{N}} \mu^{q+}_{npkk'}\left[q^1_{npk} - q^1_{npk'} + x^1_{npkk'} M^q\right] \\
&\qquad + \mu^{q-}_{npkk'}\left[q^1_{npk'} - q^1_{npk} + \left(1 - x^1_{npkk'}\right) M^q\right] \\
&\qquad + \mu^{p+}_{npkk'}\left[p^1_{npk}\left(q^1_{npk}\right) - p^1_{npk'}\left(q^1_{npk'}\right) + x^1_{npkk'} M^p\right] \\
&\qquad + \mu^{p-}_{npkk'}\left[p^1_{npk'}\left(q^1_{npk'}\right) - p^1_{npk}\left(q^1_{npk}\right) + \left(1 - x^1_{npkk'}\right) M^p\right]
\end{aligned} \tag{5.47}$$

The dualization of the non-decreasing constraints implies changes only in the master dual problem and in the subproblems corresponding to the first market mechanism. The rest of subproblems, $\mathrm{MS2}_{npk}$, $\mathrm{OPS}^{\,o}_{npk}$, GS_t and GS_h are formulated as indicated in section 5.5.1.

The subproblem corresponding to the first market mechanism can now be formulated for each scenario k, each period p and each hourly auction n:

$$\begin{aligned}
&w_{\mathrm{MS1}_{npk}}\left(\lambda_{npk}^{\ \nu}, \mu^{q+}_{npkk'}, \mu^{q-}_{npkk'}, \mu^{p+}_{npkk'}, \mu^{p-}_{npkk'}\right) = \\
&= \operatorname*{Max}_{q^1_{npk}, p^o_{npk}} \sum_{k\in \mathrm{K}} \pi_k \left[\left[p^1_{npk}\left(q^1_{npk}\right) + \frac{\sigma^1_{np}}{Q^1_{npk}} - \frac{\lambda_{npk}^{\ \nu}}{\pi_k}\right]\cdot q^1_{npk} + \sum_{c\in \mathrm{C}^{\mathrm{P}}}\left(p^c_{np} - \frac{\lambda_{npk}^{\ \nu}}{\pi_k}\right) q^c_{np}\right.\\
&\qquad\qquad \left. + \sum_{c\in \mathrm{C}^{\mathrm{D}}}\left(p^c_{np} - p^1_{npk}\left(q^1_{npk}\right)\right)\cdot q^c_{np} + \sum_{o\in \mathrm{O}^{\mathrm{F}}}\left(p^o_{npk} - p^1_{npk}\left(q^1_{npk}\right)\right)\cdot q^o_{np}\right] \\
&\qquad + \sum_{k\in \mathrm{K}}\left\{\sum_{k'>k} \mu^{q+}_{npkk'}\left[q^1_{npk} + x^1_{npkk'} M^q\right] + \mu^{q-}_{npkk'}\left[-q^1_{npk} + \left(1 - x^1_{npkk'}\right) M^q\right]\right.\\
&\qquad\qquad\qquad + \mu^{p+}_{npkk'}\left[p^1_{npk}\left(q^1_{npk}\right) + x^1_{npkk'} M^p\right] + \mu^{p-}_{npkk'}\left[-p^1_{npk}\left(q^1_{npk}\right) + \left(1 - x^1_{npkk'}\right) M^p\right] \\
&\qquad\quad \left. + \sum_{k'<k}\left[-\mu^{q+}_{npk'k} + \mu^{q-}_{npkk'}\right] q^1_{npk} + \left[-\mu^{p+}_{npk'k} + \mu^{p-}_{npkk'}\right] p^1_{npk}\left(q^1_{npk}\right)\right\} \\
&\text{s.t. } q^1_{npk} \in \mathrm{Q}^1_{npk}, \\
&\qquad p^o_{npk} \in \mathrm{P}^o_{npk}, \quad o \in \mathrm{O}^{\mathrm{F}}, \\
&\qquad x^1_{npkk'} \in \{0,1\}.
\end{aligned}$$

The master dual problem must update not only the Lagrange multipliers corresponding to the energy balance equation, but also those corresponding to the non-decreasing constraints:

$$
\begin{aligned}
&\underset{\substack{z^{\nu},\lambda_{npk}^{\nu},\\ \mu_{npkk'}^{q+\ \nu},\mu_{npkk'}^{q-\ \nu},\\ \mu_{npkk'}^{p+\ \nu},\mu_{npkk'}^{p-\ \nu}}}{\text{Min}} \quad z^{\nu}\\
&\text{s.t. } z^{\nu} \geq w\left(\lambda_{npk}^{l}\right)\\
&\quad -\sum_{k\in\mathrm{K}}\sum_{p\in\mathrm{P}}\sum_{n\in\mathrm{N}}\left(\lambda_{npk}^{\nu}-\lambda_{npk}^{l}\right)\Bigg[q_{npk}^{1\ \ l}+q_{npk}^{2\ \ l}+\sum_{c\in\mathrm{C}^{\mathrm{P}}}q_{np}^{c}+\sum_{o\in\mathrm{O}^{\mathrm{P}}}u_{npk}^{o\ \ l}\cdot q_{np}^{o}\\
&\qquad\qquad -\sum_{t\in\mathrm{T}}q_{npk}^{t\ \ l}-\left[\sum_{h\in\mathrm{H}}q_{npk}^{h\ \ l}-b_{npk}^{1\ \ l}\right]\Bigg]\\
&\quad +\sum_{k\in\mathrm{K}}\Bigg\{\sum_{k'>k}\sum_{p\in\mathrm{P}}\sum_{n\in\mathrm{N}}\left(\mu_{npkk'}^{q+\ \nu}-\mu_{npkk'}^{q+\ l}\right)\left[q_{npk}^{1\ \ l}-q_{npk'}^{1\ \ l}+x_{npkk'}^{1\ \ l}M^{q}\right]\\
&\qquad +\left(\mu_{nkk'}^{q-\ \nu}-\mu_{npkk'}^{q-\ l}\right)\left[q_{npk'}^{1\ \ l}-q_{npk}^{1\ \ l}+\left(1-x_{npkk'}^{1\ \ l}\right)M^{q}\right]\\
&\qquad +\left(\mu_{npkk'}^{p+\ \nu}-\mu_{npkk'}^{p+\ l}\right)\left[p_{npk}^{1}\left(q_{npk}^{1\ \ l}\right)-p_{npk'}^{1}\left(q_{npk'}^{1\ \ l}\right)+x_{npkk'}^{1\ \ l}M^{p}\right]\\
&\qquad +\left(\mu_{npkk'}^{p-\ \nu}-\mu_{npkk'}^{p-\ l}\right)\left[p_{npk'}^{1}\left(q_{npk'}^{1\ \ l}\right)-p_{npk}^{1}\left(q_{npk}^{1\ \ l}\right)+\left(1-x_{npkk'}^{1\ \ l}\right)M^{p}\right]\\
&\quad l=1,\ldots,\nu-1.
\end{aligned}
$$

5.6.3 Advantages and disadvantages

The Lagrangian relaxation approach presents several advantages when applied to the weekly stochastic unit-commitment problem:

- It is a decomposition approach that tolerates the generalized presence of binary variables and provides a good approximation of the solution.
- It also provides valuable information relative to the dual variables that can be used to evaluate the situation of the spot market.

The disadvantages of Lagrangian relaxation in this case coincide with those identified when applying it to the two-stage stochastic program. However, given that standard Benders' decomposition cannot efficiently cope with the commitment binary variables, we will adopt this methodology to solve the weekly stochastic unit-commitment problem. A realistic numerical example is solved in chapter 6 to illustrate the features of this approach.

5.7 Conclusion

In this chapter we have developed a methodology to obtain numerical results for two of the main problems faced by a generation company in an electricity spot market. These two problems have been formulated as mathematical programs that are particularizations of the general multistage stochastic programming approach suggested in chapter 4.

On the one hand, we have formulated the problem of deciding an offering strategy for each of the spot market mechanisms as a two-stage program. This approach is based on assuming that the relative importance of the spot market mechanisms diminishes as the moment of physical delivery gets nearer. On the other hand, we have expressed the weekly stochastic unit-commitment problem as a sequence of two-stage programs.

A detailed analysis of the structure of these two mathematical programs has lead to the identification of both complicating variables and complicating constraints. Focusing on the two-stage program, we have shown how Benders' decomposition benefits from the existence of complicating variables and from the two-stage structure of the problem. This solution approach has been explored in detail and an original technique has been proposed that provides quasioptimal feasible solutions for this particular type of problem. Additionally, we have explained how Lagrangian relaxation exploits the presence of complicating constraints by relaxing them. The application of Lagrangian relaxation to the problems considered in this thesis presents several interesting novelties with respect to more traditional uses of this technique, such as the presence of new subproblems oriented to the optimization of the company's strategy in the spot market. However, the solutions it yields fail to satisfy the energy balance equation, which is crucial to guarantee that the strategy followed by the company in the spot market can then be materialized with its generating units.

Nevertheless, Lagrangian relaxation turns out to be an adequate methodology to address the weekly stochastic unit-commitment problem for a generation company participating in a spot market. In addition to providing a good approximation for the optimal unit-commitment schedule, it yields valuable dual information relative to the current situation of the spot market.

The combination of Lagrangian relaxation (for the weekly unit-commitment problem) and Benders' decomposition (to develop offering strategies for each market mechanism) provides a powerful framework whose possibilities are evaluated in chapter 6.

5.8 References

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6 Computational results

This chapter presents several realistic numerical examples in order to evaluate the adequacy of the methodology proposed in this thesis. All these examples consider the case of a fictitious but large and representative generation company operating in the Spanish electricity spot market, which reinforces the validity of the suggested approach.

The study cases included in this chapter are organized in two main parts. The first one is oriented to the development of an optimal offering strategy for the day-ahead market. This problem is solved under a variety of assumptions and a sensitivity analysis is performed so as to determine the influence that a number of relevant factors exert on the strategies developed by the model. The results obtained confirm the soundness of the ideas developed in this thesis. The second part of the chapter corresponds to the weekly stochastic unit-commitment problem and comprises only one study case due to the huge computational effort required for its numerical resolution. The unit-commitment decision tree derived with this approach is consistent with the stochastic perspective adopted in this thesis.

6.1 Introduction

In this thesis we have proposed a multistage stochastic programming framework to represent the problem faced by a generation company in an electricity spot market. This conceptual approach, although consistent and attractive, presents difficulties when numerical results have to be obtained. For this particular purpose two different perspectives have been adopted in chapter 5.

On the one hand, we have suggested that the problem of developing an offering strategy for a certain spot market mechanism be represented as a two-stage program in which only two market mechanisms are explicitly considered. We have also shown that the best approach to solve this type of problem is Benders' decomposition, together with two ad hoc techniques that facilitate the search for a good feasible solution. In this chapter we solve a number of realistic numerical examples in which the case of a fictitious but representative generation company operating in the Spanish electricity spot market is considered. In particular, an example consisting of eleven spot market scenarios is used in order to perform a sensitivity analysis of the offering strategies developed by the model with respect to different relevant factors, such as the value that the company gives to its market share, the volume of available hydro reserves, the outage of a generating unit or the existence of long-term open positions in the company's portfolio.

On the other hand, there are some decisions that depend on the uncertain outcome of the spot market but that must be taken with a one-week perspective, such as the commitment schedule for the thermal units or the management of hydro reserves during the week. We have proposed a modeling approach for this problem based on a sequence of seven daily two-stage programs. Given the abundance of binary variables in this formulation, Lagrangian relaxation is considered the best approach to obtain numerical results. In this chapter we solve a real numerical example in which the stochastic unit-commitment problem of the generation company of study is represented by means of sixteen weekly scenarios. This yields sixteen different unit-commitment schedules, together with sixteen different hydro management strategies, thus capturing much of the flexibility with which a generation company can operate in the spot market throughout the week.

6.2 Offering strategies for the day-ahead market

In this section we present the results of a number of numerical examples in which a generation company develops offering strategies for a day-ahead market[1]. All the cases correspond to the Spanish spot market session that took place on October 24$^{\text{th}}$, 2001, which was also the session considered in the numerical examples solved in chapter 5. In this section, however, the problems solved are significantly larger. The first study case considers five spot market scenarios and is used to provide further details about the solution strategy suggested in the previous chapter. The rest of examples consist of eleven spot market scenarios and are basically identical, except for slight differences that are introduced to analyze the sensitivity of the solution with respect to a number of factors. More information about the input data used in these examples can be found in appendix C.

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[1] The input data used that defines this company's generation portfolio can be found in appendix C.

6.2.1 A five-scenario case

6.2.1.1 Characteristics of the problem

This study case is presented with the aim of providing a practical vision of the solution strategy developed in chapter 5. It is a larger problem than the one addressed in that chapter, not only due to an increase of the number of scenarios (from two to five) but also because of the detail with which residual demand curves and revenue functions are modeled (twenty segments, instead of six). Table 6.1 establishes a comparison between both numerical examples.

	Equations	Variables	Binary variables	Solution (M€)	Maximum optimality loss	Execution time
Two-scenario problem	13249	13771	474	30.112293	0.00 %	6 s.
Five-scenario problem	43921	41305	4704	32.099927	0.79 %	30 min.

Table 6.1. A comparison of two study cases.

In order to increase the number of spot market scenarios from two to five, it suffices to enlarge the range of historic days that are explored when seeking for sessions similar to the one of study (see appendix C for further details). Nevertheless, the input information not strictly related to the spot market model that was used for the two-scenario case has also been used for the five-scenario case, including generating units' data and the hourly value of the company's market share. Hence, the results obtained for both examples should be similar.

6.2.1.2 Numerical resolution

When five spot market scenarios are considered, the problem can no longer be solved using CPLEX in a straightforward manner. Moreover, the use of the standard Benders' algorithm is also useless, given that the master problem that is obtained is still a large-scale MIP problem. As a result, the ad hoc techniques suggested in chapter 5 must be used in order to derive a feasible solution. In other words, the master problem must be separated into hourly problems, even though this may cause a loss of optimality. When the master problem is separated, Benders' algorithm evolves as indicated in Table 6.2. As can be seen, the solution thus obtained is 32.040514 M€.

Iteration	Master	Subproblem	Total
IT1	38.099767	-16.576566	21.5232
IT2	36.886674	-7.348882	29.537792
IT3	33.795529	-3.08575	31.091753
IT4	32.1811	-3.912572	31.980665
IT5	31.817476	-3.651384	32.044016
IT6	31.625251	-3.80858	32.039218
IT7	31.614987	-3.805078	32.040514
IT8	31.613861	-3.799984	32.040507
IT9	31.611793	-3.805078	32.040514

Table 6.2. Evolution of Benders' algorithm with the master problem separated into hourly problems.

The solution provided by Benders' algorithm when the master problem is separated, is a feasible solution for the original problem. However, in this context, Benders' algorithm does not provide a measure of the loss of optimality caused by the separation of the master problem into hourly problems. To derive such a measure we can simply try to solve the full original problem with CPLEX, using as initial values those obtained from the master problem separated into hourly problems. In this manner, the branch-and-bound algorithm starts from a feasible solution, which reduces the size of the binary tree used to search for new solutions. In practice, CPLEX branch-and-bound algorithm is not able to improve the initial values of the binary variables. Its only contribution is to slightly improve the values of the continuous variables, now that the twenty-four auctions of the day-ahead market are being simultaneously considered. This leads to the solution indicated in Table 6.1, 32.099927 M€. This solution is better than the one obtained from the master problem separated into hourly problems in only 0.19 %. CPLEX also quantifies the maximum suboptimality of the final solution. As can be seen, this solution is suboptimal in, at most, 0.79 %. The computational effort required to obtain these final results (about 1 minute of CPU time) is minimal if compared with the effort required to derive the feasible initial solution (about 30 minutes).

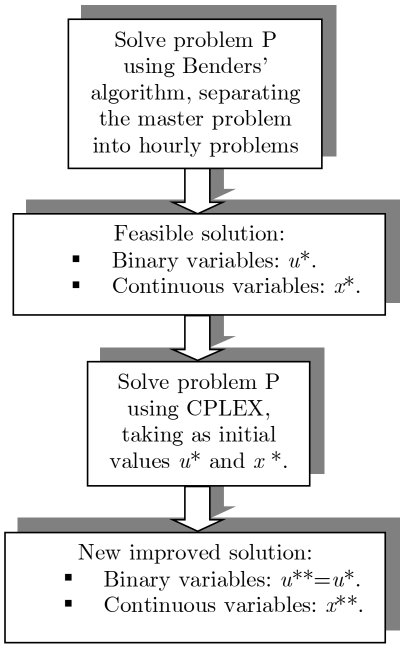


Figure 6.1. The solution process for a realistic two-stage problem.

6.2.1.3 Analysis of the results

Figure 6.2 depicts the five quantities that the company should offer to each of the day-ahead market hourly auctions, according to the results obtained. It also presents the five series of price realizations that might result, given those quantities. If these graphs are compared with the ones represented in chapter 5 for the two-scenario case, it can be seen that this study case yields higher expected prices for the day-ahead market. This can be explained by the presence of additional spot market scenarios that may correspond to situations of higher prices.

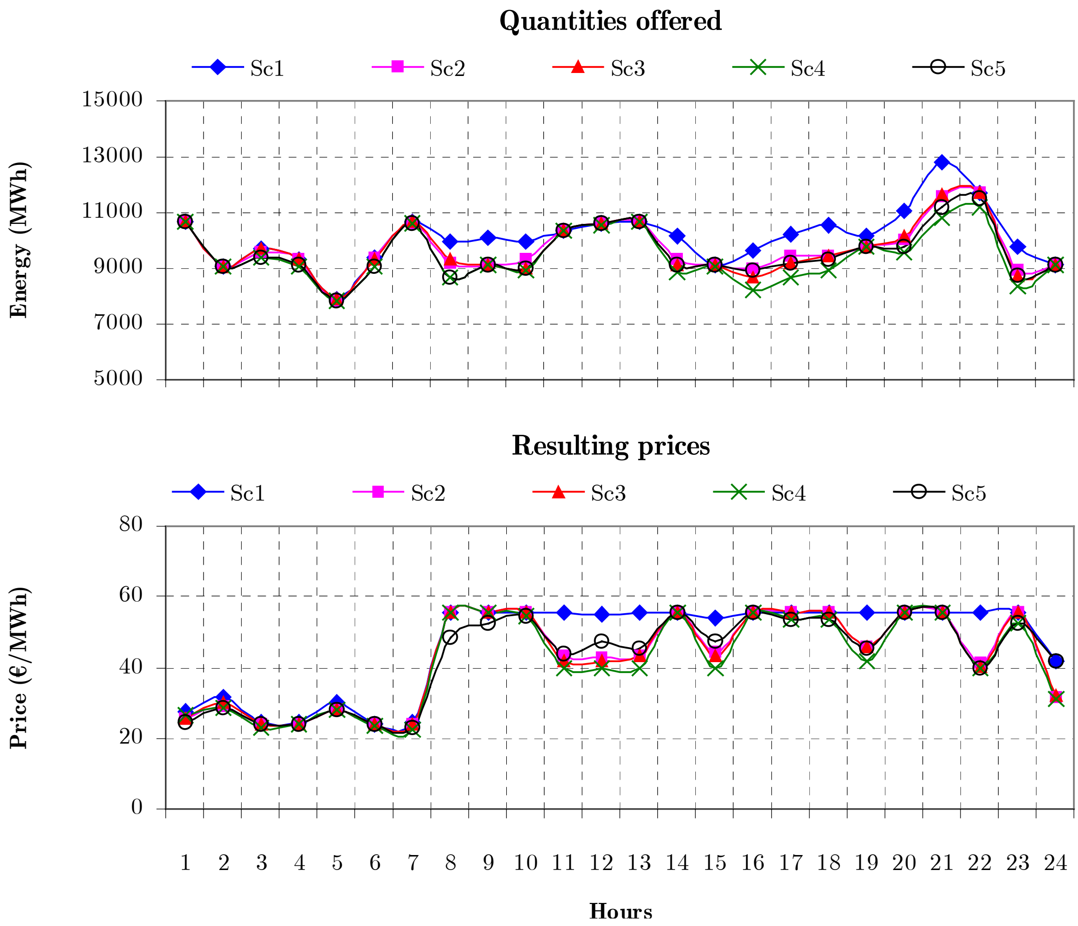


Figure 6.2. Offers for the five-scenario case.

Figure 6.2 emphasizes the fact that the offering strategies developed by the model explicitly consider the links that exist between the different hourly auctions of the same spot market session. These links are due both to generation units' constraints such as ramp rate limits and to the daily management of hydro reserves.

It is also interesting to analyze the shape of the offer curves suggested by the model. Figure 6.3 shows the offer curve built for the twelfth hourly auction, which is almost completely vertical (the details can be seen in Figure 6.4). According to this offer curve, the company is almost certain to sell about 10600 MWh in this auction. In contrast, the clearing price might vary between 40 €/MWh and 56 €/MWh due to the uncertain behavior of the rest of agents. This figures highlight the mutual influence that exists between the offers decided for different spot market scenarios, which is explicitly formulated through the non-decreasing constraints.

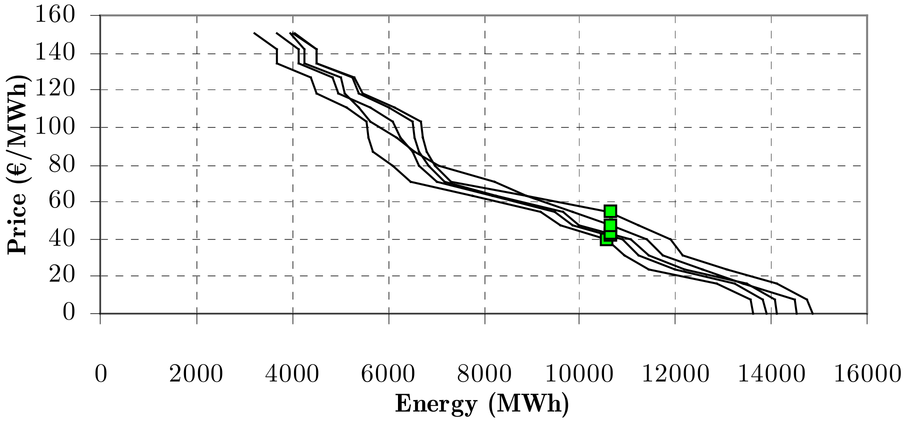


Figure 6.3. Offers for the 12th hourly auction of the day-ahead market in the five-scenario case.

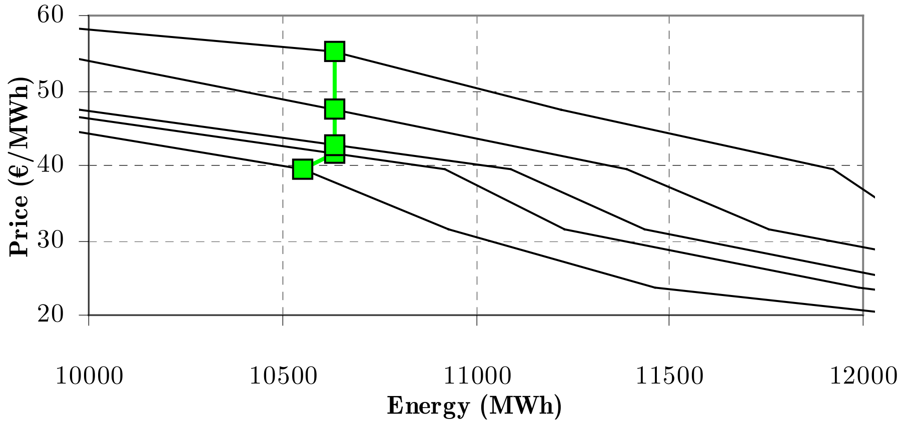


Figure 6.4. Detail of the offers for the 12th hourly auction of the day-ahead market in the five-scenario case.

Figure 6.5 represents the offer curve suggested by the model for the fifth hourly auction. This curve is almost constituted by a unique offer, due to the similarity of the possible residual demand realizations in the vicinity of the solution. For a more detailed representation, see Figure 6.6.

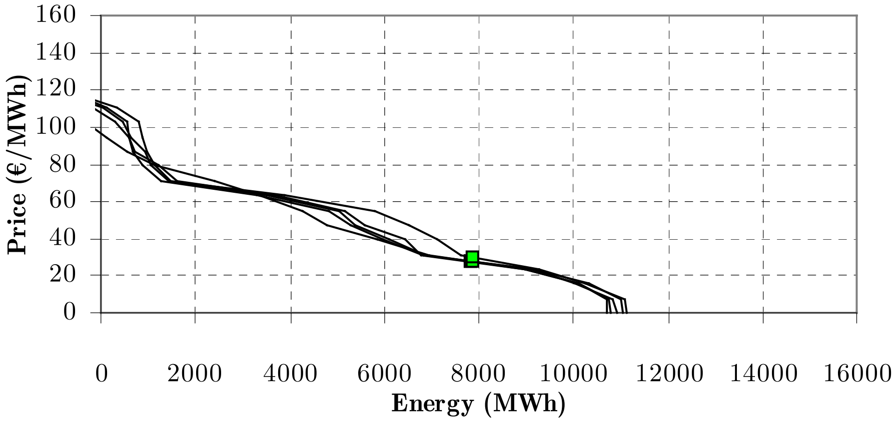


Figure 6.5. Offers for the 5th hourly auction of the day-ahead market in the five-scenario case.

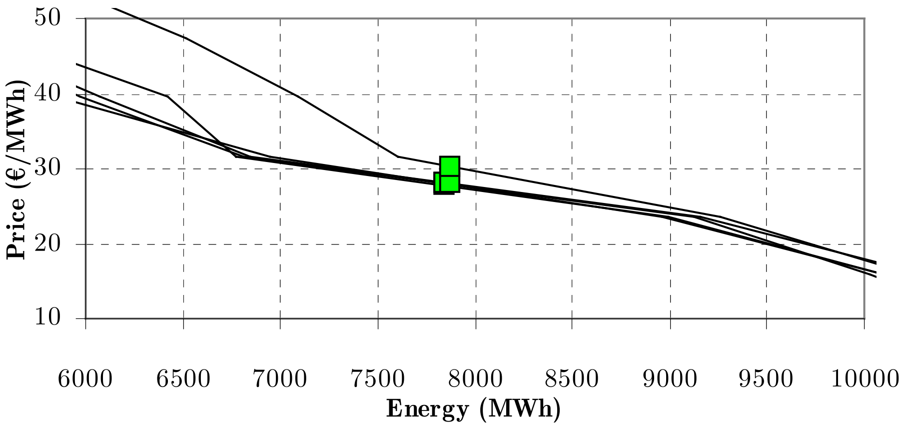


Figure 6.6. Detail of the offers for the 5th hourly auction of the day-ahead market in the five-scenario case.

Figure 6.7 and Figure 6.8 show the recourse actions taken by the model in the twelfth hourly auction of the adjustment market. Similarly, Figure 6.9 and Figure 6.10 depict the recourse actions corresponding to the fifth hourly auction. As can be seen, each recourse action corresponds to one specific realization of the five possible outcomes considered for the day-ahead market in this example. Hence, these decisions do not represent an offering strategy and are not required to constitute a non-decreasing offer curve.

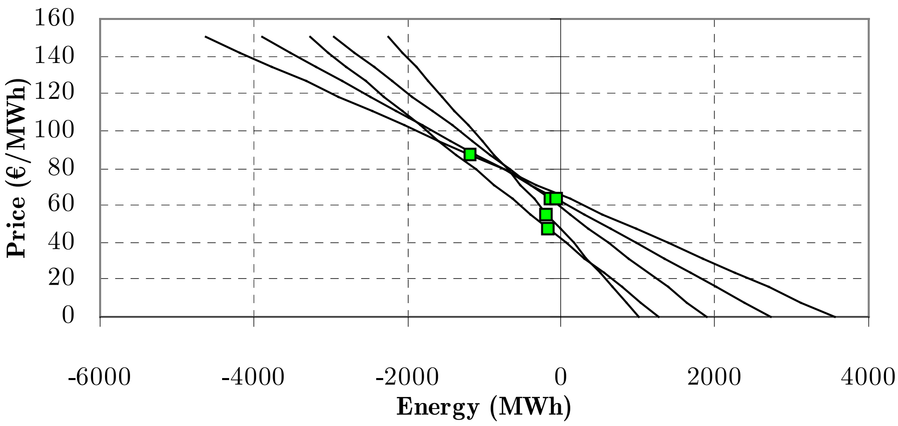


Figure 6.7. Offers for the 12th hourly auction of the adjustment market in the five-scenario case.

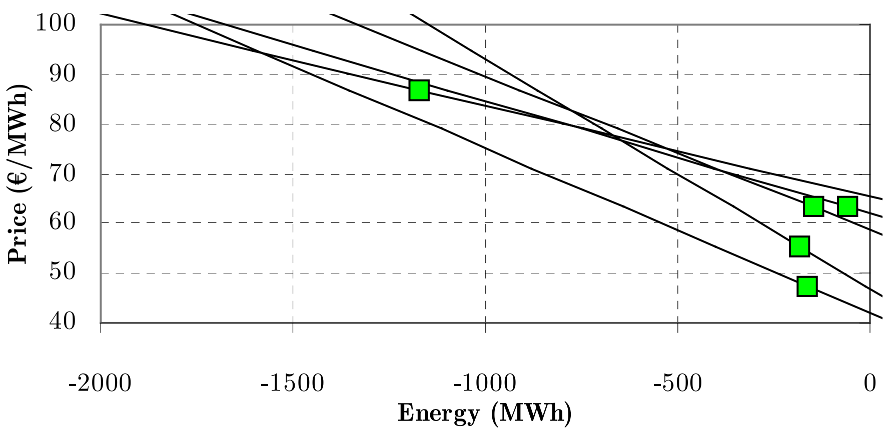


Figure 6.8. Detail of the offers for the 12th hourly auction of the adjustment market in the five-scenario case.

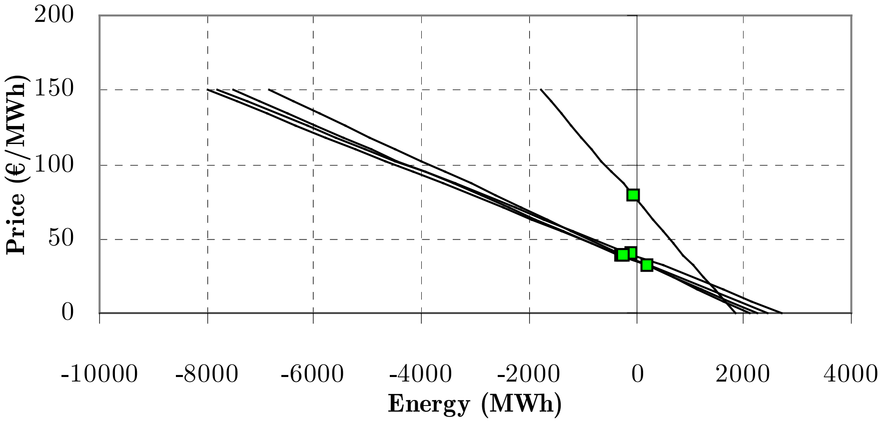


Figure 6.9. Offers for the 5th hourly auction of the adjustment market in the five-scenario case.

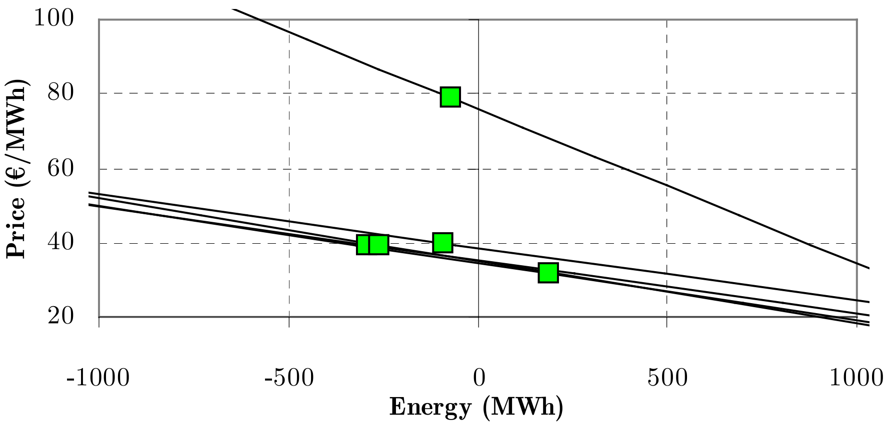


Figure 6.10. Detail of the offers for the 5th hourly auction of the adjustment market in the five-scenario case.

An important conclusion is that this numerical example confirms the validity of the modeling approach adopted to represent the spot market auctions. Indeed, the offers constructed by the model are in accordance with the residual demand curves used as input data. Moreover, whatever the shape of these residual demand curves, the model provides decisions that yield admissible offer curves for the day-ahead market. This example also indicates that Benders' algorithm, in combination with the techniques suggested in chapter 5, provides solutions that are reasonably close to the optimum.

6.2.2 An eleven-scenario case

6.2.2.1 Characteristics of the problem

The encouraging results obtained for the previous study case suggest testing the model with larger problems. We now consider a numerical example with eleven spot market scenarios. The features of the resulting MIP problem are compared in Table 6.3 with those of the previous two examples.

	Equations	Variables	Binary variables	Solution (M€)	Maximum optimality loss	Execution time
Two-scenario problem	13249	13771	474	30.112293	0.00 %	6 s.
Five-scenario problem	43921	41305	4704	32.099927	0.79 %	30 min.
Eleven-scenario problem	99758	91043	10529	32.216254	0.68 %	1 h 30 min

Table 6.3. A comparison of three study cases.

Two interesting conclusions can be derived from the analysis of Table 6.3. On the one hand, the maximum relative loss of optimality due to the use of a simplified solution strategy does not seem to increase with the size of the problem. Indeed, CPLEX estimated a maximum relative loss of optimality for the five-scenario case of 0.79 %, whereas the estimation for the eleven-scenario case amounts to 0.68 % . On the other hand, the relative change observed in the solution when passing from five to eleven scenarios is much less relevant than the one observed when passing from two to five scenarios. This seems to indicate that the accuracy with which the spot market uncertainty can be represented tends to saturate when the number of scenarios considered increases.

6.2.2.2 Numerical resolution

The process followed to obtain a numerical solution for this problem is identical to the one suggested for the five-scenario case. We first use Benders' algorithm with the master problem separated into hourly problems to derive a good feasible solution, as illustrated in Table 6.4. After that we try to solve the full original problem with CPLEX, using this feasible solution as a starting point. CPLEX is unable to improve the values of the binary variables, but slightly improves the continuous variables and provides an upper bound for the loss of optimality, as shown in Table 6.3.

Iteration	Master	Subproblem	Total
IT1	39.323263	-47.956541	-8.633278
IT2	38.607588	-37.948807	0.658781
IT3	37.417309	-25.273184	12.336544
IT4	36.352892	-14.687634	22.272742
IT5	35.447241	-9.093781	27.552025
IT6	34.413846	-5.855816	30.06107
IT7	32.507973	-4.360769	31.660443
IT8	31.969344	-3.959781	32.025566
IT9	31.601185	-3.75295	32.146506
IT10	31.357666	-3.816563	32.15053
IT11	31.356317	-3.802377	32.155819
IT12	31.331983	-3.822391	32.150632
IT13	31.331982	-3.822392	32.150631

Table 6.4. Evolution of Benders' algorithm for the eleven-scenario case.

6.2.2.3 Analysis of the results

The following figures demonstrate that the eleven hourly residual demand realizations used in this numerical example provide a framework to develop offer curves that is both relevant and flexible. Figure 6.11 provides a general perspective of the offer curves developed by the model for the 24 hourly auctions of the day-ahead market.

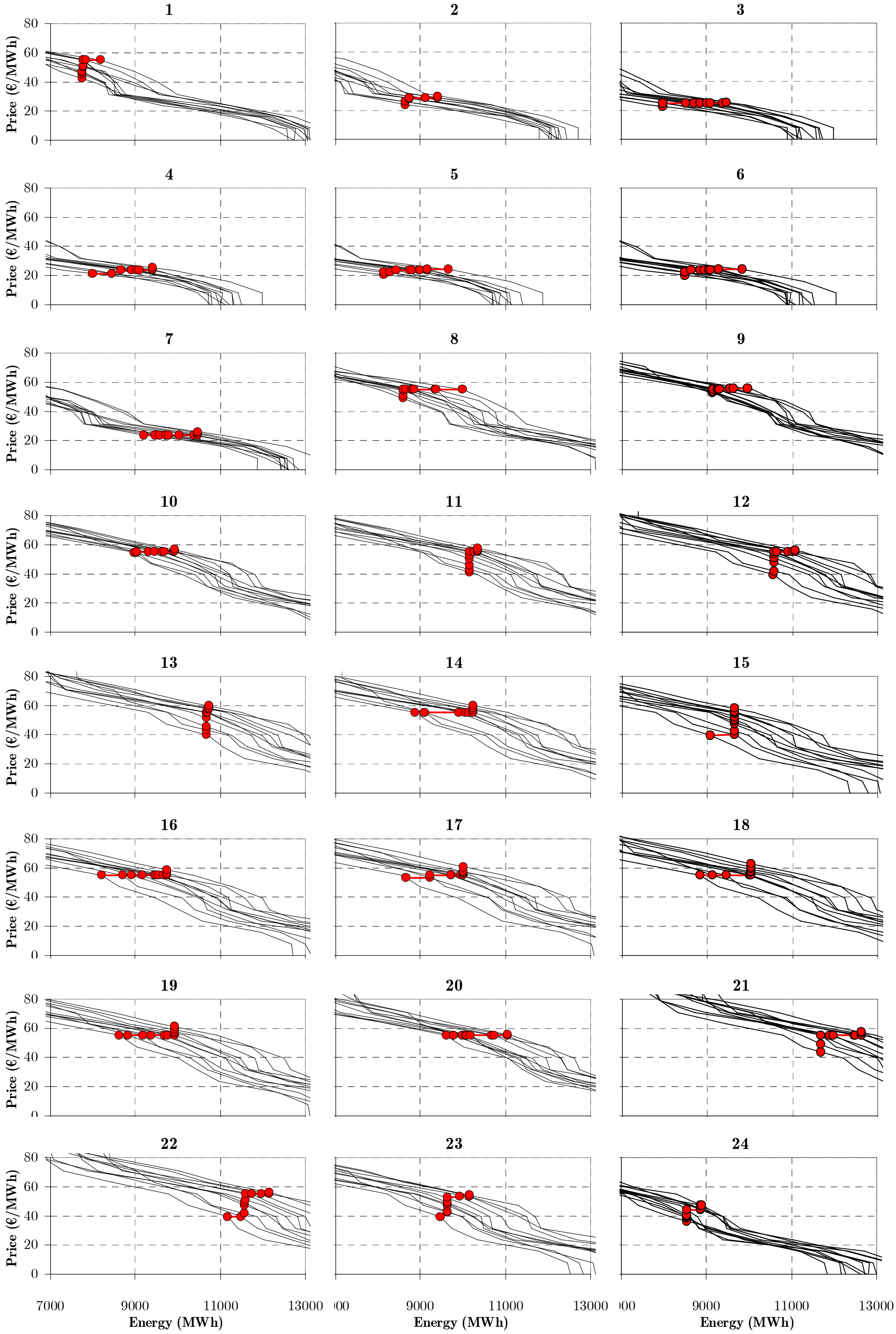


Figure 6.11. Offers for the 24 hourly auctions in the eleven-scenario case.

Figure 6.12 shows the offer curve constructed for the twelfth hourly auction, whose details can be analyzed in Figure 6.13. This curve presents a vertical segment indicating the minimum amount of energy that the company is willing to sell in this auction due the value of its market-share. It also has an horizontal segment defining a market-clearing price for which the company is willing to sell an extra amount of 500 MWh.

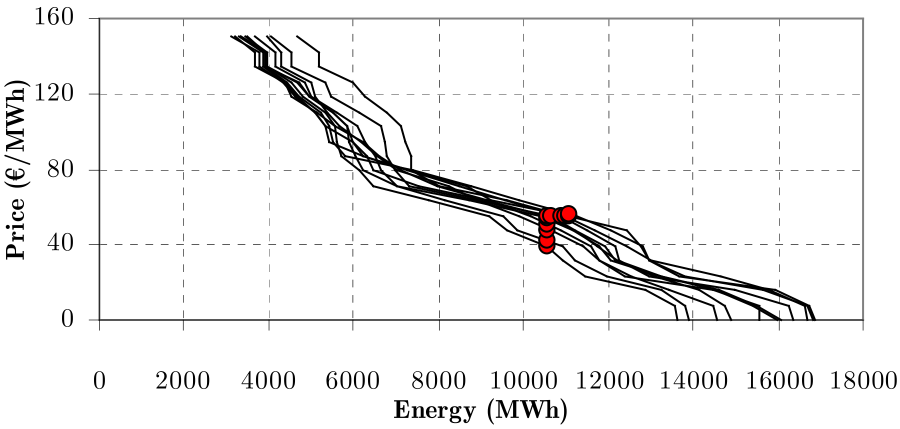


Figure 6.12. Offers for the 12th hourly auction in the eleven-scenario case.

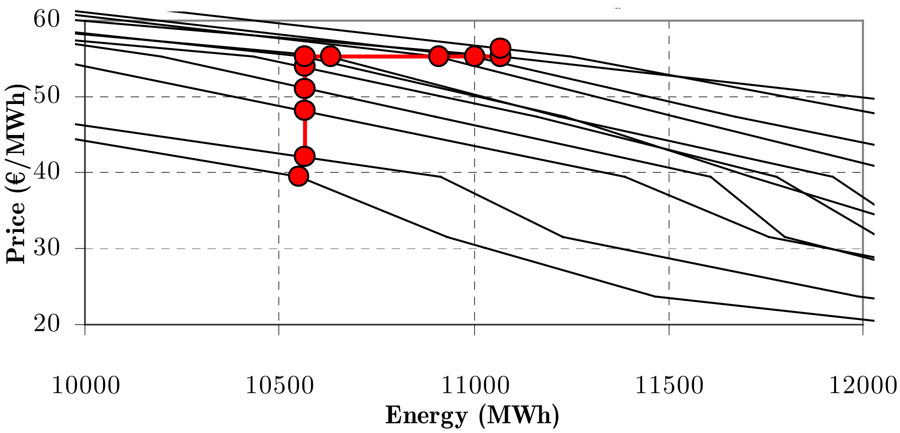


Figure 6.13. Detail of the offers for the 12th hourly auction in the eleven-scenario case.

The general features of the offer curve constructed for hour 5 contrast with those observed in the one corresponding to hour 12, as can be seen in Figure 6.14 and Figure 6.15. This curve seems to suggest that the company is not very worried about its sales in the auctions corresponding to off-peak hours. It is an almost horizontal offer curve that can lead to a variety of outcomes in terms of the quantity sold by the company with slight variations in the clearing price.

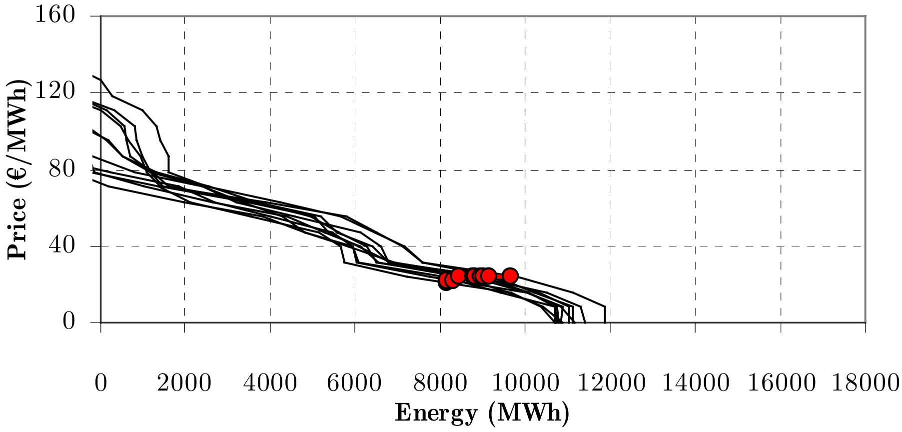


Figure 6.14. Offers for the 5th hourly auction in the eleven-scenario case.

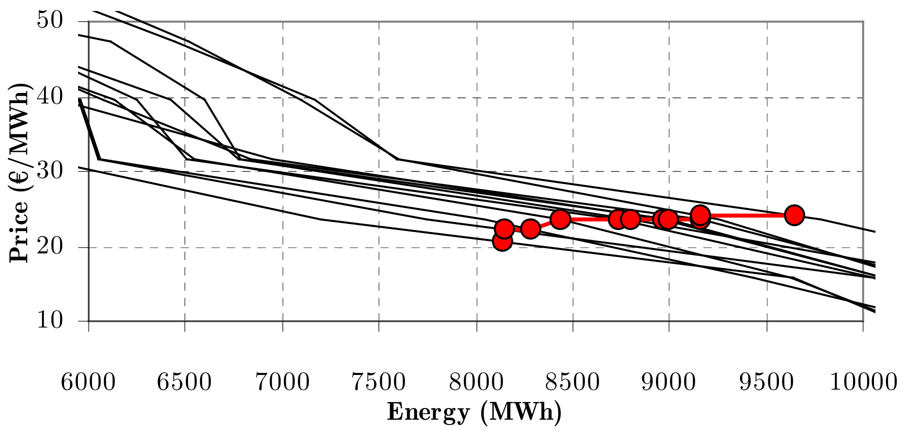


Figure 6.15. Detail of the offers for the 5th hourly auction in the eleven-scenario case.

6.2.2.4 A comparison with the real offers

Even though the generation company of study is fictitious, the actual offers that were submitted on October 24th 2001 corresponding to each generating unit are publicly available. Based on this information we can construct the offer curve that the generation company of study "submitted" that day and compare them with those developed by the model. Figure 6.16 and Figure 6.17 establish this comparison. As can be seen, the offer curves suggested by the model are quite similar to those actually observed in the day of study. Obviously, this similarity is mainly due to our choice of the hourly market-share values. If higher values were selected, the mode would offer larger amounts of energy at a lower prices and vice versa. In any case it is reassuring to verify that the solutions provided by the model can mimic those actually observed in a real spot market.

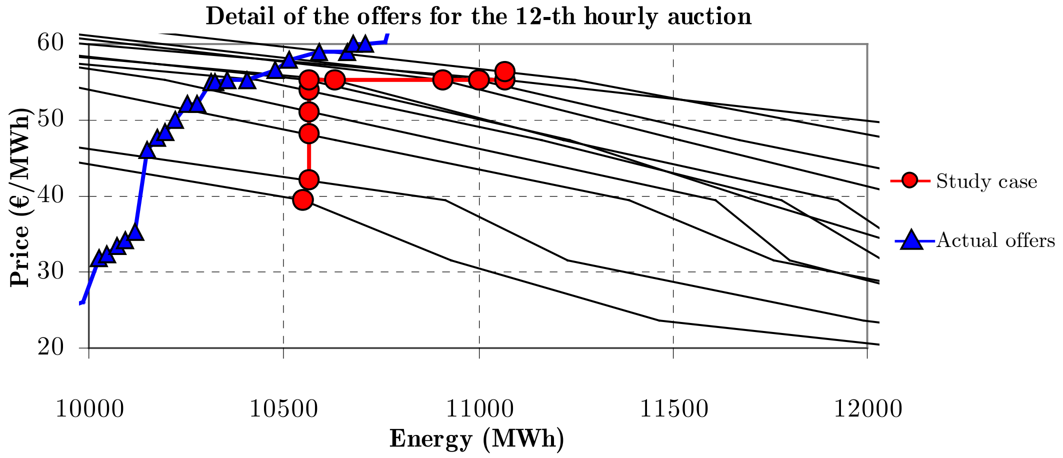


Figure 6.16. A comparison of the offers developed by the model and the real offers for the 12th hourly auction.

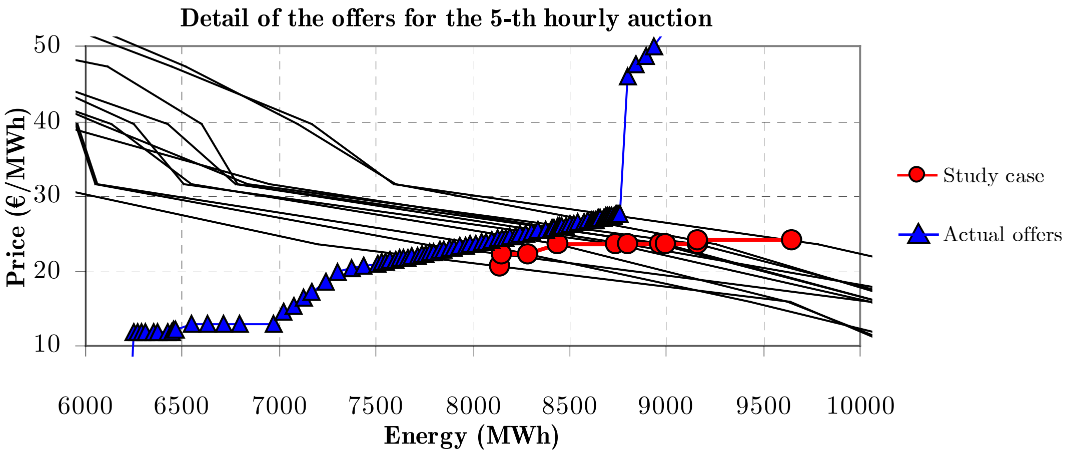


Figure 6.17. A comparison of the offers developed by the model and the real offers for the 12th hourly auction.

6.2.2.5 Influence of the non-decreasing constraints

The presence of vertical and horizontal segments in the offer curve developed for the twelfth hourly auction (Figure 6.12) seems to indicate that, if it were possible, the company would rather use an offer curve with both increasing and decreasing segments. Figure 6.18 illustrates this idea. If the non-decreasing constraints are omitted, the model suggests decisions for the day-ahead market that do not constitute a valid offer curve. In this particular case, it seems that two alternative offer curves could be constructed. The first one would simply consist of an horizontal segment. The second one would be formed by a vertical segment and an horizontal segment. In principle, it is difficult to decide which of these two alternatives is the best. Indeed, only by including the non-decreasing constraints in the model can we make the correct choice.

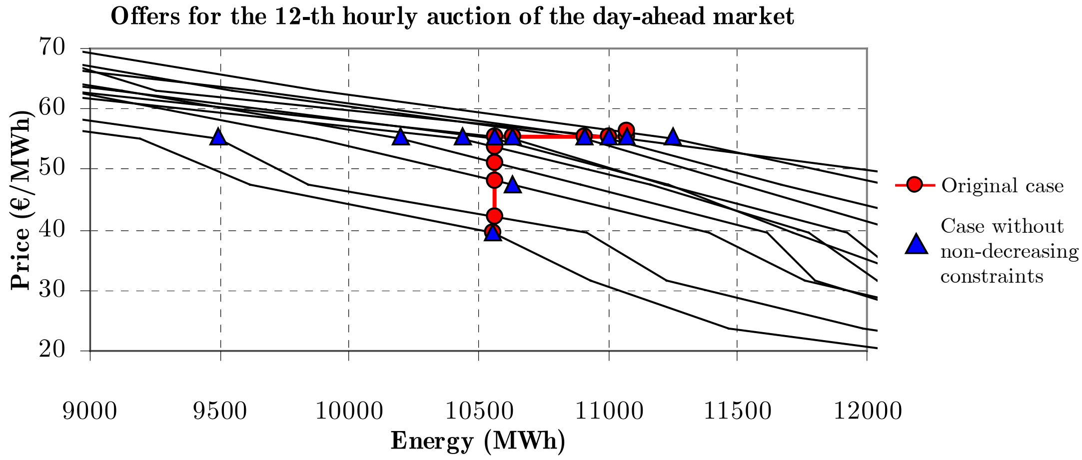


Figure 6.18. Influence of the non-decreasing constraints on the offers for the 12th hourly auction.

6.2.2.6 Influence of the number of scenarios

It is also interesting to determine to what extent does the number of scenarios affect the shape of the offer curves decided by the model. Figure 6.19 represents the offer curves obtained for the twelfth hourly auction in the three numerical examples addressed hitherto. As can be seen, the offer curve constructed for the two-scenario case deviates importantly from the offer curves corresponding to the five-scenario case and the eleven-scenario case. This is due to the inclusion of several possible high-price residual demand realizations in the latter cases. Actually, the residual demand curve that the company faced in this particular auction lied between the two curves considered for the two-scenario case, so it is likely that the offer curve obtained for this case be the most appropriate. Something similar happens with the fifth hourly auction, as shown in Figure 6.20. Nevertheless, it is clear that the offering strategy developed for the eleven-scenario case is more robust than the one adopted for the two-scenario problem, even if the scenarios contemplated for the two-scenario case happen to be very close to the realization of uncertainty.

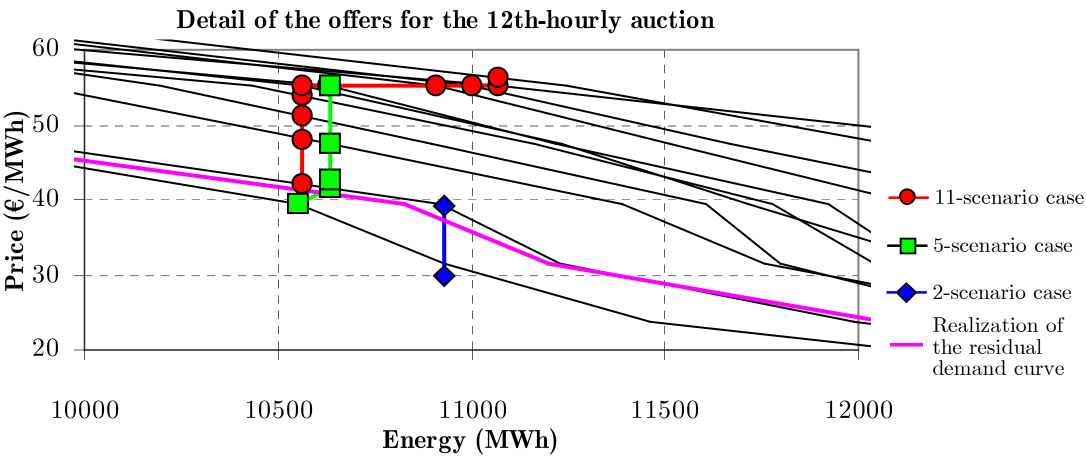


Figure 6.19. Influence of the number of scenarios on the offers for the 12th hourly auction.

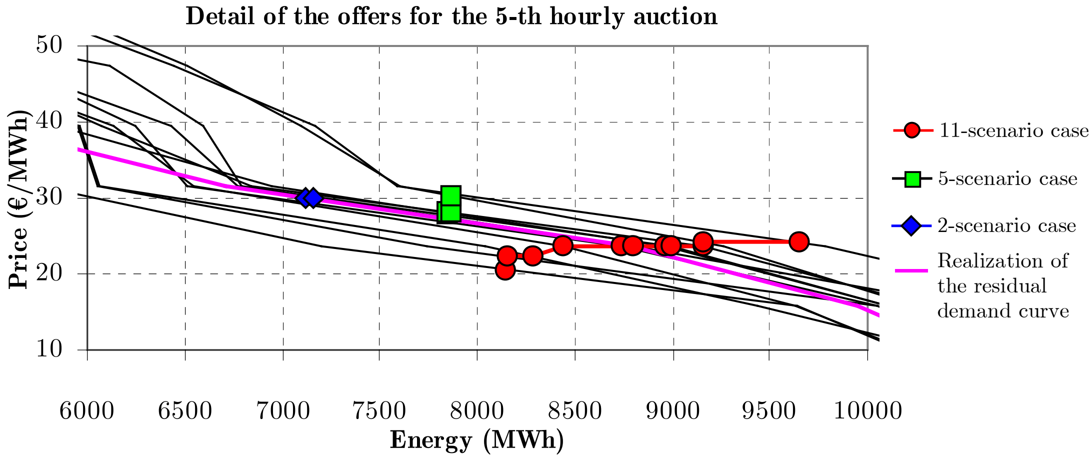
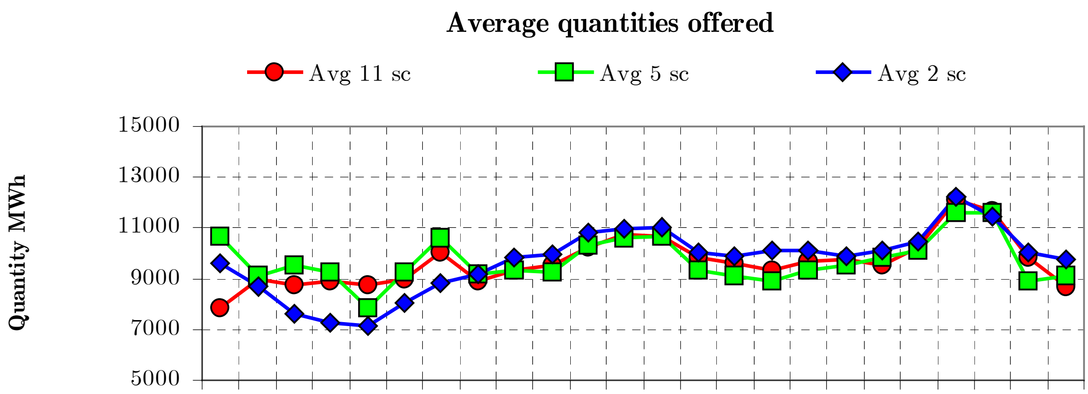


Figure 6.20. Influence of the number of scenarios on the offers for the 5th hourly auction.

In terms of expected sales and expected clearing prices (Figure 6.21), the two-scenario case is characterized by high expected sales together with low expected clearing prices in the on-peak hours as well as low expected sales and high expected clearing prices in the off-peak hours. On the other hand, the five-scenario case presents high expected sales in the off-peak hours and low expected sales in the on-peak hours. Finally, the eleven-scenario case leads to high expected clearing prices in the on-peak hours.



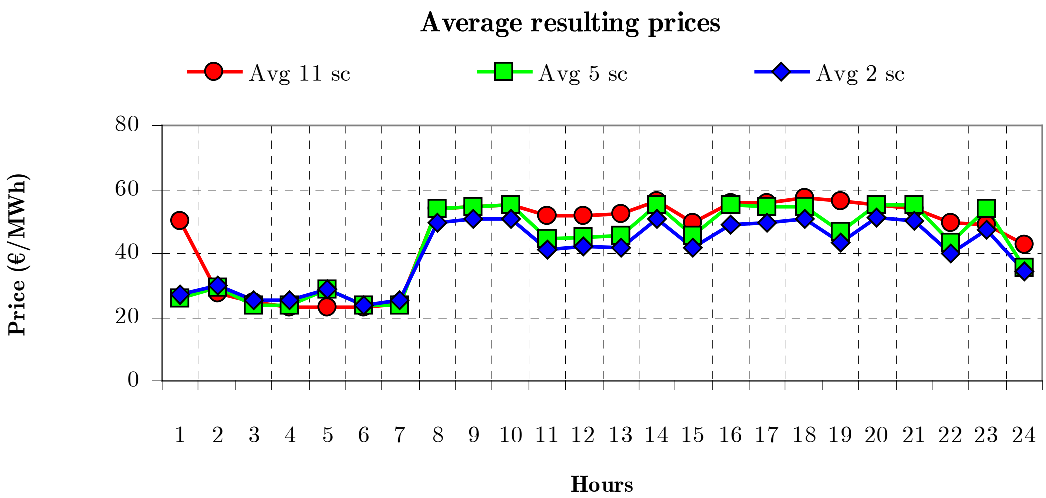


Figure 6.21. Influence of the number of scenarios on the expected results.

6.2.3 Sensitivity analysis

The generation company whose case is studied in the examples of this thesis obtains a volume of revenues in the day-ahead market of about 8 M€. It is not likely that such a company would develop its offering strategies with the model developed in this thesis without previously guaranteeing that the model adapts well to a variety of circumstances concerning its portfolio. Examples of these circumstances are the unavailability of a specific generating unit, changes in the amount of hydro reserves that can be used in the day of interest, the existence of an open position in a contract for differences, etc. It is impossible to include here the exhaustive sensitivity analysis that would be required to validate the model for commercial use. Nevertheless, it is interesting to solve a number of examples with slight differences in some of these aspects, so as to evaluate the variations observed in the solutions. The previous eleven-scenario study case is used as a reference to measure these differences.

6.2.3.1 Market-share value

One of the most relevant elements of the input data used for these numerical examples is the value that the company gives to its market-share in each of the hourly auctions that constitute the spot market. The influence of this parameter is determinant for the offering strategies developed by the model. To illustrate this influence, a case has been solved in which the value given to the company's market-share is null. Figure 6.22 compares the expected outcome of the day-ahead market in this case with that of the base case.

Figure 6.22. Influence of the market-share value on the expected day-ahead market results .

As can be seen, when the value of the company's market share is assumed to be equal to zero, the company's expected sales are significantly lower than those expected in the base case. Moreover, the profile of these expected sales does not correspond with the typical chronological evolution of demand. Indeed, the model decides to withdraw energy from the on-peak hours reaching expected production levels similar (or even lower) than those expected for off-peak hours. This causes a substantial increase on the expected market-clearing prices, rendering important benefits to the company and even more important to its competitors.

The shapes of the company's offer curves for both numerical examples are so dramatically different that they can hardly be compared. Figure 6.23 shows the offer curves corresponding to the twelfth hourly auction. The offering strategy developed for the case in which no value is given to the company's market-share aims at clearing prices almost three times higher than those expected in the base case. A similar effect is observed in the offers for the fifth hourly auction (Figure 6.24).

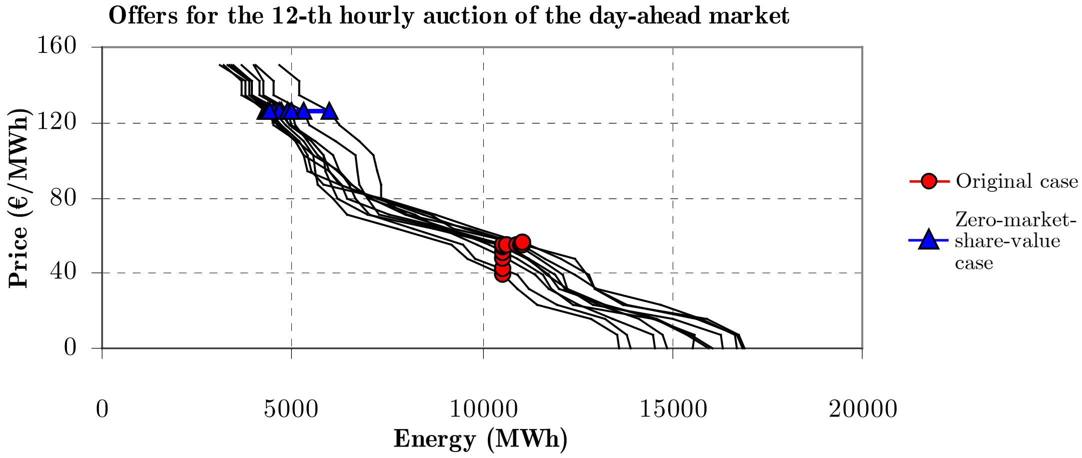


Figure 6.23. Influence of market-share value on the offers for the 12th hourly auction.

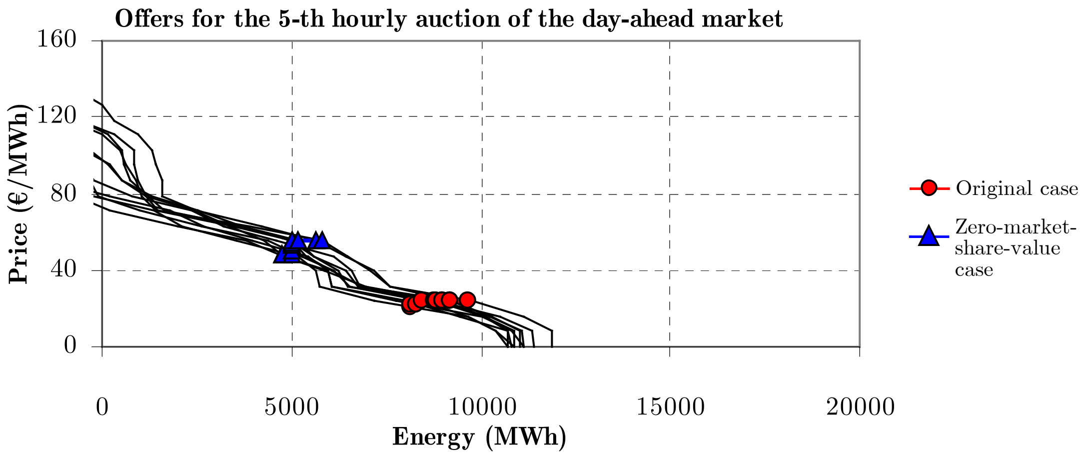


Figure 6.24. Influence of market-share value on the offers for the 5th hourly auction.

The remarkable influence that the value of the company's market share exerts on the offer curves developed by the model reveals the strategic role played by this parameter. Indeed, this value must be calculated with care, making use of medium-term models that provide optimum guidelines for the annual operation of the generation company. Given a certain value for this parameter, the model developed in this thesis is able to seize some of the short-term opportunities that may arise and at the same time pursue the company's medium and long-term objectives. An additional conclusion that can be drawn from this example is that the value of the company's market share is not an essential information in order to develop an offering strategy with the methodology suggested in this thesis. It has been shown that an offering strategy can actually be obtained without this information, although the results will typically not lead toward the maximization of the company's long-term profits.

6.2.3.2 Hydro reserves

Another result that should be provided by a medium-term model and that is assumed to enter as input data in the model developed in this thesis is the amount of reserves that each hydro unit can use during the period of interest (in this case, one day). This information conditions to a certain extent the offering strategy developed by the model. To illustrate this idea, two different situations have been considered. In the first one, an extra 50 % of hydro energy is assumed to be available in each reservoir. In contrast, the second case assumes that no hydro energy can be used. The results for these two situations are illustrated in the following figures. Figure 6.25 compares the expected outcome of these two cases with the results of the base case. As can be seen, when extra hydro reserves are available, the model expects to sell more energy during the on-peak hours. On the contrary, when no hydro reserves are available the model reduces the company's expected energy sales during the on-peak hours.

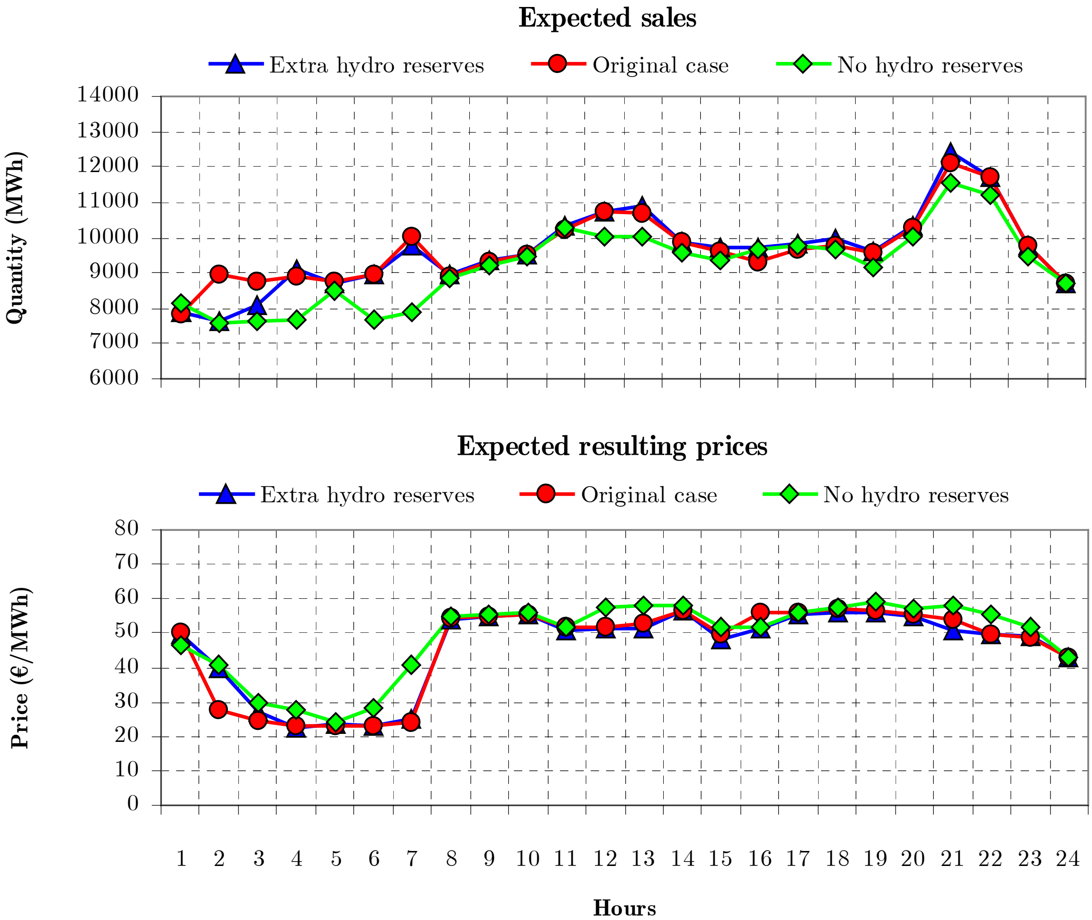


Figure 6.25. Influence of hydro reserves on the expected day-ahead market results .

The increment observed in the company's expected sales for the case with extra hydro reserves is due to a change in the company's offering strategy. As is shown in Figure 6.26, when extra hydro reserves are available, the vertical segment of the offer curve for the twelfth hourly auction suffers a translation to the right of almost 100 MWh. The horizontal segment remains invariable, which justifies that the expected sales for this particular auction are almost the same as in the base case. In contrast, when no hydro reserves are available, the offer curve experiences a translation to the left as well as a significant change in its shape.

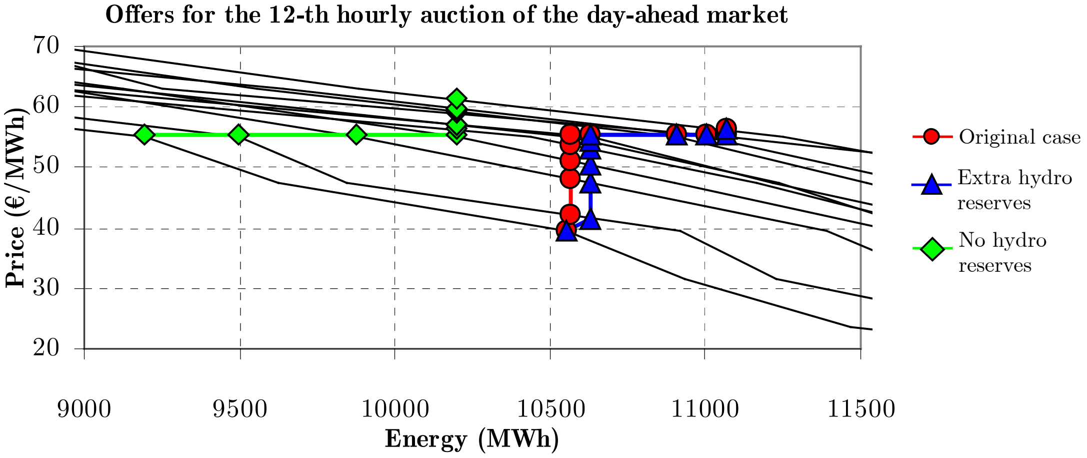


Figure 6.26. Influence of hydro reserves on the offers for the 12^{th} hourly auction.

The effect of the variations in the available hydro reserves is almost unnoticeable in the offers curve corresponding to the fifth hourly auction, as depicted in Figure 6.27.

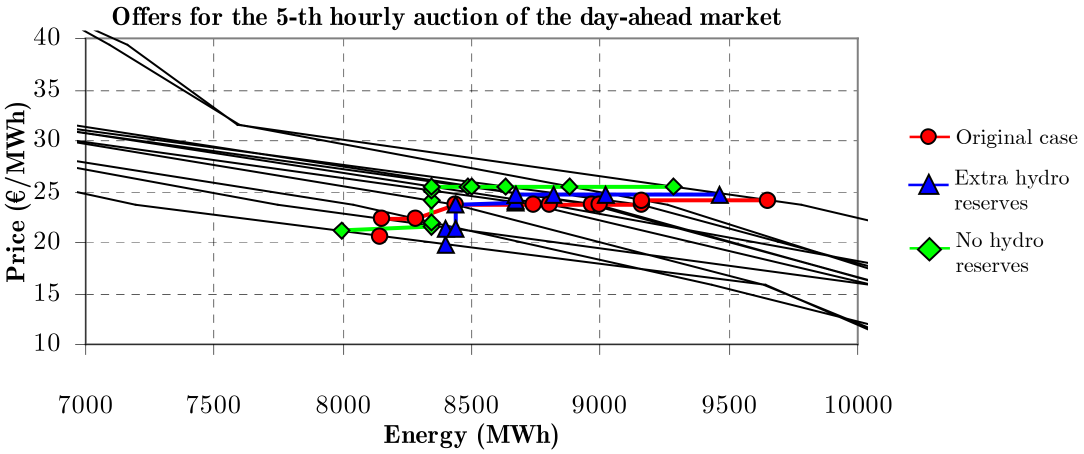


Figure 6.27. Influence of hydro reserves on the offers for the 5^{th} hourly auction.

In conclusion, a variation in the available hydro resources for the period of study leads to consistent changes in the offering strategy developed by the model for the day-ahead market. Indeed, if more hydro energy can be used, the model offers higher quantities in the hours and at the prices that are more convenient.

6.2.3.3 Unit availability

The availability of generating units is also a factor with relevant influence on the company's offering strategy. The probability of suffering the unplanned outage of a generating unit is not negligible at all and a generation company must be able to adapt its offer curves so as to minimize the negative effects of this failure.

Let us assume that the company's nuclear unit G1 has been forced to stop some hours before the day-ahead market session of study. The company must correct its offering strategy in order to take this circumstance into account. With the model developed in this thesis, it suffices to declare the nuclear unit unavailable. Figure 6.28 depicts the significant reduction that this unavailability produces in the company's expected sales, particularly in the on-peak hours. This, in turn, causes an increase in the expected clearing prices. It is interesting to notice that the decrease in the expected output observed during on-peak hours is greater than the unit's generating capacity. This means that the model not only has adapted the offering strategy to this capacity reduction but has also reallocated hydro resources, due to the interaction between the shape of the residual demand curves in the vicinity of the new solution and the value of the company's market share. This effect highlights the adequacy of our approach.

Figure 6.28. Influence of units' availability on the expected day-ahead market results.

As in other numerical examples, it is also interesting to check the effect that the unavailability of a nuclear unit has on the shape of the offer curves constructed by the model. Figure 6.29 shows the variations introduced in the offer curve for the twelfth hourly auction. The most evident change is a shift of the curve more than 1000 MWh to the left. However, the model also introduces changes to adapt the shape of the offer curve to the new relevant region of the possible residual demand realizations. This reinforces the validity of our approach.

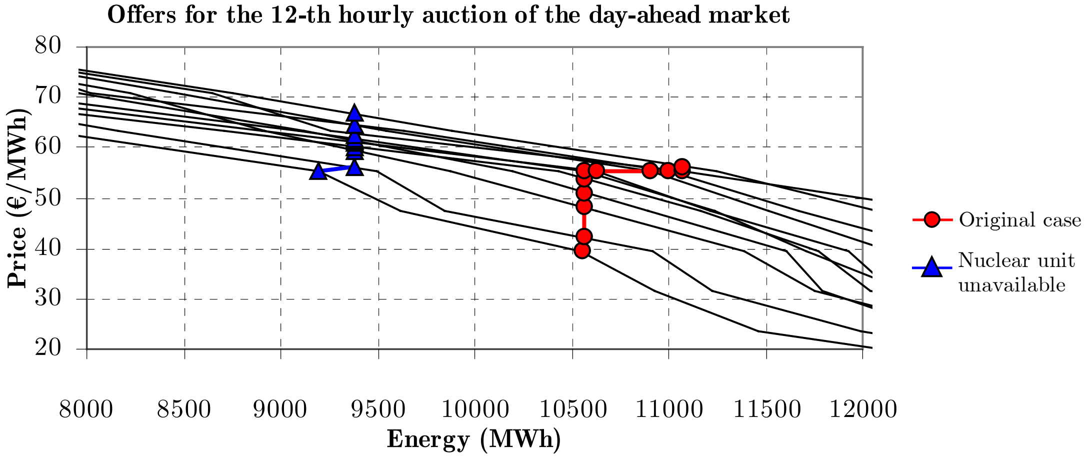


Figure 6.29. Influence of units' availability on the offers for the 12th hourly auction.

The variations observed in the offer curve for the fifth hourly auction are less significant than those identified for the previous one. A simple translation to the left is observed in Figure 6.30. The reason is that the residual demand realizations for this fifth auction are almost parallel straight lines in a range of about 2000 MWh.

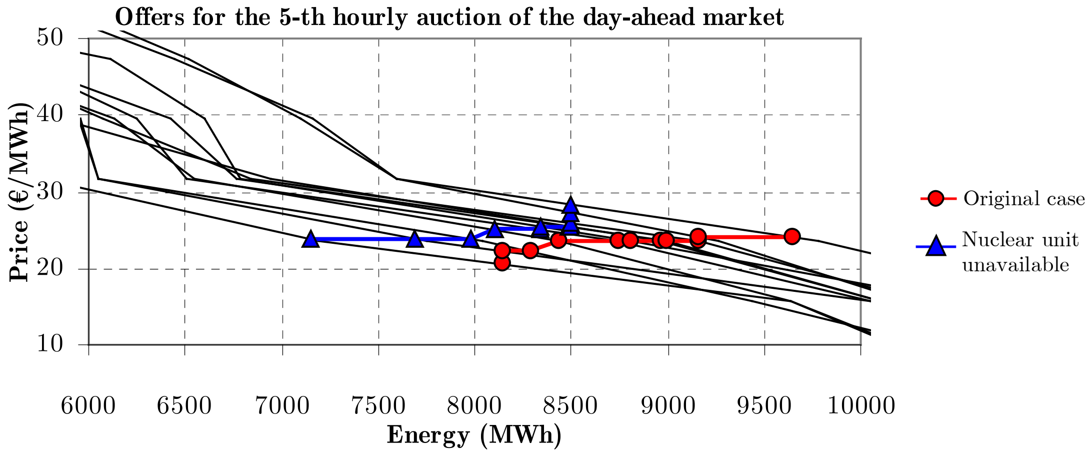


Figure 6.30. Influence of units' availability on the offers for the 5th hourly auction.

To sum up, the model has proved to be useful for those cases in which the generation company suffers the forced outage of one of its units. Not only does it suggest changes in the company's offering strategy that can be intuitively interpreted. It also introduces changes in the shape of the offer curves to adapt to the characteristics of the possible residual demand realizations in the new region of interest.

6.2.3.4 Contracts for differences

As was explained in chapter 4, the portfolio of a generation company that takes part in an electricity spot market is constituted not only by its generation units, but also by its open positions in physical bilateral contracts and electricity derivatives such as contracts for differences or options. These derivatives are frequently indexed to the spot price of electricity, thus affecting the strategy followed by the company in the spot market. To illustrate how the model developed in this thesis is able to develop offering strategies that take into account the company's open positions in derivative products, we consider the case of a contract for differences. This contract is an agreement to exchange the difference between a fixed price (50 €/MWh) and the day-ahead market clearing prices corresponding to the time interval that goes from the eleventh to the twenty-second hourly auction. The amount of energy affected by this contract is 3000 MWh in each auction.

As was indicated in chapters 4 and 5, a contract for differences reduces the incentive of a generation company to raise the spot price of electricity. This is the result observed in Figure 6.31, where the company's expected sales increase in almost all the hours affected by the CfDs, causing a reduction of the expected day-ahead market clearing prices. The influence on other hours is an indirect effect that is difficult to justify.

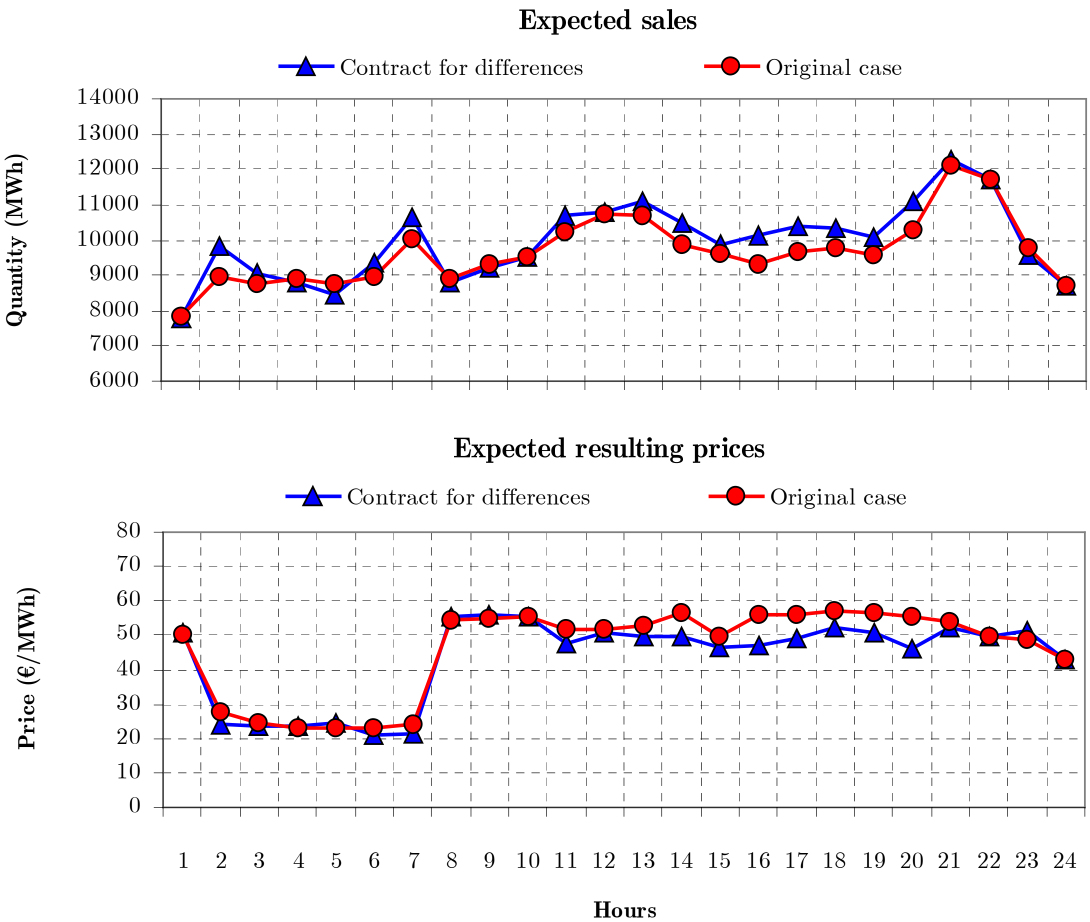


Figure 6.31. Influence of a contract for differences on the expected day-ahead market results.

The influence of the contract for differences on the offer curve for the twelfth hourly auction is very similar to the effect of an increase of the available hydro resources. A translation of the vertical segment of the offer curve is induced, implying an increase of the company's expected sales for that hourly auction. In this case we omit the analysis of the offer curve obtained for the fifth hourly auction.

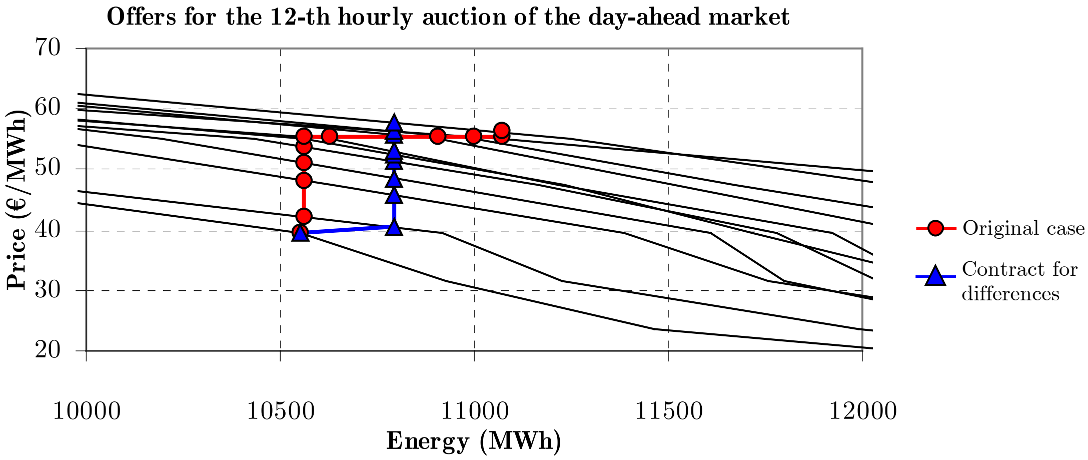


Figure 6.32. Influence of a contract for differences on the offers for the 12th hourly auction.

To summarize, the model is also able to adapt its decisions to the presence of open positions in the company's portfolio. This is an issue of great importance, particularly in those wholesale electricity markets with important trading volumes in electricity derivatives, such as the Nord Pool.

An aspect that the model does not account for is the influence that current spot prices exert on the price of electricity derivatives. Given that the spot price is usually taken as a reference, generation companies may have an incentive to artificially raise it in order to improve the conditions of these long-term contracts. This strategy, however, can be seen as a form of market power abuse and as such should not be sustainable in the long term.

6.2.4 Conclusions

In this section we have presented a number of numerical examples to show the potential both of the model developed in this thesis and of the solution strategies adopted for its numerical resolution. All the examples focus on the same generation company and on the same spot market session, thus facilitating the comparison of the results obtained.

In spite of the complexity of the large-scale MIP problems that have to be solved, the solution strategy suggested in chapter 5, based on the combination of Benders' decomposition and two ad hoc techniques, has proven to be powerful and robust enough to provide reasonably good solutions for a battery of real-size numerical examples.

6.3 Weekly unit-commitment schedules under spot market uncertainty

In this section we solve a numerical example of the weekly stochastic unit-commitment problem faced by the generation company of study. Input data is taken from the operation of the Spanish electricity spot market corresponding to the week that goes from October 22nd to October 28th 2001, as explained in appendix C.

6.3.1 A 16-scenario case

The uncertainty faced by the generation company of study in this numerical example is represented by means of a sixteen-scenario tree. This scenario tree is due to the uncertain outcome of each of the seven spot market sessions considered.

It has been justified that Lagrangian relaxation is an adequate approach to solve the mathematical program that results when the weekly stochastic program is formulated, given the generalized presence of binary variables. Figure 6.33 shows the evolution of the Lagrangian relaxation algorithm for this particular case. After 250 iterations the objective function of the master dual problem is 240.472157 M€, whereas the value of the dual function is 240.482718 M€. The relative difference between both values is 0.0044 %, which is low enough to stop the process. Each iteration takes about 8 minutes of CPU time, which gives an idea of the computational effort required (about 35 hours).

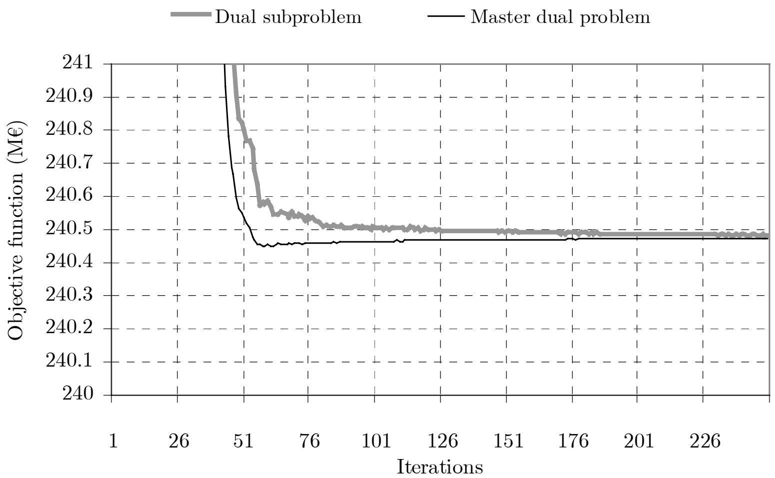


Figure 6.33. Evolution of the LR algorithm for the weekly stochastic problem.

Figure 6.34 depicts the energy sold by the company in each hourly auction of the day-ahead market for the different scenarios of the week of study.

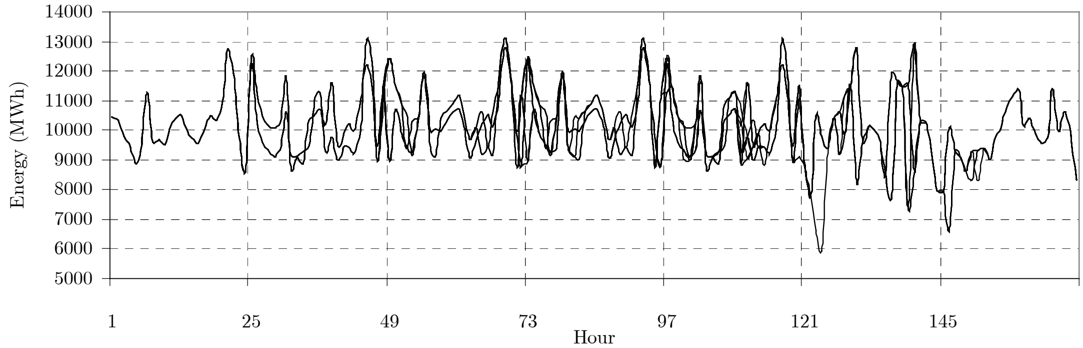


Figure 6.34. Energy sold by the company in the day-ahead market for each weekly scenario.

The high variability observed in the company's sales on Thursday and Friday illustrates the idea of stochastic optimization: depending on the outcome of the uncertain factors, the decisions taken by the model may present significant differences. Figure 6.35 presents the same information in a more intuitive fashion, explicitly representing the sixteen-scenario tree.

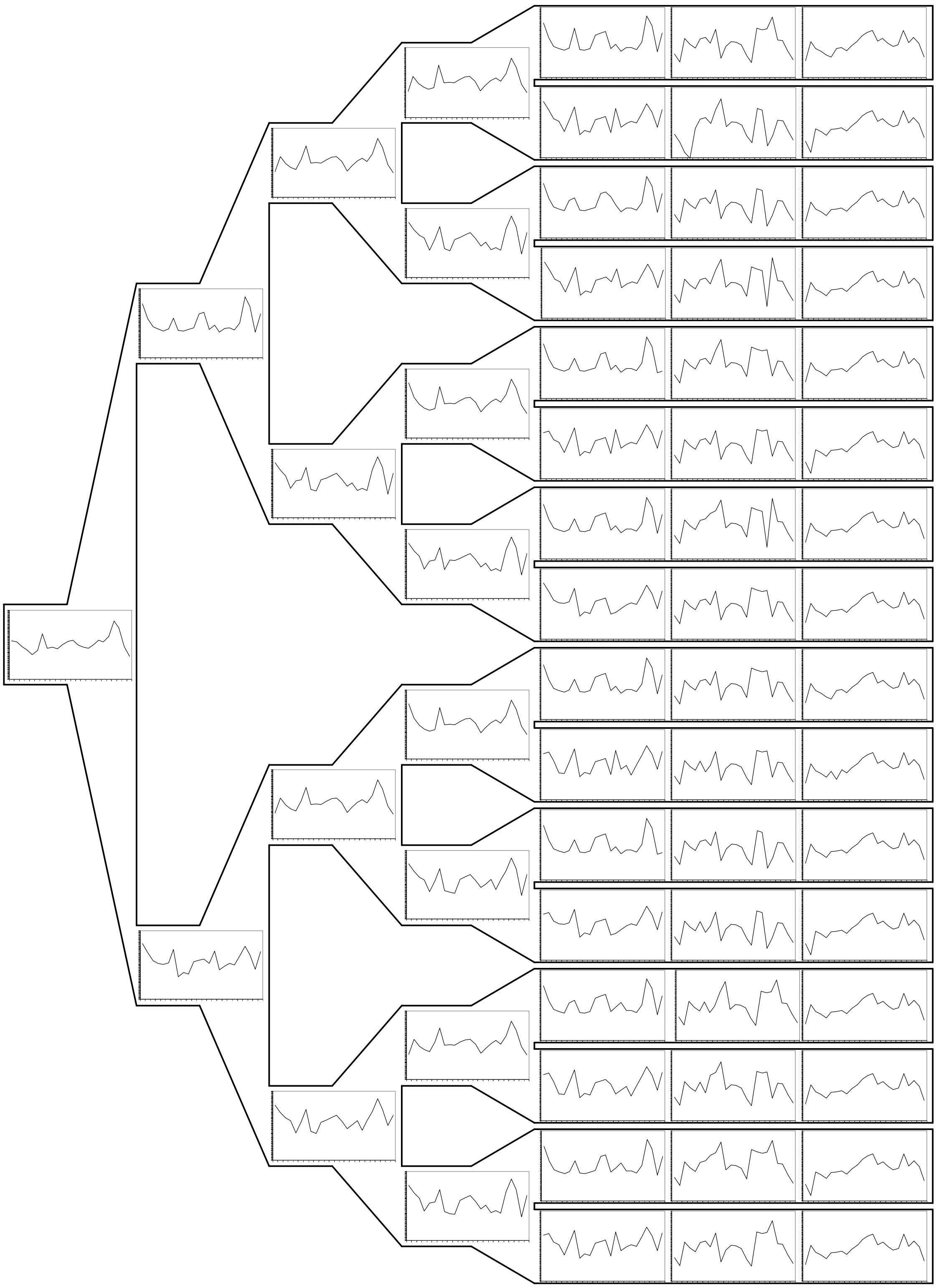


Figure 6.35. Tree constituted by the company's sales in the day-ahead market.

Figure 6.36 shows the detail of the energy sold by the company in the two possible realizations considered for the day-ahead market session corresponding to the second day (Tuesday).

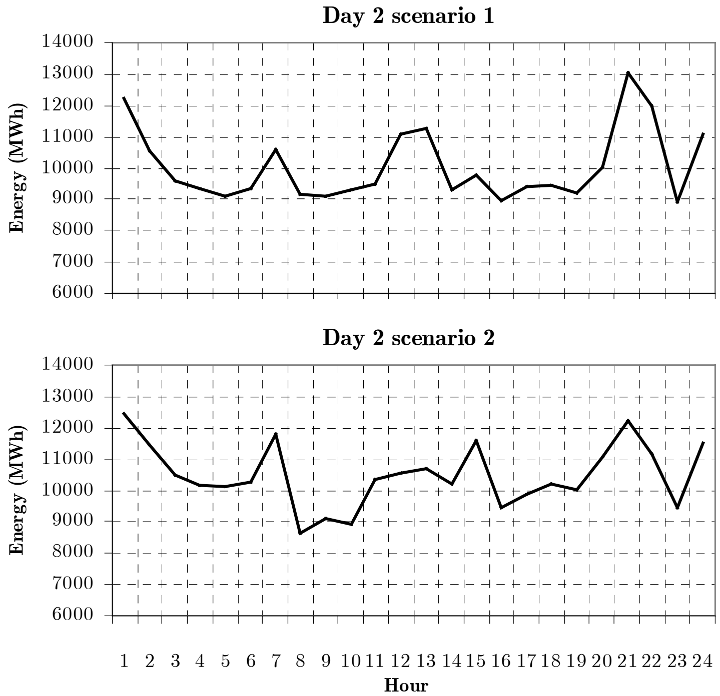


Figure 6.36. Detail of the day-ahead market company's sales for the two nodes corresponding to Tuesday.

As a result of the sales decided by the model for the day-ahead market, the weekly sequence of market clearing prices depicted in Figure 6.37 is obtained. The variability observed in the company's sales is also observed in the day-ahead market clearing prices. However, it is interesting to notice that the model focuses on several price levels. This is due to the particular features of the Lagrangian relaxation method.

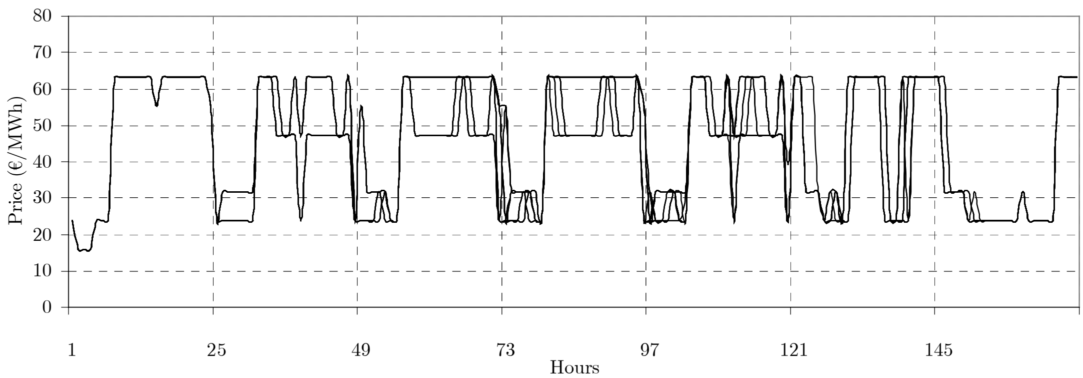


Figure 6.37. Day-ahead market clearing prices for each scenario.

Figure 6.38 presents a detail of the market-clearing prices obtained for the two possible outcomes of the day-ahead market session corresponding to Tuesday.

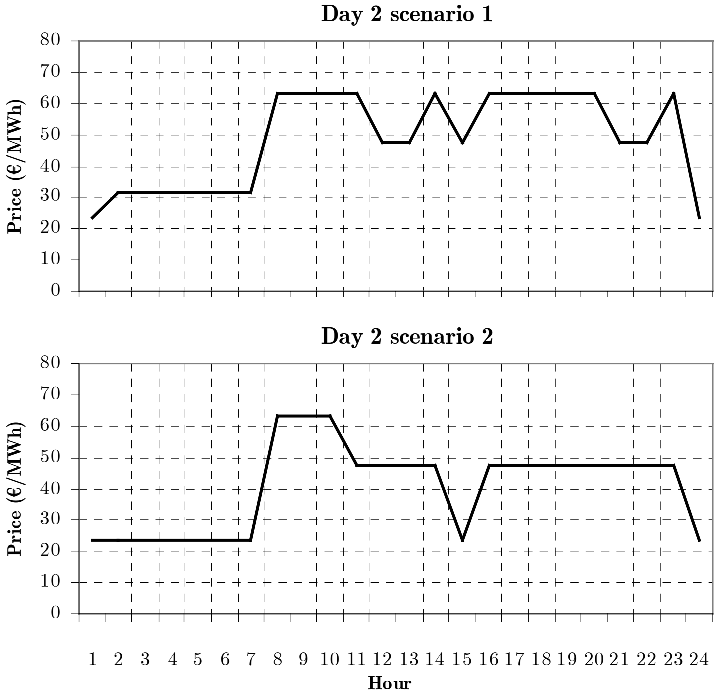


Figure 6.38. Detail of the day-ahead market clearing prices for the two nodes corresponding to Tuesday.

The results obtained for the adjustment market are similar to those reported for the day-ahead market. A great variability is observed in the company's position for the adjustment market, according to Figure 6.39. The adjustment market clearing prices also present certain variability but several predominant price levels can be detected.

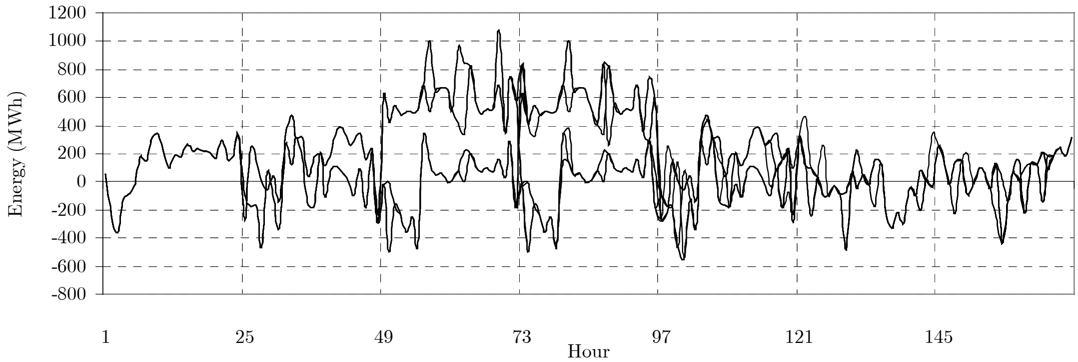


Figure 6.39. Company's sales in the adjustment market.

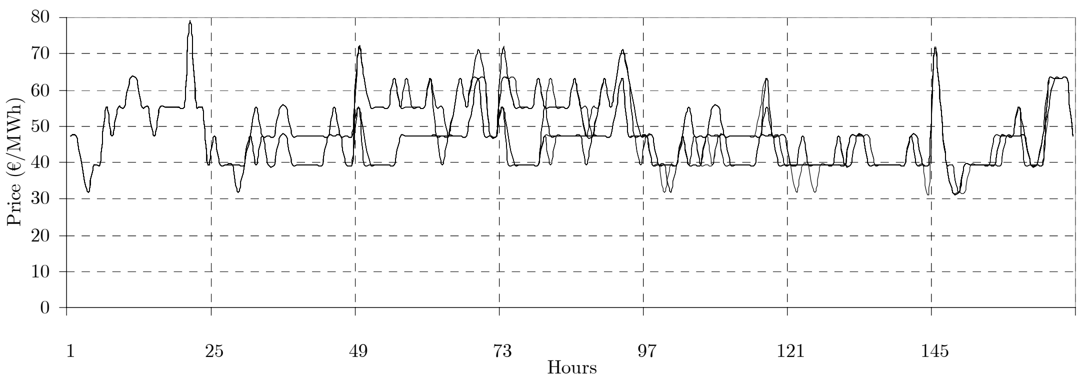


Figure 6.40. Adjustment market clearing prices for each scenario.

The generation schedule decided by the model also depends on the outcome of the spot market. In particular, the model decides a different unit-commitment schedule for each weekly scenario. To illustrate this, the two unit-commitment schedules given by the model for Tuesday are shown in Table 6.5 and Table 6.6. It can be seen that the two schedules decided for generators G24 and G32 are not the same for the two scenarios.

		1	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**	**11**	**12**	**13**	**14**	**15**	**16**	**17**	**18**	**19**	**20**	**21**	**22**	**23**	**24**
Nuclear	G1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
	G2	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
	G3	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
Coal	G4	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
	G5	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
	G6	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
	G7	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
	G8	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
	G9	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
	G10	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
	G11	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
	G12	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
	G13	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
	G14	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
	G15	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
	G16	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
	G17	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
	G18	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
	G19	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
	G20	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
	G21	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
	G22	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
	G23	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
Gas/oil	G24	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
	G25	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
	G26	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
	G27	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
	G28	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
	G29	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
	G30	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
	G31	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
	G32	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
	G33	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
	G34	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
	G35	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
	G36	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
	G37	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
	G38	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
	G39	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0

Table 6.5. Unit-commitment schedule for the first possible outcome of Tuesday's session.

		1	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**	**11**	**12**	**13**	**14**	**15**	**16**	**17**	**18**	**19**	**20**	**21**	**22**	**23**	**24**
Nuclear	G1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
	G2	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
	G3	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
Coal	G4	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
	G5	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
	G6	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
	G7	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
	G8	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
	G9	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
	G10	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
	G11	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
	G12	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
	G13	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
	G14	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
	G15	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
	G16	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
	G17	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
	G18	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
	G19	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
	G20	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
	G21	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
	G22	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
	G23	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
Gas/oil	G24	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	1	1	1	1
	G25	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
	G26	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
	G27	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
	G28	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
	G29	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
	G30	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
	G31	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
	G32	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	1	1	1	1
	G33	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
	G34	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
	G35	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
	G36	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
	G37	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
	G38	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
	G39	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0

Table 6.6. Unit-commitment schedule for the second possible outcome of Tuesday's session.

To further illustrate the flexibility of the unit-commitment schedule provided by a stochastic programming model, Figure 6.41 represents the commitment schedule decided by the model for unit G24. As can be seen, this schedule depends on the outcome of the sequence of spot market sessions. For example, unit G24 will be online on Tuesday with a probability of 50 %. If it remains offline on Tuesday then it operates on Wednesday with a probability of 50 %, and so forth. Something similar happens with the management of hydro resources. Figure 6.42 shows the hydro energy used by the model in each node of the scenario tree and illustrates its dependence on the outcome of the spot market.

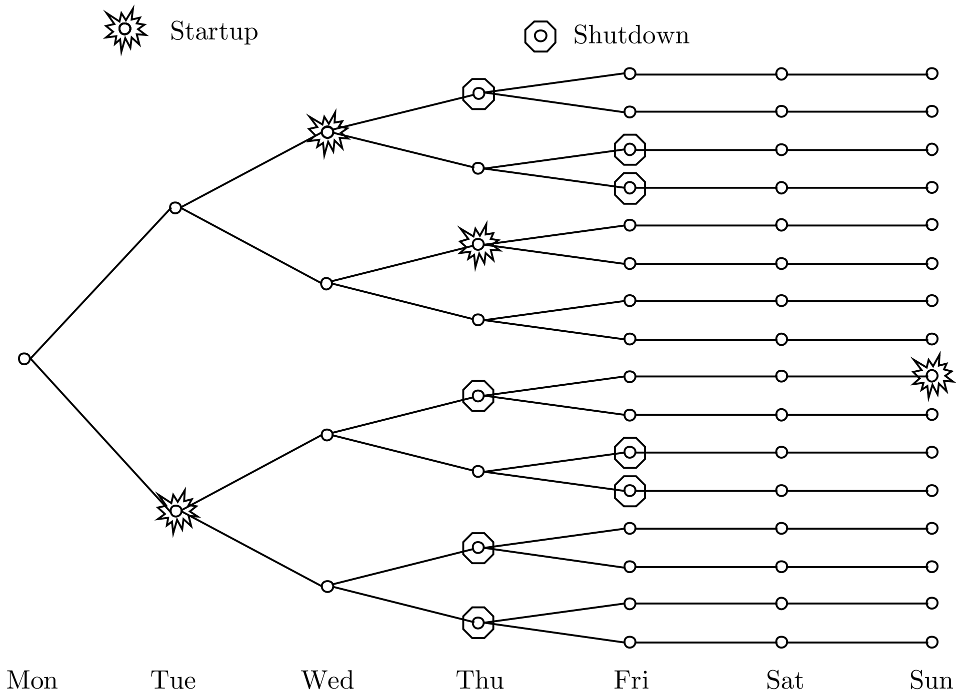


Figure 6.41. Scenario tree for the startup and shutdown of unit G24.

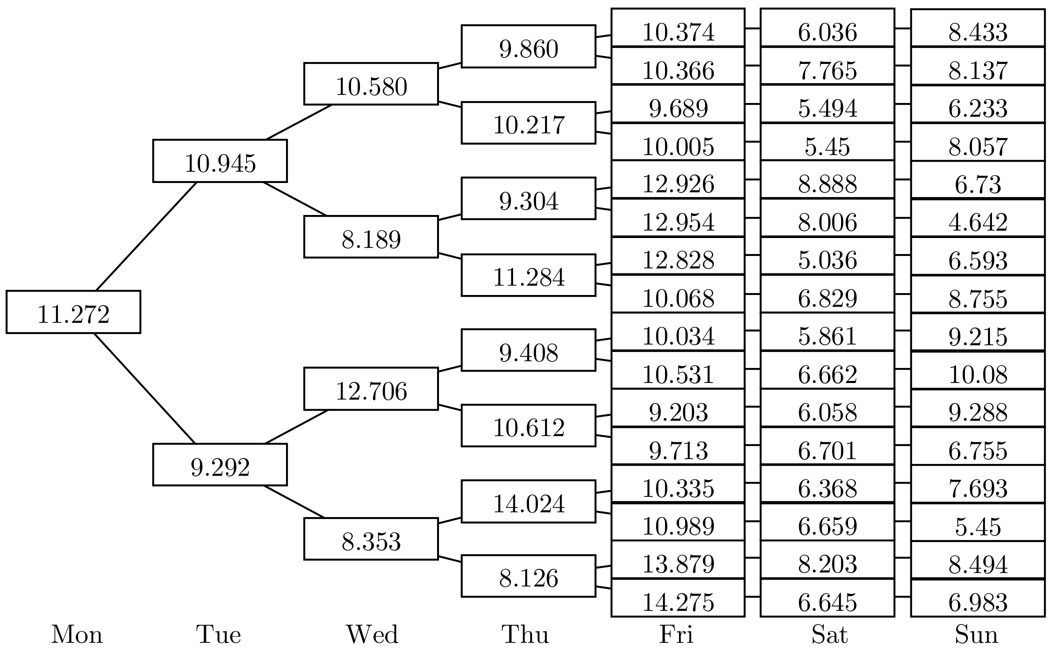


Figure 6.42. Hydro energy used in each node of the scenario tree in GWh.

These flexible unit-commitment schedules and hydro management decisions can be easily implemented in an offering strategy. For example, it can be assumed that unit G24 is online if the market-clearing price exceeds a certain value. A similar approach can be used to allocate hydro resources in the company's offer curves.

Although the energy used by the model in a certain spot market session strongly depends on the outcome of that particular session, the overall energy used during the week does not vary significantly from one scenario to another. According to Table 6.7, the company of study should use between 64.4 and 70.0 GWh of hydro energy during this week.

Scenario	Hydro energy (GWh)
1	67.500
2	68.925
3	64.430
4	66.526
5	68.254
6	65.312
7	66.147
8	67.342
9	67.788
10	69.951
11	68.431
12	67.051
13	67.337
14	66.039
15	67.619
16	64.946

Table 6.7. Hydro energy used in each weekly scenario.

The purchase of energy for pumping is also a relevant result. Table 6.8 indicates the energy purchased for pumping in each of the sixteen weekly scenarios, which ranges from 6.6 to 13.7 GWh.

Scenario	Energy pumped (GWh)
1	10.941
2	12.256
3	6.561
4	8.830
5	10.903
6	5.972
7	9.408
8	9.596
9	10.237
10	13.721
11	11.157
12	9.183
13	10.383
14	8.528
15	11.448
16	8.026

Table 6.8. Energy pumped in each weekly scenario.

6.3.2 Conclusions

This second part of the chapter has been dedicated to present the results obtained for a numerical example of the weekly stochastic unit-commitment problem. According to these results, the proposed approach suggests unit-commitment schedules that are consistent with the stochastic programming perspective adopted in this thesis. Indeed, the model does not yield a unique schedule, but rather a decision tree indicating the decisions that the company must take as uncertainty is unveiled. In this manner, the model correctly interprets that the company actually has the possibility of changing its strategy depending on the outcome of the spot market. This flexibility can be easily implemented in the company's offering strategy, which emphasizes the consistency between the two perspectives adopted in chapter 5 in order to address the problem faced by a generation company in a spot market.

The computational effort required to obtain the results for this study case does not invalidate our approach. According to our experience, weekly unit-commitment schedules should be obtained on Sunday in order to decide the operation of coal generators, which typically require a week to recover the cost of a startup. It does not seem unreasonable to make this effort once a week. The case of oil/gas generators is different, given that they can recover the cost of a startup in several hours. Hence, if the commitment schedule of oil/gas units must be revised during the week, it suffices to consider two spot market sessions and not the whole week. Obviously, these smaller problems pose less computational requirements.

7 Conclusions, contributions and future research

This last chapter is dedicated to evaluating the conclusions that result from the research conducted in this thesis. It includes a brief summary of the analysis, developments and findings that constitute the core of this work. It also specifies the original contributions that have resulted while pursuing the general objective of this thesis. Finally, several future lines of research stemming from the developments of this thesis are identified.

7.1 Summary and conclusions

This thesis has addressed the problem of developing an optimal offering strategy for a generation company operating in an electricity spot market. This issue is currently of the maximum relevance due to the regulatory reforms that have been introduced in the worldwide power industry.

The idea of constructing optimal offers for an electricity spot market is too general to be developed in a straightforward manner. The variety of spot market designs that can be found throughout the world is so wide that it is impossible to propose a general methodology valid for them all. For this reason, an overview of the most relevant spot market designs currently in operation has been included in this thesis. Based on this analysis, the rules that govern the spot market of study have been clearly defined.

The search for an optimal offering strategy requires the evaluation of the expected benefit of any candidate strategy. In particular, the company must be able to estimate the revenues that it expects to obtain in the spot market. This implies modeling the behavior of rivals, due to its relevant influence both on the company's revenues and on the price of electricity. In this thesis a literature survey has been performed in order to identify the model of competition that best suits our general objective. As a result of this analysis, a representation based on residual demand curves and revenue functions has been used to calculate the outcome of the hourly auctions that constitute the spot market of study.

Uncertainty with respect to rivals' behavior is at the root of the development of any offering strategy. It is because of this uncertainty that a generation company gets involved in a decision process more complex than simply choosing a single price for all its output or a specific level of production. However, not every representation of this uncertainty is equally amenable in order to search for an optimal offering strategy. In this thesis, uncertainty about the strategies followed both by rivals and by wholesale buyers in each of the hourly auctions that constitute the spot market has been represented by assuming that the probability distribution of the corresponding residual demand curve has finite support. In other words, it has been assumed that each hourly auction has a limited number of possible outcomes. Given that the spot market of study consists of a sequence of auctions, this approach yields a representation of the spot market in the form of a multistage stochastic program. This representation is valid not only for the case of a generation company, but also for other agents operating in an electricity spot market such as energy service providers.

The previous multistage stochastic programming framework has been enriched with a detailed model of the company's portfolio. This model considers each of the company's generation units, including their production costs and technical constraints. It also takes into account the obligations assumed by the company in previous market mechanisms, such as futures or options markets.

Although this thesis focuses on the development of strategies for market mechanisms that operate on a daily basis, the main objective of a generation company is the maximization of its long-term profit. Some sort of guideline must be incorporated into our methodology, so as to orient its results toward this long-term objective. With this purpose, an explicit valuation of the market share obtained by the company in the spot market has been suggested in order to correct the myopic incentive that a generation company has to reduce its sales and increase the spot price of electricity. Tuning of this parameter has been shown to result in realistic simulations of short-term bidding behavior, which suggests that it can be useful fore representing a company's decision to tradeoff short- and long-run benefits.

The size of the mathematical program that results when the abovementioned modeling features are put together is unmanageable for current commercial optimizers. In order to improve its numerical tractability, it has been assumed that the relative importance of the spot market mechanisms diminishes as the moment of physical delivery gets nearer. Under this assumption, the problem of choosing an optimal strategy for the spot market has turned out to have a twofold structure. On the one hand, the problem of developing optimal offers for a specific market mechanism has been expressed as a two-stage stochastic program, taking into account that recourse actions can be adopted in subsequent market mechanisms in order to correct any undesired result. On the other hand, the problem of deciding an optimal weekly unit-commitment schedule has been formulated as a sequence of two-stage stochastic programs. These two aspects of the operation of a generation company in a spot market are mutually consistent.

Both problems require the use of decomposition techniques so that realistic study cases can be formulated and solved under this framework. An analysis of the structure of both problems has been performed in order to identify the decomposition technique that best suits each of them. In the light of this analysis, Benders' decomposition appears as the most adequate approach to solve the first type of problem, given that it adapts well to its two-stage structure. In contrast, Lagrangian relaxation is the solution method chosen to address the weekly unit-commitment problem, due to the generalized presence of binary variables. The application of both decomposition techniques has been explained in detail. In particular, the formulation of the Lagrange function provides an interesting economic interpretation for the Lagrange multipliers and permits a better understanding of the problem.

The adequacy of the methodology developed in this thesis is confirmed by the results obtained for a collection of numerical examples. A variety of offering strategies have been derived for a generation company participating in a specific session of the Spanish day-ahead market under different circumstances. The sensitivity observed in the solutions proposed by our methodology with respect to a number of relevant factors is consistent. Additionally, a weekly unit-commitment schedule has been obtained for a sixteen-scenario case also based on historic sessions of the Spanish electricity spot market.

The previous summary leads to a number of relevant conclusions that can be drawn from the research conducted in this thesis. We organize them as follows:

i) Any attempt to solve one of the problems faced by an agent participating in a wholesale electricity market must be accompanied by a definition of the rules that govern the market mechanisms involved in the analysis.

ii) All the methodologies suggested in the literature to represent competition in wholesale electricity markets have advantages and shortcomings. The adequacy of a specific approach depends on the nature of the analysis that has to be performed. In particular, a representation of the spot market based on residual demand curves and revenue functions has been proposed for the problem addressed in this thesis.

iii) The explicit consideration of uncertainty with respect to rivals' behavior is crucial to guarantee the consistency of a methodology intended to develop optimal offering strategies for an electricity spot market. A flexible and powerful approach is to assume probability distributions with finite support when representing the uncertainty of residual demand curves.

iv) As a consequence of previous conclusions, the problem faced by a generation company in an electricity spot market can be formulated as a multistage stochastic program. In this context, offering strategies are simply expressed as vectors of quantities. Each quantity expresses the amount of energy that the company sells in one of the spot market auctions if a certain market situation occurs.

v) The development of an offering strategy for a generation company operating in an electricity spot market requires taking into account the company's portfolio. This implies considering not only the company's generating units, but also the obligations it has assumed in previous market mechanisms.

vi) A methodology that proposes strategies for the short-term operation of a generation company must include guidelines that orient these decisions toward the main objective of the company: the maximization of its benefits in the long term. In particular, myopic strategies such as output reductions in order to increase the price of electricity should be avoided. This difficulty can be overcome by simply introducing a term in the objective function expressing the value that the generation company gives to its market share. This term is flexible enough to permit profiting from short-term opportunities, while at the same time pursuing longer-term objectives.

vii) By assuming that the relative importance of the spot market mechanisms diminishes as the moment of physical delivery gets nearer, the problem of deciding an offering strategy for a certain market mechanism can be formulated as a two-stage stochastic program. This type of problem can be solved using Benders' decomposition under certain mild assumptions. The results obtained for a variety of realistic numerical examples confirm the validity of this approach.

viii) The weekly stochastic unit-commitment problem of a generation company operating in a spot market can be expressed as a sequence of two-stage programs, one for each day of the week. Given the structure of this problem, a decomposition scheme based on Lagrangian relaxation is a good solution approach. A realistic numerical example illustrates the features of this method.

A general conclusion is that, irrespective of the market design and the particular circumstances faced by a generation company in the new regulatory framework, three legs should sustain any methodology oriented to the development of optimal offering strategies. Firstly, a comprehensive analysis of the market rules that condition the company's operation must be performed. Secondly, a consistent and flexible model of competition should be selected that permits the explicit consideration of uncertainty and a rapid evaluation of candidate strategies. And thirdly, a solution method should be identified that yields quasi-optimal strategies in a reasonable execution time.

7.2 Original contributions

The development of this thesis has yielded a number of original contributions that can be categorized into three different types: methodological contributions, modeling contributions and algorithmic contributions.

7.2.1 Methodological contribution

The process of reasoning followed for the development of this thesis can be seen as a contribution. It starts by clearly defining the context in which the proposals of this thesis hold. In particular, an analysis of the most relevant wholesale electricity market designs has been carried out in order to specify the rules that govern the spot market of study. Additionally, the model adopted in this thesis to represent competition has also been justified according to the findings of a literature survey. Moreover, the reasons that support the approach used in this thesis to represent the uncertainty faced by generation companies in the spot market have also been carefully explained. After addressing these relevant aspects, the methodology suggested in this thesis to develop optimal offering strategies for the electricity spot market has been formulated in detail, resulting in a large-scale multistage stochastic programming problem. Finally, a strategy to obtain numerical results for this problem has been proposed that makes use of two well-known decomposition techniques. A variety of numerical examples illustrate the potential of this methodology. Although some particular hypotheses assumed in this thesis may not hold in other cases, the suggested process of reasoning can be reproduced in order to address the problems faced by a generation company operating in a substantially different spot market. Furthermore, it can also be applied to address problems related to other market mechanisms, such as futures markets, as we explain when indicating futures lines of research.

7.2.2 Modeling contributions

The following developments of this thesis constitute original contributions in the field of modeling competition in wholesale electricity markets:

i) As has been mentioned, before choosing a model to represent competition in the spot market of study, a literature survey has been performed in order to identify the advantages and shortcomings of the existing models. In this context, an original characterization of the methodologies proposed in the literature to address the short-term problems faced by generation companies in electricity spot markets has been suggested. This analysis has shown the conceptual gap that this thesis aims to fill.

ii) By using residual demand curves and revenue functions to evaluate the outcome of the spot market and by assuming that the probability distributions of these curves have finite support, an original representation of the spot market in the form of a multistage stochastic program has been derived. This modeling approach provides a consistent framework to evaluate the expected revenue of any offering strategy.

iii) A generalized model of the portfolio of a generation company from the perspective of the spot market has been suggested that considers not only the company's generation assets, but also its open positions resulting from previous market mechanisms. This guarantees a correct evaluation of the impact that each offering strategy has on the company's revenues.

iv) In order to avoid the adoption of myopic strategies based on the exercise of market power, a new term has been added to the short-term objective function of the generation company that takes into account the value of its market share. In this manner, long-term profit maximization objectives can be pursued while at the same time seizing some of the short-term opportunities that may arise.

v) It has been shown that the methodology proposed in this thesis, originally conceived to address the problem faced by generation companies in electricity spot markets, can be easily adapted to contemplate the problem of developing optimal bidding strategies for a wholesale buyer, such as an energy service provider.

7.2.3 Algorithmic contributions

In addition to proposing an original multistage stochastic formulation for the problems faced by a generation company in the spot market, this thesis suggests a solution strategy based on the assumption that the relative importance of spot market mechanisms diminishes as the moment of physical delivery gets nearer. This facilitates the search for a solution when addressing realistic study cases. Three specific contributions can be identified from this point of view:

i) Two types of mathematical programs have been suggested in order to obtain numerical results. On the one hand, the problem of developing an offering strategy for a specific market mechanism has been expressed as a two-stage stochastic program. This approach includes an explicit representation of the uncertainty faced by the company in the market mechanism of interest, considers the possibility of adopting recourse actions in subsequent market mechanisms and evaluates the impact that these decisions have on the company's portfolio. On the other hand, the problem of deciding a weekly unit-commitment schedule and distributing the company's hydro resources during the week of study has been formulated as a sequence of two-stage programs, each one corresponding to one particular day of the week. The combination of both approaches constitutes a consistent framework to assess the operation of a generation company in a spot market.

ii) A Benders' decomposition approach has been developed in order to solve the abovementioned two-stage stochastic program. Benders' master problem comprises the first-stage decisions, i.e., the development of an offering strategy for the market mechanism of interest. Benders' subproblem evaluates the possibility of taking recourse actions in subsequent market mechanisms and derives a final generation schedule that meets the obligations assumed by the company through its sales. An original ad hoc procedure based on the separation of the master problem into hourly problems (one for each auction of the day-ahead market) has been devised to quickly obtain quasi-optimal feasible offering strategies.

iii) A Lagrangian relaxation approach has been developed to tackle the weekly stochastic unit-commitment problem by dualizing the energy balance equation. In addition to constituting an adequate solution method, the formulation of the Lagrange function has provided an interesting economic interpretation for the Lagrange multipliers. The identification of the presence of new Lagrangian subproblems due to the competitive framework in which generation companies now operate is also an original result.

iv) The application of both Benders' decomposition and Lagrangian relaxation to the problems addressed in this thesis has been implemented in an algebraic modeling language so that the proposed methodology can be systematically used by a real generation company. Moreover, the routines that prepare the input data for these problems based on historic information of the Spanish electricity spot market are also implemented in a general-purpose programming language. In this manner, the daily task of developing offering strategies for a company operating in the Spanish electricity spot market can be performed in a simple and straightforward manner.

7.3 Future research

The developments of this thesis lead to a number of future lines of research whose exploration is likely to yield interesting results. This section tries to summarize them and justify their relevance. They have been organized into two main groups. The first one refers to possible advances in the field of modeling wholesale electricity markets. The second one focuses on the improvement of the algorithmic solutions suggested in this thesis.

7.3.1 Modeling advances

The analysis of competition in wholesale electricity markets is an issue that has received much attention during the last decade and that is likely to continue as a very active field of research due to its inherent complexity and to the relevant role that the electric power industry plays in the worldwide economy. Some of the research lines stemming from this thesis that would constitute interesting modeling advances can be summarized as follows:

i) As has been justified, the methodology proposed in this thesis adapts well to a specific spot market design, but it may not be adequate for others. In particular, situations such as nodal spot pricing, pay-as-bid schemes or continuous trading mechanisms are not contemplated. It would be of great interest to repeat the process followed in this thesis but assuming a substantially different spot market design.

ii) As a matter of fact, the line of reasoning followed in this thesis can also be adopted to address the problems faced by a generation company when operating in medium-term market mechanisms such as futures or options markets. Indeed, these problems seem also to adapt well to a multistage stochastic programming approach in which current decisions can be corrected by future recourse actions ultimately leading to a generation schedule. The continuous trading process typical in these markets would have to be represented with a discrete-time model. An open issue would be the choice of the competition model that best suits these problems. This decision strongly depends on the approach adopted to obtain numerical results.

iii) In this thesis, a static model of competition based on residual demand curves and revenue functions has been used to evaluate the influence that rivals' decisions exert on the company's revenues. This model overlooks the possibility of triggering an undesired reaction of a rival in a subsequent spot market session. This static perspective has been justified by assuming that a company should try to keep a steady state behavior so as to avoid sudden reactions from its competitors whose consequences are difficult to estimate. It would be interesting to explore the possibility of incorporating an explicit representation of these reactions in our modeling approach.

iv) Even though it has been justified that the methodology proposed in this thesis adapts well to the case of an wholesale energy buyer such as an energy service provider, it would be necessary to further elaborate this line of research. In particular, a detailed formulation of the portfolio of an energy service provider would be an interesting extension for this thesis.

v) In order to orient the company's short-term strategies toward the objective of long-term profit maximization, a term that evaluates the value of the company's market share has been introduced. Although the results obtained with this approach are encouraging, a more detailed analysis of the role played by this parameter would contribute to better understand its implications. Moreover, the estimation of this parameter as a result of medium-term operation-planning decisions remains an open issue.

vi) The numerical examples and solution methods included in this thesis focus mainly on the day-ahead market and the adjustment market, while leaving aside other market mechanisms such as the reserves market. Nevertheless, it has been argued that our methodology can also be applied for this purpose. The precise formulation of the two-stage problem that considers the development of an optimal offering strategy for the reserves market, given the results obtained in previous energy markets, is a relevant question that can be easily addressed given the general background provided in this thesis.

7.3.2 Algorithmic improvements

Although a significant effort has been devoted in this thesis to the development of algorithms that permit obtaining numerical results for the formulation proposed, two particular algorithmic improvements can be identified:

i) The application of Benders' decomposition has required the elimination of binary variables in the subproblem. This implies a simplification in the representation of the second stage. In particular, it has been assumed that the residual demand curves of the second-stage market mechanism are linear functions. This simplification could be avoided by enhancing Benders' decomposition approach so that binary variables can be included in the subproblem. A development of this nature has been applied to the medium-term operation-planning problem of a generation company [Cerisola '02].

ii) Even though Lagrangian relaxation provides good results for the weekly stochastic unit-commitment problem, its convergence requires a considerable computational effort. In particular, a reduction in the execution time reported for the numerical example included in this thesis would be of great interest, given that this model should be executed once a week. This improvement could be achieved with an approach that accelerates the convergence of Lagrangian relaxation for mixed linear-integer programs such as the one developed in [Cerisola '01].

7.4 References

[Cerisola '01] S. Cerisola and A. Ramos, "A finite Benders decomposition algorithm for mixed integer problems, resolution through parametric Branch and Bound," Working paper, Instituto de Investigación Tecnológica, Universidad Pontificia Comillas, 2001.

[Cerisola '02] S. Cerisola and A. Ramos, "Benders decomposition for mixed-integer hydrothemal problems by Lagrangean relaxation," presented at the 14th Power Systems Computation Conference (PSCC '02), Sevilla, Spain, June 2002.

A Modeling the electricity spot market through residual demand curves

An agent willing to optimize his strategy in an electricity spot market must be able to evaluate the influence that his decisions exert on his revenues. This requires a mathematical representation of the spot market that takes into account all the aspects that are relevant from the perspective of the agent.

In this thesis, the residual demand model is considered the most adequate approach to represent the relationship that exists between an agent's decisions and the outcome of the multiunit double auctions that frequently constitute electricity spot markets. This appendix provides the general background of this modeling approach and explains the details of its implementation in this thesis.

A.1 Introduction

The developments presented in this thesis require a formal description of the modeling approach adopted to represent competition in an electricity spot market from the perspective of a specific agent (e.g. a generation company or an energy service provider). As indicated in chapter 3, the residual demand model seems the most convenient approach for the purposes of this thesis. This appendix provides an overview of some of the basic concepts that justify the validity of this model in the context of multiunit double auctions.

The implementation of the residual demand model in a mathematical programming framework requires a consistent representation approach. In this appendix, a piecewise linear approach is described that permits the evaluation of both residual demand curves and revenue functions in a mixed-integer programming context. This piecewise linear representation also allows to compare different residual demand curves in a straightforward manner, which can be helpful in order to identify patterns in rivals' strategic behavior.

A.2 Basic concepts

A.2.1 The idea of residual demand in microeconomic theory

A.2.1.1 Equilibrium models

The idea of residual demand arises in microeconomic analysis in the case of a homogeneous good industry where a dominant firm 1 acts as a *price leader* and the rest of firms consider that the price is independent of their decisions (i.e. they act as *price takers*) [Varian '92].

In this context, each price taker $i \neq 1$ decides the output q_i that maximizes its profit, given the price p established by the dominant firm:

$$\underset{q_i}{\text{Max}} \ pq_i - c_i(q_i), \quad \forall i \neq 1. \tag{A.1}$$

As a result, the output decision taken by firm i can be expressed as a function of price in the form $q_i = S_i(p)$. This function can be seen as firm i's offer curve and obviously coincides with its marginal cost curve, $c_i'(q_i)$.

When deciding the price, in order to maximize its profits, the dominant firm 1 must take into account that the amount of product that that it can sell at each price is the result of two contributions. On one hand, the total amount of product demanded by consumers, q, which can be expressed as a function of price by means of the demand function, $q = D(p)$. On the other hand, part of this demand is covered by price takers with their output, which can be expressed as $q_{-1} = \sum_{i \neq 1} q_j = \sum_{i \neq 1} S_i(p) = S_{-1}(p)$. Consequently, the amount of product that firm 1 is able to sell at price p is given by $q_1 = R_1(p) = D(p) - S_{-1}(p)$. This function is known as firm 1's *residual demand* and its construction is illustrated in Figure A.1.

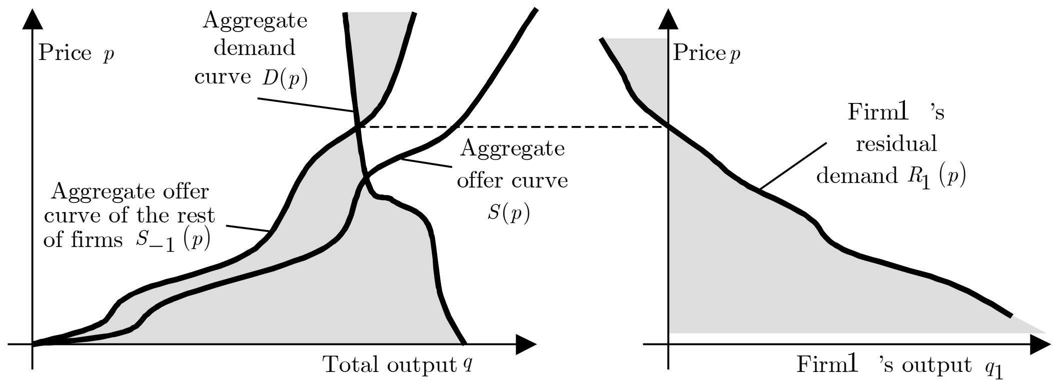


Figure A.1. Residual demand function for the price leader 1 .

The problem faced by firm 1 is then:

$$\operatorname*{Max}_{p}\ p\left[D(p)-S_{-1}(p)\right]-c_1\left(D(p)-S_{-1}(p)\right)=pR_1(p)-c_1\left(R_1(p)\right). \qquad \text{(A.2)}$$

The idea of residual demand is also useful for more advanced models of oligopoly, such as the *supply function equilibrium* model developed in [Klemperer '89]. In this model, every firm i takes its decisions in the form of a supply function $S_i(p)$. The profit maximization problem of firm i is then formulated as:

$$\operatorname*{Max}_{p}\ p\left[D(p)-S_{-i}(p)\right]-c_i\left(D(p)-S_{-i}(p)\right), \quad \forall i. \qquad \text{(A.3)}$$

This yields the following set of first-order optimality conditions:

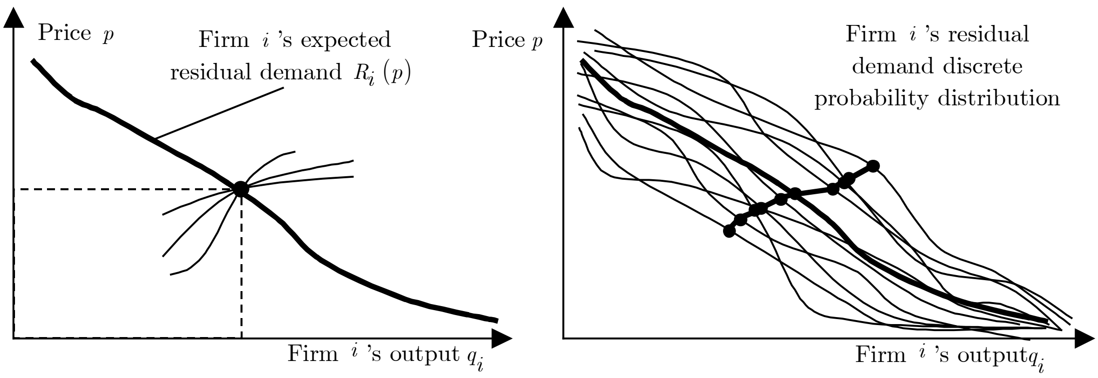
$$\left[p-c_i\left(D(p)-S_{-i}(p)\right)\right]\cdot\left[D'(p)-S_{-i}'(p)\right]+\left[D(p)-S_{-i}(p)\right]=0, \quad \forall i,p. \qquad \text{(A.4)}$$

Although equations (A.2) and (A.3) may seem similar, there is relevant difference between them: (A.3) is formulated for every firm and for every price that may result. Consequently, (A.4) constitutes a set of differential equations that is solved by any set $\mathrm{S}=\left\{S_i(p)\right\}$ of supply functions that satisfies these optimality conditions for every price p such that $D(p)\geq 0$. As uncertainty about demand increases, the number of possible clearing prices also increases and the set of solutions for this set of differential equations is reduced. Hence, in the supply function equilibrium model every firm faces an uncertain residual demand curve when expressing its strategic decisions in the form of a supply function.

A.2.1.2 The perspective of a particular firm

The previous models take the perspective of a general observer that analyzes the interaction of a number of firms and makes use of the concept of equilibrium to determine the outcome of such an industry structure. However, the idea of residual demand is also interesting from the point of view of one of the firms. Indeed, even if a specific firm is not able to unilaterally decide the market price of the good it sells, its decisions may well have a relevant influence on the final price. In that case, the firm should try to estimate the shape of its residual demand curve in order to maximize its profits. Its profit-maximization problem would then be similar to the one formulated for the price leader (A.2). The only difference is that in the price leader case, the decision has to be taken in the form of a price, whereas in this case, the firm is free to decide the form in which its strategic decisions are expressed (a price, a quantity or a supply function).

If the firm assumes that the estimated residual demand curve is correct and overlooks any source of uncertainty, it is irrelevant whether its decisions are expressed in terms of a quantity, a price or a supply function [Klemperer '86]. On the contrary, if the firm recognizes that the strategies of its rivals and/or the decisions of consumers are uncertain, its maximum expected profits might depend on the form in which the company expresses its strategic decisions. In this context, the most flexible and robust approach is to calculate the firm's profit-maximizing supply function (Figure A.2). This also includes the particular cases of deciding a fixed price for the entire firm's output or a fixed output irrespective of the market price.



Firm i 's maximum expected profit can be obtained with a variety of offering strategies, in particular, a fixed quantity or a fixed price.

To obtain firm i 's maximum expected profit a procedure to obtain optimal offering strategies must be used.

Figure A.2. The effect of considering the residual demand discrete probability distribution.

It must be emphasized that the problem of constructing the optimal supply function for a firm that faces an uncertain residual demand is different from the problem of determining the supply function equilibrium (SFE) for a number of firms. In the former case, a single offer curve has to be calculated and uncertainty can be due both to the demand curve and to rivals' behavior. On the contrary, to obtain a SFE an offer curve must be calculated for every firm and uncertainty is typically restricted to the shape of the demand curve.

A.2.2 The residual demand approach in a repetitive sealed-bid multiunit double auction

The residual demand approach can also be adopted by an agent participating in a repetitive sealed-bid homogeneous-product uniform-price multiunit double auction[1] such as the ones that constitute the spot market considered in this thesis.

If the offers (bids) rendered by all agents to a multiunit double auction are sorted in ascending (descending) order by price and their quantities are accumulated, an aggregate offer (bid) curve such as the one depicted in Figure A.3 is obtained. Only the offers and bids that are located on the left of the intersection of the aggregate offer and bid curves are accepted. Each buyer should pay, at most, the price he bid and, at least,

[1] For simplicity, henceforth this specific type of auction will be referred to as multiunit double auction.

the price of the most expensive accepted offer. Similarly, each seller should receive, at least, the price of its offer and, at most, the price of the cheapest accepted bid. In this thesis only uniform-price auctions are considered, so that all buyers pay the same price and all sellers receive the same price. This price is determined by the intersection of the aggregate offer and bid curves. In this manner a situation similar to the market equilibria analyzed in microeconomic theory is obtained.

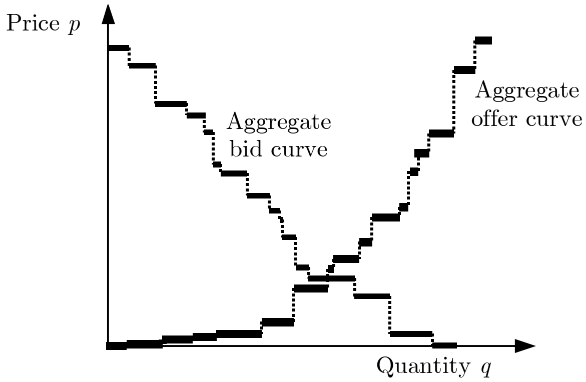


Figure A.3. Aggregate offer and bid curves for a double auction.

Therefore, the ideas presented in the previous section are applicable to the case of a multiunit uniform-price double auction. In particular, to obtain the residual demand curve for one of the sellers, the aggregate offer curve of the rest of sellers must be subtracted from the aggregate demand curve, proceeding in the way depicted in Figure A.1[2]. Due to repetition, this agent can make use of historic data to estimate the offers and bids that the rest of agents are expected to submit in the next session. In fact, it is not even necessary to estimate individual bids and offers. The residual demand curve comprises all the relevant information about the rest of the world that the agent requires in order to pursue the objective of maximizing his profit. Indeed, the residual demand curve determines which of the agent's offers are accepted. Moreover, in a uniform-pricing context, it also determines the price that the agent receives for each of these accepted offers[3].

A.2.3 From the residual demand curve to the revenue function

As has been indicated, in a multiunit uniform-price double auction, it is the residual demand curve what determines the price that a seller will receive for his accepted offers and, as a result, the revenue he will obtain. Figure A.4 depicts the revenues obtained by a selling agent in two different situations. It is quite clear that these revenues do not depend on the shape of the agent's offer curve. They just depend on the point (q, p) where his offer curve intersects with his residual demand curve. In other words, the agent's revenues are solely determined by his residual demand curve and the quantity he sells, irrespective of the specific offers that have produced these sales[4].

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[2] In rigor, the residual demand function does not exist for a seller participating in a multiunit auction. It is obvious that there is not a unique quantity that satisfies $q=R(p)$. In contrast, the inverse residual demand function $R^{-1}(q)$ does exist and will be referred to as 'residual demand curve'.

[3] This does not hold in a pay-as-bid context, where each accepted offer is remunerated at its own price.

[4] Conversely, in a pay-as-bid auction the shape of the agent's offer curve determines his revenues.

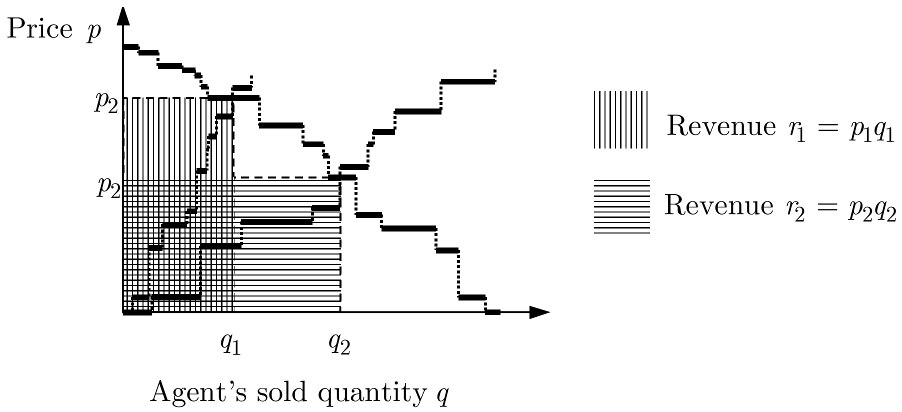

Figure A.4. The relationship between revenue and residual demand.

The agent's revenues, r, can then be expressed as a function of his sales by simply multiplying each possible quantity q_j by the corresponding price that results from the residual demand curve, $p_j = R^{-1}\left(q_j\right)$. Figure A.5 illustrates the appearance of the revenue functions that are typically obtained. It is clear that a non-linear and non-concave revenue function is likely to complicate the search for an optimal strategy. In following sections a piecewise linear approximation methodology will be suggested to overcome this difficulty.

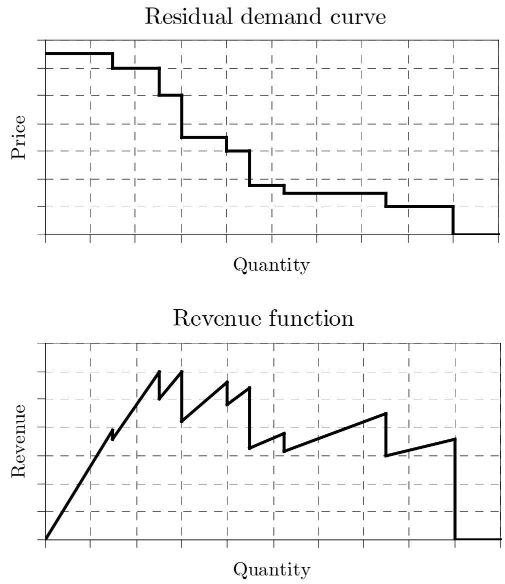

Figure A.5. A residual demand curve and the corresponding revenue function.

In conclusion, in a multiunit uniform-price double auction, a revenue function can always be derived from a residual demand curve. Hence, the terms *residual demand curve* and *revenue function* will be frequently used indistinctly to refer to a specific situation faced by an agent when participating in an auction.

A.3 Representing residual demand curves in a MIP problem

The objectives established for this thesis will be pursued making use of mathematical programming techniques. As has been shown, at least one part of the objective function, the company's revenues in the spot market auctions, take the form of non-linear and non-convex functions of the company's sales. Two alternative perspectives can be adopted to address such a mathematical programming problem: non-linear programming (NLP) or mixed-integer programming (MIP). The impressive theoretical and practical advances that have been achieved in the field of MIP and their implementation in current commercial MIP solvers provide a powerful and reliable framework for the developments of this thesis [Ceria '01].

This section suggests a method to express the revenues of a company that participates in a double auction using a MIP formulation. It is not the unique valid approach but it combines simplicity with a detailed representation. Other proposals can be found in [Mateo '01] and in [Sánchez-Úbeda '00].

A.3.1 Piecewise linear approximation of offer curves, bid curves and residual demand curves

As mentioned, the offer and bid curves involved in a double auction take the form of stepwise curves, which have certain disadvantages concerning numerical tractability. It would be significantly easier to deal with curves that were expressed in the form of vectors of offered (or bid) quantities, $(q_1,\ldots,q_J)$, at a fixed set of prices, $(p_1,\ldots,p_J)$. Figure A.6 illustrates this approach with an example in which the prices used to divide the price domain are evenly spaced. As can be seen, this yields a piecewise linear approximation of the residual demand curve that, depending on the number of components used, can be as accurate as desired[5].

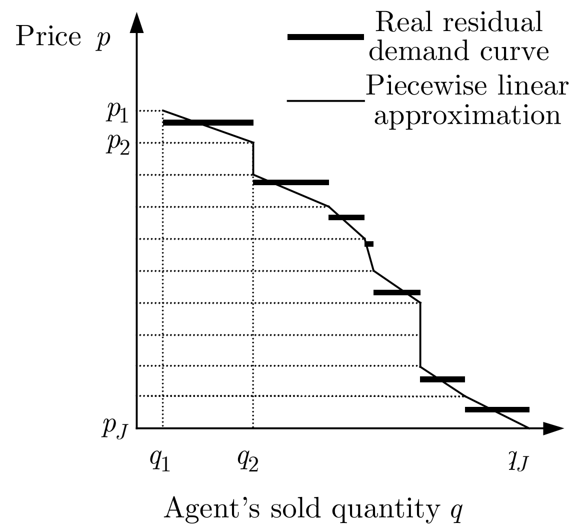

Figure A.6. Piecewise linear approximation for residual demand curves.

Figure A.7 shows another example in which the aggregate offer curve submitted to one of the hourly auctions of the Spanish day-ahead market has been approximated by a vector of 50 quantities[6].

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[5] Other approximation methods optimize the amount of information used to obtain a piecewise linear approximation for each curve [Sánchez-Úbeda '99]. However, this requires more sophisticated data processing routines (the number of components used to represent each curve can be different) and complicates the subsequent analysis (it is easier to compare a pair of vectors of the same dimension whose components provide the same information).

[6] This information is publicly available from OMEL's web site http://www.omel.es

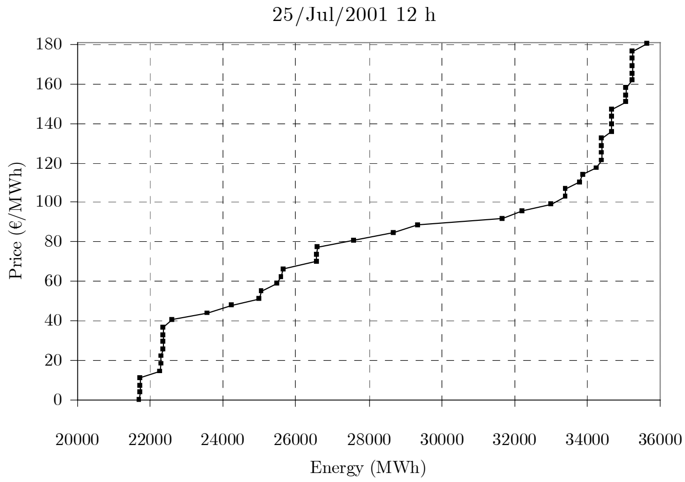

Figure A.7. PWL approximation for an aggregate offer curve in the Spanish day-ahead market.

Using a fixed number of components to approximate all curves has several advantages. First, the same amount of memory is required to store the information of every curve. Moreover, in order to compare a pair of curves it suffices to calculate the norm of the difference of both vectors. Finally, summing a pair of curves or subtracting one curve from another is equivalent to summing or subtracting vectors.

Let us assume then that the residual demand curve of a certain agent has been approximated by a vector of J components indicating the quantities $(q_1,\dots,q_J)$ that the agent is able to sell at J different prices $(p_1,\dots,p_J)$. In order to calculate the price p that results when the agent sells a certain quantity, q, it is necessary to search for the pair of components j and $j+1$ whose quantities q_j and q_{j+1} are closest to q. Price p would then be approximated as:

$$p \simeq \frac{p_{j+1}-p_j}{q_{j+1}-q_j}\cdot\left(q-q_j\right). \tag{A.5}$$

This approach is not easy to implement in a MIP formulation. An alternative is to calculate the non-positive slope δ_j of each of the $J-1$ segments defined by the piecewise linear approximation of the residual demand curve and use these slopes as input data for the MIP model:

$$\delta_j = \frac{p_{j+1}-p_j}{q_{j+1}-q_j}, \quad j<J\,. \tag{A.6}$$

In this manner, the following formulation can be used to express the auction price p as a function of the agent's sales q:

$$\begin{aligned}
&p = p_1 + \sum_{j<J}\delta_j v_j,\\
&q = \sum_{j<J} v_j,\\
&v_j \le u_j\left(q_{j+1}-q_j\right), \quad j<J,\\
&u_{j+1} \le u_j, \quad j<J-1,\\
&u_j \in \{0,1\}, v_j \in \mathbb{R}^+, j<J,
\end{aligned} \tag{A.7}$$

where p_1 is the highest price and q_1 is the corresponding quantity; u_j is a binary variable that indicates that the quantity q sold by the agent has reached segment j and v_j is the incremental quantity corresponding to segment j. Consequently, in order to calculate the auction-clearing price as a function of the agent's sales in a MIP model, it suffices to define both the vector of quantities and the vector of slopes that result from the suggested piecewise linear approximation of the residual demand curve. Figure A.8 illustrates this idea.

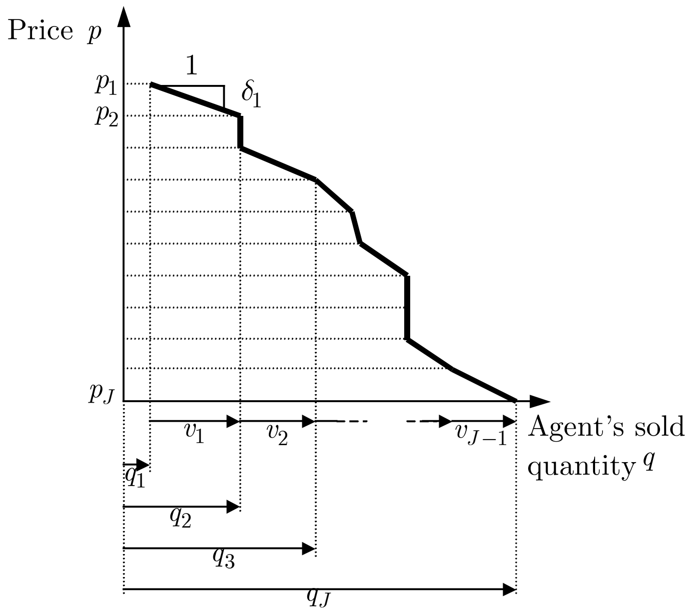

Figure A.8. Auction-clearing price as a function of the agent's sales in a MIP model.

This procedure may seem cumbersome or even useless, given that it is the revenue and not the auction-clearing price what the company is interested in. However, it will be shown that when residual demand uncertainty is explicitly considered, the auction-clearing price is a relevant variable in order to guarantee that the results provided by the model can be considered a valid offering strategy.

A.3.2 Piecewise linear approximations of revenue functions

The method that will be used in this thesis to evaluate the revenue obtained by an agent as a function of his sales is parallel to the one proposed to calculate the auction-clearing price.

Let us assume again that the residual demand curve of a certain agent has been approximated by a vector of J components indicating the quantities $(q_1,\ldots,q_J)$ that the agent is able to sell at J different prices $(p_1,\ldots,p_J)$. This yields a vector of revenues $(r_1,\ldots,r_J)=(p_1q_1,\ldots,p_Jq_J)$ that can be seen as a PWL approximation of the revenue function. Figure A.9 shows an example for a fictitious generation company in a certain auction of the Spanish day-ahead market. This company sold 12316.4 MWh in this hour and the market clearing price was 50 €/MWh.

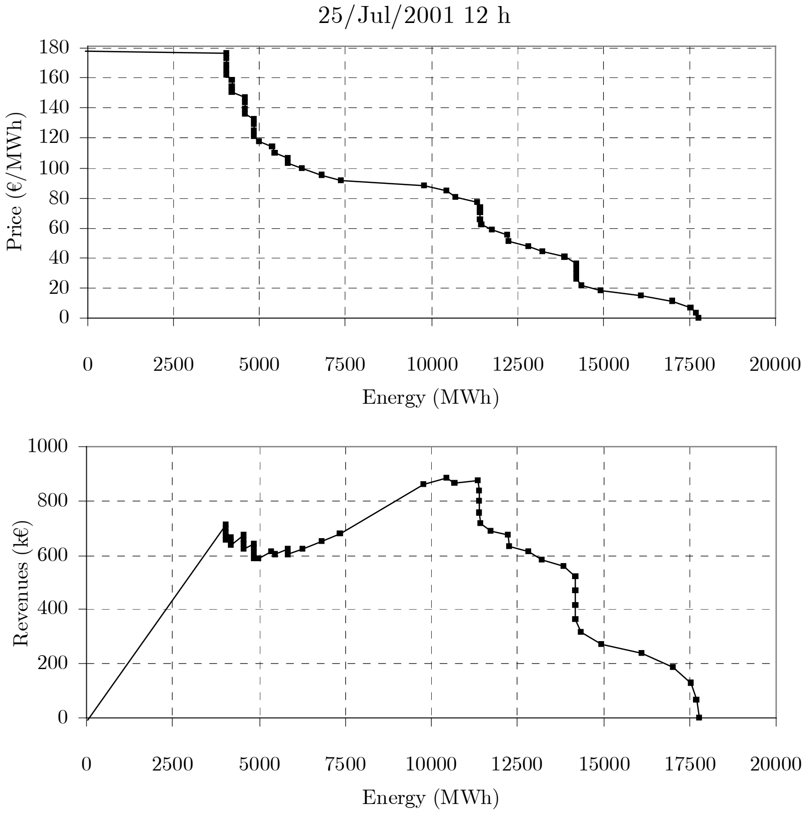

Figure A.9. From a PWL residual demand curve to the corresponding PWL revenue function.

The next step is to calculate the slope m_j of each segment j that has been obtained from the PWL approximation of the revenue function[7], according to the following expression:

$$\rho_j = \frac{r_{j+1} - r_j}{q_{j+1} - q_j}, \quad j < J. \tag{A.8}$$

Consequently, a formulation similar to the one suggested for the auction-clearing price can be used to express the agent's revenue r as a function of his sales q:

$$\begin{aligned} & r = r + \sum_{j<J} \rho_j v_j, \\ & q = \sum_j v_j, \\ & v_j \le u_j \left(q_{j+1} - q_j\right), \quad j < J, \\ & u_{j+1} \le u_j, \quad j < J-1, \\ & u_j \in \{0,1\}, v_j \in \mathbb{R}^+, j < J. \end{aligned} \tag{A.9}$$

This formulation has the advantage of being based on the same binary variables, u_j, and the same incremental quantities, v_j, that were defined in (A.7).

[7] These slopes can be interpreted as the agent's marginal revenue, an important concept in the theory of oligopoly.

A.3.3 Dealing with non-concave revenue functions

Although binary variables are useful to formulate non-linear equations in terms of mixed linear-integer expressions, their use must be limited so as to avoid increasing the complexity of the resulting MIP problem. In particular, the set of binary variables $\{u_j\}$ that has been introduced to guarantee that the segments of the residual demand curve and the revenue function are sequentially covered, might be reduced under certain circumstances.

Let us consider the case in which an agent with zero production costs tries to maximize his profit in a multiunit double auction. Assume that the agent bases his offers only on the residual demand curve he expects to face in that auction. In other words, the agent disregards the possibility of encountering a residual demand curve different form the one he has estimated. As has already been shown, under this assumption, the agent finds it irrelevant to express his decision in terms of a price, a quantity or an offer curve. Without loss of generality, it can be assumed that his decision is expressed in terms of a quantity.

When evaluating the revenue he expects to obtain in the auction, the agent can use the set of equations (A.9). Hence, he would ask the MIP model to search for the quantity that yields the maximum revenue by gradually incrementing his sales, covering the segments used to define the PWL approximation of the revenue function. However, if binary variables u_j were not used, the MIP model would not be aware of the order that should be followed when covering these segments and would prefer those segments with higher marginal revenues, as shown in Figure A.10.

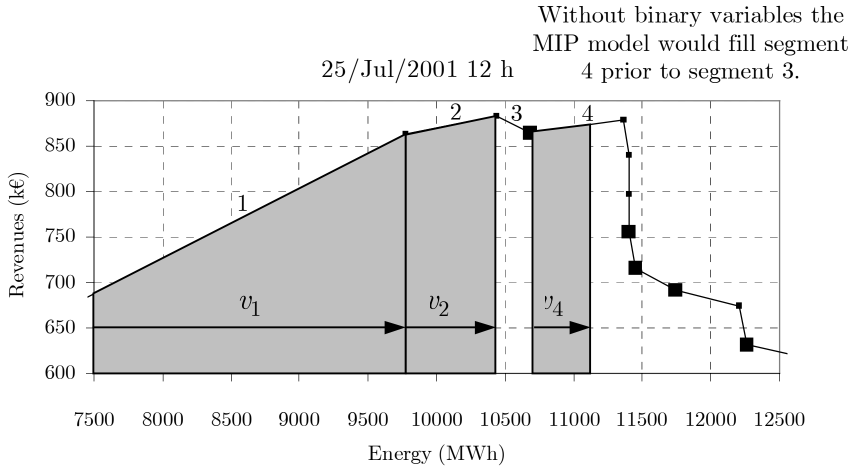

Figure A.10. Binary variables and the revenue function.

Binary variables are used to avoid this effect. Hence, they are not necessary for those pairs of segments in which the marginal revenue of the preceding segment is higher than that of the following segment. To put it in different words, binary variables are not required to guarantee that the segments that belong to the same concave section are chosen in the right order. Figure A.11 illustrates this assertion.

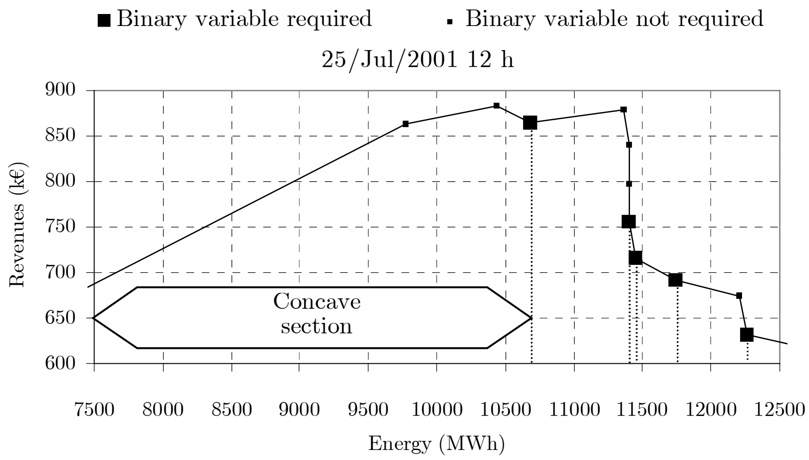

Figure A.11. Concave sections in the revenue function.

Thus, binary variables are only strictly required to establish the order in which the several concave sections that constitute the revenue function must be covered. As a result, under the specified assumptions, the number of binary variables that should be used is equal to the number of concave sections that the revenue function has.

The situation changes if the agent considers a discrete probability distribution for the residual demand curve consisting of several possible residual-demand realizations. Such an approach is explored in chapter 4 to show that, in order to guarantee that the revenue function is correctly evaluated, a binary variable is actually needed for each segment. This does not cause a dramatic increase of the computational effort required to solve the problem.

A.4 Conclusion

This appendix describes the approach adopted in this thesis to represent competition in the spot market from the perspective of a particular agent (e.g. a generation company).

After presenting basic ideas relative to the concept of residual demand, this appendix explains the piecewise linear approximations that can be used in a mixed-integer programming model to handle both residual demand curves and revenue functions. This provides a powerful platform to evaluate the influence of the agent's decisions both on the market-clearing price and on his own revenues.

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B A procedure to build spot market scenario trees

This thesis proposes a multistage stochastic programming approach to address the problem of developing optimal offering strategies for a company that participates in an electricity spot market. This line of attack combines both a correct representation of the uncertainty faced by the company and an adequate framework to obtain numerical solutions for real-size problems.

A spot-market scenario-tree structure has been suggested in which each stage corresponds to the clearing of a number of auctions that are part of a certain market mechanism. The uncertainty faced by the company in each of these auctions is represented by means of a collection of residual demand curves.

This appendix describes a practical method to construct spot-market scenario trees making use of historic information from past spot market sessions. It is based on the identification of spot market situations whose circumstances were similar to those envisaged for the current session.

B.1 Introduction

The construction of scenario trees in the context of multistage stochastic programming is not an easy task (see [Dupacová '00] for a good analysis of the matter). A frequent approach is to identify past spot market sessions that can be considered similar to the situation expected for the time scope of interest and select the scenarios from these past observations, assuming that they have equal probabilities. Although this can be seen as the simplest approach, it is also a practical solution that makes use of the historic information available at the moment of deciding an offering strategy.

This appendix describes a procedure to construct scenarios for the stochastic mathematical programs suggested in this thesis. We make use of clustering techniques in order to identify historic days that are likely to be similar to the day of study. The resulting scenario trees contemplate the timing of the generation company's decision process and comply with the non-anticipative requirements of this field of research.

B.2 The decision process of a company in the spot market

The spot market considered in this thesis is organized as a sequence of market mechanisms (see chapter 2 for further details). Each of these market mechanisms consists of a set of hourly auctions. Participants are required to submit their offers and bids for the auctions of a certain market mechanism after the previous market mechanism has been cleared. Figure B.1 illustrates this decision process, which has obviously the structure of a multistage stochastic program.

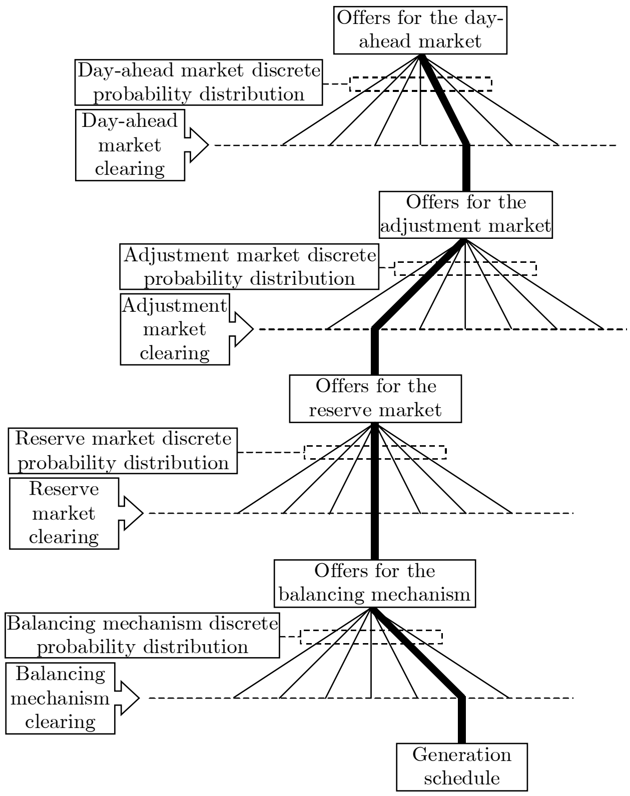

Figure B.1. The decision process of a generation company in the spot market.

According to Figure B.1, a generation company decides its offers for the hourly auctions of the adjustment market after the clearing of the day-ahead market. Similarly, the company decides its offers for the reserve market after the clearing of the adjustment market. Finally, the company introduces last-minute changes to its generation schedule through the balancing mechanism.

The volume traded in each of the market mechanisms typically diminishes as the moment of physical delivery gets nearer. For example, in the Spanish spot market, the volume of energy traded in the adjustment market is usually between a 10 and a 20 % of the volume traded in the day-ahead market. This suggests introducing a number of simplifications. In particular, when deciding the offers for the day-ahead market, the generation company might neglect the influence that the reserve market and the balancing mechanism have on its final generation schedule. Furthermore, the company might also neglect the influence of the uncertainty faced in the adjustment market by simply reducing its discrete probability distribution to the set of expected hourly residual demand curves. In other words, each possible realization of the day-ahead market would be accompanied by a single possible realization of the adjustment market. This would result in the simplified decision process depicted in Figure B.2.

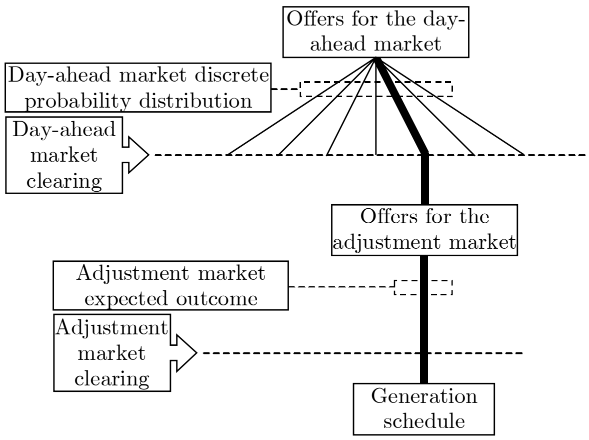

Figure B.2. A simplified version of the decision process from the perspective of the day-ahead market.

Once the day-ahead market clears, the generation company must decide its offers for the adjustment market. In this case, the influence of the balancing mechanism might be neglected. The simplified decision process shown in Figure B.3 would result:

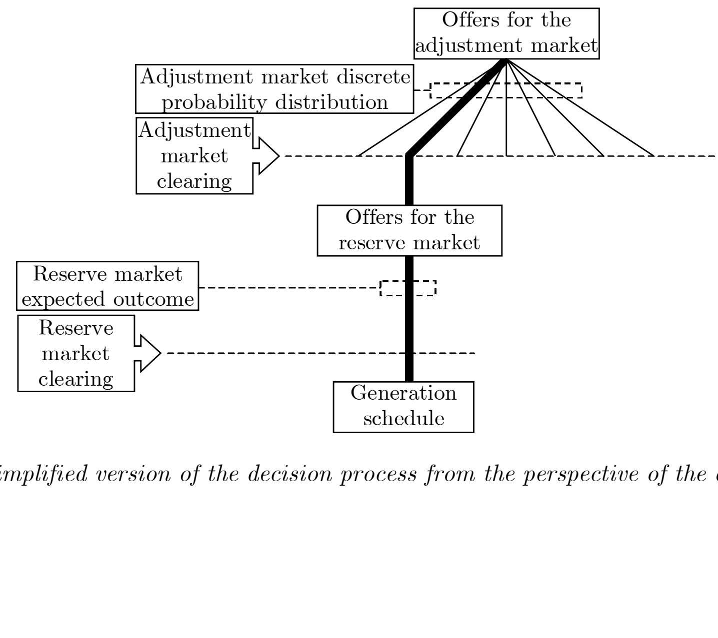

Figure B.3. A simplified version of the decision process from the perspective of the adjustment market.

Consequently, by assuming that the nearer a market mechanism is to the moment of physical delivery, the smaller its relative importance, the multistage stochastic program can be approximated by a sequence of two-stage stochastic programs. The scenario structure for each of these two-stage problems, rather than having the typical appearance of a tree, can be compared to a "fan" [Dupacová '00], as illustrated in Figure B.4:

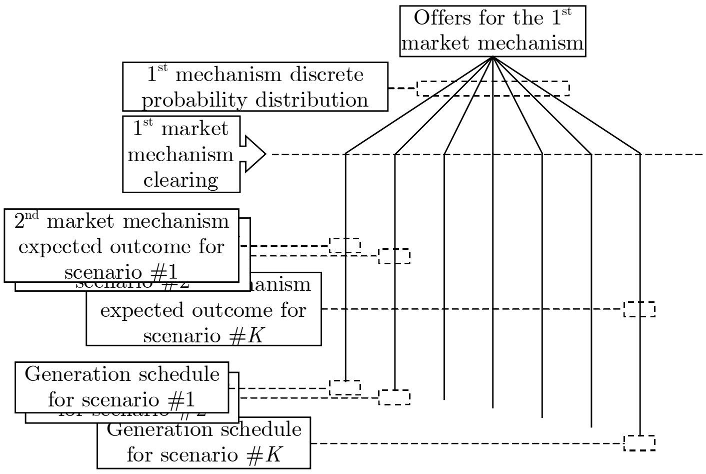

Figure B.4. The scenario structure for each of the two-stage stochastic programs.

The following sections explain in detail the method used to construct these scenario structures.

B.3 Explanatory variables

As has been indicated, the idea is to search for past spot market sessions that can be considered similar to the spot market session that the generation company is about to face. To evaluate the degree of similarity of two spot market sessions, a relevant explanatory variable (or variables) must be chosen. The choice of the explanatory variable will depend on the market mechanism that is being considered.

For example, when constructing the scenario structure for the day-ahead market problem (Figure B.2), the most reliable and relevant information that is available about the current spot market session is the chronological hourly demand curve predicted by the ISO. This prediction is typically very accurate and can be used to compare the current spot market session with past spot market sessions. This would yield a representative group of past spot market sessions whose day-ahead market information can be used to construct scenarios for the current day-ahead market session.

The situation changes when constructing the scenario structure for the adjustment market problem (Figure B.3), a task that should be commenced after the day-ahead market clears. In this context, the most reliable and relevant information about the current spot market session is the vector of 24 prices that has resulted from the day-ahead market clearing. This vector can serve to compare the current spot market session with past sessions in order to obtain a new representative group of past sessions whose adjustment market information can be used to construct the scenario structure for the current adjustment market problem.

In conclusion, the construction of the scenario structures for the two-stage stochastic programs that are suggested in this thesis to develop optimal offering strategies is based on the identification of past spot market sessions that can be considered similar to the current spot market session. This identification is carried out using a vector of explanatory variables that must be relevant and reliable.

B.4 Day-type identification by clustering techniques

One possible way of performing the mentioned day identification is to classify the whole collection of spot market sessions (including the current session) according to the values of the explanatory variables. This classification can be done using clustering techniques. The following two examples illustrate this idea.

B.4.1 An example of the construction of the scenario structure for the day-ahead market problem

Let us consider the construction of the scenario structure for the day-ahead market problem faced by a company in the Spanish spot market. In particular, the market session of July 25th 2001 is considered as the current session. As indicated, the explanatory variable that is used to identify similar past market sessions is the hourly demand curve predicted by the ISO. The past market sessions that are considered relevant range from July 9th to July 24th. The classification provided by the K-means algorithm for K=5 is presented in Table B.1. As can be seen, the five day types provided by the clustering analysis are quite reasonable: Saturdays, Sundays, Mondays and two types of weekdays other than Mondays.

July 2001

Date	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25
Day	Mo	Tu	We	Th	Fr	Sa	Su	Mo	Tu	We	Th	Fr	Sa	Su	Mo	Tu	We
Cluster	2	4	5	5	5	3	1	2	4	5	4	4	3	1	2	4	5

Table B.1. Classification of 17 days according to their 24 h demand profile.

Figure B.5 shows the reason why this particular classification has been obtained. As can be seen in the charts of the right column, the demand profile is indeed a variable that can be used to identify day types when no other information is available. The left column provides the price profiles that resulted in the day-ahead market sessions that took place on those days. In particular, the last price chart corresponds to cluster 5 and includes a highlighted representation of the price profile that was actually obtained on the day of study. It is interesting to notice that the variability observed in the series of hourly prices is much higher than the variability observed in the series of hourly demand levels. The reason is that the demand profile is the result of the decisions of a large number of relatively small agents. On the contrary, the price profile is caused by the interaction of a reduced number of generation companies.

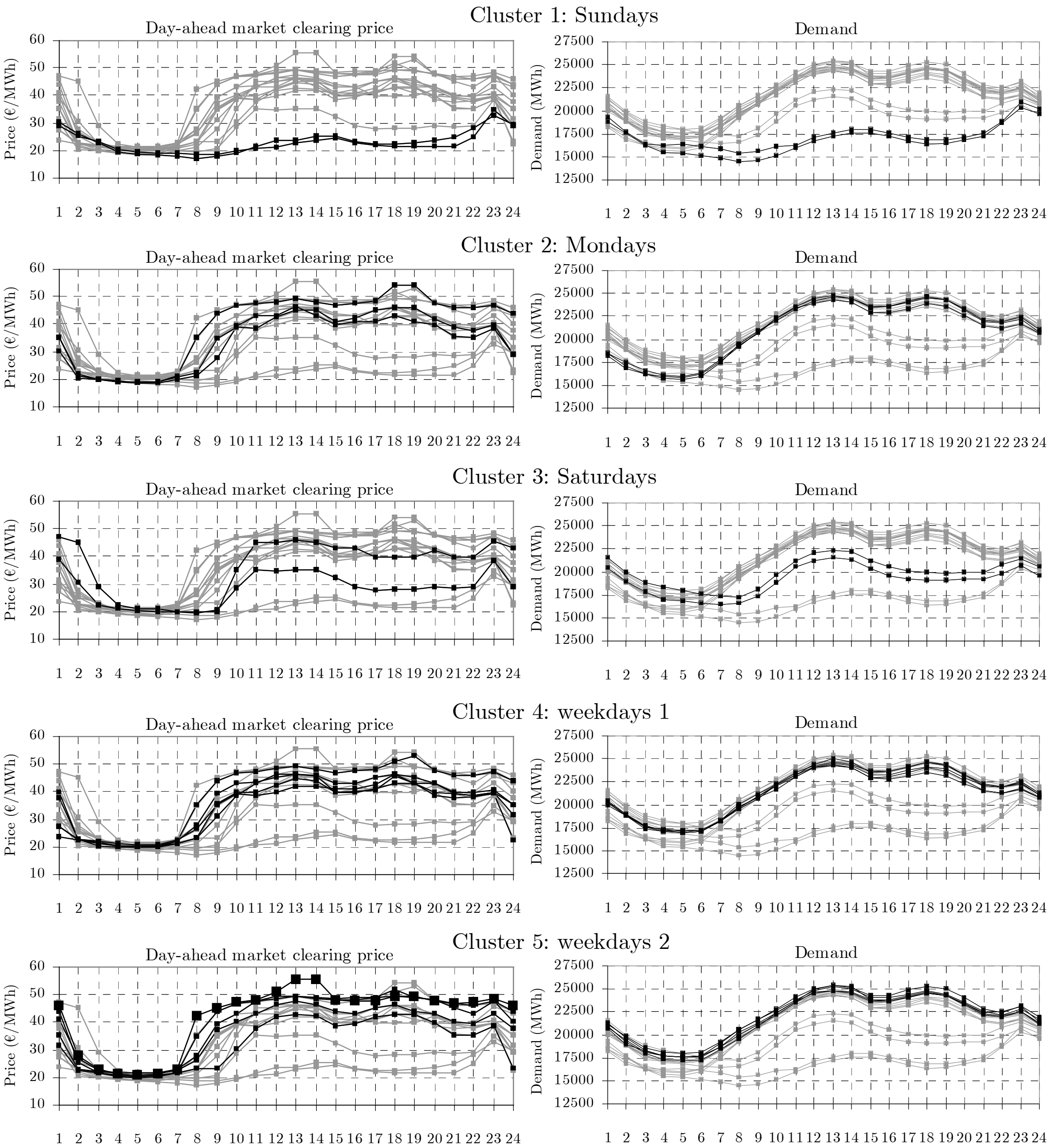

Figure B.5. Detail of the five clusters obtained in a 17-day case according to their 24 h demand profile.

The results of the clustering analysis suggest considering a four-scenario structure for the day-ahead market problem faced by the generation company on July 25th. Each of these scenarios consists of 24 residual demand curves for the day-ahead market and 24 residual demand curves for the adjustment market. These sets of 48 curves should be taken from those observed in past days belonging to the same cluster as the day of study (Figure B.6). Given that it is extremely difficult to determine whether one particular scenario is more likely to occur than others, we suggest assigning identical probabilities to all the scenarios.

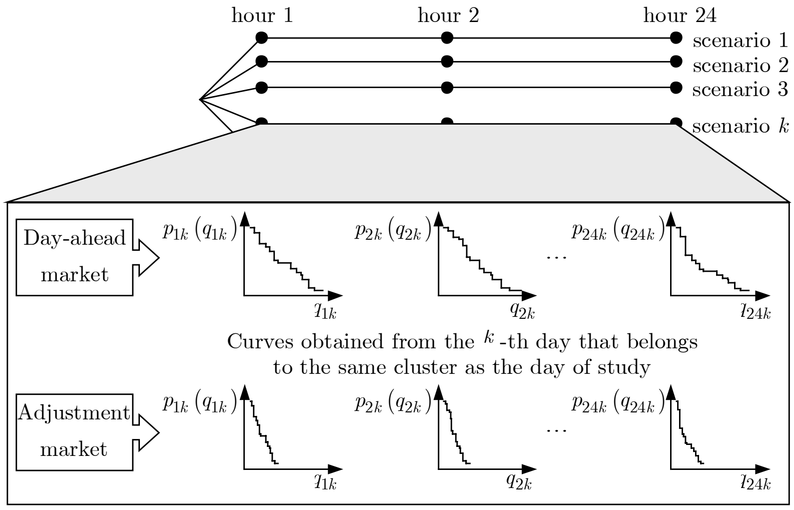

Figure B.6. Scenario structure for the day-ahead market problem.

B.4.2 An example of the construction of the scenario structure for the adjustment market problem

Let us consider now an example of the construction of the scenario structure for the two-stage program that focuses on the development of an optimal offering strategy for the adjustment market. As has been mentioned, this problem should be addressed after the clearing of the day-ahead market. Therefore, the explanatory variable that can be used to search for similar past spot market sessions is the vector of 24 prices that has resulted from the clearing of the day-ahead market. Table B.2 presents the results of the clustering analysis performed with the same range of days as in the previous example but considering only four clusters. As can be seen, in this case the four requested clusters do not yield intuitive day types. Cluster 1 corresponds to low-price days (Sundays and a Saturday), cluster 2 includes one type of weekdays, cluster 3 comprises the other type of weekdays and cluster 4 is an outlier. This analysis suggests considering four scenarios for the adjustment market problem faced by the company on July 25th.

July 2001

Date	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25
Day	Mo	Tu	We	Th	Fr	Sa	Su	Mo	Tu	We	Th	Fr	Sa	Su	Mo	Tu	We
Cluster	2	2	3	3	2	1	1	2	2	2	2	2	4	1	3	3	3

Table B.2. Classification of 17 days according to their 24 h day-ahead market price profile.

Figure B.7 presents the detail of the clusters obtained. The charts on the left column depict the 24 h day-ahead market profile that has been used to classify the range of days. The right column represents the corresponding 24 h adjustment market price profiles. It is evident that the price of the day-ahead market is a reasonable explanatory variable of the outcome of the adjustment market.

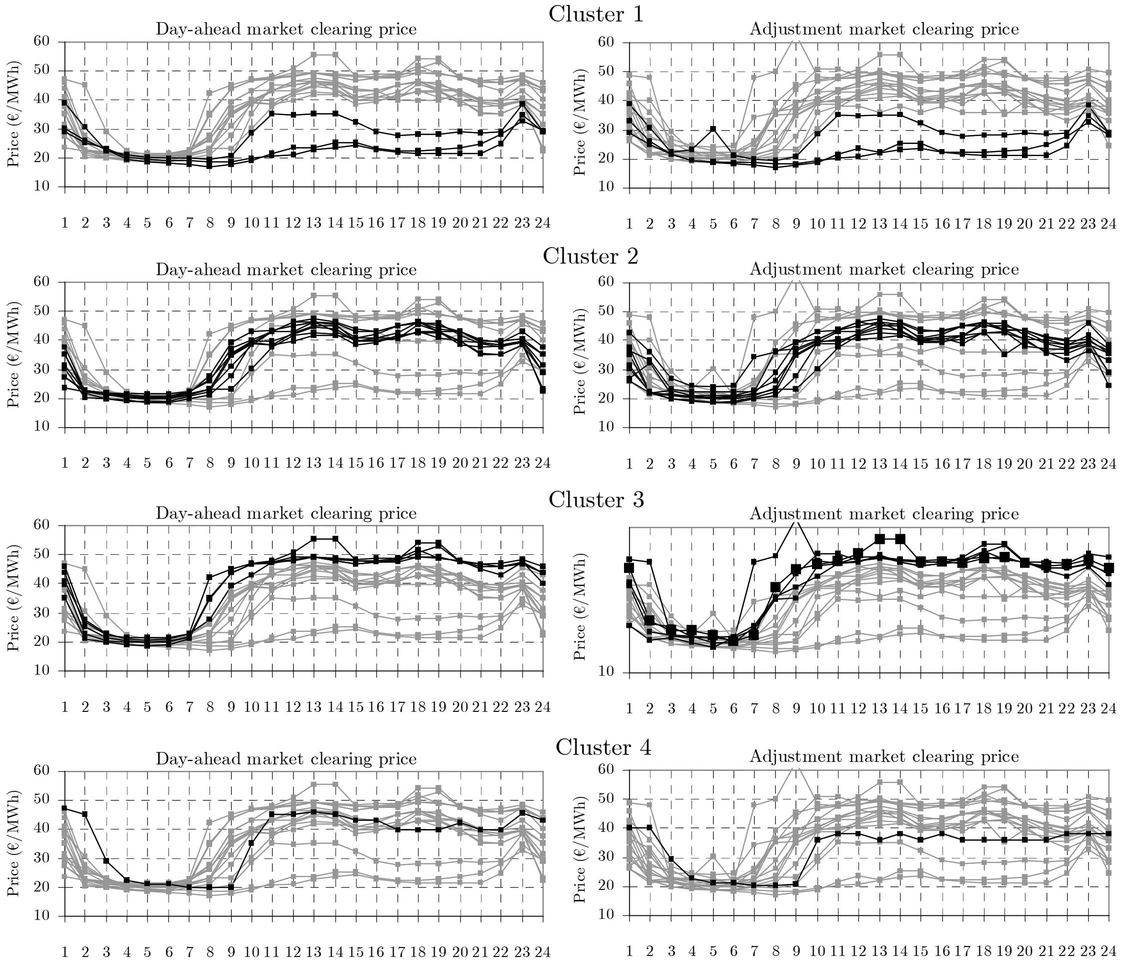

Figure B.7. Detail of the four clusters obtained for a 17-day case according to their 24 h day-ahead market price profile.

In conclusion, the scenario structures required to solve the proposed sequence of two-stage problems can be constructed based on the results observed in past market sessions that are considered similar to the current market session. The identification of these similar past market sessions can be performed by classifying a range of historic market sessions making use of clustering techniques according to the values of an explanatory variable. The scenarios contemplated in the corresponding two-stage problem can be taken from the set of past sessions that belong to the same cluster as the session of study.

B.5 Relevant vs. Significant results

A typical requirement in any statistical analysis is that enough historic information is available in order to obtain estimations with an adequate degree of significance. This is true if the process of interest is random, i.e. its realization depends on a number of unknown factors that are assumed to remain invariable (e.g. the annual inflows of a certain river basin). However, the residual demand curves faced by an agent in the spot market auctions are the result of the strategies of a limited number of agents. These strategies suffer several changes throughout the year and are not usually kept invariable for more than a month. Hence, a great part of the historic information relative to the operation of the spot market may not be relevant at all.

The suggested method to construct scenario structures allows a balance between significance and relevance through the adjustment of two parameters: the range of historic days used to search for past spot market sessions similar to the session of study and the number of clusters into which these days are classified. The values of these two parameters must be chosen based on the experience of the analyst. This can be seen as a disadvantage, particularly if the results present a great dependence on these parameters, but it provides the user with a means to add his/her personal touch to the process.

One alternative is to start by selecting the range of historic days, which would typically range from three to six weeks. Less than three weeks would not provide a significant variety of scenarios, whereas more than six weeks would probably contaminate the analysis with historic information that is not longer relevant. In fact, it is the tradeoff between relevance and significance what must guide this decision. Once the range of historic days has been selected, the number of clusters that is more convenient can be determined by performing several trials.

B.6 An extension to obtain weekly spot market scenarios

In addition to proposing a method to decide optimal offers for a generation company operating in a spot market, this thesis suggests a procedure to decide the unit-commitment schedule of a generation company including an explicit representation of the spot market uncertainty.

Traditional stochastic unit-commitment models typically consider the uncertainty of demand [Nowak '99]. More recent approaches tend to focus on the stochasticity caused by uncertain electricity spot prices under the assumption that prices are exogenously fixed [Takriti '00]. Irrespective of the source of uncertainty, Lagrangian relaxation (LR) is usually considered an appropriate solution method due to the particular structure of the unit-commitment problem: a non-convex problem with complicating constraints. Indeed, LR is also the approach adopted to solve the stochastic unit-commitment problem formulated in this thesis.

The stochastic unit-commitment problem can be seen as a multistage stochastic program. The time scope considered is usually a week, which is the horizon that many thermal units require to recover the cost of a startup.

We have assumed in a previous section that the relative importance of a market mechanism diminishes as it gets nearer to the moment of physical delivery. From the perspective of a whole week, this suggests focusing on the day-ahead market and the adjustment market while neglecting the reserve market and the balancing mechanism. In other words, the weekly multistage stochastic program can be seen as a sequence of two-stage programs, each one having the structure depicted in Figure B.1. This yields the scenario tree represented in Figure B.8.

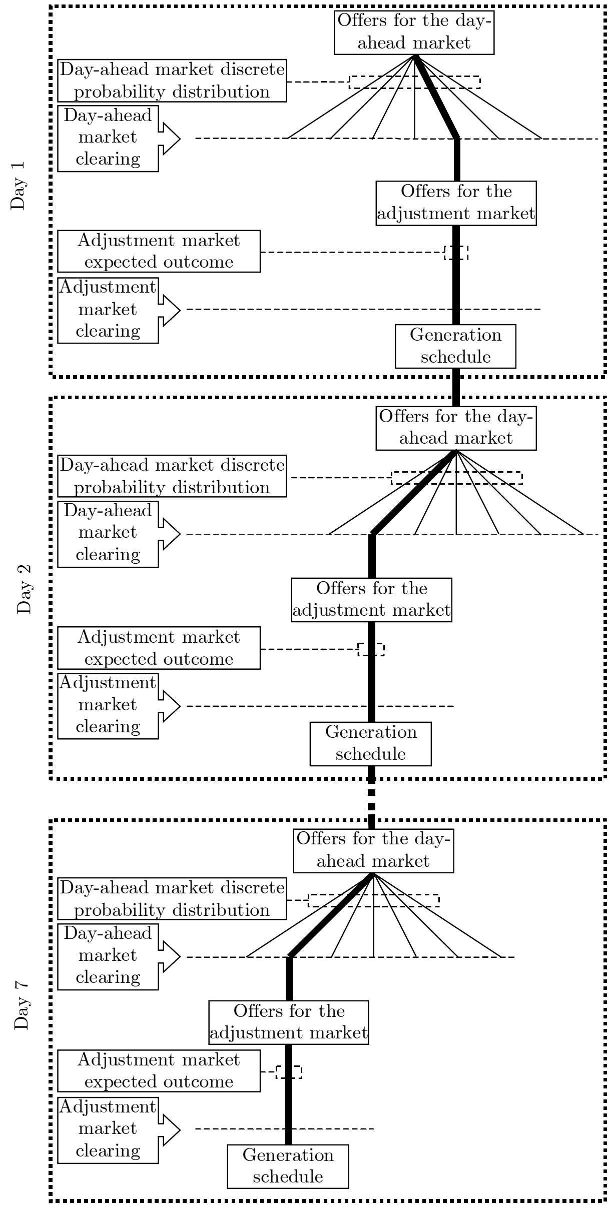

Figure B.8. The decision process from the perspective of the weekly unit-commitment problem.

The method we propose to construct scenarios for the weekly stochastic unit-commitment problem is an extension of the one suggested for the two-stage problems. The idea is to replicate the decision process that the company follows during the week. On Monday, the company will use historic spot market data (typically obtained from recent Mondays) in order to decide an optimal offering strategy for the day-ahead market. After the clearing of the day-ahead market, the company will try to introduce adjustments in its final schedule by submitting offers to the adjustment market. The company will then have to decide the optimal generation schedule that meets the obligations assumed in different market mechanisms for this day. An identical decision process is repeated during the rest of the week.

As a consequence, in order to construct the scenario tree for the whole week, a number of historic spot market sessions has to be identified that are similar to each of the sessions of the week of study. In other words, the clustering analysis suggested in a previous section must be performed seven times. For example, let us consider the construction of the scenario structure for the stochastic unit commitment problem faced by a generation company in the Spanish spot market in the week of July 23rd – July 29th 2001. The range of past data considered in this case comprises the previous two weeks. If a four-cluster analysis is performed, the results shown in Table B.3 are obtained.

July 2001

Date	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29
Day	Mo	Tu	We	Th	Fr	Sa	Su	Mo	Tu	We	Th	Fr	Sa	Su	Mo	Tu	We	Th	Fr	Sa	Su
Cluster	2	4	4	4	4	3	1	2	4	4	4	4	3	1	2	4	4	4	4	3	1

Table B.3. Classification of 21 days into four clusters according to their 24 h demand profile.

According to these results, the scenario structure that should be used in principle is the following:

- Two possible market situations for Monday, July 23rd (cluster 2).
- Eight possible market situations for Tuesday, July 24th (cluster 4).
- Eight possible market situations for Wednesday, July 25th (cluster 4).
- Eight possible market situations for Thursday, July 26th (cluster 4).
- Eight possible market situations for Friday, July 27th (cluster 4).
- Two possible market situations for Saturday, July 28th (cluster 3).
- Two possible market situations for Sunday, July 29th (cluster 1).

However, the size of the scenario trees that typically result if this sequence of possible daily outcomes is expanded becomes unmanageable. In the previous example, the scenario tree would have $2 \cdot 8 \cdot 8 \cdot 8 \cdot 8 \cdot 2 \cdot 2 = 32768$ terminal nodes.

Hence, a strategy to reduce the size of the scenario tree would be required. This is an issue that frequently arises in the field of stochastic programming and for which solutions have recently been proposed from the perspective of probability metrics [Heitsch '01, Dupacová '01]. Nevertheless, it is not the purpose of this thesis to develop a state-of-the-art method to construct scenario trees for the multistage stochastic programs addressed. In order to avoid this kind of problems, weekly scenario trees will be constructed using only very recent historic information. In this manner, the size of the resulting scenario trees will allow their direct usage and no reduction technique will have to be applied. Figure B.9 shows an example in which only one possible outcome is considered for the spot market session celebrated on Monday and two possible realizations are considered for the rest of spot market sessions. As can be seen, this leads to a 64-scenario tree.

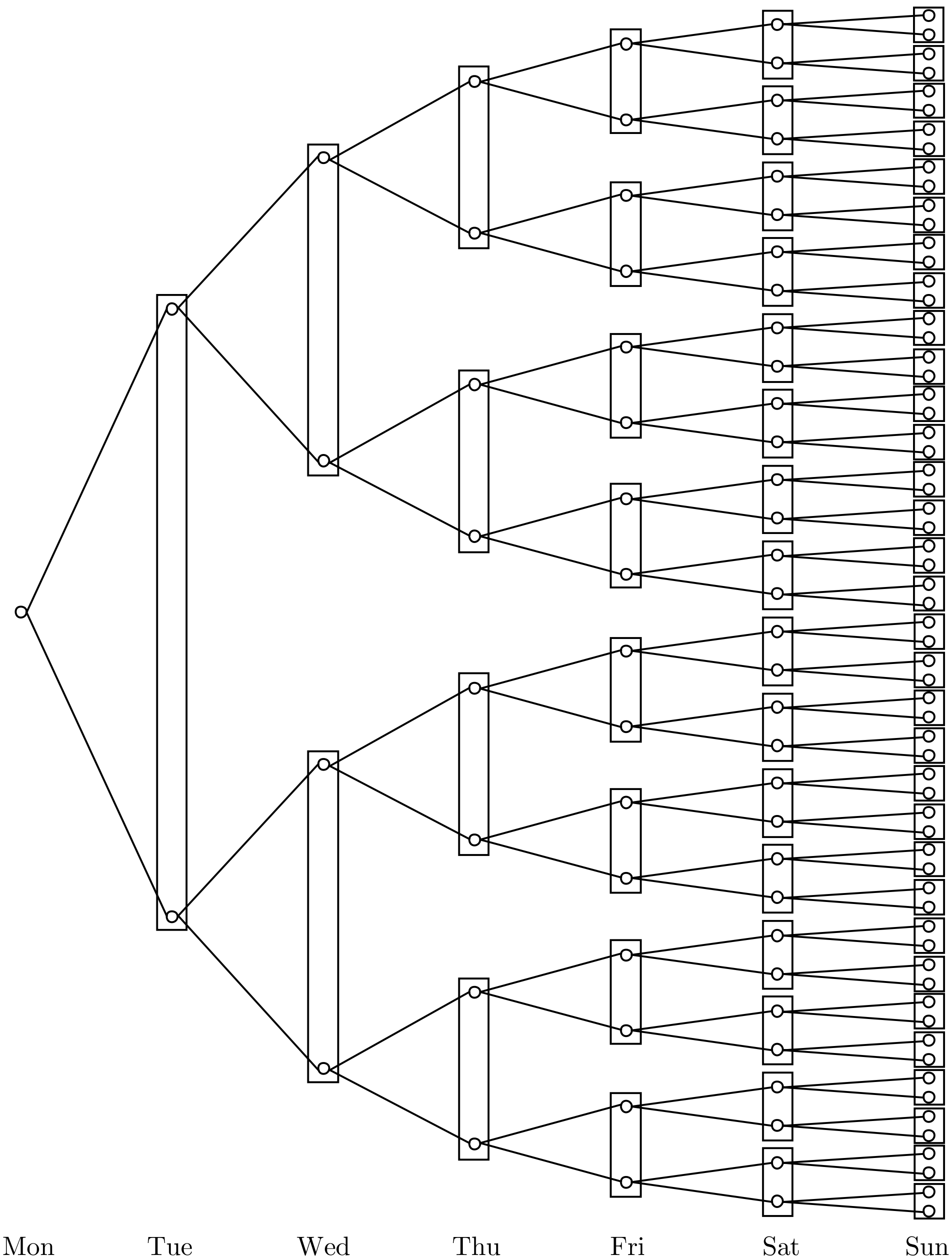

Figure B.9. A scenario tree for the weekly unit-commitment problem.

The constraints that guarantee non-decreasing offer curves must be enforced only between nodes sharing their immediate ancestor node. For example, if the scenario tree depicted in Figure B.9 is considered, the non-decreasing constraints are enforced between both Tuesday nodes. In contrast, these constraints are enforced between the two upper Wednesday nodes and also between the two lower Wednesday nodes, but not between nodes that do not share their ancestor node.

B.7 Conclusion

This appendix describes a simple and practical procedure that can be used to construct daily scenarios for the two-stage stochastic programming models with which optimal offers are developed for each of the spot market mechanisms. An extension of this procedure to provide weekly scenario trees is also presented. It must be emphasized that this procedures are by no means a contribution of this thesis. However, the inherent complexity of more sophisticated methods requires a developmental effort that goes well beyond the purposes of this thesis. On the contrary, the procedures proposed in this appendix provide a simple and practical way of obtaining manageable scenario trees that can be used to test the ideas presented in this thesis.

B.8 References

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C Input data for the study cases

In this thesis a number of numerical examples are solved in order to illustrate the methodology proposed to address the problem of developing optimal offering strategies for a company operating in an electricity spot market. These examples consider the case of a fictitious but representative generation company of study that participates in the Spanish electricity spot market. The problems solved correspond to specific past spot market sessions for which all the information is already publicly available.

This appendix provides a general overview of the input data used in these numerical examples. It includes a full description of the generation portfolio of the company of study and relevant information about the real outcome of the spot market sessions of study. More importantly, it explains in detail how the spot market scenarios that are considered in these examples were constructed using the day-type identification technique suggested in appendix B. In this manner, the results obtained from these examples can be more easily interpreted and the relevance of this thesis is increased, given that the problems solved are based on real situations.

C.1 Introduction

This appendix describes the input data used for the numerical examples included in this thesis. The information is organized in four parts. The first section is dedicated to the generation company whose case is studied in all the examples. The second section focuses on the actual sessions of the Spanish electricity market that are explored in these examples. The third section describes how we prepared the input data for the examples that are formulated as two-stage programs. Finally, the fourth section explains how the scenario tree for the weekly stochastic unit-commitment problem was built. All the information relative to the Spanish electricity spot market is publicly available at OMEL's website [OMEL].

C.2 Generation units' data

All the numerical examples included in this thesis refer to the same generation company. It is a large fictitious generation company owning 38 thermal units and 16 hydro units that have been selected from the Spanish generation system. The characteristics estimated for the thermal units are shown in Table C.1, whereas those assumed for the hydro units are specified in Table C.2.

Type	Unit t	$\bar{q}_t$ MW	$\underline{q}_t$ MW	k_t p.u.	o_t €/MWh	l_t MW	f_t €/Tcal	α_t Tcal/MWh	β_t Tcal/h	s_t €
Nuclear	G1	963	963	0.955	0.07		3.546	1.000	0.000	50000
	G2	968	968	0.955	0.07		3.546	1.000	0.000	50000
	G3	1009	1009	0.955	0.07		3.546	1.000	0.000	50000
Coal	G4	550	180	0.95	0.035		5.422	2.092	97.597	13975
	G5	550	180	0.95	0.035		5.452	2.147	34.146	11420
	G6	534	180	0.95	0.1	276	5.452	2.139	36.845	11420
	G7	330	160	0.945	0.045		8.601	2.296	34.156	6768
	G8	350	175	0.945	0.045		8.601	2.263	41.539	3913
	G9	350	175	0.945	0.045		8.601	2.204	67.606	11324
	G10	141	65	0.93	0.065		8.601	2.298	14.765	17912
	G11	141	70	0.93	0.065		8.601	2.251	43.453	8668
	G12	220	80	0.905	0.065		10.254	2.116	68.295	6714
	G13	313	150	0.905	0.045		8.806	2.316	27.190	16674
	G14	160	80	0.885	0.065		8.136	2.460	23.005	7784
	G15	160	80	0.885	0.065		8.806	2.412	32.950	10621
	G16	80	44	0.93	0.065		9.058	2.133	18.612	6011
	G17	350	180	0.945	0.04		8.283	2.424	-1.883	16091
	G18	350	180	0.945	0.04		8.283	2.424	-1.883	16091
	G19	350	180	0.945	0.04		8.283	2.424	-1.883	16091
	G20	350	230	0.945	0.07		7.783	2.394	-10.104	18285
	G21	350	230	0.945	0.07		7.783	2.519	-13.391	18285
	G22	350	230	0.945	0.07		7.783	2.394	-10.104	18285
	G23	350	230	0.945	0.07		7.783	2.394	-10.104	18285
Oil/gas	G24	350	100	0.93	0.1		17.678	2.249	32.506	4358
	G25	172	55	0.93	0.1		10.597	2.358	28.530	6011
	G26	172	55	0.93	0.1		10.597	2.358	28.530	6011
	G27	70	22	0.9	0.1		18.984	2.613	14.100	6011
	G28	160	48	0.93	0.1		18.984	2.331	28.200	6011
	G29	120	40	0.93	0.1		18.878	2.115	24.081	6011
	G30	350	100	0.95	0.1		18.878	2.283	12.332	8944
	G31	350	100	0.93	0.1		17.678	2.249	32.506	6011
	G32	350	100	0.93	0.1		17.678	2.249	32.506	4358
	G33	220	66	0.95	0.1		18.984	2.239	6.447	11306
	G34	533	160	0.955	0.1	300	18.984	2.099	59.505	11012
	G35	150	45	0.93	0.1		18.419	2.091	31.592	6011
	G36	300	60	0.95	0.1	180	18.419	2.224	5.612	4081
	G37	148	43	0.93	0.1		18.984	0.233	28.200	6011
	G38	520	100	0.955	0.1	300	19.072	2.113	40.586	6269

Table C.1. Characteristics of the thermal units.

	Unit h	$\bar{q}_h$ MW	$\bar{b}_h$ MW	η_h p.u.	W_{h0} (1 day) MWh	W_{h0} (1 week) MWh
Hydro	G39	340	308	0.7		
	G40	989				
	G41	197			4320	25720
	G42	605				
	G43	115			820	2820
	G44	272				
	G45	210	194	0.7		
	G46	84	76	0.7		
	G47	213	207	0.7	660	1840
	G48	88	80	0.7	180	800
	G49	587			3040	15720
	G50	416	416	0.7	1000	4760
	G51	134			500	2510
	G52	70			110	550
	G53	360	333	0.7	1080	4690
	G54	277			540	3840

Table C.2. Characteristics of the hydro units.

C.3 The spot market sessions of study

All the numerical examples provided in this thesis are based on real sessions of the Spanish electricity spot market. In this section we provide the results that were actually observed in the spot market sessions of study. This information can be used as a benchmark to evaluate the solutions obtained with our model.

C.3.1 Daily cases

The majority of the study cases included in this thesis are oriented to the development of optimal offering strategies for the day-ahead market, taking into account the company's portfolio as well as the possibility of adopting recourse actions in the adjustment market. All these numerical examples are based on the session of the Spanish electricity spot market that took place on October 24th 2001. Figure C.1 represents the trading volumes and the clearing prices observed in that session both in the day-ahead and in the on-day market. As can be seen, the volume traded in the on-day market is significantly lower than that observed in the day-ahead market.

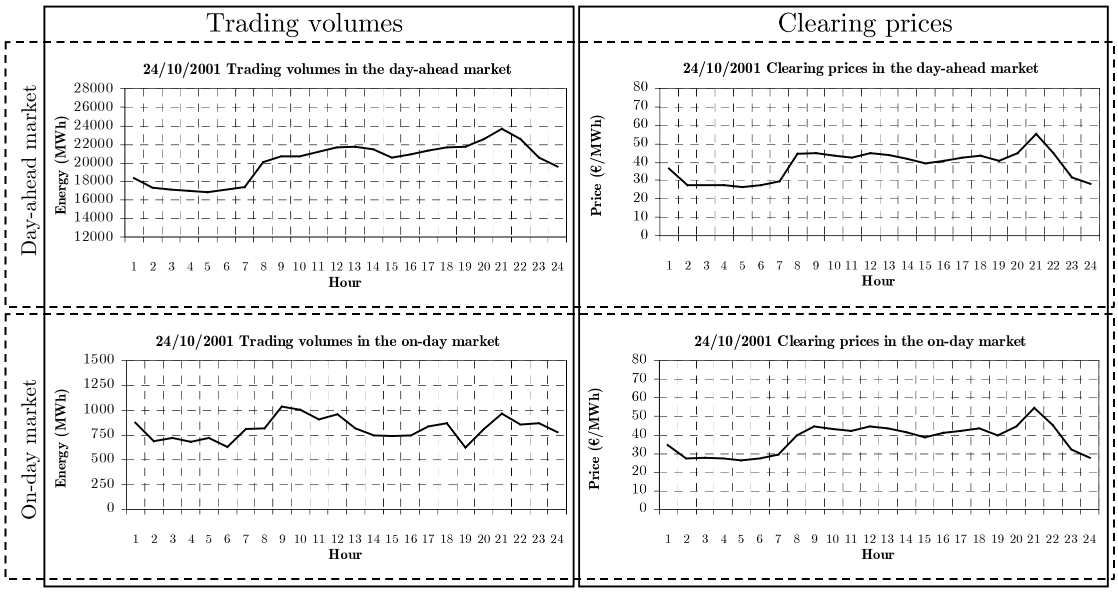

Figure C.1. Trading volumes and clearing prices on October 24th 2001.

Figure C.2 depicts the energy that the company of study sold in both market mechanisms. It must be noticed that the company can adopt a net selling or net buying position in the hourly auctions of the on-day market. Additionally, Figure C.3 specifies the type of generation units that were supposed to provide the energy sold (or purchased, in the case of pumping) in the day-ahead market.

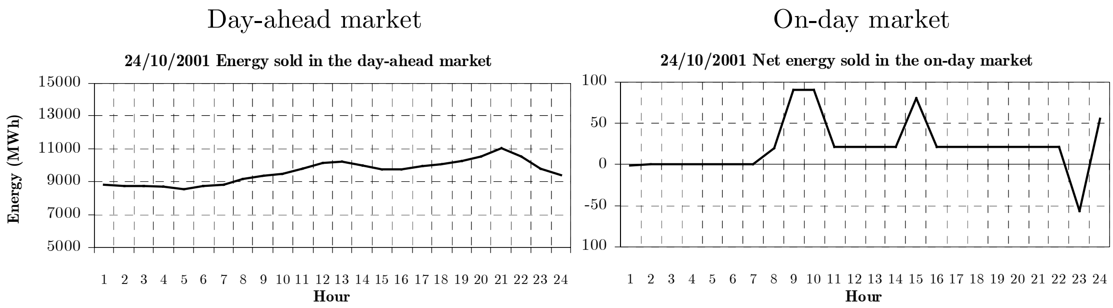

Figure C.2. Energy sold by the company on October 24th 2001.

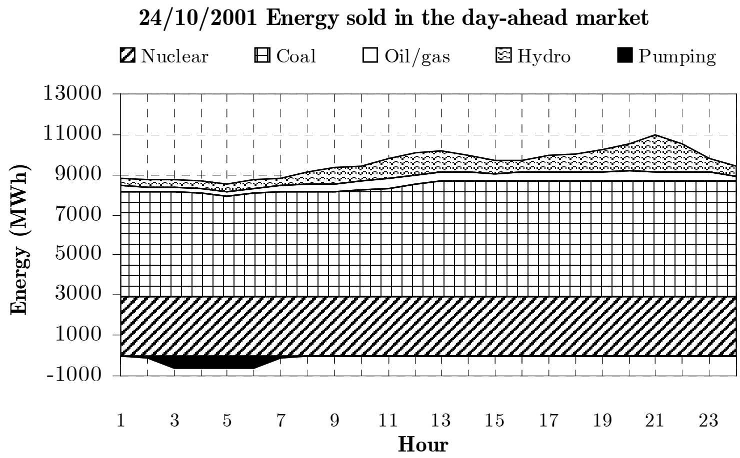

Figure C.3. Energy sold by the company in the day-ahead market on October 24th 2001.

As is justified in chapter 5, commitment decisions are assumed to be taken on a weekly basis. Hence, in the numerical cases that we solve to obtain an optimal offering strategy for a particular market mechanism, we consider commitment decisions as input data. Table C.3 contains the commitment schedule that is used as input data for the numerical examples corresponding to October 24th, 2001.

		1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24
	G1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
Nuclear	G2	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
	G3	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
	G4	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
	G5	0	0	0	0	0	0	0	0	0	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
	G6	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
	G7	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
	G8	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
	G9	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
	G10	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
	G11	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
	G12	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
	G13	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
Coal	G14	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
	G15	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
	G16	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
	G17	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
	G18	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
	G19	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
	G20	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
	G21	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
	G22	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
	G23	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
	G24	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
	G25	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
	G26	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
	G27	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
	G28	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
	G29	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
	G30	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
Oil/gas	G31	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
	G32	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
	G33	0	0	0	0	0	0	0	0	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	0
	G34	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
	G35	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
	G36	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
	G37	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	1	1	1	1	0
	G38	0	0	0	0	0	0	0	0	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	0

Table C.3. Unit-commitment schedule for October 24th 2001.

C.3.2 Weekly cases

In addition to suggesting a procedure to develop optimal offering strategies for each of the spot market mechanisms, this thesis proposes a methodology to formulate and solve the weekly unit-commitment problem with explicit consideration of the spot market uncertainty. In chapter 6 a numerical example is solved in order to illustrate the potential of this approach. This study case is based on the sessions of the Spanish electricity spot market that took place from Monday October 22$^{\text{nd}}$ 2001 to Sunday October 28$^{\text{th}}$ 2001. The trading volumes and clearing prices observed both in the day-ahead market and in the adjustment market during this week are represented in Figure C.4.

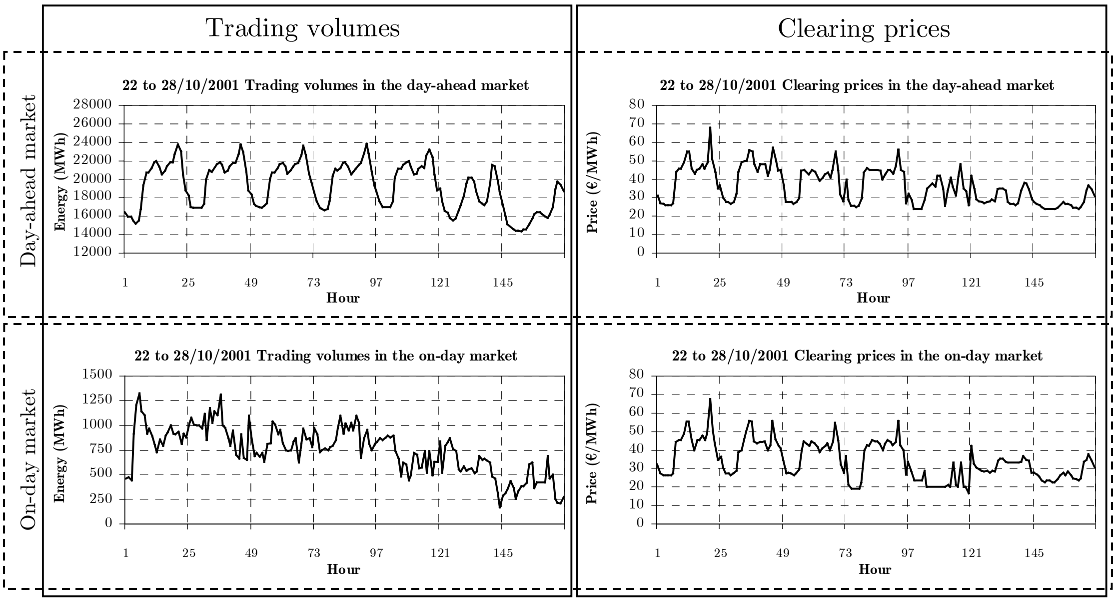

Figure C.4. Trading volumes and clearing prices from October 22nd to October 28th 2001.

The solution of the weekly stochastic unit-commitment problem provides a good approximation for the company's optimal unit-commitment schedule as well as a strategy to distribute the company's hydro resources throughout the week. This solution can be compared with the company's actual operation in the Spanish electricity spot market. Figure C.5 shows the company's sales both in the day-ahead market and in the on-day market. Figure C.6 represents the sales of the company in the day-ahead market identifying the different generation technologies that are expected to produce these sales, whereas Table C.4 provides the daily energy sold by the company in the day-ahead market according for each type of generating unit.

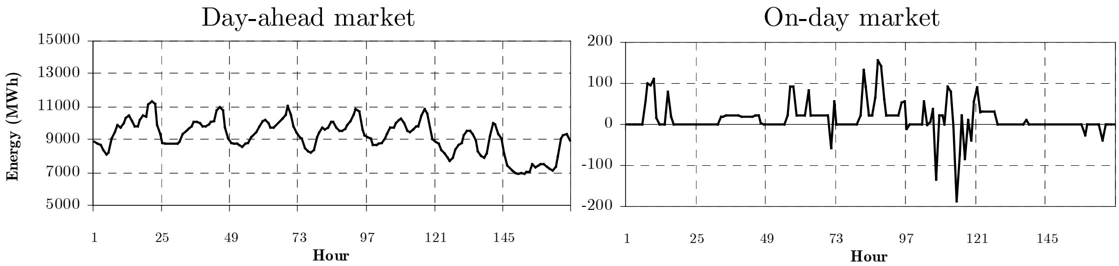

Figure C.5. Energy sold by the company from October 22nd to October 28th 2001.

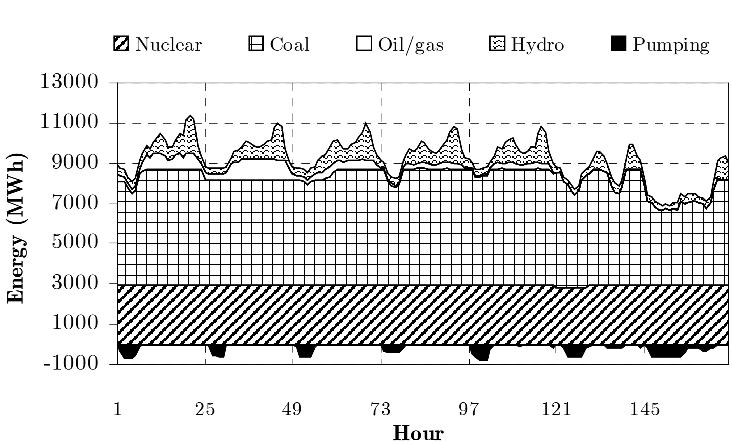

Figure C.6. Energy sold by the company in the day-ahead market from October 22nd to October 28th 2001.

	Mo	Tu	We	Th	Fr	Sa	Su	Total
Nuclear	70531	70531	71113	71056	71202	69612	71088	**495133**
Coal	132442	124867	131345	134575	135784	128230	101382	**888625**
Oil/gas	13244	18774	8833	5203	5447	0	0	**52268**
Hydro	11064	10791	12032	10577	9773	6239	5410	**65885**
Pumping	-3104	-2356	-2756	-2129	-3954	-4780	-7795	**-26873**
Total	**224177**	**222607**	**220567**	**219282**	**218252**	**200068**	**170085**	**1475038**

Table C.4. Energy sold by the company in the day-ahead market for each type of unit from October 22nd to October 28th 2001, in MWh.

The commitment schedule of each thermal unit depends on the ratio between its startup cost and its variable cost. For instance, coal units usually follow a weekly cycle, given that their startup cost is high if compared with their variable cost. In contrast, oil/gas units frequently follow a daily cycle. This idea is confirmed by Table C.5, in which we specify the hours of startup and shutdown of each of the company's thermal units during the week of study.

		Hour of startup							**Hour of shutdown**						
		Mo	**Tu**	**We**	**Th**	**Fr**	**Sa**	**Su**	**Mo**	**Tu**	**We**	**Th**	**Fr**	**Sa**	**Su**
Nuclear	G1														
	G2														
	G3														
Coal	G4														
	G5			10											
	G6														
	G7	6													
	G8														
	G9														
	G10														
	G11														
	G12														
	G13														
	G14	5													
	G15														
	G16														
	G17														
	G18	5													
	G19														
	G20														
	G21														
	G22														
	G23														
Oil/gas	G24	9	8						22						
	G25														
	G26														
	G27														
	G28					19							24		
	G29														
	G30														
	G31				8	8						23	23		
	G32														
	G33	10	9	9	9	9	11		23	24	24	24	24	24	
	G34														
	G35														
	G36														
	G37	20	19	19	19				24	24	24	24			
	G38	8		9	9	9				24	24	23	23		

Table C.5. Hours of startup and shutdown for the company's thermal units from October 22nd to October 28th 2001.

Another important aspect of the weekly operation of a generation company is the management of its hydro resources. Table C.6 shows the daily energy that the company produced with each of its hydro units during the week of study.

	Mo	Tu	We	Th	Fr	Sa	Su	Total
G39								
G40								
G41	4264.4	4273.5	4286.4	4078	3861.9	3265.3	2890.5	**26920**
G42								
G43		27	816					**843**
G44								
G45	50	50	50	50				**200**
G46	224	224		168				**616**
G47	360	360	660		240			**1620**
G48		220	176	176	132			**704**
G49	3148.1	2764.8	2982.5	2896.3	2762	1470	1366	**17389.7**
G50	850	800	900	1226	682.9	450	250	**5158.9**
G51	505.7	485.3	501.7	454.8	350.4	236.3	172.6	**2706.8**
G52	90	97.4	106.7	74.7	75	36	35.7	**515.5**
G53	990	810	1008.9	810	1070	360	225	**5273.9**
G54	581.4	678.6	543.8	642.8	598.8	421.2	470.5	**3937.1**
Total	**11063.6**	**10790.6**	**12032**	**10576.6**	**9773**	**6238.8**	**5410.3**	**65884.9**

Table C.6. Energy produced by the company's hydro units from October 22nd to October 28th 2001, in MWh.

C.4 Spot market data for the two-stage stochastic problems

In this section we explain the process followed when preparing the spot market data for the numerical examples included in this thesis. We have already mentioned that the information for these study cases corresponds to the session of the Spanish electricity spot market that took place on October 24th 2001. The scenario trees that were used to represent uncertainty were constructed based on spot market results observed in previous days. As indicated in appendix B, the number of scenarios can be adjusted by modifying the range of historic days that are considered when searching for similar spot market sessions.

C.4.1 Two-scenario case

This simple numerical example is the only one that can be solved without using decomposition techniques. As a matter of fact, it was conceived to check that the algorithms suggested in this thesis had been correctly implemented in GAMS language. The detailed description of the process followed to prepare the input data for this case serves as a paradigm for the rest of numerical examples.

C.4.1.1 Clustering analysis

Following the ideas presented in appendix B, a clustering analysis is performed with the aim of identifying two recent past days that can be considered relevant to estimate the outcome of the October 24th session. The days ranging from October 19th to October 24th are classified into four clusters according to their 24-hour demand profile, as shown in Table C.7. Based on these results, the spot market data of Friday October 19th and Tuesday October 23rd are identified as two relevant days for estimating the probability distribution of the spot market session celebrated on October 24th. In fact, in this particular study case, only these two days are used to estimate this probability distribution.

October 2001

Date	19	20	21	22	23	24
Day	Fr	Sa	Su	Mo	Tu	We
Cluster	4	3	1	2	4	4

Table C.7. Classification of 5 days according to their 24 h demand profile.

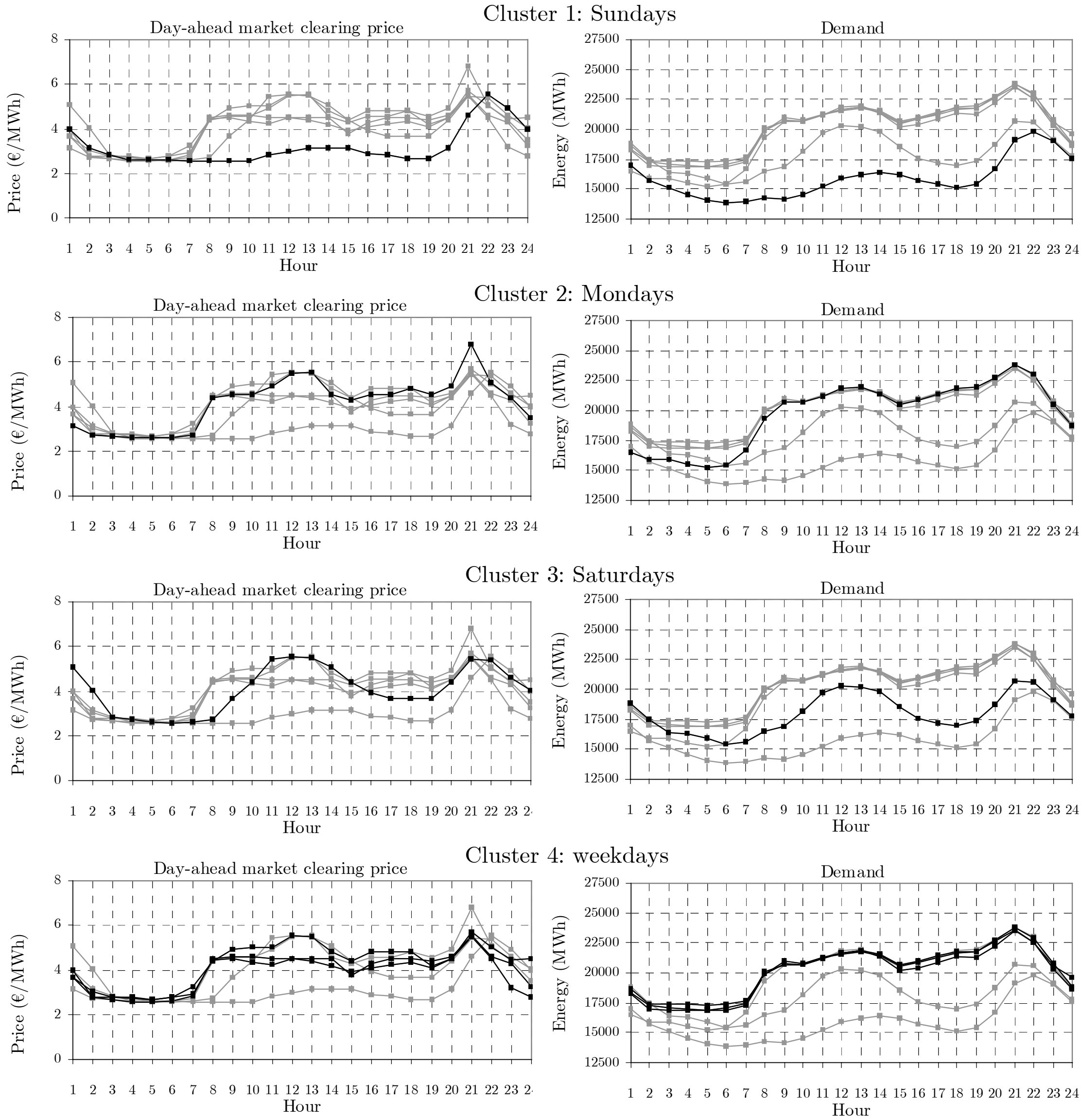

Figure C.7. Detail of the four clusters obtained in a 6-day case according to their 24 h demand profile.

C.4.1.2 Relevant historic spot market data for the current spot market session

After identifying two relevant historic days that can be used to prepare the input data for this two-scenario case, we specify the particular information that has to be processed. Firstly, the residual demand curves that the generation company of study faced in the spot market auctions celebrated in these two days must be obtained. Figure C.8 shows how the residual demand curves faced by this company on October 19th and on October 23rd both in the day-ahead market and in the first session of the on-day market constitute the two spot market scenarios considered for this study case. One revenue function can be obtained from each of these residual demand curves. In particular, the revenue functions corresponding to the adjustment market are approximated by a concave piecewise linear function, as described in chapter 4.

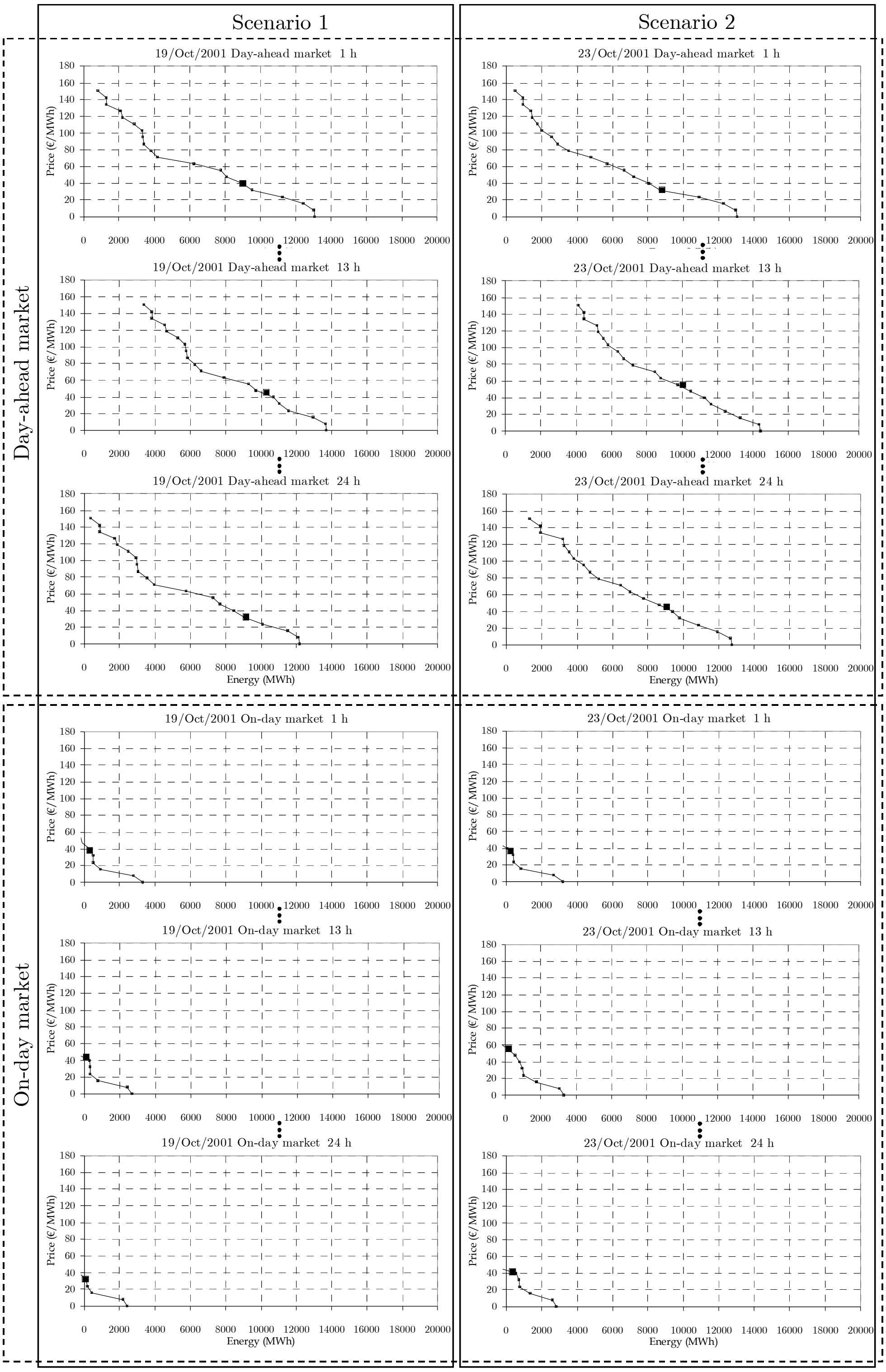

Figure C.8. Historic residual demand curves constitute the spot market scenarios

We have assumed that all the scenarios have the same probability of occurring. Hence, in this two-scenario case the probability of each scenario is 0.5.

The trading volumes observed in past auctions are also relevant, given that our objective function includes a term evaluating the future value of the company's current market share. Figure C.9 shows the trading volumes observed in each of the hourly auctions of the two considered historic days.

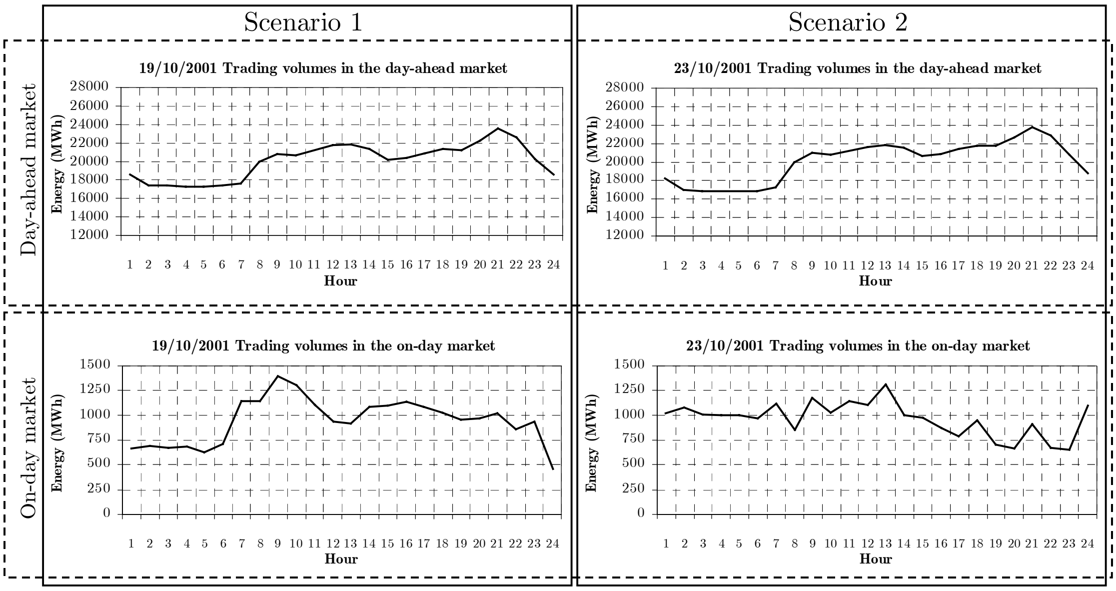

Figure C.9. Trading volumes for each of the spot market scenarios

To sum up, a two-stage two-branch scenario tree is constructed making use of the results observed in two past spot market sessions that are likely to be similar to those of the current session. These results include both residual demand curves and trading volumes.

C.4.2 Five-scenario case

In this section we explain how a five-scenario tree was constructed in order to develop optimal offers for the day-ahead market session celebrated on October 24th 2001. With the aim of identifying five historic days that are similar to the day of study, we perform a clustering analysis that considers the sequence of 9 daily spot market sessions that took place prior to the day of study, as indicated in Table C.8.

October 2001

Date	15	16	17	18	19	20	21	22	23	24
Day	Mo	Tu	We	Th	Fr	Sa	Su	Mo	Tu	We
Cluster	2	4	4	4	4	3	1	2	4	4

Table C.8. Classification of 10 days according to their 24 h demand profile.

Figure C.10 illustrates the features of this clustering analysis, in which four clusters have been considered.

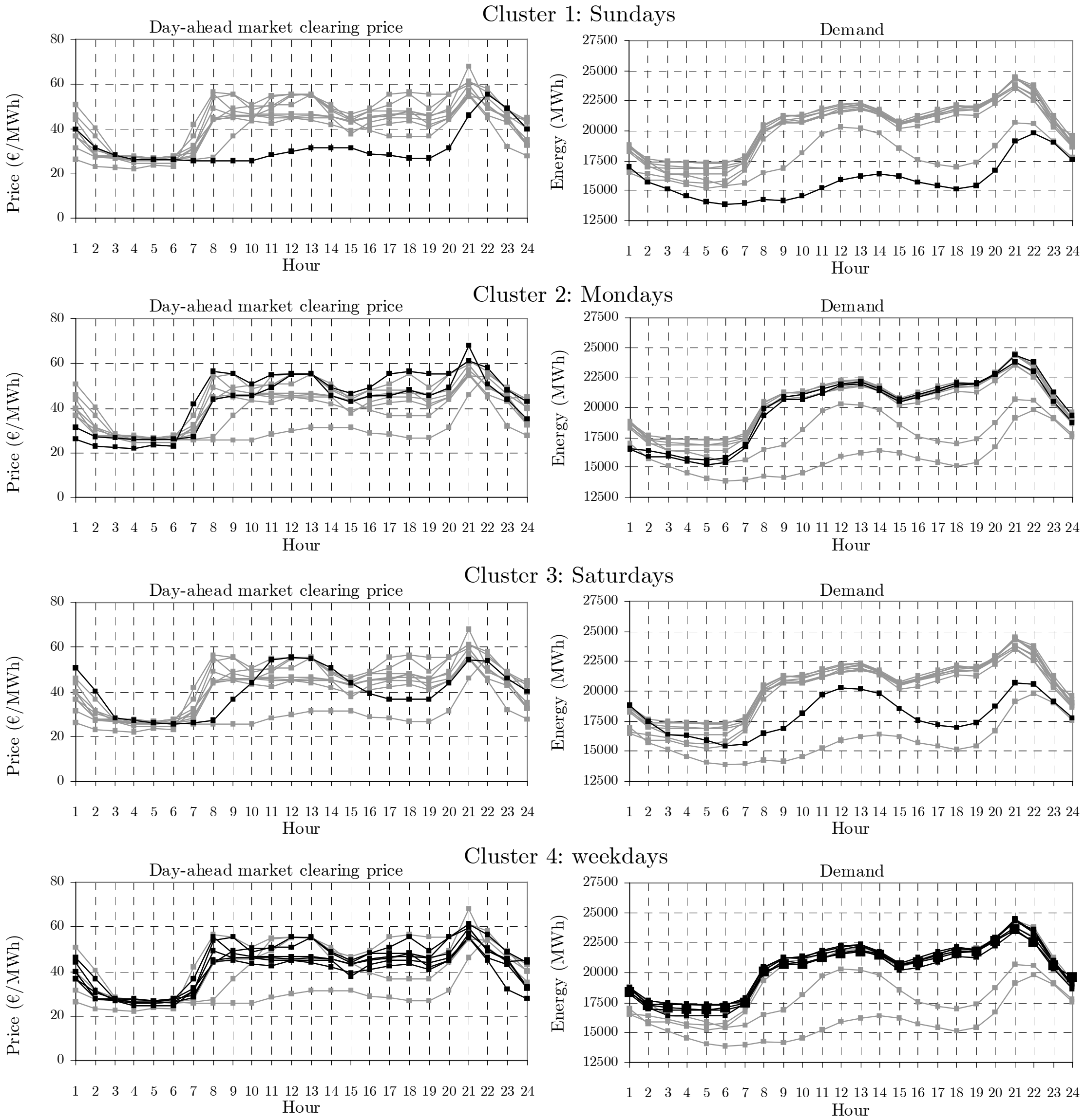

Figure C.10. Detail of the four clusters obtained in a 10-day case according to their 24 h demand profile.

C.4.3 Eleven-scenario case

To prepare the input data for the eleven-scenario case, eleven historic days that are likely to be similar to the day of study have to be identified. Table C.8 shows the days selected by the clustering analysis when four clusters are considered. Figure C.11 provides the details of this particular clustering analysis.

October

Date	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24
Day	Mo	Tu	We	Th	Fr	Sa	Su	Mo	Tu	We	Th	Fr	Sa	Su	Mo	Tu	We	Th	Fr	Sa	Su	Mo	Tu	We
Cluster	2	4	4	4	4	3	1	2	4	4	4	1	3	1	2	4	2	4	4	3	1	2	4	4

Table C.9. Classification of 24 days according to their 24 h demand profile.

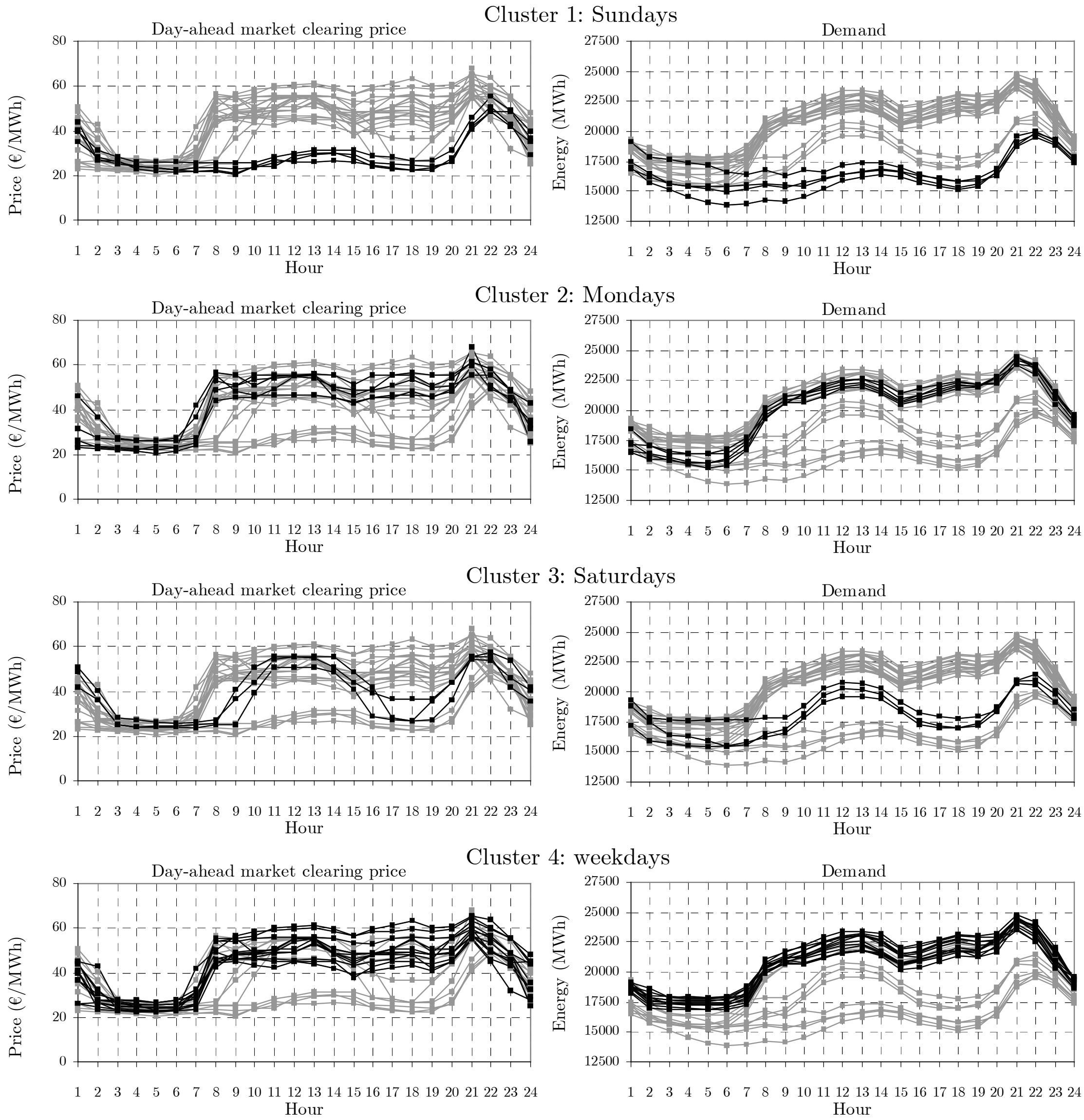

Figure C.11. Detail of the four clusters obtained in a 24-day case according to their 24 h demand profile.

C.5 Spot market data for the weekly stochastic UC problem

As indicated in appendix B, when a company faces the problem of deciding an optimal weekly unit-commitment schedule, it must anticipate the decision process that it will follow during the week. In other words, the company must consider that on Monday it will face an uncertain day-ahead market session with several possible outcomes. To estimate these possible outcomes a number of similar historic days will have to be identified. After the clearing of the day-ahead market, a session of the adjustment market will take place and the company will have the opportunity to correct any undesired result obtained in the day-ahead market. If we neglect subsequent market mechanisms, after the clearing of the adjustment market, the company has to decide a generation schedule to meet the obligations assumed for that day. The next six days evolve in the same manner, so that a number of similar historic days has to be identified for each of them. In order to identify these historic days we use clustering analysis, as we explain for the 16-scenario problem solved in chapter 6.

C.5.1 16-scenario case

As mentioned, in order to construct the scenario tree for the weekly stochastic unit-commitment problem, a number of historic relevant days has to be identified for each day of the week. Let us assume that the search for relevant historic days is restricted to the previous week. If a clustering analysis is performed in order to classify the days of the week of study and the days of the previous week into five clusters, the results shown in Figure C.12 are obtained.

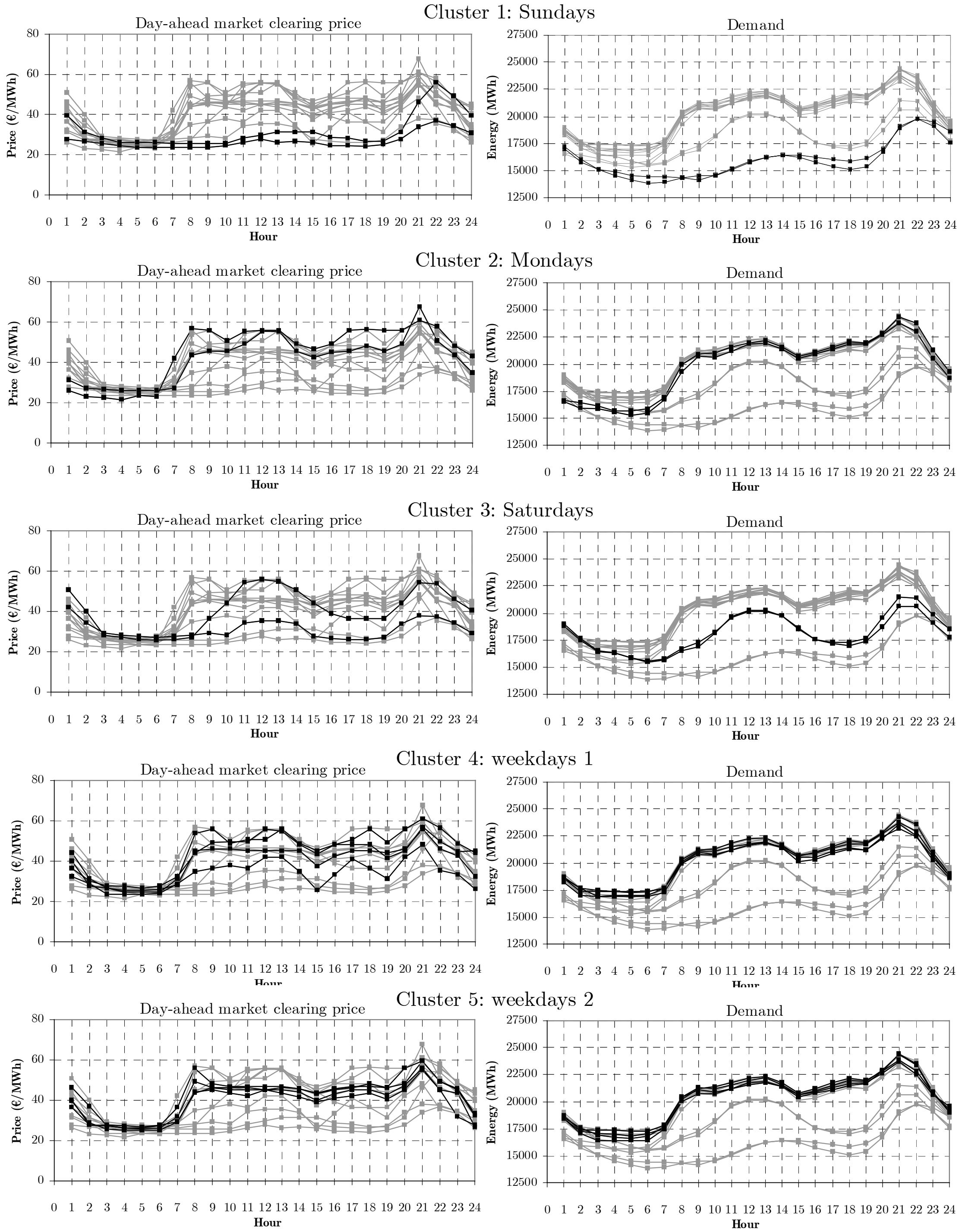

Figure C.12. Detail of the five clusters obtained in a 14-day case according to their 24 h demand profile.

The results of this clustering analysis are summarized in Table C.10:

Cluster	Day type	Current week	Previous week
1	Sundays	Sunday	Sunday
2	Mondays	Monday	Monday
3	Saturdays	Saturday	Saturday
4	Weekdays 1	Tuesday, Friday	Tuesday, Friday
5	Weekdays 2	Wednesday, Thursday	Wednesday, Thursday

Table C.10. Classification of two weeks into five clusters according to their 24 h demand profile.

Hence, for the Sunday of the week of study only one possible outcome will be considered: the previous Sunday. The same happens with the Monday and the Saturday of the week of study. In contrast, for each of the weekdays of the week of study two possible outcomes are considered. This yields the scenario-tree structure depicted in Figure C.13.

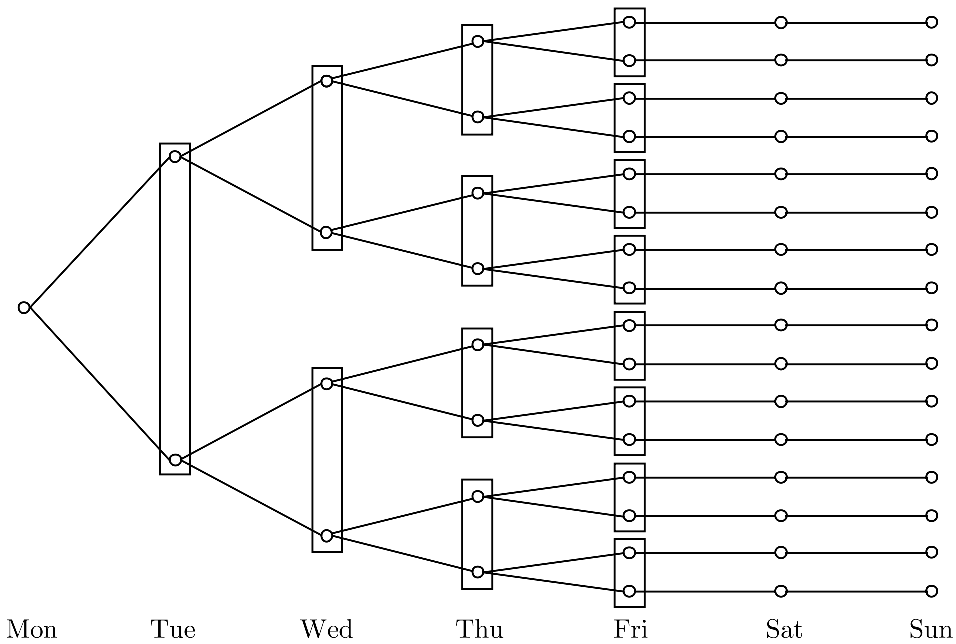

Figure C.13. A sixteen-scenario tree for the weekly unit-commitment problem.

This scenario tree correctly represents the decision process that the company is expected to follow throughout the week if the company considers that the presence of relevant historic data is restricted to the previous week and assumes that five types of day can occur during the week. Obviously, if more historic information were used or a different number of day types were proposed, the resulting scenario tree would be different. As indicated in appendix B, the size of a weekly scenario tree constructed in this manner is typically unmanageable and requires the use of scenario reduction techniques. Nevertheless, the sixteen-scenario structure developed for this example can be addressed in a straightforward manner with the LR decomposition approach described in chapter 5.

C.6 Conclusion

This appendix provides a qualitative description of how the input data for the numerical examples studied in this thesis were prepared. The objective is to give a general overview, so that the reader can identify the most relevant aspects of the information comprised in these examples and can interpret more easily the results obtained. Particular attention has been paid to the characteristics and operation of the generation units owned by the company of study. Additionally, we have summarized the actual results observed in the sessions of the Spanish electricity spot market that constitute the core of these examples. Finally, we explain in detail the manner in which the scenario trees included in these examples were prepared, following the ideas developed in appendix B.

C.7 References

[OMEL] OMEL, "http://www.omel.es."